Understanding quantum transport in quasi-periodic lattice systems

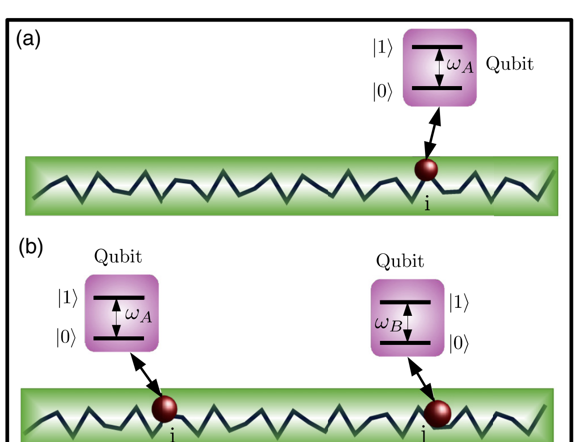
Bijay Kumar Agarwalla
Department of Physics
IISER PUNE, India

IISER PUNE

Collaboration:

Madhumita Saha (IISER Pune)
Prasanna B Venkatesh (IIT Gandhinagar)

Phys. Rev. A 103, 023330 (2021)
arXiv: 2202.14033



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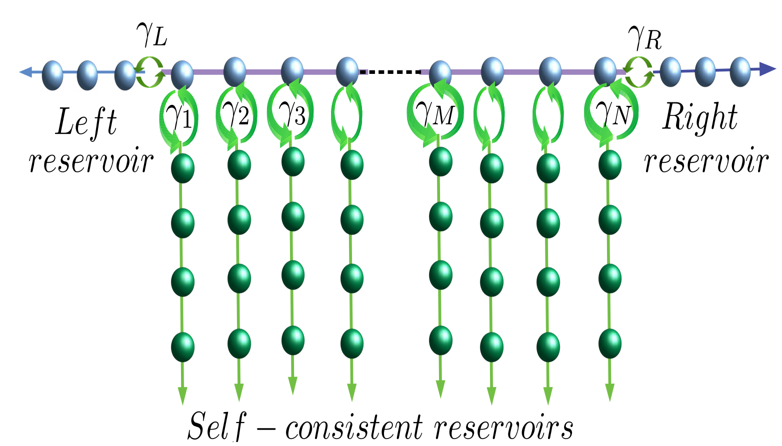


Quasi-Periodic Systems

- Lattice with disordered on-site potential (uncorrelated)

Anderson Localization, Mobility Edge in 3-d

P.W. Anderson Phys. Rev. 109, 1492 (1958)

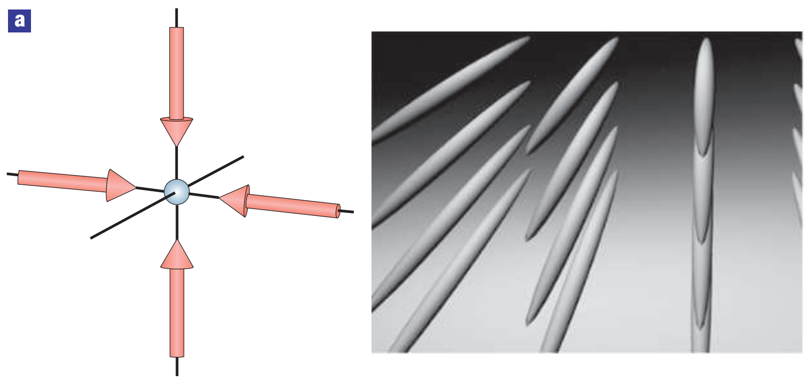
- Quasi-Periodic Systems

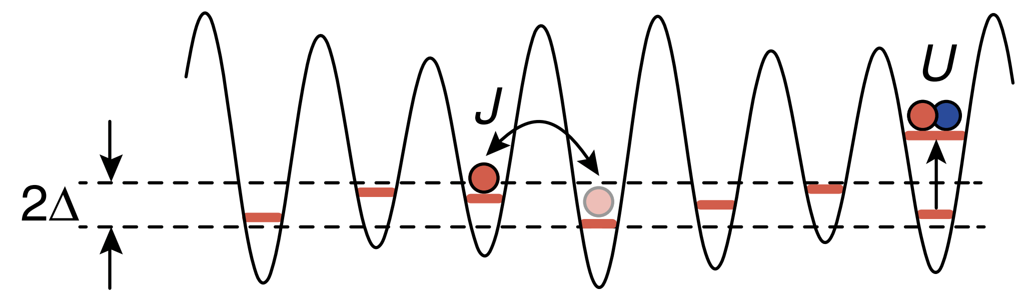
Neither periodic nor disordered systems: e.g. Aubre-André-Harper
Mobility Edge in 1d

S. Aubry and G. André, Ann. Isr. Phys. Soc. 3, 18 (1980);
S. Ganeshan, K. Sun, and S. Das Sarma, Phys. Rev. Lett. 110, 180403 (2013).
S. Ganeshan, J. H. Pixley, and S. Das Sarma, Phys. Rev. Lett. 114, 146601 (2015).

Quasi-Periodic Systems

- Multiple Experimental Realizations





I. Bloch, Nature Physics **1**, 23(2005); G. Roati et.al., Nature **453**, 895(2008);
H.P. Lüschen et.al., Phys. Rev. Lett. **120**,160404 (2018)

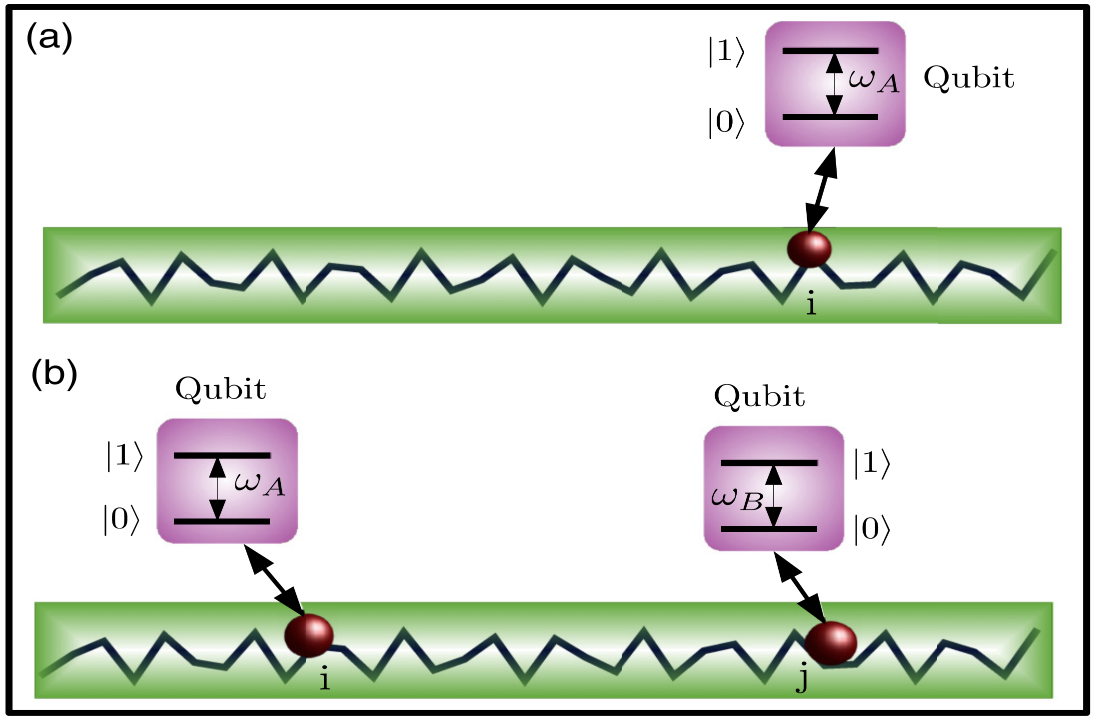
- Connections to Many Body Localization

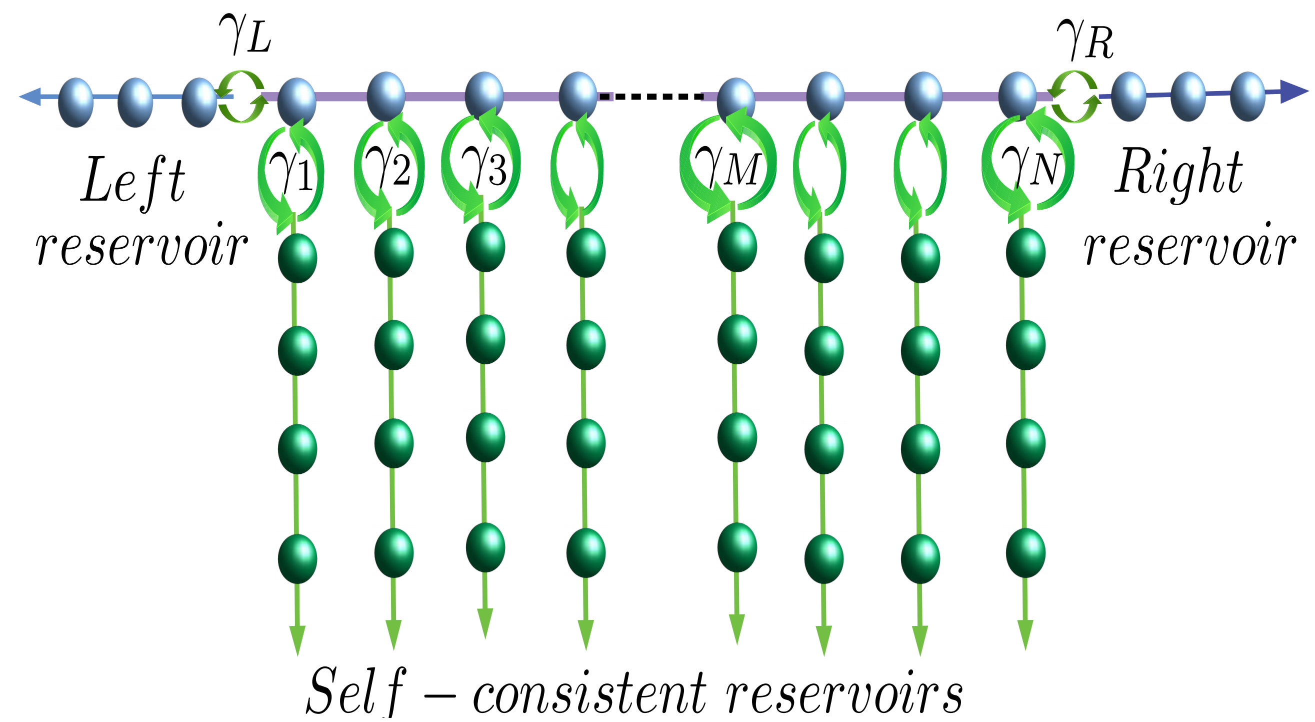
M. Schreiber et.al, Science **49**, 842 (2015); R. Modak and S. Mukerjee, Phys. Rev. Lett. **115**, 230401 (2015).

- Open version of quasi-periodic systems

H.P. Lüschen et.al., Phys. Rev. X **7**, 011034 (2017); J. Sutradhar et.al., Phys. Rev. B **99**, 224204 (2019);
A. Purkayastha et.al., Phys. Rev. B 97, 174206 (2018); Phys. Rev. B 96, 180204(R) (2017)

This Talk



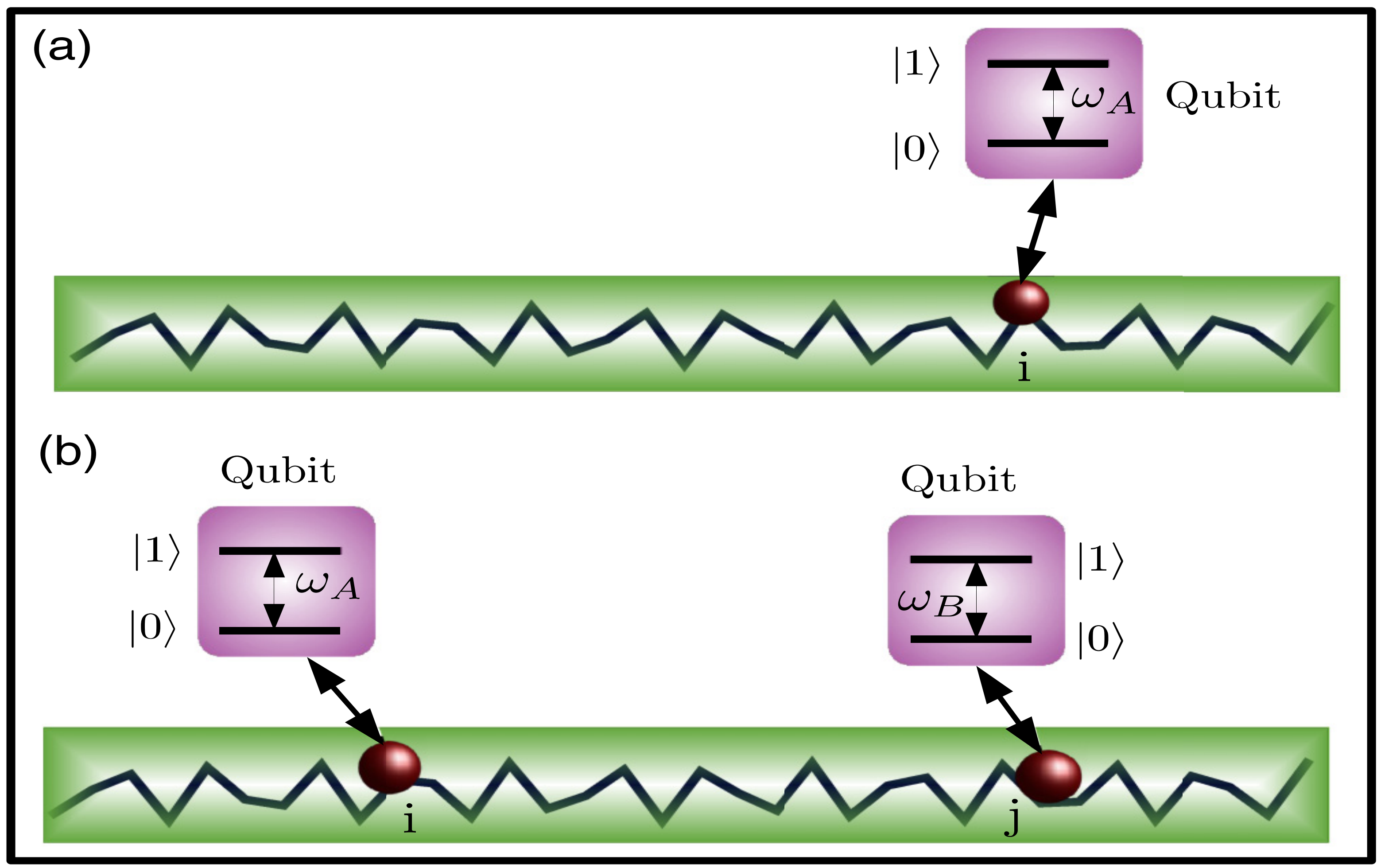


- Couple Qubit(s) to quasi-periodic GAAH lattice
- Read out nature of SPEs
- Read out transport properties

Phys. Rev. A 103, 023330 (2021)

- Transport properties in presence of voltage probes— ***Environment assisted transport***

arXiv: 2202.14033


(a)
|1⟩
|0⟩
ω_A
Qubit
i
(b)
Qubit
|1⟩
|0⟩
ω_A
Qubit
|1⟩
|0⟩
ω_B
i
j

Properties of AAH and GAAH

$$\hat{H} = \sum_{n=1}^{N} \left(\mu + \frac{2\lambda \cos[2\pi bn + \phi]}{1 + \alpha \cos[2\pi bn + \phi]} \right) \hat{c}_n^\dagger \hat{c}_n + \sum_{n=1}^{N-1} (\hat{c}_n^\dagger c_{n+1} + \hat{c}_{n+1}^\dagger \hat{c}_n)$$

Hopping Strength

$J = 1$

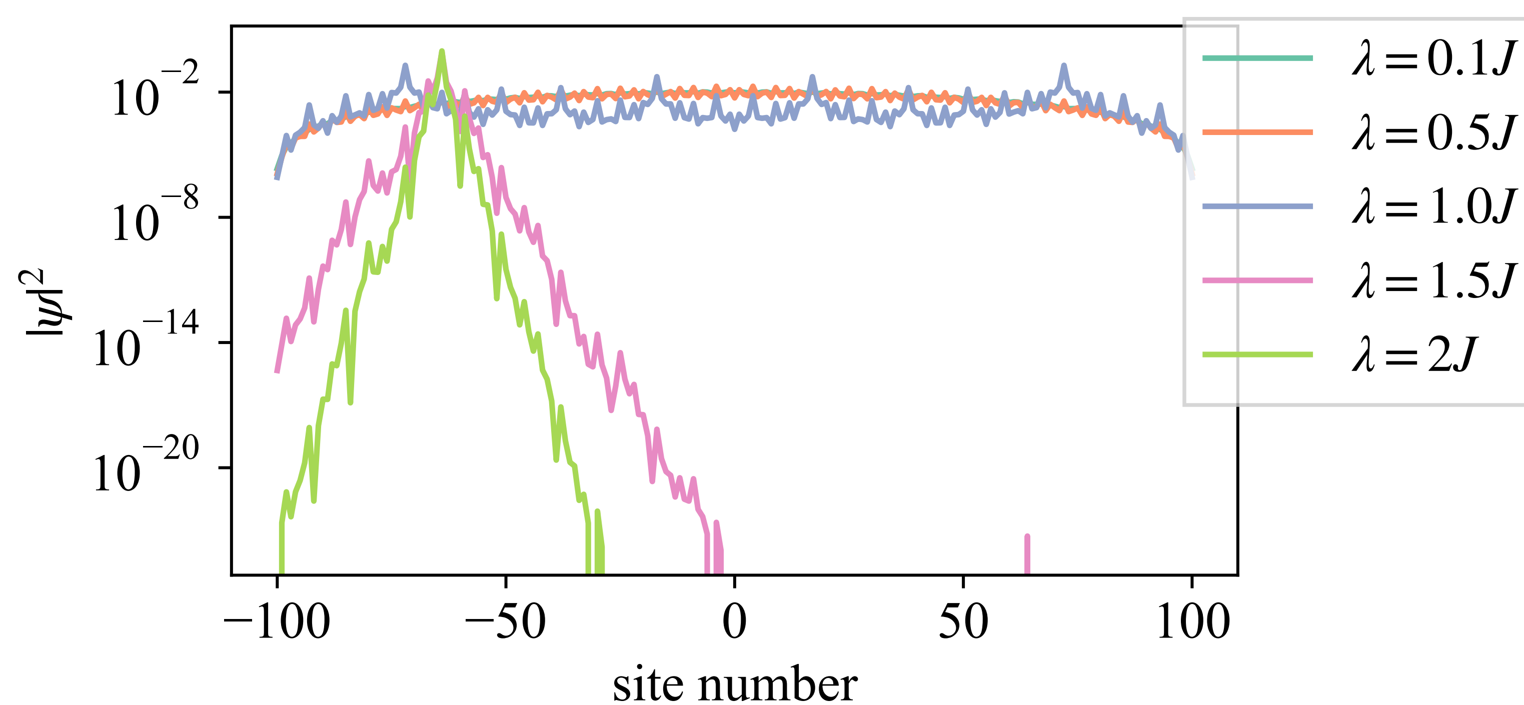
SPEs $\hat{H}\psi_k = \omega_k \psi_k$

AAH Model $\alpha = 0$

$\lambda < J$ All SPEs Extended

$\lambda = J$ All SPEs Critical

$\lambda > J$ All SPEs Localized



Properties of AAH and GAAH

$$\hat{H} = \sum_{n=1}^{N} \left(\mu + \frac{2\lambda \cos[2\pi bn + \phi]}{1 + \alpha \cos[2\pi bn + \phi]} \right) \hat{c}_n^\dagger \hat{c}_n + \sum_{n=1}^{N-1} (\hat{c}_n^\dagger c_{n+1} + \hat{c}_{n+1}^\dagger \hat{c}_n)$$

Hopping Strength

$$J = 1$$

GAAH Model $\alpha > 0,\ \lambda > 0$

$$\hat{H}\psi_k = \omega_k \psi_k$$ SPEs

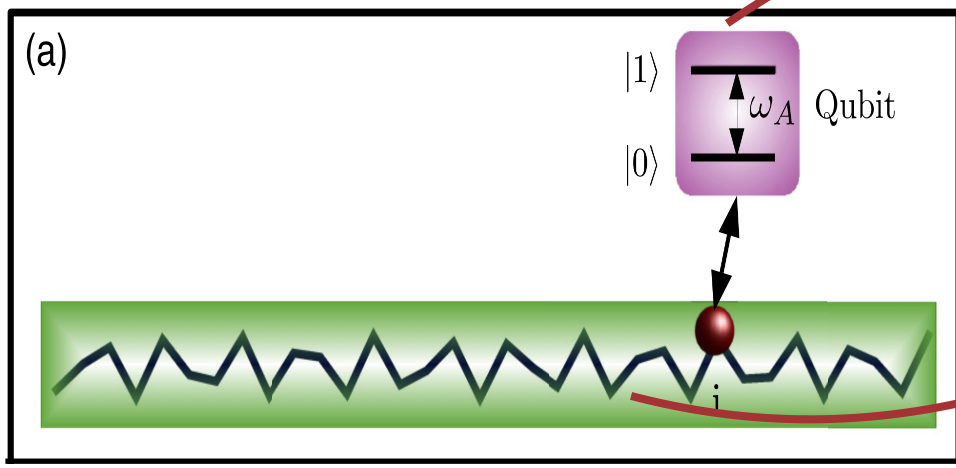
Mobility Edge

$$E = \mu + 2(J - \lambda)/\alpha$$

$\omega_k < E$ Extended

$\omega_k > E$ Localized

Single Qubit Coupled to GAAH Chain



$$\hat{H}_{1q} = \hat{H} + \frac{\omega_A}{2}\hat{\sigma}_z^i + g\hat{\sigma}_z^i(\hat{c}_i^\dagger + \hat{c}_i)$$

$$\hat{H} = \sum_{n=1}^{N}\left(\mu + \frac{2\lambda\cos[2\pi bn + \phi]}{1 + \alpha\cos[2\pi bn + \phi]}\right)\hat{c}_n^\dagger\hat{c}_n + \sum_{n=1}^{N-1}(\hat{c}_n^\dagger c_{n+1} + \hat{c}_{n+1}^\dagger\hat{c}_n)$$

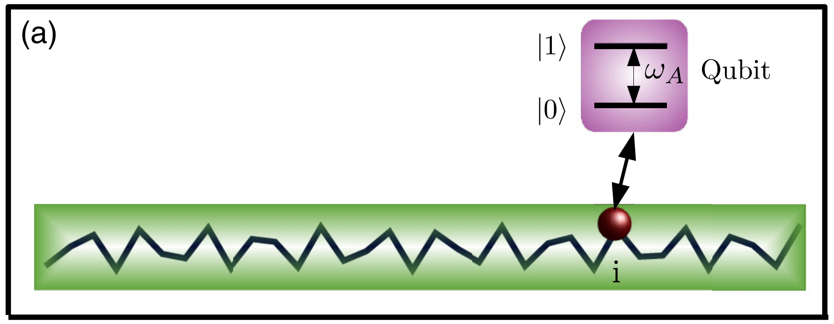
$$\hat{H} = \sum_{k=1}^{N}\omega_k\hat{\eta}_k^\dagger\hat{\eta}_k \qquad \hat{c}_i = \sum_{k=1}^{N} S_{i,k}\hat{\eta}_k \qquad g_k^i = gS_{i,k}$$

$$\hat{H}_{1q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \sum_{k=1}^{N}\omega_k\hat{\eta}_k^\dagger\hat{\eta}_k + \sum_{k=1}^{N}\hat{\sigma}_z^i\left(g_k^i\hat{\eta}_k^\dagger + g_k^{i*}\hat{\eta}_k\right)$$

Spin-Boson Dephasing Coupling - can be exactly solved

Dephasing Spin-Boson model: Solution

$$\hat{c}_i = \sum_{k=1}^{N} S_{i,k} \hat{\eta}_k$$



$$\hat{H}^{SB}_{1q} = \frac{\omega_A}{2} \hat{\sigma}_z^i + \sum_{k=1}^{N} \omega_k \hat{\eta}_k^\dagger \hat{\eta}_k + \sum_{k=1}^{N} \hat{\sigma}_z^i \left(g_k^i \hat{\eta}_k^\dagger + g_k^{i*} \hat{\eta}_k \right)$$

$$\hat{\rho}(0) = \hat{\rho}_s(0) \otimes (e^{-\beta \hat{H}} / Z_\beta) \quad Z_\beta = \mathrm{Tr}_B [e^{-\beta \hat{H}}]$$

$$\langle \hat{\sigma}_-^i(t) \rangle = \langle \hat{\sigma}_-^i(0) \rangle e^{-i\omega_A t - \Gamma_i(t)},$$

$$\Gamma_i(t) = 4 \sum_{k=1}^{N} \left| g_k^i \right|^2 \coth\left(\frac{\beta \omega_k}{2} \right) \frac{1 - \cos(\omega_k t)}{\omega_k^2}.$$

$$g_k^i = g S_{i,k}$$

$$\Gamma_i(t) = \Gamma_{i,\mathrm{vac}}(t) + \Gamma_{i,\mathrm{th}}(t)$$

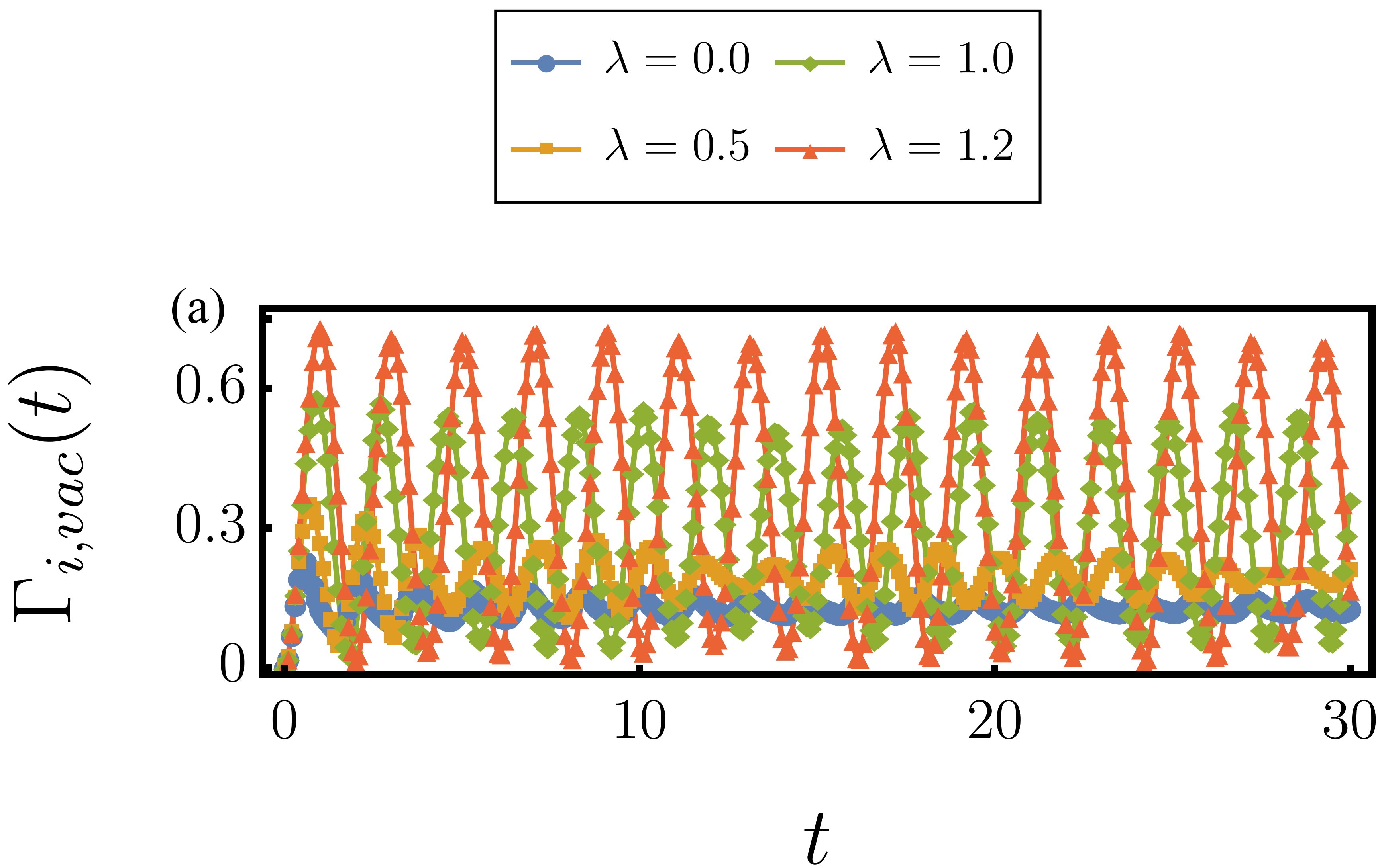
$$\Gamma_{i,\mathrm{vac}}(t) = \sum_{k=1}^{N} \frac{|\alpha_k^i|^2}{2}$$

$$\Gamma_{i,\mathrm{th}}(t) = \sum_{k=1}^{N} \bar{n}(\beta, \omega_k) |\alpha_k^i|^2$$

Results: Single Qubit Dephasing Dynamics

$$\langle \hat{\sigma}_-^i(t) \rangle = \langle \hat{\sigma}_-^i(0) \rangle e^{-i\omega_A t - \Gamma_i(t)}$$

$$\Gamma_i(t) = 4 \sum_{k=1}^{N} \left|g_k^i\right|^2 \coth\left(\frac{\beta\omega_k}{2}\right) \frac{1 - \cos(\omega_k t)}{\omega_k^2}$$



AAH Model
N = 610

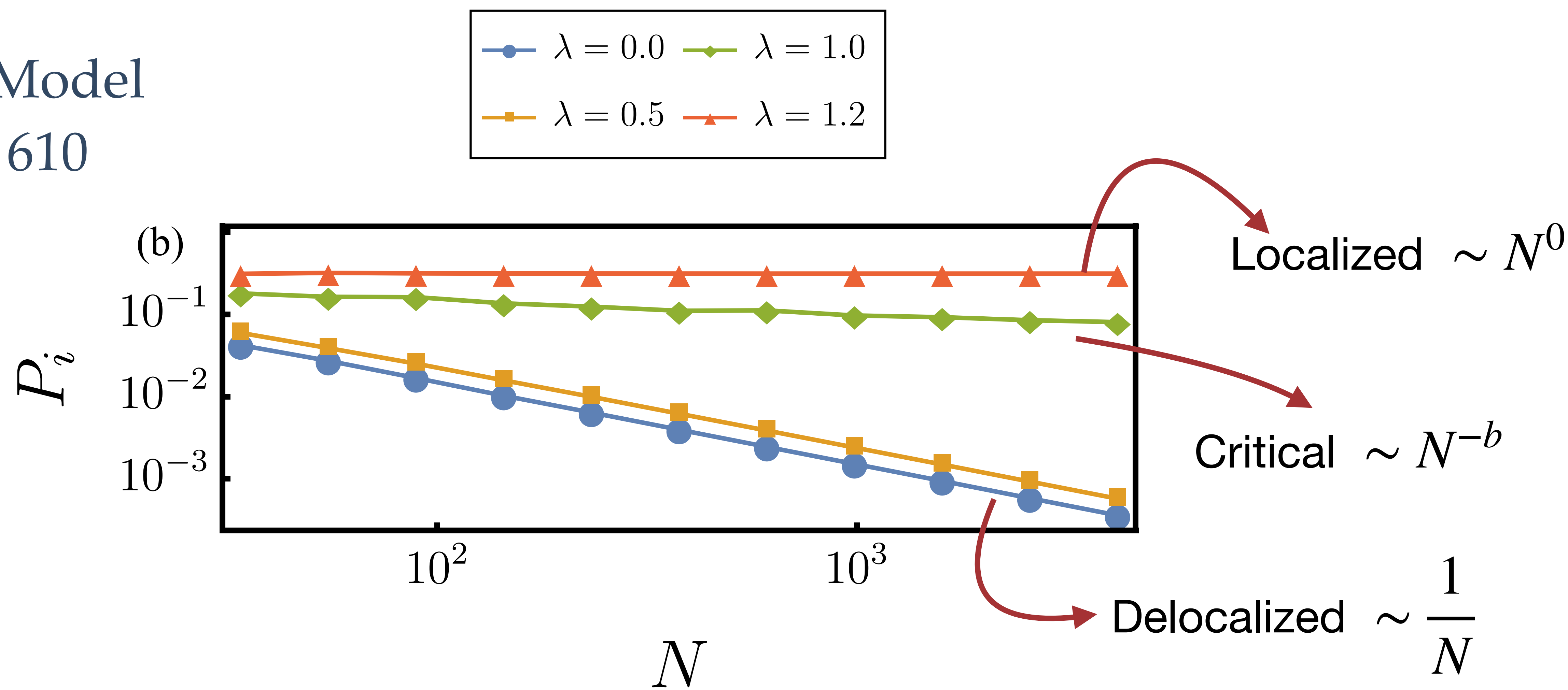
Single Qubit Dephasing Dynamics

$$\langle \hat{\sigma}_-^i(t) \rangle = \langle \hat{\sigma}_-^i(0) \rangle e^{-i\omega_A t - \Gamma_i(t)}$$

$$\Gamma_i(t) = 4 \sum_{k=1}^{N} |g_k^i|^2 \coth\left(\frac{\beta \omega_k}{2}\right) \frac{1 - \cos(\omega_k t)}{\omega_k^2}$$

$$P_i \equiv \sum_{k=1}^{N} |g_k^i|^4$$

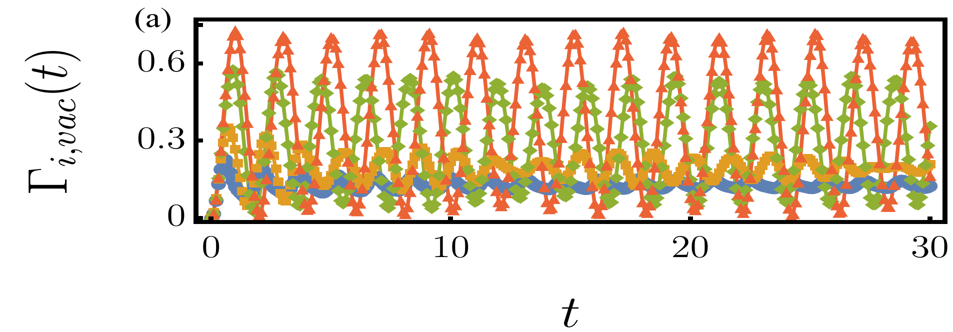
AAH Model
N = 610



Non-Markovianity as SPE indicator

$$\langle \hat{\sigma}^{i}_{-}(t) \rangle = \langle \hat{\sigma}^{i}_{-}(0) \rangle e^{-i\omega_A t - \Gamma_i(t)}$$

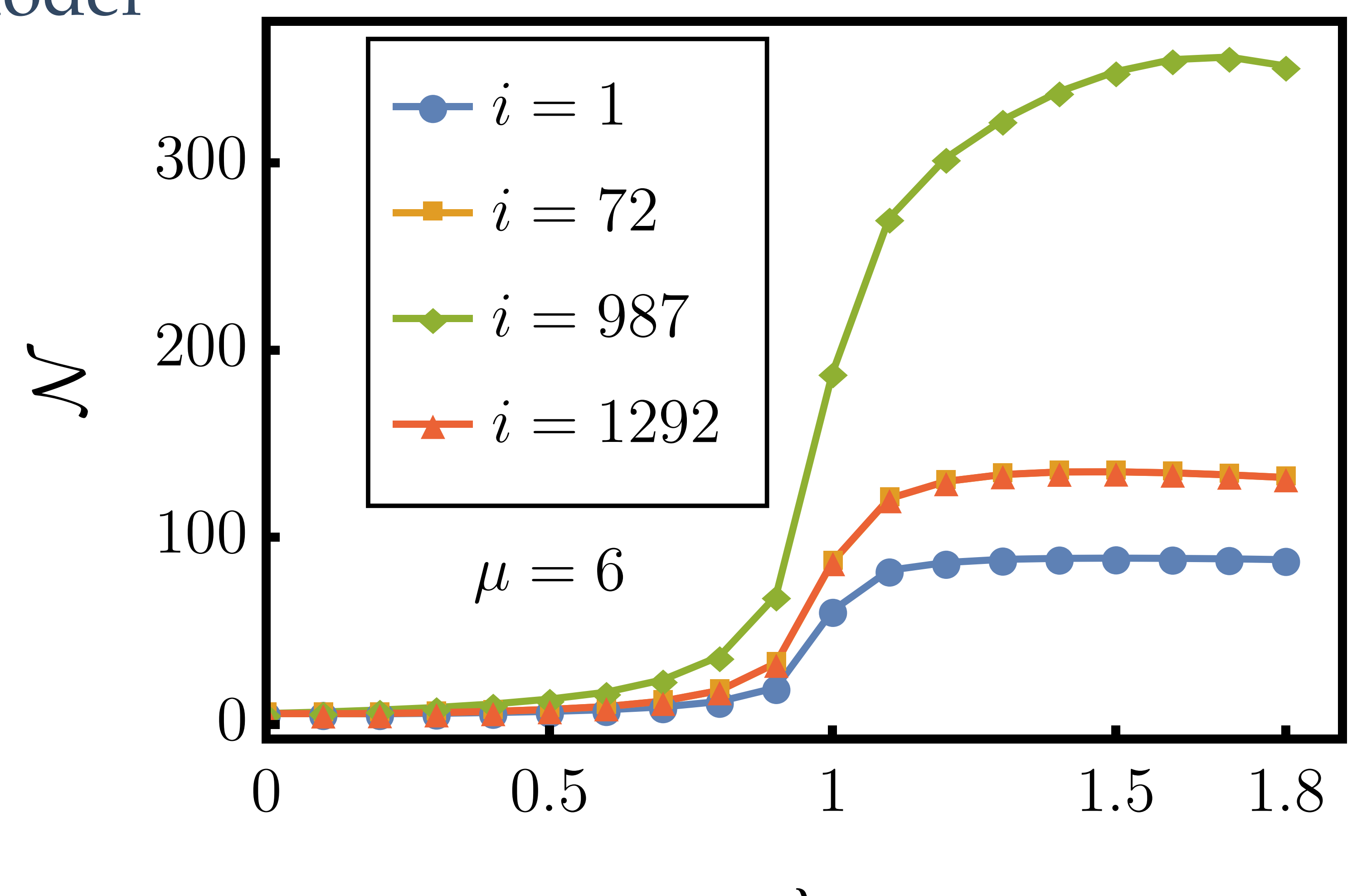
$$\Gamma_i(t) = 4 \sum_{k=1}^{N} \left| g^i_k \right|^2 \coth\left(\frac{\beta \omega_k}{2}\right) \frac{1 - \cos(\omega_k t)}{\omega_k^2}$$



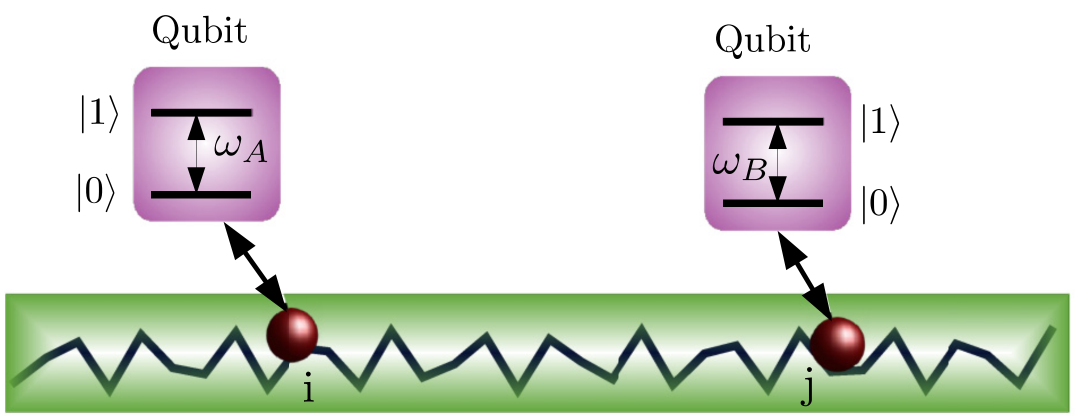
Sum is over all intervals $[t_p^i, t_p^f]$ wherein $\dot{\Gamma}(t) < 0$

$$\mathcal{N} = \sum_{p=1}^{N_{\max}} \left(e^{-\Gamma(t_p^f)} - e^{-\Gamma(t_p^i)} \right)$$

AAH Model



Two Qubits Coupled to GAAH



Motivation: non-local probe for transport

localized initial state width evolution

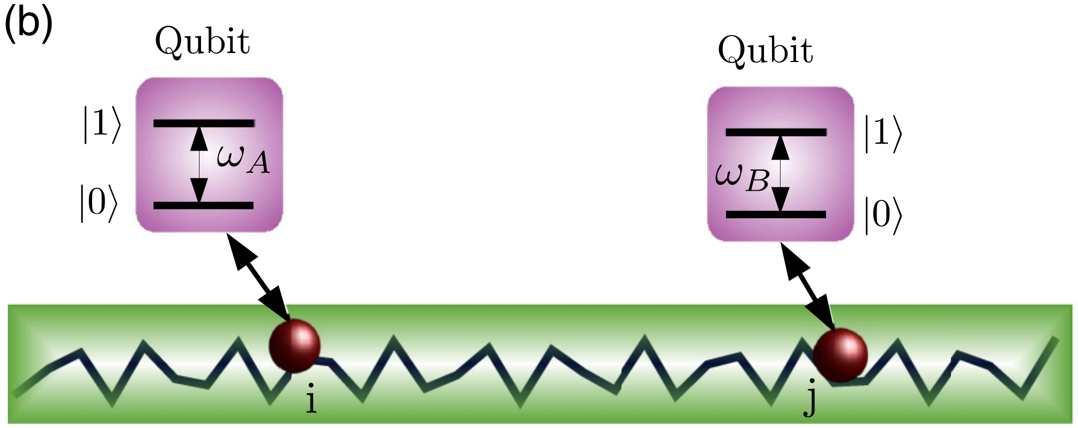
$$w(t) \sim t^{\eta} \begin{cases} \eta = 1 & \text{Ballistic} \\ \eta = 0.5 & \text{Diffusive} \\ \eta = 0 & \text{Localized} \end{cases}$$

$$\hat{H}_{2q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \frac{\omega_B}{2}\hat{\sigma}_z^j + \sum_{k=1}^{N} \omega_k \hat{\eta}_k^\dagger \hat{\eta}_k$$

$$+ \sum_{k=1}^{N} \hat{\sigma}_z^i \left(g_k^i \hat{\eta}_k^\dagger + g_k^{i*} \hat{\eta}_k\right) + \sum_{k=1}^{N} \hat{\sigma}_z^j \left(g_k^j \hat{\eta}_k^\dagger + g_k^{j*} \hat{\eta}_k\right)$$

Two Qubits Dynamics Solution

(b)



$$\hat{H}^{SB}_{2q} = \frac{\omega_A}{2}\hat{\sigma}^i_z + \frac{\omega_B}{2}\hat{\sigma}^j_z + \sum_{k=1}^{N}\omega_k\hat{\eta}^\dagger_k\hat{\eta}_k + \sum_{k=1}^{N}\hat{\sigma}^i_z\left(g^i_k\hat{\eta}^\dagger_k + g^{i*}_k\hat{\eta}_k\right) + \sum_{k=1}^{N}\hat{\sigma}^j_z\left(g^j_k\hat{\eta}^\dagger_k + g^{j*}_k\hat{\eta}_k\right)$$

$$\mathrm{Cov}(\hat{\sigma}^i_-\hat{\sigma}^j_+) = e^{-i(\omega_A-\omega_B)t}\Big(\langle\hat{\sigma}^i_-(0)\hat{\sigma}^j_+(0)\rangle e^{-\Gamma_{ij}(t)} - \langle\hat{\sigma}^i_-(0)\rangle\langle\hat{\sigma}^j_+(0)\rangle\left\langle e^{-i\Delta\Omega_-(t)\hat{\sigma}^j_z + i\Delta\Omega_+(t)\hat{\sigma}^i_z}\right\rangle e^{-[\Gamma_i(t)+\Gamma_j(t)]}\Big)$$

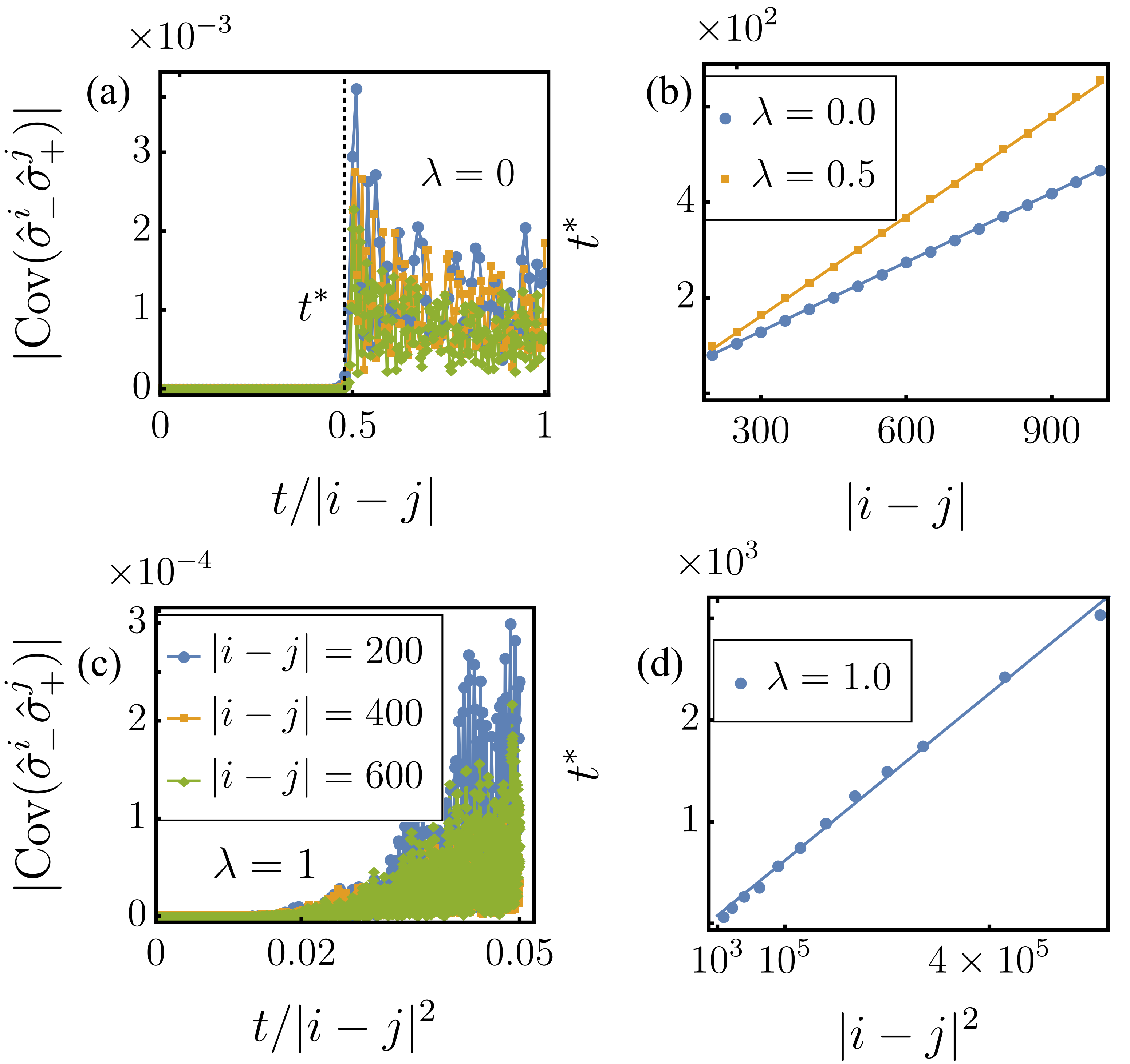
$$\Delta\Omega_\pm(t) = \sum_{k=1}^{N}\frac{4}{\omega_k^2}\left([\sin(\omega_k t) - \omega_k t]\,\mathrm{Re}\left[g^i_k g^{j*}_k\right] \pm [1-\cos(\omega_k t)]\,\mathrm{Im}\left[g^i_k g^{j*}_k\right]\right).$$

Gen. Lamb Shift

$$\Gamma_{ij}(t) = 4\sum_{k=1}^{N}\left|g^i_k - g^j_k\right|^2 \coth\left(\frac{\beta\omega_k}{2}\right)\frac{1-\cos(\omega_k t)}{\omega_k^2}$$

Correlated Dephasing

Transport Readout: Two qubit correlations



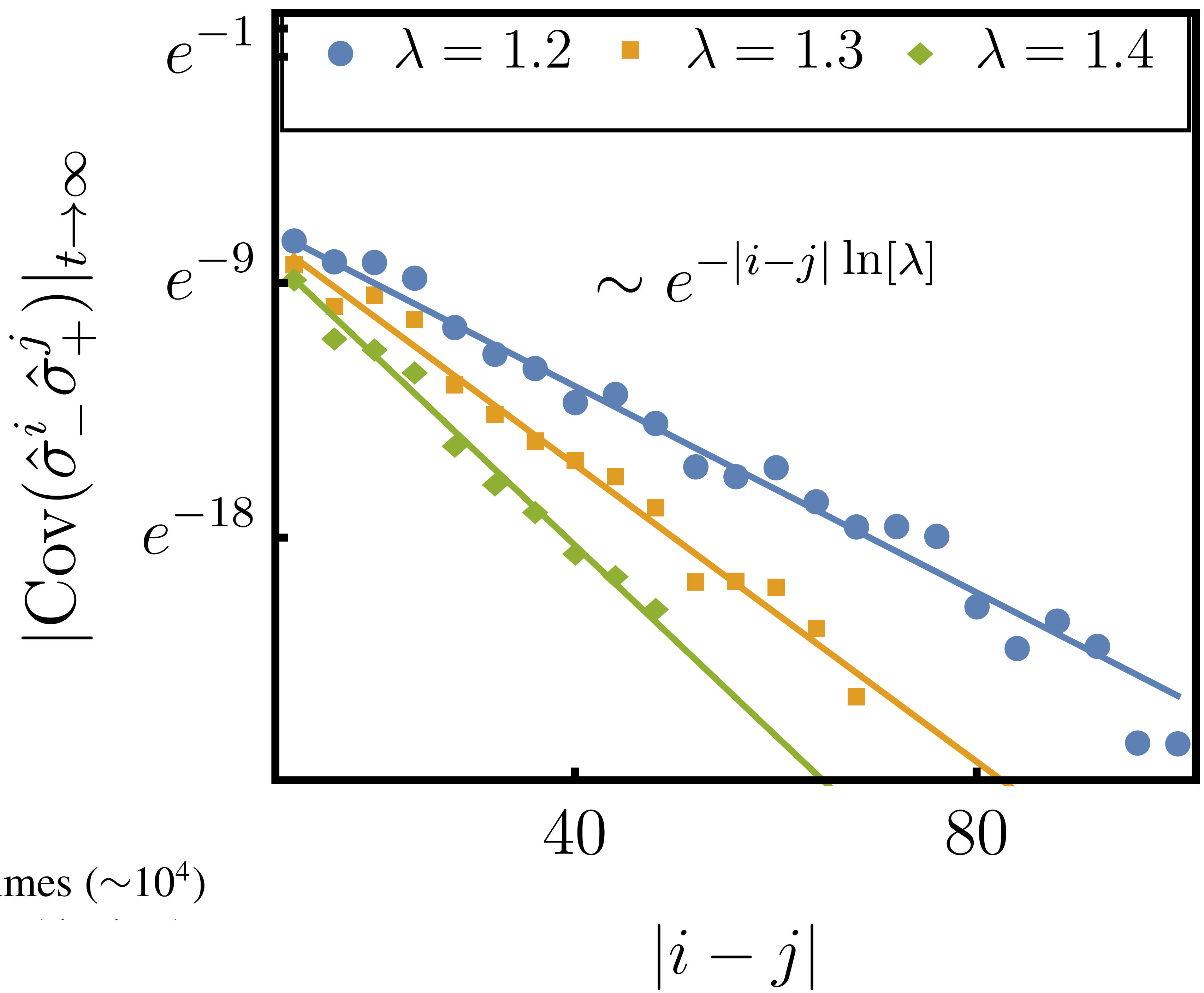
Ballistic

Diffusive

$N = 400$, 800, and 1200

$i = N/4 \quad j = 3N/4$

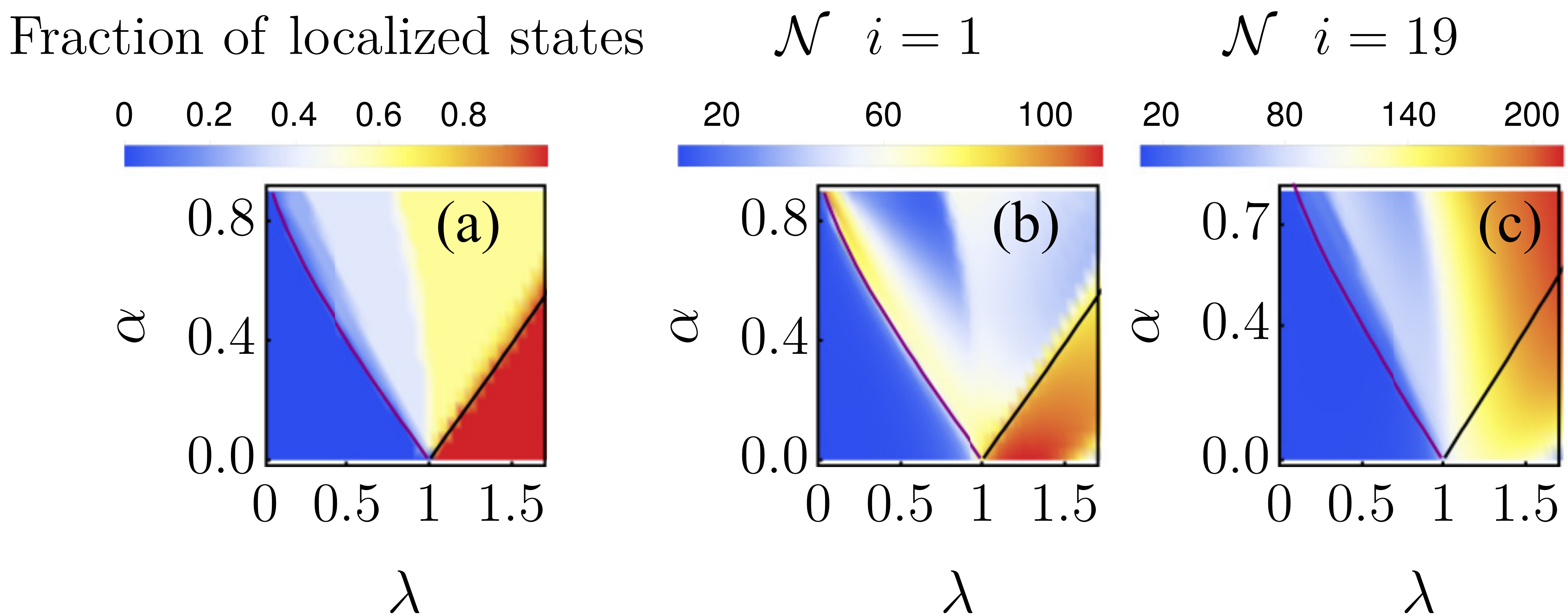
Correlations: Localized Regime



long times ($\sim 10^4$)

$\xi = 1/\ln[\lambda]$

Localization Length Readout

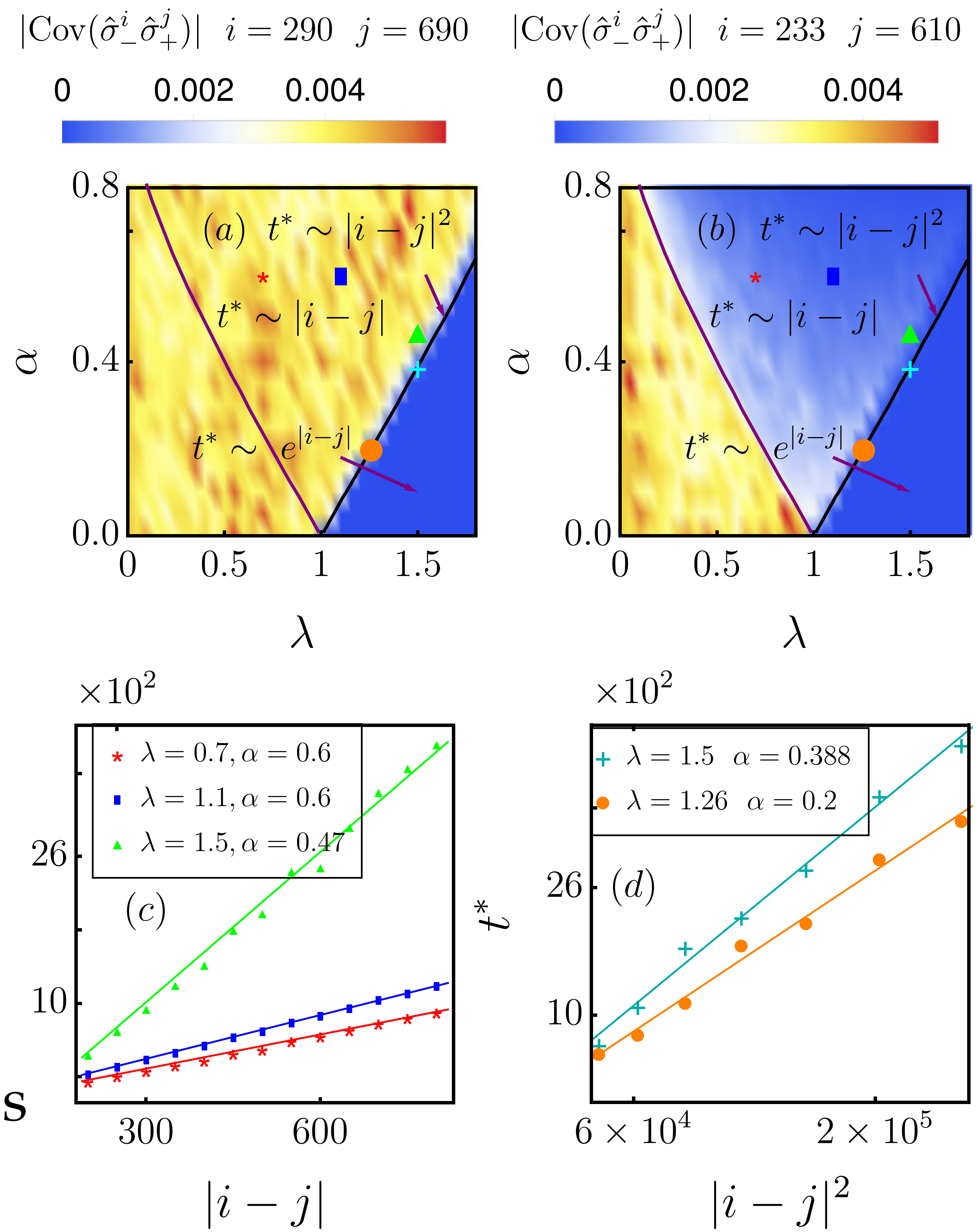


with mobility edge: site dependence of $\mathcal{N}$

$E = \mu + 2(J - \lambda)/\alpha$

Generalized AAH Model: Two qubit Correlations

long-time correlations



site-dependence with mobility edge

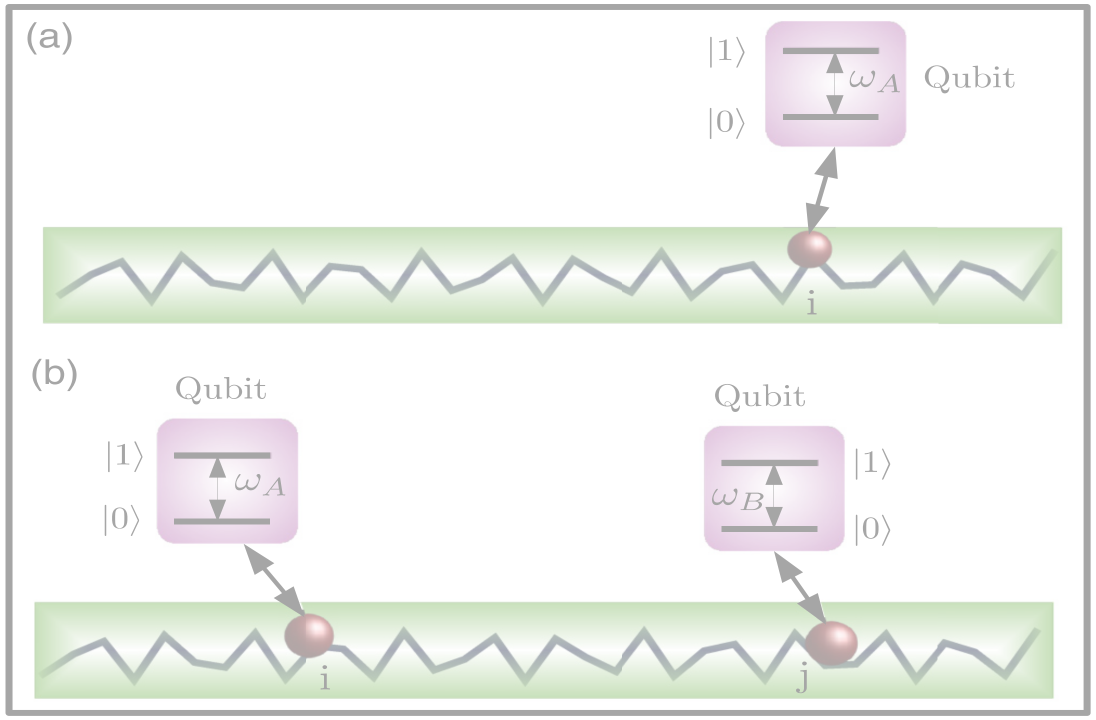
slope increases with λ

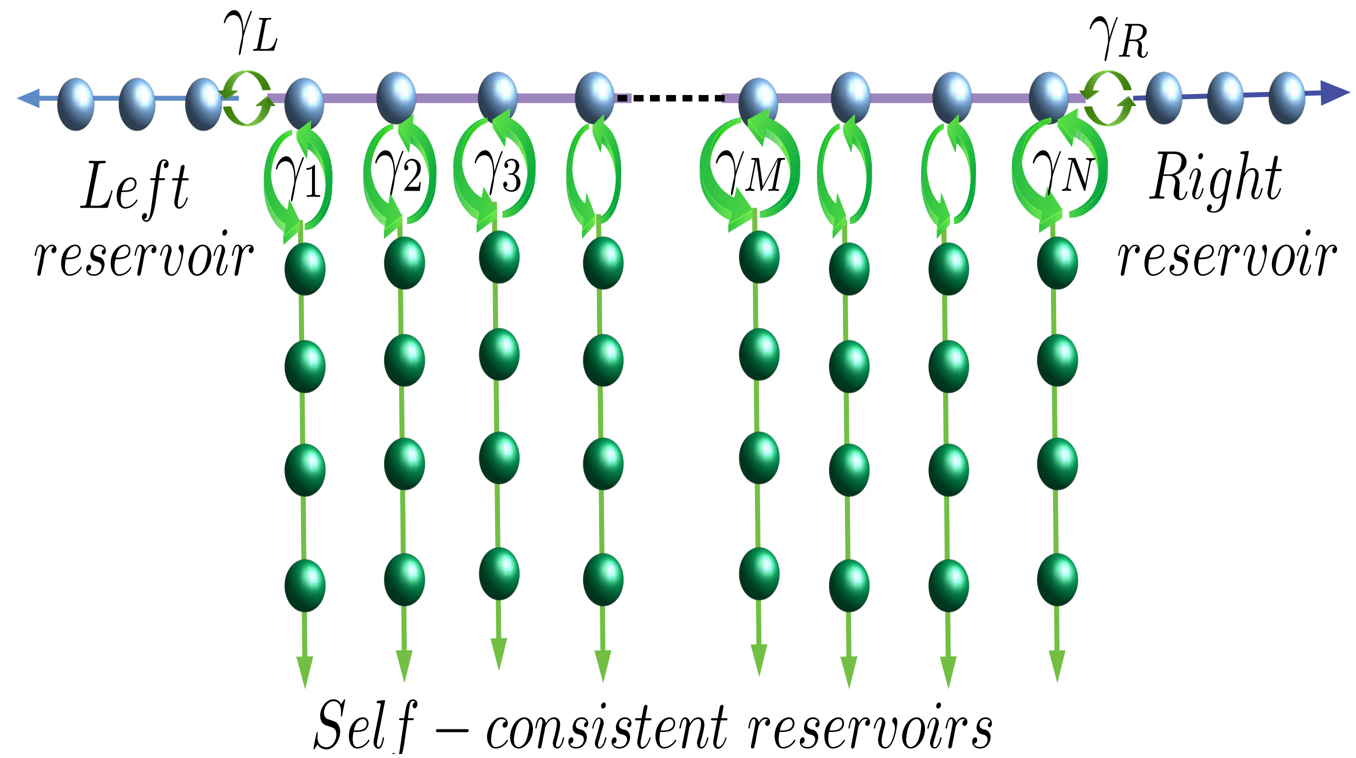
mixture of critical+localized

Summary Part 1

- Readout of GAAH chain by coupling to qubits
- Single Qubit: Non-markovianity of dephasing, nature of SPEs
- Two Qubits: Transport properties from correlations
- Experimental Implementation: Single qubit good prospect with ultracold atoms.
- Non-dephasing coupling: AAH and GAAH Bath Thermodynamics, readout of current (direct signature of transport)
- Back-action of qubit on chain?

This Talk





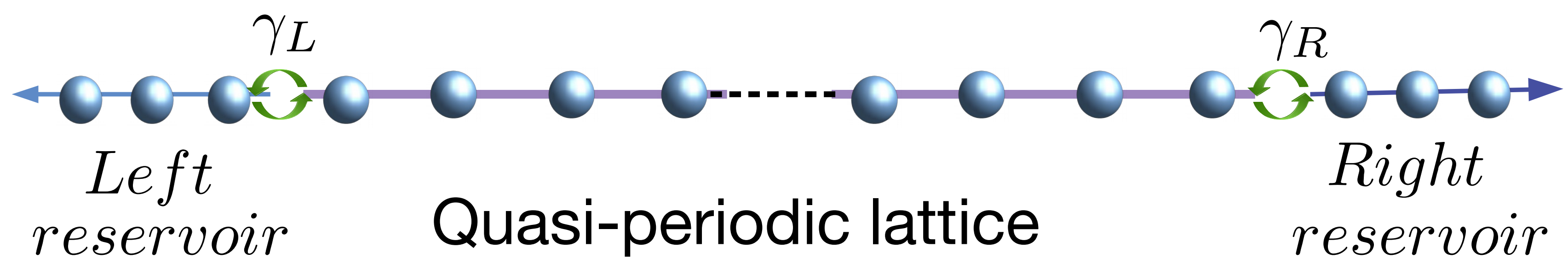
- Couple Qubit(s) to quasi-periodic GAAH lattice
- Read out nature of SPEs
- Read out transport properties

Phys. Rev. A 103, 023330 (2021)

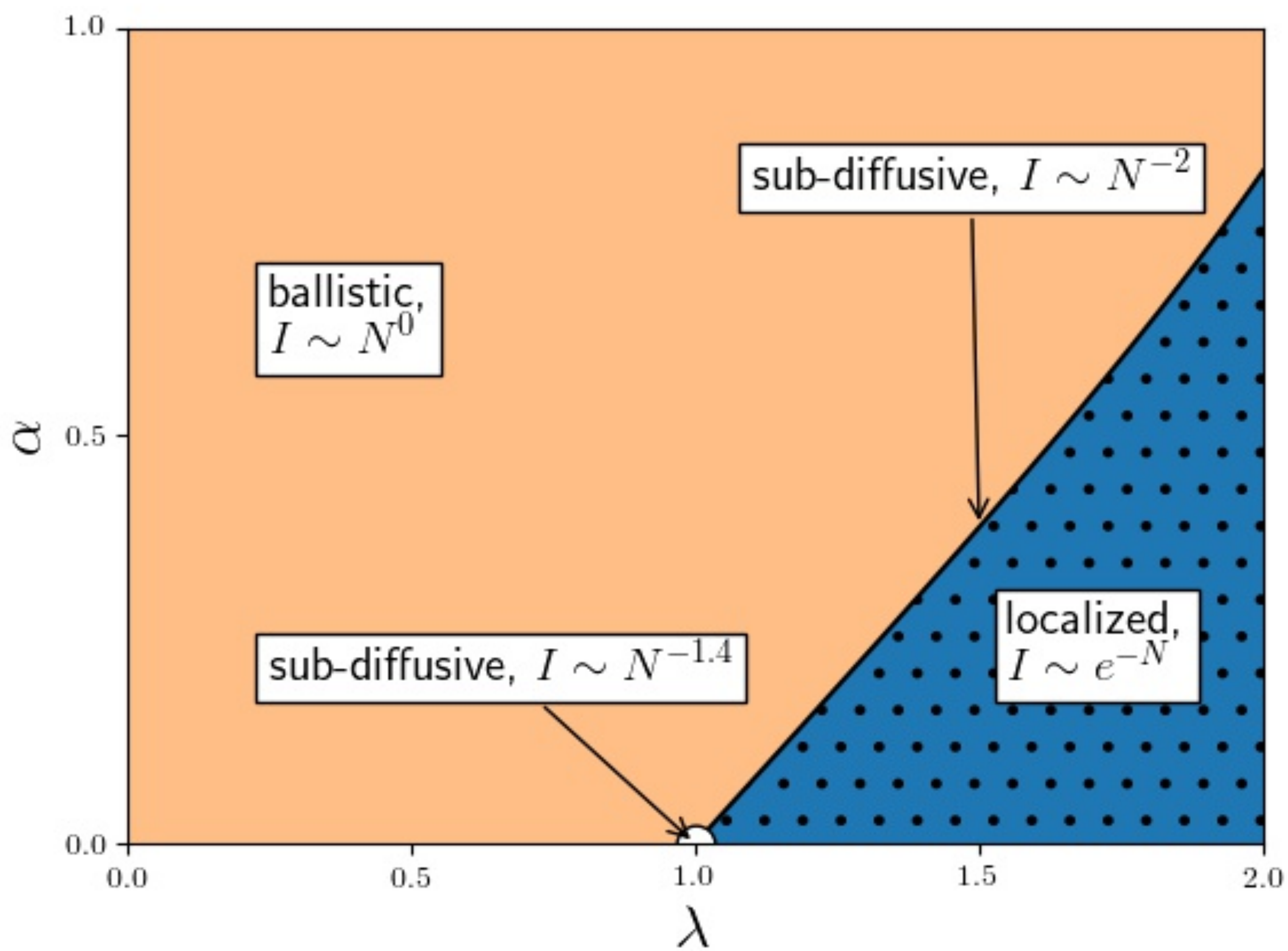
- Transport properties in presence of voltage probes— *Environment assisted transport*

arXiv: 2202.14033

Known results without the probes for GAAH



$$\hat{H} = \sum_{n=1}^{N} \left(\mu + \frac{2\lambda \cos[2\pi bn + \phi]}{1 + \alpha \cos[2\pi bn + \phi]} \right) \hat{c}_n^\dagger \hat{c}_n + \sum_{n=1}^{N-1} (\hat{c}_n^\dagger c_{n+1} + \hat{c}_{n+1}^\dagger \hat{c}_n)$$



Non-equilibrium phase diagram: Scaling of current with system size

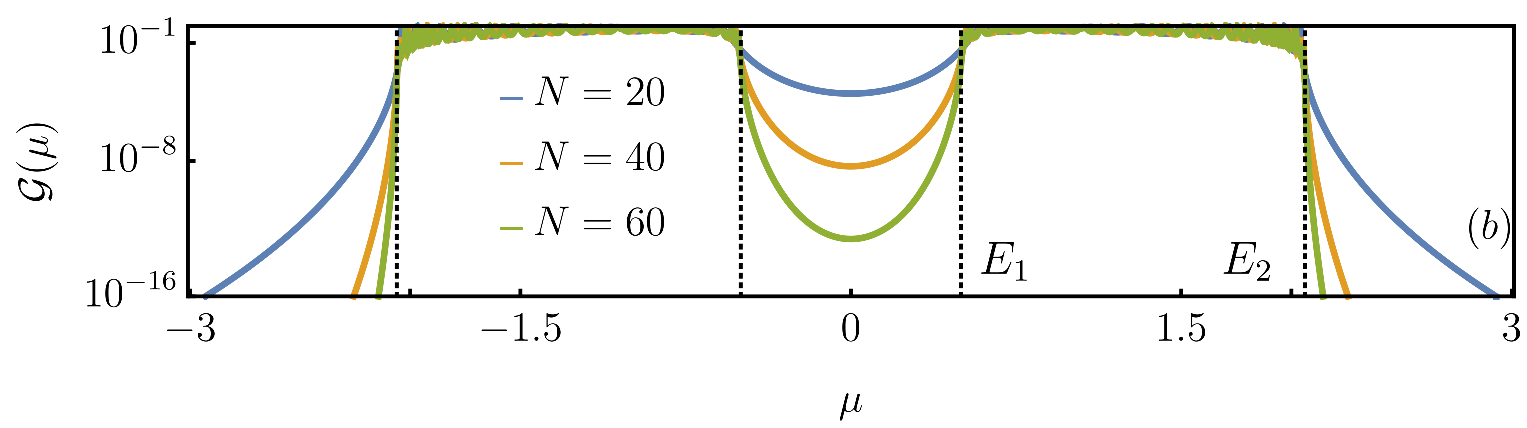
Phys. Rev. B **96**, 180204(R) (2017)

Universal subdiffusive behavior at band edges from transfer matrix exceptional points

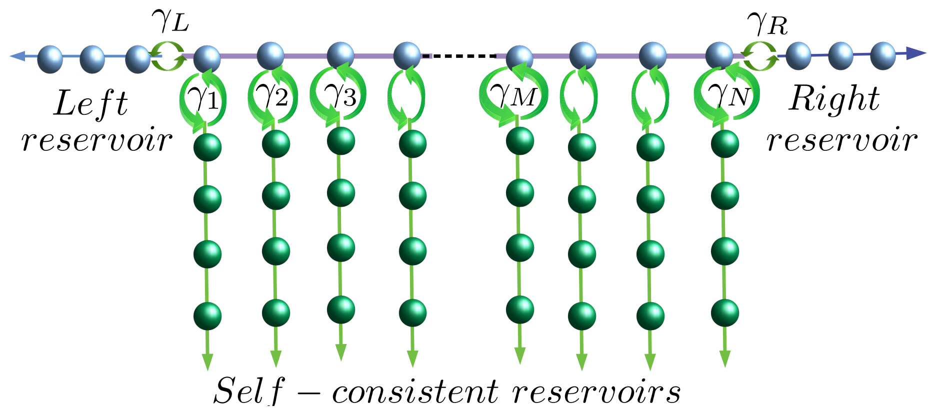
Madhumita Saha,[1,*] Bijay Kumar Agarwalla,[1,†] Manas Kulkarni,[2,‡] and Archak Purkayastha[3,4,§]
[1] *Department of Physics, Indian Institute of Science Education and Research Pune, Dr. Homi Bhabha Road, Ward No. 8, NCL Colony, Pashan, Pune, Maharashtra 411008, India*
[2] *International Centre for Theoretical Sciences, Tata Institute of Fundamental Research, Bangalore 560089, India*
[3] *School of Physics, Trinity College Dublin, Dublin 2, Ireland*
[4] *Centre for complex quantum systems, Aarhus University, Nordre Ringgade 1, 8000 Aarhus C, Denmark*
(Dated: May 5, 2022)

arXiv: 2205.02214

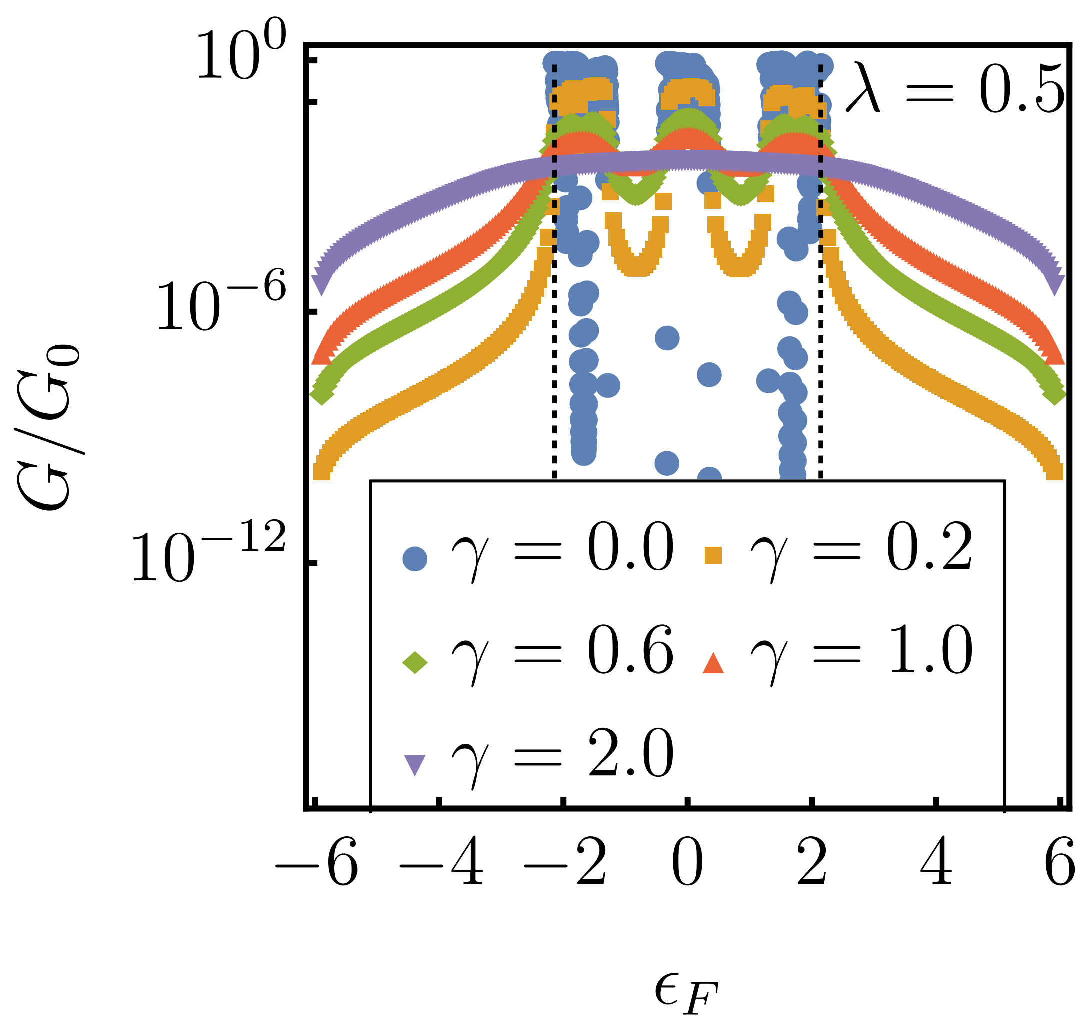
	$T_q(\mu)$	$\mathcal{G}(\mu)$
μ in system bands	$\lambda_\pm = e^{\pm ik}$ symmetry broken regime	$\mathcal{G}(\mu) \sim N^0$ ballistic transport
μ outside system bands	$\lambda_\pm = e^{\pm \kappa}$ symmetric regime	$\mathcal{G}(\mu) \sim e^{-N/\xi}$ 'localized' localization length ξ
μ at band edges	$\lambda_\pm = 1$ exceptional point	$\mathcal{G}(\mu) \sim N^{-2}$ subdiffusive transport

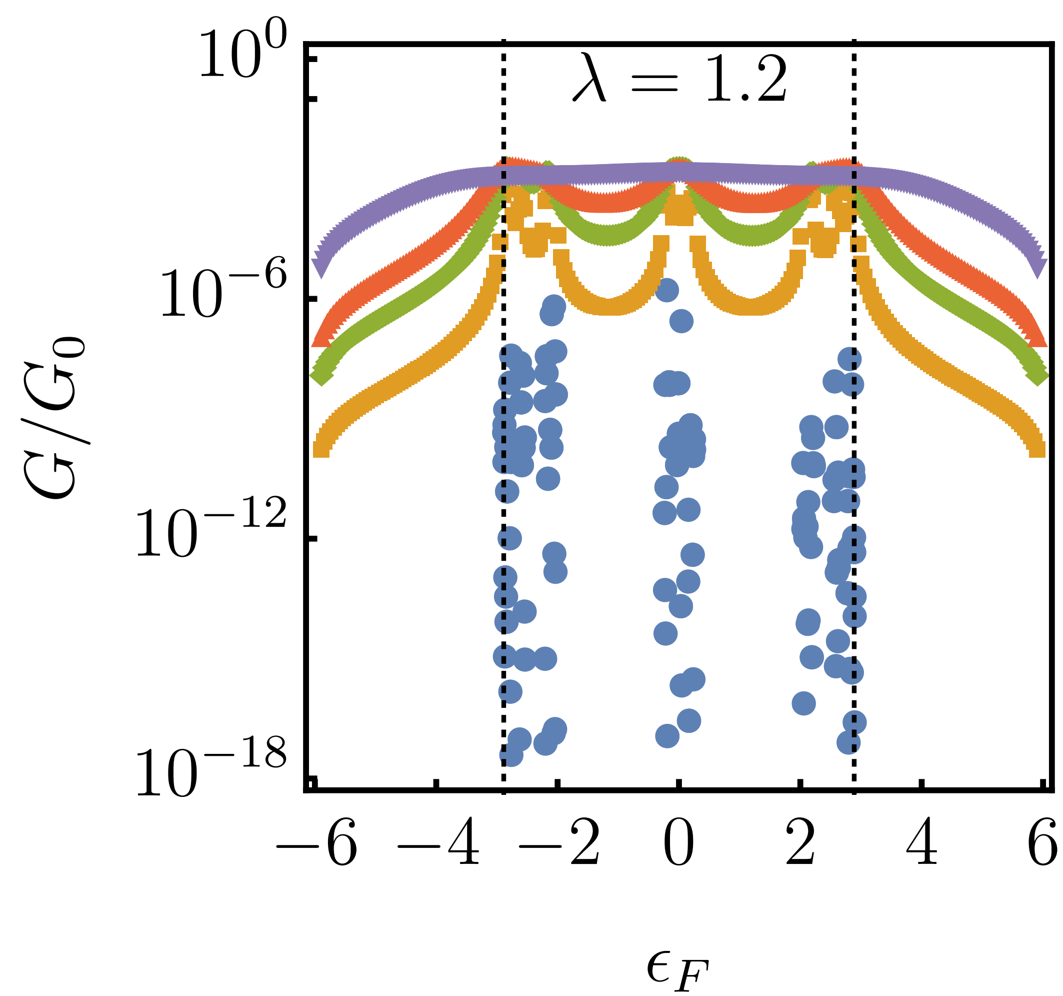


Electrical Conductance: Zero temperature



Results for AAH model

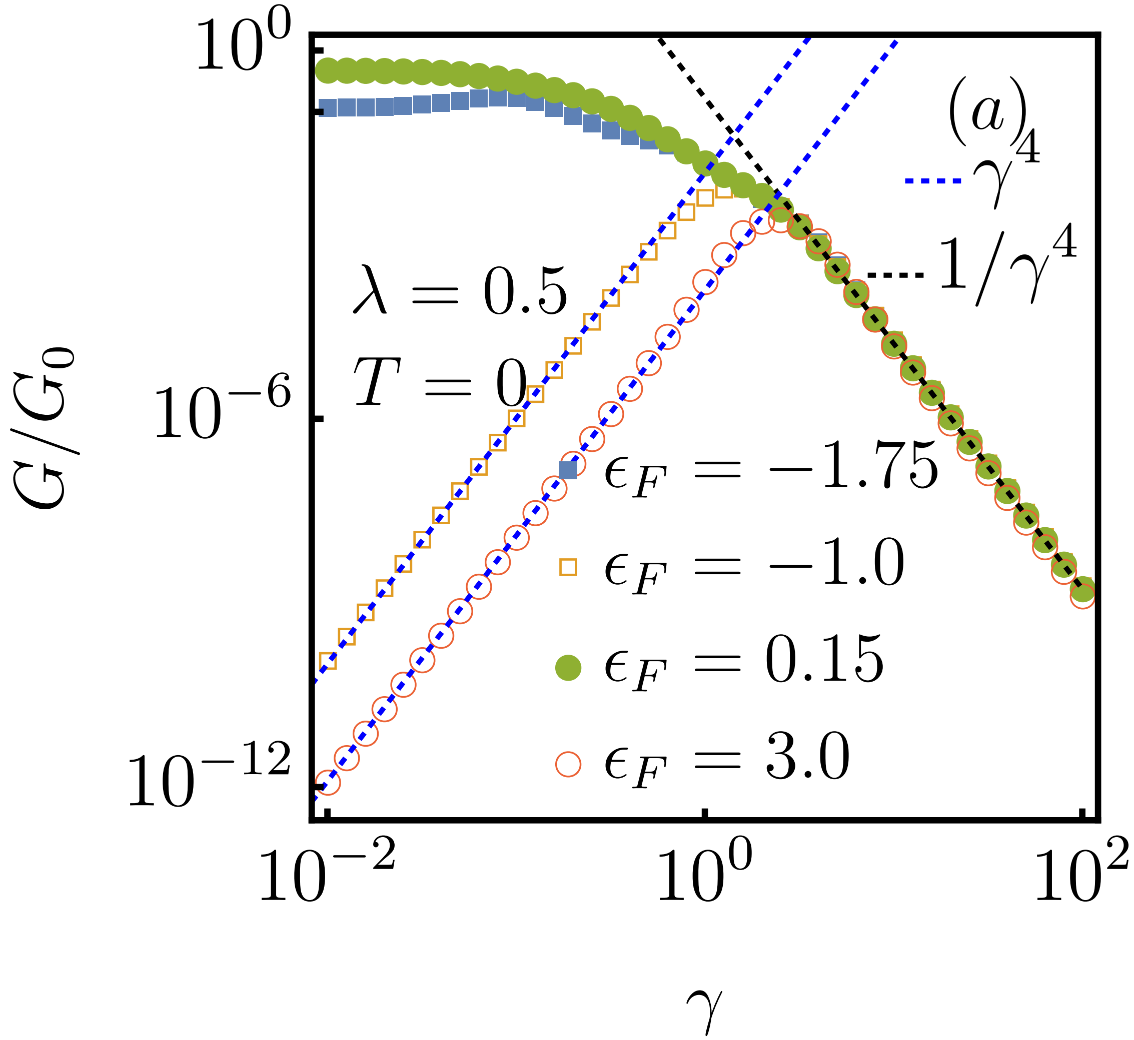


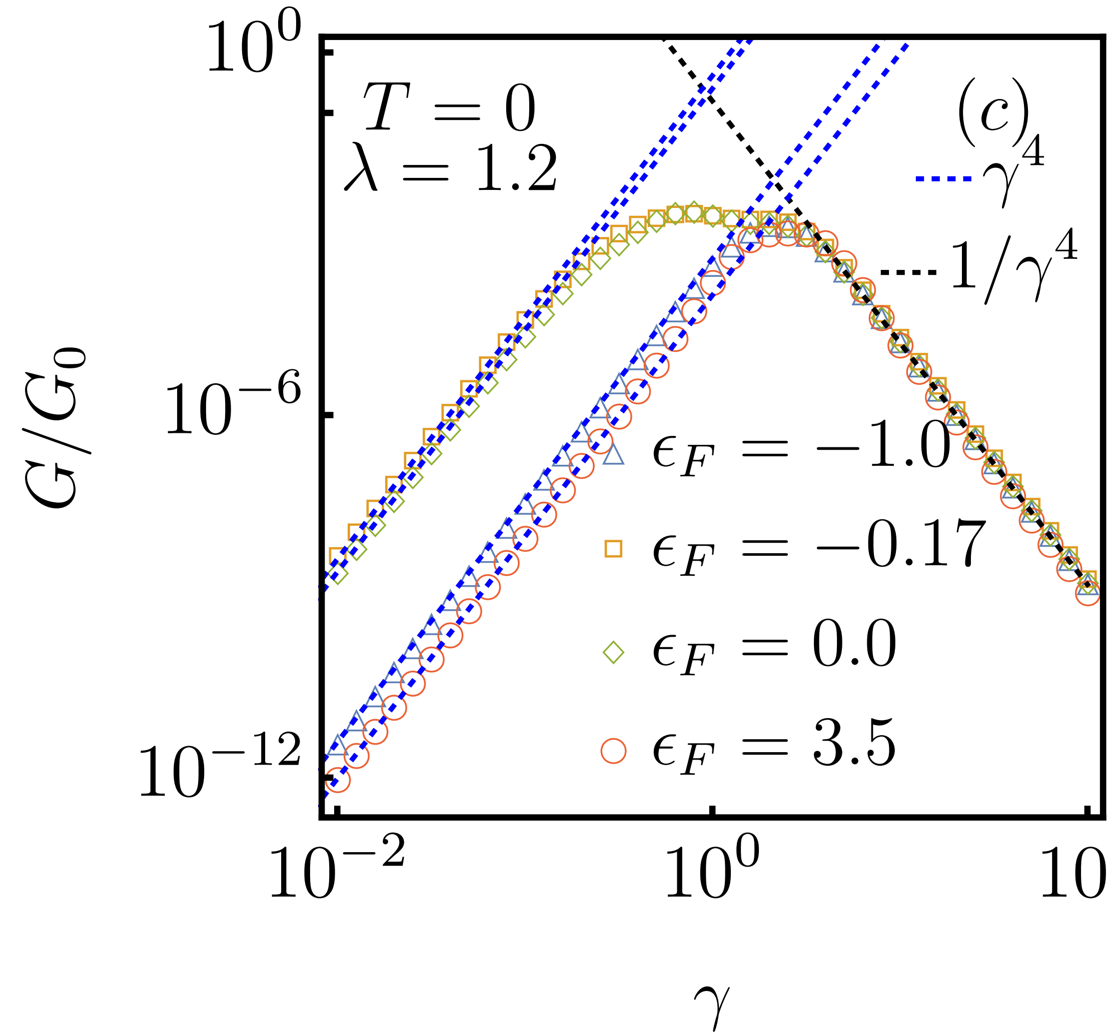


Probe coupling strength γ

Universal decay $1/\gamma^4$ and enhancement as γ^4

Results for AAH model



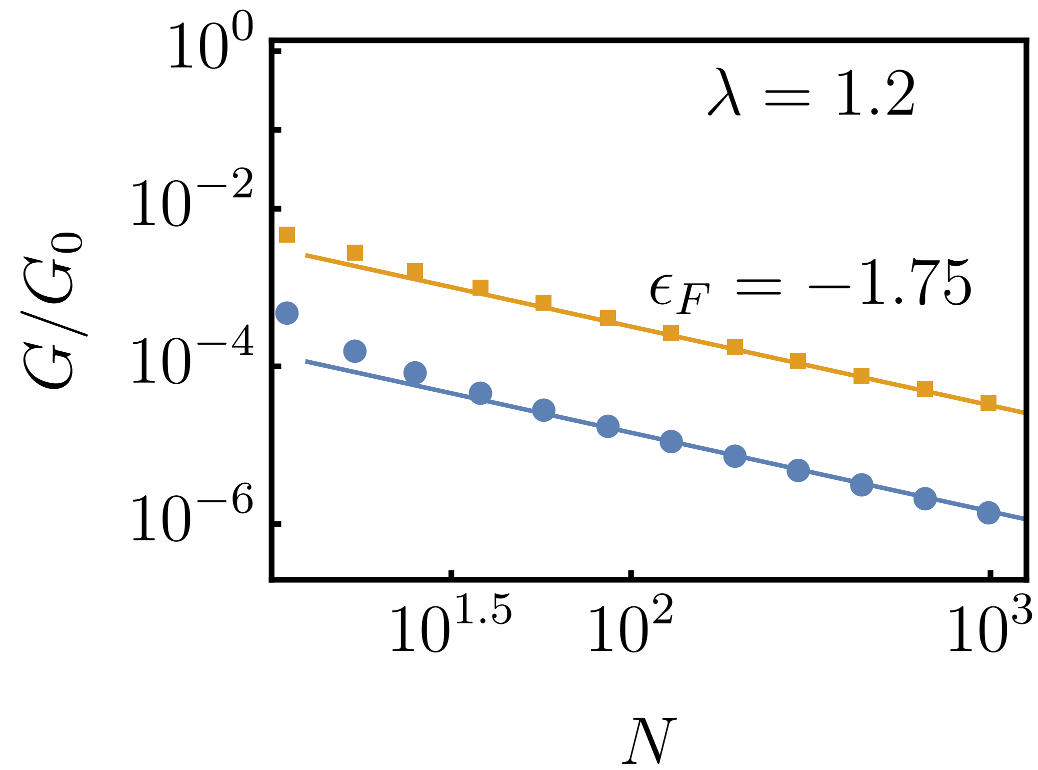


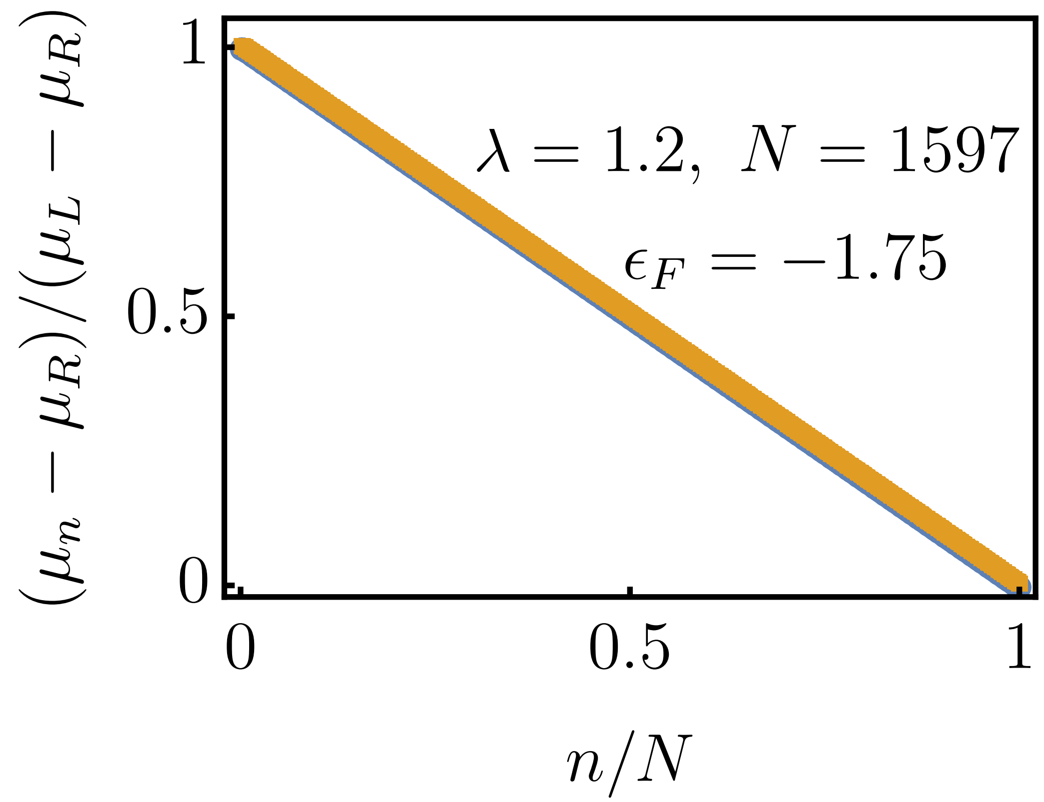
conductance for large coupling $\sim \dfrac{1}{N\gamma^4}$

Diffusive scaling with system size for conductance

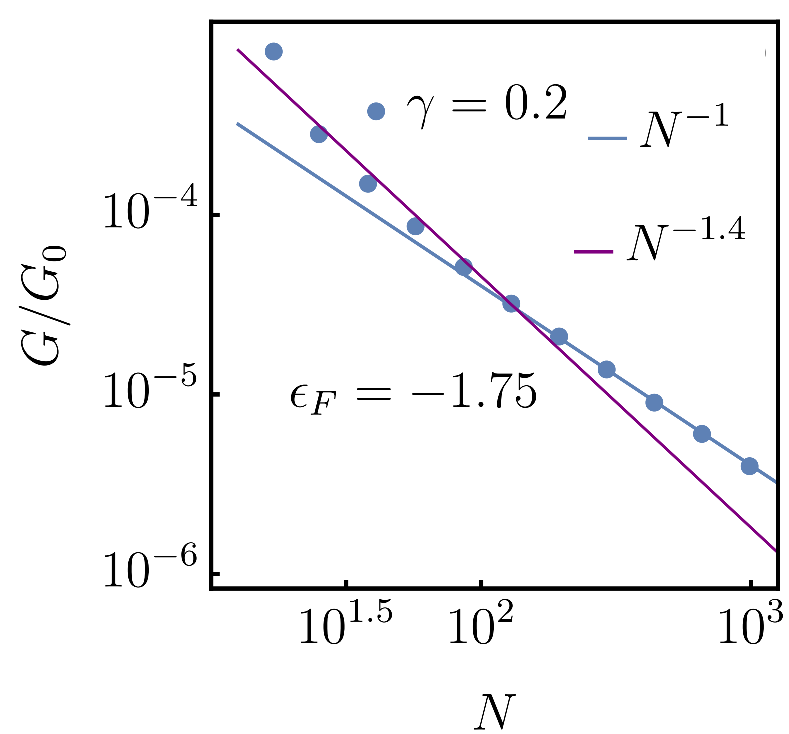
Diffusive behaviour in presence of probes

Localised regime





Critical regime



Summary Part 2

- Environment assisted quantum transport at all localised regimes
- Power law scaling in conductance with probe coupling
- For sufficiently strong coupling transport eventually becomes diffusive at all regimes

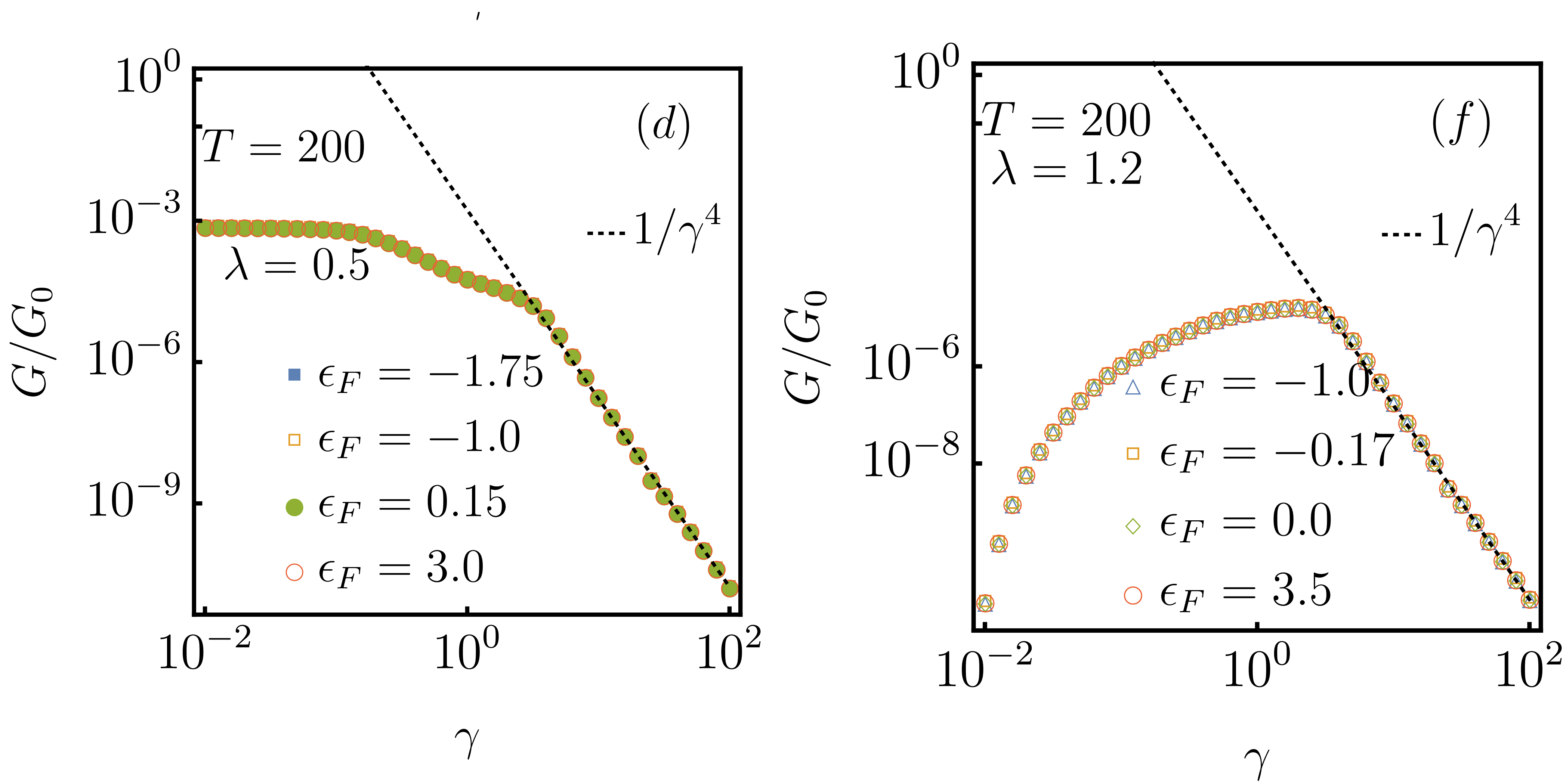
Thank you!

Max Planck India Mobility Grant

SERB, India

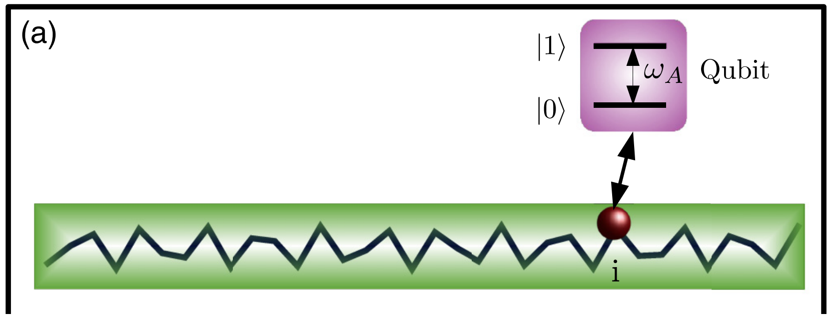
Shastri Indo-Canada research grant

Results at finite temperature



Dephasing Spin-Boson Solution

$$\hat{c}_i = \sum_{k=1}^{N} S_{i,k}\hat{\eta}_k$$



$$g_k^i = gS_{i,k}$$

$$\hat{H}_{1q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \sum_{k=1}^{N}\omega_k\hat{\eta}_k^\dagger\hat{\eta}_k + \sum_{k=1}^{N}\hat{\sigma}_z^i\left(g_k^i\hat{\eta}_k^\dagger + g_k^{i*}\hat{\eta}_k\right)$$

$$\hat{\rho}(0) = \hat{\rho}_s(0)\otimes\left(e^{-\beta\hat{H}}/Z_\beta\right) \quad Z_\beta = \mathrm{Tr}_B[e^{-\beta\hat{H}}]$$

Solution:

$$\hat{\sigma}_-^i(t) = e^{-i\omega_A t}\hat{\sigma}_-^i(0)\otimes\prod_{k=1}^{N}\hat{D}_k\left(\alpha_k^i\right)$$

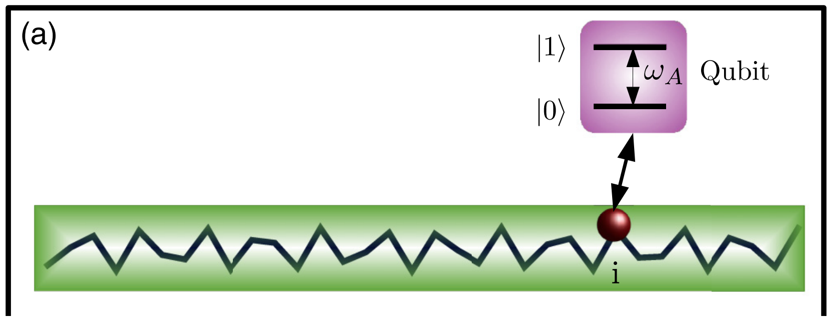
$$\hat{\sigma}_z(t) = \hat{\sigma}_z(0)$$

$$\alpha_k^i = \frac{2g_k^i(1-e^{i\omega_k t})}{\omega_k}$$

$$\hat{D}_k(\alpha) = \exp[\alpha\hat{\eta}_k^\dagger(0) - \alpha^*\hat{\eta}_k(0)]$$

Dephasing Spin-Boson Solution

$$\hat{c}_i = \sum_{k=1}^{N} S_{i,k}\hat{\eta}_k$$



$$g_k^i = gS_{i,k}$$

$$\hat{H}_{1q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \sum_{k=1}^{N}\omega_k\hat{\eta}_k^\dagger\hat{\eta}_k + \sum_{k=1}^{N}\hat{\sigma}_z^i\left(g_k^i\hat{\eta}_k^\dagger + g_k^{i*}\hat{\eta}_k\right)$$

$$\langle\hat{\sigma}_-^i(t)\rangle = \langle\hat{\sigma}_-^i(0)\rangle e^{-i\omega_A t - \Gamma_i(t)},$$

$$\Gamma_i(t) = 4\sum_{k=1}^{N}\left|g_k^i\right|^2 \coth\left(\frac{\beta\omega_k}{2}\right)\frac{1-\cos(\omega_k t)}{\omega_k^2}.$$

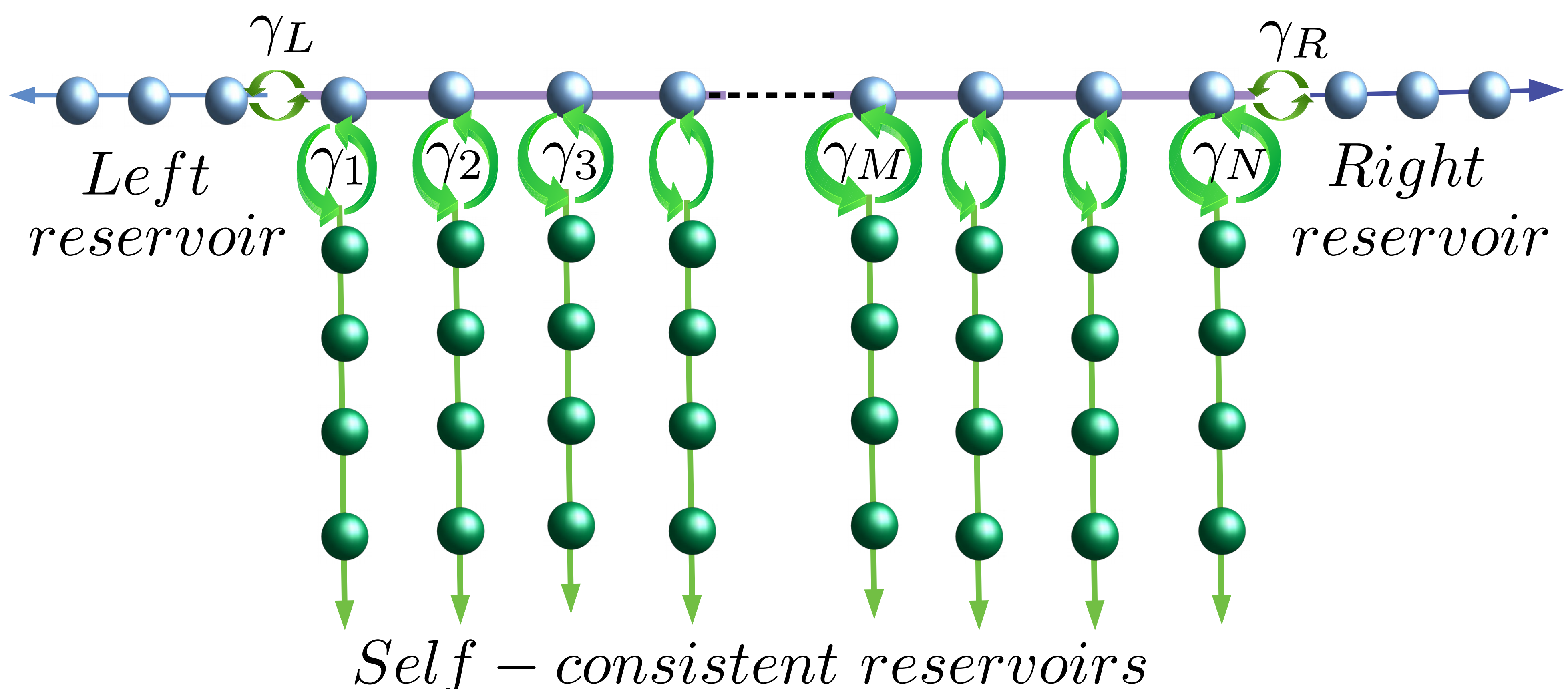
$$\Gamma_i(t) = \Gamma_{i,\text{vac}}(t) + \Gamma_{i,\text{th}}(t)$$

$$\Gamma_{i,\text{vac}}(t) = \sum_{k=1}^{N}\frac{|\alpha_k^i|^2}{2}$$

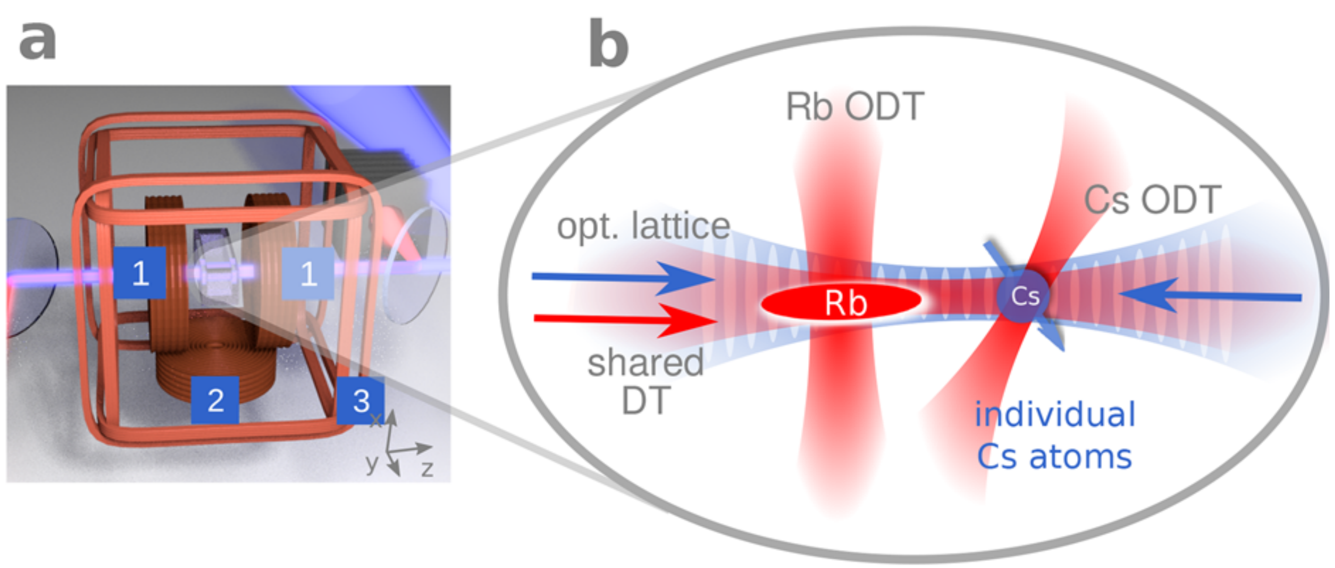
$$\Gamma_{i,\text{th}}(t) = \sum_{k=1}^{N}\bar{n}(\beta,\omega_k)|\alpha_k^i|^2$$

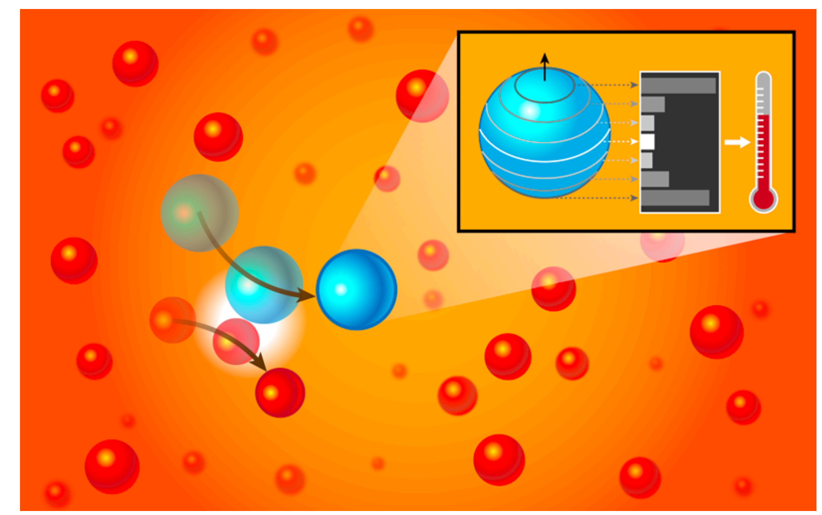
Second Set-up

Quasi-periodic lattice + Buttiker probes



arXiv: 2202.14033





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