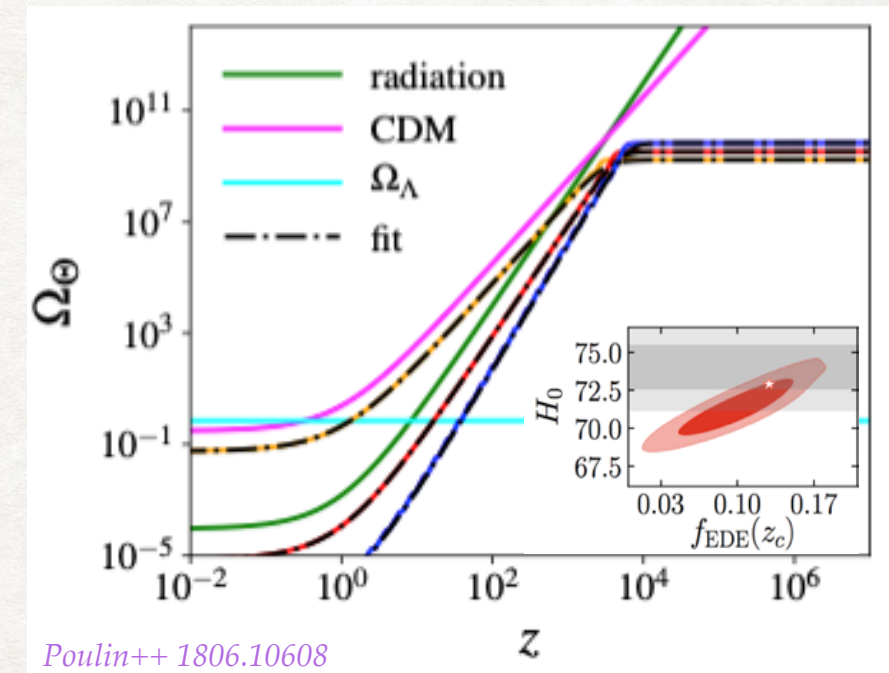
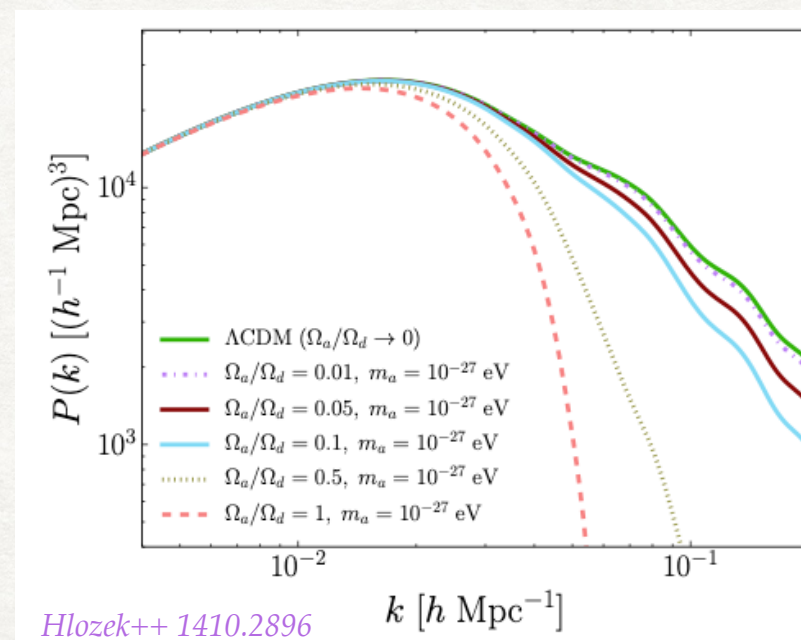
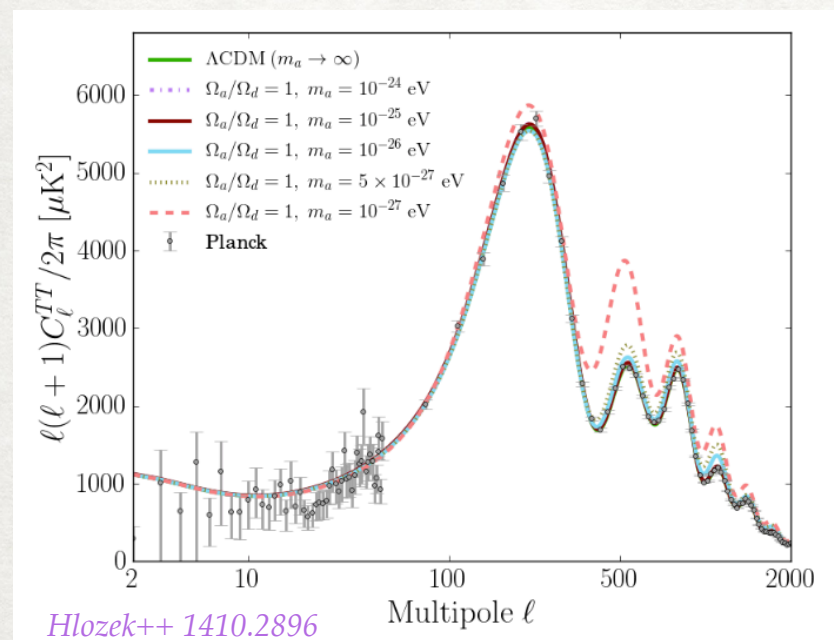


Cosmological signatures of axion-like particles



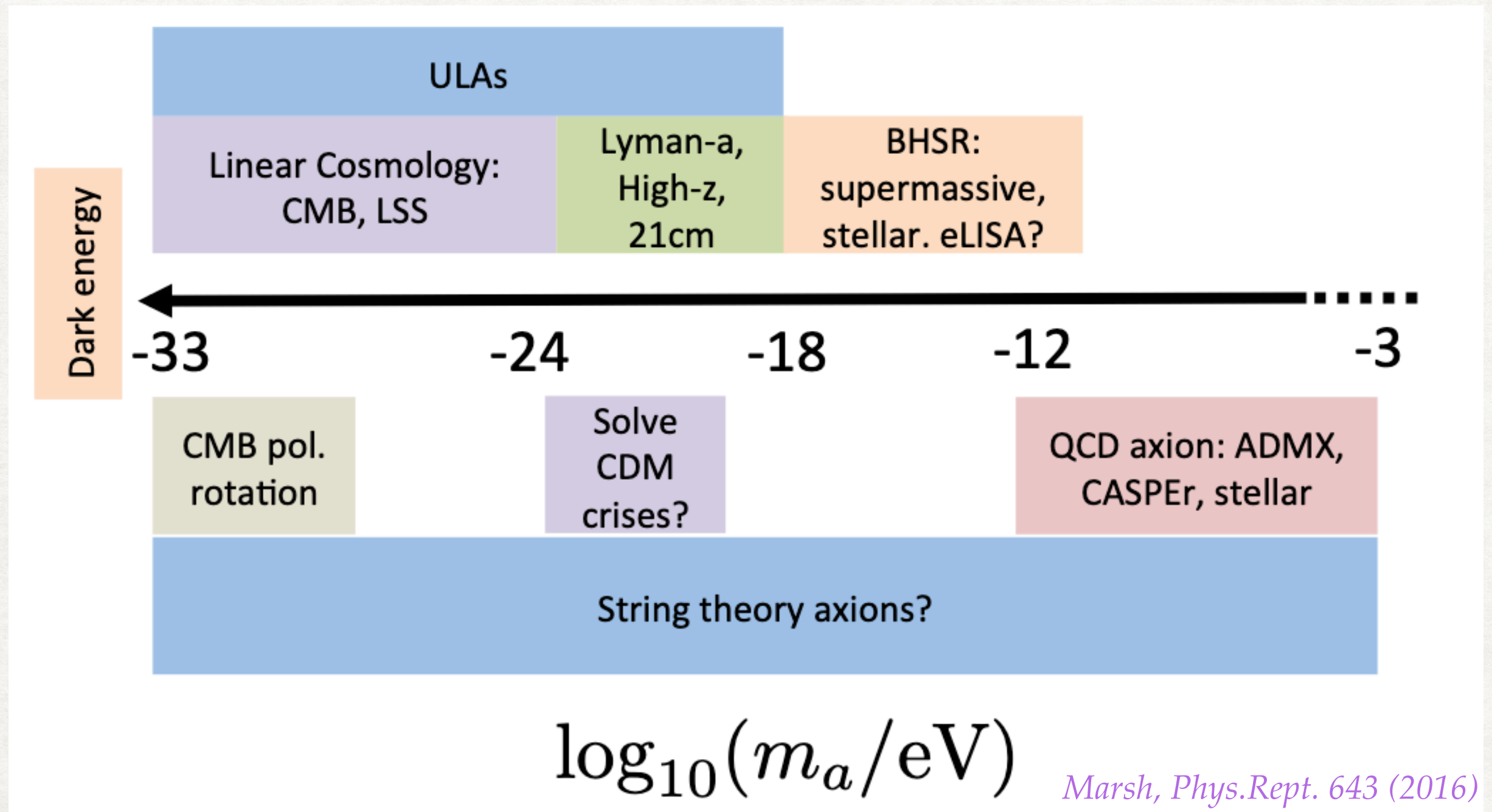
Vivian Poulin

Laboratoire Univers et Particules de Montpellier
CNRS & Université de Montpellier

ICTS, "Less travelled path of Dark Matter"
November, 11 2020



The Axion landscape



Theory of cosmological ALP in a nutshell

Marsh, 1510.07633

- Typical mass range interesting for us $[10^{-33}, 10^{-18}]$
- Potential has the form: $V(\phi) = \Lambda^4(1 - \cos(\phi/f))$
- f is the spontaneous symmetry breaking scale, Λ the non-perturbative physics scale. Both scales enter the axion mass $m_a \approx \Lambda^2/f$.
- The energy density and pressure of the field are given by:
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V_n(\phi), \quad P_\phi = \frac{1}{2}\dot{\phi}^2 - V_n(\phi)$$
- The KG equation governs the field dynamics
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV_n(\phi)}{d\phi} = 0$$
- In the limit $\phi/f \ll 1$, $V(\phi) = m_a^2\phi^2/2$ such that $\ddot{\phi} + 3H\dot{\phi} + m_a^2\phi = 0$.
- This is exactly solvable in the limit where H is dominated by a single component.

Producing a cosmological population of ALP

$$\Omega_a h^2 = \frac{\rho_a}{\rho_{c,0}} h^2 = F \Omega_{\text{cdm}} h^2 \simeq 0.12 F$$

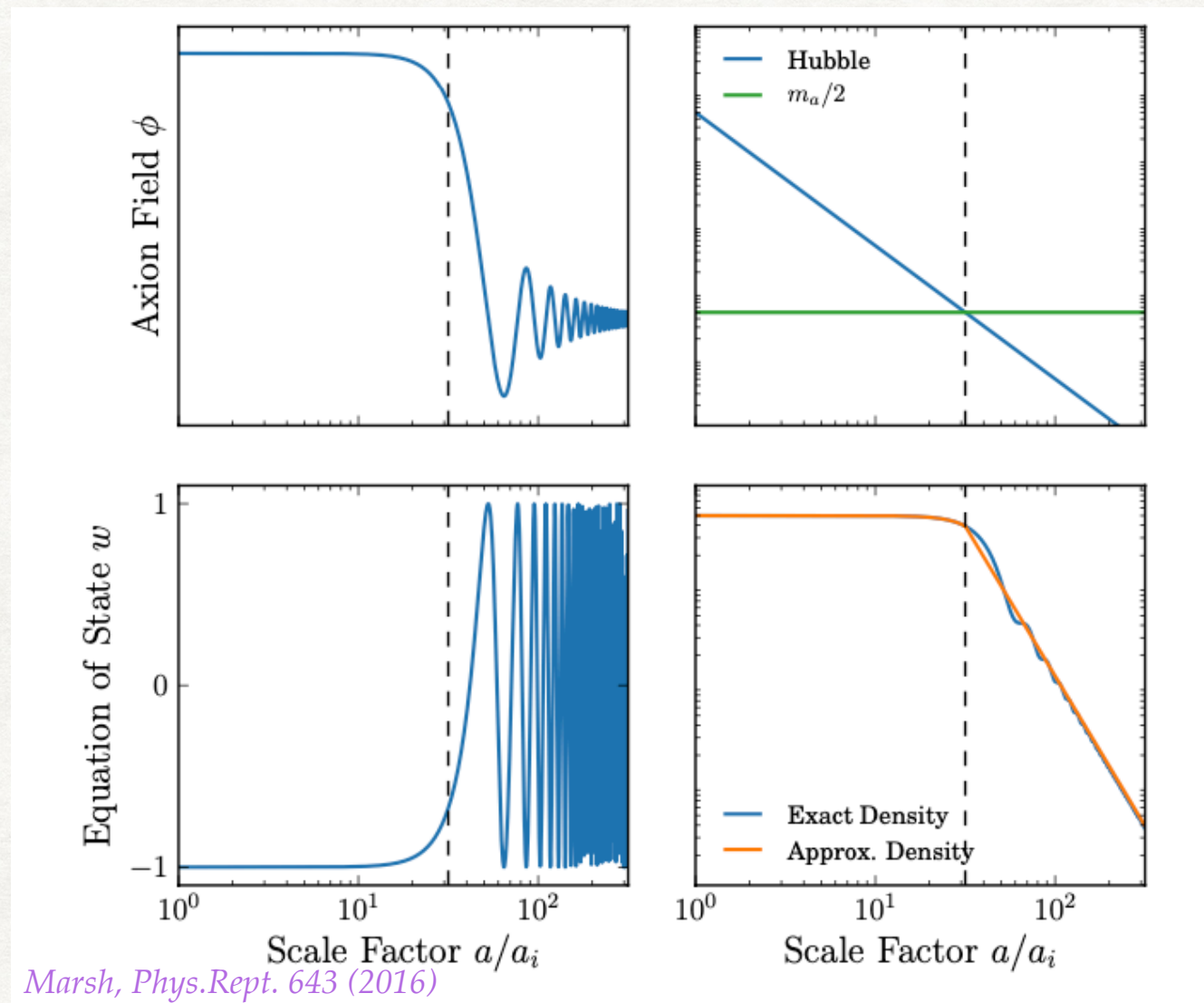
Lecture by D.J.E. Marsh

- Thermal population produced from the thermal bath typically from $\pi\pi \rightarrow \pi a$. The axion-SM coupling can be arbitrary in the general case.
- Production from the decay of a mother particle X with $m_X > m_a$. The axion would contribute to the radiation bath N_{eff} an amount (if X particles dominate energy density):

$$\Delta N_{\text{eff}} = \frac{43}{7} \left(\frac{10.75}{g_{\star S}(T_r)} \right)^{1/3} \frac{B_a}{1 - B_a}$$

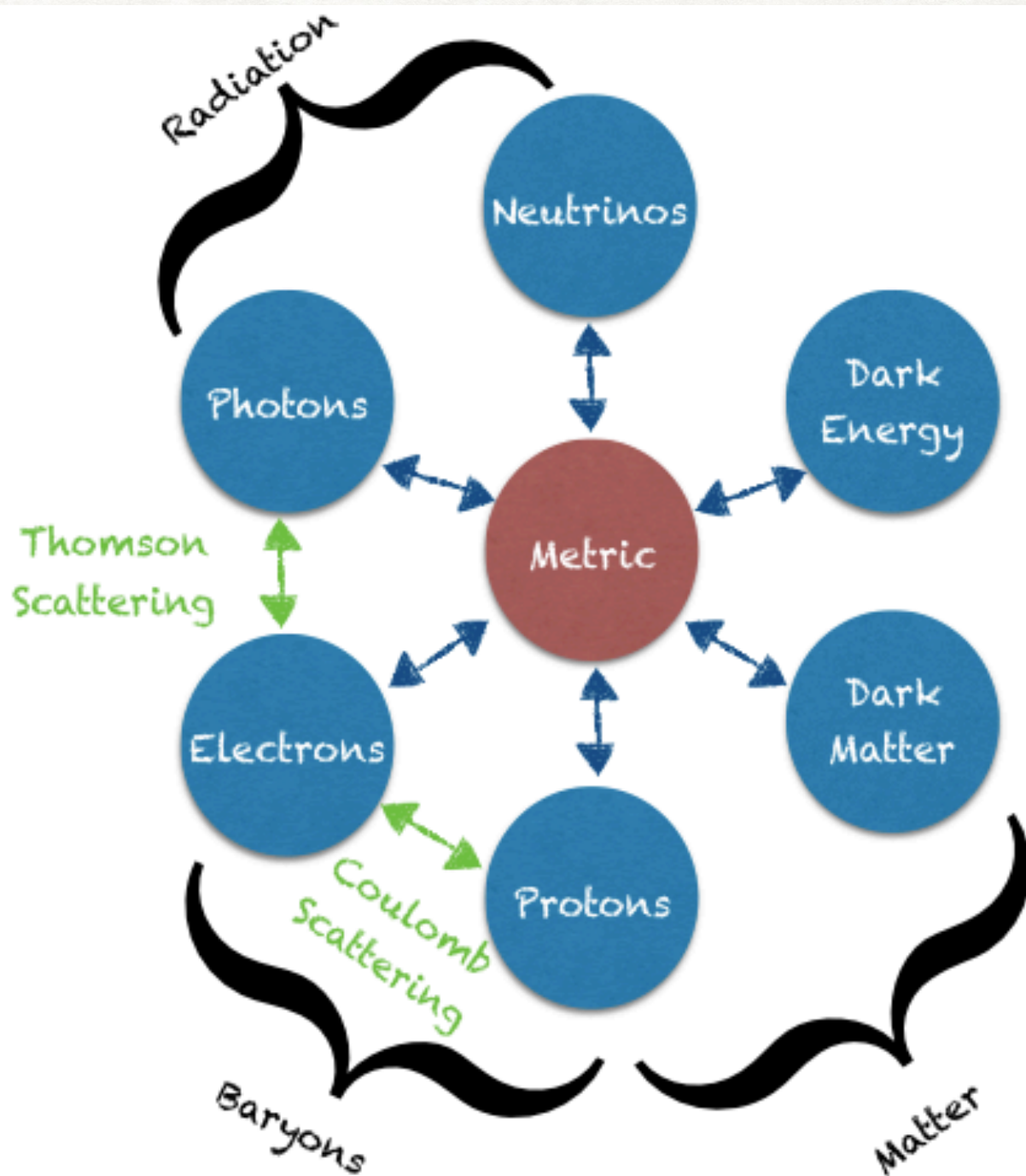
- Production from the decay of topological defects: if $f_a < H_I/2\pi$, Peccei-Quinn symmetry is unbroken during inflation; leads to horizon patches with different ϕ_i and formation of “topological defects”. These are unstable and leads to the creation of axion particles.
- The “misalignment mechanism”: population from the coherent initial displacement of the axion field $\phi_i \neq 0$. This is the scenario we focus in the remainder of this lecture.

The misalignment mechanism



- $H \gg m_a$: the field is frozen $w \equiv P_\phi/\rho_\phi = -1$ (nb: $\dot{\phi}_i = 0$); $H < m_a$: field ϕ oscillates around 0 and $\rho \propto a^{-3}$.
- Once the fields start oscillating: Virial theorem on the "cycle-average" field $\langle \dot{\phi}^2/2 \rangle = \langle V_\phi \rangle \Rightarrow \langle w \rangle \simeq 0$
- Cosmological density is estimated as $\rho_{\text{today}} \simeq \rho_{\text{osc}} (a/a_{\text{osc}})^{-3} \simeq (m_a^2 \phi_i^2/2)(a/a_{\text{osc}})^{-3}$ with $H(a_{\text{osc}}) \sim m_a$
- ALP are a good DM candidate if $H(a_{\text{osc}}) \sim m_a > H(a_{\text{eq}}) \simeq 10^{-28} \text{eV}$

Cosmological probes are sensitive to ALP perturbations



$$\delta g_{\mu\nu}(\vec{x}, t) = 8\pi G \delta T_{\mu\nu}(\vec{x}, t)$$

- Focus on the 4 scalar degrees of freedom
 $\{\delta_x \equiv \delta\rho_x/\rho_x, \theta_x \equiv kv_x, \delta P_x, \sigma_x\}$

$$\bar{\rho} \delta = -\delta T_0^0$$

$$(\bar{\rho} + \bar{P})\theta = \sum_i \partial_i \delta T_i^0$$

$$\delta P = \frac{1}{3} \sum_i \delta T_i^i$$

$$(\bar{\rho} + \bar{P})\nabla^2 \sigma = - \sum_{i,j} \left(\partial_i \partial_j \frac{1}{3} \nabla^2 \delta_{ij} \right) \delta T_j^i.$$

See e.g. Baumann "cosmology" lectures
 Dodelson, "Modern Cosmology"

ULA perturbations

Ma&Bertschinger astro-ph/9506072, Hu astro-ph/9801234

- Let's work in the “newtonian” gauge: $ds^2 = a^2(\tau) \{ - (1 + 2\Psi)d\tau^2 + (1 - 2\Phi)dx^i dx_i \}$
- From the perturbed KG equations we have: $\delta\phi'' + 2\mathcal{H}\delta\phi' + (k^2 + m_a^2 a^2)\delta\phi = (\Psi' + 3\Phi')\phi' - 2m_a^2 a^2 \phi\Psi$
- And we can use the definition of $T_{\mu\nu}$ to relate $\delta\phi(\vec{x}, t) \equiv \phi(\vec{x}, t) - \bar{\phi}(t)$ to $\{\delta\phi, \theta_\phi, \delta P_\phi, \sigma_\phi\}$. We obtain:

$$\delta\rho_\phi = a^{-2}(\dot{\phi}\delta\phi - \dot{\phi}^2\Psi) + m_a\phi\delta\phi$$

$$\delta p_\phi = \delta\rho_\phi - 2m_a\phi\delta\phi$$

$$(\rho_\phi + p_\phi)\theta_\phi = a^{-2}k^2\dot{\phi}\delta\phi$$

$$\sigma_\phi = 0$$

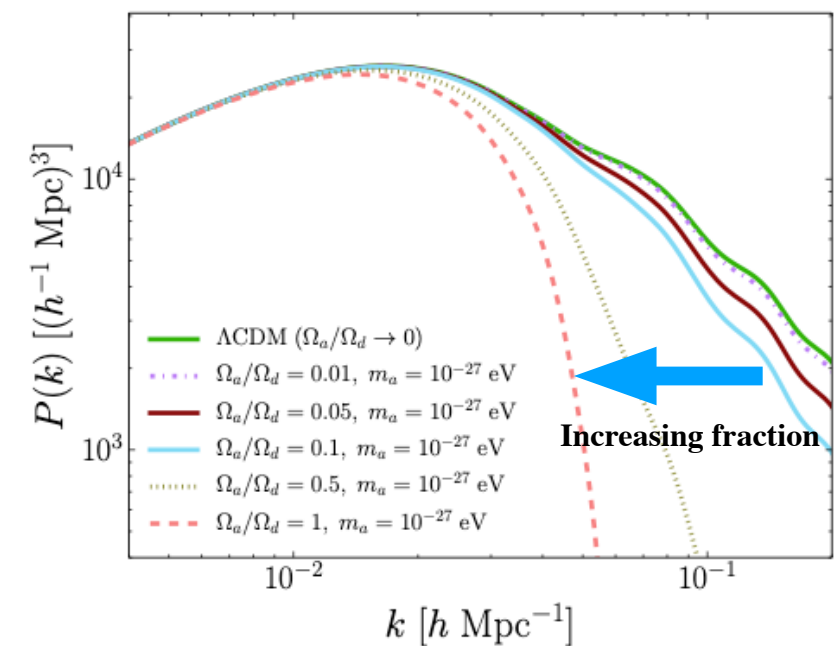
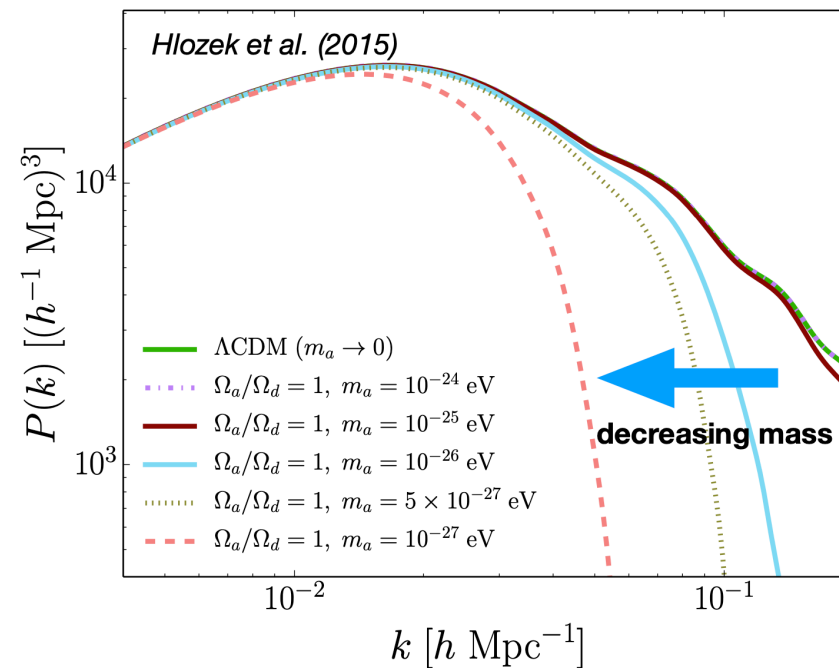
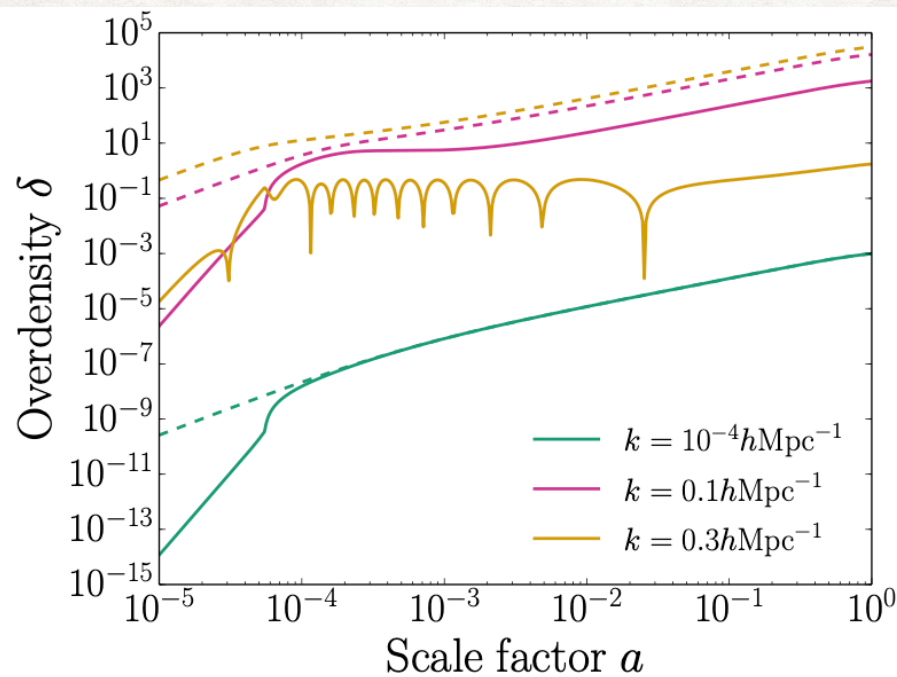
- Alternatively we can write fluid equations from $\nabla\delta T^\mu_\nu = 0$ as

$$\delta'_\phi = - (1 + w_\phi)(\theta_\phi - 3\Phi') - 3\mathcal{H}\left(\frac{\delta p_\phi}{\delta\rho_\phi} - w_\phi\right)\delta_\phi \qquad \theta'_\phi = -\frac{a'}{a}(1 - 3w_\phi)\theta_\phi - \frac{w'_\phi}{1 + w_\phi}\theta_\phi + \frac{\delta p_\phi/\delta\rho_\phi}{1 + w_\phi}k^2\delta_\phi + k^2\Psi$$

- Key quantities: The equation of state w and the sound-speed $c_s^2 \equiv \frac{\delta p_\phi}{\delta\rho_\phi}$.

The cycle-average perturbations

- It is possible to get an approximate expression for the sound speed from the 'cycle-average' $c_{\text{eff}}^2 \equiv \langle \delta P_\phi \rangle / \langle \delta \rho_\phi \rangle$
- Famous result: $c_{\text{eff}}^2 = \frac{k^2/4m_a^2 a^2}{1 + k^2/4m_a^2 a^2}$. There is a power suppression on scales $k^2 \gg 4a^2 m_a^2$.
See Hu astro-ph/9801234, Hlozek++ 1410.2896, VP++ 1806.10608
- On small scales the equation for δ_a (+Poisson equation) becomes: $\ddot{\delta}_a + 2H\dot{\delta}_a + (k^2 c_{\text{eff}}^2 / a^2 - 4\pi G \rho_a) \delta_a = 0$
- Axion jeans scale: $k_j = (16\pi G a \rho_{a,0})^{1/4} m_a^{1/2} \simeq 70 a^{1/4} \left(\frac{\Omega_a h^2}{0.12} \right)^{1/4} \left(\frac{m_a}{10^{-22} \text{eV}} \right)^{1/2} \text{Mpc}^{-1}$



The Halo Mass Function

- The number of halos in the ULA DM scenario is reduced: connection to the missing satellites problem?

$$\frac{dn}{d \ln M} = -\frac{1}{2} \frac{\rho_m}{M} f(\nu) \frac{d \ln \sigma^2}{d \ln M}$$

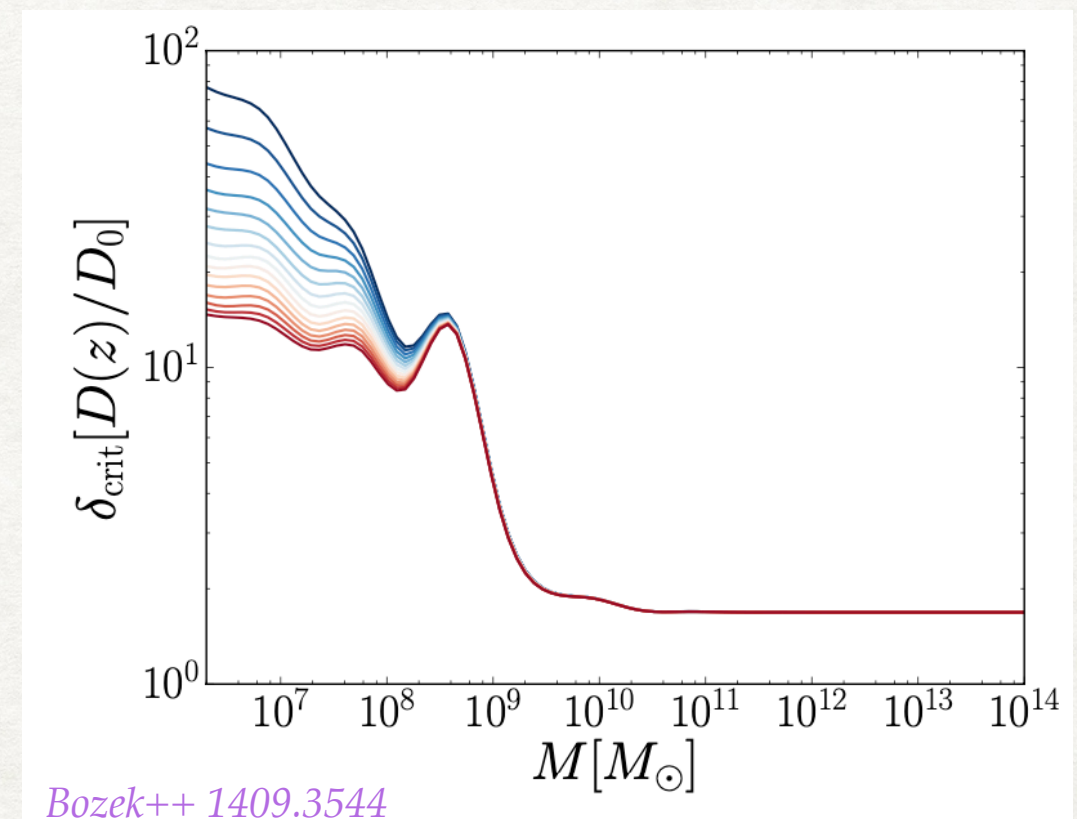
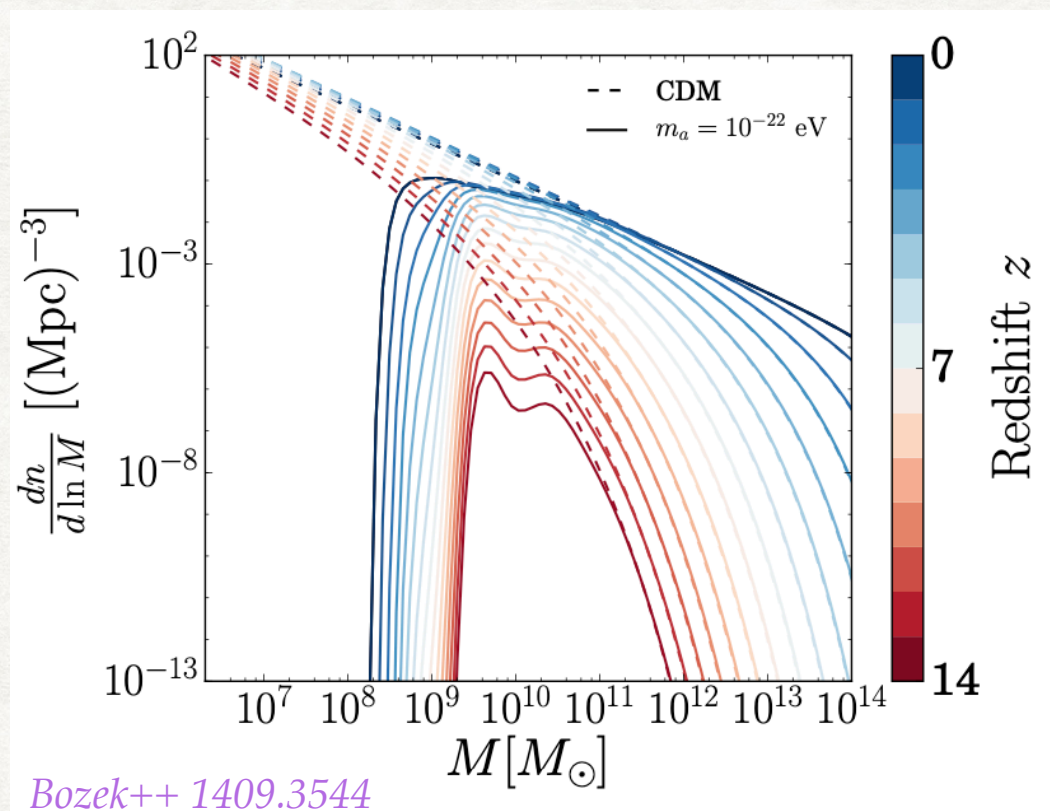
$$f(\nu) = A \sqrt{\frac{2}{\pi}} \sqrt{q} \nu (1 + (\sqrt{q} \nu)^{-2p}) \exp \left[-\frac{q \nu^2}{2} \right]$$

$$\nu \equiv \frac{\delta_{\text{crit}}}{\sigma}$$

(From CDM simulation)

Press-Schechter ApJ 1974, Sheth&Tormen astro-ph/9901122

- The scale-dependent growth can be incorporated via a scale-dependent collapse threshold $\delta_{\text{crit}}(M, z) = 1.686 \mathcal{G}(M, z)$



See also recent update Kulkarni&Ostriker 2011.02116

- This impacts also the reionization of the universe by the first stars and the UV luminosity function, 21cm....

The ULA halo density profile

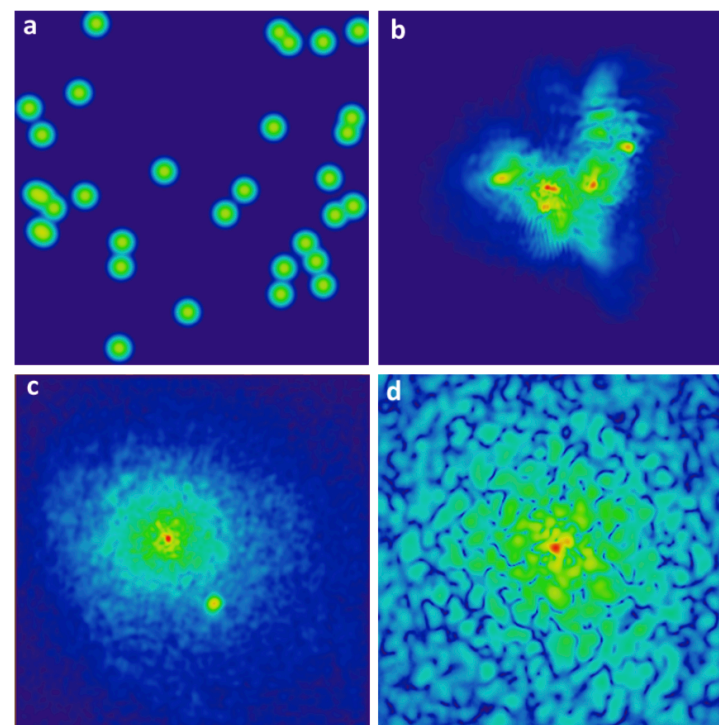
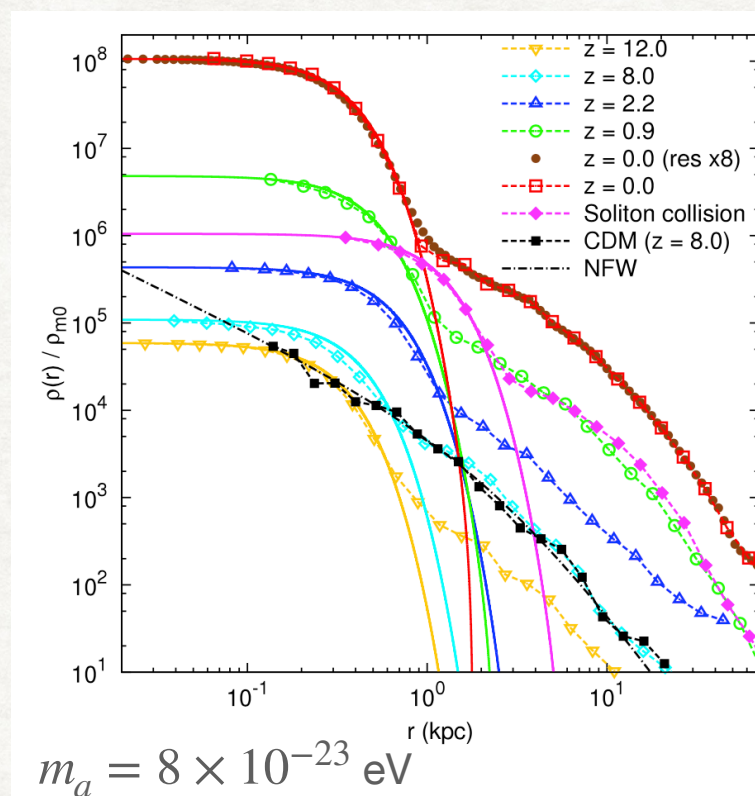
- The wave-like nature of the ULA shows up on scale $\lambda_{\text{dB}} = 1/mv \simeq 10^{-2} \left(\frac{m_a}{10^{-22} \text{ eV}} \right)^{-1} \left(\frac{v}{100 \text{ km/s}} \right)^{-1} \text{ kpc}$
- Density profile comes from solving the ‘Schrödinger-Poisson’ equation $\left[i \frac{\partial}{\partial \tau} + \frac{\nabla^2}{2} - aV \right] \psi = 0$ $\psi = \mathcal{X}(r)e^{-i\gamma t}$ $\rho = \mathcal{X}^2$
- A ‘soliton’ forms (stationary wave of constant energy)

$$\rho(r) = \Theta(r_\epsilon - r)\rho_{\text{sol}}(r) + \Theta(r - r_\epsilon)\rho_{\text{NFW}}(r)$$

$$\rho_{\text{sol}}(r) = \frac{\rho_{\text{sol}}(0)}{(1 + (r/r_{\text{sol}})^2)^8}$$

$$r_{\text{sol}} = 22 \left(\frac{\rho_{\text{sol}}(0)}{\rho_{\text{crit}}} \right)^{-1/4} \left(\frac{m_a}{10^{-22} \text{ eV}} \right)^{-1/2} \text{ kpc}$$

$$r_{\text{sol}} \sim r_{\text{dB}} \sim r_J$$

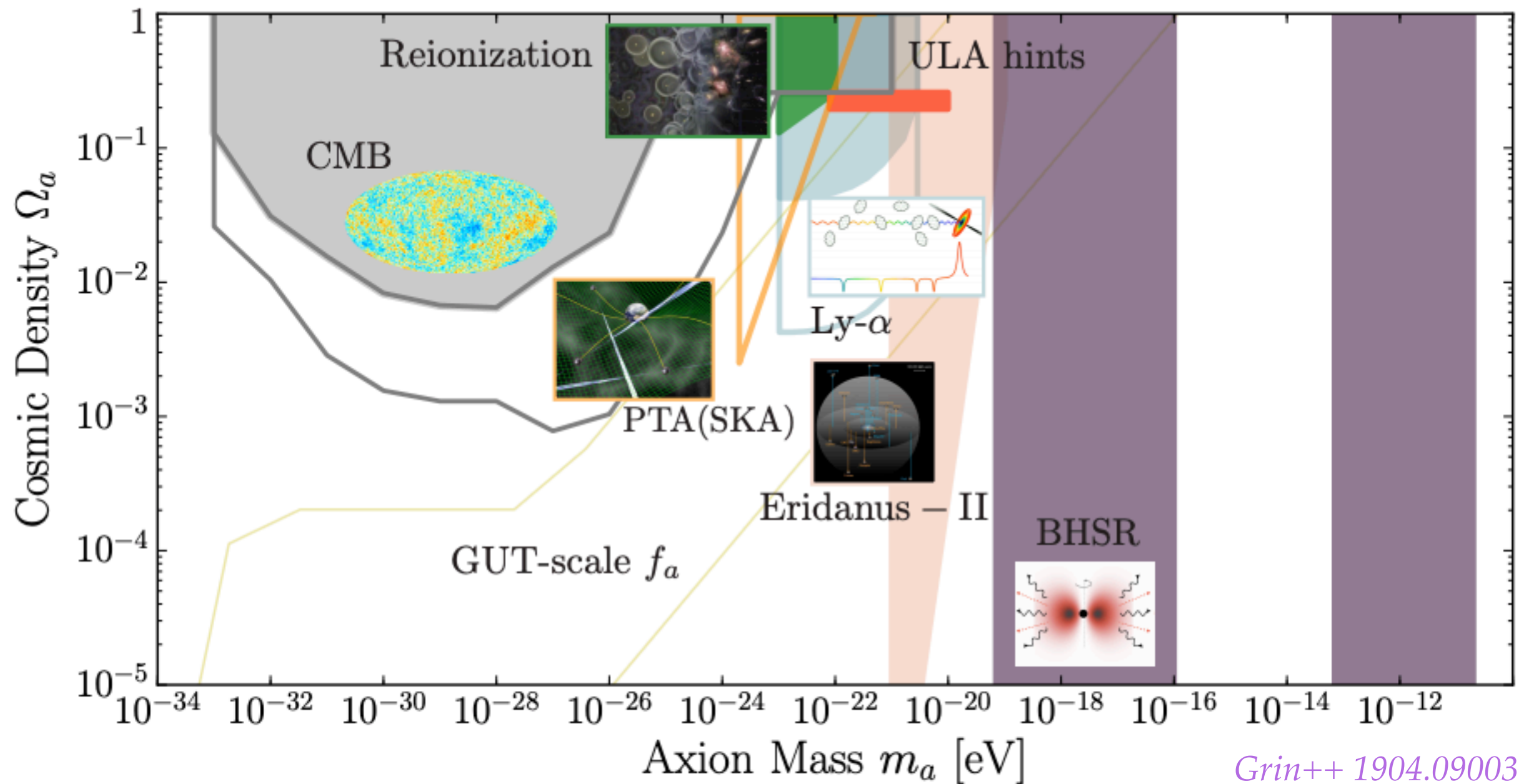


Schive++ 1407.7762

- ULA might solve the ‘core-cusp’ problem of CDM for $m_a \lesssim 10^{-22} \text{ eV}$! *Marsch & Pop, 1502.03456*

Bounds on the ALP mass & fraction

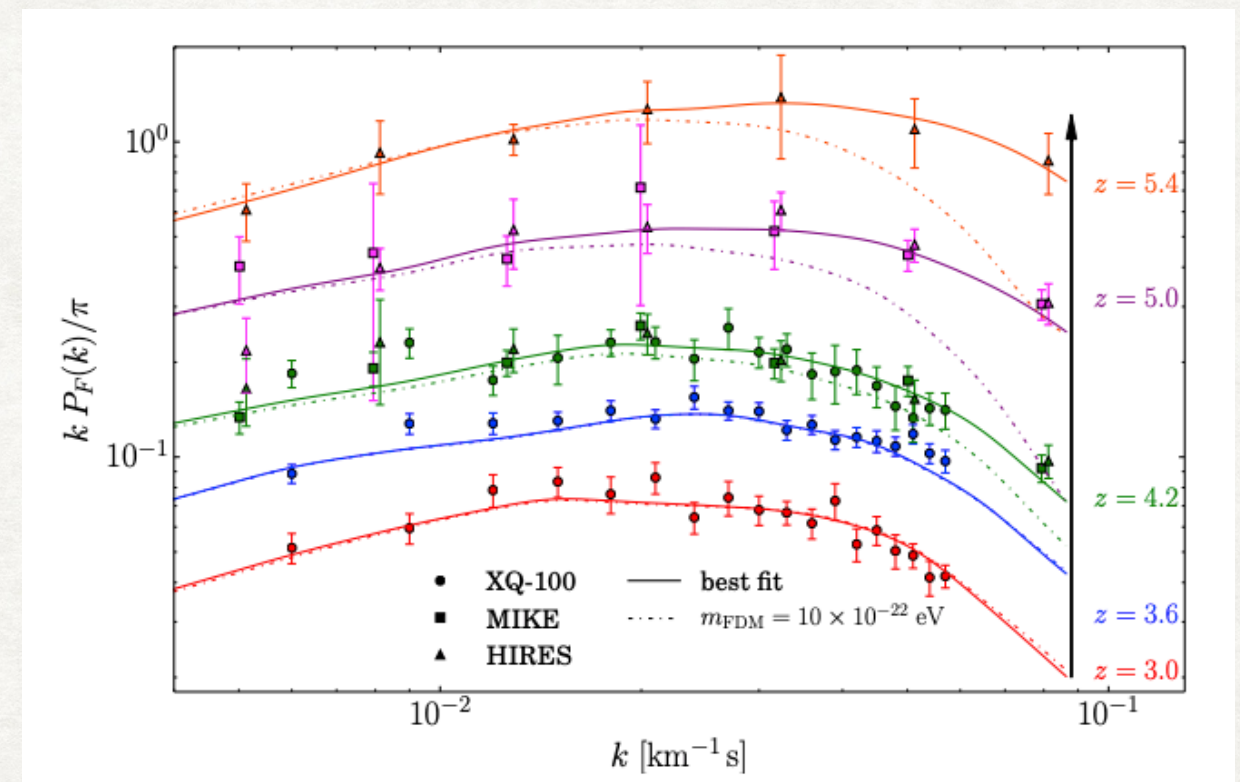
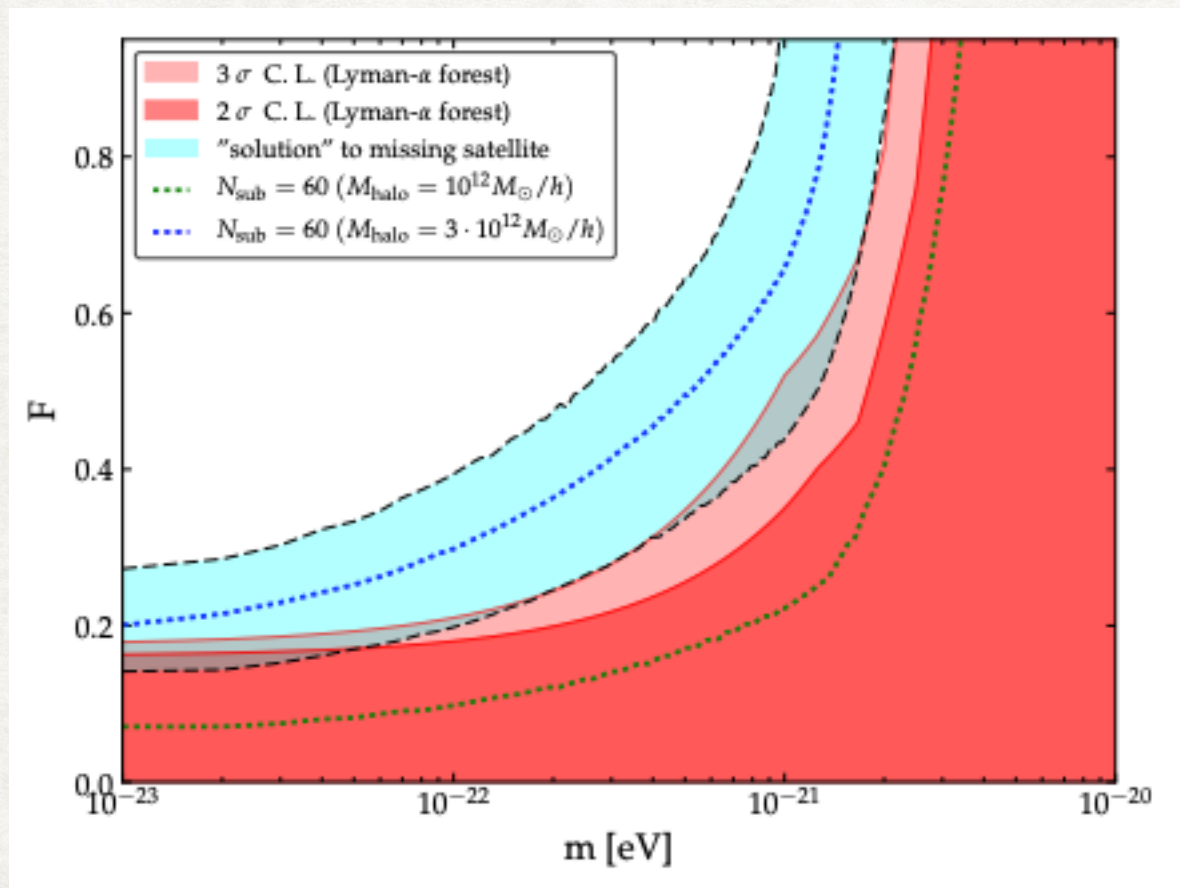
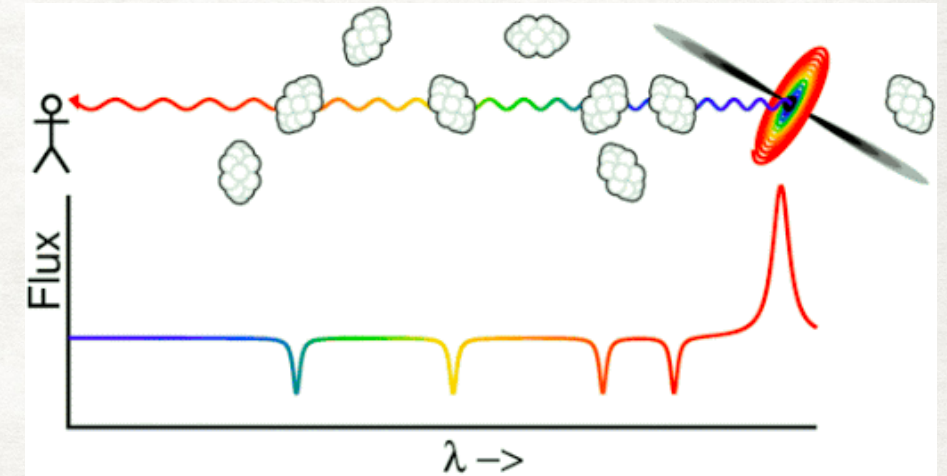
- There exists various ways of putting bounds on the ALP mass from large-scale structure observations: e.g. galaxy clustering and weak lensing power spectrum, Ly- α power spectrum, sub halos number count and more!



Grin++ 1904.09003

The Ly- α bound

- Absorption of the QSOs light by neutral hydrogen leads to the Lyman- α forest.
- Measures properties (T and density) of the IGM at $z \sim 3-6$.
- This provides strong constraint on the ULA mass and density (some uncertainty on the IGM temperature history).



Irsic++ 1703.04683

Most stringent bounds from Ly α to date: $m_a > 2 \times 10^{-20}$ eV

Rogers&Peiris 2007.12705

Constraints from the CMB

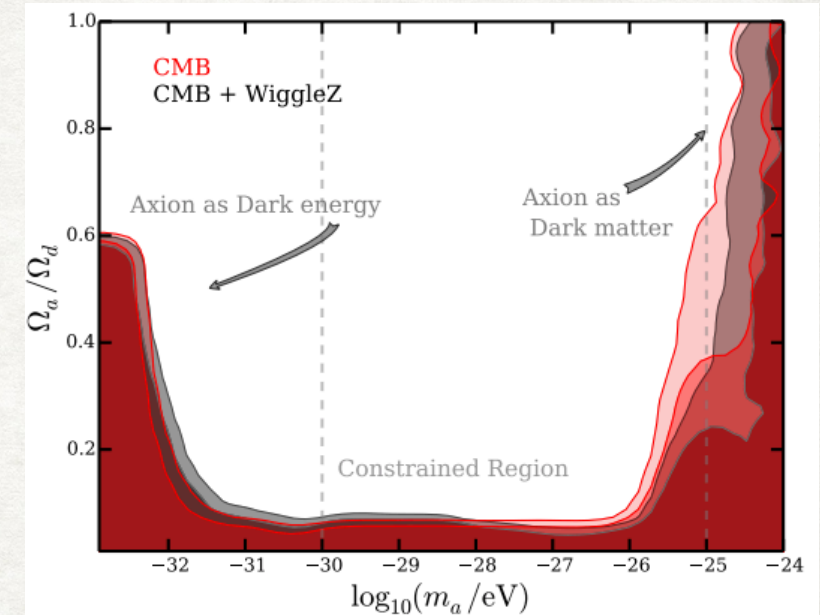
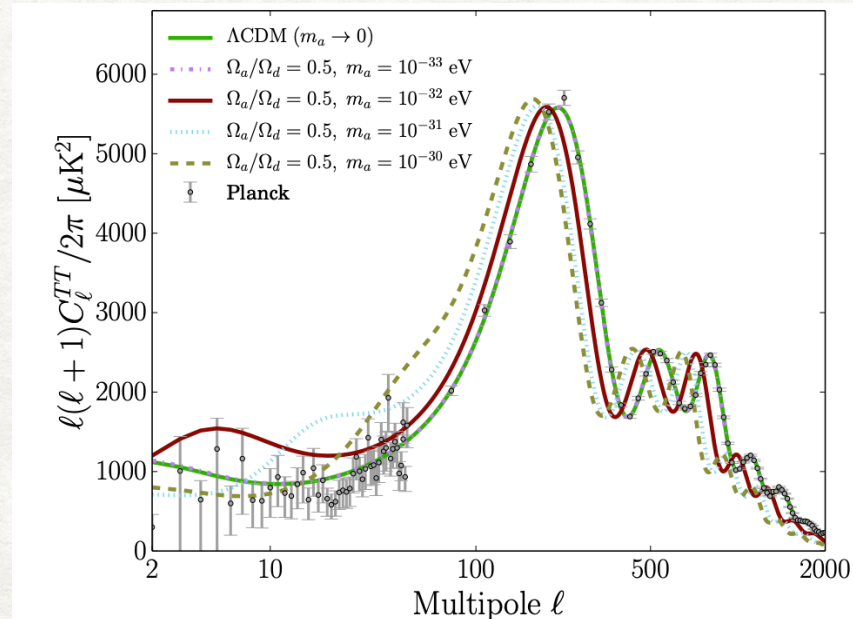
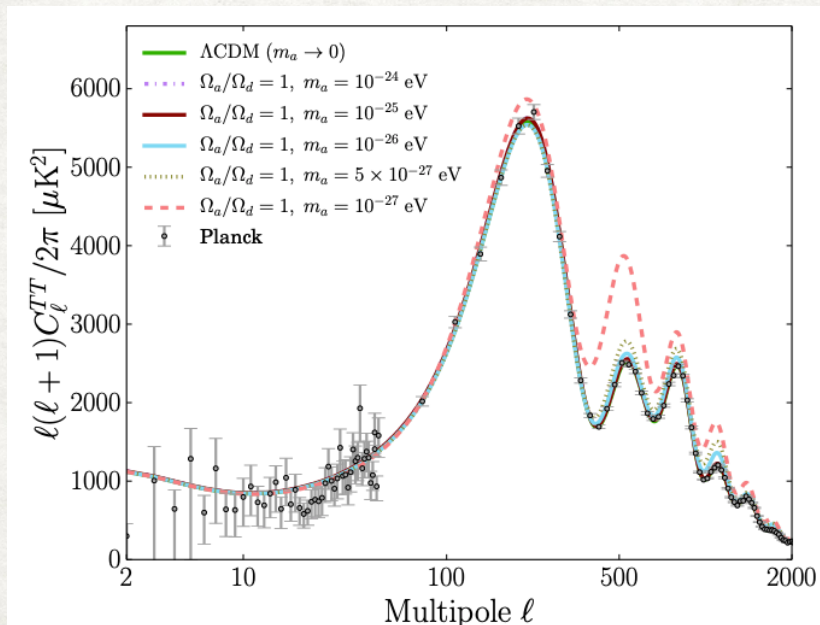
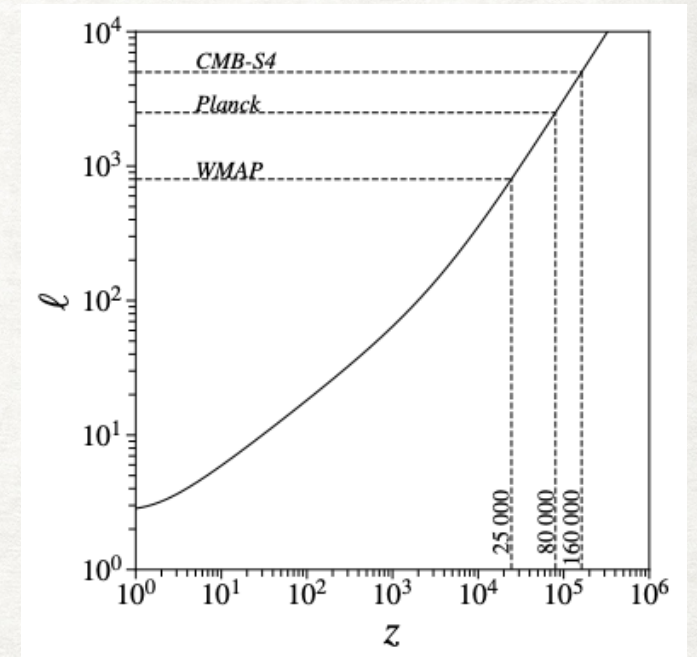
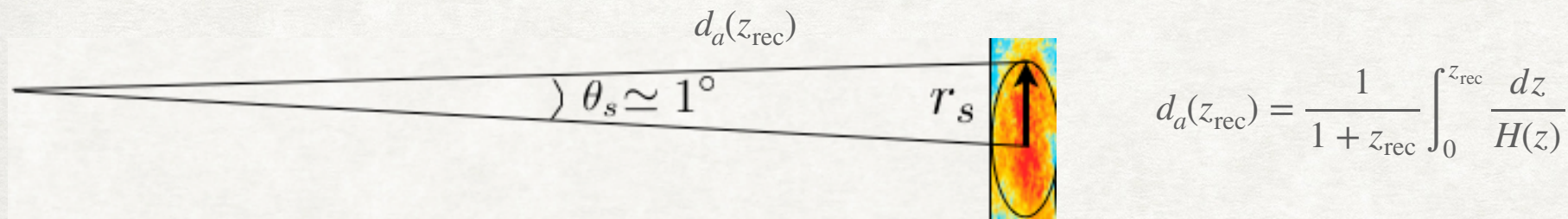
- The CMB does not provide the best constraints on the ALP mass, but is able to probe small fraction of ultra-light ALP

- CMB probes linear scales and therefore $10^{-33} \text{ eV} \lesssim m_a \lesssim 10^{-24} \text{ eV}$

- Field becoming dynamical before $z_{\text{rec}} \sim 1000$: affect (mostly) the time-evolution of potential well (integrated sachs-wolfe effect)

$$C_\ell^{TT} \simeq C_\ell^{\text{SW}} + C_\ell^{\text{doppler}} + C_\ell^{\text{ISW}} \quad C_\ell^{(\text{ISW})} = \frac{2}{\pi} \int \frac{dk}{k} k^3 \left[\int_{\eta_*}^{\eta_0} d\eta (\dot{\Psi} - \dot{\Phi}) j_\ell(k(\eta_0 - \eta)) \right]^2$$

- Field becoming dynamical after $z_{\text{rec}} \sim 1000$: affect (mostly) the angular-diameter distance between us and the CMB.



Initial conditions and iso-curvature

e.g. Langlois C. R. Physique 4 (2003) 953–959, Hlozek++ 1708.05681

- Initial condition (i.e. condition right after inflation) are decomposed between one adiabatic mode and a number of iso-curvature modes.

- The adiabatic mode verifies $\delta(n_i/n_j) = 0$ which can be shown to give $\delta_i = \frac{3}{4}(1 + w_i)\delta_\gamma$. For a frozen axion $w'_a \simeq -1 \Rightarrow \delta_a \simeq 0$.

(Nb: in practice numerical code do not start at $\tau = 0$ and include a small correction valid for $k\tau \ll 1$).

- ‘iso-curvature’ modes: variations in the particle number ratios at constant total density perturbation (i.e. curvature) $S_{i,j} = \frac{\delta n_i}{n_i} - \frac{\delta n_j}{n_j}$

- In the case $f_a > H_I/2\pi$: the axion exists as a massless field during inflation and develop perturbations $\delta\phi = \frac{H_I}{2\pi} \equiv M_{\text{pl}} \sqrt{\frac{A_s r_T}{8}}$

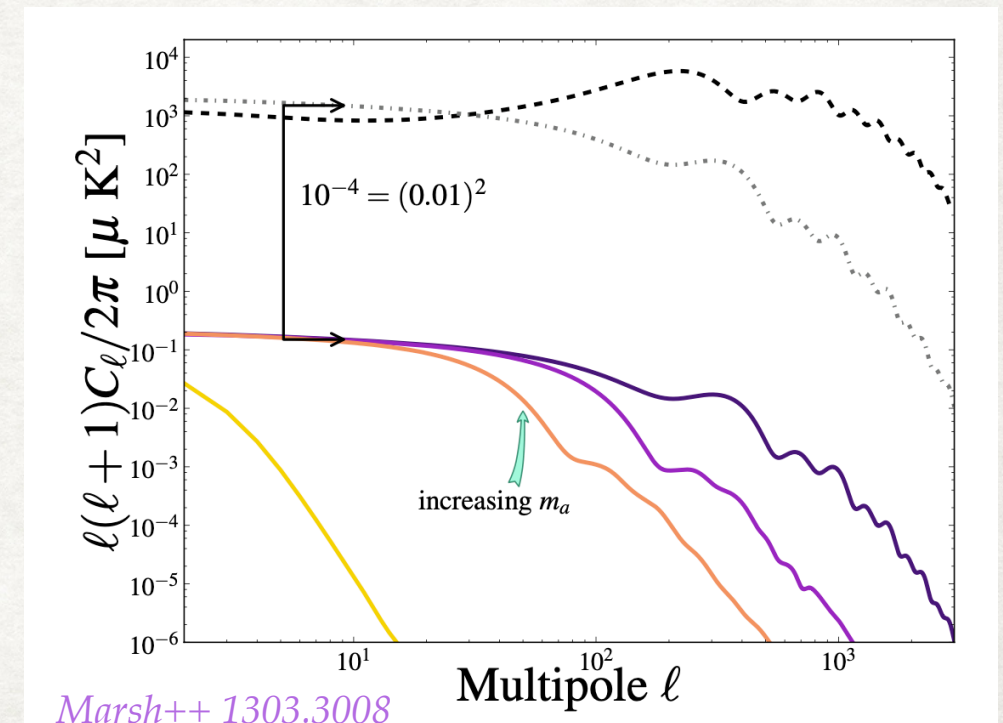
- $r_T \equiv A_T/A_s$ is the tensor-to-scalar ratio, it is bounded $r < 0.06$ (95%C.L., Bicep+Planck) which implies $H_I/2\pi \lesssim 10^{13}\text{GeV}$

- This translates into a spectrum of fluctuations for the axion

$$P_{\delta_a}(k) = A_I \left(\frac{k}{k_0} \right)^{n_{\text{iso}}-1} \text{ with } A_I = \frac{H_i^2}{\pi^2 \bar{\phi}_i^2}$$

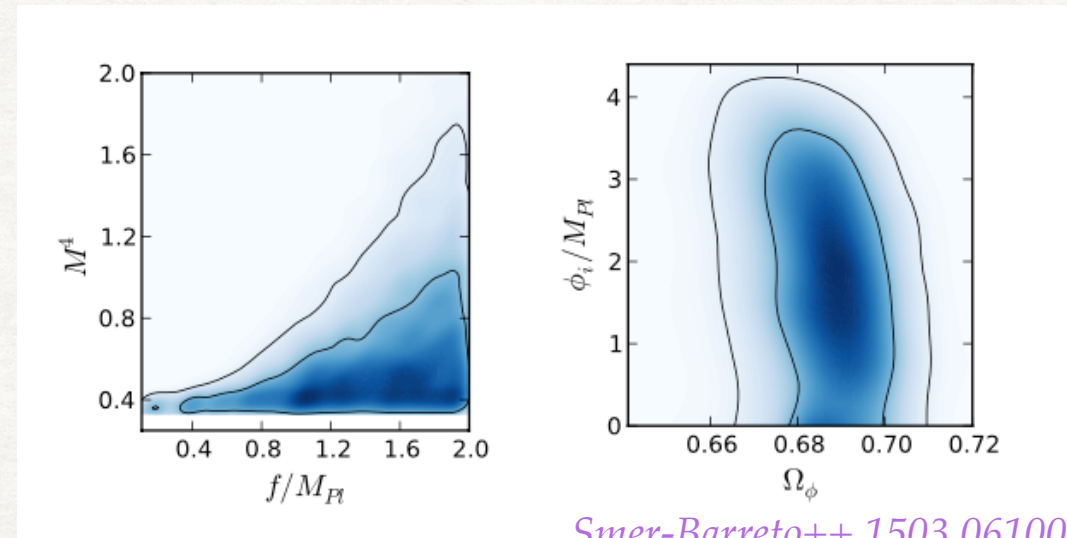
- Hence the detection of a non-zero r implies the existence of a non-zero A_I .
Current constraints: $A_I/A_{\text{ad}} < 0.038/F^2$ (95% Planck)
- Detecting r would constrain ALP as DM! Alternatively detecting ALP iso-curvature would constrain the energy scale of inflation.

e.g. Visinelli++ 1403.4594



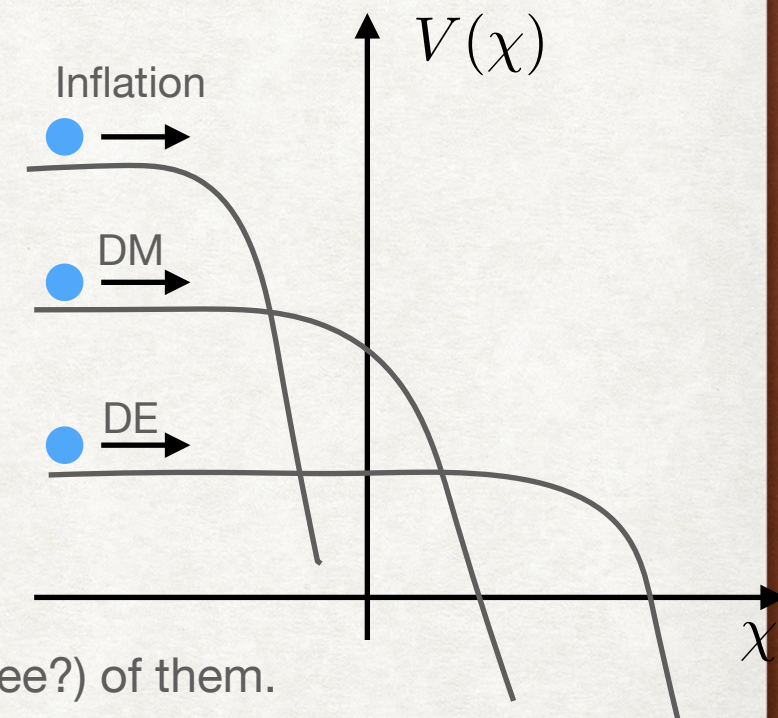
ALP as a dark energy candidate

- ALP with masses $m_a \sim 10^{-33}$ eV are frozen by Hubble friction until today and therefore viable DE candidate.
- Assume $V = M^4(1 + \cos(\phi/f))$, energy density set by ϕ_i .
- Degeneracy between M^4 and f from requirement $H_0 \gtrsim m_a$

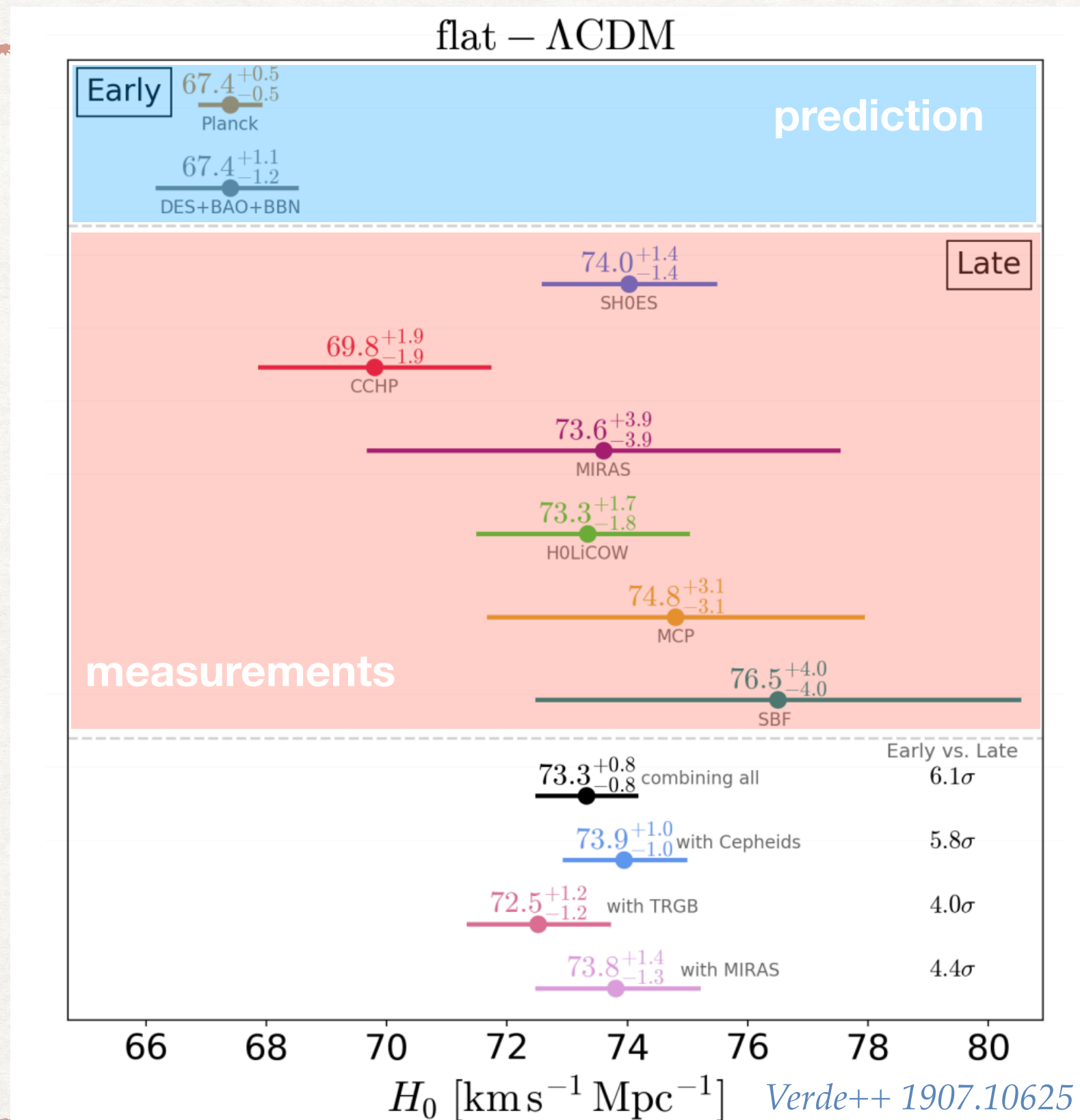


Smer-Barreto++ 1503.06100

- However, this model does not explain why the accelerated expansion occurs now, or in other words why $\Omega_m \sim \Omega_{DE}$.
- One possible paradox: the 'axiverse' many ALP fields which had impact at various epochs.
- First field could be the inflation: 'natural inflation' similar to $m^2\phi^2$ inflation: testable prediction $r > 0.01$ for next generation CMB experiment
- Another field could be Dark Matter (not the same! Otherwise inflation never ends).
- Another field could be Dark Energy. In Kamionkowski++1409.0549 : 1/100 chance.
- What if there were **more of such era to be discovered**? We already have seen two (three?) of them.



The H_0 tension now reaches 4 – 6 σ



How does CMB data measure H_0 ?

- The 'sound horizon' r_s , a standard ruler in the sky: distance travelled by sound wave until recombination.
- Planck measures θ_s and, given a model, can extract r_s .
- H_0 appears *only* in the angular diameter distance d_A .

$$\theta_s \equiv \frac{r_s(z_*)}{d_A(z_*)} = \frac{\int_{\infty}^{z_*} dz \, c_s(z) / \sqrt{\rho_{\text{tot}}(z)}}{\int_0^{z_*} dz / \sqrt{\rho_{\text{tot}}(z)}}$$

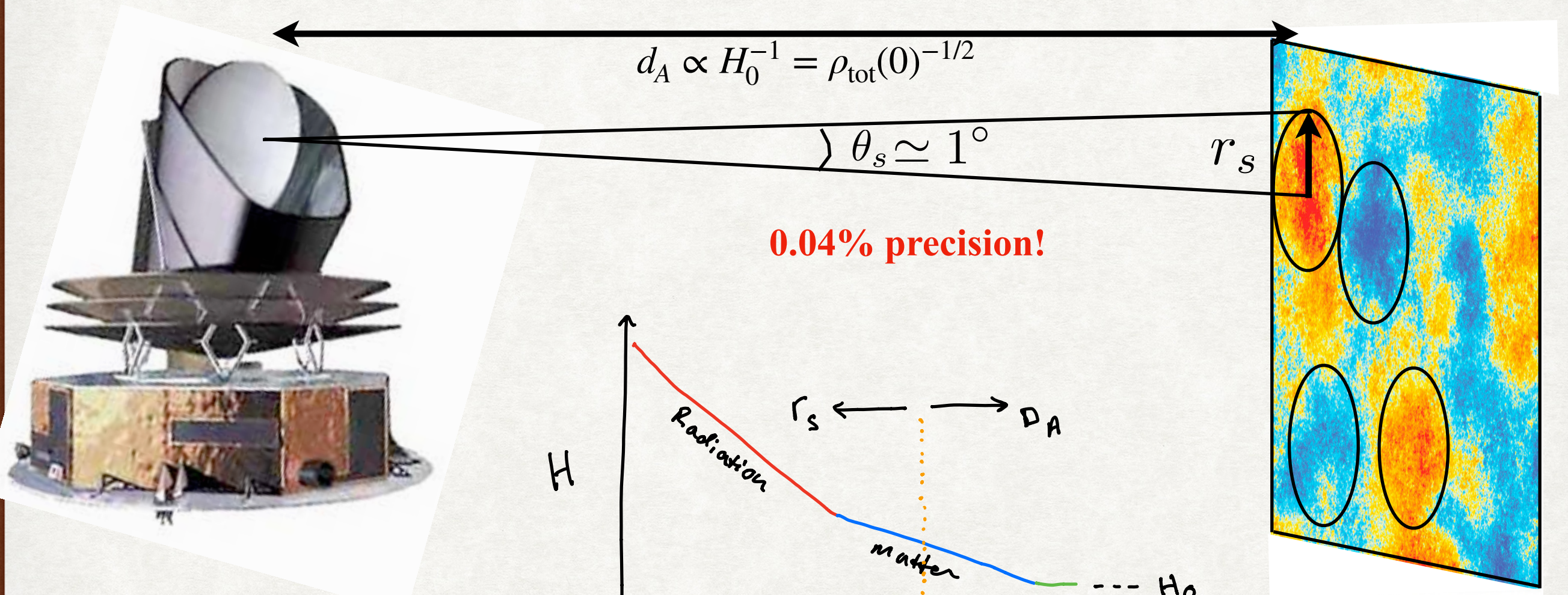


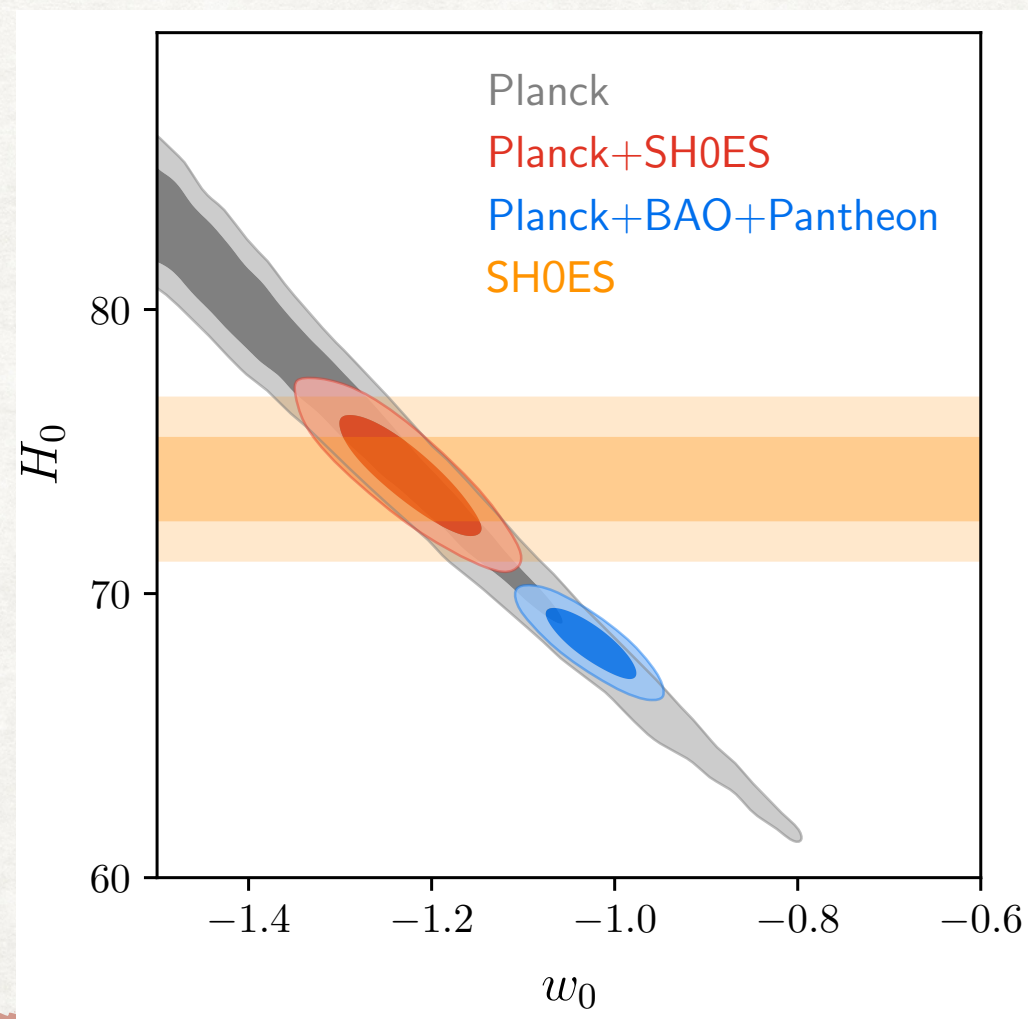
illustration: T. Smith

Geometrical degeneracy in Planck!

- A higher H_0 can be compensated by a lower $H(z > 0)$ such as to keep $d_A(z_*)$ fixed

$$d_A(z_*) = \frac{1}{1+z_*} \int_0^{z_*} \frac{dz}{100 \sqrt{\omega_M(1+z)^3 + \Omega_{DE}(z)h^2}}$$

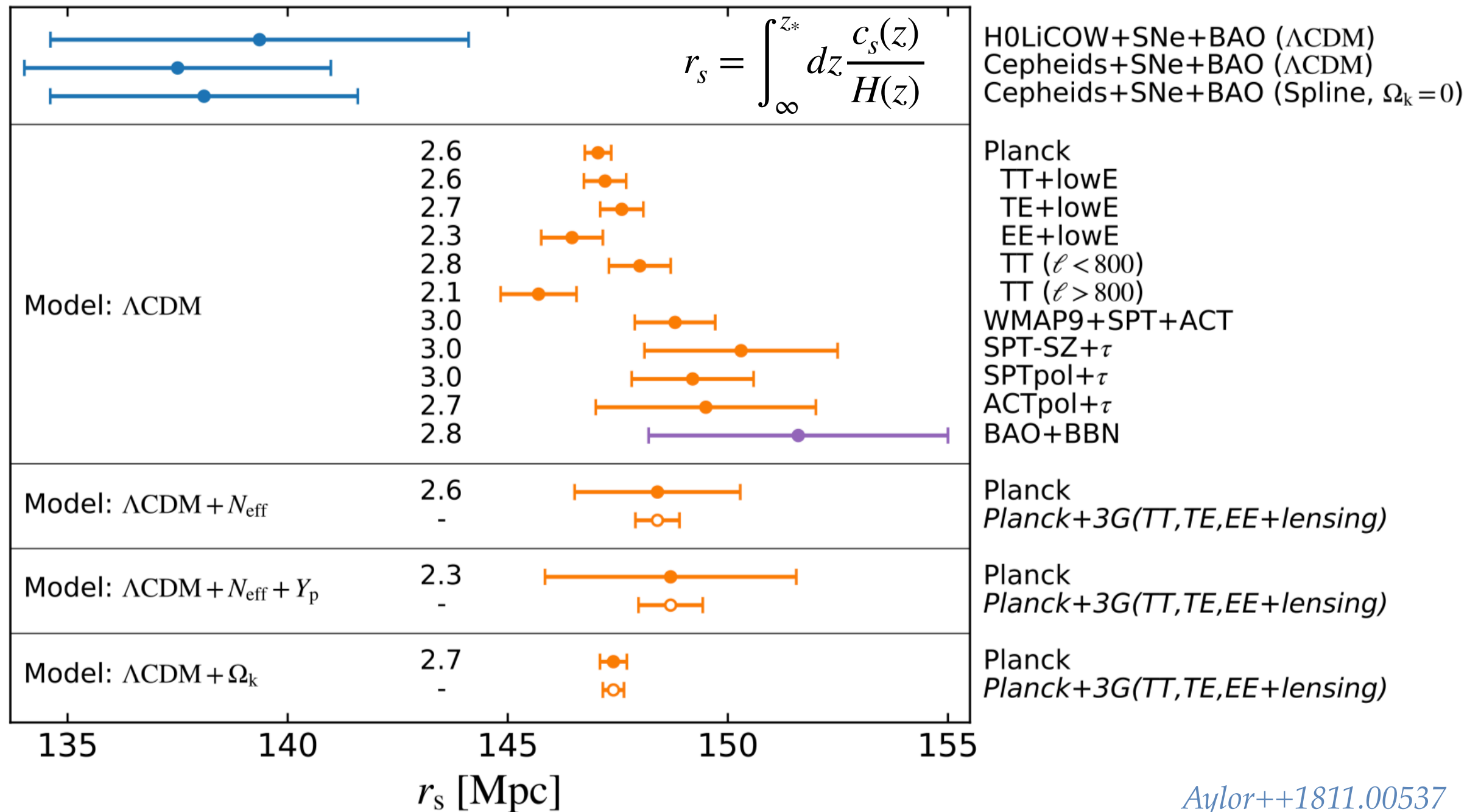
- ‘phantom dark energy’ $w < -1$, DE phase transition, DE-DM interaction, decaying/annihilating DM, and many more...
[\[http://arxiv.insert_your_favorite_model_here.com\]](http://arxiv.insert_your_favorite_model_here.com)
- Planck can easily accommodate a higher H_0 : problem with BAO and Pantheon



H_0 tension or r_s tension?

One can deduce the co-moving sound horizon r_s from H_0 and BAO

r_s from CMB needs to **decrease by ~ 10 Mpc**



A sketch of the physics at play

- Could the CMB be closer to us than Λ CDM tells us? This is what a higher H_0 suggests.
- Therefore, could spot in the CMB be smaller? This is what new physics must achieve.

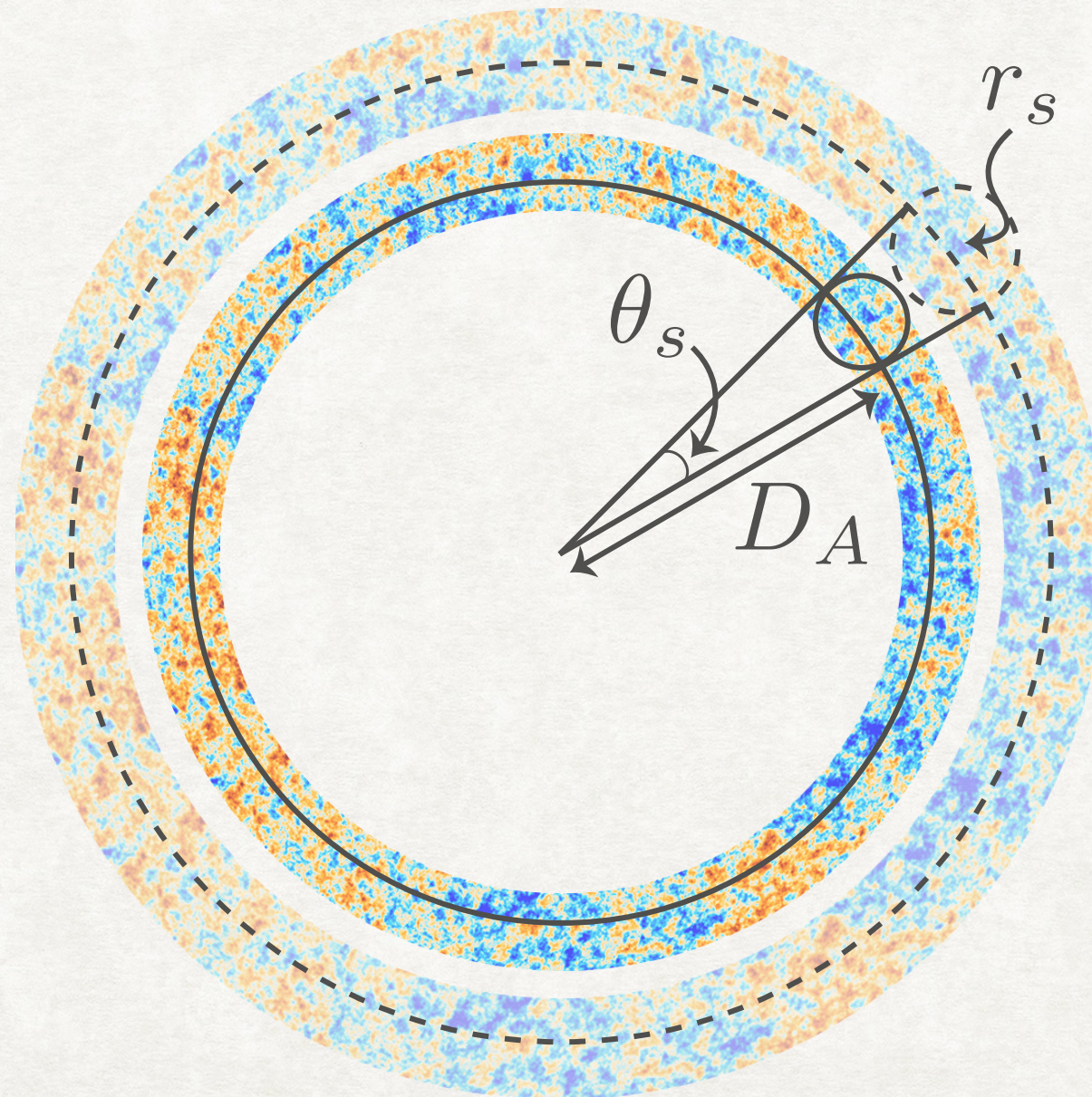


illustration: T. Smith

Early-time resolution to the H_0 tension

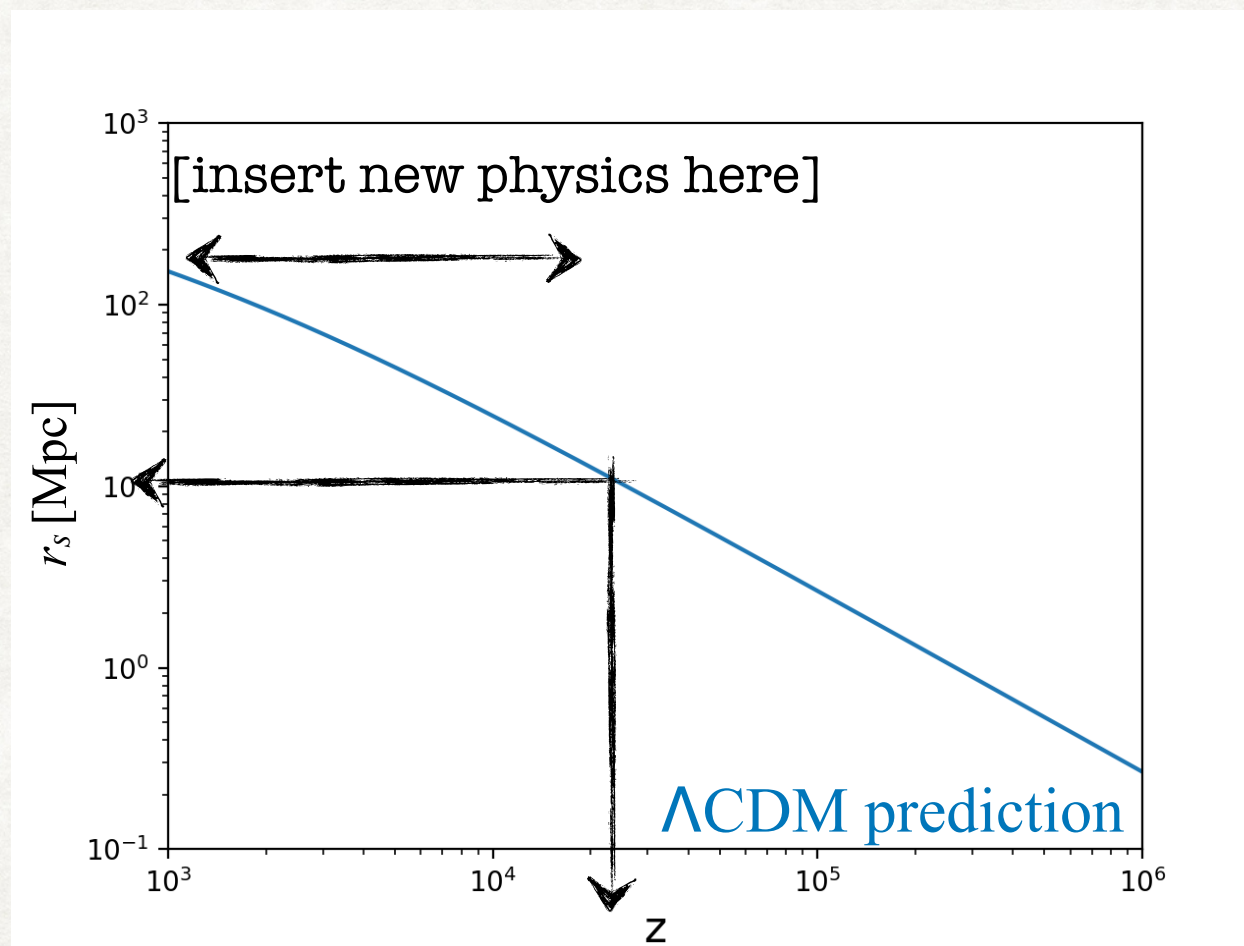
affect z_* : modified recombination physics?

affect c_s : DM-photon scattering? DM-b scattering?

$$r_s = \int_{\infty}^{z_*} dz \frac{c_s(z)}{\sqrt{\rho_{\text{tot}}(z)}}$$

increase $\rho(z)$: Neff? Early Dark Energy?
Modified Gravity?

- r_s does not reach **10Mpc before ~ 25000** in Λ CDM



GOAL: decreasing r_s by 10Mpc while keeping r_s/r_d and r_s/r_{eq} fixed

See the 'Hubble Hunter's guide' [Knox&Millea 1908.03663](#)

Scalar field and Early Dark Energy

- Initially **slowly-rolling field** (due to Hubble friction) that later **dilutes faster than matter**

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV_n(\phi)}{d\phi} = 0 \quad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V_n(\phi), \quad P_\phi = \frac{1}{2}\dot{\phi}^2 - V_n(\phi)$$

- Oscillating (toy) potential:

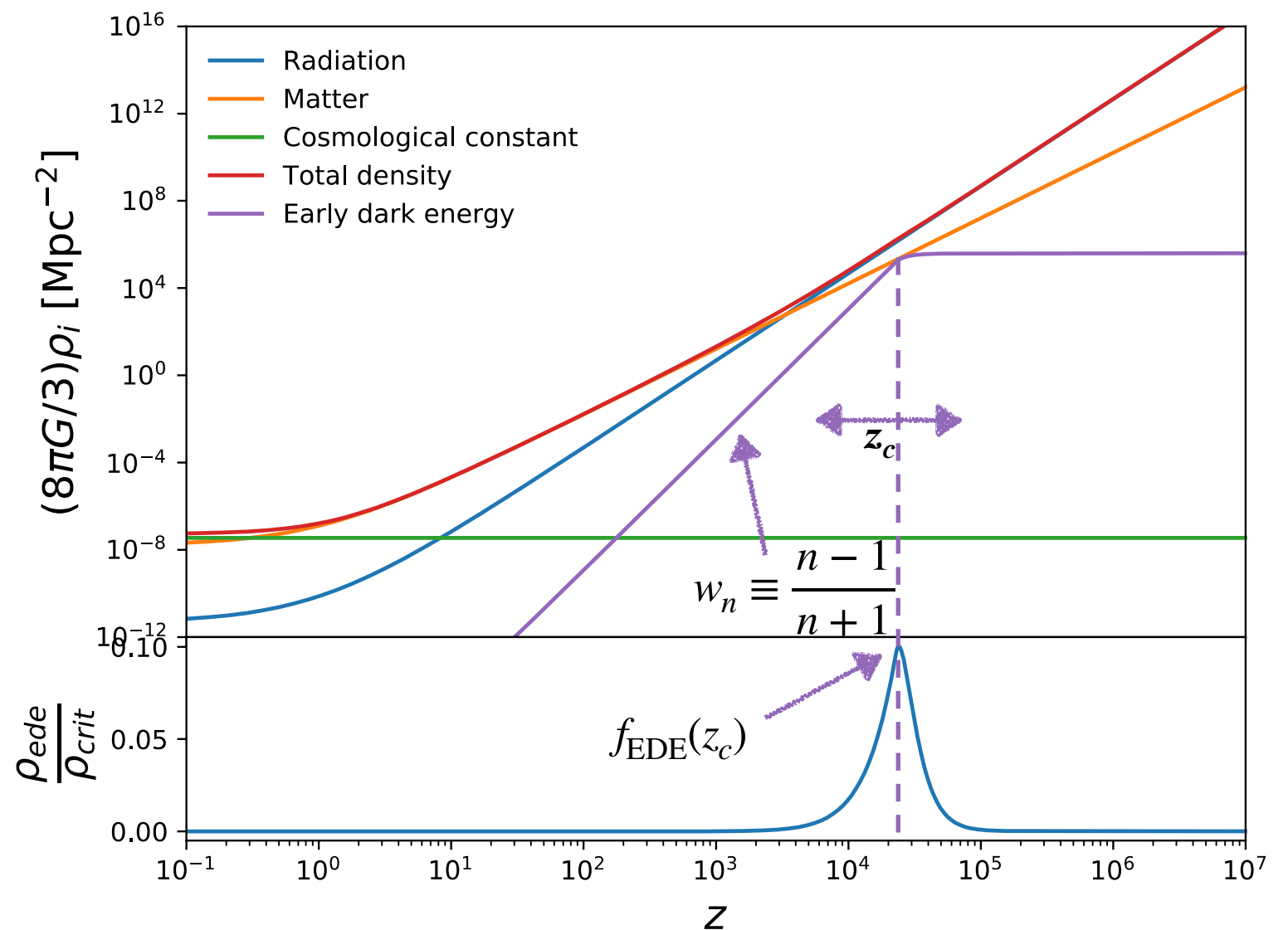
$$V(\phi) \propto (1 - \cos(\phi/f))^n \xrightarrow{\phi \ll f} \left(\frac{\phi}{f}\right)^{2n}$$

VP++ 1806.10608 & 1811.04083
Smith, VP++ 1908.06995

- Specified by $f_{\text{EDE}}(z_c)$, z_c , $w(n)$, $c_s^2(k, \tau)$

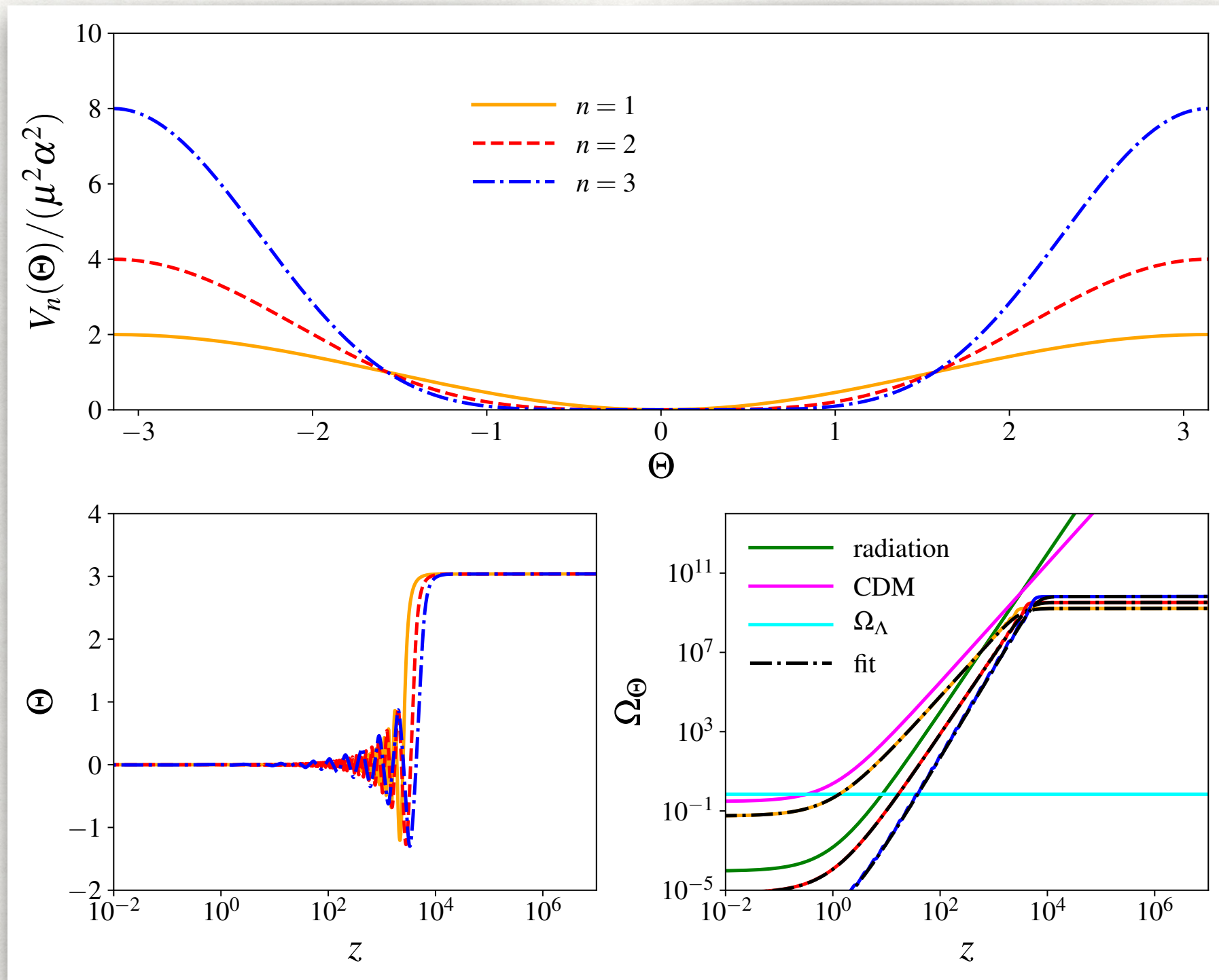
$$\begin{cases} z > z_c \Rightarrow w_n = 1 \\ z < z_c \Rightarrow w_n = (n-1)/(n+1) \end{cases}$$

$n = 1$: matter, $n = 2$: radiation, etc.



plot by T. Karwal

Homogeneous evolution



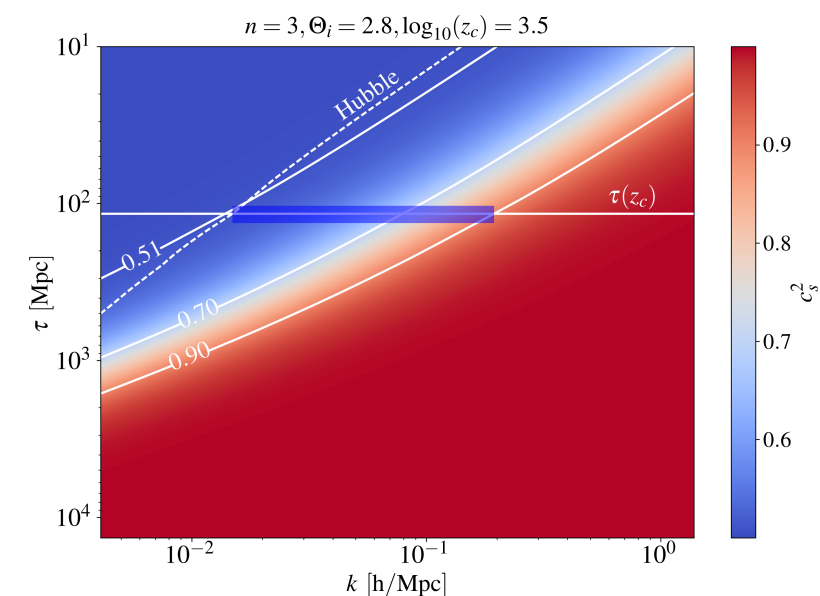
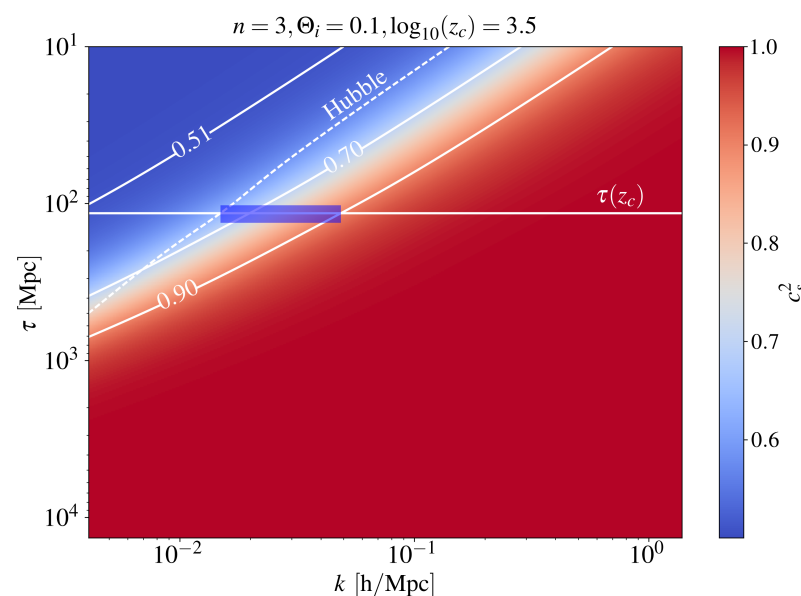
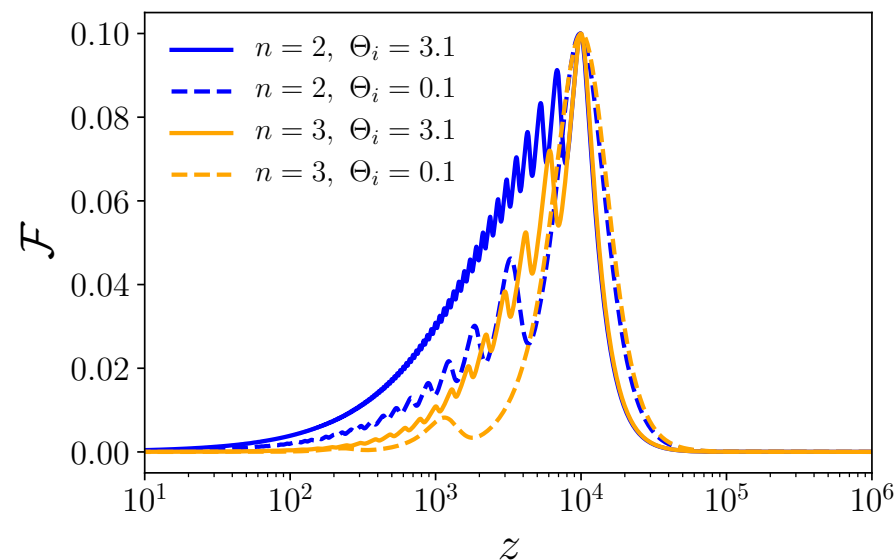
VP++ 1806.10608

Effective sound speed

- Θ_i affects the **oscillation frequency** $\varpi(a)$ and **asymmetry** of the energy injection as well as the **range of modes** having $c_s^2 \rightarrow 1$

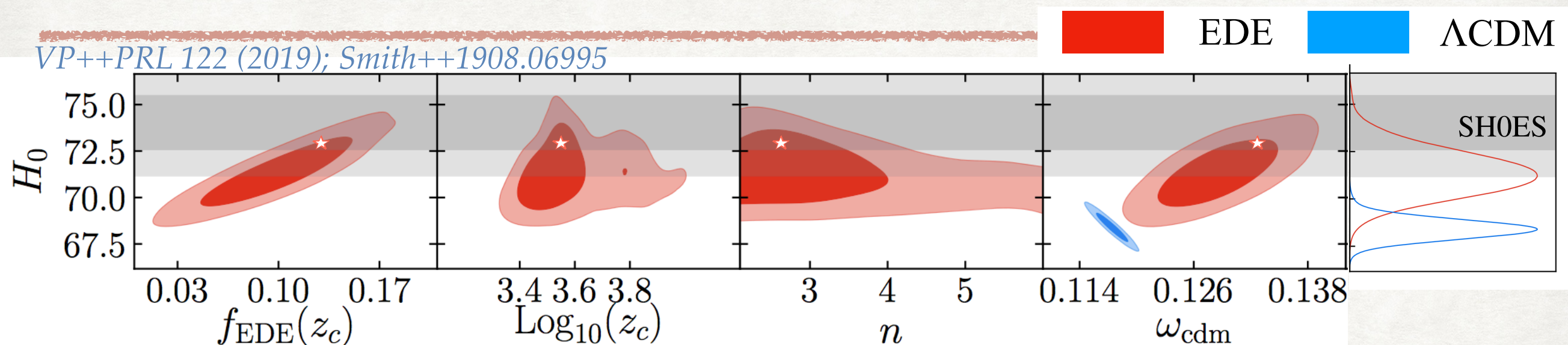
$$c_s^2 = \frac{2a^2(n-1)\varpi^2(a) + k^2}{2a^2(n+1)\varpi^2(a) + k^2}$$

$$\varpi(a) \simeq 3H(z_c) \frac{\sqrt{\pi}\Gamma(\frac{1+n}{2n})}{\Gamma(1+\frac{1}{2n})} 2^{-(1+n)/2} \frac{\Theta_{\text{osc}}^{n-1}(a)}{\sqrt{|E_{n,\Theta\Theta}(\Theta_i)|}}$$



EDE Can Resolve The Hubble Tension

VP++PRL 122 (2019); Smith++1908.06995

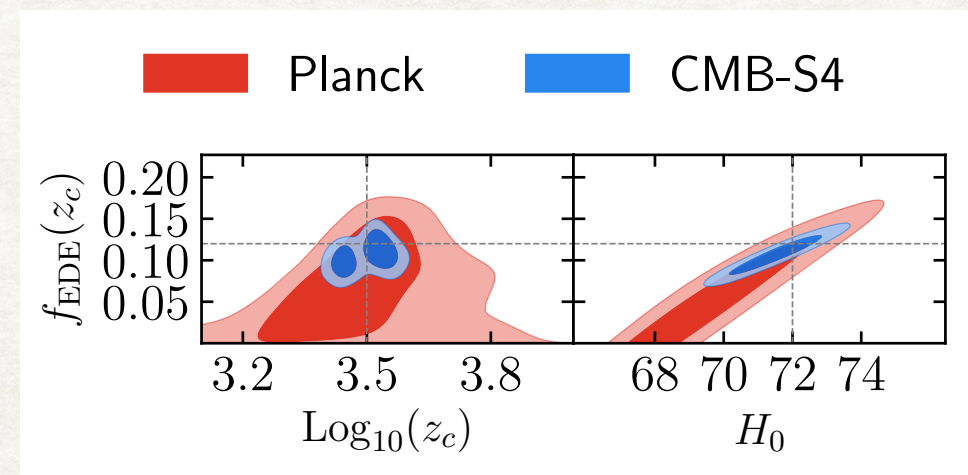
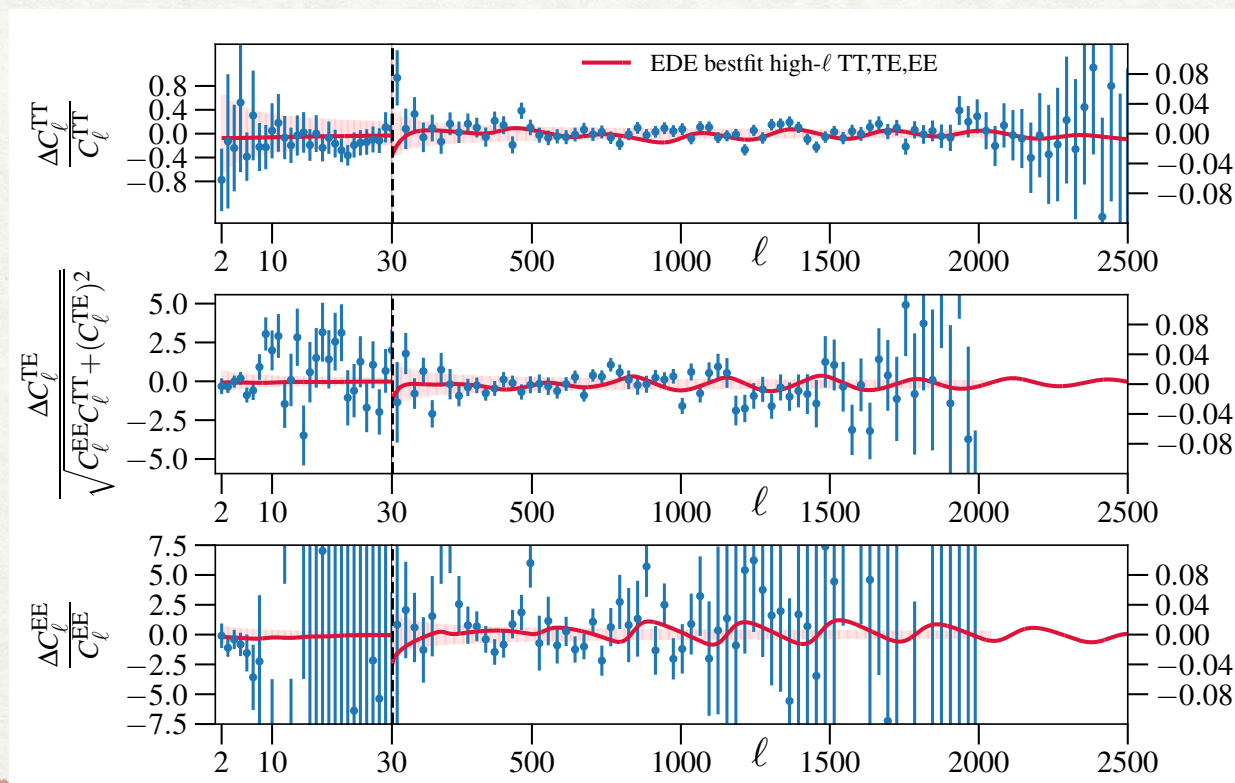


- Planck high- ℓ TT,TE, EE+lowTEB+lensing+BAO+Pantheon+SH0ES 19

$$f(z_c) = 0.10 \text{ (0.13)} \pm 0.03$$

$$\text{Log}_{10}(z_c) = 3.56 \text{ (3.53)}^{+0.05}_{-0.1}$$

$$H_0 = 71.5 \text{ (72.8)} \pm 1.2 \text{ km/s/Mpc}$$

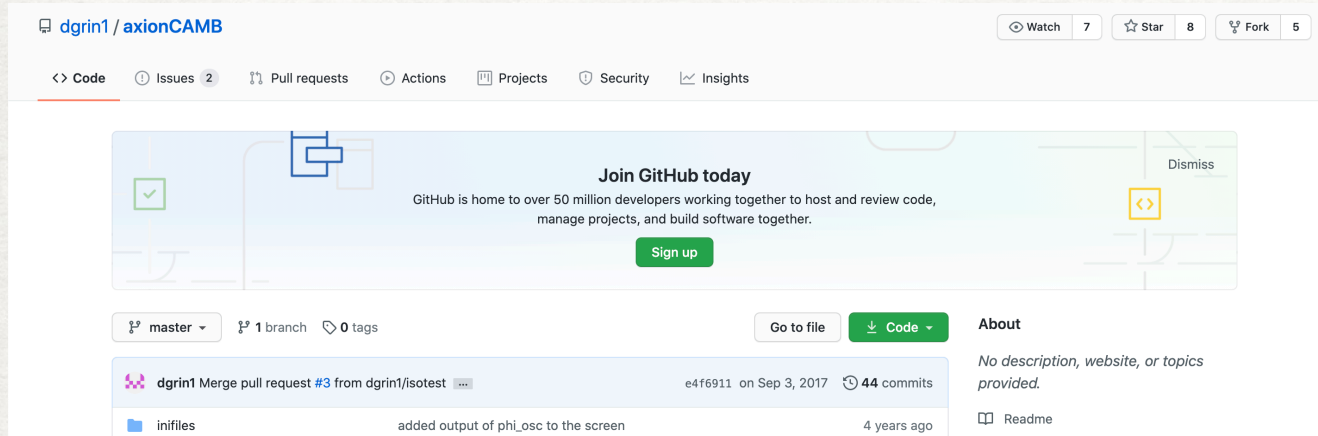


Smith ++ 1908.06995

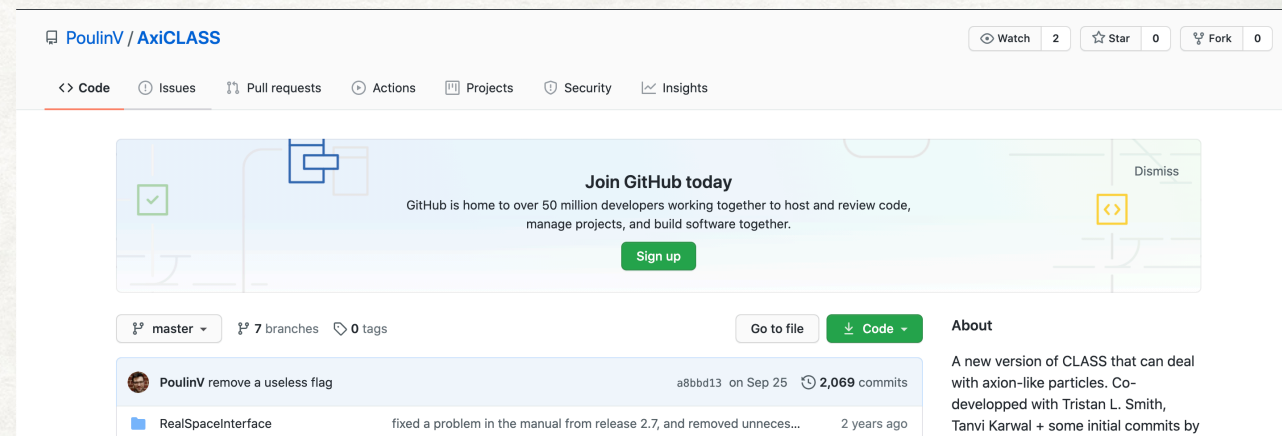
- CMB-S4 will certainly detect (exclude) $f_{\text{EDE}}(z_{\text{eq}}) \sim 10\%$

Exercices!

- Numerical codes are public!



■ axionCAMB *Hlozek++ 1708.05681*



■ axiCLASS *VP++ 1806.10608 & 1811.04083
Smith, VP ++ 1908.06995*

- You can study the impact of axion dark matter produced via the 'misalignment mechanism' on CMB and LSS
- Try to reproduce some of the figures from this lecture.

Conclusions

- ALP can play **many roles in Cosmology**: from the inflaton to Dark Energy, including Dark Matter. What if all these new “dark” sectors were connected to each other?
- As a DM candidate: It could explain the ‘small-scale crisis’ of CDM.
- As a DE candidate: It could explain the old and new cosmological constant problem.
- As an inflaton candidate: r detectable by next generation experiment.
- As an Early Dark Energy candidate: it could relax the H_0 tension.
- CMB & matter power spectrum, Ly- α forest, small-scale structure of DM: many ways to test the ALP signatures. Have we already discovered it?