Online conference ”Multi scale analysis”
ICTS, TIFR Bangalore, February 15-19, 2021

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An example of boundary homogenization: the homogenization of the Neumann's brush problem

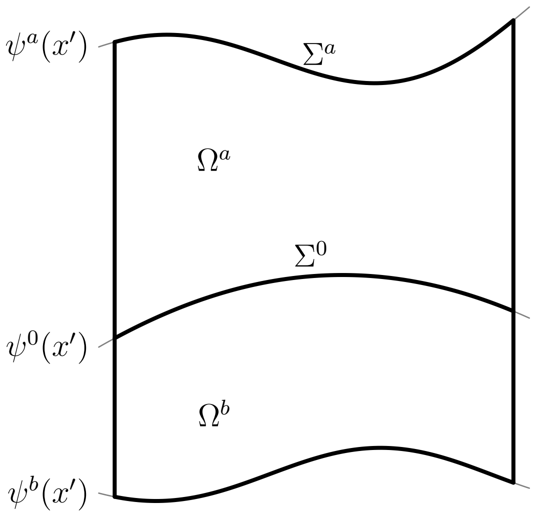
In these three lessons, I will study a boundary homogenization problem: the homogenization of the Neumann's brush problem. The lessons will be based on joint work with Antonio Gaudiello (Naples, Italy) and Olivier Guibé (Rouen, France).

I will consider a sequence of domains Ω_ε, all included in a fixed domain Ω, which have the form of brushes (in dimension $N = 3$) or of combs (in dimension $N = 2$) (see the attached figures). Each domain Ω_ε is an open subset of $\mathbf{R}^N$ with $N \geq 2$ which is made of teeth distributed over a basis. When ε tends to zero, the basis remains fixed, while the teeth vary in sizes and forms. The teeth are assumed to be vertical, cylindrical, and of fixed height, and do vary with ε: for each domain Ω_ε, the cross sections of the teeth are open sets of $\mathbf{R}^{N-1}$; their forms and sizes can vary from one tooth to another one, and they are not assumed to be smooth; moreover the teeth can be adjacent, i.e. the cross sections can share parts of their boundaries; no periodicity is assumed on the way in which they are distributed ; and the diameter of each cross section is assumed to be less than or equal to ε. Finally the sequence of the characteristic functions of the cross sections of the teeth is assumed to have an $L^\infty(\mathbf{R}^{N-1})$ weak-star limit $\theta = \theta(x')$ as ε tends to zero (this latest assumption is an innocuous one). In these lessons, I will moreover assume that this density θ satisfies $\theta(x') \geq \theta_0$ for some $\theta_0 > 0$, even if this assumption can be removed at the price of substantial efforts.

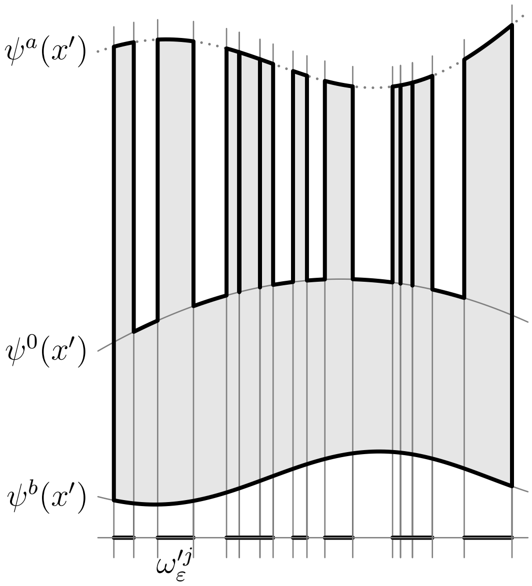
For this sequence of domains I will study the asymptotic behavior, as ε tends to zero, of the sequence of the unique solutions u_ε to a given second order linear elliptic (α-coercive) partial differential equation with bounded coefficients, with a given zeroth order term with a bounded coefficient which is bounded from below by a constant $\gamma > 0$, and with a given source term in L^2, when the homogeneous Neumann boundary condition is imposed to u_ε on the whole of the boundaries of the domains Ω_ε.

This is a classical homogenization problem since the pioneering work presented by R. Brizzi and J.-P. Chalot in their Ph.D. Thesis in 1978, but here the problem takes place in a geometry which is much more general that the ones which have been considered since that time.

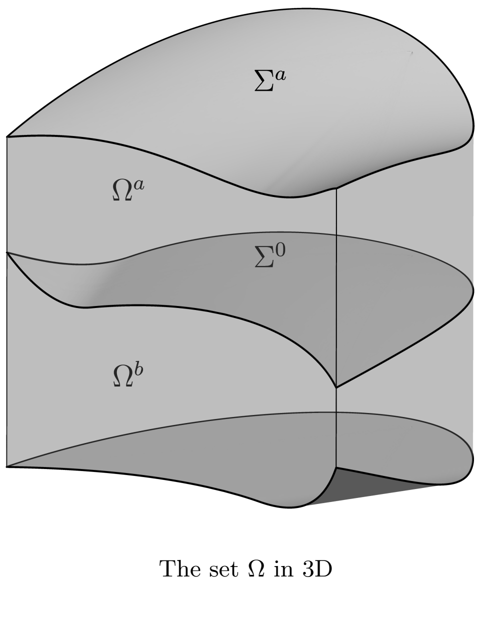
I will state and prove an homogenization result (or in other terms pass to the (weak) limit in this problem as ε tends to zero), as well as a corrector result for this problem (or in other terms provide an explicit representation of u_ε and of its gradient) as ε tends to zero.



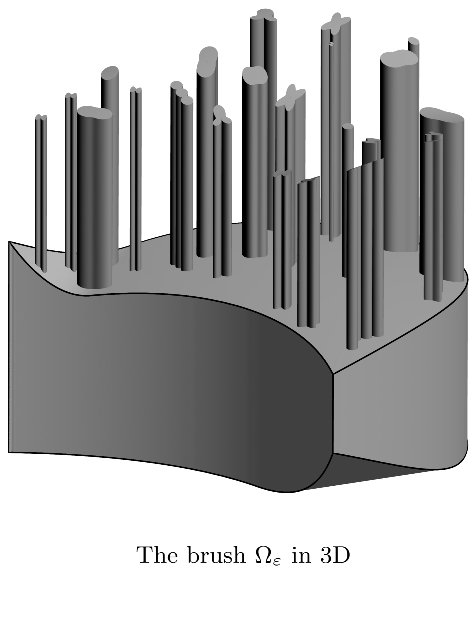
The set Ω in 2D



The comb Ω_ε in 2D



The set Ω in 3D



The brush Ω_ε in 3D