Flocks in fluids
Simha-Ramaswamy instability and beyond

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Active Matter and Beyond
ICTS, Bangalore
7 November, 2023

Active matter

Microscopic free energy supply
Conversion to work

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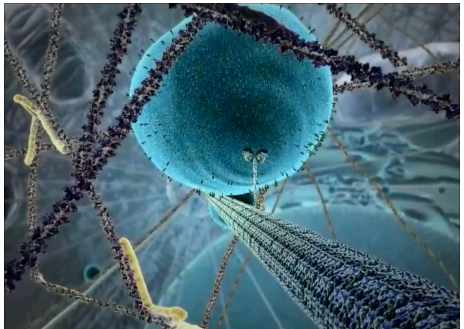
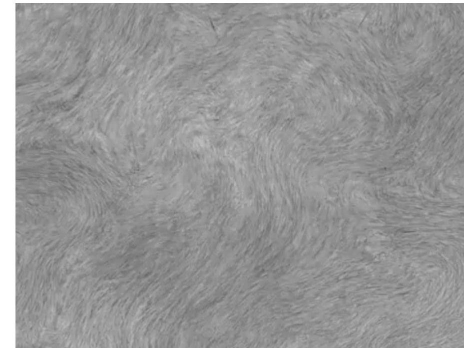
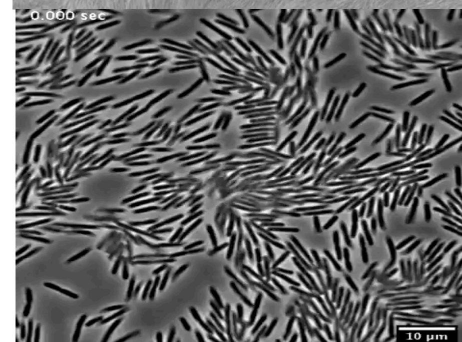


Collective behaviour





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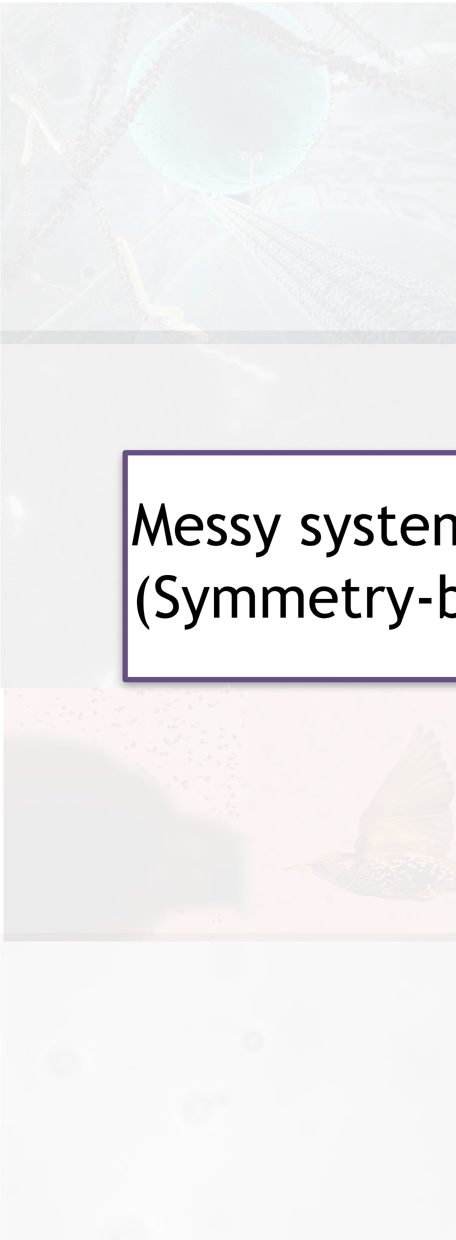
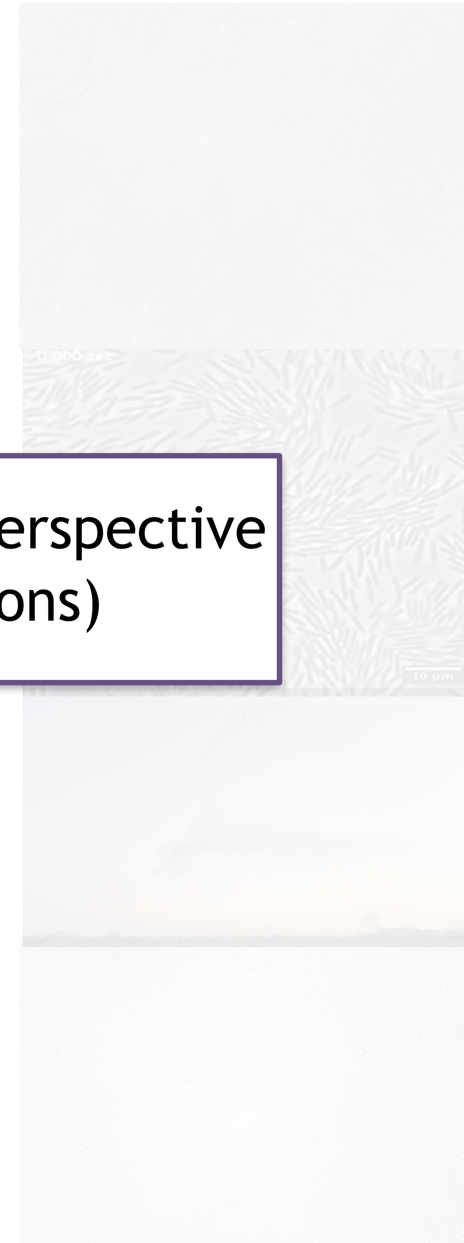


Messy systems →condensed matter physics perspective
(Symmetry-breaking, ordering, phase transitions)



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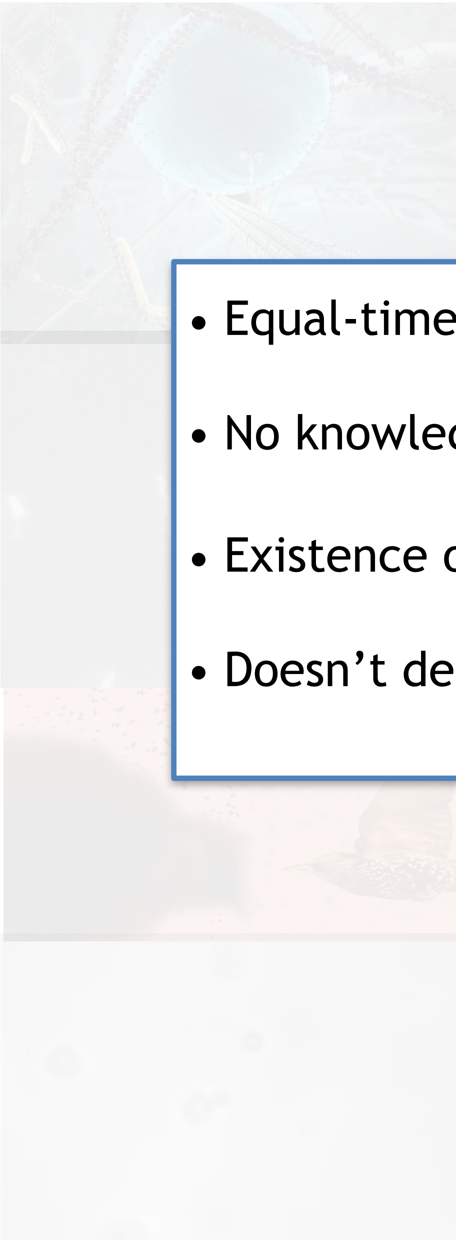
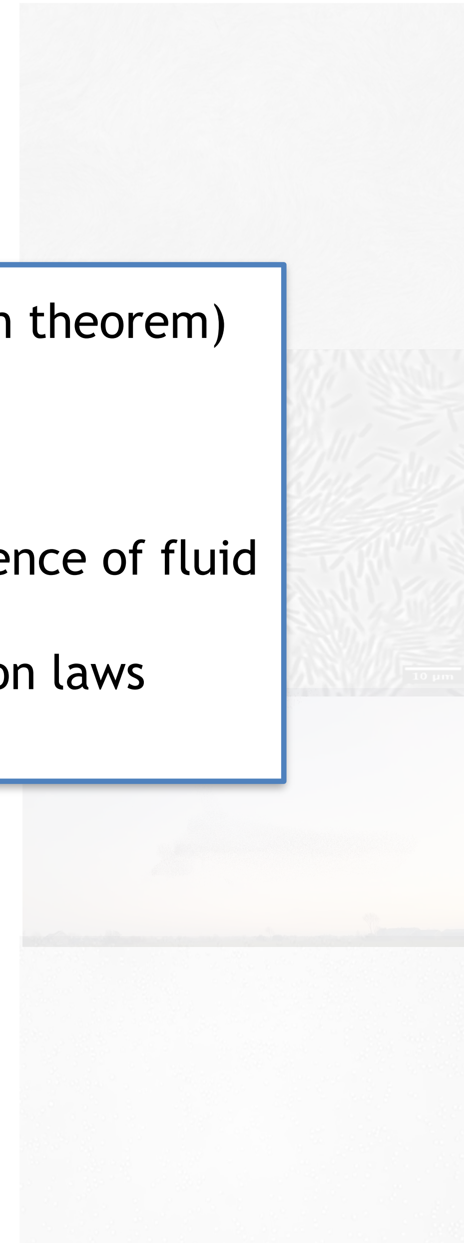


Microscopic free energy supply
Conversion to work

Equilibrium

- Equal-time correlators →free energy (equipartition theorem)
- No knowledge of dynamics required
- Existence of ordered state → independent of presence of fluid
- Doesn't depend on momentum or other conservation laws

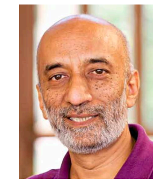


Active Liquid Crystalline Fluids

Uniaxial order in momentum-conserved fluids?

Hydrodynamic Fluctuations and Instabilities in Ordered Suspensions of Self-Propelled Particles

R. Aditi Simha* and Sriram Ramaswamy†
Centre for Condensed-Matter Theory, Department of Physics, Indian Institute of Science, Bangalore 560 012, India



(Received 18 August 2001; published 15 July 2002)

We construct the hydrodynamic equations for suspensions of self-propelled particles (SPPs) with spontaneous orientational order, and make a number of striking, testable predictions: (i) Nematic SPP suspensions are always absolutely unstable at long wavelengths. (ii) SPP suspensions support novel propagating modes at long wavelengths, coupling orientation, flow, and concentration. (iii) In a wave number regime accessible only in low Reynolds number systems such as bacteria, polar-ordered suspensions are invariably convectively unstable. (iv) The variance in the number N of particles, divided by the mean $\langle N \rangle$, diverges as $\langle N \rangle^{2/3}$ in polar-ordered SPP suspensions.

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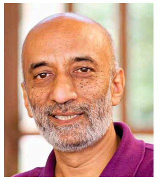

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(iii) In a wave number regime accessible only in low Reynolds number systems such as bacteria, polar-ordered suspensions are invariably convectively unstable.

Simha-Ramaswamy instability of uniaxial active suspensions

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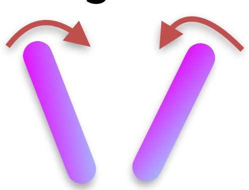
Proof by contradiction: Assume an ordered state; show it is unstable

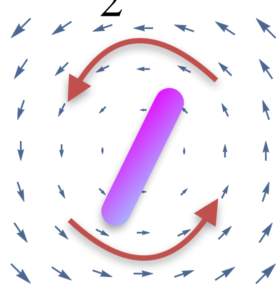
$$\partial_t Q = -\gamma \frac{\delta F}{\delta Q} + \mathbf{Q} \cdot \mathbf{\Omega} - \mathbf{\Omega} \cdot \mathbf{Q} - \lambda A$$

Passive alignment



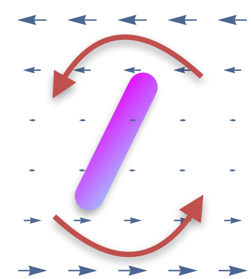
Co-rotation

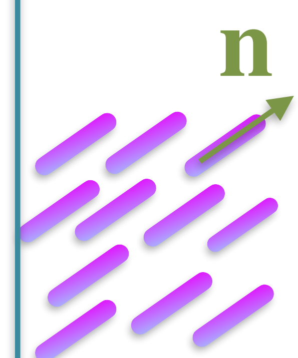
$$\mathbf{\Omega} = \frac{1}{2}\left[\nabla \mathbf{v} - (\nabla \mathbf{v})^T\right]$$



Strain-rate alignment

$$A = \frac{1}{2}\left[\nabla \mathbf{v} + (\nabla \mathbf{v})^T\right]$$





Director: $\mathbf{n}$ (unit vector)

Degree of ordering $\mathbf{Q} = \left\langle \mathbf{n}\mathbf{n} - \frac{\mathbf{I}}{d} \right\rangle$

Apolar order parameter

Simha & Ramaswamy, PRL 2002

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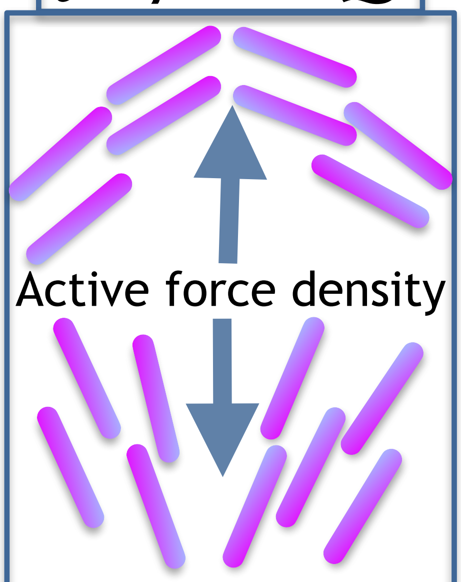
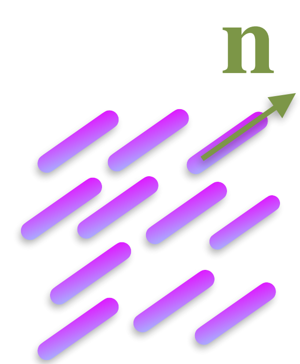
$$\eta \nabla^2 \mathbf{v} = \nabla \Pi + \zeta \Delta\mu \nabla\cdot Q$$

Pressure gradient
Incompressible
Flow.
Enforces
$\nabla\cdot\mathbf{v} = 0$





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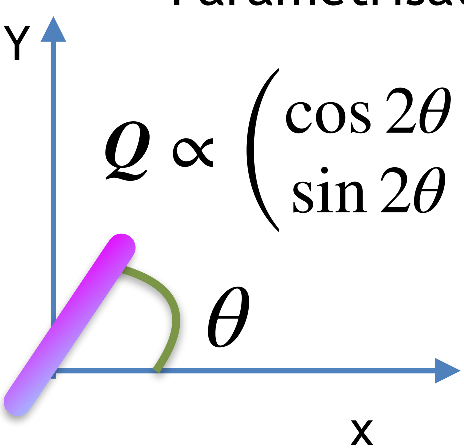
Parametrisation

$$Q \propto \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$



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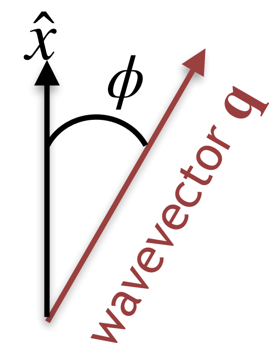

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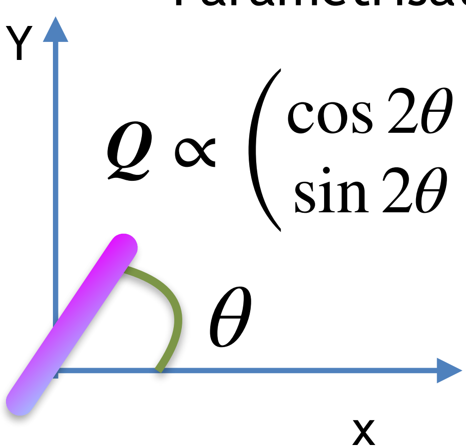


Eigenfrequency of orientational fluctuations

$$\omega = i\frac{\zeta\Delta\mu}{2\eta}\cos 2\phi(1 - \lambda\cos 2\phi)$$

Positive growth rate above or below $\phi = \pi/4$

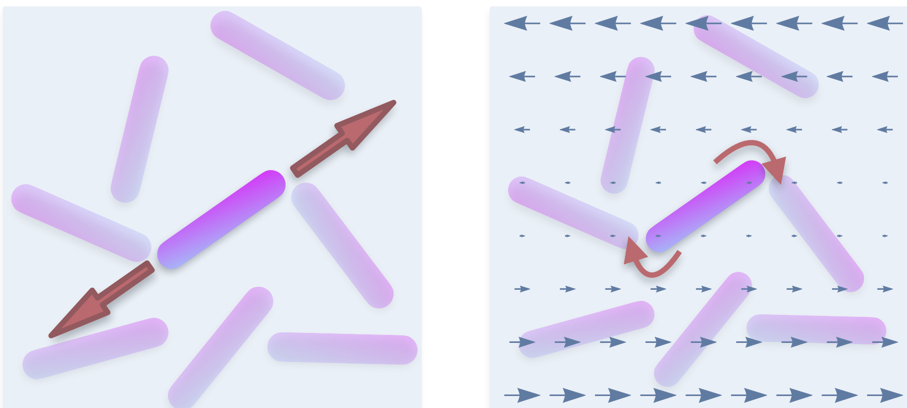
Growth rate independent of wavenumber

Viscosity/activity → Unique timescale

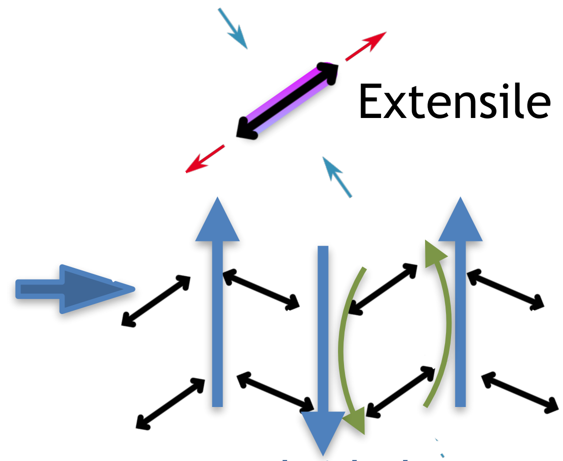
Active Liquid Crystalline Fluids

Uniaxial order in momentum-conserved fluids?

$$\zeta\Delta\mu > 0$$



Active fluid flow   Flow alignment



Destabilises bend

$$\phi \approx 0$$

Simha & Ramaswamy, PRL 2002;

Ramaswamy & Rao, NJP 2007

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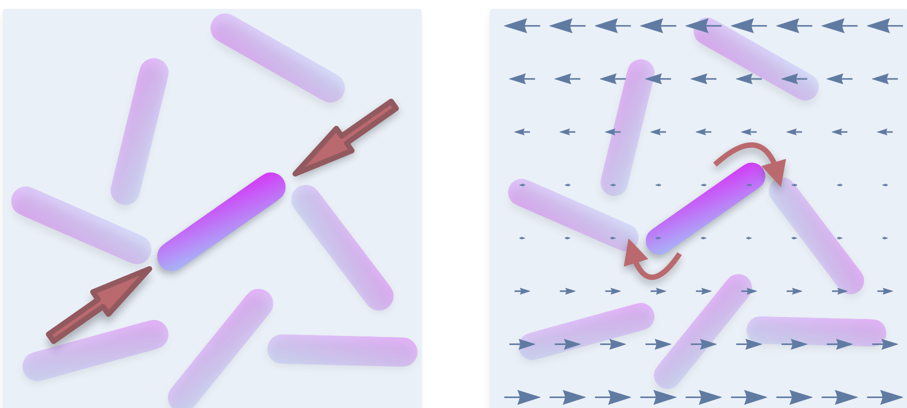
$\zeta \Delta \mu < 0$



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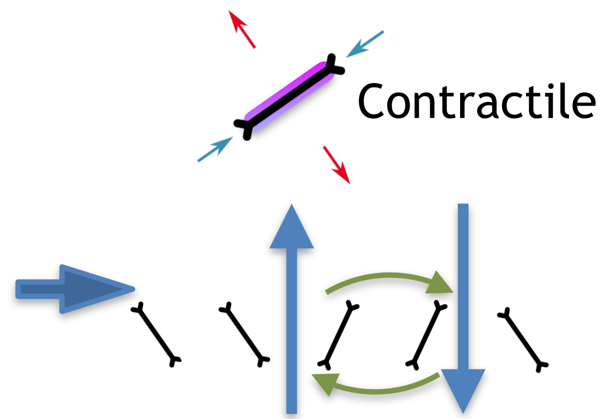

Fluid Flow
Destabilises splay

$$\phi \approx \frac{\pi}{2}$$

Ramaswamy & Rao, NJP 2007

Eigenfrequency of orientational fluctuations

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Simha-Ramaswamy instability for uniaxial suspensions

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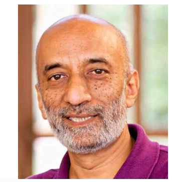

Momentum conservation + Number/ mass conservation

→ Nematic phase doesn't exist

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* With inertia: Polar flock saved by Toner-Tu waves→See **Chatterjee et al. PRX 2021**

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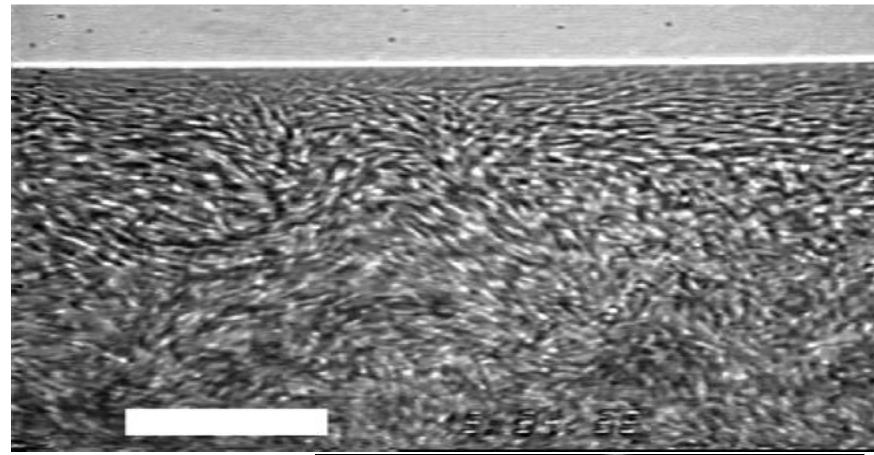

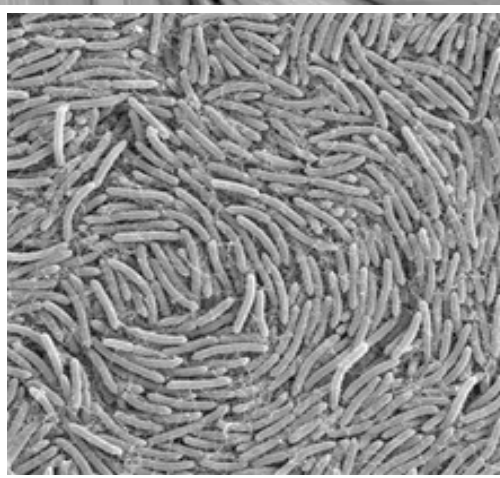

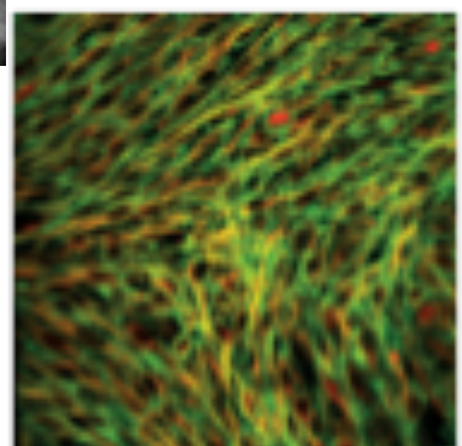

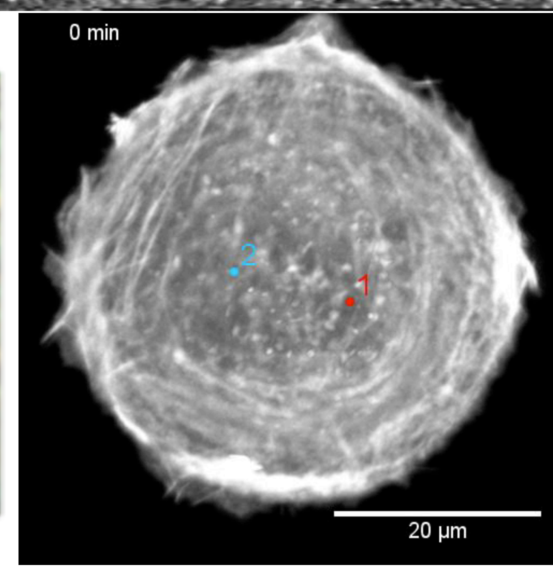

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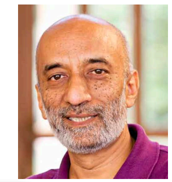

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How to have uniaxial suspensions in Stokesian active fluids?

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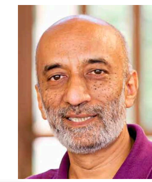

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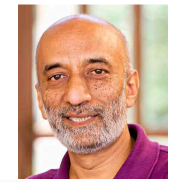

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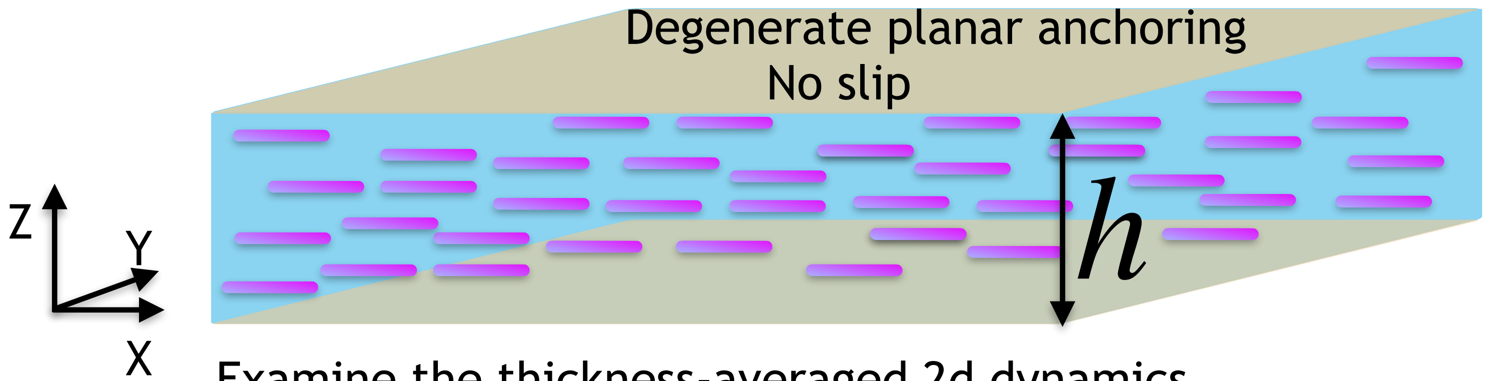
Nematic order in active fluids on a substrate or in a confined geometry



Examine the thickness-averaged 2d dynamics

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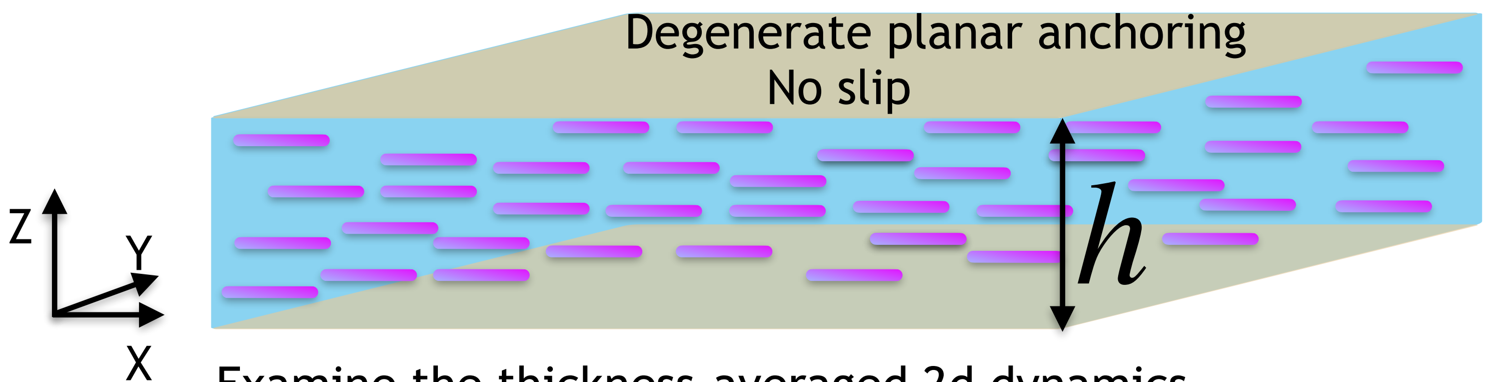

Examine the thickness-averaged 2d dynamics

Expectation   Voituriez, Joanny, Prost, EPL 2005

Hydrodynamics cut-off at scale $\sim h$

Average with lubrication approximation

$$\left(v_z = 0, \partial_z^2 \gg \partial_x^2, \partial_y^2 \right)$$

2d incompressibility $\nabla \cdot \mathbf{v} = 0$

(∇ $\to$ 2d gradient, $\mathbf{v}$ $\to$ z-averaged velocity)

Effective friction: $\Gamma \propto \eta / h^2$

Active growth rate $\sim q^2$

Fights against elasticity

$$\omega = i q^2 \left[\frac{\zeta \Delta\mu}{2\Gamma} \cos 2\phi (1 - \lambda \cos 2\phi) - \gamma K \right]$$

Stable at small $\Delta\mu$ but unstable for

$\Delta\mu \gg 2\gamma K \Gamma / |\zeta|$

($\Delta\mu \gtrsim 2\gamma K \Gamma / |\zeta|$ when $|\lambda| < 1$)

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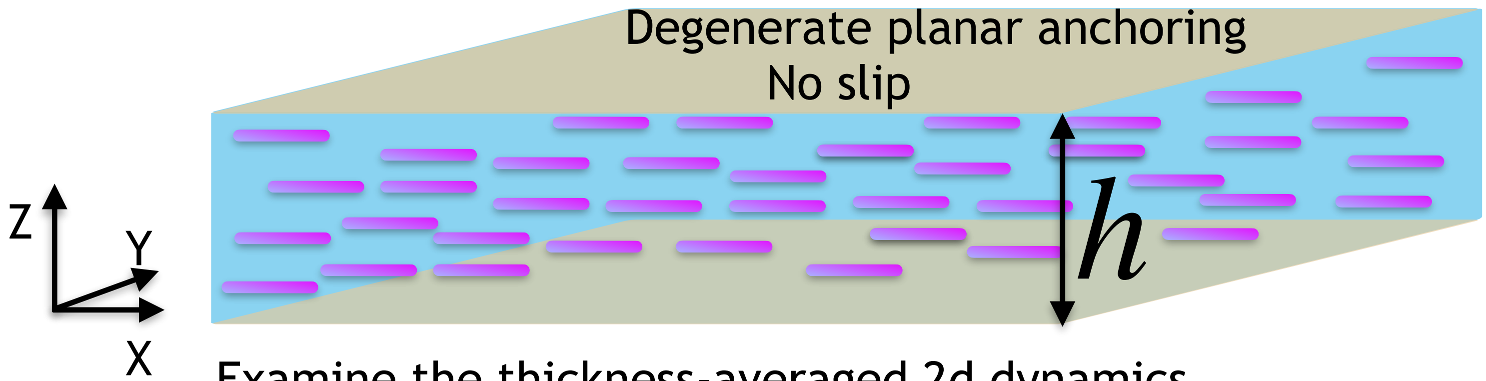

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Reality

Maitra et al., PNAS 2018

Not extensive in the z direction → Need more careful gradient expansion

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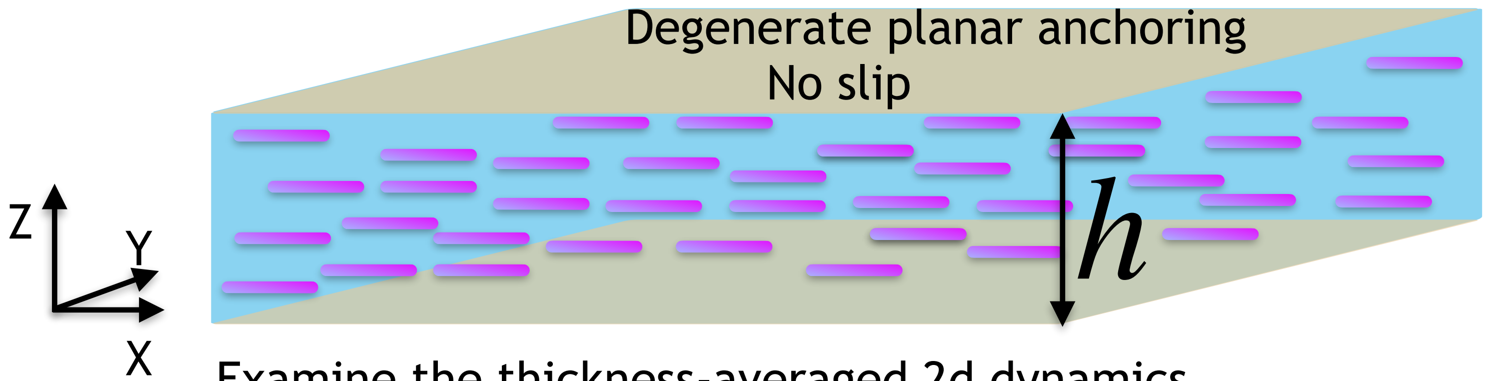

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As viscosity leads to friction $\eta \nabla_3^2 \mathbf{v}_3 \approx \eta \partial_z^2 \mathbf{v}_3 \to -\eta \mathbf{v}/h^2 = -\Gamma \mathbf{v} \; \left(\partial_z^2 \propto 1/h^2\right)$

Higher order in gradient active stresses can affect $q \to 0$ dynamics

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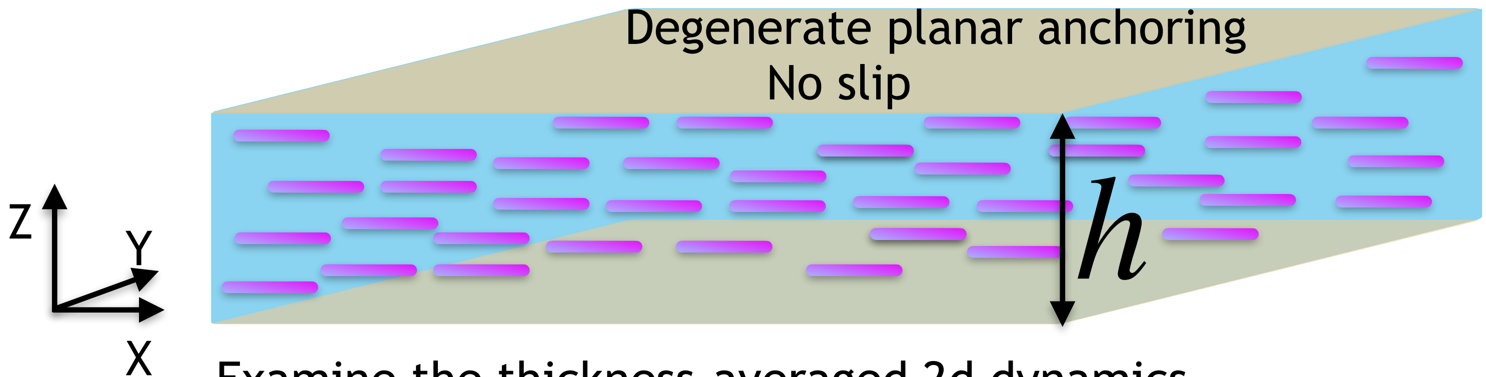

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$$\boldsymbol{\sigma}_3 \propto \left[\nabla_3(\mathbf{Q}_3 \cdot \nabla_3 \cdot \mathbf{Q}_3)\right]^{ST} \implies \mathbf{f}^a \propto \nabla_3^2\left(\mathbf{Q}_3 \cdot \nabla_3 \cdot \mathbf{Q}_3\right)$$

Projected z-averaged active force $\left(\partial_z^2 \propto 1/h^2\right) \rightarrow \mathbf{Q} \cdot \nabla \cdot \mathbf{Q}$

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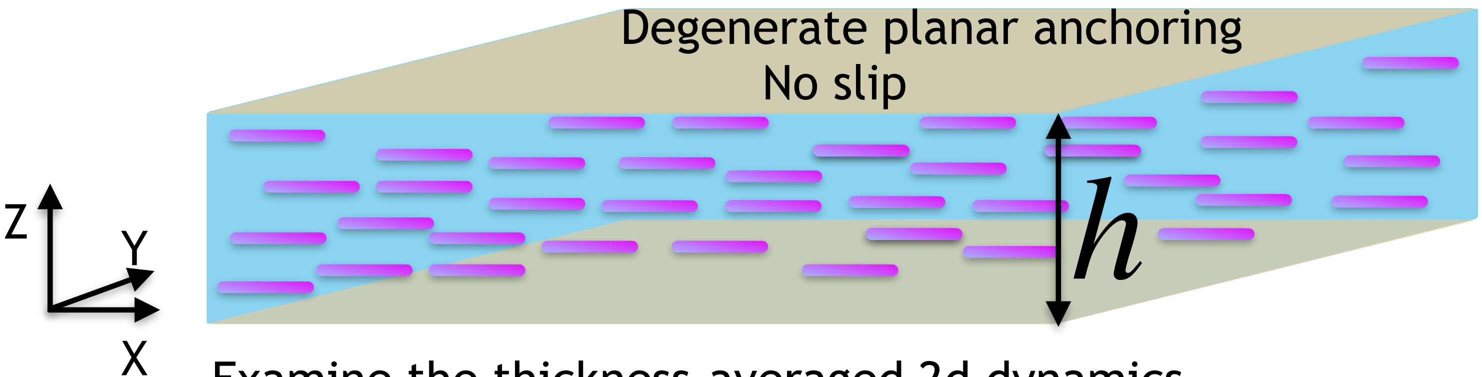

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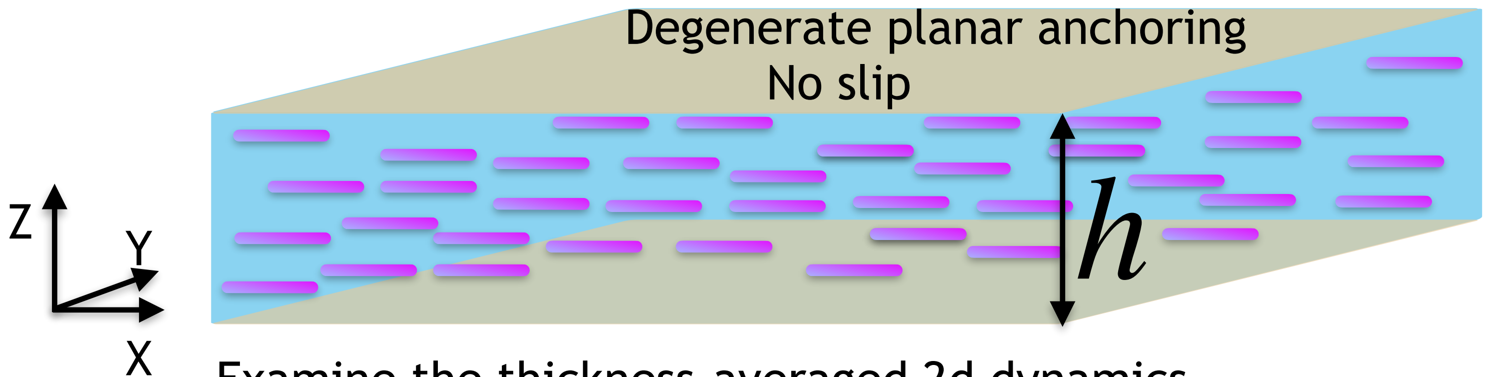

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Different angular symmetry from the usual active force $\propto \nabla \cdot \mathbf{Q}$

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What does this mean?

Different active force along and transverse to the ordering direction

Different active force for splay $\mathbf{n}(\nabla\cdot\mathbf{n})$ and bend $\mathbf{n}\cdot\nabla\mathbf{n}$ distortions

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Active force in systems not conserving momentum: $\mathbf{F}^a \propto \mathbf{p}$ (polarisation)

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Active force in systems not conserving momentum: $\mathbf{F}^a \propto \mathbf{p}$ (polarisation)

A general polarisation in a nematic: $\mathbf{p} = a\mathbf{n}(\nabla \cdot \mathbf{n}) + b\mathbf{n} \cdot \nabla \mathbf{n}$

Active Nematic Fluids

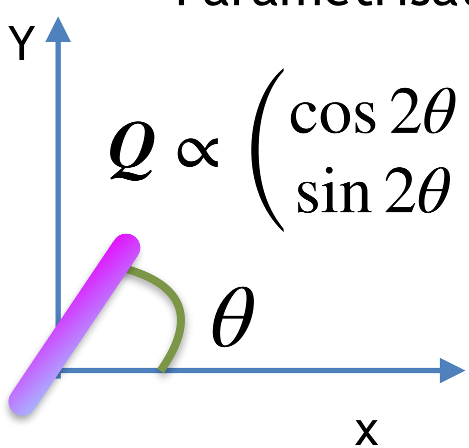
Nematic order in active fluids on a substrate or in a confined geometry

$$\partial_t Q = -\gamma \frac{\delta F}{\delta Q} + \mathbf{Q}\cdot\mathbf{\Omega} - \mathbf{\Omega}\cdot\mathbf{Q} - \lambda A$$

$$\Gamma \mathbf{v} = -\nabla\Pi - \zeta_1 \Delta\mu \nabla\cdot Q + \zeta_2 \Delta\mu \mathbf{Q}\cdot(\nabla\cdot\mathbf{Q})$$

Parametrisation

$$Q \propto \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$



Ordered state: Along $\hat{x}$

$$\theta_0 = 0$$

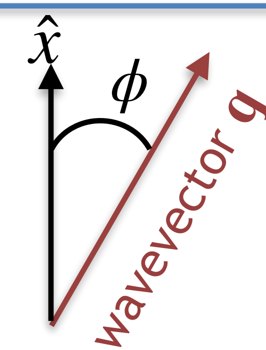
Spatio-temporal fluctuations:

$\theta(\mathbf{r}, t)$ about this state

Active Nematic Fluids

Nematic order in active fluids on a substrate or in a confined geometry

$$\partial_t \theta = \underbrace{\gamma K \nabla^2 \theta}_{\text{Elasticity}} + \underbrace{\frac{1-\lambda}{2}\partial_x v_y - \frac{1+\lambda}{2}\partial_y v_x}_{\text{Flow coupling}}$$



$$\Gamma \mathbf{v} = -\nabla \Pi - \underbrace{(\zeta_1 - \zeta_2)\Delta\mu \partial_x \theta \hat{y}}_{\text{Active force for bend}} - \underbrace{(\zeta_1 + \zeta_2)\Delta\mu \partial_y \theta \hat{x}}_{\text{Active force for splay}}$$

Eigenfrequency: $\omega = -iq^2 \left[\gamma K - \frac{\Delta\mu}{2\Gamma}(\zeta_1 \cos 2\phi - \zeta_2)(1 - \lambda \cos 2\phi)\right]$

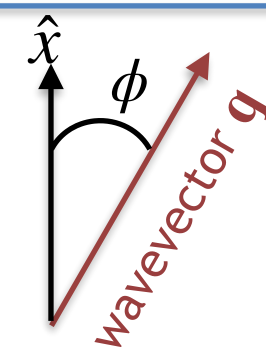
Active Nematic Fluids

Nematic order in active fluids on a substrate or in a confined geometry

$$\partial_t \theta = \gamma K \nabla^2 \theta + \frac{1-\lambda}{2}\partial_x v_y - \frac{1+\lambda}{2}\partial_y v_x$$

Elasticity

Flow coupling



$$\Gamma \mathbf{v} = -\nabla \Pi - (\zeta_1 - \zeta_2)\Delta\mu \partial_x \theta \hat{y} - (\zeta_1 + \zeta_2)\Delta\mu \partial_y \theta \hat{x}$$

Active force for bend

Active force for splay

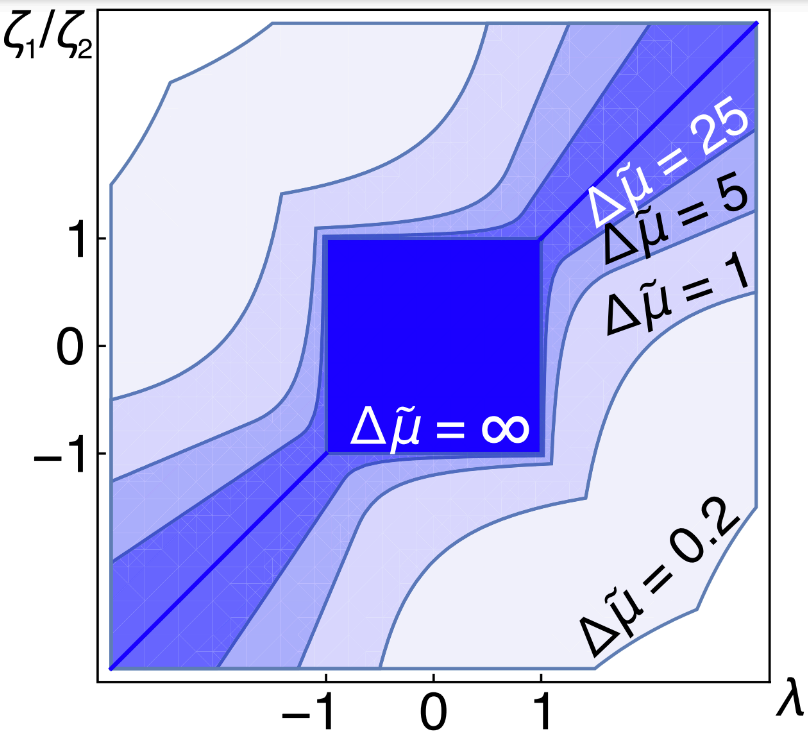
Eigenfrequency: $\omega = -iq^2 \left[\gamma K - \frac{\Delta\mu}{2\Gamma}(\zeta_1 \cos 2\phi - \zeta_2)(1 - \lambda \cos 2\phi) \right]$

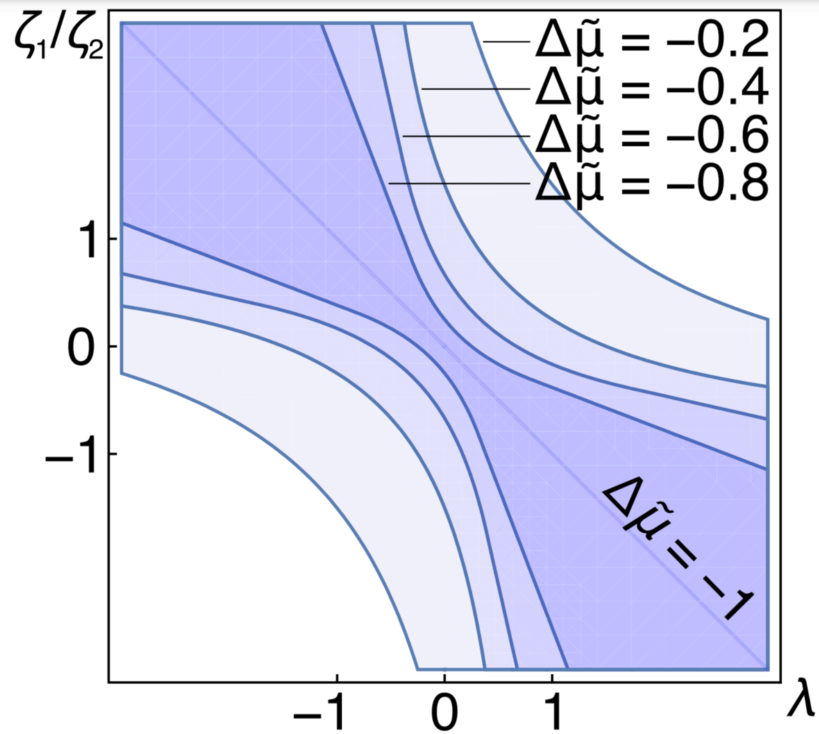
Important: $\Delta\mu$ dependent part does not change sign around $\phi = \pi/4$

Depending on ζ_1/ζ_2 and λ, may never change sign $\rightarrow$ always stable

Active Nematic Fluids

Nematic order in active fluids on a substrate or in a confined geometry



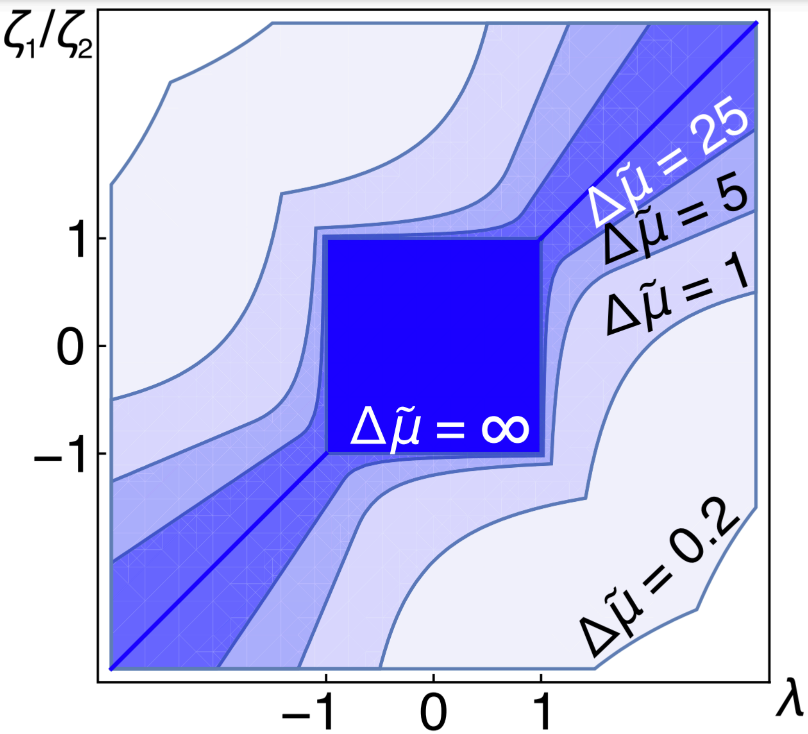


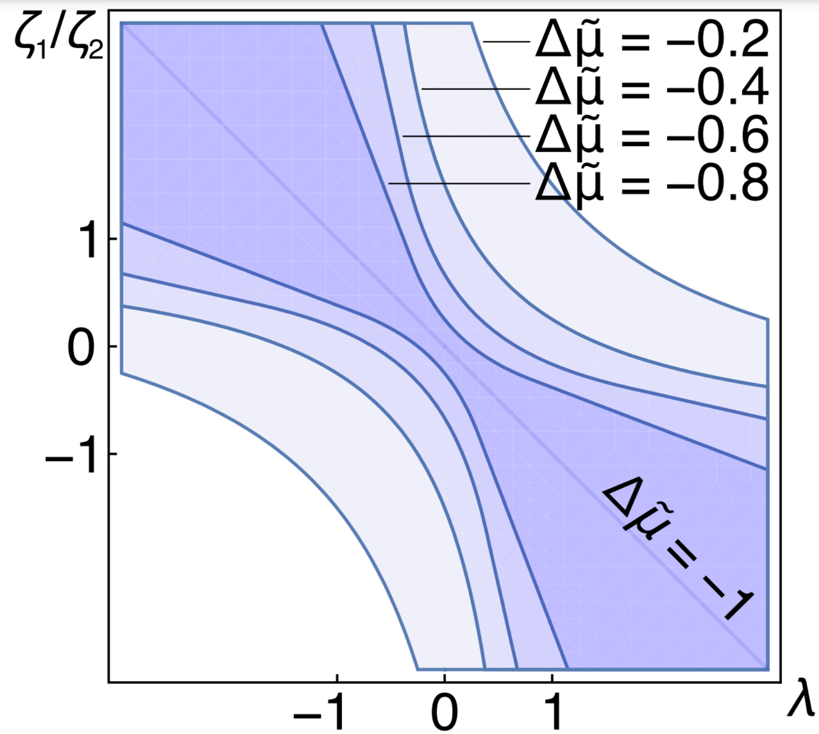
Stable even for $\Delta\mu \to \infty$ in flow-tumbling systems

Quasi-long-range ordered nematic

Active Nematic Fluids

Nematic order in active fluids on a substrate or in a confined geometry





Quasi-long-range ordered nematic at arbitrary activity with $\mathbf{f}^a \propto \mathbf{Q} \cdot \nabla \cdot \mathbf{Q}$

Active Nematic Fluids

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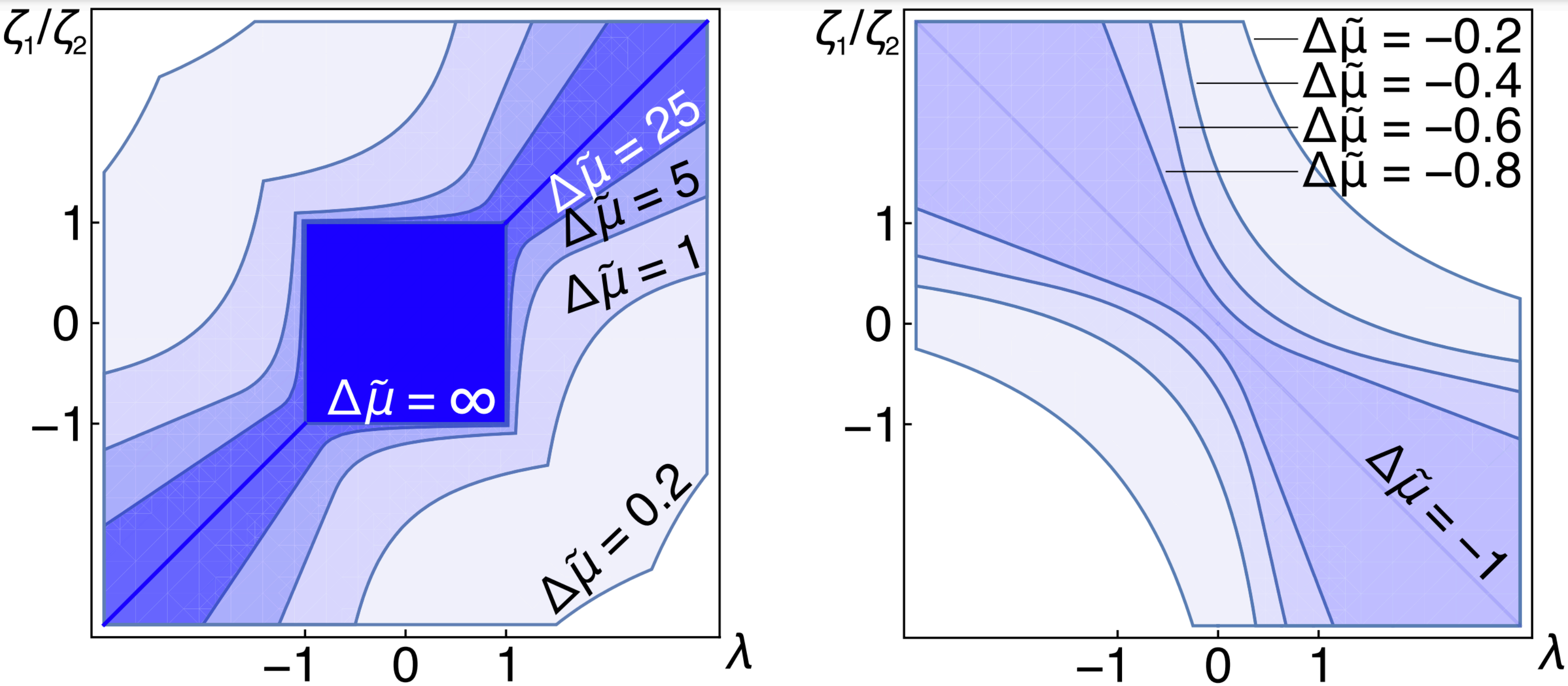

Quasi-long-range ordered nematic at arbitrary activity with $\mathbf{f}^a \propto \mathbf{Q} \cdot \nabla \cdot \mathbf{Q}$

Large scales: $\mathbf{f}^a \propto \mathbf{Q} \cdot \nabla \cdot \mathbf{Q}$ dominates over $\nabla \cdot \mathbf{Q}$

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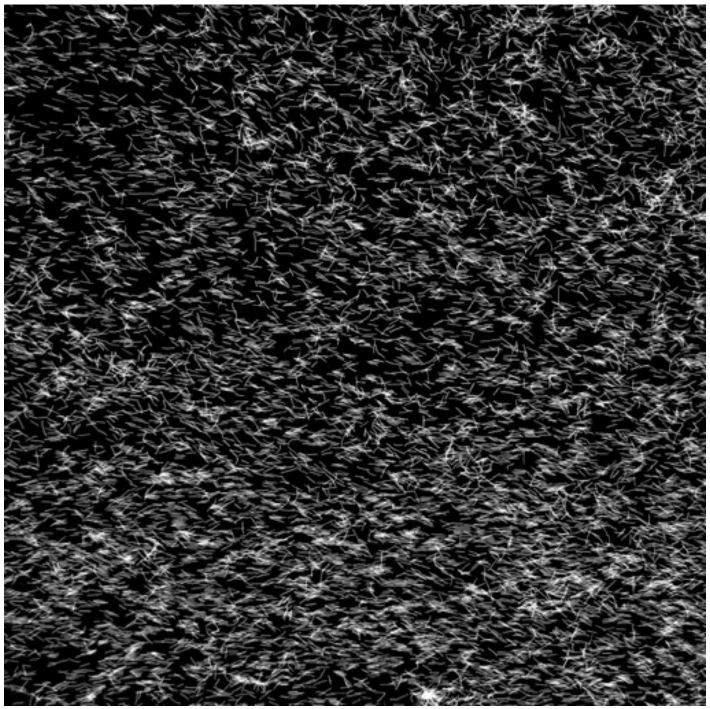

Nishiguchi et al. PRE 2018

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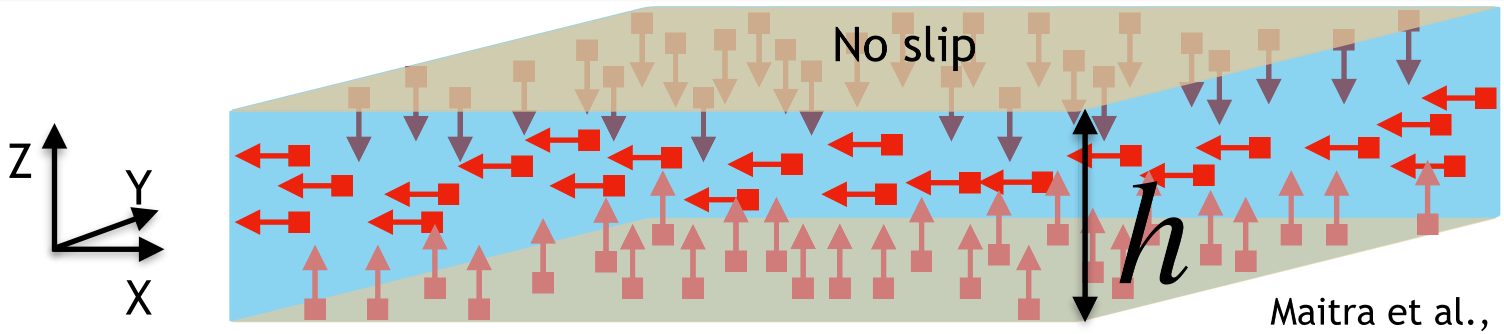
Active Polar Fluids

Polar order in active fluids on a substrate or in a confined geometry



Maitra et al., PRL 2020

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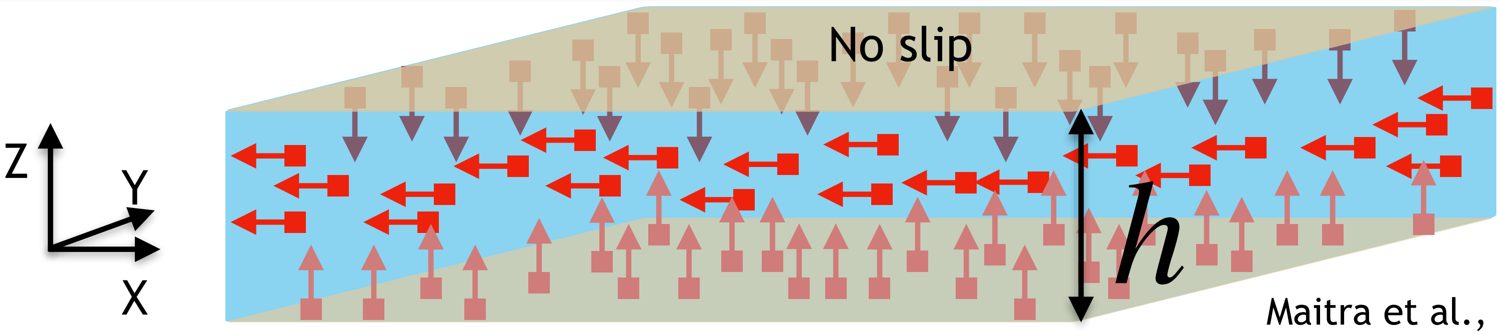

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Active polar order in bulk <u>Stokesian</u> fluid → orientation fluctuation ≡ apolar

Generically unstable → no order

Active Polar Fluids

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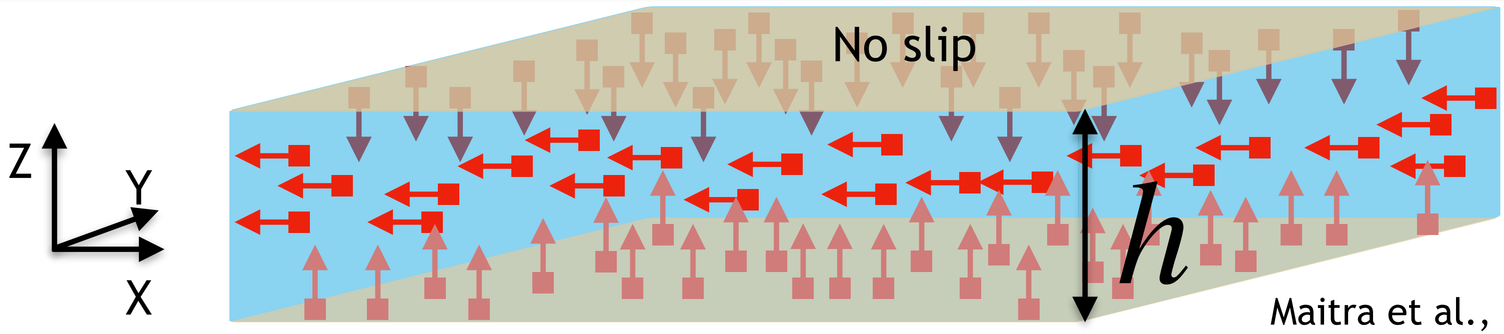

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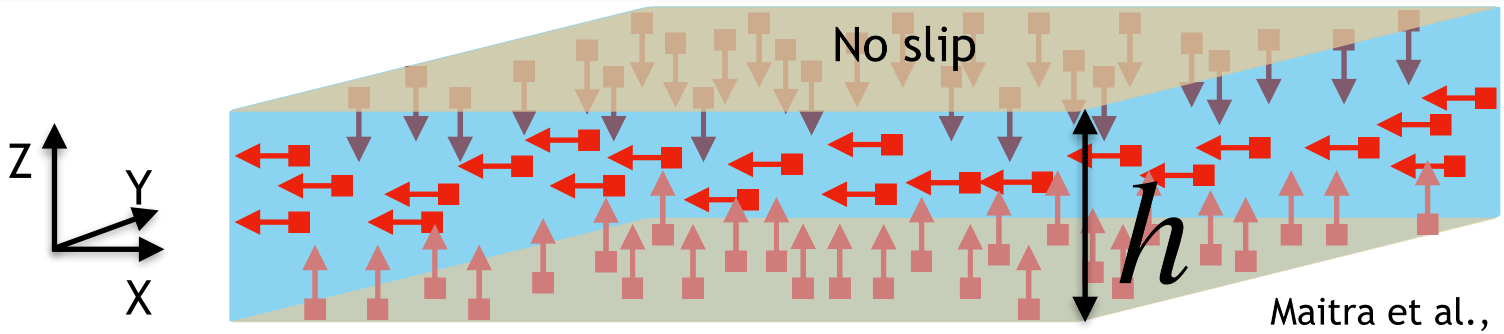

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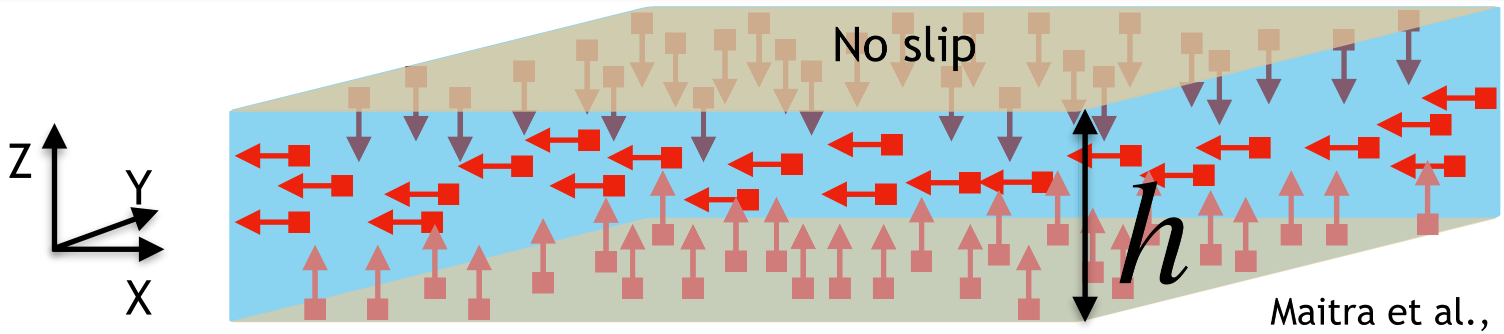
Equivalent to active apolar order

Polar Terms

$$\eta \nabla_3^2 \mathbf{v}_3 = \nabla_3 \Pi + \zeta \Delta\mu \nabla_3 \cdot (\mathbf{pp}) + \zeta_p \Delta\mu \nabla_3^2 \mathbf{p}$$

Active Polar Fluids

Polar order in active fluids on a substrate or in a confined geometry



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Polar passive Flow-alignment

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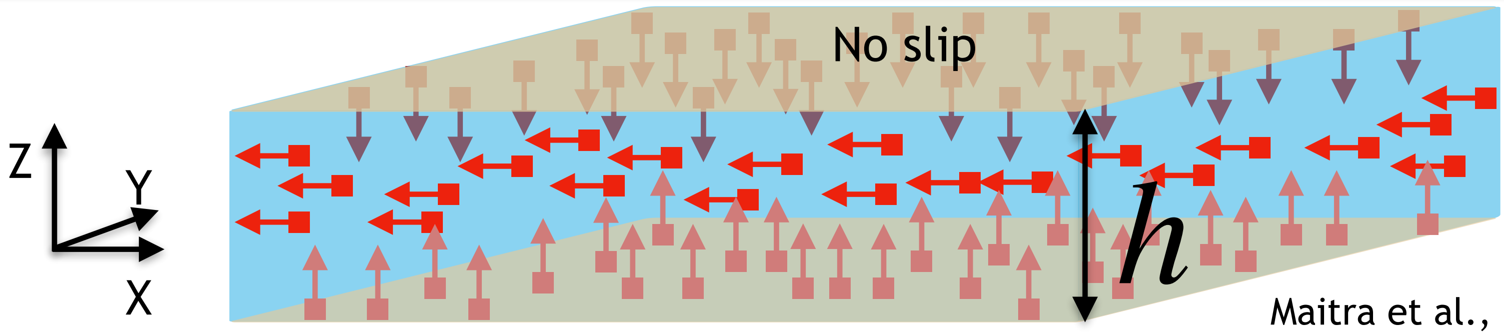

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$\lambda_p \nabla_3^2 \mathbf{v}_3$ → Polar passive Flow-alignment

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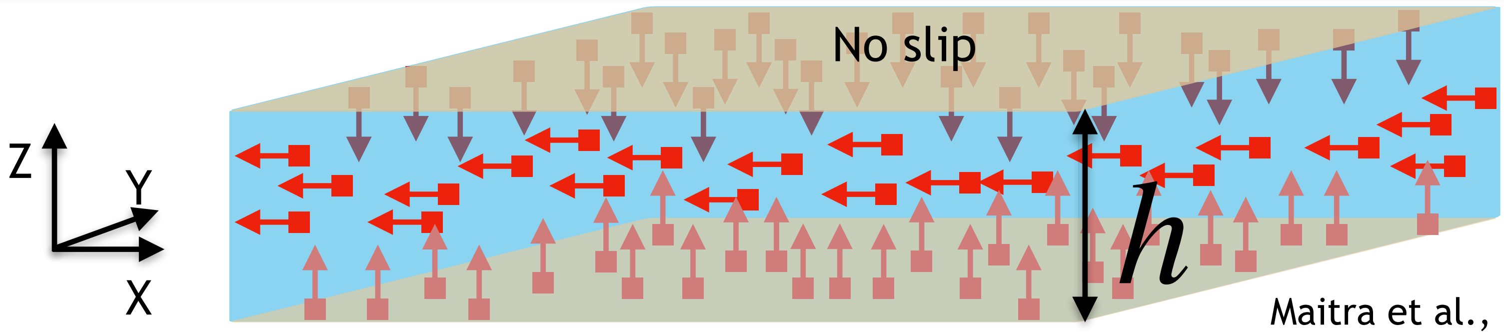
$$\eta \nabla_3^2 \mathbf{v}_3 = \nabla_3 \Pi + \zeta \Delta\mu \nabla_3 \cdot (\mathbf{pp}) + \zeta_p \Delta\mu \nabla_3^2 \mathbf{p}$$

$\zeta_p \Delta\mu \nabla_3^2 \mathbf{p}$ → Active polar force Fore-aft asymmetry; motility

Active Polar Fluids

Polar order in active fluids on a substrate or in a confined geometry



Maitra et al., PRL 2020

Thickness average using lubrication approximation

Active Polar Fluids

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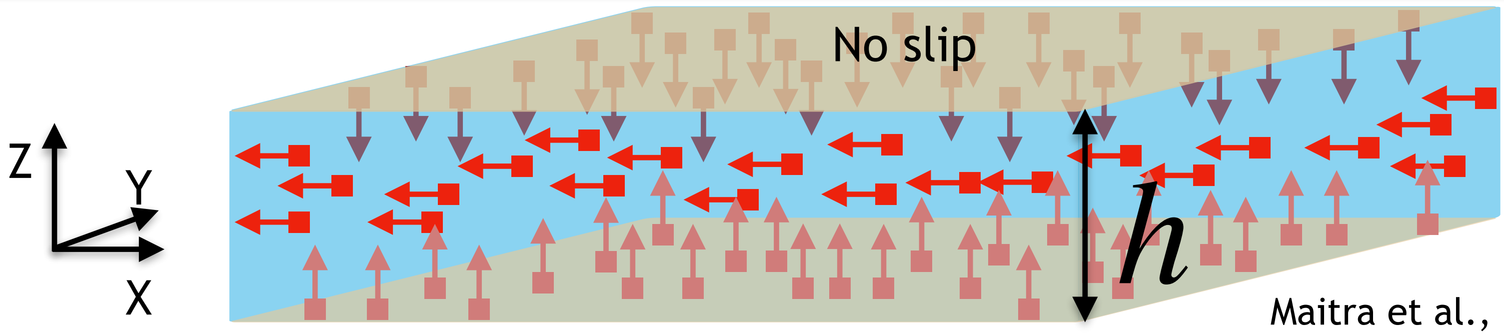


Maitra et al., PRL 2020

Thickness average using lubrication approximation

$$\dot{\mathbf{p}} = \Lambda\mathbf{v} - \gamma\frac{\delta F}{\delta \mathbf{p}}\,; \qquad \Gamma\mathbf{v} = -\nabla\Pi + \upsilon\mathbf{p} - \Lambda\frac{\delta F}{\delta \mathbf{p}}$$

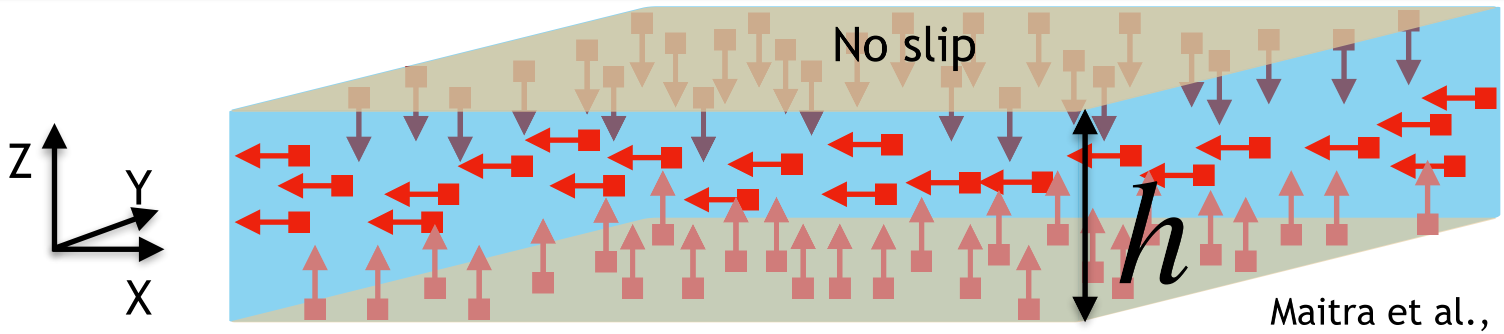
Steady state: $\mathbf{p}_0 = p_0\hat{x}$

$\mathbf{v}_0 = (wp_0/\Lambda)\hat{x}.$

$$w \equiv \frac{\Lambda\upsilon}{\Gamma + \Lambda^2}$$

Active Polar Fluids

Polar order in active fluids on a substrate or in a confined geometry



Maitra et al., PRL 2020

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Long-range interaction due to incompressibility

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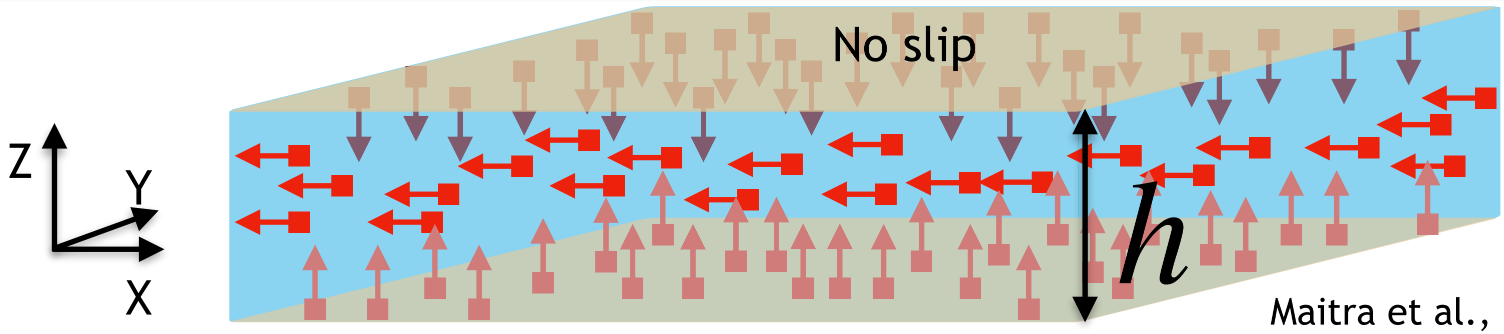

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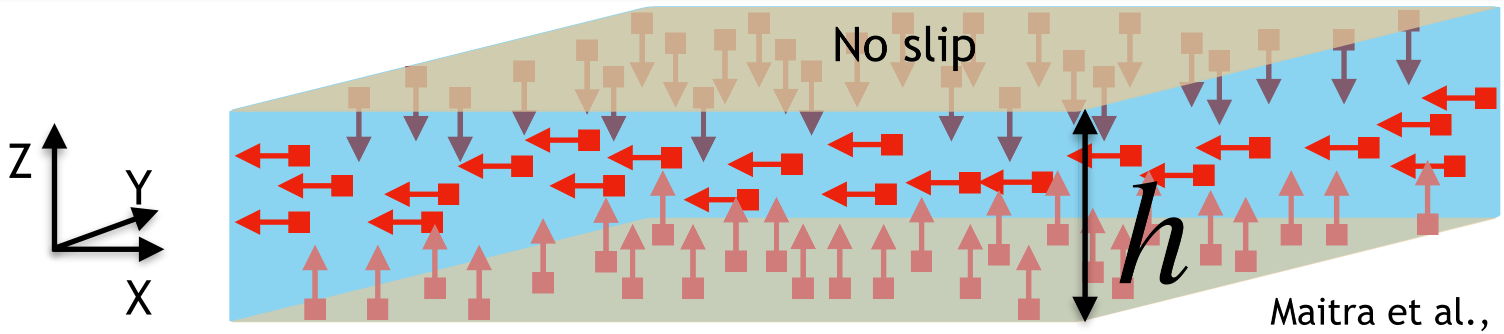

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Long-range order

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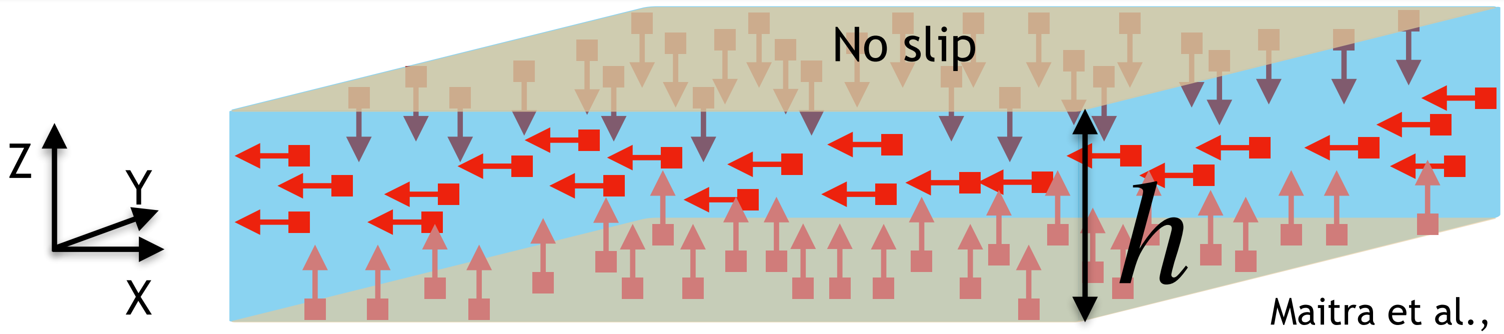

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Exact mapping to equilibrium dipolar magnet (Kashuba, PRL 1994)
(Advection irrelevant)

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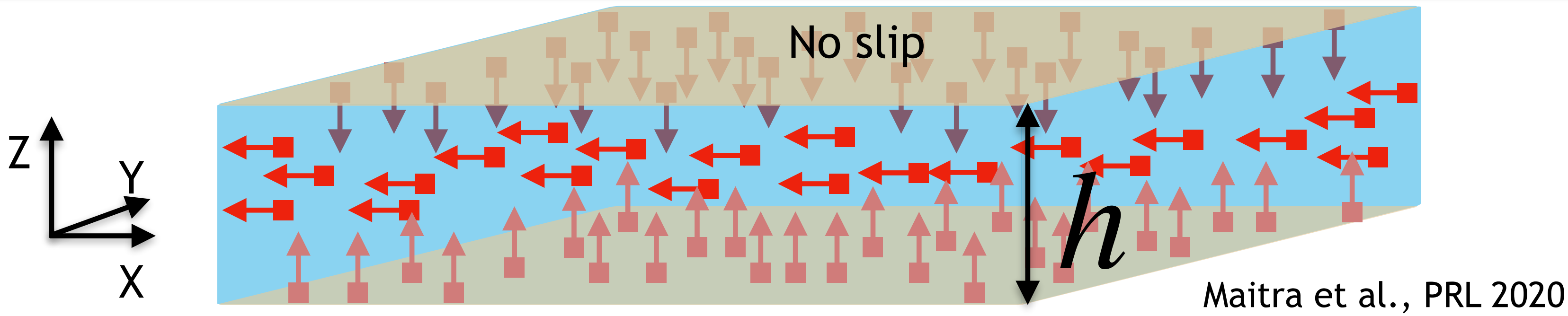

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Dynamical exponent from dynamical RG (Chen, Lee, Maitra, Toner, arXiv 2023)

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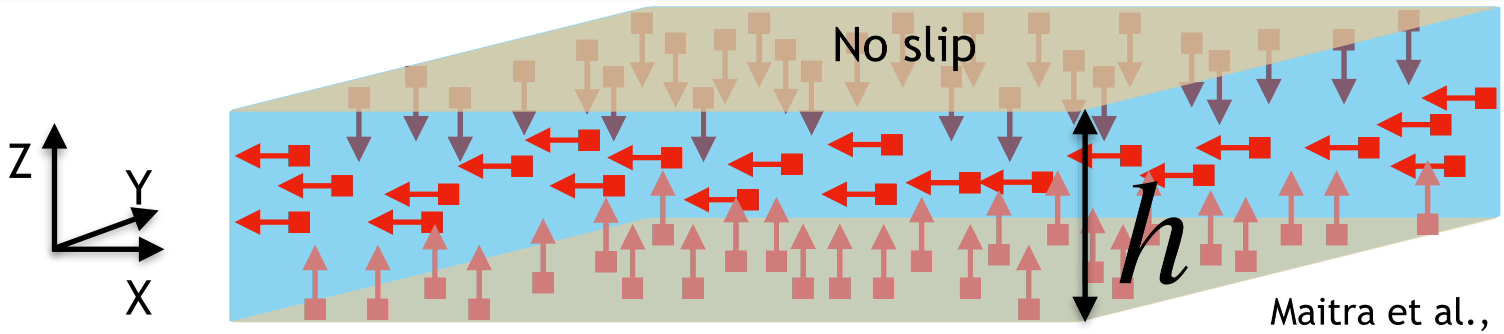

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Surprising variant of Anderson mechanism

Active Polar Fluids

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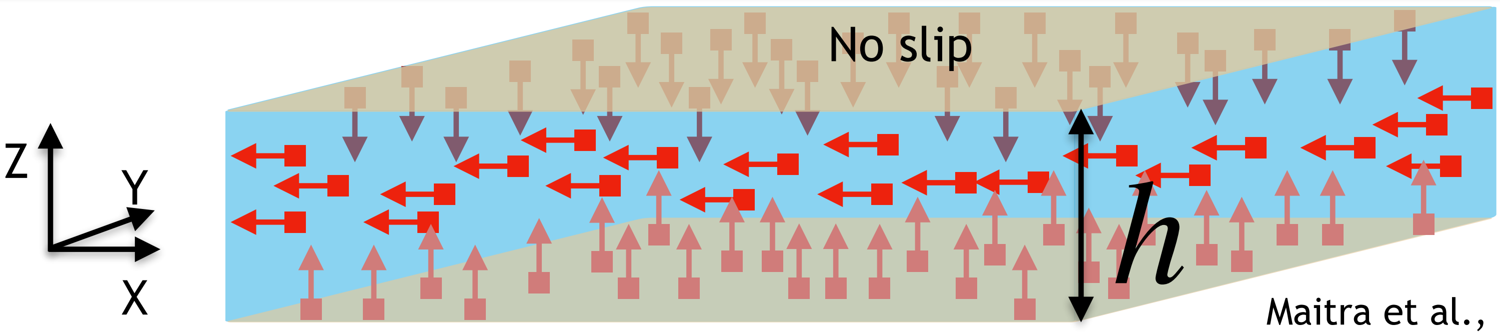

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LRO even on disordered substrate (Maitra, PRE 2020; Chen, Lee, Maitra, Toner, PRL 2022)

Exponents on disordered substrates via RG (Chen, Lee, Maitra, Toner, PRE 2022)

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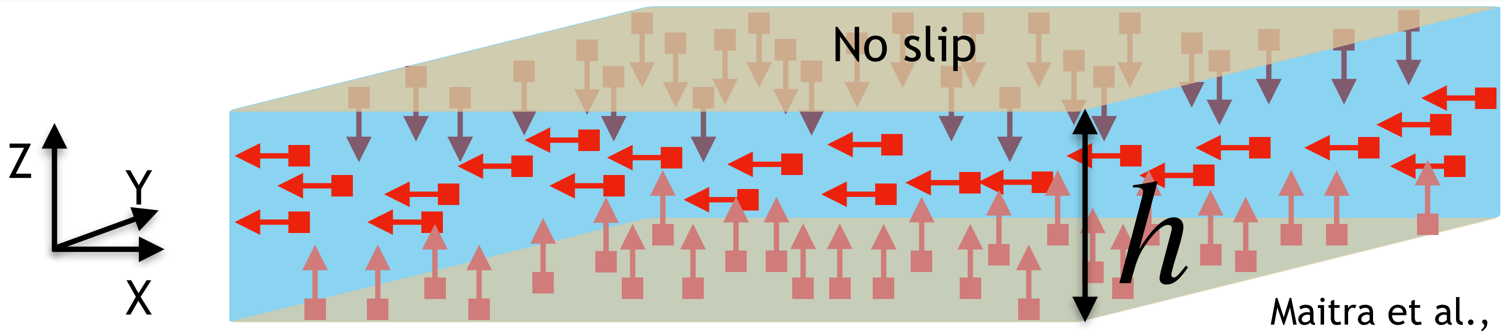

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Exponents on disordered substrates via RG (Chen, Lee, Maitra, Toner, PRE 2022)

Self-advection important. LRO in disordered media not possible in passive systems

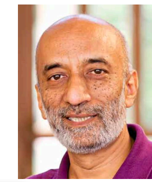
Active Liquid Crystalline Fluids

Hydrodynamic Fluctuations and Instabilities in Ordered Suspensions of Self-Propelled Particles

R. Aditi Simha* and Sriram Ramaswamy†

Centre for Condensed-Matter Theory, Department of Physics, Indian Institute of Science, Bangalore 560 012, India

(Received 18 August 2001; published 15 July 2002)



Momentum conservation + Number/ mass conservation

→ Nematic phase doesn't exist

→ Polar phase doesn't exist in the Stokesian regime *

How to have uniaxial suspensions in Stokesian active fluids?

~~Momentum conservation~~

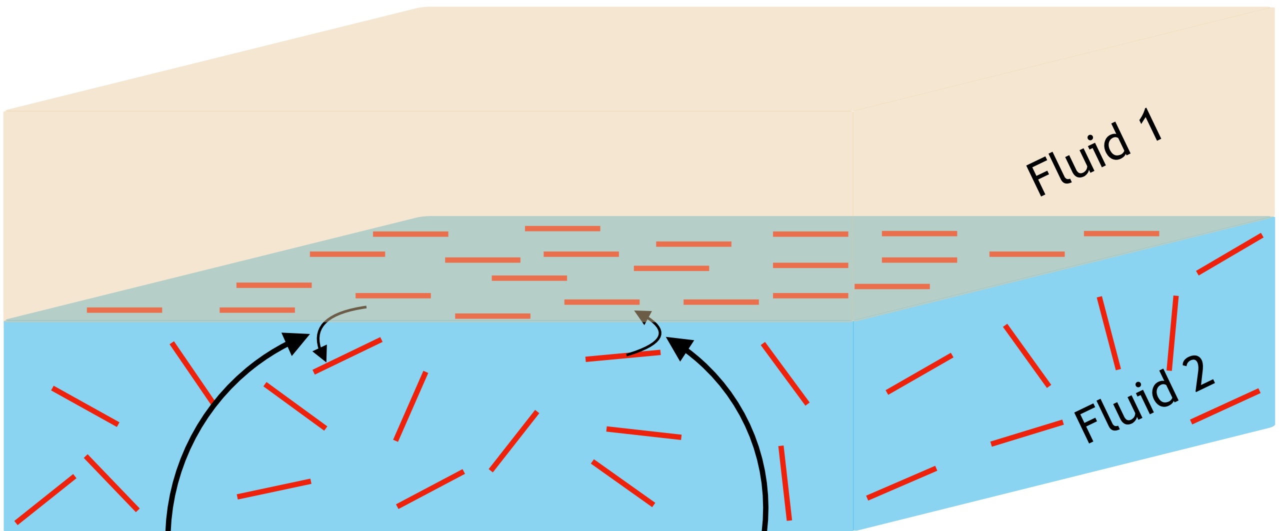
Or

~~Mass/ number conservation~~

* With inertia: Polar flock saved by Toner-Tu waves→See **Chatterjee et al. PRX 2021**

Active Uniaxial Surface Order

Uniaxial active ordering at fluid-fluid or fluid-air interfaces



Particle exchange between bulk and interface

Maitra, Nat. Phys. 2023

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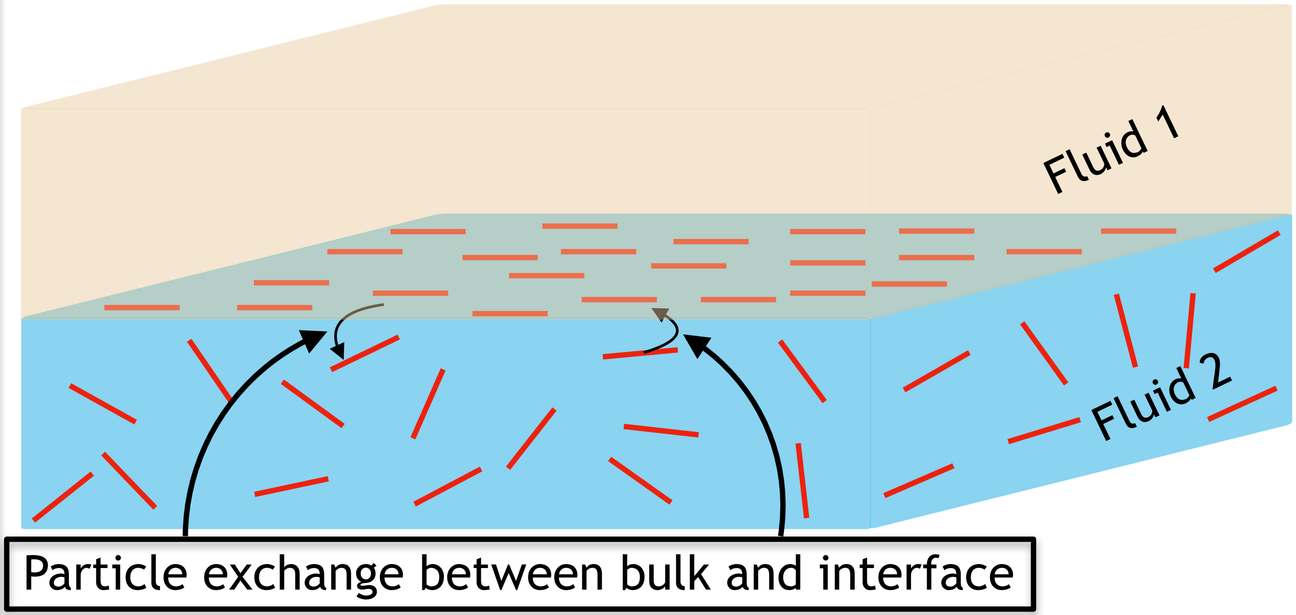

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Momentum conserved systems

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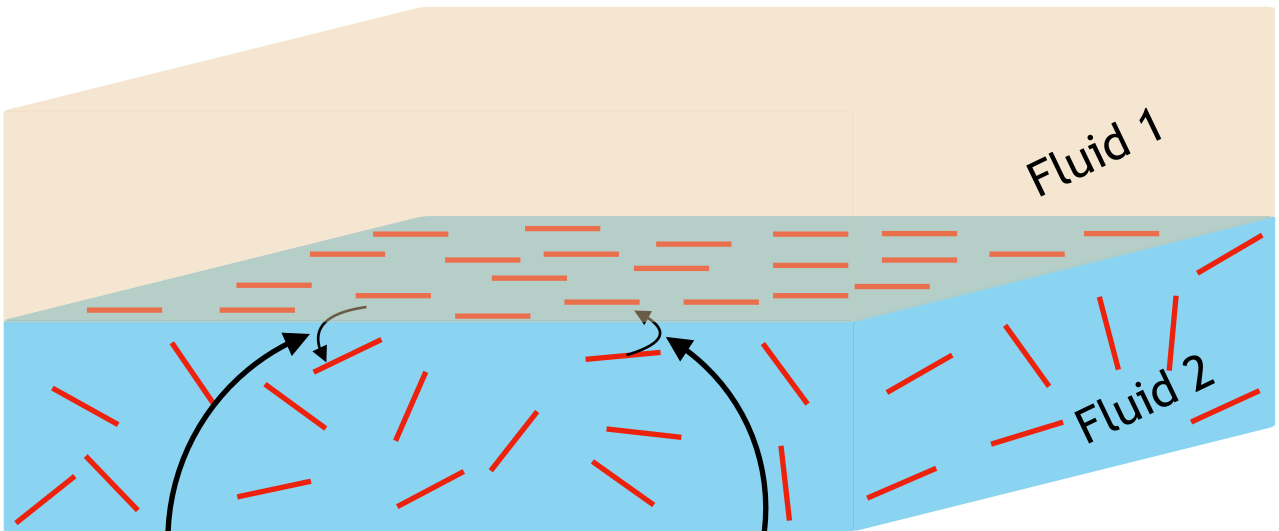

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Momentum conserved systems

Interface provides a degenerate set of planar easy axes

Ordered wetting layer

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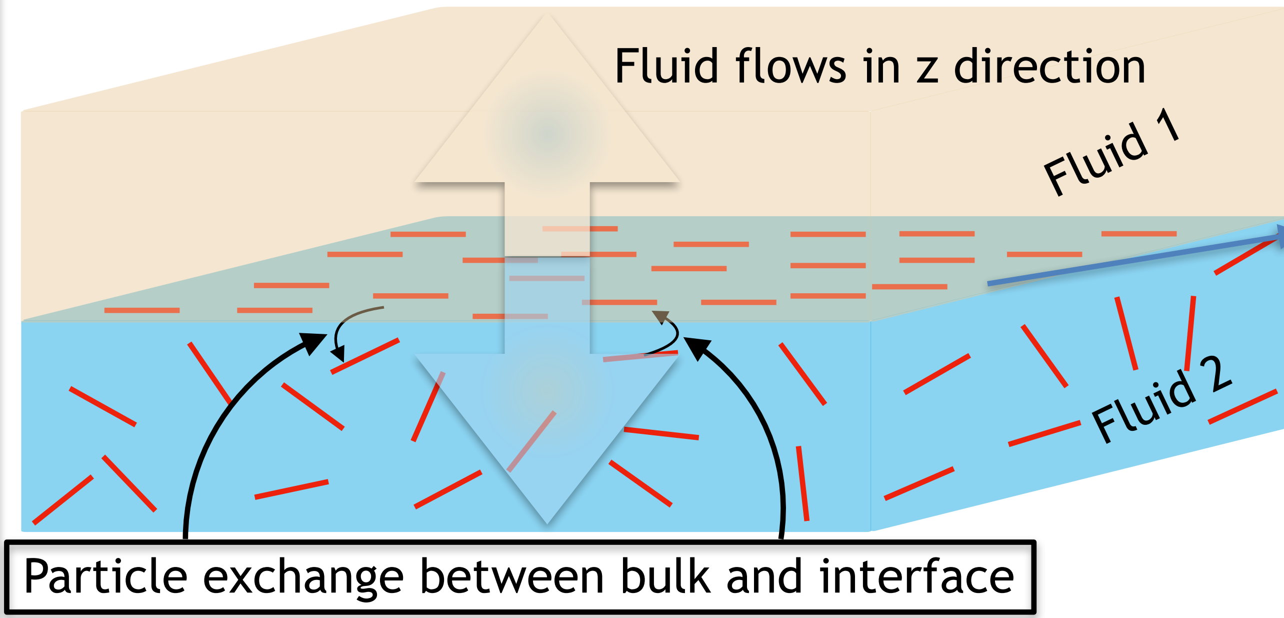

In the fluid,
$\nabla_3 \cdot \mathbf{V} = 0;$
At the interface,
$\partial_z V_z \neq 0 \implies \nabla \cdot \mathbf{v}^s \neq 0$

Maitra, Nat. Phys. 2023

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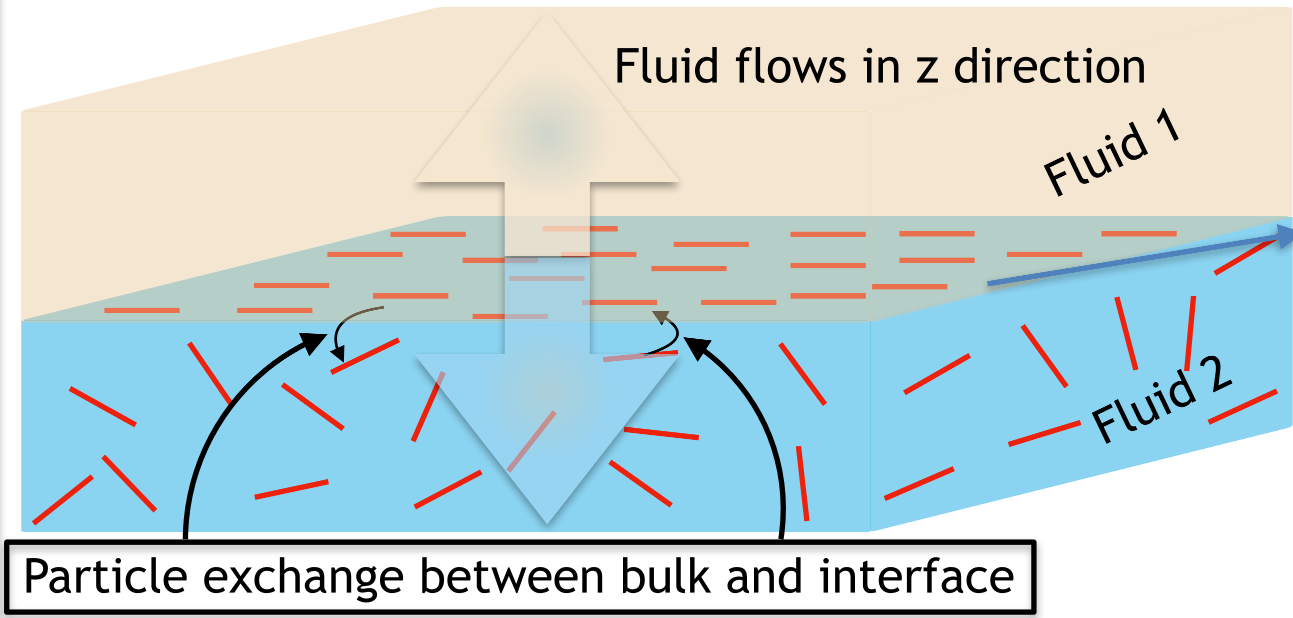

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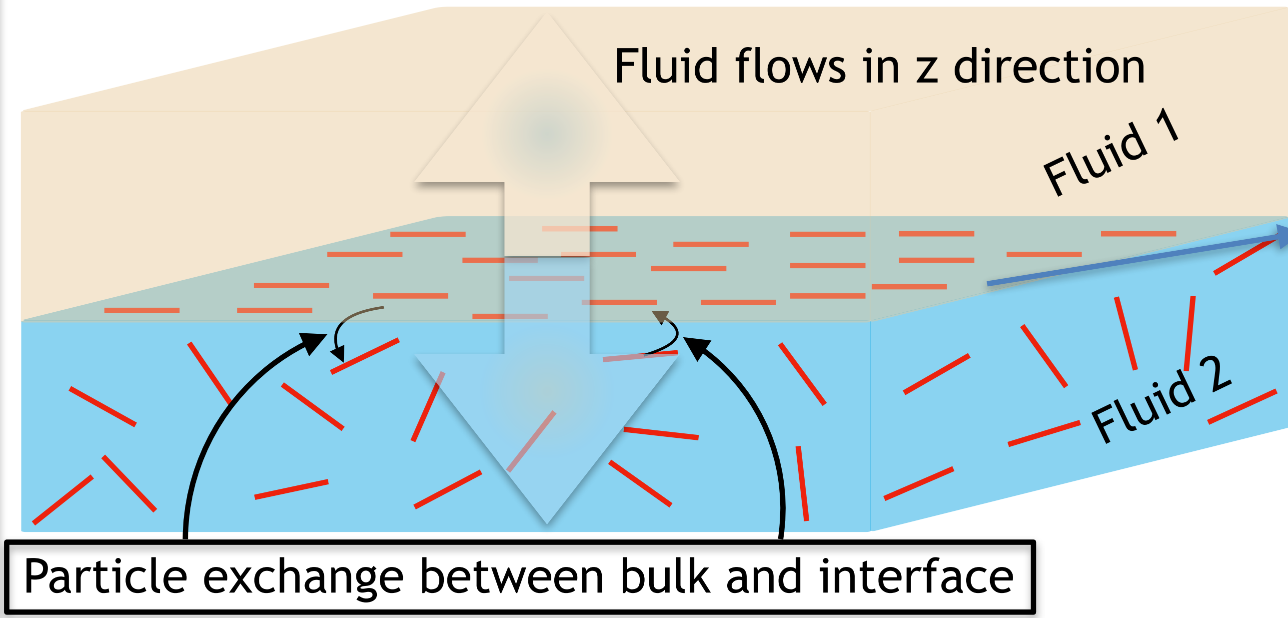

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Maitra, Nat. Phys. 2023

Interfacial order parameter $\mathbf{Q}$ and interfacial active surface stress $\boldsymbol{\sigma}^s \propto \mathbf{Q}$

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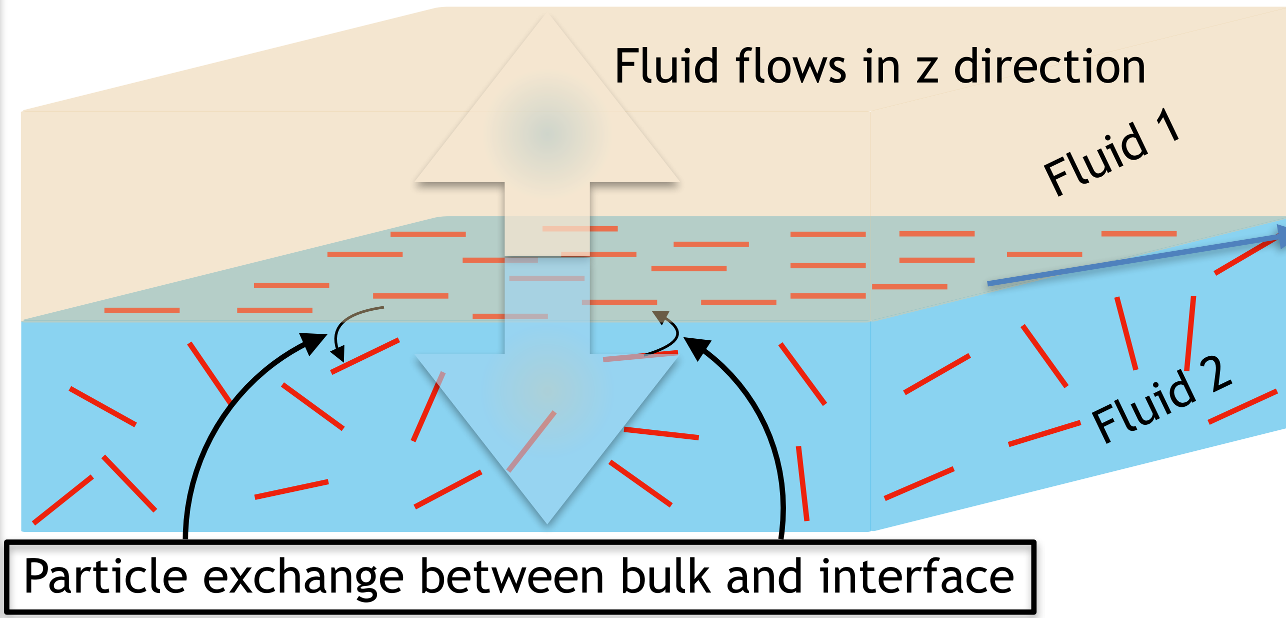

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Interfacial velocity $\mathbf{v}^s = \mathbf{M} \cdot \mathbf{F}^s = \mathbf{M} \cdot (\nabla \cdot \boldsymbol{\sigma}^s)$

$$\mathbf{M} = \frac{1}{4\eta |q|^3} \begin{pmatrix} q_x^2 + 2q_y^2 & -q_x q_y \\ -q_x q_y & 2q_x^2 + q_y^2 \end{pmatrix} \quad \eta \equiv (\eta_1 + \eta_2)/2$$

Active Uniaxial Surface Order

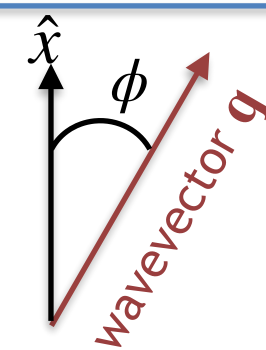
Uniaxial active ordering at fluid-fluid or fluid-air interfaces

Interfacial velocity $\mathbf{v}^s = \mathbf{M} \cdot \mathbf{F}^s;\ \mathbf{M} \sim \mathcal{O}\left(q^{-1}\right)$

$$\mathbf{F}^s = -i\zeta\Delta\mu\left(q_y\theta\hat{x} + q_x\theta\hat{y}\right)$$

Active Uniaxial Surface Order

Uniaxial active ordering at fluid-fluid or fluid-air interfaces



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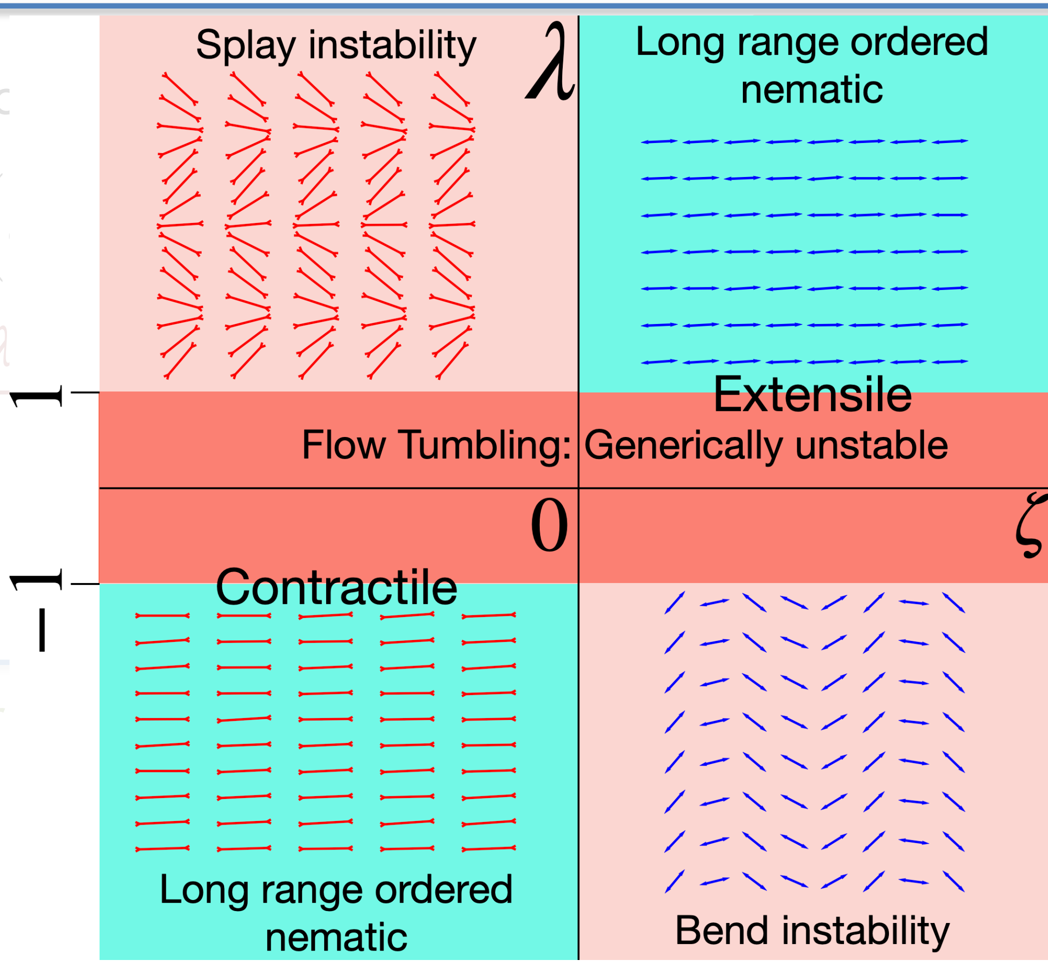

Stable at $\Delta\mu \to \infty$ for $\zeta\lambda > 0$ and $|\lambda| > 1$

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No relevant nonlinearity → Linear theory is exact

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No extraordinary transition from surface to bulk order

The ordered wetting layer never acquires macroscopic thickness

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Simha-Ramaswamy instability in the bulk; screening of fluctuations at the surface

Gravitational Jeans-like bulk instability, Coulomb-like screening at the surface

The active stress that destroys bulk order, anomalously stabilises surface order

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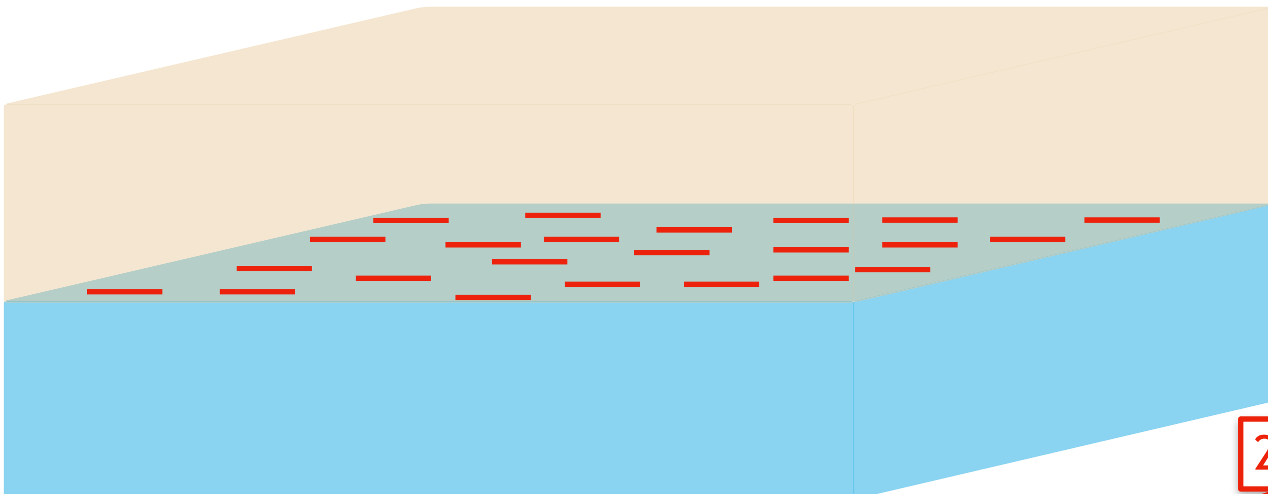

2d Number conservation

Apolar order when active units are confined to interface

Maitra, Nat. Phys. 2023

Generically unstable → apolar state doesn’t exist

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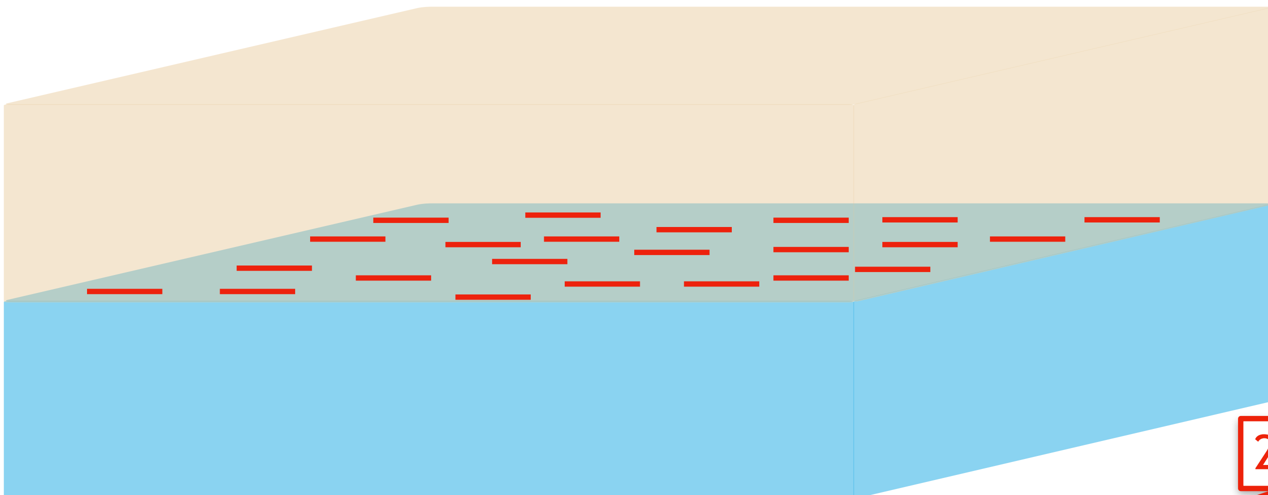

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Different from Simha-Ramaswamy instability of incompressible active nematic fluids

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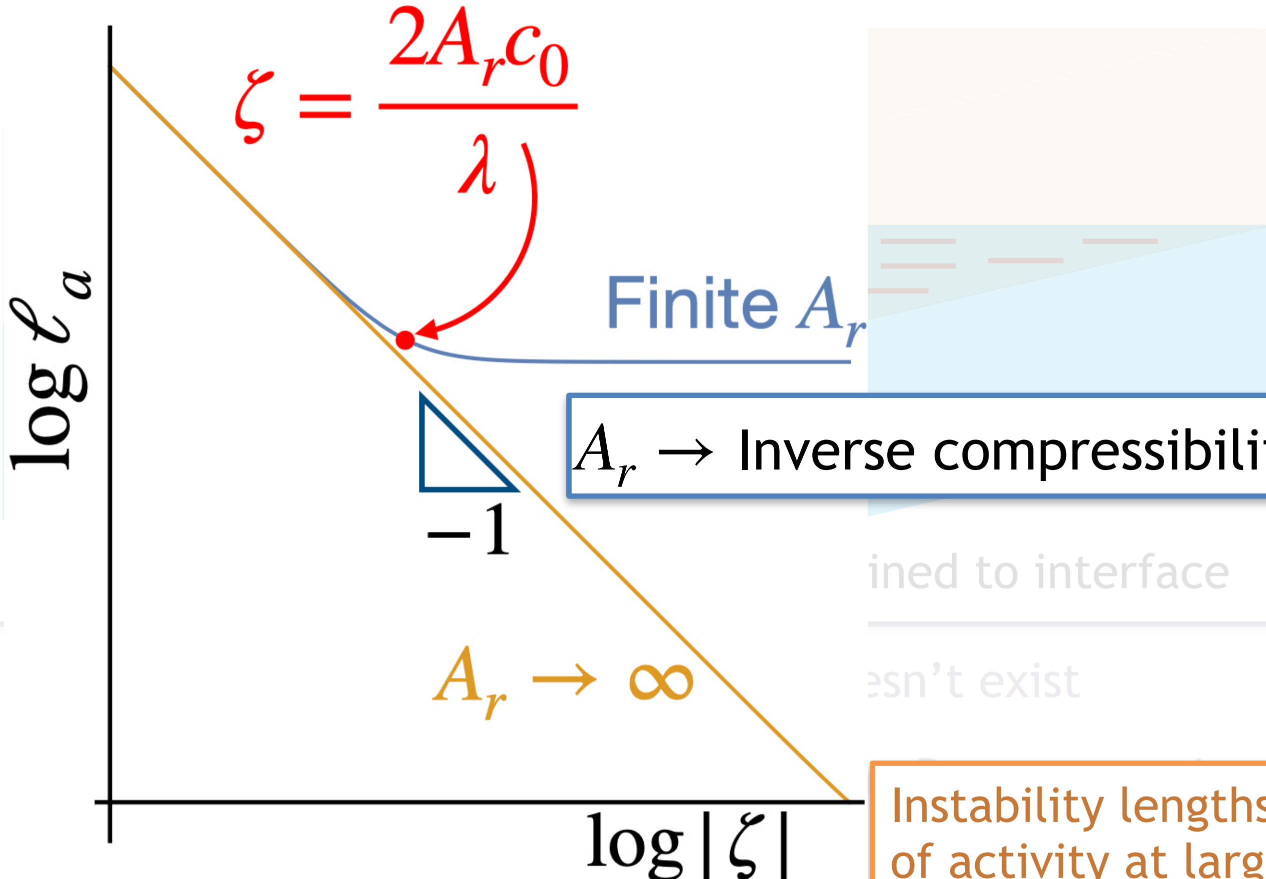

$A_r \to$ Inverse compressibility

Maitra, Nat. Phys. 2023

Instability lengthscale independent of activity at large activity.

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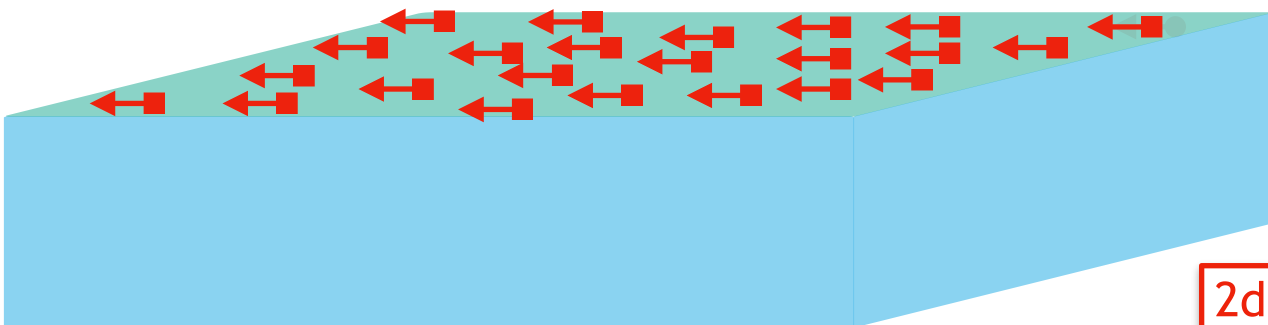

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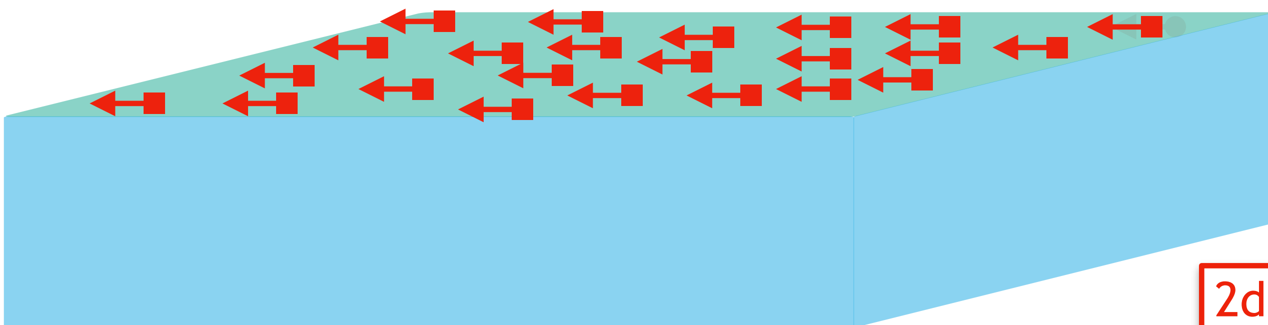

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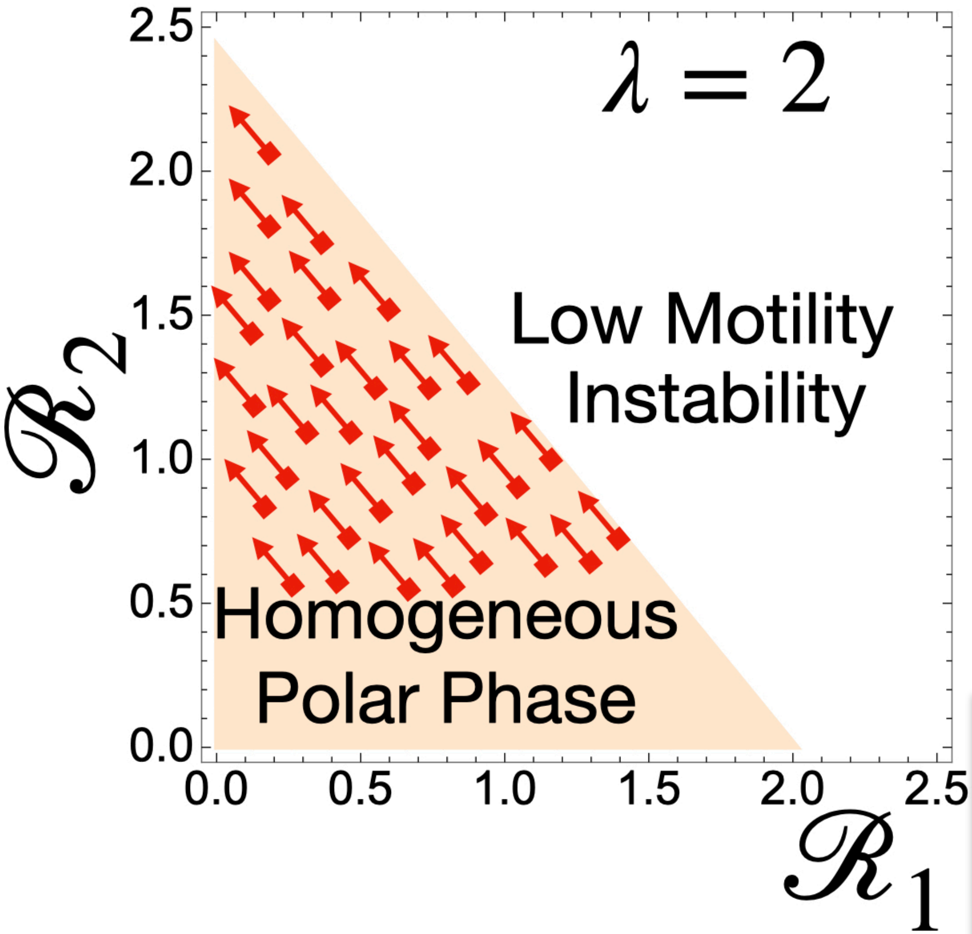

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When motility is high→ polar LRO

$\mathcal{R}_1 \rightarrow$ 1/motility

$\mathcal{R}_2 \rightarrow$ measure of compressibility

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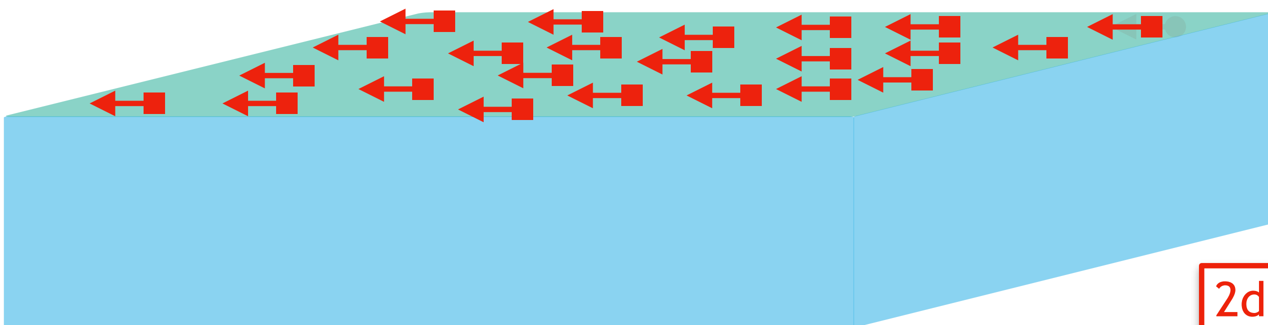

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Linear theory is exact: no relevant nonlinearity.

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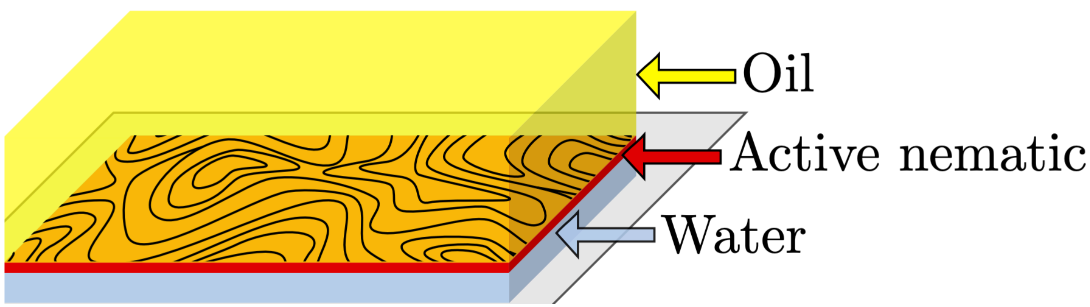
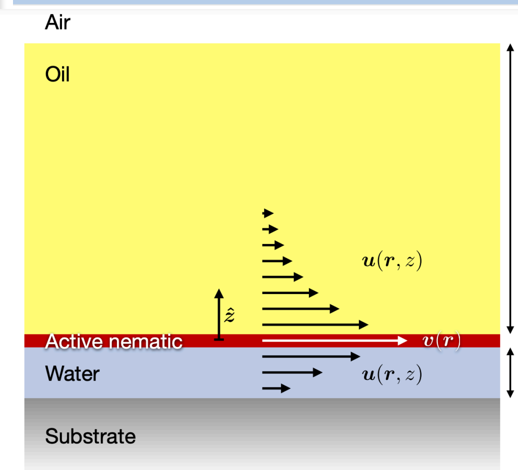
Natural in experiments





Instability in this geometry not generic. Depends on experimental details.

Martinez-Prat et al., PRX 2021

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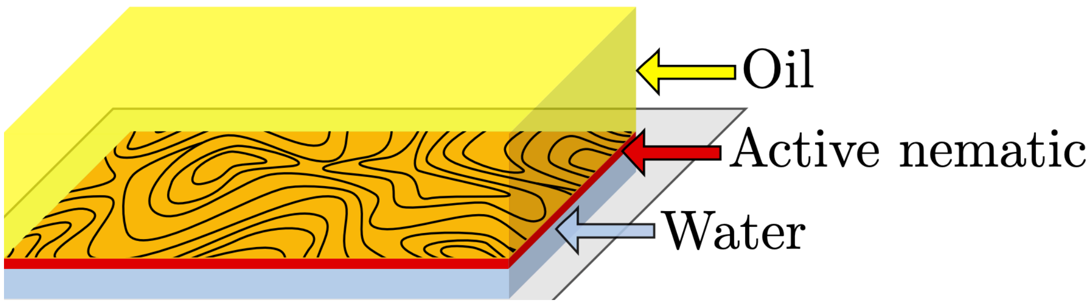

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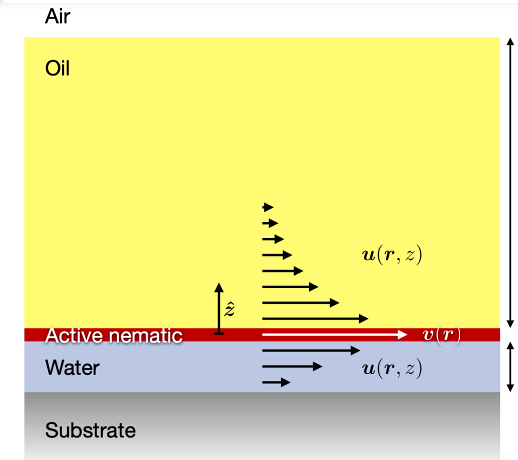

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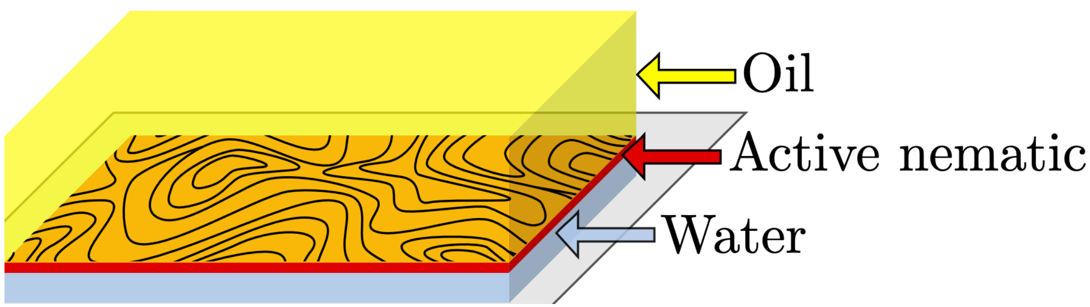

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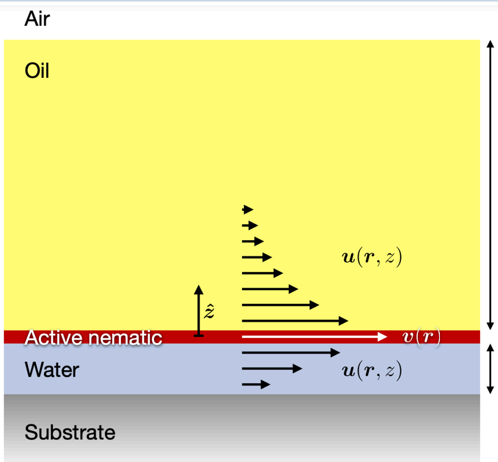

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Possibility of protocortex in protocells (liquid-liquid phase-separation)

Active Liquid Crystalline Fluids

Summary

Simha-Ramaswamy instability $\rightarrow$ No order in Stokesian bulk fluids

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QLRO nematic phase at arbitrary activity in fluids on substrates

Simha-Ramaswamy instability doesn't preclude ordering at any activity in most experimental active fluids

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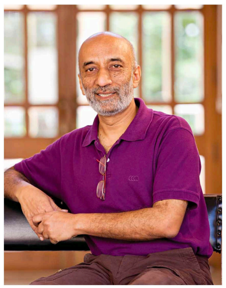
Collaborators



S. Ramaswamy

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M. C. Marchetti

J. Toner

J. Lintuvuori

C-F. Lee

L. Chen

P. Srivastava

Thank you