Azimuthal asymmetries in back to back J/ψ and $\pi^{\pm}$ production at EIC

Khatiza Banu, Asmita Mukherjee, Amol Pawar and Sangem Rajesh

Department of Physics, IIT BOMBAY

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Outline

- Gluon TMDs
- Azimuthal Asymmetries
- TMD parameterizations
- Results and discussions

Gluon TMDs

gluon pol.

nucleon pol.	U	Circularly	Linearly
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

At leading Twist

- TMD-PDF $f(x, k_t, Q^2)$
- The gluon correlator

$$\Phi^{\mu\nu}(x, q_T) = \int \frac{d\xi^- d^2\xi_T}{M_p(2\pi)^3} e^{iq\cdot\xi} \left\langle P \right| Tr[F^{+\mu}(0) U^{[C]}(0,\xi) F^{+\nu}(\xi) U^{[C]}(\xi, 0)] \left| P \right\rangle \Big|_{\xi^+=0}$$

Gluon field strength tensor

Wilson line

Linearly polarized gluon TMD $h_1^{\perp g}(x, q_T^2)$

- It can be probed in Semi Inclusive Deep Inelastic Scattering (SIDIS), and Drell-Yan processes.
- By extracting $\cos 2\phi_T$ azimuthal asymmetry one can probe $h_1^{\perp g}(x, q_T^2)$.
- It follows positivity bound, $\frac{q_T^2}{2M_p^2}|h_1^{\perp g}(x, q_T^2)| \leq f_1^g(x, q_T^2)$ Mulders and Rodrigues (2001).

Sivers gluon TMD $f_{1T}^{\perp g}(x, q_T^2)$

- correlates the hadron spin and polarization of gluon
- T-odd function
- By extracting $\sin(\phi_S - \phi_T)$ azimuthal asymmetry one can probe $f_{1T}^{\perp g}(x, q_T^2)$.
- It follows positivity bound, $\frac{|q_T|}{M_p}|f_{1T}^{\perp g}(x, q_T^2)| \leq f_1^g(x, q_T^2)$ Mulders and Rodrigues (2001).

Azimuthal Asymmetries

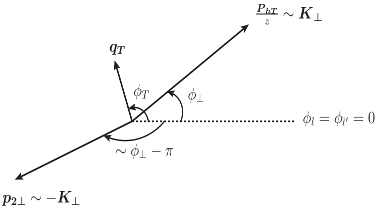
Kinematics

$$e^-(l) + p^{\uparrow}(P) \to e^-(l') + J/\psi(\mathrm{P}_\psi) + \pi^\pm(\mathrm{P}_\pi) + X$$

- $\gamma^* + g \to J/\psi + g$
- $g \to \pi^\pm$
- virtual photon and Proton along $\pm z$ axis
- Leptonic plane $\Rightarrow$ measuring azimuthal angles

$Q^2 = -q^2,\ s = (P+l)^2,\ x_B = \frac{Q^2}{2P\cdot q}$
$y = \frac{P\cdot q}{P\cdot l},\ z_1 = \frac{P\cdot P_\psi}{P\cdot q},\ z_2 = \frac{P\cdot P_g}{P\cdot q},$
$z_h = \frac{P\cdot P_h}{P\cdot q},\ z = \frac{P\cdot P_h}{P\cdot P_g} = \frac{z_h}{z_2}$
$\boldsymbol{q}_T = \boldsymbol{P}_{\psi\perp} + \frac{\boldsymbol{P}_{\pi\perp}}{z},\quad \boldsymbol{K}_\perp = \frac{\boldsymbol{P}_{\psi\perp} - \frac{\boldsymbol{P}_{\pi\perp}}{z}}{2}$

- back to back scattering $\Rightarrow$ $|\mathbf{q}_T|^2 \ll |\mathbf{K}_\perp|^2 \sim M_\psi^2 \Rightarrow$ TMD factorization.
- ϕ_T and $\phi_\perp$



Total cross-section

- In back to back lepto-production of J/ψ and $\pi^{\pm} \to$ TMD factorization is expected to follow

$$
\begin{aligned}
\mathrm{d}\sigma^{ep\to e+J/\psi+\pi+X} = {} & \frac{1}{2s}\,\frac{\mathrm{d}^3\boldsymbol{l}'}{(2\pi)^3 2E_{l'}}\,\frac{\mathrm{d}^3\boldsymbol{P}_\psi}{(2\pi)^3 2E_\psi}\,\frac{\mathrm{d}^3\boldsymbol{P}_\pi}{(2\pi)^3 2E_\pi}\int \mathrm{d}x_g\,\mathrm{d}^2\boldsymbol{k}_{\perp g}\,\mathrm{d}z\,(2\pi)^4\,\delta^4(q+k-P_\psi-P_g) \\
& \times \frac{1}{Q^4}\,L^{\mu\mu'}(l,q)\,\Phi_g^{\alpha\alpha'}(x,\boldsymbol{k}_{\perp g})\,\mathcal{M}_{\mu\alpha}^{\gamma^* g\to J/\psi+g}\,\mathcal{M}_{\mu'\alpha'}^{*\gamma^* g\to J/\psi+g}\,D(z)J(z).
\end{aligned}
$$

Pisano, Boer, Brodsky, Buffing and Mulders (2013)

Total cross-section

$$d\sigma^{ep\to e+J/\psi+\pi+X} = \frac{1}{2s}\frac{d^3\boldsymbol{l}'}{(2\pi)^3 2E_{l'}}\frac{d^3\boldsymbol{P}_\psi}{(2\pi)^3 2E_\psi}\frac{d^3\boldsymbol{P}_\pi}{(2\pi)^3 2E_\pi}\int dx_g\, d^2\boldsymbol{k}_{\perp g}\, dz\,(2\pi)^4\,\delta^4(q+k-P_\psi-P_g)$$
$$\times\frac{1}{Q^4}\boxed{L^{\mu\mu'}(l,q)}\,\boxed{\Phi_g^{\alpha\alpha'}(x,\boldsymbol{k}_{\perp g})}\,\mathcal{M}_{\mu\alpha}^{\gamma^* g\to J/\psi+g}\mathcal{M}_{\mu'\alpha'}^{*\gamma^* g\to J/\psi+g}\,D(z)J(z).$$

$$L^{\mu\nu} = e^2\frac{Q^2}{y^2}\Big[-(1+(1-y)^2)g_T^{\mu\nu}+4(1-y)\epsilon_L^\mu\epsilon_L^\nu+4(1-y)\left(\hat{l}_\perp^\mu\hat{l}_\perp^\nu+\frac{1}{2}g_T^{\mu\nu}\right)$$
$$+2(2-y)\sqrt{1-y}\left(\epsilon_L^\mu\hat{l}_\perp^\nu+\epsilon_L^\nu\hat{l}_\perp^\mu\right)\Big],$$

Boer, D'Alesio, Murgia, Pisano and Taels (2020)

$$\Phi_U^{\mu\nu}(x,\boldsymbol{k}_{\perp g}) = \frac{1}{2x}\left\{-g_T^{\mu\nu}f_1^g(x,\boldsymbol{k}_{\perp g}^2)+\left(\frac{k_{\perp g}^\mu k_{\perp g}^\nu}{M_p^2}+g_T^{\mu\nu}\frac{\boldsymbol{k}_{\perp g}^2}{2M_p^2}\right)h_1^{\perp g}(x,\boldsymbol{k}_{\perp g}^2)\right\}$$

$$\Phi_T^{\mu\nu}(x,\boldsymbol{k}_{\perp g}) = \frac{1}{2x}\left\{-g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma}k_{\perp g\rho}S_{T\sigma}}{M_p}f_{1T}^{\perp g}(x,\boldsymbol{k}_{\perp g}^2)+i\epsilon_T^{\mu\nu}\frac{k_{\perp g}\cdot S_T}{M_p}g_{1T}^g(x,\boldsymbol{k}_{\perp g}^2)\right.$$
$$\left.+\frac{k_{\perp g\rho}\epsilon_T^{\rho\{\mu}k_{\perp g}^{\nu\}}}{2M_p^2}\frac{k_{\perp g}\cdot S_T}{M_p}h_{1T}^{\perp g}(x,\boldsymbol{k}_{\perp g}^2)-\frac{k_{\perp g\rho}\epsilon_T^{\rho\{\mu}S_T^{\nu\}}+S_{T\rho}\epsilon_T^{\rho\{\mu}k_{\perp g}^{\nu\}}}{4M_p}h_{1T}^g(x,\boldsymbol{k}_{\perp g}^2)\right\}$$

Mulders and Rodrigues (2001)

Total cross-section

$$d\sigma^{ep\to e+J/\psi+\pi+X} = \frac{1}{2s}\frac{d^3\boldsymbol{l}'}{(2\pi)^3 2E_{l'}}\frac{d^3\boldsymbol{P}_\psi}{(2\pi)^3 2E_\psi}\frac{d^3\boldsymbol{P}_\pi}{(2\pi)^3 2E_\pi}\int dx_g\, d^2\boldsymbol{k}_{\perp g}\, dz\,(2\pi)^4\,\delta^4(q+k-P_\psi-P_g)$$
$$\times\frac{1}{Q^4}L^{\mu\mu'}(l,q)\Phi_g^{\alpha\alpha'}(x,\boldsymbol{k}_{\perp g})\boxed{\mathcal{M}_{\mu\alpha}^{\gamma^* g\to J/\psi+g}\mathcal{M}_{\mu'\alpha'}^{*\gamma^* g\to J/\psi+g}}D(z)J(z).$$

- Color Singlet Model:
  $c\bar{c}\to$ same spin, orbital and color state as that of final J/ψ. Braaten, Fleming, Yuan (1996)
- Color Evaporation model:
  $c\bar{c}\to$ colored
  color is bleached by final-state soft interactions. Amundson, Eboli, Georges, Halzen (1997)
- NRQCD:
  $c\bar{c}\to$ can be color singlet or color octet. Bodwin, Braaten, Lepage (1994)
- Difference between CEM and NRQCD $\to$ CEM all the color configurations are equiprobable, for NRQCD they are not

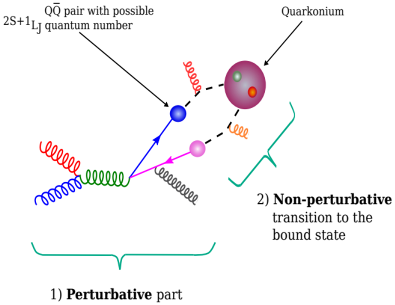


Image: Rajesh

Total cross-section

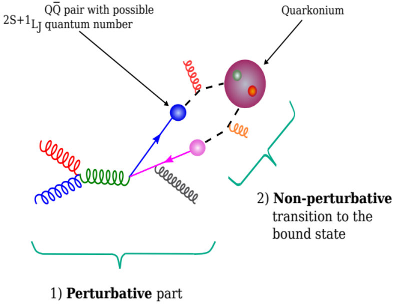
$$d\sigma^{ep\to e+J/\psi+\pi+X} = \frac{1}{2s}\frac{d^3\boldsymbol{l}'}{(2\pi)^3 2E_{l'}}\frac{d^3\boldsymbol{P}_\psi}{(2\pi)^3 2E_\psi}\frac{d^3\boldsymbol{P}_\pi}{(2\pi)^3 2E_\pi}\int dx_g\, d^2\boldsymbol{k}_{\perp g}\, dz\,(2\pi)^4\,\delta^4(q+k-P_\psi-P_g)$$
$$\times\frac{1}{Q^4}L^{\mu\mu'}(l,q)\Phi_g^{\alpha\alpha'}(x,\boldsymbol{k}_{\perp g})\boxed{\mathcal{M}_{\mu\alpha}^{\gamma^* g\to J/\psi+g}\mathcal{M}_{\mu'\alpha'}^{*\gamma^* g\to J/\psi+g}}D(z)J(z).$$

In our work, we implement the NRQCD to calculate the J/ψ production matrix elements
The relative momentum $k^2 \ll M_c^2 \Rightarrow$ non-relativistic approximation of QCD.

$$\mathcal{M}^{ab\to J/\psi} = \sum_n \mathcal{M}[ab\to c\bar{c}\left({}^{2S+1}L_J^{(1,8)}\right)]$$
$$\langle 0|\mathcal{O}^{J/\psi}\left({}^{2S+1}L_J^{(1,8)}\right)|0\rangle$$

- $\mathcal{M}[ab\to c\bar{c}\left({}^{2S+1}L_J^{(1,8)}\right)]$: High energy perturbative part.
- $\langle 0|\mathcal{O}^{J/\psi}\left({}^{2S+1}L_J^{(1,8)}\right)|0\rangle$: Non perturbative Long Distance Matrix Element. $c\bar{c}\left({}^{2S+1}L_J^{(1,8)}\right)$ to J/ψ

Bodwin, Braaten, Lepage (1994)



Pic: Rajesh

Total cross-section

$$d\sigma^{ep\to e+J/\psi+\pi+X} = \frac{1}{2s}\frac{d^3\boldsymbol{l}'}{(2\pi)^3 2E_{l'}}\frac{d^3\boldsymbol{P}_\psi}{(2\pi)^3 2E_\psi}\frac{d^3\boldsymbol{P}_\pi}{(2\pi)^3 2E_\pi}\int dx_g\, d^2\boldsymbol{k}_{\perp g}\, dz\,(2\pi)^4\,\delta^4(q+k-P_\psi-P_g)$$
$$\times\frac{1}{Q^4}L^{\mu\mu'}(l,q)\Phi_g^{\alpha\alpha'}(x,\boldsymbol{k}_{\perp g})\boxed{\mathcal{M}_{\mu\alpha}^{\gamma^* g\to J/\psi+g}\mathcal{M}_{\mu'\alpha'}^{*\gamma^* g\to J/\psi+g}}D(z)J(z).$$

$$\mathcal{M}\Big(\gamma^* g\to c\bar{c}[^{2S+1}L_J^{(1,8)}]g\Big) = \sum_{L_z S_z}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\Psi_{LL_z}(\mathbf{k})\langle LL_z;SS_z|JJ_z\rangle$$
$$\times\ Tr[O(q,p_g,\mathrm{P}_\psi,k)\mathcal{P}_{SS_z}(\mathrm{P}_\psi,k)]$$

- $k^2 \ll M_c^2 \ \rightarrow \mathcal{M}(k) = \mathcal{M}(k)|_{k=0} + \dots$
- $\mathcal{M}(k)|_{k=0}$ - S wave scattering amplitude, $k\mathcal{M}'(k)|_{k=0}$ - P wave scattering amplitude
- $\Psi_{LL_z} \rightarrow$ Long Distance Matrix Elements

  D'Alesio, Murgia, Pisano, and Taels (2019).

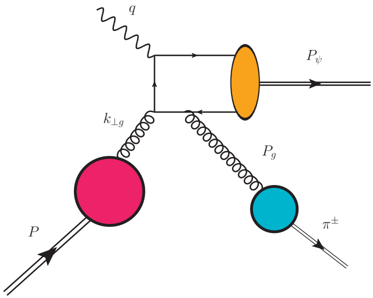
$$\boldsymbol{\mathcal{P}}_{SS_z}(\mathrm{P}_\psi,k) = \sum_{s_1 s_2}\Big\langle\frac{1}{2}s_1;\frac{1}{2}s_2\Big|SS_z\Big\rangle v\Big(\frac{\mathrm{P}_\psi}{2}-k,s_1\Big)\bar{u}\Big(\frac{\mathrm{P}_\psi}{2}+k,s_2\Big)$$
$$=\frac{1}{4M_\psi^{3/2}}(-\not{P}_\psi+2\not{k}+M_\psi)\Pi_{SS_z}(\not{P}_\psi+2\not{k}+M_\psi)+\mathcal{O}(k^2)$$

Kishore, Mukherjee and Rajesh (2020)

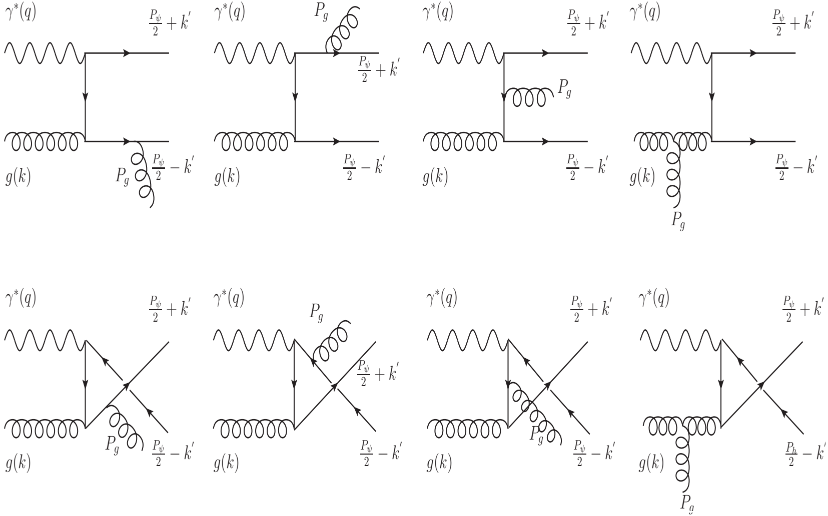
Total cross-section

$$d\sigma^{ep\to e+J/\psi+\pi+X} = \frac{1}{2s}\frac{d^3\boldsymbol{l}'}{(2\pi)^3 2E_{l'}}\frac{d^3\boldsymbol{P}_\psi}{(2\pi)^3 2E_\psi}\frac{d^3\boldsymbol{P}_\pi}{(2\pi)^3 2E_\pi}\int dx_g\, d^2\boldsymbol{k}_{\perp g}\, dz\,(2\pi)^4\,\delta^4(q+k-P_\psi-P_g)$$
$$\times\frac{1}{Q^4}L^{\mu\mu'}(l,q)\Phi_g^{\alpha\alpha'}(x,\boldsymbol{k}_{\perp g})\mathcal{M}_{\mu\alpha}^{\gamma^* g\to J/\psi+g}\mathcal{M}_{\mu'\alpha'}^{*\gamma^* g\to J/\psi+g}\boxed{D(z,Q^2)}J(z).$$

- we consider the back-to-back J/ψ and $\pi^\pm$ production
- J/ψ and $\pi^\pm \Rightarrow$ large transverse momentum
- $k_{\perp\pi} \ll K_\perp \Rightarrow D(z,k_{\perp\pi}^2,Q^2) \to \int d^2k_{\perp\pi} D(z,k_{\perp\pi}^2,Q^2) \approx D(z,Q^2)$
- This allows us to approximate the pion formation as a collinear process



$g\gamma^* \to J/\psi\ g$



$\cos 2\phi_T$ Azimuthal Asymmetry

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}z_h\mathrm{d}^2\boldsymbol{q}_T\mathrm{d}^2\boldsymbol{K}_\perp} \equiv \mathrm{d}\sigma(\phi_S,\phi_T) = \mathrm{d}\sigma^U(\phi_T,\phi_\perp) + \mathrm{d}\sigma^T(\phi_S,\phi_T)\,.$$

D'Alesio, Murgia, Pisano, and Taels (2019)

$$\mathrm{d}\sigma^U = \mathcal{N}\int \mathrm{d}z\Big[\big(\mathcal{A}_0 + \mathcal{A}_1\cos\phi_\perp + \mathcal{A}_2\cos 2\phi_\perp\big) f_1^g(x,\boldsymbol{q}_T^2) + \big(\mathcal{B}_0\cos 2\phi_T + \mathcal{B}_1\cos(2\phi_T - \phi_\perp) + \mathcal{B}_2\cos 2(\phi_T - \phi_\perp) + \ldots\big)\frac{\boldsymbol{q}_T^2}{M_p^2}\, h_1^{\perp\, g}(x,\boldsymbol{q}_T^2)\Big] D(z)\,.$$

$$\mathrm{d}\sigma^T = \mathcal{N}|\boldsymbol{S}_T|\int \mathrm{d}z\Big[\sin(\phi_S - \phi_T)\big(\mathcal{A}_0 + \mathcal{A}_1\cos\phi_\perp + \mathcal{A}_2\cos 2\phi_\perp\big)\frac{|\boldsymbol{q}_T|}{M_p} f_{1T}^{\perp\, g}(x,\boldsymbol{q}_T^2) + \cos(\phi_S - \phi_T)\big(\mathcal{B}_0\sin 2\phi_T\ldots\big)\frac{|\boldsymbol{q}_T|^3}{M_p^3}\, h_{1T}^{\perp\, g}(x,\boldsymbol{q}_T^2) + \big(\mathcal{B}_0\sin(\phi_S + \phi_T) + \ldots\big)\frac{|\boldsymbol{q}_T|}{M_p} h_{1T}^{g}(x,\boldsymbol{q}_T^2)\Big] D(z)\,,$$

$$A^{W(\phi_S,\phi_T)} \equiv 2\,\frac{\int \mathrm{d}\phi_S\,\mathrm{d}\phi_T\,\mathrm{d}\phi_\perp\, W(\phi_S,\phi_T)\,\mathrm{d}\sigma(\phi_S,\,\phi_T,\,\phi_\perp)}{\int \mathrm{d}\phi_S\,\mathrm{d}\phi_T\,\mathrm{d}\phi_\perp\,\mathrm{d}\sigma(\phi_S,\phi_T,\phi_\perp)}$$

$$W(\phi_T) = \cos 2\phi_T \rightarrow A^{\cos 2\phi_\perp} = \frac{\boldsymbol{q}_\perp^2}{M_p^2}\frac{\int dz\,\mathcal{B}_0\,D(z)\,h_1^{\perp g}(x,\boldsymbol{q}_\perp^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,\boldsymbol{q}_\perp^2)}$$

$$W(\phi_\perp,\phi_T) = \cos 2(\phi_T - \phi_\perp) \rightarrow A^{\cos 2(\phi_\perp - \phi_S)} = \frac{\boldsymbol{q}_\perp^2}{M_p^2}\frac{\int dz\,\mathcal{B}_2\,D(z)\,h_1^{\perp g}(x,\boldsymbol{q}_\perp^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,\boldsymbol{q}_\perp^2)}$$

$$W(\phi_S,\phi_T) = \sin(\phi_S - \phi_T) \rightarrow A^{\sin(\phi_S - \phi_T)} = \frac{|\boldsymbol{q}_\perp|}{M_p}\frac{\int dz\,\mathcal{A}_0\,D(z)\,f_{1T}^{\perp g}(x,\boldsymbol{q}_\perp^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,\boldsymbol{q}_\perp^2)}$$

$\cos 2\phi_T$ Azimuthal Asymmetry

Saturation of positivity bound $\rightarrow$ Upperbound

Upperbound $\rightarrow$ maximum value of azimuthal asymmetry for given parameterization

$$\frac{\boldsymbol{q}_T^2}{2M_p^2} h_1^{\perp\, g}(x, \boldsymbol{q}_T^2) = f_1^g(x, \boldsymbol{q}_T^2)$$

$$A^{\cos 2\phi_T} = \frac{\boldsymbol{q}_T^2}{M_p^2} \frac{\int \mathrm{d}z\, \mathcal{B}_0\, D(z)\, h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)}{\int \mathrm{d}z\, \mathcal{A}_0\, D(z)\, f_1^g(x, \boldsymbol{q}_T^2)} \qquad A^{\cos 2\phi_T} \rightarrow |A^{\cos 2\phi_T}|_{Max} = 2\frac{|\int \mathrm{d}z\, \mathcal{B}_0\, D(z)\, f_1^g(x, \boldsymbol{q}_T^2)|}{\int \mathrm{d}z\, \mathcal{A}_0\, D(z)\, f_1^g(x, \boldsymbol{q}_T^2)}$$

$$A^{\cos 2(\phi_T - \phi_\perp)} = \frac{\boldsymbol{q}_T^2}{M_p^2} \frac{\int \mathrm{d}z\, \mathcal{B}_2\, D(z)\, h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)}{\int \mathrm{d}z\, \mathcal{A}_0\, D(z)\, f_1^g(x, \boldsymbol{q}_T^2)} \qquad A^{\cos 2(\phi_T - \phi_\perp)} \rightarrow U = 2\frac{|\int \mathrm{d}z\, \mathcal{B}_2\, D(z)\, f_1^g(x, \boldsymbol{q}_T^2)|}{\int \mathrm{d}z\, \mathcal{A}_0\, D(z)\, f_1^g(x, \boldsymbol{q}_T^2)}$$

TMD parameterizations

Gaussian Parameterization

- Drell-Yan and SIDIS $\Rightarrow$ transverse momentum spectra $\rightarrow$ roughly Gaussian in nature. Stefano Melis (2014)
- TMDs = Collinear pdf (x-dependent) $\otimes$ transverse momentum dependent (q_T-dependent) part.
- The transverse momentum-dependent part is Gaussian in nature.

$f_1^g(x, Q_f)$ gluon pdf CT18NLO set

$D(z, Q_f)$ pion fragmentation function NNFF10_PIsum_lo

$Q_f = \sqrt{m_\psi^2 + Q^2}$ scale

$$f_1^g(x, \mathbf{q}_T^2) = f_1^g(x, Q_f)\frac{1}{\pi\langle q_T^2\rangle} e^{-q_T^2/\langle q_T^2\rangle}$$

$$h_1^{\perp g}(x, q_T^2) = \frac{M_p^2 f_1^g(x, Q_f)}{\pi\langle q_T^2\rangle^2}\frac{2(1-r)}{r} e^{1-\frac{q_T^2}{r\langle q_T^2\rangle}}$$

D. Boer, C. Pisano, (2012)

$$\frac{|\boldsymbol{q}_T|}{M_P} f_{1T}^{\perp g}(x, q_T) = -\frac{\sqrt{2e}}{\pi}\mathcal{N}_g(x) f_{g/p}(x)\sqrt{\frac{1-\rho}{\rho}} q_T \frac{e^{-\boldsymbol{q}_T^2/\rho\left\langle q_T^2\right\rangle}}{\left\langle q_T^2\right\rangle^{3/2}}$$

$$\frac{|\boldsymbol{q}_T|}{M_p}|f_{1T}^{\perp g}(x, \boldsymbol{q}_T^2)| \leq f_1^g(x, \boldsymbol{q}_T^2), \quad \frac{\boldsymbol{q}_T^2}{2M_p^2}|h_1^{\perp g}(x, \boldsymbol{q}_T^2)| \leq f_1^g(x, \boldsymbol{q}_T^2)$$

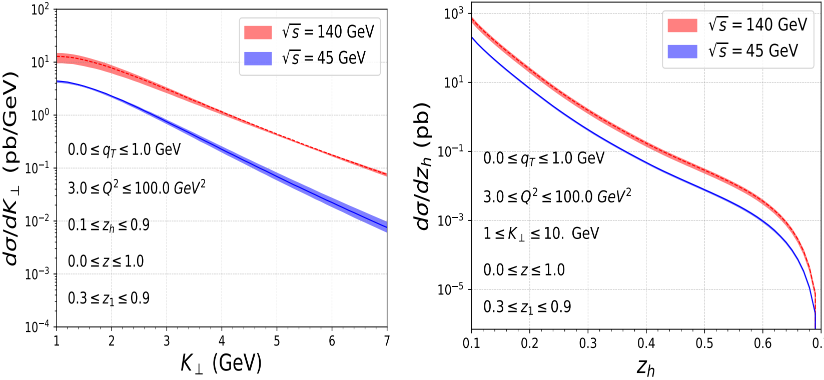
$$\mathcal{N}_g(x) = N_g x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}.$$

$$N_g = 0.25, \quad \alpha = 0.6, \quad \beta = 0.6, \quad \rho = 0.1$$

D'Alesio, Flore, Murgia, Pisano, Taels (2019)

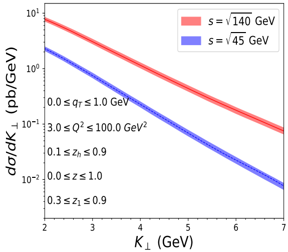
Numerical Results and Discussion

Unpolarized Scattering cross-section



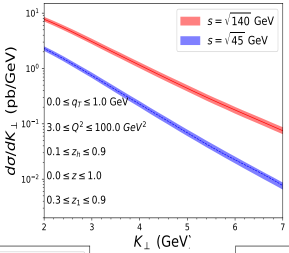
- $K_\perp$ increases $x_g \to 1$, pdf $\to 0$
- to avoid fragmentation and soft gluon emission we set $0.3 < z_1 < 0.9$
- restrictions on $z_1 \Rightarrow d\sigma/dz_h \to 0$ as $z_h \to 0.7$
- band is uncertainty in scale $\frac{Q_f}{2} < Q_f < 2Q_f$

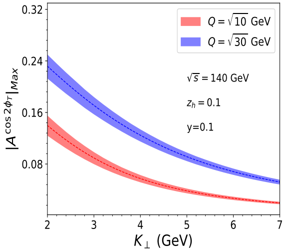
Unpolarized Scattering cross-section for LDME sets

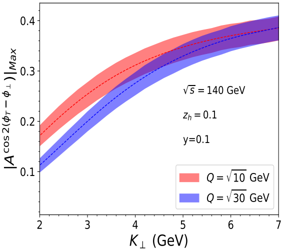


- the 1-σ error band due to CMSWZ LDMES Chao, Ma, Shao, Wang, and Zhang (2012)

Unpolarized Scattering cross-section for LDME sets

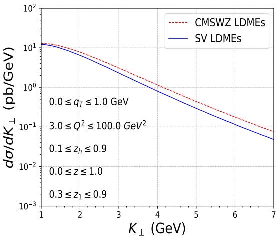






- The 1-σ error band in the azimuthal asymmetries
- $|A^{\cos 2\phi_T}|_{Max}$ error is $\approx 4\%$ and $|A^{\cos 2(\phi_T - \phi_\perp)}|_{Max}$ error is $\approx 4 - 5\%$

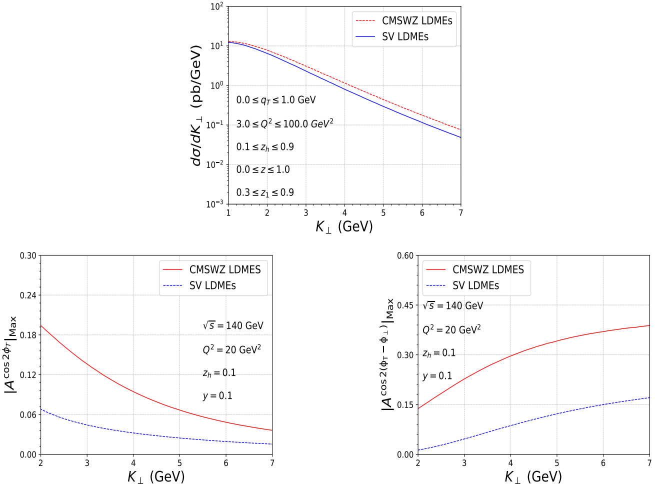
Effect of LDMEs



- Difference between unpolarized cross-section due to LDME sets

Chao, Ma, Shao, Wang, and Zhang (2012)
Sharma and Vitev (2013)

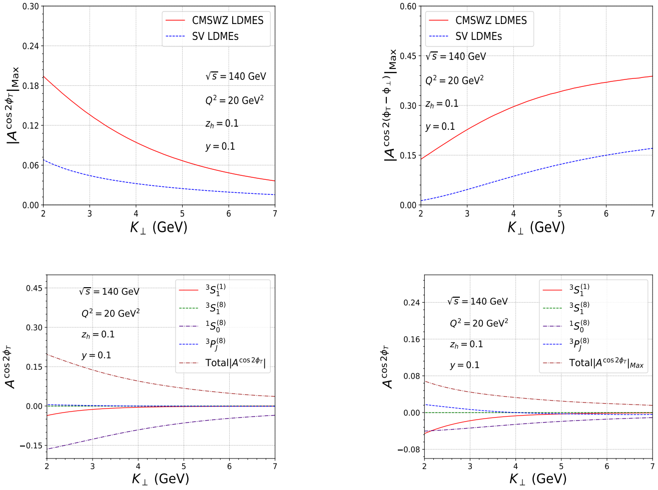
Effect of LDMEs



- Difference between upper bound due to LDME sets
- Maximum difference for $|A^{\cos 2\phi_T}|_{Max}$ is 14% and $|A^{\cos 2(\phi_T - \phi_\perp)}|_{Max}$ is 22%

Chao, Ma, Shao, Wang, and Zhang (2012)
Sharma and Vitev (2013)

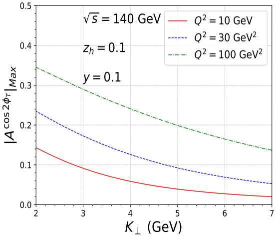
Effect of LDMEs



- State wise contribution to upper bound for $A^{\cos 2\phi T}$
- For both sets the dominant state is ${}^1S_0^{(8)}$ state
- For the SV set of LDMEs, the dominant state is smaller compared to the CMSWZ set

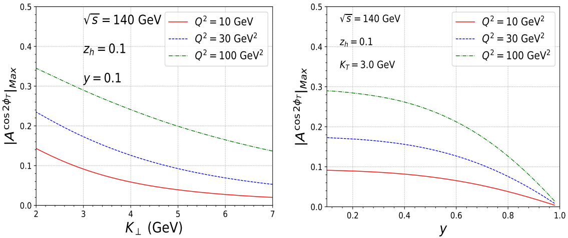
Chao, Ma, Shao, Wang, and Zhang (2012) Sharma and Vitev (2013)

Upperbound



- $K_\perp$ increases $x_g \to 1$, $\mathrm{pdf} \to 0 \Rightarrow |A^{\cos 2\phi_T}|_{Max}$ decreases
- $|A^{\cos 2\phi_T}|_{Max}$ increases as Q increases

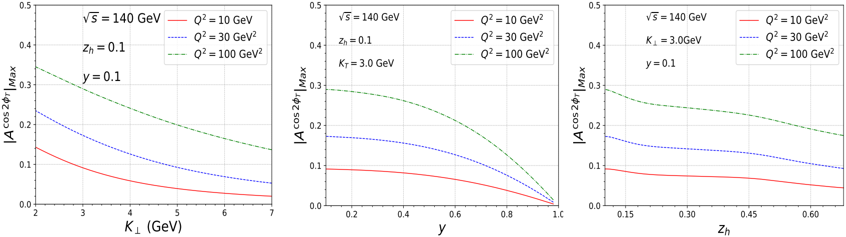
Upperbound



- In y variation $\mathcal{B}_0 \to 0$ as $y \to 1$

$$|A^{\cos 2\phi_T}|_{Max} = 2\frac{|\mathcal{B}_0|}{\mathcal{A}_0}$$

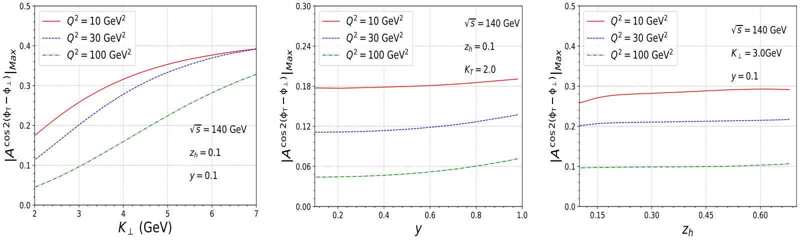
Upperbound



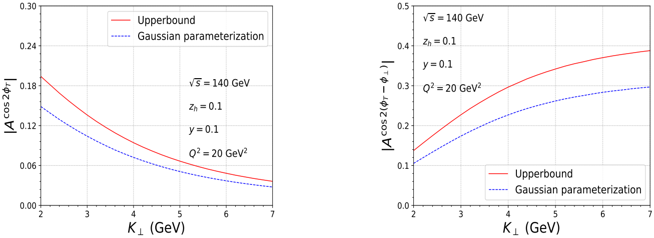
- As z_h increases $|A^{\cos 2\phi_T}|_{Max}$ decreases

Upperbound

- As $K_\perp$ increases $\mathcal{B}_2$ increases $\Rightarrow |A^{\cos 2(\phi_T - \phi_\perp)}|_{Max}$ increases
- $|A^{\cos 2(\phi_T - \phi_\perp)}|_{Max} \approx$ Constant for y and z_h variation
- $|A^{\cos 2(\phi_T - \phi_\perp)}|_{Max}$ decreases as Q increases

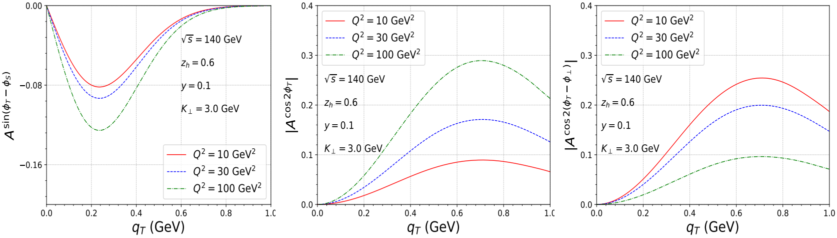


Comparison between Gaussian parameterization and Upper-bound



- $|A^{\cos 2\phi_T}|_{Max}$ maximum difference is 5%
- $|A^{\cos 2(\phi_T - \phi_\perp)}|_{Max}$ maximum difference is 10%

q_T dependence of the Asymmetries



- $\boldsymbol{q}_T = \boldsymbol{P}_{\psi\perp} + \frac{\boldsymbol{P}_{\pi\perp}}{z}$ dependence shows a peak for all asymmetries
- Sivers peak $\rightarrow$ $q_T \approx 0.25$ GeV
- $A^{\cos 2(\phi_T-\phi_\perp)}$ & $A^{\cos 2\phi_T}$ peak $\rightarrow$ $q_T \approx 0.7$ GeV
- Sivers Asymmetry increases as Q increases

Conclusion

- We estimate the $\cos 2\phi_T$, $\cos 2(\phi_T - \phi_\perp)$ and Sivers asymmetry in electroproduction of J/ψ and $\pi^\pm$ at the future EIC.
- $\cos 2\phi_T$, $\cos 2(\phi_T - \phi_\perp)$ are sizeable $\approx 20 - 30\%$ and Sivers asymmetry $\approx 10 - 15\%$
- These kinematical regions will be accessible at EIC
- Back-to-back production of J/ψ and $\pi^\pm$ can be a promising channel to probe linearly polarized gluon TMD and the gluon Sivers TMD at EIC.

Thank you

Backup Slides

Gluon Correlator

Gluon Helicity $\quad |+\rangle \quad |+\rangle \quad |-\rangle \quad |-\rangle$

$\Phi^{\mu\nu}$

$$
\begin{matrix} \langle+| \\ \langle+| \\ \langle-| \\ \langle-| \end{matrix}
\left\{
\begin{matrix}
f_1^g + g_{1L}^g & \frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g - i f_{1T}^{\perp g}) & -\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} + i h_{1L}^{\perp g}) & -i\frac{q_T e^{-3i\phi}}{M_p} h_{1T}^{\perp g} \\
\frac{q_T e^{i\phi}}{M_p}(g_{1T}^g + i f_{1T}^{\perp g}) & f_1^g - g_{1L}^g & -i\frac{q_T e^{-i\phi}}{M_p} h_1^g & -\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} - i h_{1L}^{\perp g}) \\
-\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} - i h_{1L}^{\perp g}) & i\frac{q_T e^{i\phi}}{M_p} h_1^g & f_1^g - g_{1L}^g & -\frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g + i f_{1T}^{\perp g}) \\
i\frac{q_T e^{3i\phi}}{M_p} h_{1T}^{\perp g} & -\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} + i h_{1L}^{\perp g}) & -\frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g - i f_{1T}^{\perp g}) & f_1^g + g_{1L}^g
\end{matrix}
\right\}
$$

$$M_p^2\, \Phi^{ii} = 2(P^+)^2 \int_0^1 dx_g x_g f_1^g(x_g, Q^2) = \langle P, S|T^{++}|P, S\rangle$$

Gluon Correlator

Gluon Helicity $|+\rangle$ $|+\rangle$ $|-\rangle$ $|-\rangle$

$\Phi^{\mu\nu}$

$$
\begin{array}{c}
\langle +| \\ \langle +| \\ \langle -| \\ \langle -|
\end{array}
\left\{
\begin{array}{cccc}
f_1^g + g_{1L}^g & \frac{q_T e^{-i\phi}}{M_p} g_{1T}^g & -\frac{q_T^2 e^{-2i\phi}}{M_p^2} h_1^{\perp g} & 0 \\
\frac{q_T e^{i\phi}}{M_p} g_{1T}^g & f_1^g - g_{1L}^g & 0 & -\frac{q_T^2 e^{-2i\phi}}{M_p^2} h_1^{\perp g} \\
-\frac{q_T^2 e^{-2i\phi}}{M_p^2} h_1^{\perp g} & 0 & f_1^g - g_{1L}^g & -\frac{q_T e^{-i\phi}}{M_p} g_{1T}^g \\
0 & -\frac{q_T^2 e^{-2i\phi}}{M_p^2} h_1^{\perp g} & -\frac{q_T e^{-i\phi}}{M_p} g_{1T}^g & f_1^g + g_{1L}^g
\end{array}
\right\}
$$

Matrix element for J/ψ and $\pi^{\pm}$ production

$$\mathcal{M}\left(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}]g\right) = \sum_{L_z S_z} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Psi_{LL_z}(\mathbf{k}) \langle LL_z; SS_z | JJ_z \rangle$$
$$\times Tr[O(q, p_g, \mathrm{P}_\psi, k) \mathcal{P}_{SS_z}(\mathrm{P}_\psi, k)]$$

- expand $\mathcal{M}\left(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}]g\right)$ as series expansion of k
- zeroth order term is S wave scattering amplitude
  first order terms in k is P wave scattering amplitude

$$\mathcal{M}[^{2S+1}S_J^{(1,8)}] = \frac{1}{\sqrt{4\pi}} R_0(0)\, Tr[O(q, p_g, \mathrm{P}_\psi, k) \mathcal{P}_{SS_z}(\mathrm{P}_\psi, k)]\Big|_{k=0}$$

$$\mathcal{M}[^{2S+1}P_J^{(8)}] = -i\sqrt{\frac{3}{4\pi}} R_1'(0) \sum_{L_z S_z} \varepsilon^{\alpha}_{L_z}(\mathrm{P}_\psi) \langle LL_z; SS_z | JJ_z \rangle$$
$$\times \frac{\partial}{\partial k^\alpha} Tr[O(q, p_g, \mathrm{P}_\psi, k) \mathcal{P}_{SS_z}(\mathrm{P}_\psi, k)]\Big|_{k=0}$$

Kishore, Mukherjee and Rajesh (2020)

S and P amplitude

$$\sum_{L_z S_z} \langle LL_z; SS_z | JJ_z \rangle \varepsilon^{\alpha}_{S_z}(\mathrm{P}_\psi) \varepsilon^{\beta}_{L_z}(\mathrm{P}_\psi) = \sqrt{\frac{1}{3}} \left(g^{\alpha\beta} - \frac{\mathrm{P}^{\alpha}_{\psi} \mathrm{P}^{\beta}_{\psi}}{M^2_\psi} \right),$$

$$\sum_{L_z S_z} \langle LL_z; SS_z | JJ_z \rangle \varepsilon^{\alpha}_{S_z}(\mathrm{P}_\psi) \varepsilon^{\beta}_{L_z}(\mathrm{P}_\psi) = -\frac{i}{M_\psi} \sqrt{\frac{1}{2}} \epsilon_{\delta\zeta\xi\varrho} g^{\xi\alpha} g^{\varrho\beta} \mathrm{P}^{\delta}_{\psi} \varepsilon^{\zeta}_{J_z}(\mathrm{P}_\psi),$$

$$\sum_{L_z S_z} \langle LL_z; SS_z | JJ_z \rangle \varepsilon^{\alpha}_{S_z}(\mathrm{P}_\psi) \varepsilon^{\beta}_{L_z}(\mathrm{P}_\psi) = \varepsilon^{\alpha\beta}_{J_z}(\mathrm{P}_\psi).$$

$$\varepsilon^{\alpha}_{J_z}(\mathrm{P}_\psi) \mathrm{P}_{\psi\alpha} = 0,$$

$$\sum_{J_z} \varepsilon^{\alpha}_{J_z}(\mathrm{P}_\psi) \varepsilon^{*\beta}_{J_z}(\mathrm{P}_\psi) = \left(-g^{\alpha\beta} + \frac{\mathrm{P}^{\alpha}_{\psi} \mathrm{P}^{\beta}_{\psi}}{M^2_\psi} \right) = \mathcal{Q}^{\alpha\beta}.$$

The $\varepsilon^{\alpha\beta}_{J_z}(\mathrm{P}_\psi)$ is polarization tensor corresponding to $J = 2$ which is symmetric in the Lorentz indices and follows the relations ,

$$\varepsilon^{\alpha\beta}_{J_z}(\mathrm{P}_\psi) = \varepsilon^{\beta\alpha}_{J_z}(\mathrm{P}_\psi) \; \varepsilon^{\alpha}_{J_z\,\alpha}(\mathrm{P}_\psi) = 0 \; \varepsilon^{\alpha}_{J_z}(\mathrm{P}_\psi) \mathrm{P}_{\psi\alpha} = 0$$

$$\varepsilon^{\alpha\beta}_{J_z}(\mathrm{P}_\psi) \varepsilon^{*\mu\nu}_{J_z}(\mathrm{P}_\psi) = \frac{1}{2}[\mathcal{Q}^{\alpha\mu}\mathcal{Q}^{\beta\nu} + \mathcal{Q}^{\alpha\nu}\mathcal{Q}^{\beta\mu}] - \frac{1}{3}[\mathcal{Q}^{\alpha\beta}\mathcal{Q}^{\mu\nu}]$$

D. Boer and Pisano (2012)

S and P amplitude

$$\mathrm{S-Wave\ amplitudes}$$

$$\mathcal{M}[^3S_1^{(1)}](\mathrm{P}_\psi, p_g) = \frac{1}{4\sqrt{\pi M_\psi}} R_0(0) \frac{\delta_{ab}}{2\sqrt{N_c}} Tr\left[\sum_{i=1}^{3} O_i(0)(\not{P}_\psi + M_\psi)\not{\epsilon}_{S_z}\right],$$

$$\mathcal{M}[^3S_1^{(8)}](\mathrm{P}_\psi, p_g) = \frac{1}{4\sqrt{\pi M_\psi}} R_0(0) \frac{\sqrt{2}}{2} d_{abc} Tr\left[\sum_{i=1}^{3} O_i(0)(\not{P}_\psi + M_\psi)\not{\epsilon}_{S_z}\right]$$

$$\mathcal{M}[^1S_0^{(8)}](\mathrm{P}_\psi, p_g) = \frac{1}{4\sqrt{\pi M_\psi}} R_0(0) i \frac{\sqrt{2}}{2} f_{abc} Tr\Big[(O_1(0) - O_2(0) - O_3(0) + 2O_4(0)) (\not{P}_\psi + M_\psi)\gamma_5\Big]$$

$$\mathrm{P-Wave\ amplitudes}$$

$$\mathcal{M}[^3\mathcal{P}_J^{(8)}](\mathrm{P}_\psi, p_g) = \frac{\sqrt{2}}{2} f_{abc} \sqrt{\frac{3}{4\pi}} R_1'(0) \sum_{L_z S_z} \varepsilon^{\alpha}_{L_z}(\mathrm{P}_\psi) \langle LL_z; SS_z | JJ_z \rangle Tr[(O_{1\alpha}(0) + O_{6\alpha}(0) - O_{3\alpha}(0) + 2O_{4\alpha}(0))\mathcal{P}_{SS_z}(0) + (O_1(0) + O_6(0) - O_3(0) + 2O_4(0))\mathcal{P}_{SS_z\alpha}(0)].$$

Kishore, Mukherjee and Rajesh (2020)

Total differential cross-section

Integrated over the final momenta,

$$\frac{\mathrm{d}^3\ell'}{(2\pi)^3 2E_{\ell'}} = \frac{\mathrm{d}Q^2\mathrm{d}y}{16\pi^2}.$$

$$\frac{\mathrm{d}^3\mathrm{P}_\psi}{(2\pi)^3 2E_\psi} = \frac{\mathrm{d}z\mathrm{d}^2\mathbf{P}_{\psi\perp}}{(2\pi)^3 2z}, \quad \frac{\mathrm{d}^3\mathrm{P}_\pi}{(2\pi)^3 2E_j} = \frac{\mathrm{d}\bar{z}\mathrm{d}^2\mathbf{P}_{j\perp}}{(2\pi)^3 2\bar{z}}$$

The four-momentum delta function

$$\delta^4\big(q + p_g - \mathrm{P}_\psi - \mathrm{P}_\pi\big) = \frac{2}{ys}\delta\big(1 - z - \bar{z}\big)\delta\left(x - \frac{\bar{z}(M^2 + \mathbf{P}_{\psi\perp}^2) + z\mathbf{P}_{j\perp}^2 + z\bar{z}Q^2}{z(1-z)ys}\right) \delta^2\big(\mathbf{p}_T - \mathbf{P}_{j\perp} - \mathbf{P}_{\psi\perp}\big),$$

Back to back J/ψ and $\pi^\pm$ in transverse plane

$$q_t \equiv \mathbf{P}_{\psi\perp} + \mathbf{P}_{j\perp}\,, \quad \mathbf{K}_t \equiv \frac{\mathbf{P}_{\psi\perp} - \mathbf{P}_{j\perp}}{2}$$

LDMEs

	$\langle 0\vert\mathcal{O}_8^{J/\psi}(^1S_0)\vert 0\rangle$	$\langle 0\vert\mathcal{O}_8^{J/\psi}(^3S_1)\vert 0\rangle$	$\langle 0\vert\mathcal{O}_1^{J/\psi}(^3S_1)\vert 0\rangle$	$\langle 0\vert\mathcal{O}_8^{J/\psi}(^3P_0)\vert 0\rangle/m_c^2$
Ref. Sharma (2013)	1.8 ± 0.87 $\times 10^{-2}\text{GeV}^3$	0.13 ± 0.13 $\times 10^{-2}\text{GeV}^3$	1.2×10^2 $\times 10^{-2}\text{GeV}^3$	1.8 ± 0.87 $\times 10^{-2}\text{GeV}^3$
Ref. Chao (2012)	8.9 ± 0.98 $\times 10^{-2}\text{GeV}^3$	0.30 ± 0.12 $\times 10^{-2}\text{GeV}^3$	1.2×10^2 $\times 10^{-2}\text{GeV}^3$	0.56 ± 0.21 $\times 10^{-2}\text{GeV}^3$

Spectator model

Parameter	Replica 11	Parameter	Replica 11
A	6.0	κ_2 (GeV^2)	0.414
a	0.78	σ (GeV)	0.50
b	1.38	Λ_X (GeV)	0.448
C	346	κ_1 (GeV^2)	1.46
D (GeV)	0.548		

all 4-momenta

$$p_g^\mu = xP^\mu + (p_g \cdot P + M^2 x)n^\mu + \boldsymbol{p}_T^\mu \approx xP^\mu + \boldsymbol{p}_T^\mu,$$

$$\mathrm{P}_\psi^\mu = \frac{\mathbf{P}_{\psi\perp}^2 + M_\psi^2}{2zP \cdot q} P^\mu + z(P \cdot q)n^\mu + \boldsymbol{P}_{\psi\perp}^\mu,$$

$$\mathrm{P}_\pi^\mu = \frac{\mathbf{P}_{J\perp}^2}{2(1-z)P \cdot q} P^\mu + (1-z)(P \cdot q)n^\mu + \mathbf{P}_{J\perp}^\mu,$$

$$q^\mu = -x_B n_-^\mu + \frac{Q^2}{2x_B} n_+^\mu \approx -x_B P^\mu + (P \cdot q)n_+^\mu,$$

$$l^\mu = \frac{(1-y)x_B}{y} P^\mu + \frac{(P \cdot q)}{y} n^\mu + \frac{\sqrt{1-y}}{y} Q\hat{l}_\perp^\mu$$

$$l'^\mu = l^\mu - q^\mu$$

$$Q^2 = -q^2, \quad s = (P + l)^2$$

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_\psi}{P \cdot q}$$

TikZ Arrow at 45-Degree Angle