Dynamics of Heavy Quarks In and Out of Equilibrium

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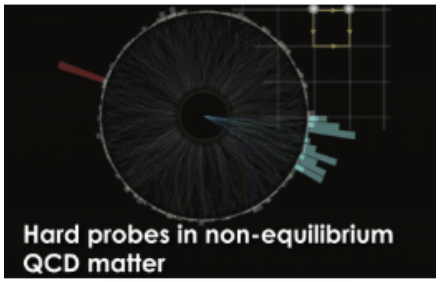


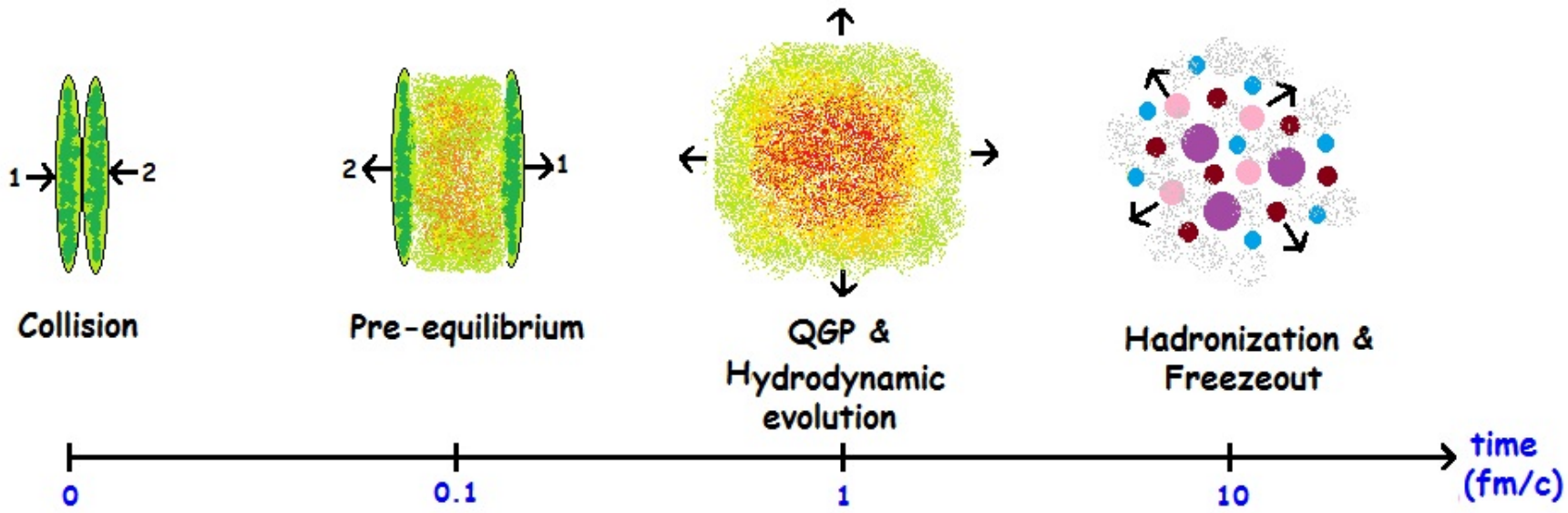
ICTS

Outline

1. Introduction and Motivation
2. Transport coefficients of a heavy quark in hot viscous QCD matter
3. Open Quantum System approach for memory effect
4. Conclusion and Future Work

Introduction and Motivation

- Heavy quarks (c, b) produced in early-stage of the heavy-ion collisions introduce **large mass** scale in the QGP medium $(M \gg T)$.
- Witness entire QGP evolution: From creation to hadronization. Important **probes** to study the QCD medium properties
- Their thermalization and energy loss mechanisms provide a direct **scan** of the microscopic transport properties of the deconfined medium.



Introduction and Motivation

Typical **energy scales**

- **Hard Scale**: HQ Mass (M_c, M_b) $= 1.3 - 4.2$ GeV (Initial production in hard scattering)
- **Soft Scales**:
  1. Temperature (T) $= 200 - 600$ MeV (Thermal matter interactions)
  2. Binding Energy (E_b) $= 600$ MeV (J/ψ dissociation)
  3. QCD Intrinsic Scale (Λ_{QCD}) $= 200$ MeV (Non-perturbative limit)

Interaction scale limits

1. **Soft**: $q \sim gT$ (Multiple soft collisions, Brownian motion)
2. **Intermediate**: $q \sim T$ (Interaction time $\approx$ relaxation time)
3. **Hard**: $q >> gT$ (Rare, hard scatterings & gluon radiation, pQCD)

HQ transport

Ideal case

- Non-equilibrated heavy quark traversing equilibrated plasma.
- Brownian motion of heavy quark (non-relativistic) within QGP medium
- *Boltzmann transport equation* for the phase space density $f_{HQ}(\mathbf{x}, \mathbf{p}, t)$ of the heavy quark.
- Homogeneous plasma ($\partial f_{HQ}/\partial \mathbf{x} = \mathbf{0}$) with no external force ($\mathbf{F} = 0$)

B. Svetitsky, *Phys. Rev. D, 37(9), 1988*

$$p^\mu \partial_\mu f_{HQ}(x, \mathbf{p}) = C[f_{HQ}](x, \mathbf{p})$$

$$L.H.S. \implies \left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{E_p} \frac{\partial}{\partial \mathbf{x}} + \mathbf{F} \frac{\partial}{\partial \mathbf{p}} \right) f_{HQ}(\mathbf{x}, \mathbf{p}, t) \equiv \frac{\partial f(\mathbf{p}, t)}{\partial t}$$

- $w(p, k)$: collision rate changing HQ momentum from p to $p - k$,

$$R.H.S. \implies C[f_{HQ}](x, \mathbf{p}) = \int d^3k [w(p+k, k)\, f_{HQ}(x, p+k) - w(p, k)\, f_{HQ}(x, p)]$$

Soft scattering

- Simplifying integro-differential Boltzmann equation
  $\rightarrow$ Soft scattering approximation (low momentum transfer, $|\mathbf{k}| << \mathbf{p}$).
- Expanding $w(p+k,k)\, f_{HQ}(x,p+k)$ around k:

$$w(p+k,k)\, f_{HQ}(x,p+k) \approx w(p,k)\, f(x,p) + k\frac{\partial}{\partial p}(wf) + \frac{1}{2}\, k_i\, k_j\, \frac{\partial^2}{\partial p_i\, \partial p_j}(w\, f)$$

$\Longrightarrow$ *Fokker-Planck equation*:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left(A_i(\mathbf{p}) f + \frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p})] f \right)$$

- $A_i = \int d^3k\, w(p,k) k_i = A(p)\, p_i$ and $B_{ij} = \int d^3k\, w(p,k) k_i k_j$ related to the drag and diffusion coefficient, respectively.

Transport coefficients definition

- A_i and B_{ij} depends only on the initial momentum ($\mathbf{p}$)

$\rightarrow$ **Drag:**

$$A_i = \langle (p - p')_i \rangle = p_i A(p^2)$$

$$\boxed{A(p^2) = \frac{p_i A_i}{p^2} = \langle \mathbf{1} \rangle - \frac{\langle \mathbf{p} \cdot \mathbf{p}' \rangle}{p^2}}$$

$\rightarrow$ **Diffusion:** $B_{ij} \equiv \frac{1}{2} \langle (p - p')_i (p - p')_j \rangle$

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) B_0(p^2) + \left(\frac{p_i p_j}{p^2} \right) B_1(p^2)$$

Transverse $\rightarrow$ $$\boxed{B_0(p^2) = \frac{1}{2} \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) B_{ij} = \frac{1}{4} \left[\langle p'^2 \rangle - \frac{\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle}{p^2} \right]}$$

Longitudinal $\rightarrow$ $$\boxed{B_1(p^2) = \left(\frac{p_i p_j}{p^2} \right) B_{ij} = \frac{1}{2} \left[\frac{\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle}{p^2} - 2 \langle \mathbf{p} \cdot \mathbf{p}' \rangle + p^2 \langle \mathbf{1} \rangle \right]}$$

Alternate approach

- **Langevin equations**:

$$dx_i = p_i E\, dt,$$

$$dp_i = -A\, p_i dt + C_{ij}\rho_j\sqrt{dt}$$

- A: drag force
  C_{ij}: covariance matrix describing the stochastic force in terms of independent Gaussian distributed random variables ρ_j,
  $< \rho_i\, \rho_j >= \delta(t_i - t_j)$ and $< \rho_j >= 0$.

$$P(\rho) = \frac{e^{-\rho^2/2}}{(2\pi)^{3/2}}$$

- Relation between C_{ij} and diffusion coefficients:

$$C_{ij} = \sqrt{2B_0}\, P_{ij}^{\perp} + \sqrt{2B_1} P_{ij}^{||}$$

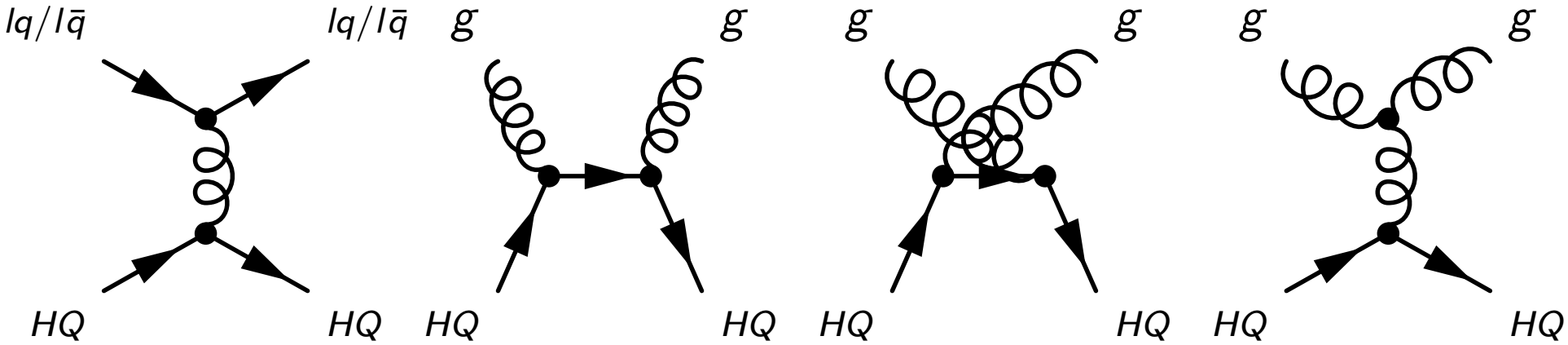
- *Transverse* proj.: $P_{ij}^{\perp} = \delta_{ij} - \frac{p_i p_j}{p^2}$, *Longitudinal* proj.: $P_{ij}^{||} = \frac{p_i p_j}{p^2}$

HQ transport

Collisional process

- HQ interact with the medium via two-body collisions defined by the scattering matrix element.
- $HQ(p) \; + \; lq/l\bar{q}/g(q) \rightarrow HQ(p') \; + \; lq/l\bar{q}/g(q')$

$$\langle F(p) \rangle_{col} = \frac{1}{2E_p \gamma_{HQ}} \int \frac{d^3q}{(2\pi)^3 E_q} \int \frac{d^3q'}{(2\pi)^3 E_{q'}} \int \frac{d^3p'}{(2\pi)^3 E_{p'}} \sum |\mathcal{M}|^2_{2\to 2}$$
$$\times (2\pi)^4 \; \delta(p+q-p'-q') \; f(E_q) \; (1 \pm f(E_{q'})) \; F(p)$$



- Similar, diagrammatic approach to evaluate gluon emission from heavy quarks.

Radiative process

- $HQ(p) + lq/l\bar{q}/g(q) \rightarrow HQ(p') + lq/l\bar{q}/g(q') + g(k')$

$$\langle F(p)\rangle_{rad} = \frac{1}{2E_p\gamma_{HQ}} \int \frac{d^3\mathbf{q}}{(2\pi)^3 E_q} \int \frac{d^3\mathbf{q}'}{(2\pi)^3 E_{q'}} \int \frac{d^3\mathbf{p}'}{(2\pi)^3 E_{p'}} \int \frac{d^3\mathbf{k}'}{(2\pi)^3 E_{k'}}$$
$$\times \sum |\mathcal{M}|^2_{2\rightarrow 3}\ (2\pi)^4\ \delta^{(4)}(p+q-p'-q'-k')\ f_k(E_q)\ (1 \pm f_k(E_{q'}))$$
$$\times (1 + f_k(E_{k'}))\ \theta_1(E_p - E_{k'})\ \theta_2(\tau - \tau_F)\ F(\mathbf{p})$$

S. Mazumdar *et al., Phys. Rev. D 89, 014002 (2014)*

- Gluon emission from the heavy quark induced by QGP medium (after scattering by light quarks, antiquarks and gluons).
- Radiated soft gluon momentum: $k' = (E_{k'}, \mathbf{k}'_\perp, k'_z) \rightarrow 0$

$$|\mathcal{M}|^2_{2\rightarrow 3} = |\mathcal{M}|^2_{2\rightarrow 2} * \frac{12g_s^2}{{k'_\perp}^2} \left(1 + \frac{m^2_{HQ}}{s} e^{2y_{k'}}\right)^{-2}$$

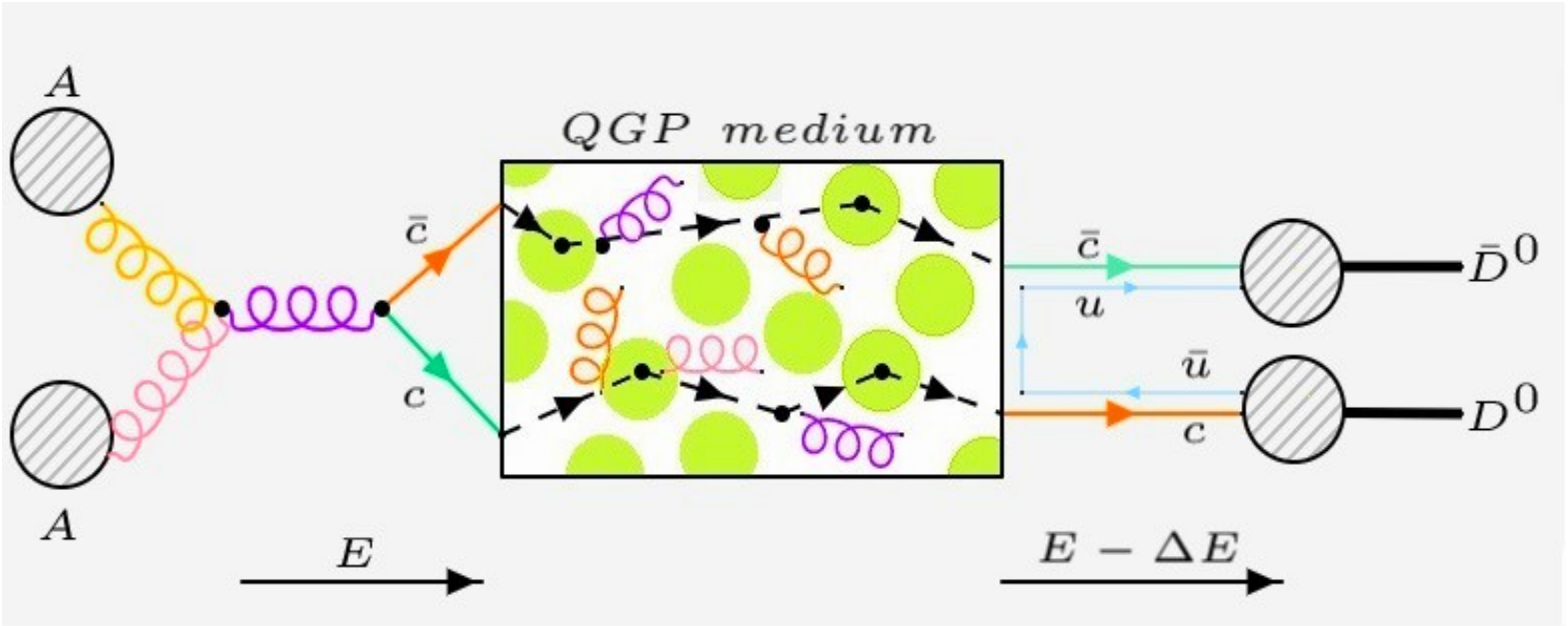
R. Abir *et al., Phys. Rev. D 85, 054012 (2012)*

[i] Thermal medium (non-pert) interaction: EQPM

- Non-perturbative input into the analysis through effective models.
- **EQPM**: **E**ffective fugacity **Q**uasi-**P**article **M**odel
- In-medium interactions of QGP encoded into particle $\rightarrow$ **quasiparticle** $\rightarrow$ Model based on mapping of the EoS with Lattice QCD EoS

V. Chandra et al., *Phys. Rev. C, 76 (054909), 2007*

- System as the grand canonical ensemble with each particle species (chemical constituent) assigned a partial pressure (fugacity).



[i] Thermal medium (non-pert) interaction: EQPM

- Introduction of temperature dependent effective fugacity z_k in the distribution functions of quasiparticles $k \equiv (lq, l\bar{q}, g)$.

$$f_k^0 = \frac{z_k \exp\{-\beta E_k\}}{1 \pm z_k \exp\{-\beta E_k\}}$$

- Temperature dependence of effective fugacity $z_{k=q,g}$ is parametrized using (2+1) flavor LQCD EoS at T_c =170 MeV as follows,

$$z_k = a_k \ \exp\left\{-\frac{b_k}{(T/T_c)^2} - \frac{c_k}{(T/T_c)^4} - \frac{d_k}{(T/T_c)^6}\right\}$$

- Quasiparticle dispersion relation: $\tilde{q_k}^\mu = q_k^\mu + \delta\omega_k \, u^\mu$
- Collective excitations of quasi-partons: $\delta\omega_k = T^2 \, \partial_T\{\ln(z_k)\}$

$$\omega_{q/g} = E_{q/g} + T^2 \, \partial_T\{\ln(z_k)\}$$

- Effective strong coupling constant $\alpha_{s(eff)}$ is introduced through EQPM based Debye mass.

S. Mitra et al., *Phys. Rev. D, 96 (094003), 2017*

[ii] Viscous hydrodynamic corrections

- Energy-momentum tensor for the dissipative (viscous) hydrodynamics,

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- Near local thermal equilibrium, the quasi-parton distribution function becomes,

$$f_k = f_k^0 + \delta f_k \text{ where } \frac{\delta f_k}{f_k^0} << 1$$

- Viscous hydrodynamic evolution of QGP to study its transport properties with EQPM using covariant kinetic theory approach.

S. Bhadury et al., *J. Phys. G 47 (2020) 8, 085108*

- Solving the effective Boltzmann equation for quasiparticles with RTA and using Chapman-Enskog iterative method.

$$\tilde{q}_k^{\mu}\, \partial_{\mu} f_k(x, \tilde{q}_k) + F_k^{\mu}\, (u \cdot \tilde{q}_k)\, \partial_{\mu}^{(q)} f_k = -\,(u \cdot \tilde{q}_k)\, \frac{\delta f_k}{\tau_R}$$

[ii] Viscous hydrodynamic corrections

- Considering boost-invariant Bjorken (longitudinal) expansion of QGP.
  $\rightarrow$ In the fluid rest frame $\implies u^{\mu} \equiv (1, \overrightarrow{0})$
- First-order shear and bulk viscous corrections to the quasiparticle distribution function. **A. Shaikh** *et al., Phys. Rev. D 104, 034017 (2021)*

$$\delta f_k = f_k^0(1 \pm f_k^0)\{\phi_k(bulk)^{(1)} + \phi_k(shear)^{(1)}\}$$

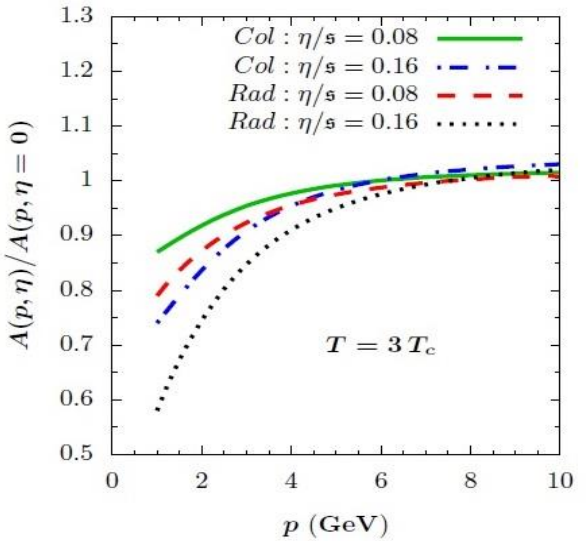
$$\phi_k(bulk)^{(1)} = \frac{s}{\beta_\Pi \omega_k T \tau}\left(\frac{\zeta}{s}\right)\left[\omega_k^2 c_s^2 - \frac{|\overrightarrow{q_k}|^2}{3} - \omega_k \delta\omega_k\right]$$

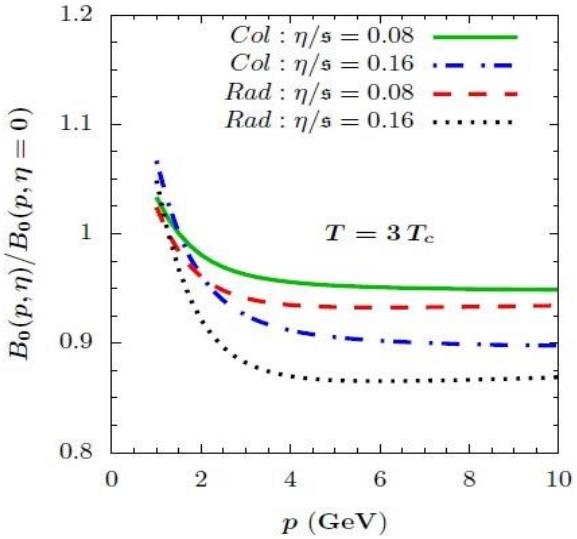
$$\phi_k(shear)^{(1)} = \frac{s}{\beta_\pi \omega_k T \tau}\left(\frac{\eta}{s}\right)\left[\frac{|\overrightarrow{q_k}|^2}{3} - (q_k)_z^2\right]$$

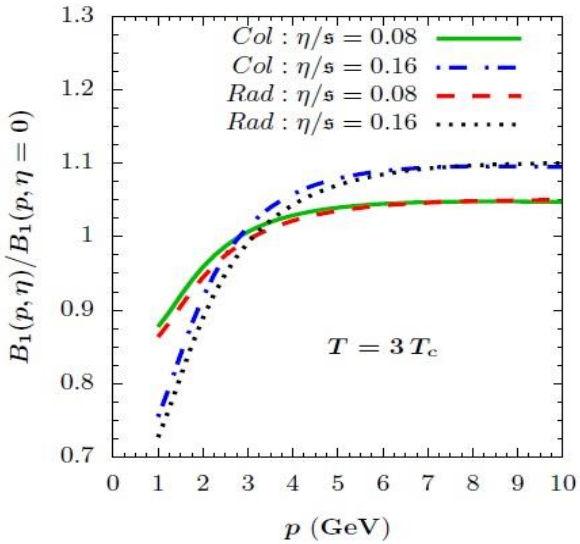
- Investigate the effect of leading order viscous corrections to the HQ radiative process within the EQPM framework.

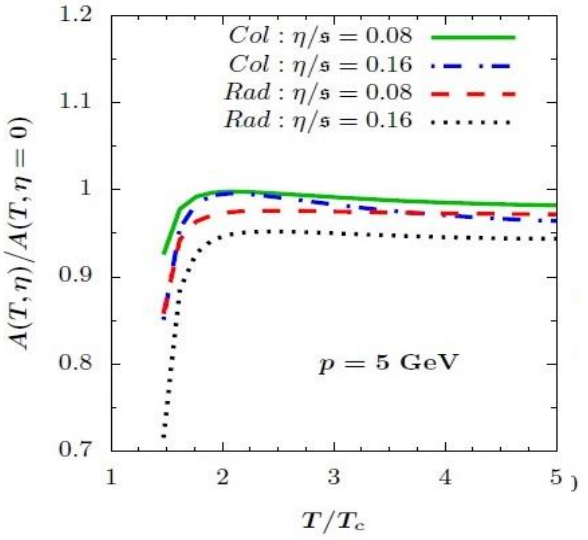
Results for **shear** viscous correction - (i)

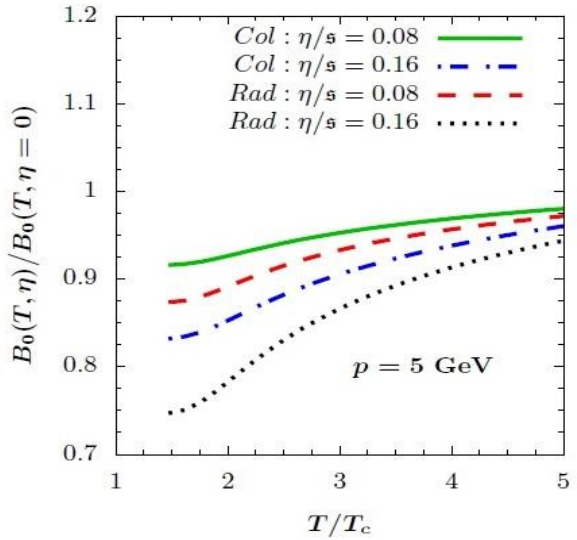
$N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$

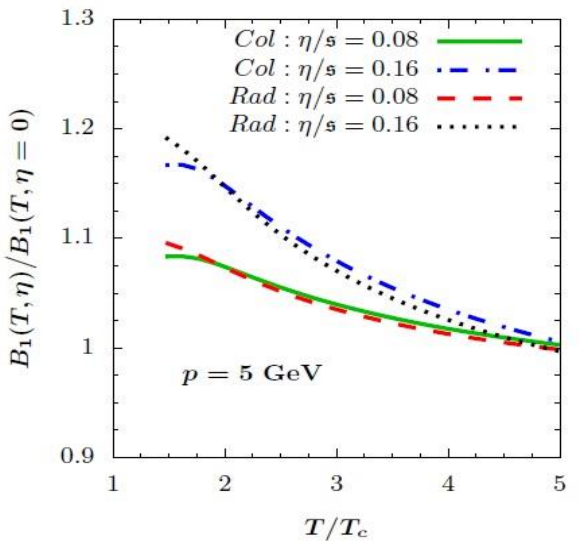








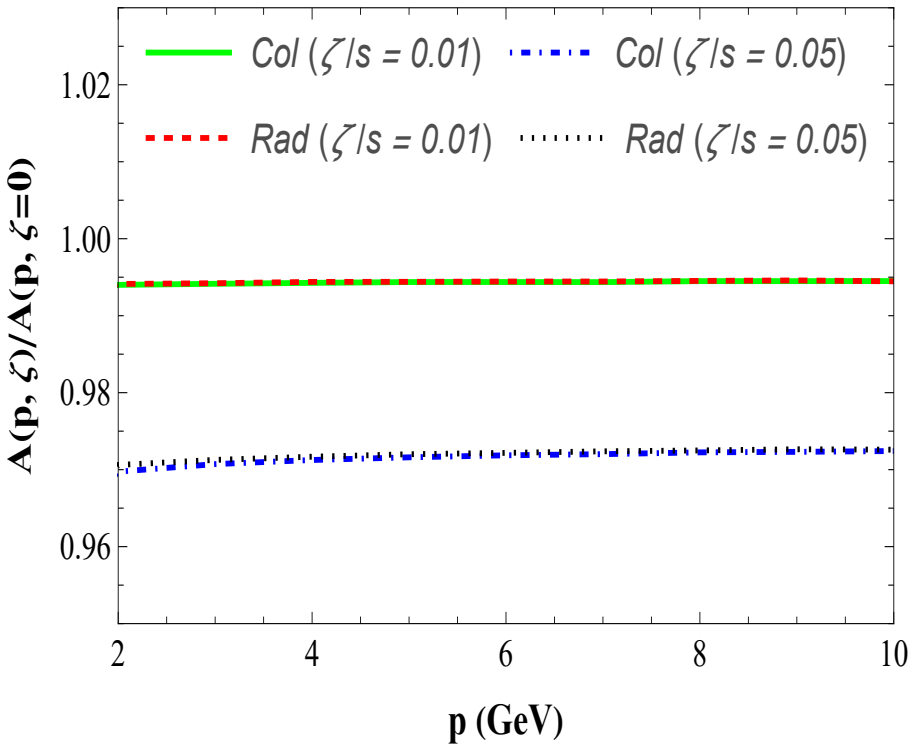


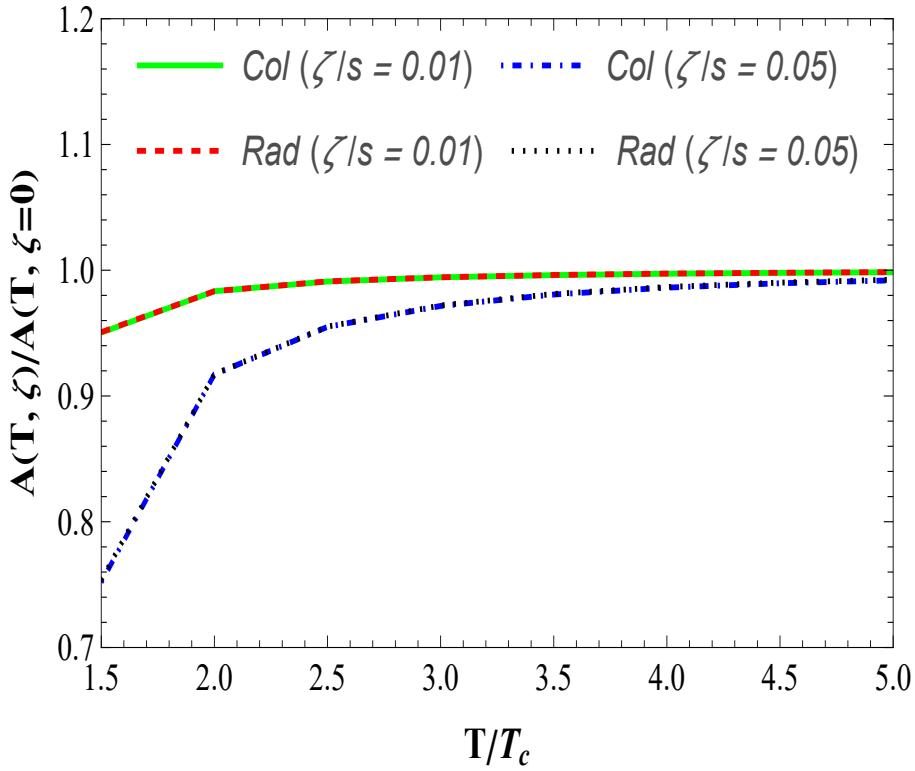


Results for **bulk** viscous correction

$$N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$

$$m_b = 4.2\,\text{GeV}$$



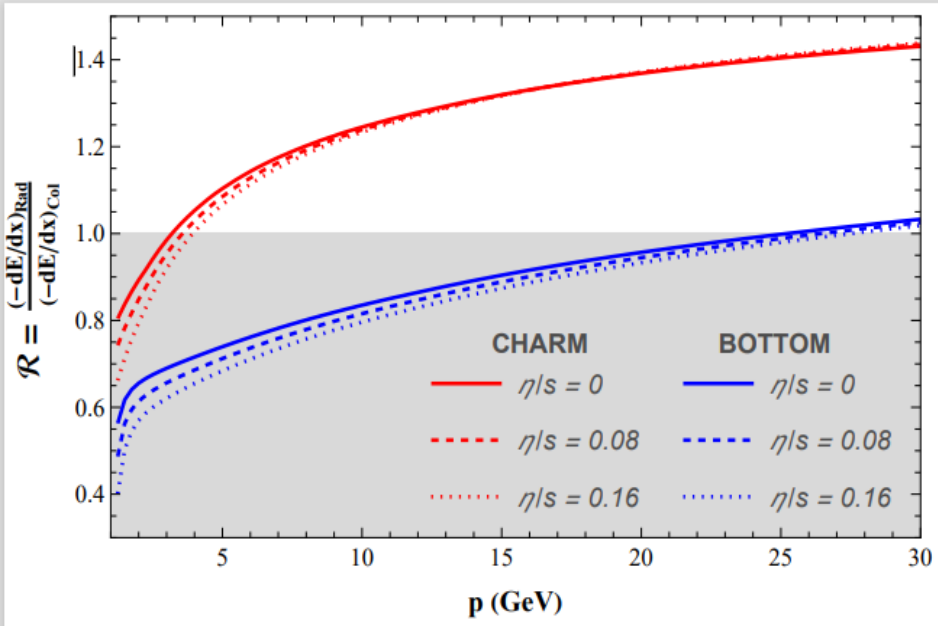


Similar results for charm → **A.Shaikh** *et al.*, *PoS CHARM2020 (2021) 060*

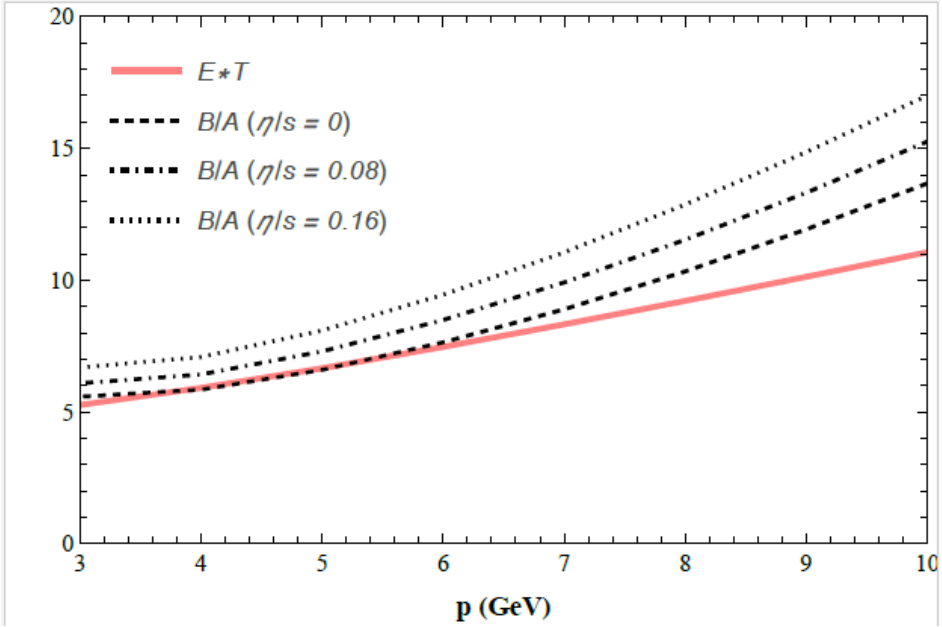
- Shear and bulk viscosity effects at first order are prominent for low initial HQ momentum and low T.

Results for **shear** viscous correction - (ii)

Diff. energy loss: $-dE/dx = p\,A(p,T)$



F-D theorem: $B/A = E\,T$



A. Shaikh, S. Dash, B. K. Nandi, *Eur. Phys. J. C 83 (2023) 10, 959*

- Radiative dominance for the bottom quark occurs at 10x initial momentum compared to charm.
- Fluctuation-Dissipation theorem holds valid for the bottom quark within the EQPM framework for low momentum and in the absence of shear viscous corrections.

Transition from Markovian to Non-Markovian

Markovian "memory-less" limit

- The medium (QGP) relaxes instantaneously ($\tau_{medium} \approx 0$).
- Heavy quarks undergo a series of independent, uncorrelated kicks (described by the Langevin or Boltzmann equations).
- Only holds if the heavy quark relaxation time $\tau_{rel} >> \tau_{medium}$ In a realistic, strongly coupled QGP, these scales often overlap.

Beyond Markovian approximation $\Longrightarrow$ Non-Markovian "memory" effect

- Internal color dynamics takes time to dissipate.
- A heavy quark's current state depends on its past interactions.
- Memory effects dominant in the initial stages of a collision (out of equilibrium system).

HQ transport coefficients (drag, diffusion) are no longer just constants; they become time-dependent.

M. Ruggieri et al., *Phys.Rev.D 106 (2022) 3, 034032*

Open Quantum System (OQS)

HQ (system) in QGP (environment)

- Total Hamiltonian: $H_{total} = H_S + H_E + H_{int}$
  H_{int}: Interaction exchanging energy, momentum, and colour between the heavy quark and the medium.
- Reduced Density Matrix: $\rho_S(t) = \mathrm{Tr}_E[\rho_{total}(t)]$
  Captures the quark's evolution and trace out plasma d.o.f.
- Wigner distribution of the density matrix,

$$f(t, \vec{r}, \vec{p}) = \int d^3u \; e^{-i\vec{p}.\vec{u}} \left\langle \vec{r} + \vec{u}/2 \right| \overline{\rho}(t) \left| \vec{r} - \vec{u}/2 \right\rangle$$

Open Quantum System

- The evolution equation,

$$\frac{\partial F(t,p)}{\partial t} = -\mathcal{N}\int \frac{d^3k}{(2\pi)^3}\left(W(p,k,t)F(t,p) - W(p+k,k,t)F(t,p+k)\right)$$

- "Rate equation" and transport coefficient

$$W(\vec{p},\vec{k},t) = 2\ \mathcal{R}e\ \left\{\int_0^t dt'\ \tilde{\Gamma}_{00}\ e^{i\left[\frac{p^2-(\vec{p}-\vec{k})^2}{2M}\right]t'}\right\}$$

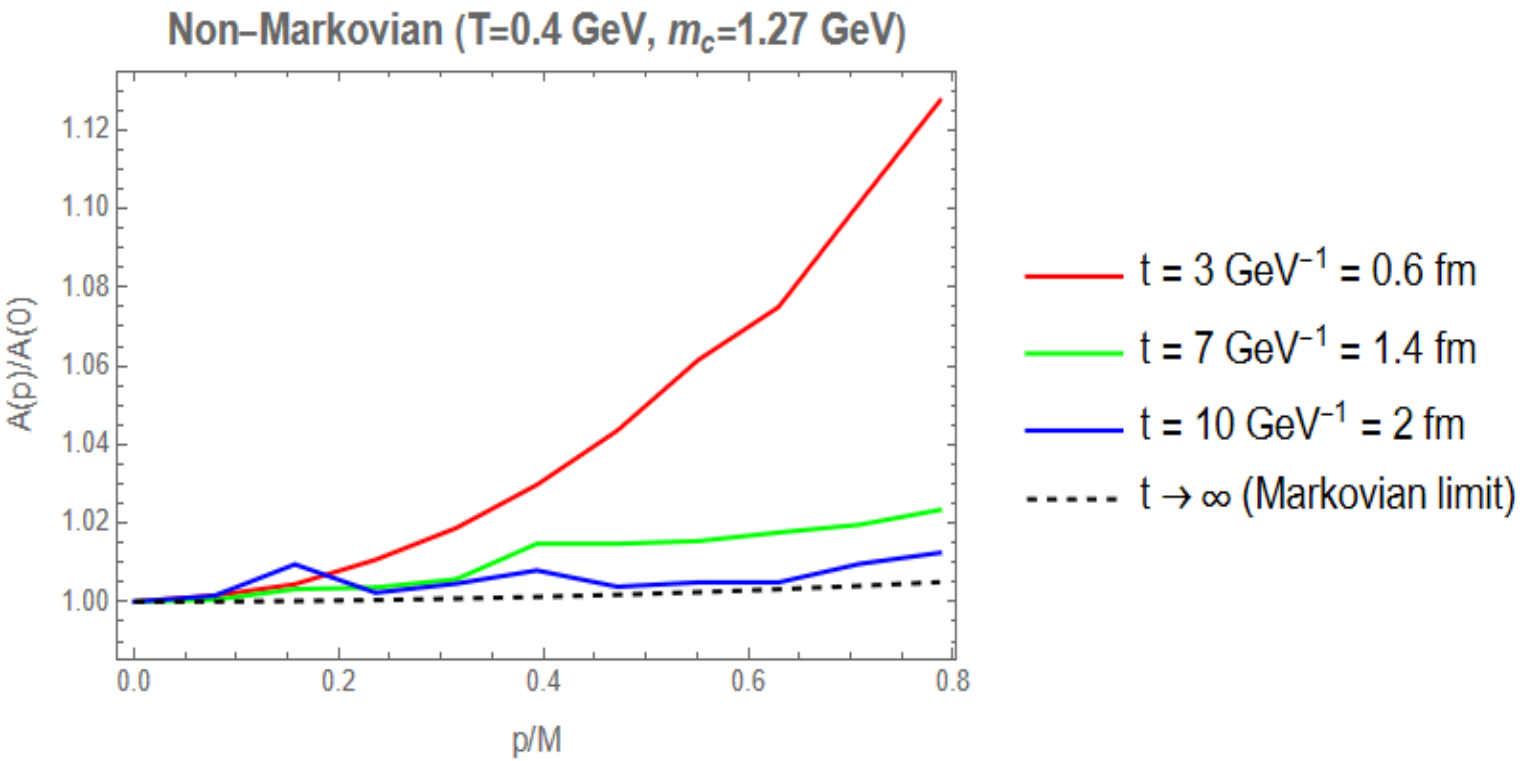
Drag: $A_i(p,t) = \int d^3k\ k_i\ W(\vec{p},\vec{k},t)$

$$= \frac{2\pi}{p}\int_0^\infty dk\ k^3 \int_0^\pi d\theta\ \sin(2\theta)\int_{-\infty}^\infty \frac{d\omega}{(2\pi)}\tilde{\Gamma}_{00}(k,\omega)\left[\frac{\sin(\omega' t)}{\omega'}\right]$$

- To calculate the E-E correlator, HTL (Hard Thermal Loop) approximated gluon propagator is used to include effects of Debye screening.

Preliminary Result

- Early times show more memory effect
- Late times approach Markovian limit



A. Shaikh, Vyshakh B. R., R. Sharma, *In progress*

Summary

Conclusion

- Heavy quark transport coefficients is studied in a **viscous QCD** medium, including the **collisional and radiative processes**
- **Shear viscous corrections** are substantial for low p
- **Bulk viscous corrections** are prominent for low T
- **Radiative dominance** of energy loss mechanism for charm quark occurs at almost one order of magnitude less in initial momentum as compared with the bottom quark
- Studied the memory effect on HQ drag coefficient using OQS

Future work

- Diffusion coefficient
- Evolution of $< p_{HQ}^2 >$ using Wigner distribution