Lecture I: Flavour Physics + QCD

1. Introduction to QCD, symmetries + RG running
2. Introduction to EFT, RG running of the operators + resummation of large logarithms
3. Form factor: parameterisations + non-perturbative calculations.
4. QCD factorization in B decays

Introduction to QCD: symmetries and the RGE.

Motivation for colour as a symmetry $\Omega^- (s, s, s)$

To construct antisymmetric wavefunction need a quantum number to distinguish between quarks.

QCD Lagrangian: $\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$

$$(D_\mu)_{ab} = \partial_\mu \delta_{ab} + ig\, T^c A^c_{ab}$$

$$\mathcal{L} = \underbrace{\bar{u}\, i\not{D} u + \bar{d}\, i\not{D} d}\ \underbrace{(- m_u \bar{u} u - m_d \bar{d} d.)}_{\text{can be neglected.}}$$

⇒ Conformal symmetry:

Lagrangian is invariant under $x \to \lambda x$, $q_f \to q_f \lambda^{-3/2}$, $A_\mu \to A_\mu / \lambda$

However it is broken at the quantum level by Λ_{QCD}.

⇒ Flavour symmetry:

Lagrangian is invariant for each flavour under $U(1)_V$

$q_f \to e^{i\alpha} q_f$ and the conserved current $= J^f_\mu = \bar{q}_f \gamma_\mu q_f$

Further, restricting to u, d quarks, these can be rotated amongst themselves $\underline{q'_{f'} = \Omega_{f'f}\, q_f}$ $\quad \Omega = \exp(i \sum \sigma_i \alpha^i)$

$SU(2)_V$.

⇒ Chiral symmetry: $q'_{f'_{L/R}} = \Omega^{L/R}_{f'f}\, q_{f_{L/R}}$

Vector rotation when $\alpha_L^i = \alpha_R^i$ and axial $\alpha_L^i = -\alpha_R^i$

$\underline{SU(2)_V}$ $\underbrace{SU(2)_A\ U(1)_V}$ $\underbrace{U(1)_A}$.

broken spontaneously by the quark condensate.
goldstone boson ⇒ pion
mass non-zero as symmetry not exact.

anomalous, not a symmetry of the quantum theory.

See Buras Notes
hep-ph/9806471

QCD renormalization / β function / RGE

To deal with divergences in loop corrections to Greens functions, need to regularize to have explicit parameterisation of singularities + then renormalize to render them finite.

Regularization generally done by dimensional regularization
$$D = 4 - 2\varepsilon.$$

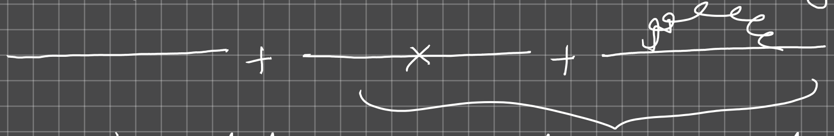
Renormalization to remove divergences : MS/$\overline{\text{MS}}$ scheme.
$$A^a_{0\mu} = Z_A^{1/2} A_\mu, \quad q_0 = Z_q^{1/2} q, \quad g_0 = Z_g g \underbrace{\mu^\varepsilon}, \quad m_0 = Z_m m.$$

The scale μ is introduced so that g remains dimensionless.

$$\mathcal{L} = \bar{q}_0 i \not{\partial} q_0 - m_0 \bar{q}_0 q_0$$

$$\underbrace{\bar{q} i \not{\partial} q - m \bar{q} q}_{\text{renormalized kinetic + mass terms}} + \underbrace{(Z_q - 1)\bar{q} i \not{\partial} q - (Z_q Z_m - 1) m \bar{q} q}_{\text{Counterterms.}}$$

Counterterms $\sim (Z-1)$ can be treated as interaction terms contributing to Greens functions in perturbation theory



Z_i chosen to cancel divergence. divergence cancels.

$$\frac{d g_0}{d\ln\mu} = 0.$$

$$\frac{d}{d\ln\mu} g(\mu) = \beta(\varepsilon, g(\mu)) \rightarrow -\varepsilon g \underbrace{- g \frac{1}{Z_g} \frac{d Z_g}{d\ln\mu}}_{\beta(g)}$$

$$\frac{d}{d\ln\mu} m_0 = 0$$

$$\frac{d}{d\ln\mu} m(\mu) = -\underbrace{\gamma_m(g)}_{\frac{1}{Z_m}\frac{dZ_m}{d\ln\mu}} m(\mu)$$

The β function and the anomalous dimension of the mass (γ_m) therefore control the scale dependence of the QCD coupling and the quark mass.

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} \qquad \gamma_m(g) = \gamma_{m,0} \frac{g^2}{16\pi^2} + \gamma_{m,1} \frac{g^4}{(16\pi^2)^2}$$

RGEs can be integrated + we can find explicit results at a specific order in perturbation theory, e.g. at 2 loops

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\ln\left(\ln(\mu^2/\Lambda^2)\right)}{\ln(\mu^2/\Lambda^2)}\right)$$

$$m(\mu) = m(m) \left(\frac{\alpha_s(\mu)}{\alpha_s(m)}\right)^{\gamma_{m0}/2\beta_0} \left(1 + \left(\frac{\gamma_{m1}}{2\beta_0} - \frac{\beta_1 \gamma_{m0}}{2\beta_0^2}\right) \frac{\alpha_s(\mu) - \alpha_s(m)}{4\pi}\right)$$

Introduction to EFT:

Reference: Notes on EFT
Ira Rothstein
hep-ph/0308266

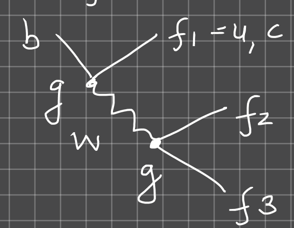
Interesting physics at all scales.
Theories become more + more general.
Calculations in more general theories are more complex

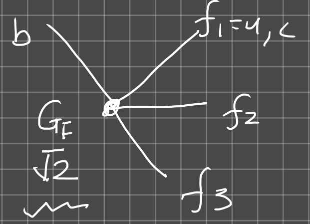
EFT: Simplest framework to capture essential physics for a given problem in manner which can be corrected to arbitrary precision.

In QFT multiple scales $\Rightarrow$ taking into account all virtual states is a complicated problem $\Rightarrow$ perturbation theory may break down due to large logs. EFT provides a solution, general set of frameworks to deal with multiscale problem. Organizational scheme: remove modes in theory which are irrelevant at scale being probed.

The most commonly used EFT in B physics is a Fermi-like theory containing 4-fermion operators, all physics above m_b integrated out.

For a generic b decay at tree level.

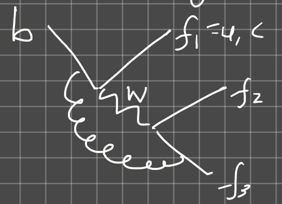




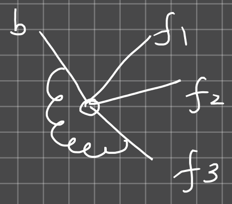
$$\frac{-ig^2}{(p_i - p_{f_1})^2 - M_W^2} \Rightarrow \frac{+ig^2}{M_W^2}$$

At scales much below the W mass we can ignore external momenta in the propagator $\Rightarrow$ W boson no longer dynamic and it can be absorbed into an effective coupling $G_F = \sqrt{2}\frac{g^2}{M_W^2}$ called the Wilson coefficient. $[G_F] = -2$.

Full theory



EFT.



This correction is finite + contains a Large log. $\sim \lambda_1 \frac{\alpha_s(\mu)}{4\pi}\ln\left(\frac{M_W^2}{m_b^2}\right)$

Must resum these logs.

$$\lambda_1 \frac{\alpha_s(\mu)}{4\pi}\left(\frac{1}{\epsilon} - \ln\left(\frac{m_b^2}{\mu^2}\right)\right) + K$$

Solution to RGE

$$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{1-\beta_0 \frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right)}$$

Expand in $\alpha_s(M_Z)$

$$= \alpha_s(M_Z)\left[1 + \underbrace{\sum_{n=1}^{\infty}\left(\beta_0 \frac{\alpha_s}{2\pi}\ln\frac{M_Z}{\mu}\right)^n}\right]$$

For a generic Green's function

$$G_{\log} = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n \sum_{m=0}^{n} C_m^{(n)} \ln\delta^m \qquad \delta = \frac{M_X}{M_Y}$$

$$LL \rightarrow \sum_n\left(\frac{\alpha_s}{4\pi}\ln\delta\right)^n$$

$$NLL \rightarrow \sum_n\left(\frac{\alpha_s}{4\pi}\right)^n \ln\delta^{n-1}$$

Matching: Demand that IR physics of full theory reproduced by effective theory.

$$C_{4F}(\mu) = G_F\left(1+\lambda_1\left(\frac{\alpha_s(\mu)}{4\pi}\ln\left(\frac{M_W^2}{m_b^2}\right)+\ln\left(\frac{m_b^2}{\mu^2}\right)+\rho\right)\right)$$

$$\left(\mu\frac{\partial}{\partial\mu}+\gamma_{O_{4F}}\right)O_{4F}^R = 0.$$

$$\gamma_{O_F} = \mu\frac{\partial}{\partial\mu}\left(-\delta_{O_{4F}} + \frac{4\,\delta_2}{2}\right)$$

$$\rightarrow \frac{4}{3}\frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}$$

$$\rightarrow -\frac{1}{\varepsilon}\lambda_1\frac{\alpha_s}{4\pi}$$

The solution to the RG equation:

$$O_{4F}^R(\mu) = O_{4f}^R(\mu_0)\exp\left(\int_{g(\mu_0)}^{g(\mu)}\frac{dg}{g}\frac{\gamma_{O_{4F}}(g)}{\beta(g)}\right)$$

$$O_{4F}^R(M_W) = O_{4F}(m_b)\left(\frac{\ln(M_W^2/\Lambda^2)}{\ln(M_b^2/\Lambda^2)}\right)^{A_1/(2\beta_0)}$$

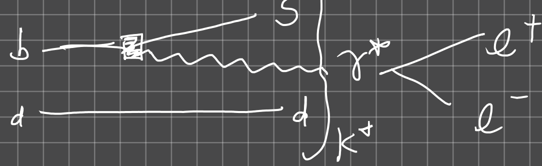
$$\gamma_{O_F} = \frac{\alpha_s}{4\pi}A_1$$

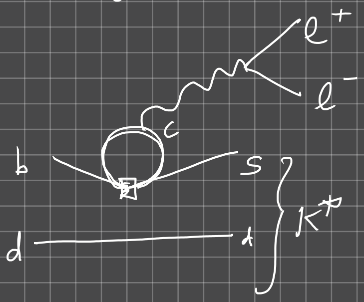
This result is at Leading log order (LL)

For NLL need running at 2-loops.

For accuracy at α^n need to run at order $n+1$ + match at $O(\alpha^n)$.

$B \rightarrow K^* l^+ l^-$.





For full list of operators see hep-ph/9612313 and 0811.1214