RENORMALIZED SOLUTIONS FOR ELLIPTIC EQUATIONS WITH L^1 DATA

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In this talk starting from the work of Boccardo-Gallouët we give an overview of elliptic equation with L^1 data. In general we cannot expect to have the existence of a weak solution having a finite energy. To overcome this difficulty different notions of solutions have been introduced : "duality" solutions, entropy solutions, SOLA (solutions obtained as limit of approximation), renormalized solutions. In this talk we focus on the notion of renormalized solutions, which has been introduced by DiPerna and Lions for first order equations and then by Lions and Murat for elliptic equations with L^1 data.

We consider quasilinear elliptic problems

$$\begin{cases} -\mathrm{div}(A(x,u)\nabla u) = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain of $\mathbb{R}^N$, $N \geq 2$, the datum f belongs to $L^1(\Omega)$ and the matrix field $A(x,s)$ satisfies ellipticity condition and a local condition with respect to s. In this case we will give the definition of a renormalized solution and prove existence and uniqueness results. Generalizations to another boundary conditions (Neumann or Robin conditions) will be adressed in relation with some homogenization problems with L^1 data.

Key words and phrases. Nonlinear elliptic equations, renormalized solutions, existence results, uniqueness results.

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