LECTURE 2
Phenomenology

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f(R) Gravity

f(R) gravity

$$\boxed{S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\,[R+f(R)] + \int d^4x\sqrt{-g}\mathcal{L}_m\,[\chi_i, g_{\mu\nu}]}$$

(S.Carroll, V.Duvvuri, M.Trodden & M.S.Turner, Phys.Rev.D70 043528 (2004), S.Capozziello, S.Carloni & A.Troisi, astro-ph/0303041)

$$\begin{cases} (1+f_R)\,R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,(R+f) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\,f_R = \frac{T_{\mu\nu}}{M_P^2} \\ \nabla_\mu T^{\mu\nu} = 0 \end{cases}$$

$$f_R \equiv \frac{df}{dR}$$

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The Einstein equations are fourth order !

f(R) g

The following term:

$$g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\mu\nabla_\nu\left(-\delta g^{\mu\nu} + g^{\mu\nu}g_{\alpha\beta}\delta g^{\alpha\beta}\right)$$

in GR is a boundary term, but in this new action it comes multiplied by f_R and gives rise to:

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The Einstein equations are fourth order !

The trace-equation becomes:

dynamical !

$$(1 - f_R)\, R + 2f - 3\Box f_R = \frac{T}{M_P^2}$$

Background Cosmology

Background

Hence there is an additional dynamical DOF, dubbed the scalaron, which obeys the following eom:

$$\Box f_R = \frac{1}{3}\left(R + 2f - Rf_R\right) - \frac{\kappa^2}{3}(\rho - 3P) \equiv \frac{\partial V_{\rm eff}}{\partial f_R}$$

By design, the f(R) theories we consider must have $f \ll R$ and $f_R \ll 1$ at high curvatures to be consistent with our knowledge of the high redshift universe.
In this limit, the extremum of the effective potential lies at the GR value $R = \kappa^2\left(\rho - 3P\right)$.
Whether this extremum is a minimum or a maximum is determined by the second derivative of the effective potential at the extremum:

$$m_{f_R}^2 \equiv \frac{\partial^2 V_{\rm eff}}{\partial f_R^2} = \frac{1}{3}\left[\frac{1+f_R}{f_{RR}} - R\right] \approx \frac{1+f_R}{3f_{RR}} \approx \frac{1}{3f_{RR}}$$

We will get back to this characteristic lengthscale of the model several times.

Einstein frame

Let's look at it from another angle! Introducing an auxiliary field we can write a dynamically equivalent action:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\;[(\Phi + f(\Phi)) + (1+f_\Phi)(R-\Phi)] + \int d^4x\sqrt{-g}\,\mathcal{L}_{\mathrm{m}}[\chi_i, g_{\mu\nu}]\,, \qquad f_\Phi \equiv \frac{df}{d\Phi}$$

If $f_{\Phi\Phi} \neq 0$, the field equation for Φ gives R= Φ which reduces the above action to the original one.

T. Chiba, Phys. Lett. B 575 (2003) G. Magnano and L. M. Sokolowski, Phys. Rev. D 50, 5039 (1994)

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Now, let us perform a *conformal transformation*:

$$\tilde{g}_{\mu\nu} = e^{2\omega(x^\alpha)} g_{\mu\nu} \qquad \text{with} \qquad e^{-2\omega}(1+f_R) = 1$$

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Defining $\phi \equiv \frac{2\omega}{\beta\kappa}$, we get the following action:

$$\tilde{S} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\,\tilde{R} + \int d^4x\sqrt{-\tilde{g}}\,\left[-\frac{1}{2}\tilde{g}^{\mu\nu}(\tilde{\nabla}_\mu\phi)\tilde{\nabla}_\nu\phi - V(\phi)\right] + \int d^4x\sqrt{-\tilde{g}}\,e^{-2\beta\kappa\phi}\mathcal{L}_{\rm m}[\chi_i, e^{-\beta\kappa\phi}\tilde{g}_{\mu\nu}]$$

for a scalar field with the following potential:

$$V(\phi) = \frac{1}{2\kappa^2}\frac{Rf_R - f}{(1+f_R)^2}$$

$$\boxed{\beta = \sqrt{\frac{2}{3}}}$$

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Ei

This is the Einstein frame ! The gravitational action has the standard Einstein-Hilbert form,
there is an explicit additional scalar DOF which is coupled to matter.
This frame is physically equivalent to the one of the original action, i.e. the Jordan frame. The latter is defined by the fact that matter fields follow geodesics of the metric. f(R) has a universal coupling
and hence allows for a uniquely defined Jordan frame.

Let's look at it from another angle! Intro
dynamically eq

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Designer approach

From Einstein eqs.: $$\frac{\ddot{a}}{a} - (1 + f_R)\mathcal{H}^2 + a^2\frac{f}{6} + \frac{1}{2}\ddot{f}_R = -\frac{\kappa^2}{6}a^2(\rho + 3P)$$

in conformal time!

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Let us define:

$$y \equiv \frac{f(R)}{H_0^2} \ , \quad E \equiv \frac{H^2}{H_0^2}$$

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$$y \equiv \frac{f(R)}{H_0^2} \;, \quad E \equiv \frac{H^2}{H_0^2}$$

and fix the desired expansion history to that of a flat universe containing matter, radiation and dark energy:

$$E = \Omega_m a^{-3} + \Omega_r a^{-4} + \rho_{\rm eff}/\rho_c^0 \equiv E_m + E_r + E_{\rm eff}$$

$$E_{\rm eff} = (1 - \Omega_m - \Omega_r)\exp\left[-3\ln a + 3\int_a^1 w_{\rm eff}(a)d\ln a\right]$$

$$\rho_c^0 \equiv 3H_0^2/\kappa^2$$

$$E_i \equiv \rho_i/\rho_{cr}^0$$

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All standard background functions in the Friedmann equation can be expressed in terms of E and its derivatives. The resulting equation is a 2nd order ODE for f(R) in terms of derivatives wrt ln(a):

$$y'' - \left(1 + \frac{E'}{2E} + \frac{R''}{R'}\right)y' + \frac{R'}{6H_0^2E}y = -\frac{R'}{H_0^2E}E_{\rm eff}$$

Designer approach

In order to set the initial conditions, let us consider the general and the particular solutions at early times, when the effects of the effective dark energy on the expansion are negligible. At a certain early value a_i , the homogeneous part of the eq. is satisfied by a power law ansatz $y \propto a_i^p$. Substituting this ansatz in and solving the quadratic equation for p yields:

$$p_\pm = \frac{1}{2}\left(-b \pm \sqrt{b^2 - 4c}\right)$$

$$b = \frac{7 + 8r_i}{2(1 + r_i)} \ , \quad c = -\frac{3}{2(1 + r_i)} \ , \quad r_i = \frac{a_{eq}}{a_i}$$

The decaying mode solution corresponding to p_- leads to a large f(R) at early times which makes it unacceptable, and we set its amplitude to zero. The particular solution at a_i can be found by substituting $y_p = A_p E_{\rm eff}(a_i)$ in the ODE. One then finds:

$$A_p = \frac{-6c}{-3w'_{\rm eff} + 9w^2_{\rm eff} + (18 - 3b)w_{\rm eff} + 9 - 3b + c}$$

Put together, the initial conditions at a_i are

$$y_i = Ae^{p_+ \ln a_i} + y_p$$
$$y'_i = p_+ Ae^{p_+ \ln a_i} - 3[1 + w_{\rm eff}(a_i)]y_p$$

and A is the remaining arbitrary constant that can be used to parametrize different f(R) models with the same expansion history.

Designer approach

There is a family of f(R) models for each expansion history.

As a label for each family, it is common to use the boundary condition at a=0, B0, defined as the today value of:

$$B = \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

Which is the characteristic lengthscale of the scalaron in units of the horizon scale.

Viability conditions

L. Pogosian, A. Silvestri, Phys. Rev. D77 (2008) 023503

Conditions of viability:

- $f_{RR} > 0$ to have a stable high-curvature regime
- $1 + f_R > 0$ to have a positive effective Newton constant
- $f_R < 0$ negative, monotonically increasing function of R that asymptotes to zero from below

----

- $|f_R^0| \leq 10^{-6}$ must be small at recent epochs to pass local tests of gravity

Dolgov, Kawasaki, Phys.Lett.B 573 (2003), I.Navarro, K. van Acoyelen, JCAP 0702:022, 2007
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$$\Downarrow$$

$$w_{\text{eff}} \simeq -1$$

Dolgov, Kawasaki, Phys.Lett.B 573 (2003), I.Navarro, K. van Acoyelen, JCAP 0702:022, 2007

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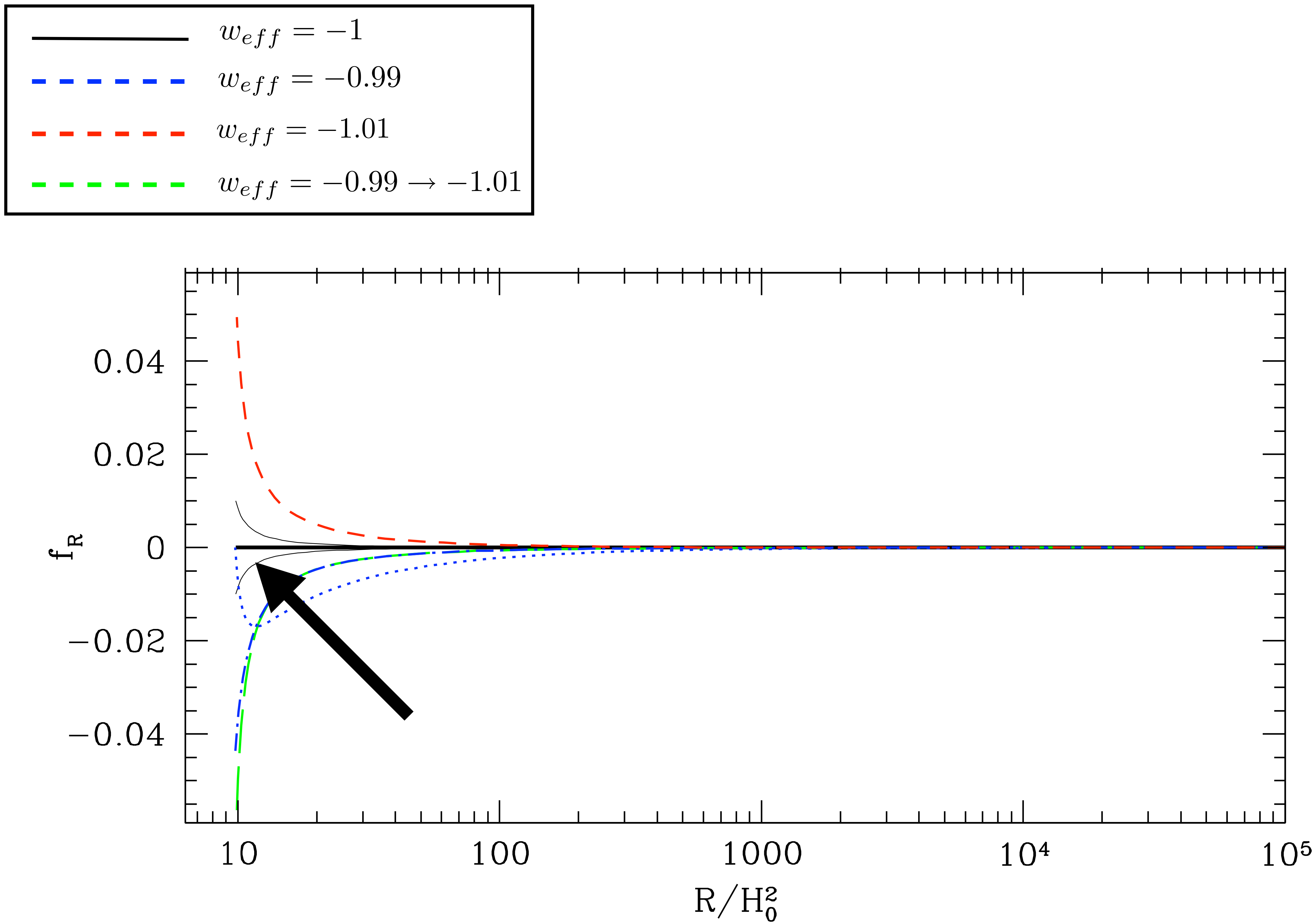
Designer approach



Take home message

Generally theories beyond ΛCDM have enough freedom to reproduce 'any' desired expansion history, being higher order in nature.

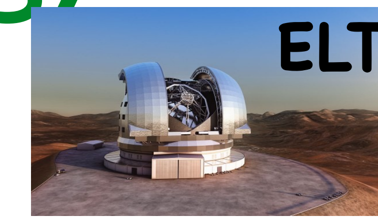
In other words, at the level of background expansion history there is a degeneracy among different approaches to the phenomenon of cosmic acceleration.

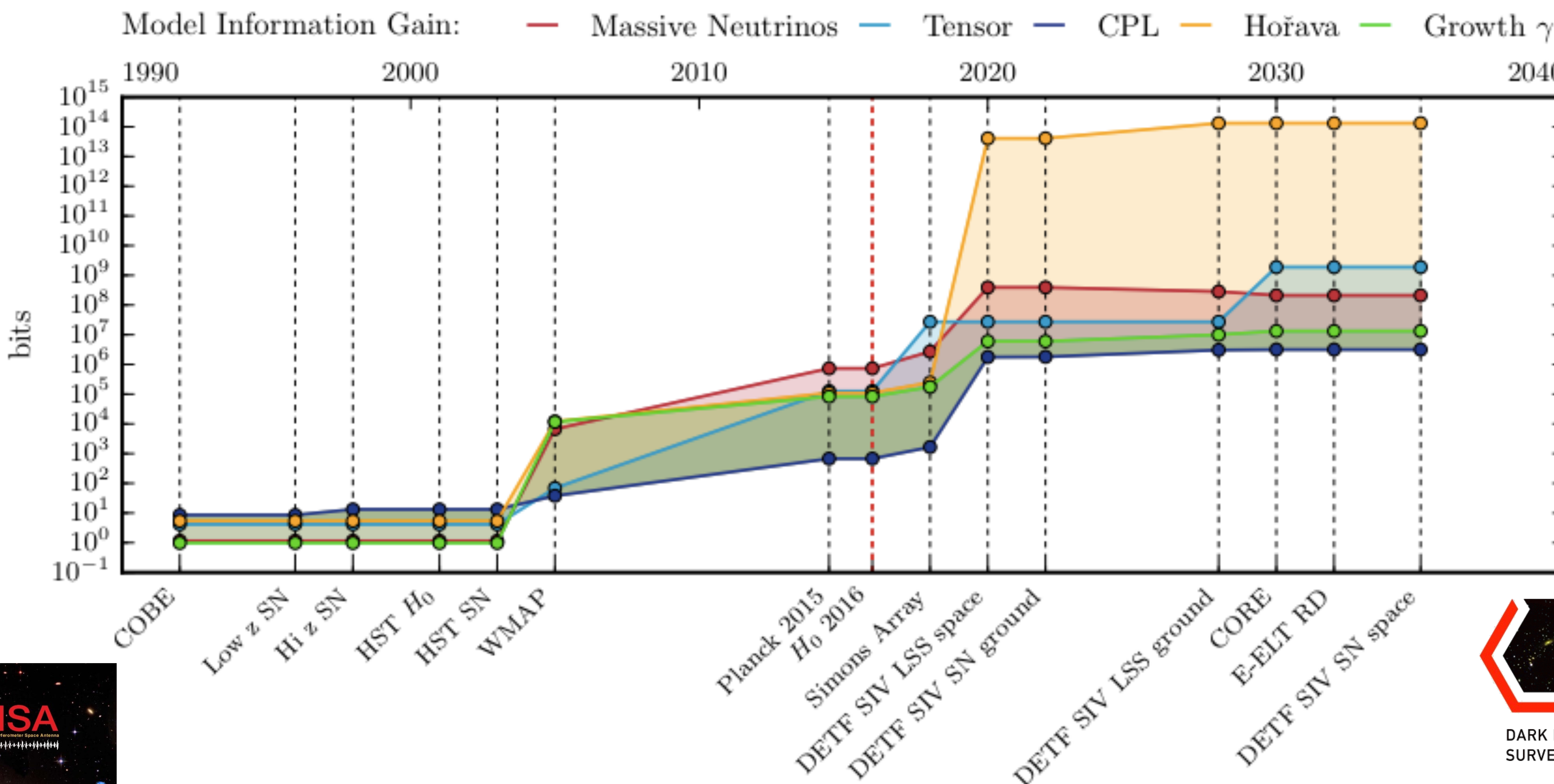
In the past decade it has become increasingly clear that we need to go beyond geometrical probes in order to disentangle the theoretical landscape of cosmic acceleration.

In other words, the growth of structure is expected to be a powerful testbed,
in particular through combinations of different observables.

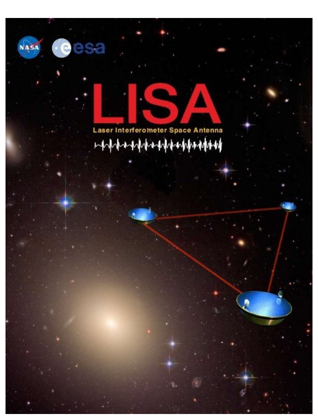
Information Gain in Cosmology

EUCLID
Mapping the geometry of the dark Universe





Model cumulative Information Gain



DARK ENERGY SURVEY

ATACAMA COSMOLOGY TELESCOPE ACTPol

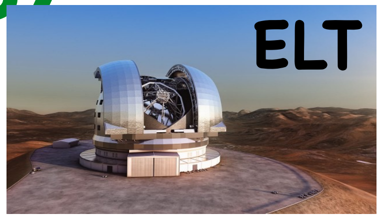
SKA SQUARE KILOMETRE ARRAY

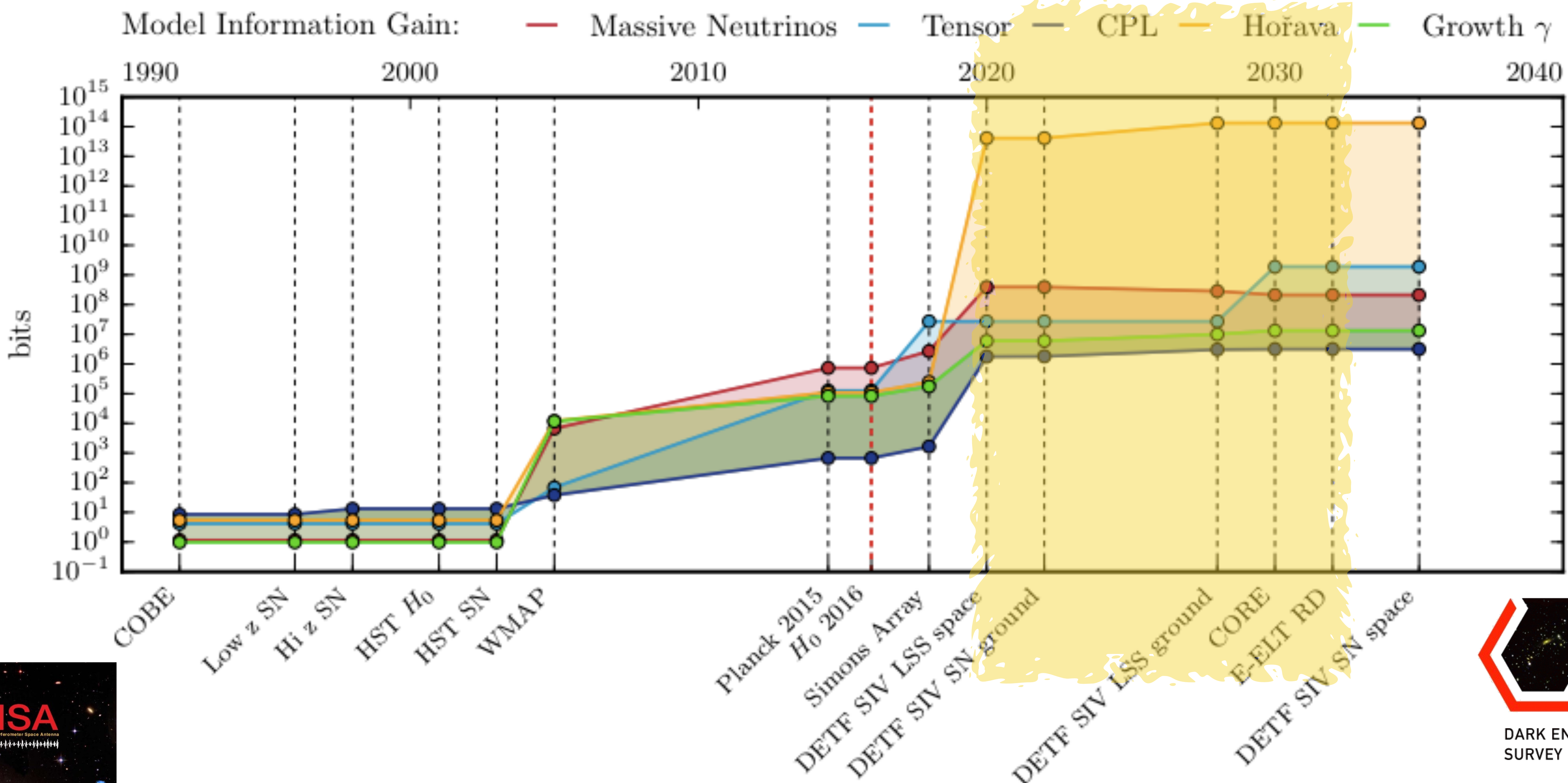
GIANT MAGELLAN TELESCOPE

LSST Large Synoptic Survey Telescope

Raveri, Martinelli, Zhao, Wang, **CosmicFish**/arXiv:1606.0627

Information Gain in Cosmology





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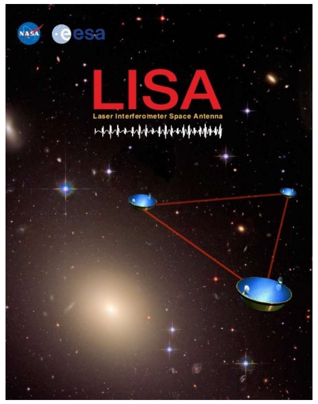

DARK ENERGY SURVEY

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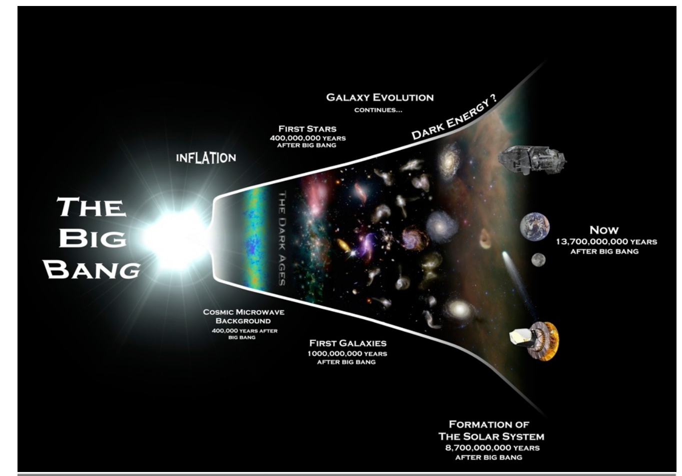
Structure Formation

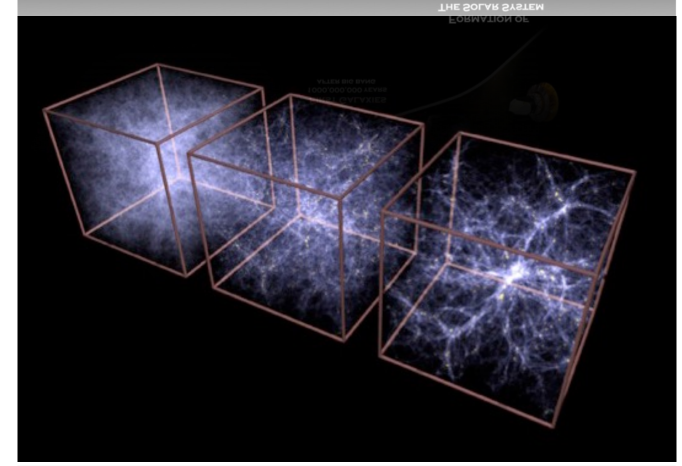
Perturbed metric and LSS

$$ds^2 = -a^2(\tau)\left[(1+2\Psi(\tau,\vec{x}))\,d\tau^2 - (1-2\Phi(\tau,\vec{x}))\right]$$

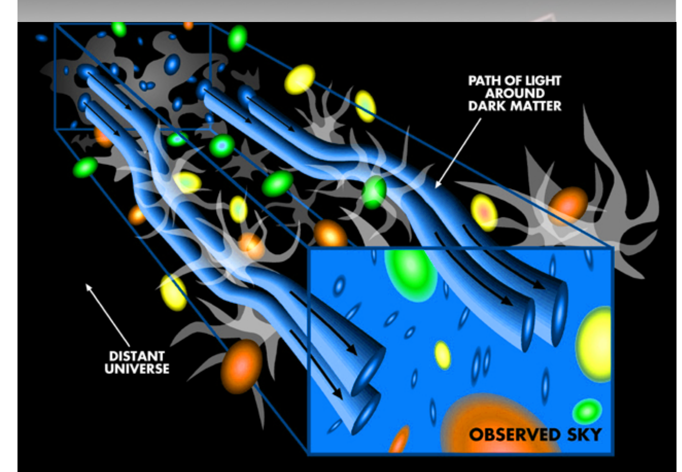
expansion history: $a(\tau)$



non-relativistic dynamics
(growth of structure, pec. vel.): $\Psi(\tau,\vec{x})$



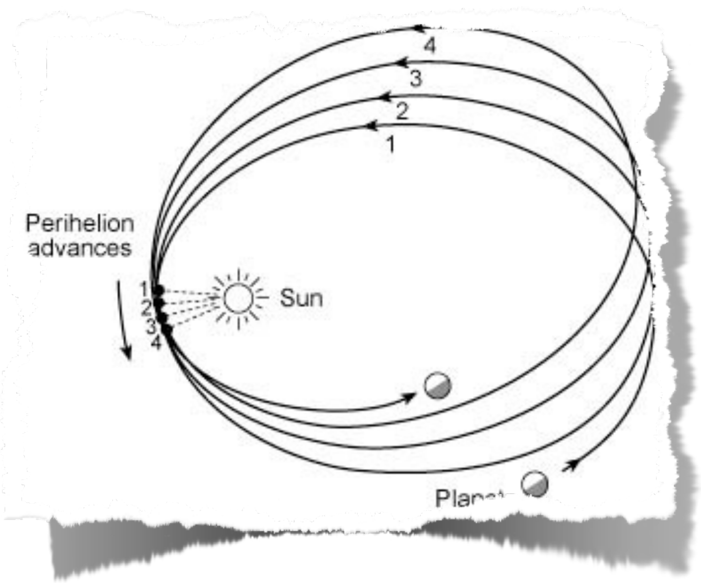
relativistic dynamics
(weak lensing, ISW): $(\Phi+\Psi)\,(\tau,\vec{x})$



Cosmic Functions of Interest

Ψ : **non-relativistic tracers**

orbits of planets



galaxies: growth of structure and peculiar velocities

Newtonian potential

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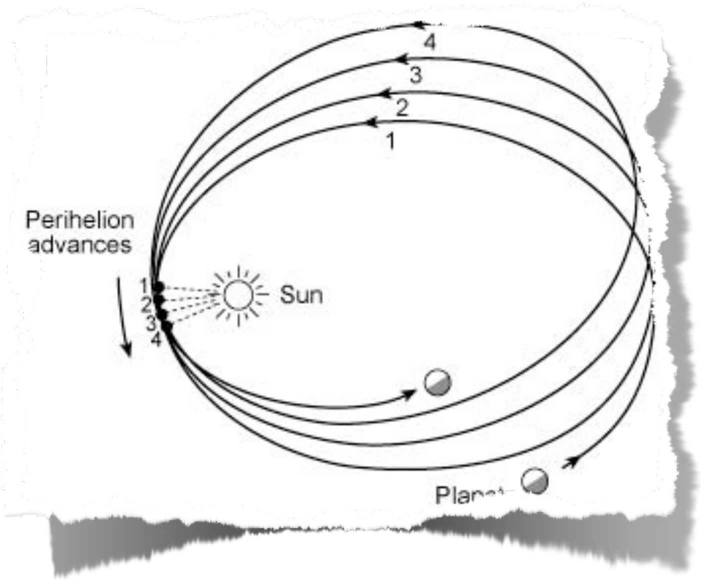

galaxies: growth of structure and peculiar velocities

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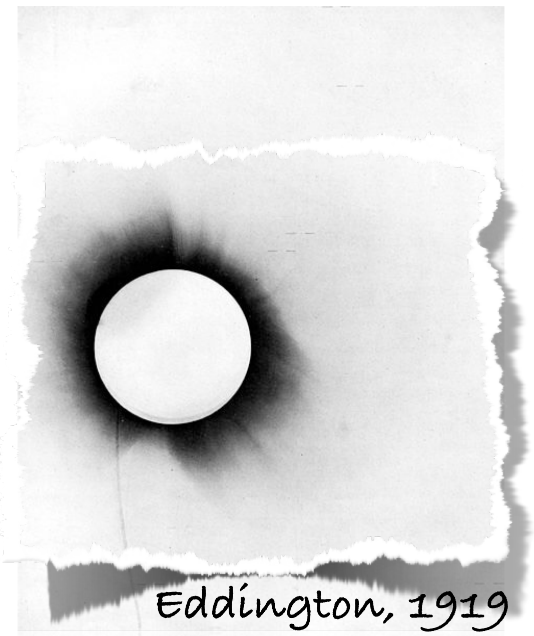
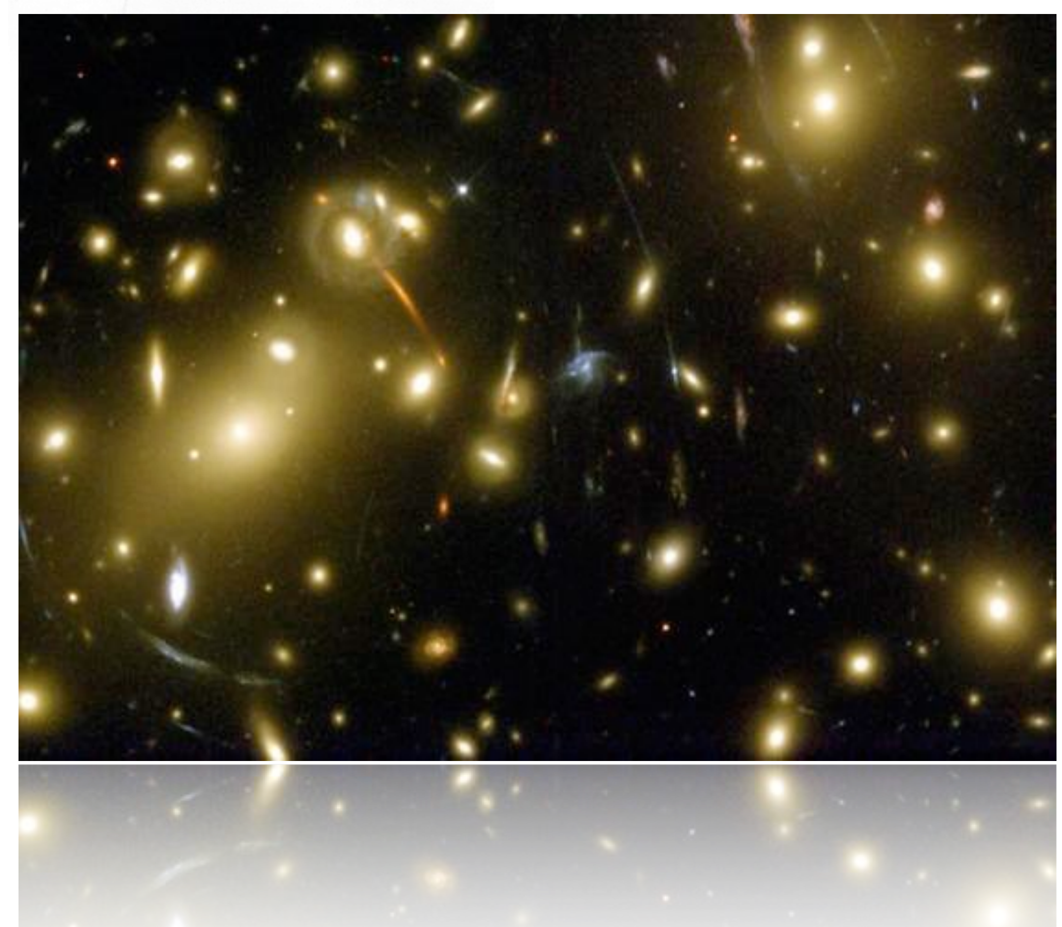
Cosmic Functions of Interest

$\Phi + \Psi$: **massless particles**

light deflection, time delay



weak lensing,
Integrated Sachs-Wolfe effect in CMB



lensing potential

LSS in LCDM

Einstein eqs. tell us how the metric potentials relate to matter:

$$\Phi = \Psi = -\frac{a^2}{2k^2}\frac{\rho\delta}{2M_P^2}$$

Now let us close the system with the fluid equations for matter. Let us focus on <u>CDM</u> since we are interested in <u>late times clustering</u>:

$$\begin{aligned}\dot{\delta} &= -\theta + 3\dot{\Phi} \\ \dot{\theta} &= -\mathcal{H}\theta + k^2\Psi\end{aligned} \quad \xrightarrow[k \gg aH]{} \quad \ddot{\delta} + \mathcal{H}\delta + k^2\Psi = 0 \quad \longrightarrow \quad \ddot{\delta} + \mathcal{H}\delta - \frac{a^2}{2M_P^2}\rho\delta = 0$$

$$\ddot{\delta} + \mathcal{H}\delta - \frac{3}{2}\Omega_m(a)\delta = 0$$

scale independent !

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scale independent !

the growing mode goes like:

$$D_1(a) = \frac{5\Omega_m}{2}\frac{H(a)}{H_0}\int_0^a \frac{da'}{\left[a'\frac{H(a')}{H_0}\right]^3}$$

which gives a <u>consistency relation btw expansion history and growth of structure</u>

The growth rate:

$$f \equiv \frac{d\ln\delta}{d\ln a} \sim \Omega_m(a)^{6/11}$$

LSS in a nutshell

LSS in a nutshell

LCDM: $\boxed{w = -1 \qquad \Phi = \Psi \qquad \Psi = -\frac{a^2}{k^2}\frac{\rho\Delta}{2M_P^2}}$

LSS in a nutshell

LCDM:

$$\boxed{w = -1 \qquad \Phi = \Psi \qquad \Psi = -\frac{a^2}{k^2}\frac{\rho\Delta}{2M_P^2}}$$

- relativistic and non-relativistic probes respond to the same metric potential

- the growth of structure is scale-independent

. . . in f(R) gravity

LSS in f(R)

Let us start from the Einstein equations in f(R) gravity:

$$\left(1+f_R\right) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R+f\right) + \left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu\right) f_R = \frac{T_{\mu\nu}}{M_P^2}$$

LSS in f(R)

Let us start from the Einstein equations in f(R) gravity:

$$(1+f_R)\,R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\,(R+f)+(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)\,f_R=\frac{T_{\mu\nu}}{M_P^2}$$

One does the usual expansion to linear order in scalar perturbations. The tricky term is the last one on the l.h.s.. In the 00 case it gives:

$$\begin{aligned}
&\delta\left[\left(\delta_0^0\Box-\nabla^0\nabla_0\right)f_R\right]\\
&=\delta\left(\nabla^i\nabla_i f_R\right)\\
&=\delta\left(g^{ij}\nabla_j\nabla_i f_R\right)\\
&=\delta\left[\left(g^{ij}\partial_i\partial_j-g^{ij}\Gamma^\alpha_{ij}\partial_\alpha\right)f_R\right]\\
&=\delta\left[\left(g^{ij}\partial_i\partial_j-g^{ij}\Gamma^0_{ij}\partial_0-g^{ij}\Gamma^k_{ij}\partial_k\right)f_R\right]\\
&=g^{(0)ij}\partial_i\partial_j\delta f_R-\delta g^{ij}\Gamma^{(0)0}_{ij}\partial_0 f_R-g^{(0)ij}\Gamma^{(0)0}_{ij}\partial_0\delta f_R\\
&=-\frac{k^2}{a^2}\delta f_R-\frac{3}{2}\mathcal{H}\delta\dot{f}_R+\frac{\dot{f}_R}{a^2}\left(6\mathcal{H}\Psi+3\dot{\Phi}\right)
\end{aligned}$$

$$F \equiv 1 + f_R$$

$\mathbf{E_{00}}$:

$$\left[k^2\Phi + 3\mathcal{H}\left(\dot{\Phi} + \mathcal{H}\Psi\right)\right] + \frac{3}{2}\dot{\mathcal{H}}\frac{\delta f_R}{F} - \frac{3}{2}\mathcal{H}\frac{\delta \dot{f}_R}{F}$$
$$-\frac{k^2}{2}\frac{\delta f_R}{F} + \left(3\dot{\Phi} + 6\mathcal{H}\Psi\right)\frac{\dot{F}}{2F} = -\frac{a^2}{2M_P^2}\frac{\rho}{F}\delta$$

$\mathbf{E_{0i}}$:

$$\left[\dot{\Phi} + \mathcal{H}\Psi\right] - \frac{1}{2}\frac{\delta \dot{f}_R}{F} + \frac{1}{2}\mathcal{H}\frac{\delta f_R}{F} + \frac{1}{2}\frac{\dot{F}}{F}\Psi = \frac{a^2}{2M_P^2}\frac{(\rho + P)}{F}\theta$$

$\mathbf{E_{ii}}$:

$$\left[2\ddot{\Phi} + 2\mathcal{H}\left(\dot{\Psi} + 2\dot{\Phi}\right) + 2\left(2\dot{\mathcal{H}} + \mathcal{H}^2\right)\Psi + \frac{2k^2}{3}(\Phi - \Psi)\right]$$
$$+\left(2\mathcal{H}b^2 + \dot{\mathcal{H}}\right)\frac{\delta f_R}{F} - \frac{\delta \ddot{f}_R}{F} - \mathcal{H}\frac{\delta \dot{f}_R}{F} - \frac{2k^2}{3}\frac{\delta f_R}{F} + 2\frac{\ddot{F}}{F}\Psi$$
$$+\frac{\dot{F}}{F}\left(2\dot{\Phi} + 2\mathcal{H}\Psi + \dot{\Psi}\right) = \frac{1}{F}\frac{a^2}{M_P^2}\delta P$$

$\mathbf{E_{ij}}$:

$$k^2(\Phi - \Psi) - k^2\frac{\delta f_R}{F} = \frac{3a^2}{2M_P^2}\frac{(\rho + P)}{F}\sigma\,.$$

$$F \equiv 1 + f_R$$

$\mathbf{E_{00}}$:

$$\left[k^2\Phi + 3\mathcal{H}\left(\dot{\Phi} + \mathcal{H}\Psi\right)\right] + \frac{3}{2}\dot{\mathcal{H}}\frac{\delta f_R}{F} - \frac{3}{2}\mathcal{H}\frac{\delta \dot{f}_R}{F}$$

$$-\frac{k^2}{2}\frac{\delta f_R}{F} + \left(3\dot{\Phi} + 6\mathcal{H}\Psi\right)\frac{\dot{F}}{2F} = -\frac{a^2}{2M_P^2}\frac{\rho}{F}\delta$$

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$$\left[\dot{\Phi} + \mathcal{H}\Psi\right] - \frac{1}{2}\frac{\delta \dot{f}_R}{F} + \frac{1}{2}\mathcal{H}\frac{\delta f_R}{F} + \frac{1}{2}\frac{\dot{F}}{F}\Psi = \frac{a^2}{2M_P^2}\frac{(\rho + P)}{F}\theta$$

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Poisson eq.:

$$k^2\Phi - k^2\frac{\delta f_R}{F} + \frac{3}{2}\left[\left(\dot{\mathcal{H}} - \mathcal{H}^2\right)\frac{\delta f_R}{F} + \left(\dot{\Phi} + \mathcal{H}\Psi\right)\frac{\dot{f}_R}{F}\right] = -\frac{a^2}{2M_P^2}\rho\Delta$$

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$$k^2\left(\Phi - \Psi\right) - k^2\frac{\delta f_R}{F} = \frac{3a^2}{2M_P^2}\frac{(\rho + P)}{F}\sigma\,.$$

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entering the QS regime!

n eq.:

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On Quasi-Static approximation

Often employed on sub-horizon scales. It significantly simplifies the work because it reduces the Einstein equations, and any equation for additional scalar d.o.f., to algebraic relations in Fourier space. What does it effectively correspond to? Is it always a good approximation?

in LCDM

- sub-horizon scales: $k \gg aH$

$\Downarrow$

- time derivatives of metric potentials negligible w.r.t. space derivatives

in DE/MG

- sub-horizon scales: $k \gg aH$

and

- time derivatives negligible w.r.t. space derivatives for both metric potentials and additional scalars, i.e.

$$\delta\ddot{\phi} \ll c_s^2 k^2 \delta\phi$$

LSS in f(R)

$$k^2\Psi = -\frac{a^2}{2M_P^2}\frac{1}{F}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{F}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{F}}\rho\Delta$$

$$\frac{\Phi}{\Psi} = \frac{1+2\frac{k^2}{a^2}\frac{f_{RR}}{F}}{1+4\frac{k^2}{a^2}\frac{f_{RR}}{F}}$$

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time and scale dependent
rescaling of Newton constant

$$\frac{k^2}{a^2}\frac{f_{RR}}{F} = \frac{k^2}{a^2}\frac{1}{m^2}$$

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β

$$\boxed{\frac{G_{\rm eff}}{G} = \frac{1+\frac{4}{3}\frac{k^2}{a^2m^2}}{1+\frac{k^2}{a^2m^2}} \qquad \frac{\Phi}{\Psi} = \frac{1+\frac{2}{3}\frac{k^2}{a^2m^2}}{1+\frac{4}{3}\frac{k^2}{a^2m^2}}}$$

LSS in Brans-Dicke

This can be easily generalized to the Brans-Dicke class of models described by the action:

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_P^2}{2} \tilde{R} - \frac{1}{2} g^{\tilde{\mu}\nu} (\tilde{\nabla}_\mu \phi) \tilde{\nabla}_\nu \phi - V(\phi) \right] + S_i \left(\chi_i, e^{-\kappa \alpha_i(\phi)} \tilde{g}_{\mu\nu} \right)$$

$$k^2 \Psi = -\frac{a^2}{2M_P^2} e^{-\kappa\alpha} \frac{1 + \left(1 + \frac{1}{2}\alpha'^2\right) \frac{k^2}{a^2 m^2}}{1 + \frac{k^2}{a^2 m^2}} \rho \delta$$

$$\frac{\Phi}{\Psi} = \frac{1 + \left(1 - \frac{1}{2}\alpha'^2\right) \frac{k^2}{a^2 m^2}}{1 + \left(1 + \frac{1}{2}\alpha'^2\right) \frac{k^2}{a^2 m^2}}$$

LSS in Brans-Dicke

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> 1 **growth always enhanced**

$$k^2\Psi = -\frac{a^2}{2M_P^2}e^{-\kappa\alpha}\frac{1+\left(1+\frac{1}{2}{\alpha'}^2\right)\frac{k^2}{a^2m^2}}{1+\frac{k^2}{a^2m^2}}\rho\delta$$

$$\frac{\Phi}{\Psi} = \frac{1+\left(1-\frac{1}{2}{\alpha'}^2\right)\frac{k^2}{a^2m^2}}{1+\left(1+\frac{1}{2}{\alpha'}^2\right)\frac{k^2}{a^2m^2}}$$

Signatures on LSS in f(R)

The continuity and Euler equation for matter fields remain unchanged (i.e. we are working in the Jordan frame). On sub-horizon scales, the growth equation will become:

$$\ddot{\delta} + \mathcal{H}\delta + k^2\Psi = 0 \quad \longrightarrow \quad \ddot{\delta} + \mathcal{H}\delta - \frac{a^2}{2M_P^2}\frac{1}{F}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{F}}{1+4\frac{k^2}{a^2}\frac{f_{RR}}{F}}\rho\delta = 0$$

Signatures on LSS in f(R)

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We notice:

- the relation between expansion and growth is broken
- the growth will now be scale-dependent
- there will be two regimes for the growth:

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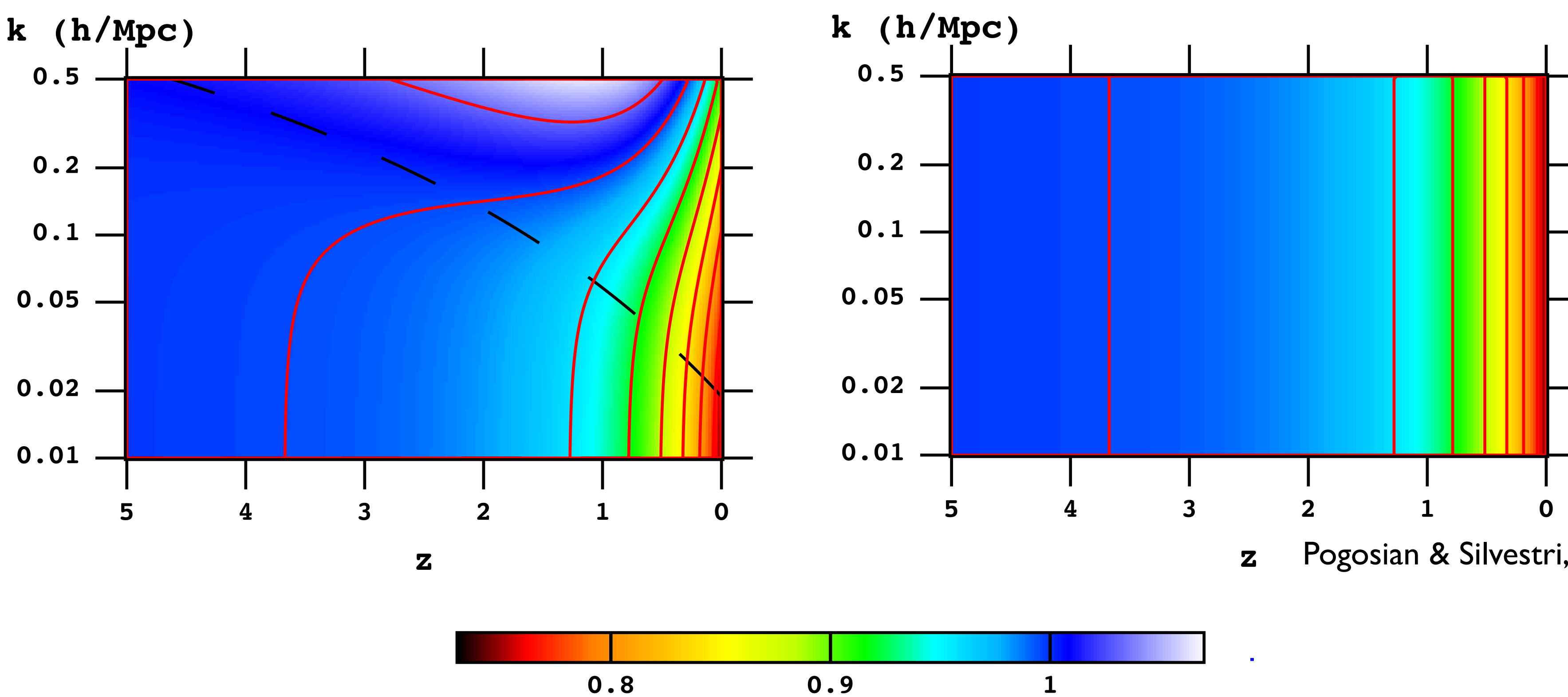
We notice:

- the relation between expansion and growth is broken
- the growth will now be scale-dependent
- there will be two regimes for the growth:
    - on scales larger than 1/m the dynamics is close to the standard one
    - on scales smaller than 1/m the dynamics is modified, growth is enhanced and metric potentials start evolving

Signatures on LSS in f(R)

Growth of linear cosmological perturbations in f(R) gravity

$$\boxed{\begin{array}{c} w_{\text{eff}} = -1 \\ |f_R^0| = 10^{-4} \end{array}}$$



$$\frac{\Delta_m(a,k)/a}{\Delta_m(a_i,k)/a_i}$$

Pogosian & Silvestri, Phys. Rev. D77, 023503 (2008)

Overall we observe a scale-dependent pattern of growth. The dynamics of perturbations is richer, and different observables are described by different functions, not by a single growth factor.

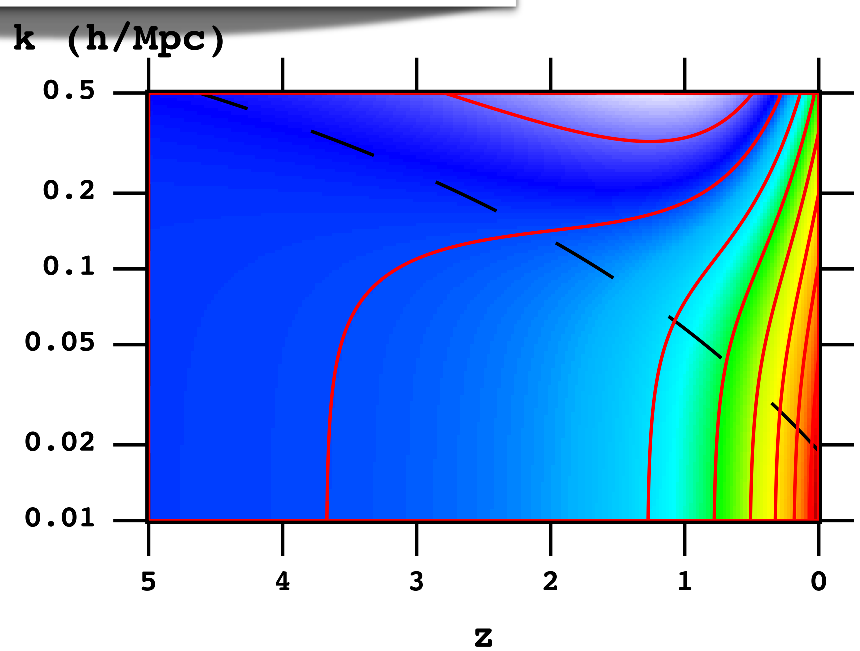
Hence the growth of structure offers a way of testing gravity that is complementary to distance measurements

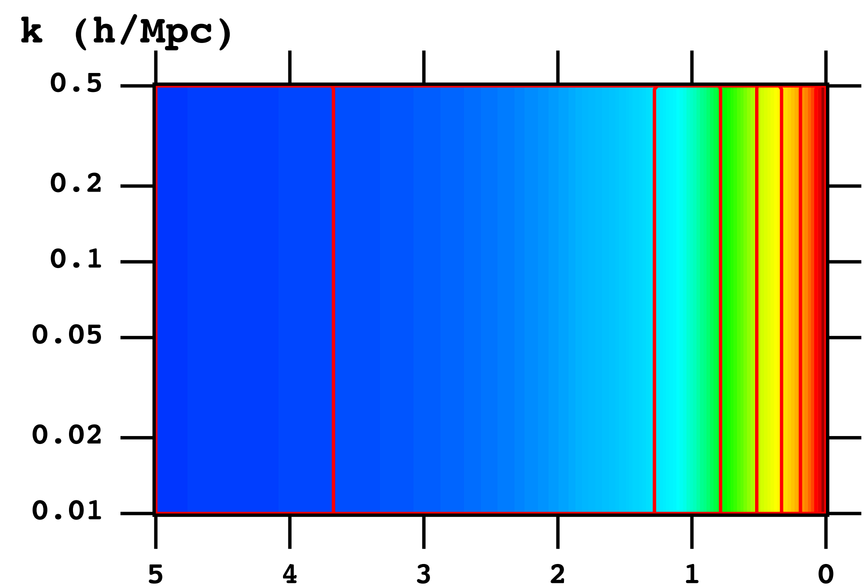
Signatures on LSS in f(R)

full dynamics, no QS

…near cosmological perturbations in f(R) gravity

$$w_{\rm eff} = -1$$
$$|f_R^0| = 10^{-4}$$





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z Pogosian & Silvestri, Phys. Rev. D77, 023503 (2008)

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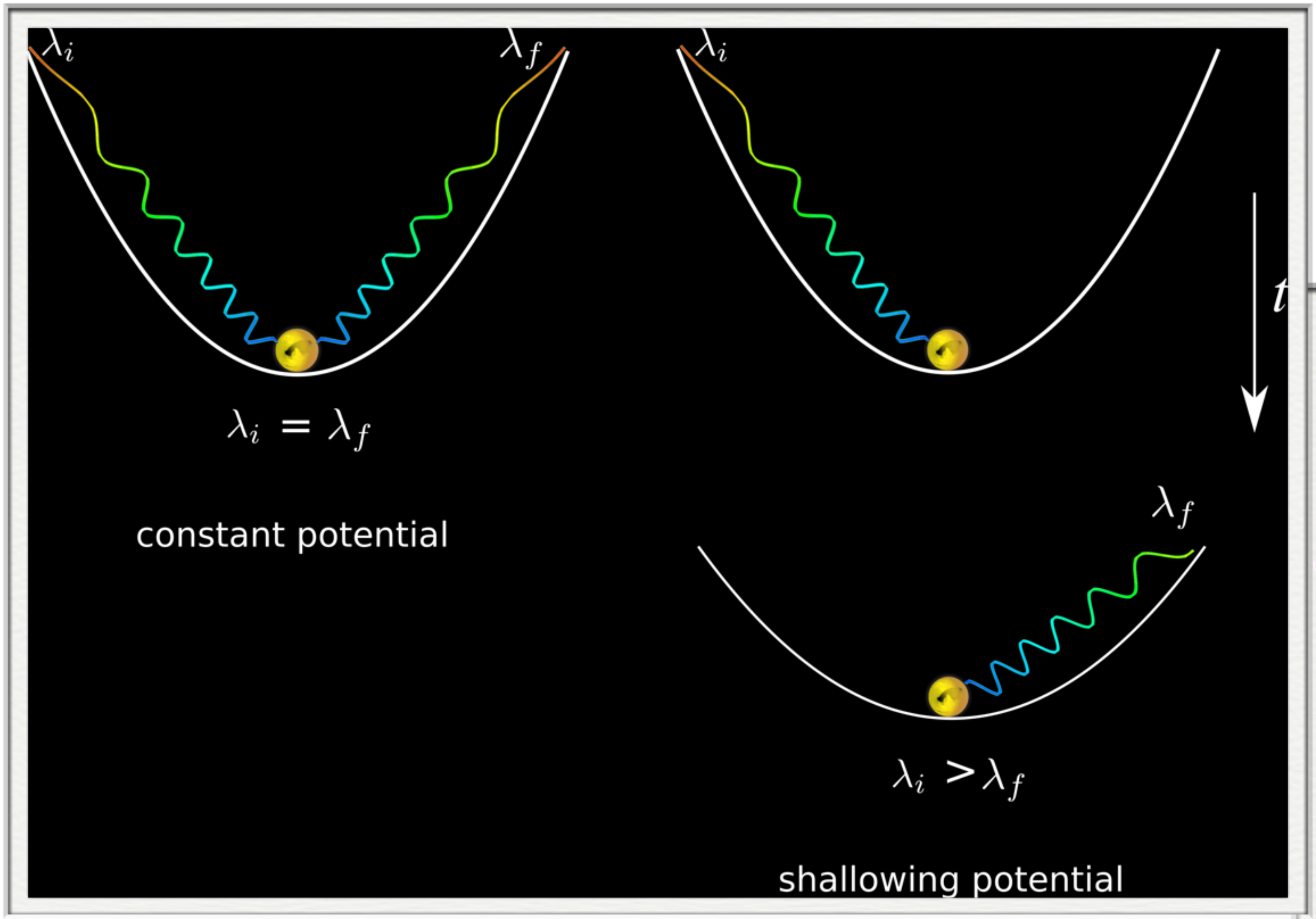
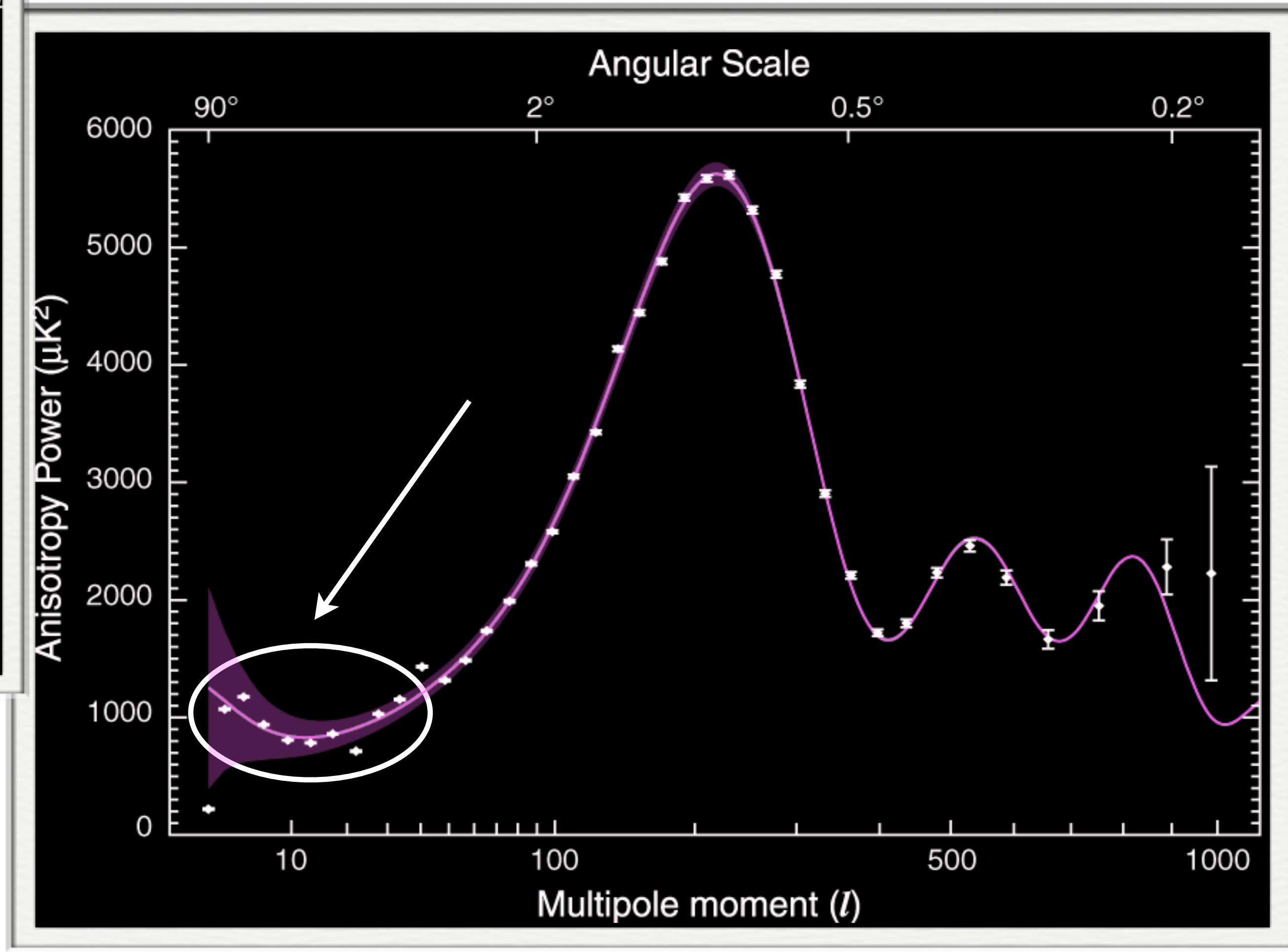
Hence the growth of structure offers a way of testing gravity that is complementary to distance measurements

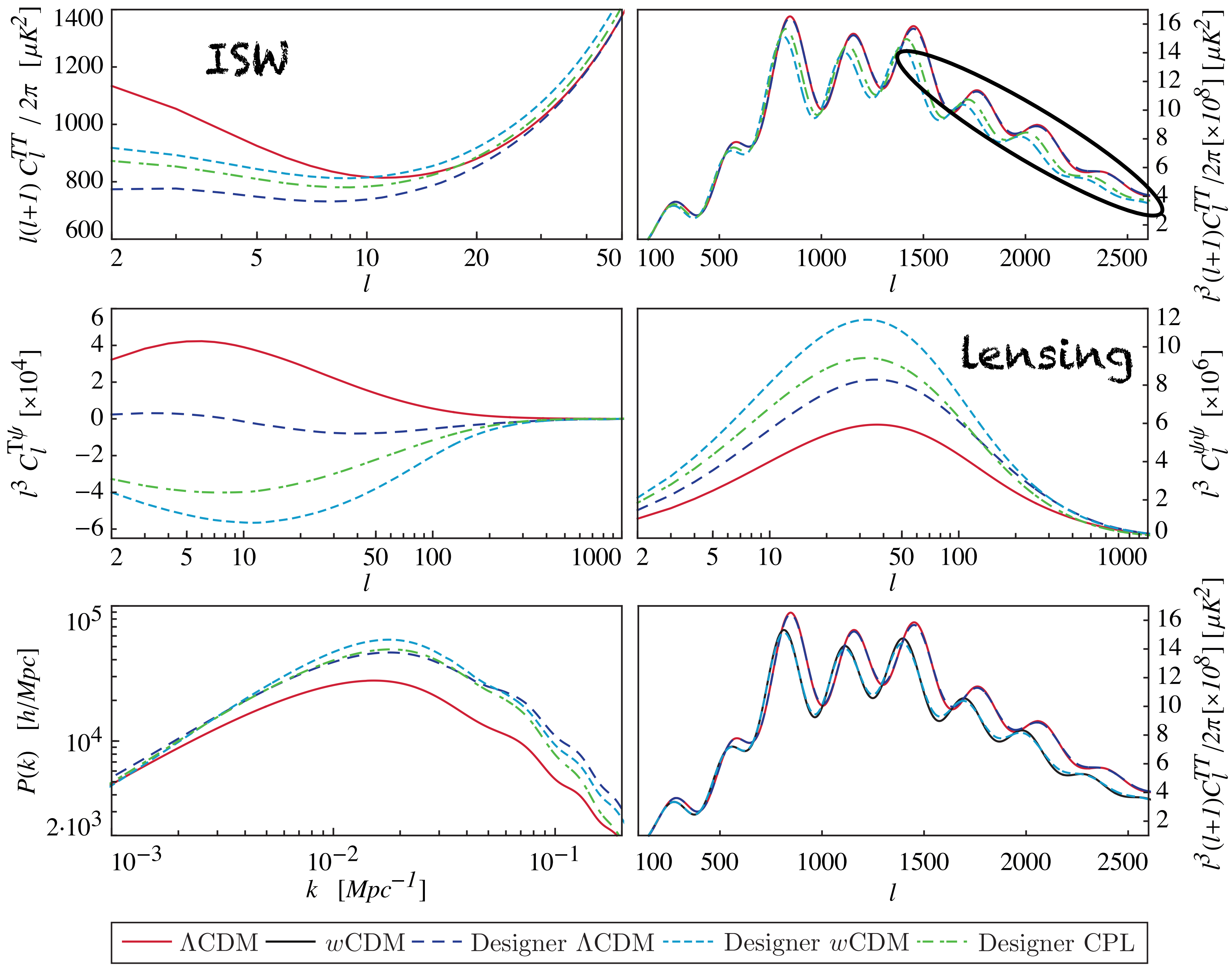
Integrated Sachs-Wolfe effect

$$\Theta_l(k,\eta_0)^{\mathrm{ISW}} = \int e^{-\tau}\left[\dot{\Phi}(k,\eta)+\dot{\Psi}(k,\eta)\right] j_l[k(\eta_0-\eta)]d\eta$$





f(R)



‘Effective Field Theory of DE: an implementation in CAMB’
Phys. Rev. D 89, 103530 (2014)
by Hu, Raveri, Frusciante, A.S.

Signatures on LSS in f(R)

Signatures on LSS in f(R)

$$\boxed{w_{\mathrm{eff}} \approx -1 \qquad \Phi \neq \Psi \qquad \Psi \neq -\frac{a^2}{k^2}\frac{\rho\Delta}{2M_P^2}}$$

Signatures on LSS in f(R)

$$\boxed{w_{\text{eff}} \approx -1 \qquad \Phi \neq \Psi \qquad \Psi \neq -\frac{a^2}{k^2}\frac{\rho\Delta}{2M_P^2}}$$

- relativistic and non-relativistic probes **DO NOT** respond to the same metric potential

- the growth of structure is **scale-dependent**

Signatures on LSS in f(R)

We have learned that in general models of DE/MG the consistency relation btw expansion history and growth of structure as measured by WL, GC, RSD, characteristic of LCDM and smooth DE, is broken.

Overall we observe a scale-dependent pattern of growth

The dynamics of perturbations is richer, and different observables are described by different functions, not by a single growth factor!

Clustering will have a different rate, and as a consequence of the difference between the two metric potentials, lensing and dynamical mass will be different.

Take home message for LSS

...we shall go beyond the expansion history, and combine distance and growth measurements to test:

the relation between matter and gravitational potential $\Psi \leftrightarrow \Delta$

the relation between the gravitational potential and the curvature of space

...and this will be possible with future surveys that will map the evolution of matter perturbations and gravitational potentials from the matter dominated epoch until today.

GW phenomenology

The Einstein equation for transverse-traceless tensors in f(R) gravity reads:

$$\ddot{h}_{ij} + \left(2 + \frac{\dot{f}_R}{\mathcal{H}(1+f_R)}\right)\mathcal{H}\dot{h}_{ij} + k^2 h_{ij} = 0$$

this comes from the non-minimal coupling/rescaling of Newton constant. It is an additional friction term.

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this comes from the non-minimal coupling/rescaling of Newton constant. It is an additional friction term.

see end of Lecture 3 !

It affects the luminosity distance of gravitational waves!