



INTERNATIONAL
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THEORETICAL
SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Black Holes, Quantum Mechanics & Space-time

Abdus Salam Memorial Lecture
Jamia Millia Islamia, New Delhi

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ICTS-TIFR



10 March 2023

Abdus Salam, ICTP and String Theory in India

- Abdus Salam is one of the creators of the Standard Model of elementary particles. In 1979 he shared the Nobel Prize for Physics with Sheldon Glashow and Steven Weinberg.
- Besides his extensive contributions to physics, he created the International Centre for Theoretical Physics (ICTP) in Trieste, Italy in 1964. ICTP is a unique institution with the mission of promoting science in the developing world.



- ICTP has contributed to the growth of science in India and in particular to the area of String theory. Salam gave enormous support and encouragement to the fledgling string community in India since we began working in 1984.
- ICTP became a place to go every summer and meet with peers from all over the world. Here we not only went to listen to lectures by peers from the developed world, but also as lecturers. In the days without the internet these meetings contributed to our being part of the global community of scholars.
- I vividly remember the excitement at ICTP during the spring and summer schools, and how Salam (till his health permitted) participated in most of them and added life to these meetings by his very generous and sensitive personality and a splendid sense of oriental humour.

- Another point worth mentioning is that Salam would support our travel to go and present our work at major conferences elsewhere. This gave visibility to the work we were doing in India all through the 1980s. Without doubt Salam and ICTP made a solid contribution to the growth of String Theory in India, which has today flowered into one of the strongest communities in this subject in the world.
- To commemorate the contribution of Abdus Salam to science in India one of the three academic blocks of ICTS is named after him, and every year ICTS hosts a “Abdus Salam Memorial Lecture” that is delivered by a distinguished scientist.

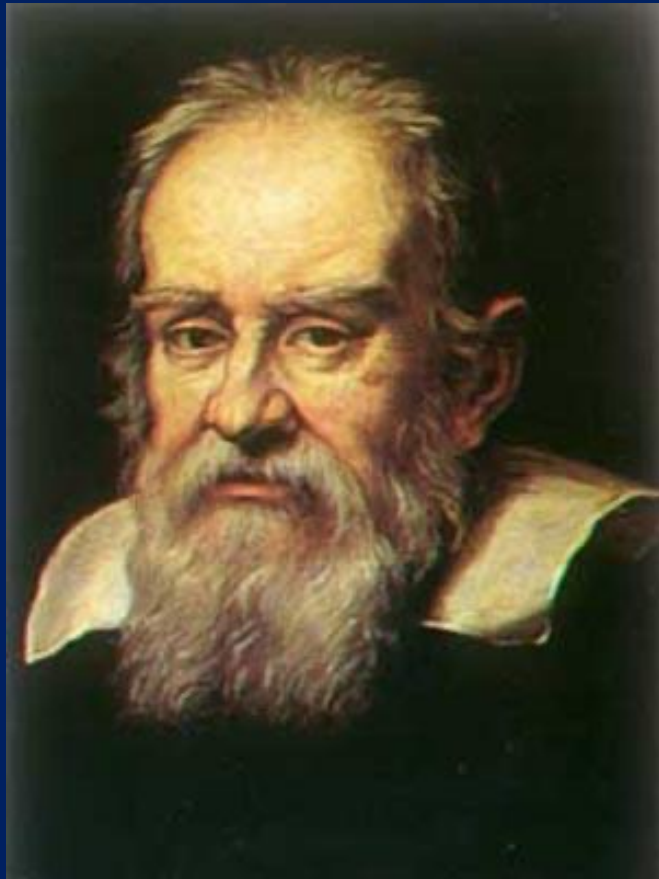
Newtonian Framework of Physics and the Law of Universal Gravitation

Galileo (1564-1642)

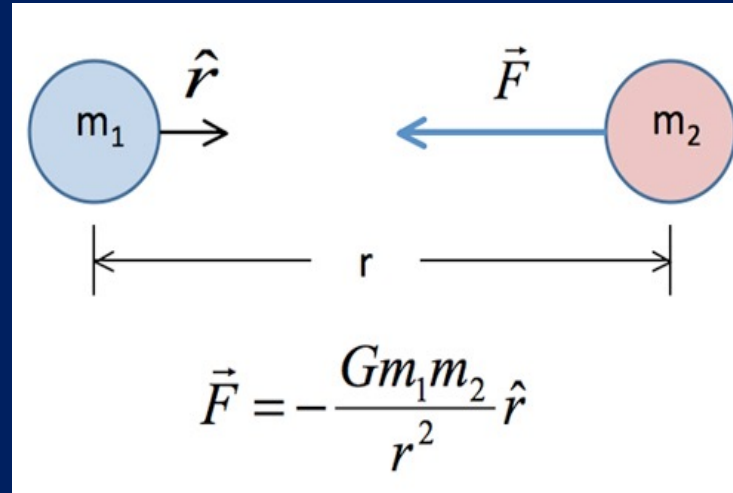
A pioneer of the modern scientific method

Discovered a new law of gravity

Gravity acts in the same way on all bodies: they all fall in the same way independent of their mass: $m_{\text{inertial}} = m_{\text{gravitational}}$ ($1/10^{13}$ precision, today) Plays a key role in Newton's theory of gravity and Einstein's theory of General Relativity.



Newton's law of Universal Gravitation



- Force acts instantaneously at a distance
- Always attractive
- Force (on particle 1 due to particle 2) = $m_1 \times \text{Acceleration}_1$

In this simple case leads to the famous elliptical orbits of planets described by Kepler.

Newton (1713) "I have not yet been able to discover the cause of these properties of gravity from phenomena and I feign no hypothesis. It is enough that gravity does really exist and acts according to the laws I have explained, and that it abundantly serves to account for all the motions of celestial bodies."

Electricity and Magnetism and Electromagnetic waves

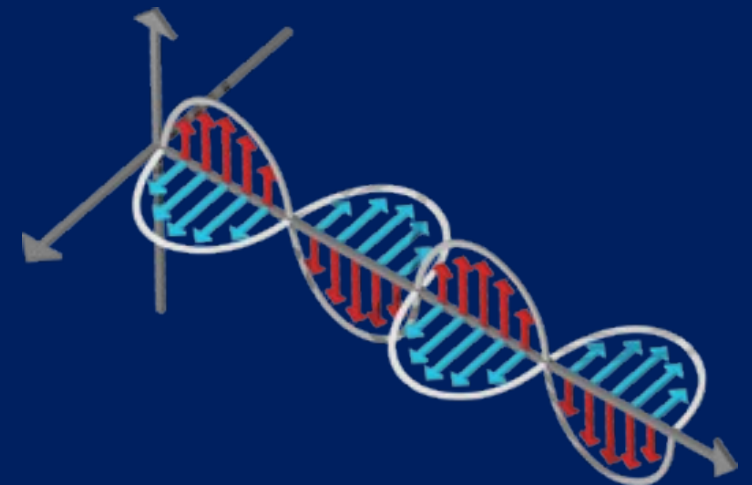
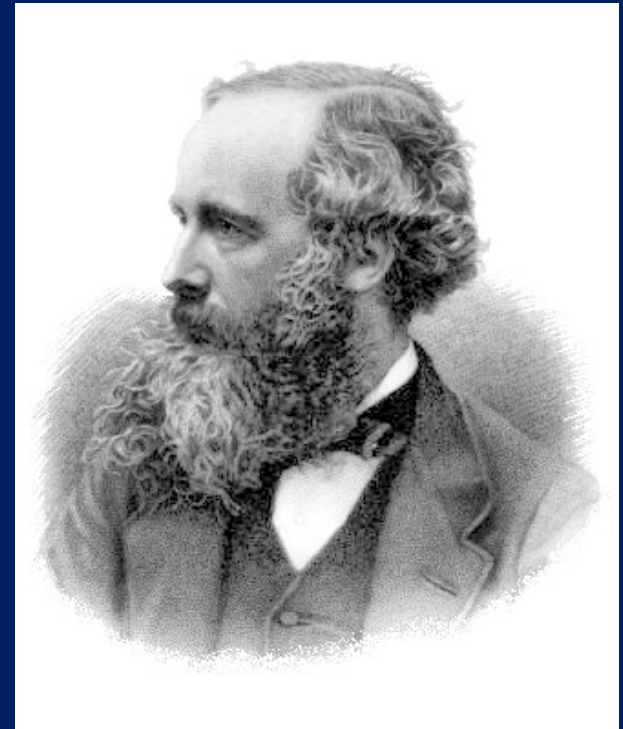
Electric and Magnetic Fields and Waves

Michael Faraday had demonstrated that light is polarized by a magnetic field.

James Clerk Maxwell unified electricity and magnetism, predicted the existence of electromagnetic waves and identified light as an electromagnetic wave of oscillating electric and magnetic fields moving with a speed c (in vacuum)
 $c = 3.1 \times 10^5$ kms/sec (1865).

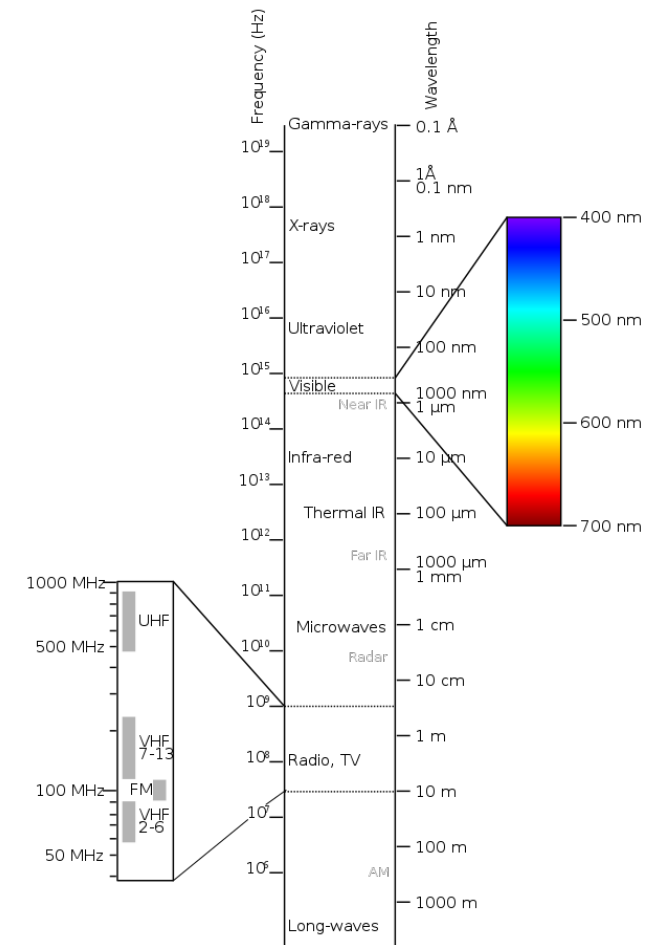
Unifies electricity, magnetism and optics.

Force between charged particles is not instantaneous. It is transmitted by the electromagnetic field at the speed of light which is finite.



Experimental tests of Maxwell's theory

- In 1887 Heinrich Hertz demonstrated existence of radio waves that were predicted by Maxwell's theory with properties the same as visible light except the wavelength is 10^4 times longer.
- In 1895 Wilhelm Conrad Roentgen made the serendipitous discovery of X-Rays, which are electromagnetic waves of wavelength 10^{-2} times shorter than visible light.



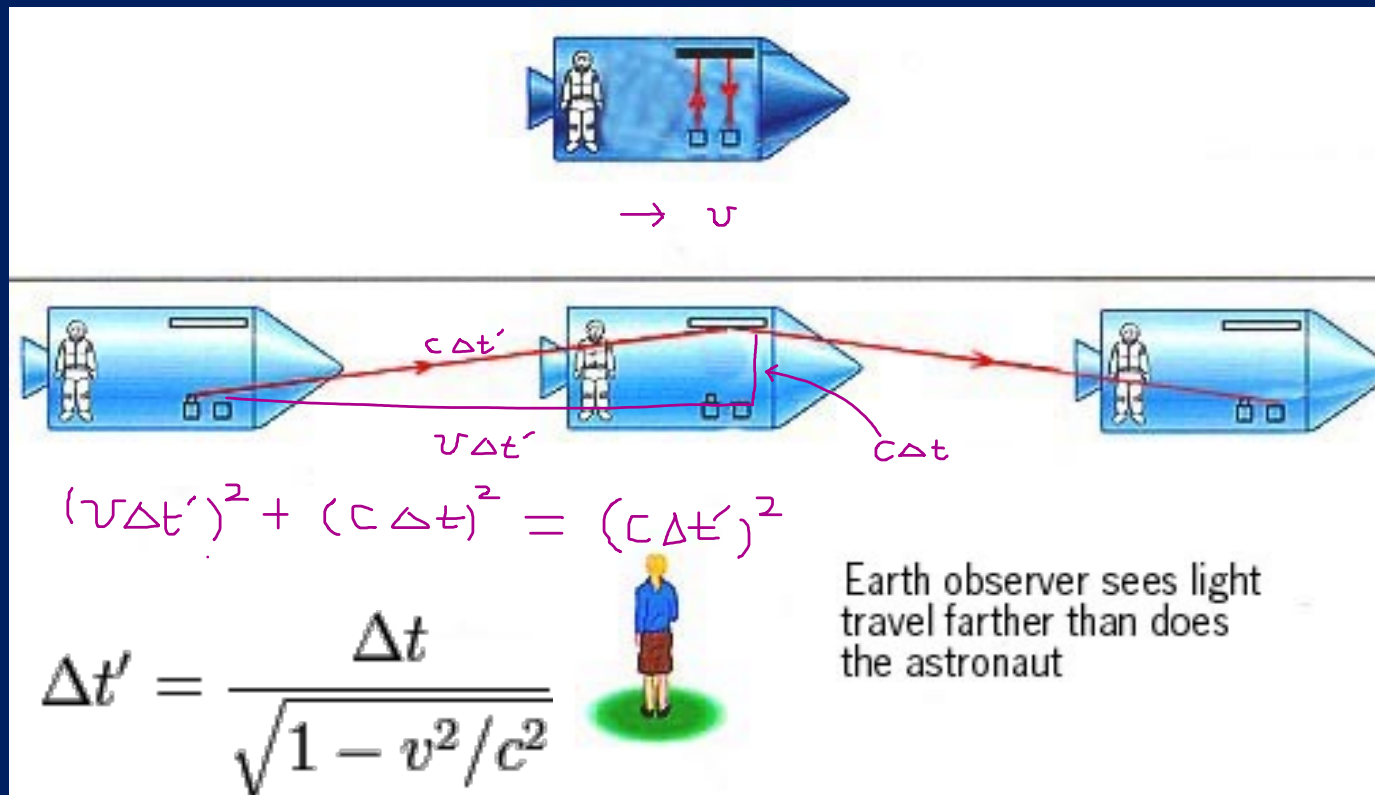
Special Relativity

Lorentz, Poincare, Einstein: Special Relativity (1905)

Implications of Maxwell's theory:

Speed of light is the same whether you run towards it or away from it. Space and time adjust themselves to ensure this!

Time intervals between events depend on your state of motion; things happen (according to us) more slowly for a moving observer than for us.



4-dimensional Space-Time Geometry

Hermann Minkowski (1864-1909)



“Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.”

Introduced Geometry in Special Relativity

- Time is a 4th component of a 4-dim. space-time $x_4 = ct$; a new geometry described by an infinitesimal Lorentz invariant distance

$$ds^2 = (dx_4)^2 - (d\mathbf{x})^2$$

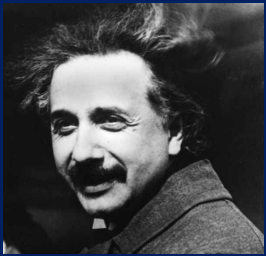
Recall Euclid's geometry:

$$ds^2 = (dx_4)^2 + (d\mathbf{x})^2$$

Is the rotational invariant distance.

General Relativity

General Relativity-1



Einstein

Given that the speed of light is the same for observers in relative motion there can be no action at a distance which is simultaneous for all inertial observers in relative motion.

Hence Newton's law of gravitation needs to be modified.

Waging one of the great struggles in modern scientific history in 1915 Einstein completed the General Theory of Relativity, which is a theory of gravity in which the force of gravity is not instantaneous. It is communicated at a finite speed c (the same as the speed of light) by changes in the geometry of space-time.

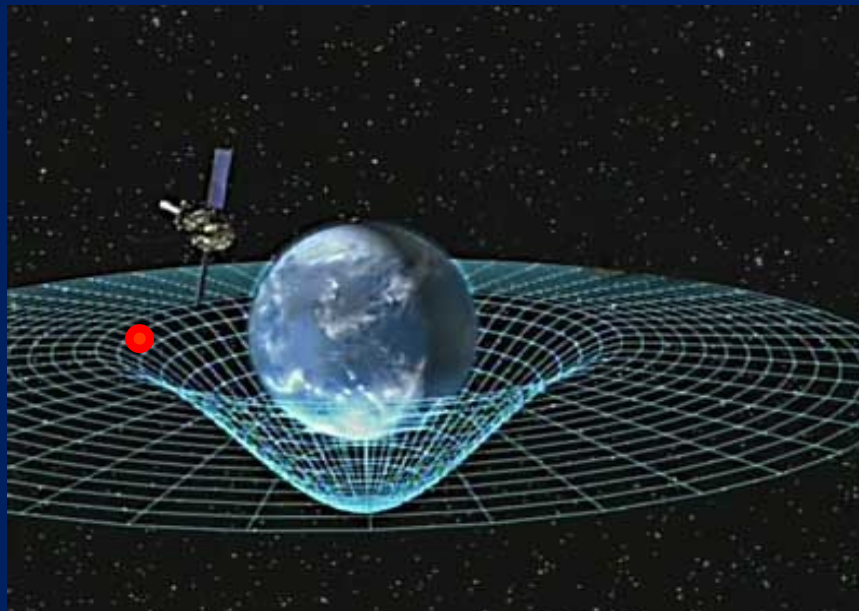
Here the role of the electromagnetic field is played by the 'metric' of spacetime.

General Relativity-2

GR describes the changes of the geometry of space-time caused by massive objects to which other objects respond.

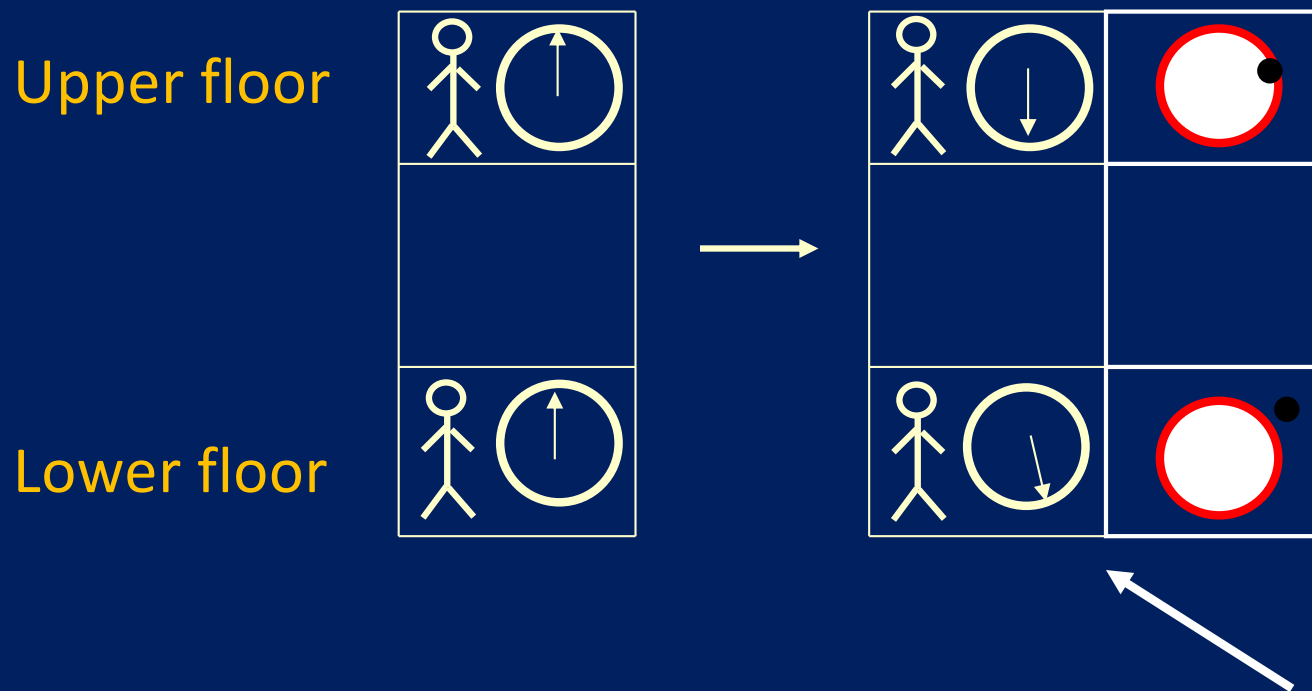
Space-time is like an 'elastic' grid, communicative and causal...but very very stiff... 10^{20} times stiffer than steel!

In a curved space-time an object follows a path that maximizes the proper time in the frame of the object.



Time in General Relativity

Einstein's theory implies that time flows different for two observers in a gravitational field



Time flows more slowly
(A part in 10^{15}) for a height of 100 meters.

This effect is relevant for GPS!

J Maldacena

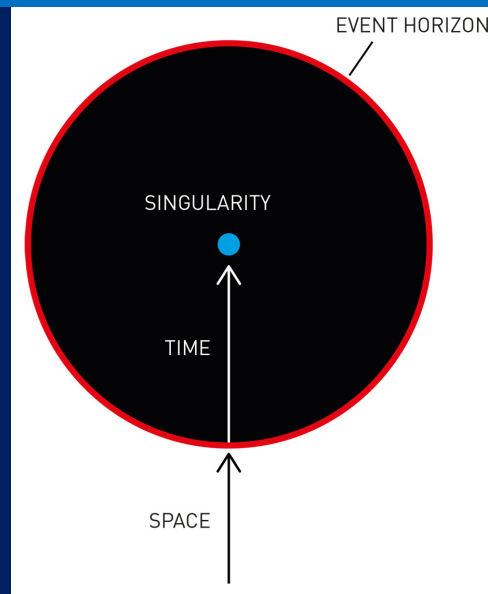
Black Holes

A prediction of General Relativity
&
Found in nature 100 years later

Black Holes-1



Karl Schwarzschild
1916



nobelprize.org

They are solutions of Einstein's equations which divide space-time into 2 parts.

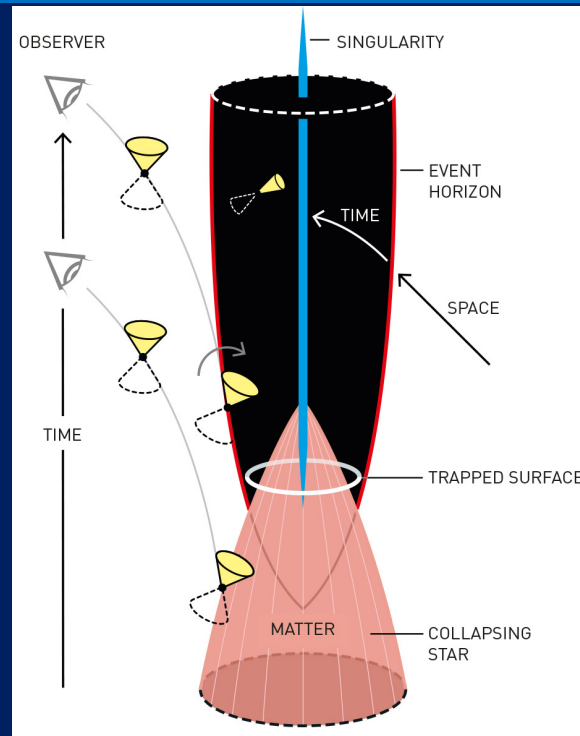
Interior and exterior separated by a surface called the horizon, which is a one-way gate. Even light entering it cannot get out. Hence the name BH.

The BH space-time has a curvature singularity in the interior. Inside the horizon the radial direction is time-like, and time ends at the singularity.

Black Holes-2



Roger Penrose



nobelprize.org

- Penrose (1973) developed topological arguments to show that BH formation is a robust prediction of the General Theory of Relativity.
- The horizon is a null surface of radius $r_s = 2GM/c^2$ (Schwarzschild radius), G is Newton's constant. $r > r_s$ describes the exterior and $r < r_s$ the interior.

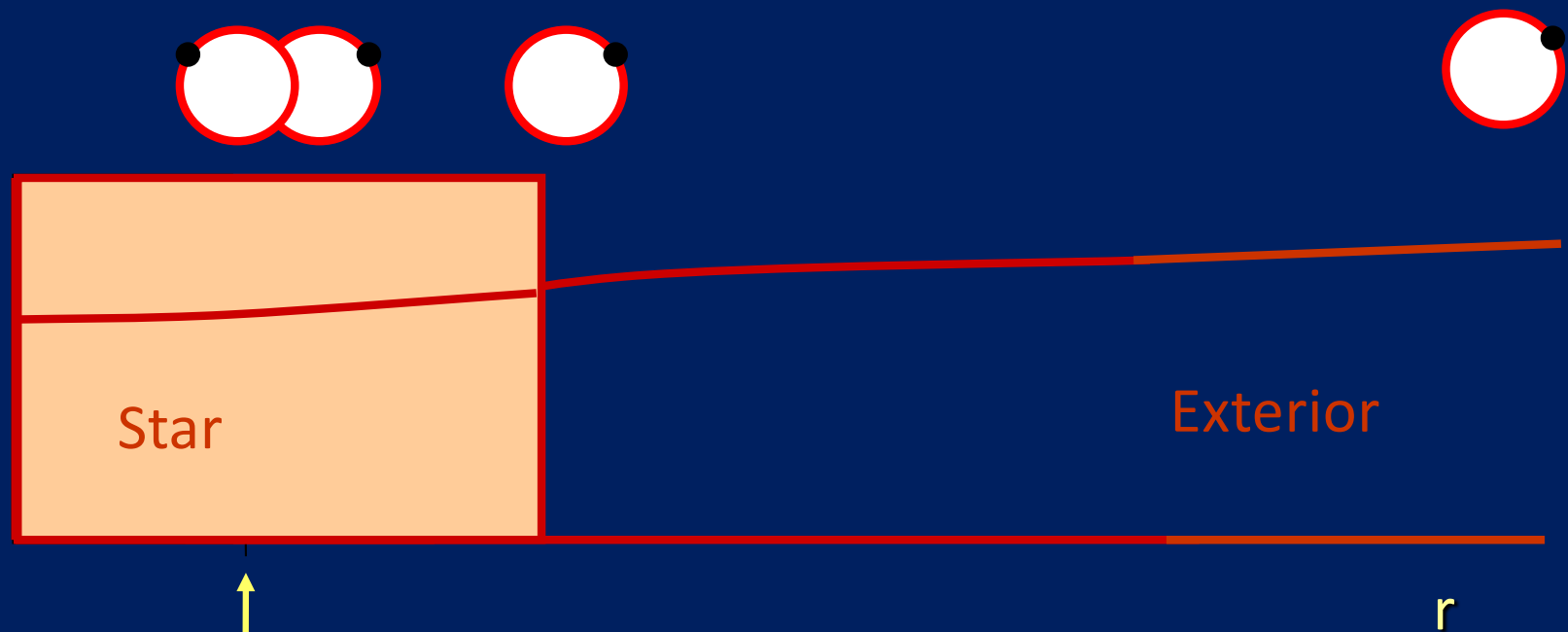
The horizon has area: $A_h = 4\pi r_s^2 = 16\pi G^2 M^2 / c^4$

Flow of time in the presence of a massive Star & a Black Hole

Flow of time

$$\sqrt{1 - \frac{r_h}{r}}$$

$$r_h = 2G \frac{M}{c^2}$$



Black hole radius

Black Holes exist in Nature

They are an observational reality!



The first image of a supermassive black hole and its shadow in the Messier 87 galaxy in the Virgo cluster. (Event Horizon Telescope collaboration - 2019). The key point is that no light was observed coming along the line of sight.

BHs have been indirectly observed in the galactic center (Ghez and Genzel) and also by the observation of gravitational waves by LIGO.

Two Black Holes 1.3 Billion Years Ago (Give or Take)

Black Hole #1
36X more massive than the Sun
210 km in diameter

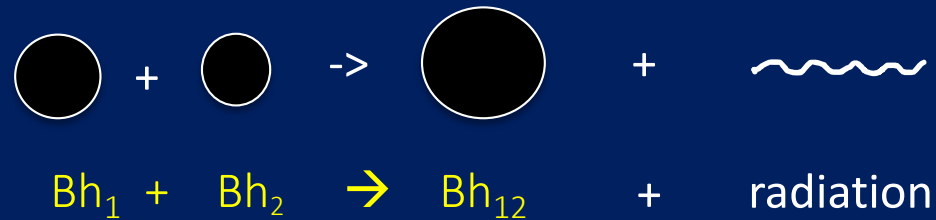
Black Hole #2
29X more massive than the Sun
170 km in diameter

Post merger the result
62 solar masses. Hence
 $3 \text{ solar masses} \times c^2$

Black Hole Entropy in Classical General Relativity

Black Hole Entropy in Classical GR -1

Area theorems of GR (Hawking, 1971)



Horizon area always increases $A_{12} > A_1 + A_2$

A BH cannot be split apart and hence the reverse process is impossible.

Bekenstein (1973):

- Take a hot object and throw it into a black hole, it disappears decreasing the entropy of the universe, and that violates the 2nd law of thermodynamics.
- A contradiction is avoided if one attributes a thermodynamic entropy to a black hole proportional to the area of the horizon.

Black Hole Entropy in Classical GR- 2

$$S = b \times \text{Area}_h = b \times 4\pi r_s^2 = b \times 4\pi(2GM)^2$$

where b is an undetermined constant

The area theorems imply the 2nd law for the generalized coarse-grained entropy:

$$S_{\text{gen}} = S_{\text{BH}} + S_{\text{matter}}$$

S_{matter} is the entropy of matter/radiation outside the BH horizon

(J. Bekenstein, A. Wall)

When the hot water bucket falls into the BH, S_{matter} decreases but S_{BH} increases by more.

Quantum Mechanics, Hawking Radiation and Information Puzzle

Hawking Temperature and Entropy

In one of the celebrated calculations of the second half of the 20th century Hawking (1974) discovered that BHs radiate due to quantum effects. He calculated the temperature of the BH and found his famous formula which is on his tombstone:



$$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$

$$T_{\text{sun}} = 3.6 \times 10^{-7} \text{ K}$$

$$T_{\text{earth}} = 0.1 \text{ K}$$

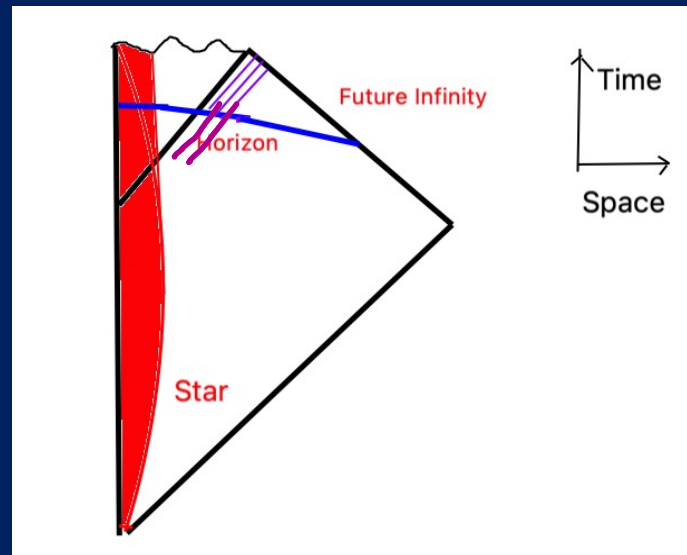
$$T_{M=10^{18} \text{ kg}} = 7000 \text{ K}$$

Using the first law of thermodynamics $dM = T dS$ the BH entropy is (note $b=1/4$)

$$S_{BH} = \frac{k_B c^3}{4 \hbar G} A$$

$$\frac{c^3}{\hbar G} = \frac{1}{l_p^2}$$

Universal Black Hole Temperature & Entropy



Penrose diagram of a BH formed by collapsing matter.

The formula for BH temperature and entropy is universal and independent of how the BH was formed.

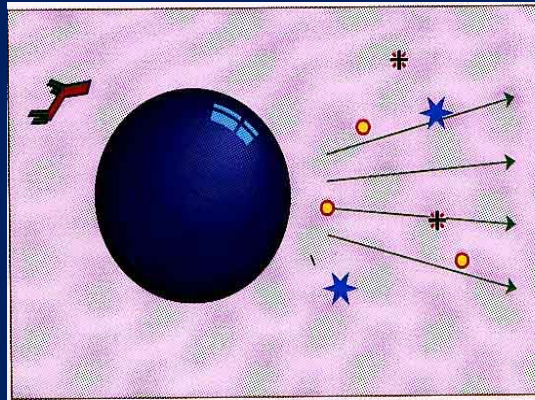
This is because the answer at late times depends only on very high energy Hawking quanta (or radiation) that emanate from near the horizon of the BH, after the collapsing matter has crossed the horizon.

The radiation appears to be thermal because we are NOT keeping track of the part that is on the other side of the horizon and is entangled with the radiation we are observing.

Information Loss is in conflict with Quantum Mechanics!

Like all hot bodies black holes radiate.

A black hole forms in various ways, but it always evaporates in the same way into thermal radiation leading to information loss as there is no memory of its initial state.



This violates a fundamental principle of unitarity in quantum mechanics:
A state cannot evolve to a mixed state in a closed system.

Hence, we have a serious problem

GR leads to information loss and QM cannot allow it!

Some people called it a crisis, but it presented an opportunity for progress

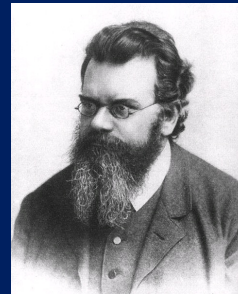
Resolving the Information Puzzle - Black Hole Microstates

Attempts to resolve the information puzzle 1990 onwards.....

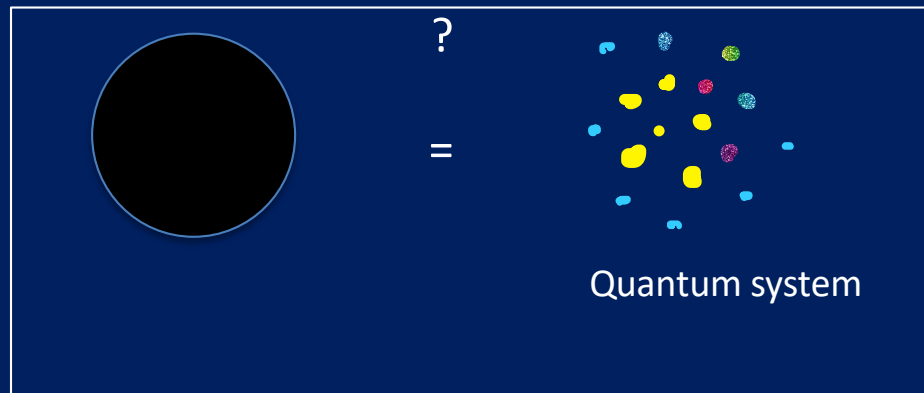
Since a BH is a thermodynamic system, can information loss for black holes be understood as due to an averaging process of many internal states, like in usual treatments of quantum statistical mechanics? Like the burning of a piece of wood?

Is there a theory of quantum gravity in which black hole entropy is given by Boltzmann's formula ?

$$S_{\text{BH}} = k_B \log(N)$$



What are the internal states of the BH, that would account for BH entropy?



Black hole micro-states

In 1996 A. Strominger and C. Vafa provided the first concrete evidence in a calculable supersymmetric model that the black hole space-time is a sort of a hydrodynamic description of more basic underlying quantum system of D-branes in string theory.

They showed that BH entropy can be calculated using Boltzmann's formula.

D-branes which are domain walls of string theory were discovered by Polchinski.

Hawking radiation and BH thermodynamics can be calculated in the framework of statistical mechanics in this constituent model of the black hole!

Microscopic model of Hawking radiation was developed post 1996 over many years:

J. David, G. Mandal, S. R. Wadia - Physics Reports (2002)

Higher order corrections to BH entropy were computed by A. Dabholkar, A. Sen, S. Murthy and others in the framework of string theory.

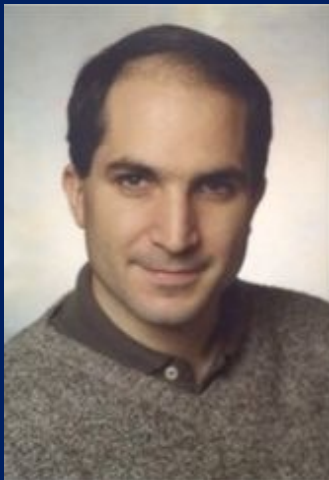
These development gave enormous evidence that string theory is the correct theory of quantum gravity and led to -

the holographic description of quantum gravity.

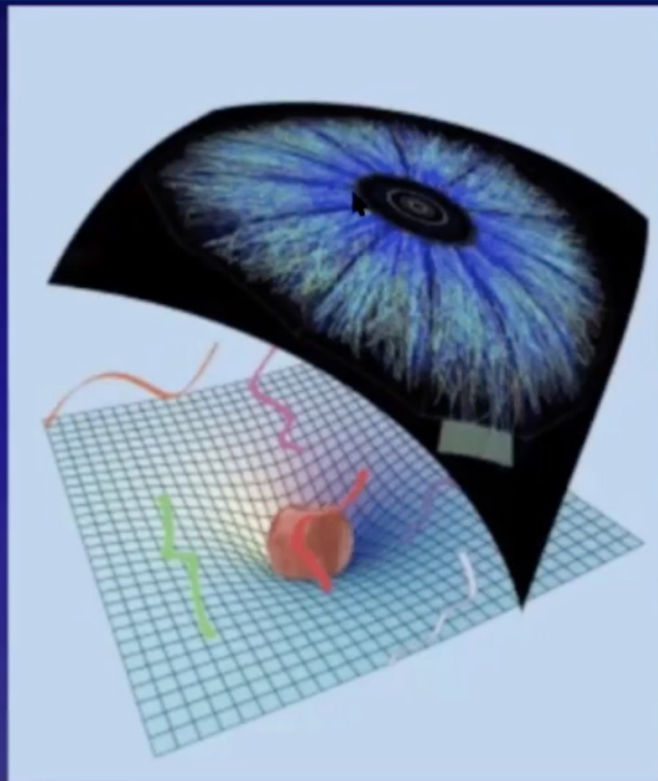
Holography and non-perturbative Quantum Gravity

Another approach to resolve the information puzzle is the AdS/CFT holography proposed by Maldacena (1997). It says that gravity and string theory in an asymptotically AdS (hyperbolic) space-time are holographically coded in a unitary QFT on its boundary.

Hence in principle one can track BH formation and evaporation in the QFT and that by the holographic correspondence would be a unitary process.



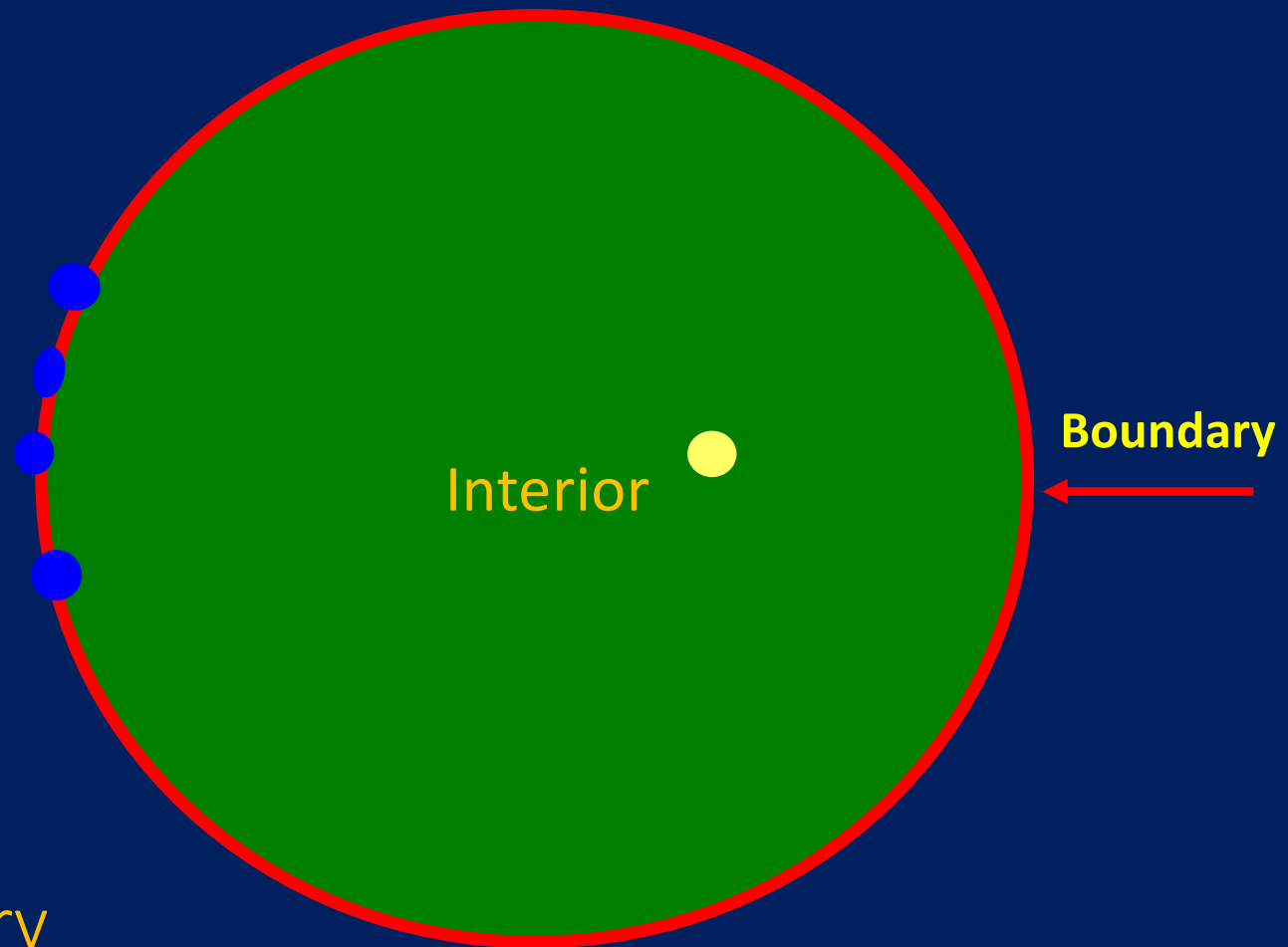
Juan Maldacena



Gravity in the interior \rightarrow
described by interacting particles on the boundary.

In the interior
is a string theory
(gravity) in
anti-de Sitter
space-time.

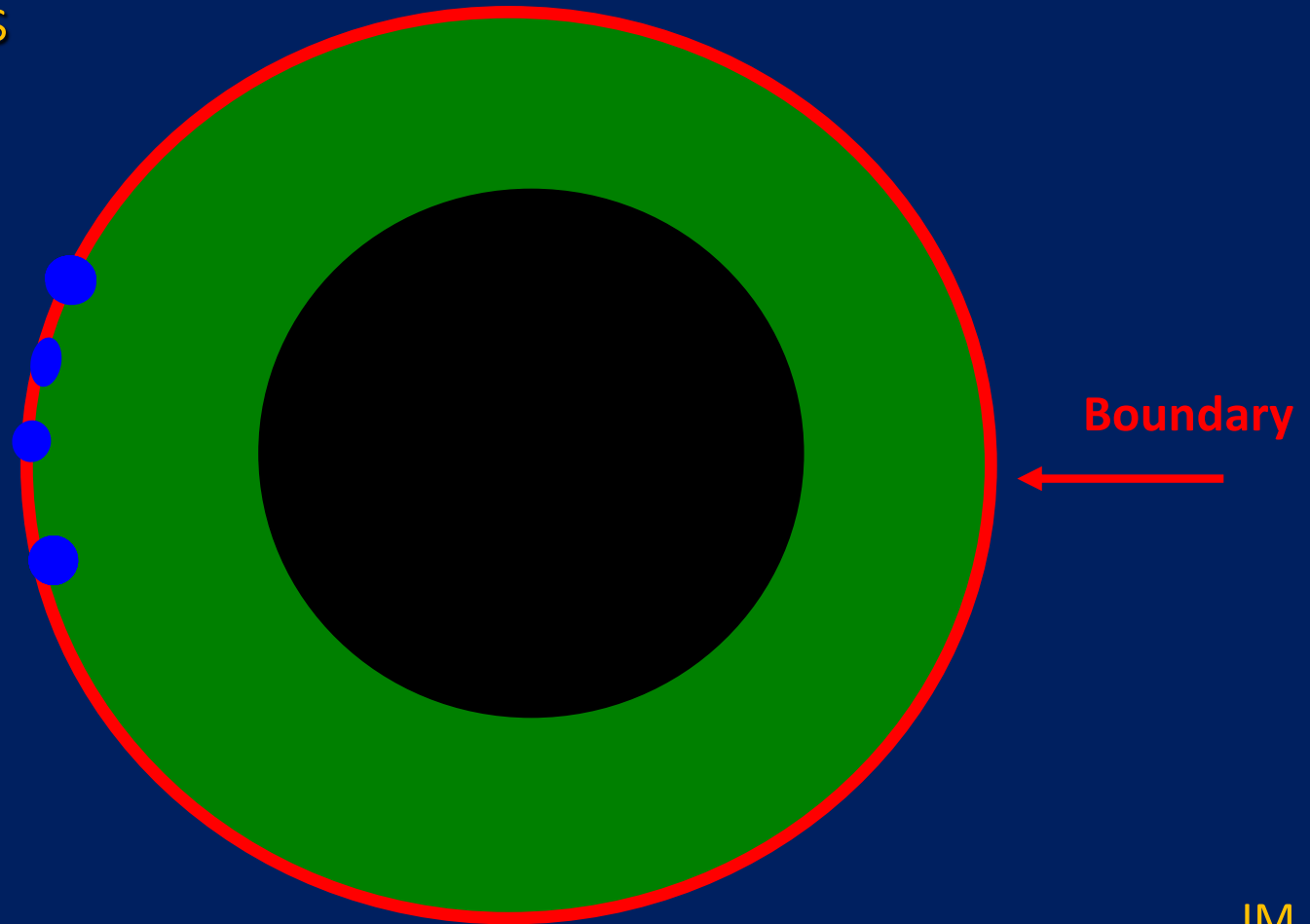
On the boundary
of AdS lives a unitary
QFT



JM

Black holes in AdS

Temperature and entropy
→ motion of particles
on the boundary
(Witten)



In principle
solves information
puzzle because

BH formation is thermalization of the theory on the boundary.
Hawking conceded that there is in principle no information loss.

JM

Entanglement Entropy in Gravity and Information Puzzle

How does gravity solve the Information Puzzle?

There is recent progress in answering this question. It involves understanding 'Information loss' as 'quantum information loss' and uses concepts from quantum information theory. In particular, the concept of Quantum Entanglement and its measure by the fine grained or entanglement entropy (Von Neumann, 1932) and its geometrization.

Work along these lines was initiated by S. Ryu and T. Takayanagi (2006), Hayden and Preskill (2007), Maldacena and Susskind, S. Mathur, A. Almheiri, D. Marolf, J. Polchinski, J. Sully (AMPS), S. Raju, K. Papadodimas, Dong, D. Harlow, N. Engelhardt, A. Wall and many others.

A good diagnostic is the Page curve (1993) of the time evolution of the entanglement entropy of an entangled composite quantum system. It is a direct consequence of the unitarity of quantum mechanics.

In what follows I will give a rough sketch of recent developments.

Entangled states and density matrix

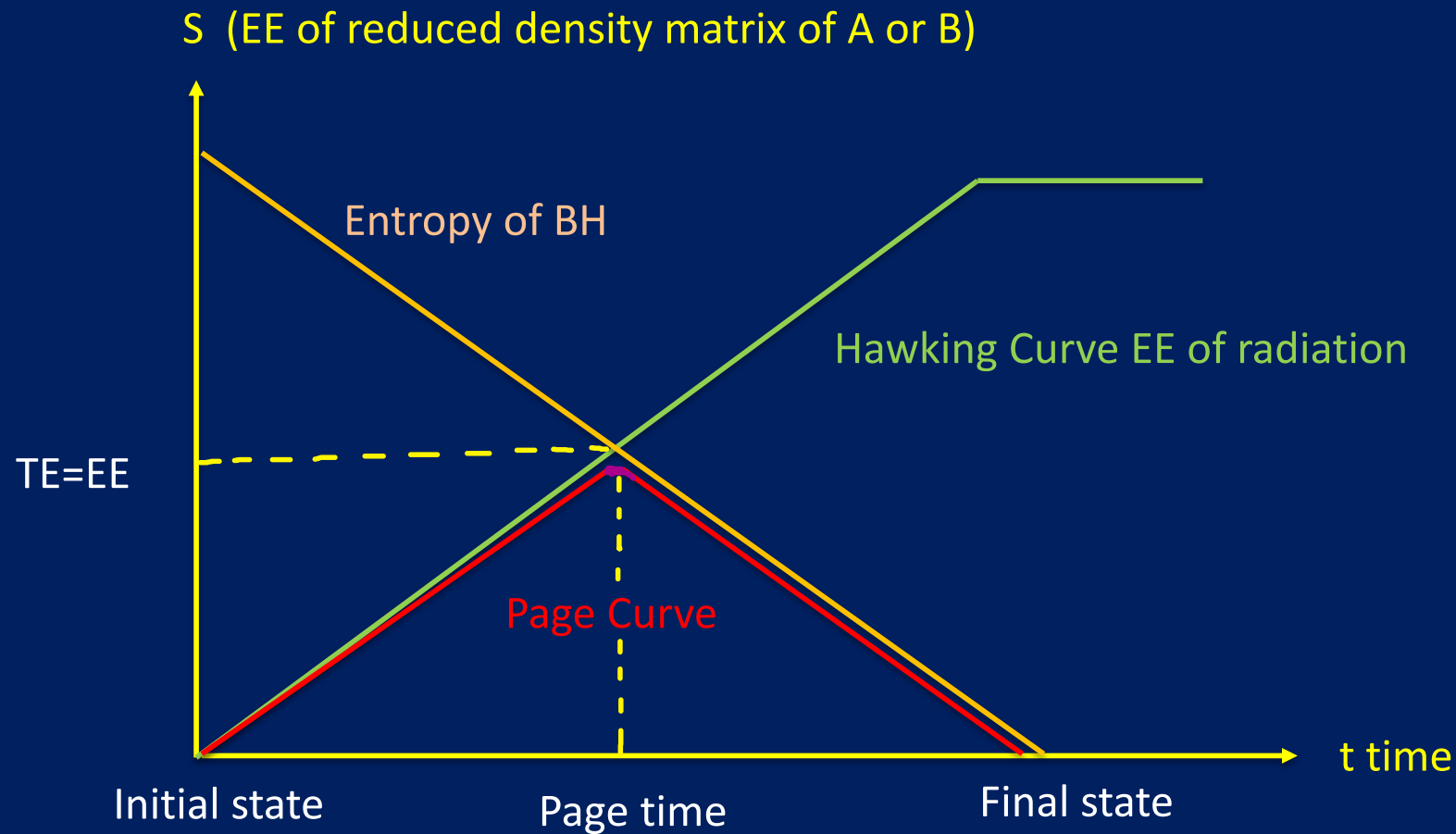
- Entanglement is a property of QM that distinguishes it from classical mechanics and accounts for its very non-intuitive consequences. Here we illustrate this very simple point in the case of two entangled two state systems.
- A and B are 2 state systems $A: \{|0\rangle_A, |1\rangle_A\}$, $B: \{|0\rangle_B, |1\rangle_B\}$, and $C = A \times B$ is the composite system
- We can consider separable product states like $|0\rangle_A \times |1\rangle_B$ in which both systems A and B are in a definite state.
- However, the state $|AB\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A \times |1\rangle_B - |1\rangle_A \times |0\rangle_B)$ is not a product of two states. It is an 'entangled' state of the system C. It does not belong to A or B.
- A state of a system in QM can be equivalently described by a 'density matrix' e.g. $\rho(A \times B) = |AB\rangle \langle AB|$. Since $|AB\rangle$ is normalized we have $\rho(A \times B)^2 = \rho(A \times B)$, a projection operator.
- In case we do not want to observe system B we trace over all the states of system B.
 $\text{tr}_B \rho(A \times B) = \rho(A)$ is the reduced density matrix of A and it turns out that $\rho(A)^2 \neq \rho(A)$

Entanglement Entropy (Von Neumann, 1932)

- Von Neumann entropy: $S = -\text{tr} \rho \ln \rho$ and $\text{tr} \rho = 1$
- $S = -\text{tr} \rho \ln \rho = 0$, iff ρ is a 'pure state'. A non-zero S is a measure of deviation from a maximally entangled pure state. $S(\rho(A \times B)) = 0$ but $S(\rho(A)) = \ln 2$
- $S(\rho(A)) = S(\rho(B))$, EE of the reduced density matrices are equal.
- $S(\rho) = S(U^\dagger \rho U)$, where U is a unitary matrix. Hence $S(\rho)$ is constant under unitary time evolution.
- $S(\rho) \leq \log N$, where N is the dim of the Hilbert space.
 $\log N = S_{\text{th}}$ is the thermodynamic entropy, which has no information
- $S_{\text{th}} - S_{\text{vn}}(\rho) \geq 0$, is a measure of information loss.
- For a bipartite system $A \times B$, $S(A \cup B) \leq S(A) + S(B)$, which is very different from the thermodynamic entropy, where the entropy of the combined system is always \geq the entropy of the parts.

Hawking and Page curves for the BH-Radiation entangled system

Imagine evaporating BH is system A and Hawking radiation is system B



Hawking curve violates unitarity

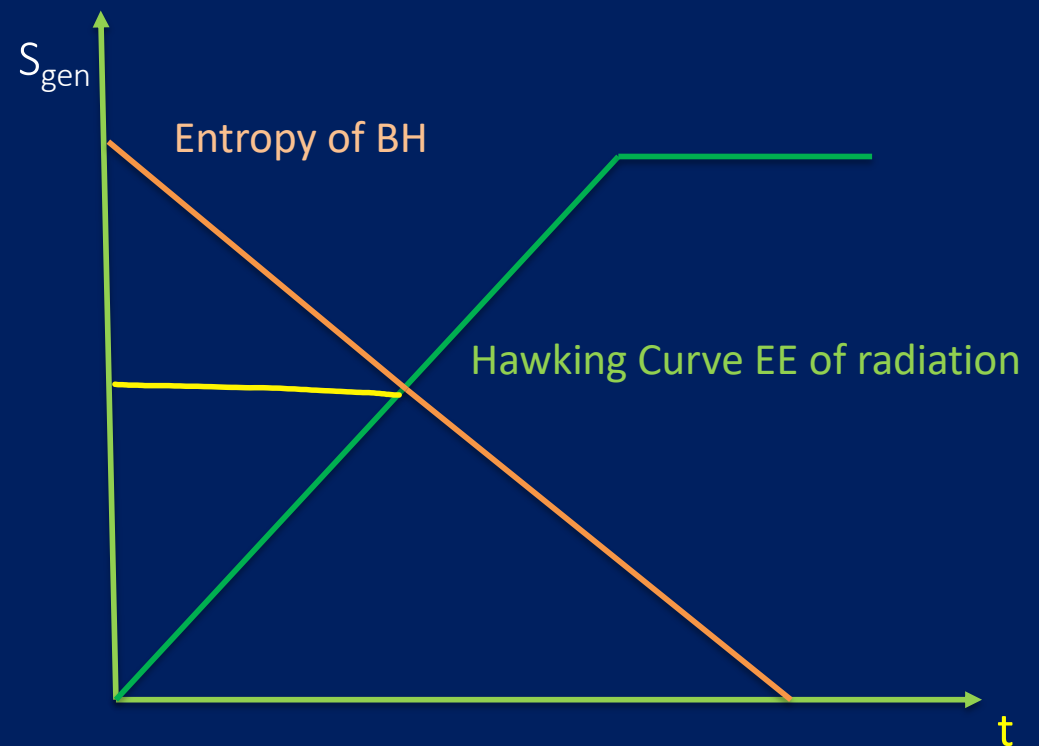
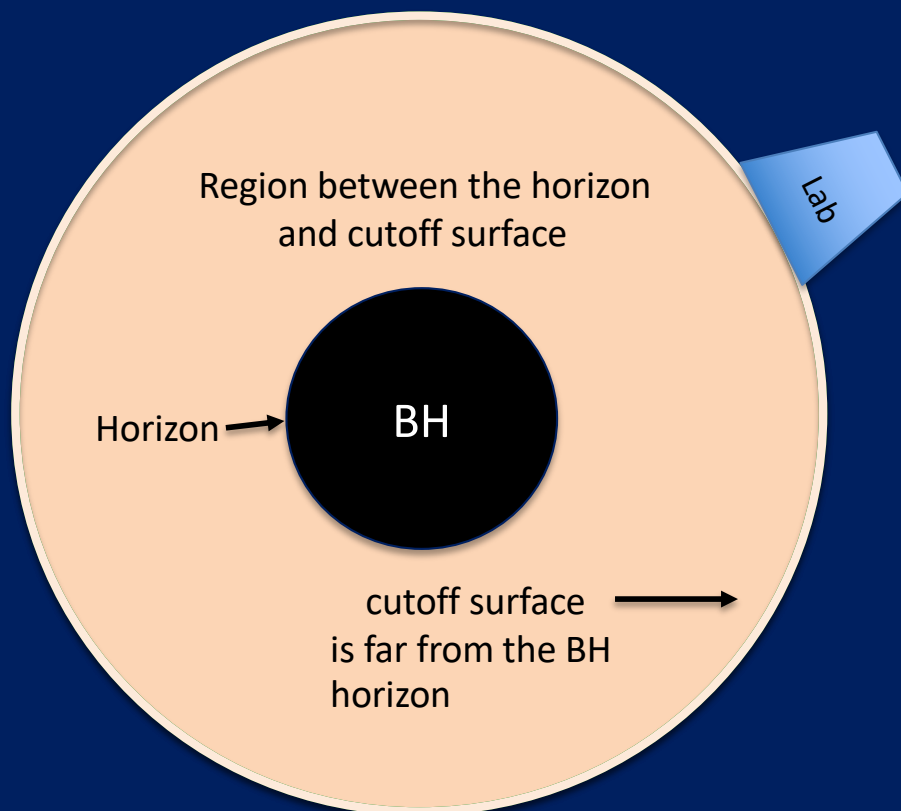
Page curve is consistent with Unitarity

Entanglement Entropy formula that computes the Hawking curve of an evaporating BH

Recall the Bekenstein-Hawking formula for the generalized entropy. It can be shown that it is a formula for EE. The area term is the EE of the Hawking quanta near the inside and outside the horizon (Sorkin 1982) and S_{matter} is the EE of matter/radiation outside the horizon.

$$S_{\text{gen}} = \frac{A}{4G} + S_{\text{matter}} \quad (\text{units } c=\hbar=1) \quad (\text{N. Engelhardt and A. Wall, 2015})$$

(The UV divergences between the two terms cancel, and the Newton constant is finite).



The EE of radiation at the Page time exceeds the TE of the BH and that is a violation of unitarity

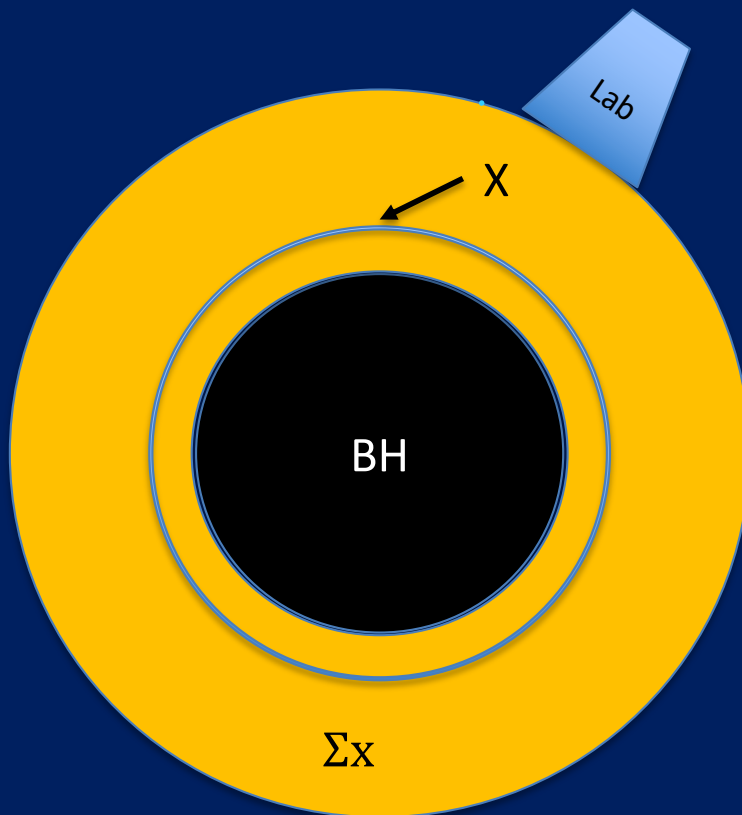
What is the correct EE formula in gravity?

It is the same formula as before except the 'surface' under consideration is not the horizon!
The 'surface' is obtained by looking at the extrema of

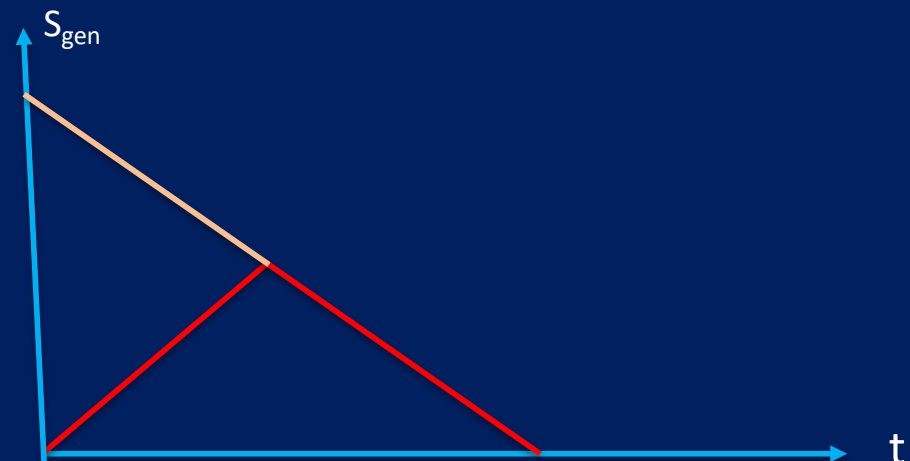
$$S(X(t))_{\text{gen}} = \left[\frac{A(X(t))}{4G} + S(\Sigma_X(t))_{\text{matter}} \right]$$

Pennington (2019)
Almheiri, Engelhardt, Marolf,
Maxfield (2019)

where $X(t)$ is a surface or a union of surfaces, $\Sigma_X(t)$ is the region bounded by $X(t)$ and the cut-off surface. (If X is the horizon, then we get Hawking's result).



The key point is that $X(t)$ can also be partly or fully inside the horizon and can be a union of surfaces...around islands, as required by the variational principle.



Summary

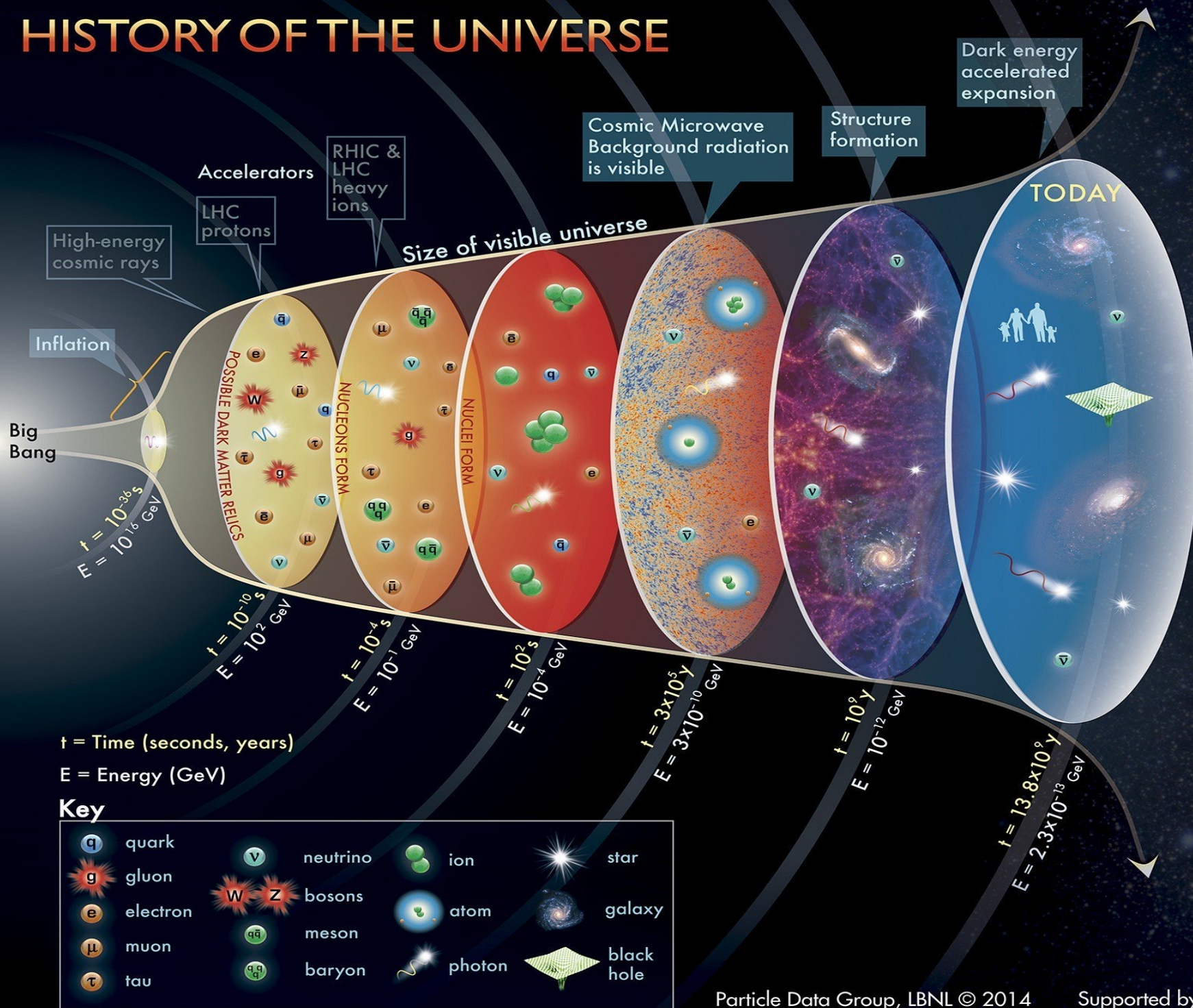
We have reviewed a particular promising development in theoretical physics which brings us closer to a reconciliation of Quantum Mechanics and General Relativity within semi-classical gravity. This has been enabled by a new gravity formula for the entanglement entropy. Explicit verification of the Page curve has been done in low dimensional systems.

In this talk I have not included the following topics:

1. Discussion of the interior of the BH and its imprint on the Hawking radiation.
2. Wormhole geometries and entanglement entropy.
3. Explicit dynamical evolution models of BH formation and evaporation in lower dimensional systems like the SYK model of random spins which is holographic to 2-dim gravity.
(A. Dhar, A. Gaikwad, L. Joshi, A. Kaushal, G. Mandal, SRW; Maldacena, Milekhin)
4. Algebraic quantum field theory and Von Neumann algebras for defining finite entanglement entropy.

One of the reasons to study BHs is to understand the principles and degrees of freedom of quantum gravity and develop a framework to discuss the quantum nature of the cosmological event horizon which also has a characteristic temperature and Hawking radiation.

HISTORY OF THE UNIVERSE



Brian Swimme: The Universe is a Green Dragon

“Most amazing is this realization that everything that exists in the universe came from a common origin. The material of your body and the material of my body are intrinsically related because they emerged from and are caught up in a single energetic event. Our ancestry stretches back through the life forms and into the stars, back to the beginnings of the primeval fireball. This universe is a single multiform energetic unfolding of matter, mind, intelligence, and life. And all of this is new. None of the great figures of human history were aware of this. Not Plato, or Aristotle, or the Hebrew Prophets, or Confucius, or Thomas Aquinas, or Leibniz, or Newton, or any other world-maker. We are the first generation to live with an empirical view of the origin of the universe. We are the first humans to look into the night sky and see the birth of stars, the birth of galaxies, the birth of the cosmos as a whole. Our future as a species will be forged within this new story of the world.”

Thank you!