Frustrated magnetism on complex networks

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Raman Research Institute

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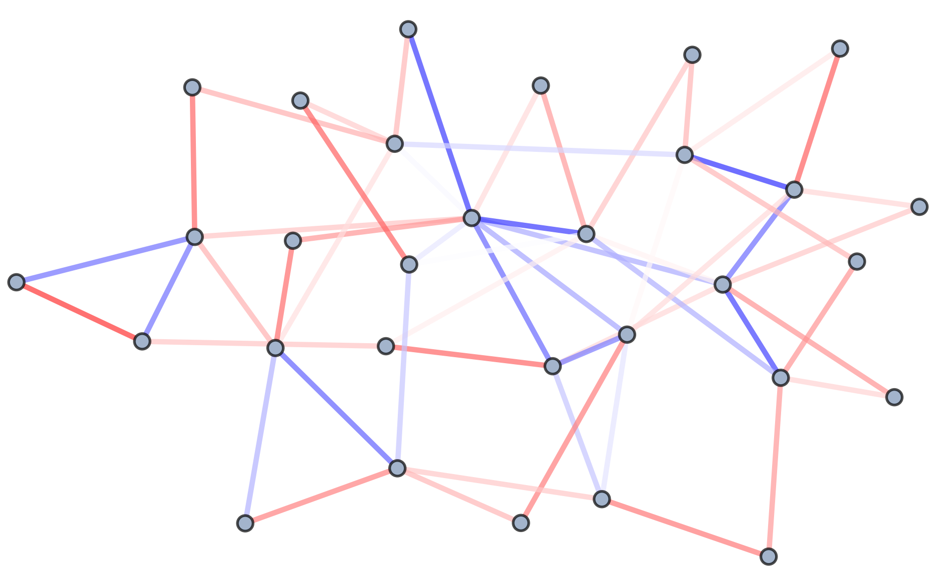
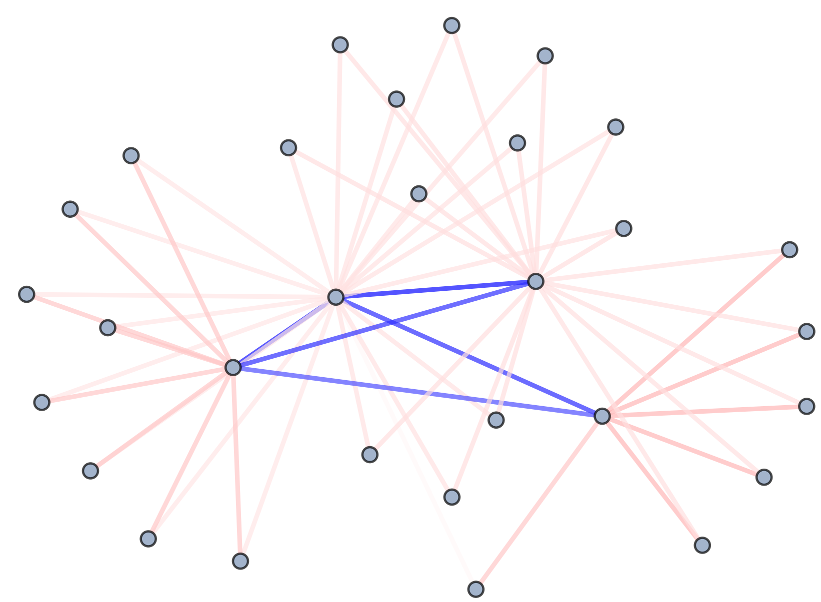
IISER TVM





विज्ञान एवं प्रौद्योगिकी विभाग
DEPARTMENT OF
SCIENCE & TECHNOLOGY
सत्यमेव जयते

STABILITY OF QUANTUM MATTER **IN** & OUT OF EQUILIBRIUM AT VARIOUS SCALES, JAN'24

RRI

Why networks?

- Why limit to regular lattices?

  Networks can host new many-body physics (e.g., from tuning dimensionality)

- Novel classical phenomena: explosive percolation, self-organized criticality

  Network topology controls disease spreading, synchronization

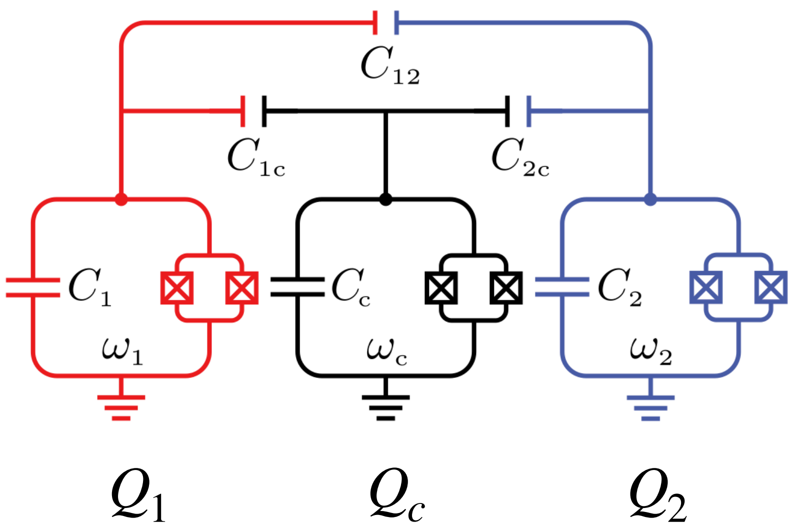
  Small-world property, community structure

  Strogatz, Nature 410, 268 (2001)

  D'Souza *et al.*, Adv. Phys. 68, 123 (2019)

  Sousa da Mata, Braz. J. Phys. 50, 658 (2020)

- Possible to synthesize arbitrary network of quantum spins



Superconducting qubits (Pedram's talk)

Superconducting circuits Lamata *et al.*, Adv. Phys. X 3, 1457981 (2018)

Trapped ions Korenblit *et al.*, New J. Phys. 14 095024 (2012)

Rydberg atoms Nguyen *et al.*, PRX 8, 011032 (2018)

Why magnetism on networks?

Networks enable variable degrees of frustration — ingredient of spin liquid

Antiferromagnetic Heisenberg model: $\hat{H} = \sum_{\langle i,j \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$

Savary, Balents, Rep. Prog. Phys. '16
Zhou, Kanoda, Ng, RMP, '17
Knolle, Moessner, '18

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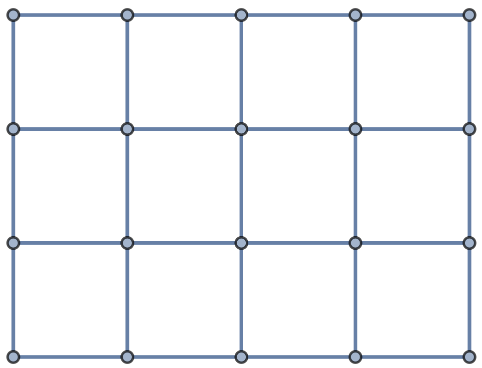
Regular bipartite



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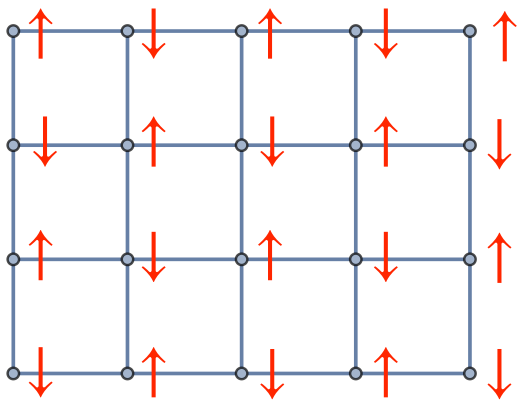

$S_{\text{total}} = 0$

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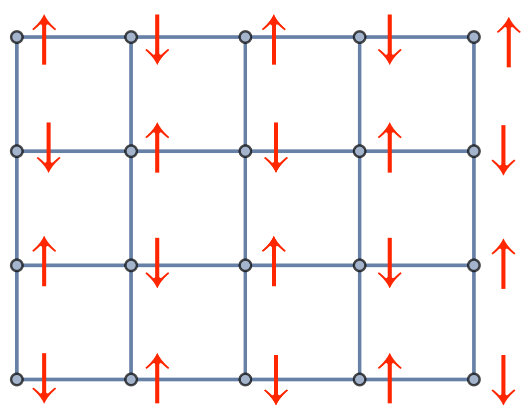

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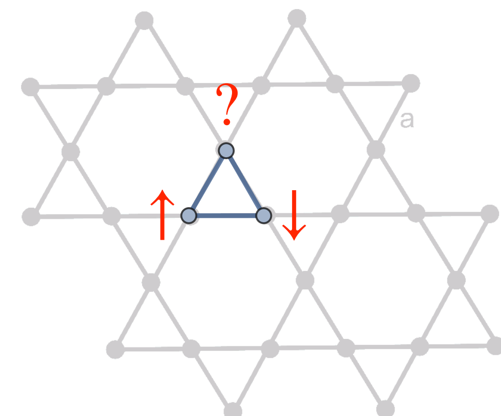
Regular nonbipartite



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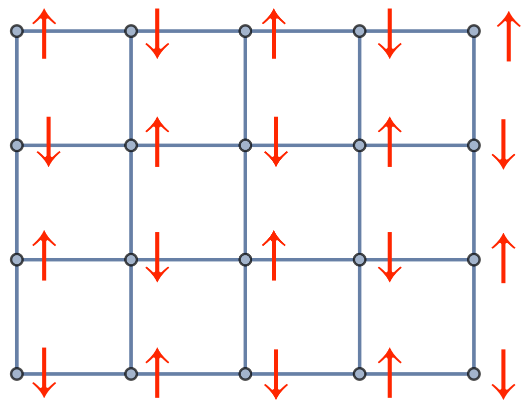

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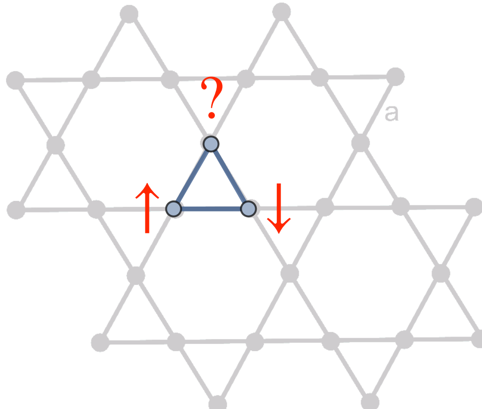

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Spin liquid

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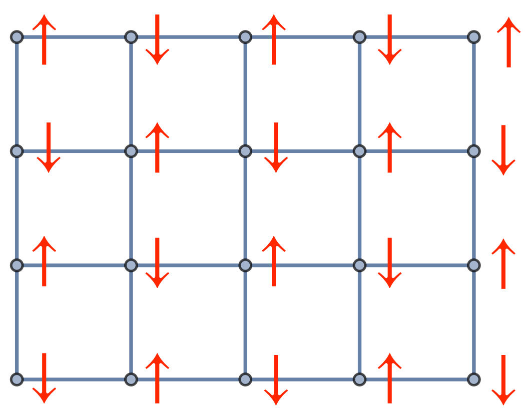

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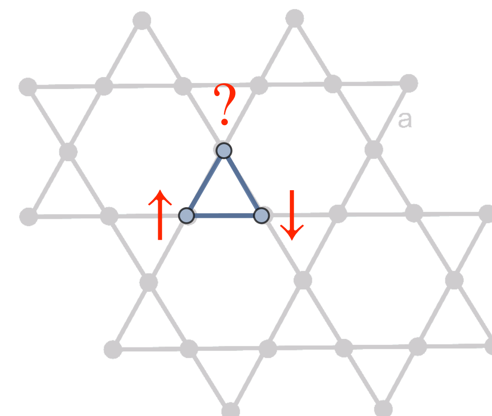

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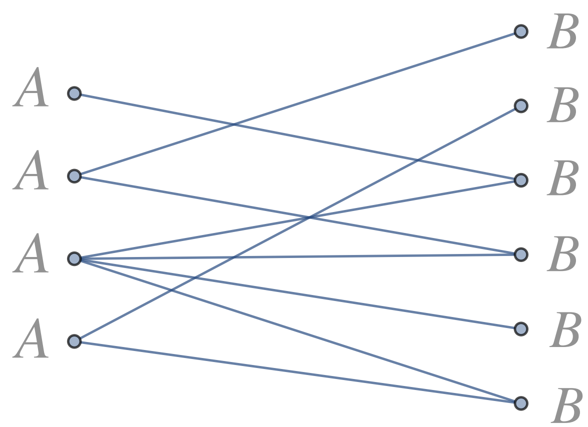

$$S_{\text{total}} = \frac{|N_A - N_B|}{2}$$

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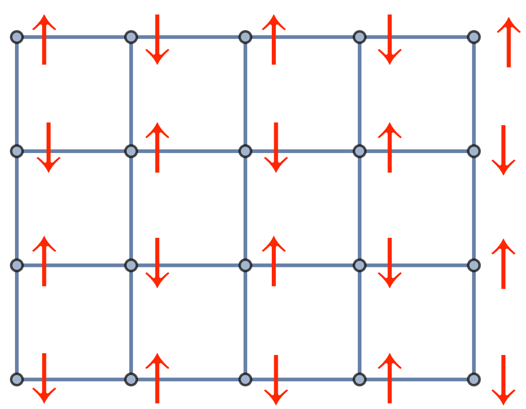

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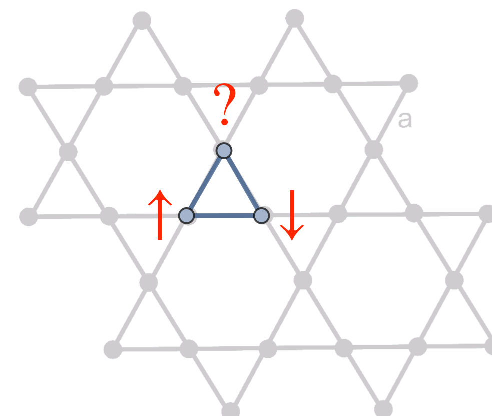

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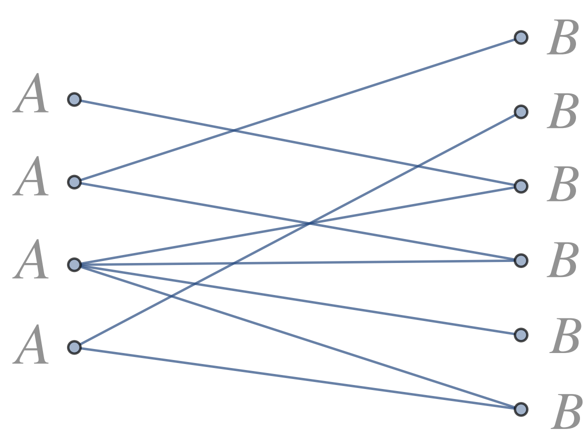

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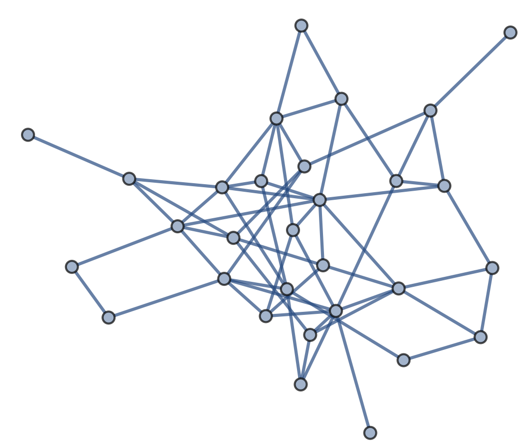
General nonbipartite



Frustrated

$S_{\text{total}} = ?$

Ordering ?

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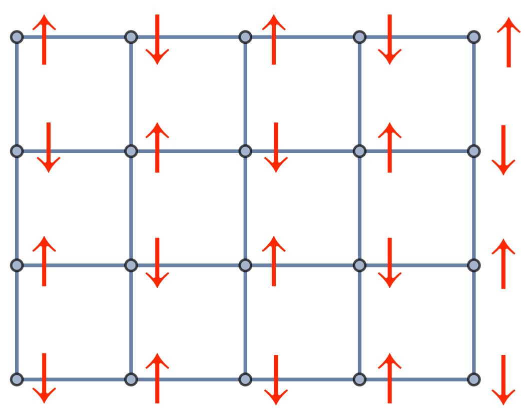

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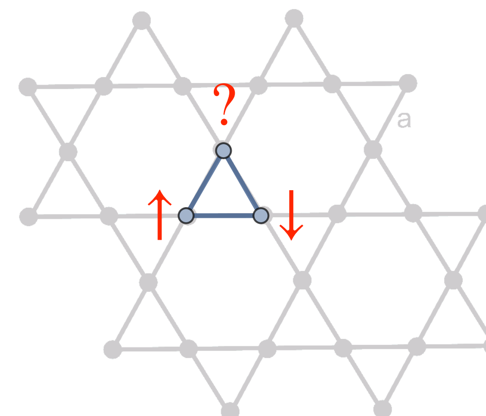

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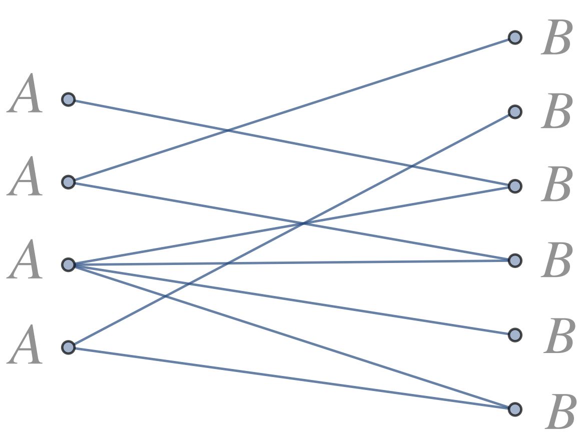

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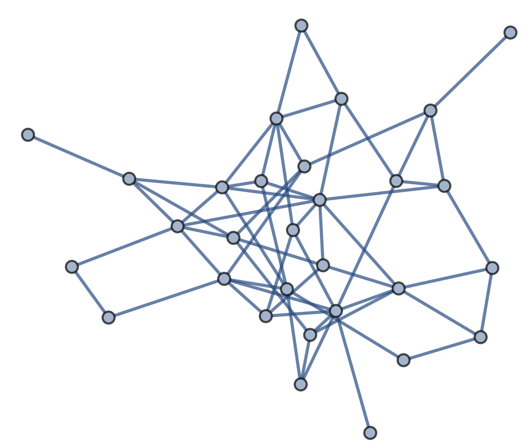


Frustrated

$S_{\text{total}} = ?$

Ordering ?

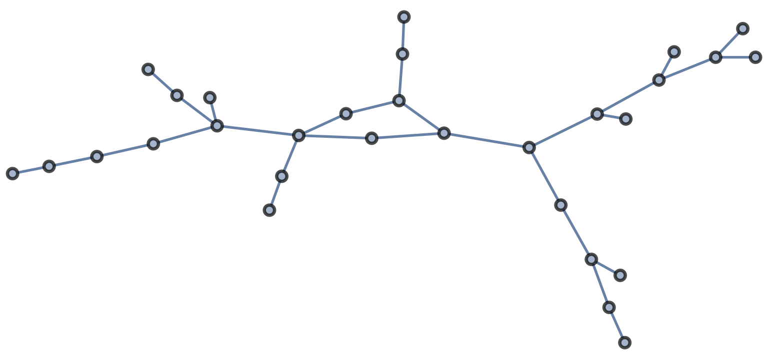
Q: How does network topology determine magnetic order? **Here: What sets S_{total} ?**

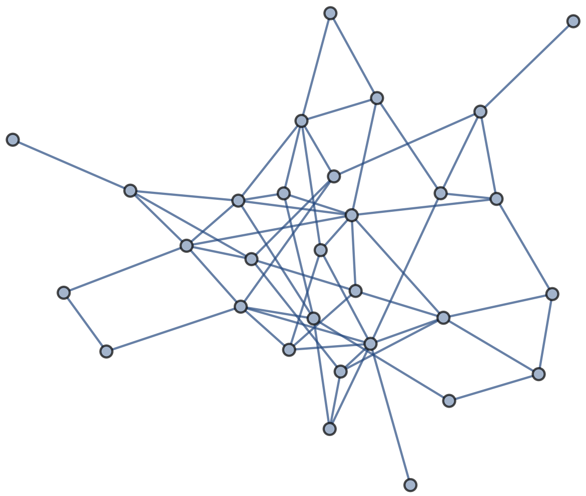
Random graphs

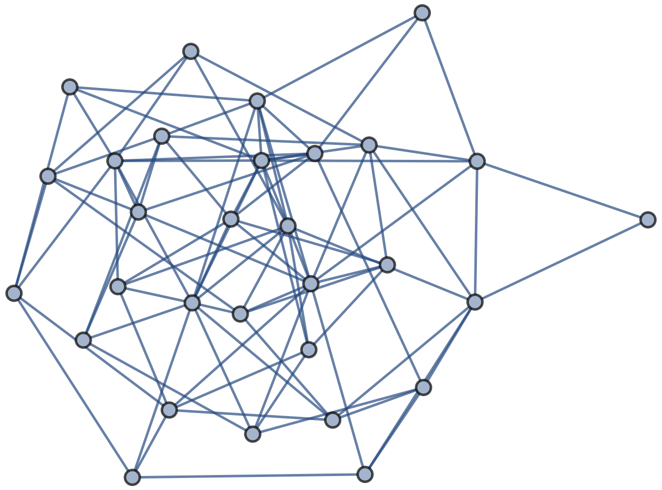
Random (connected) graphs

Random network of N spins & N_e bonds — generically nonbipartite

$N = 30, N_e = 30$ $N = 30, N_e = 60$ $N = 30, N_e = 90$







Avg # of neighbors (degree) $\bar{k} = 2N_e/N = 2, 4, 6$

Random (connected) graphs

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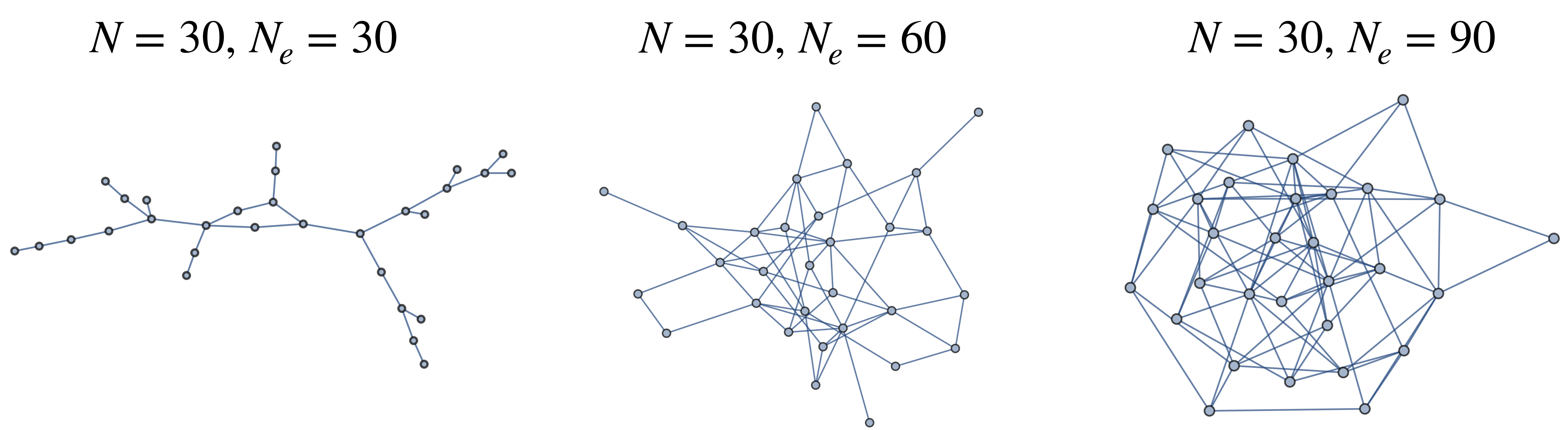

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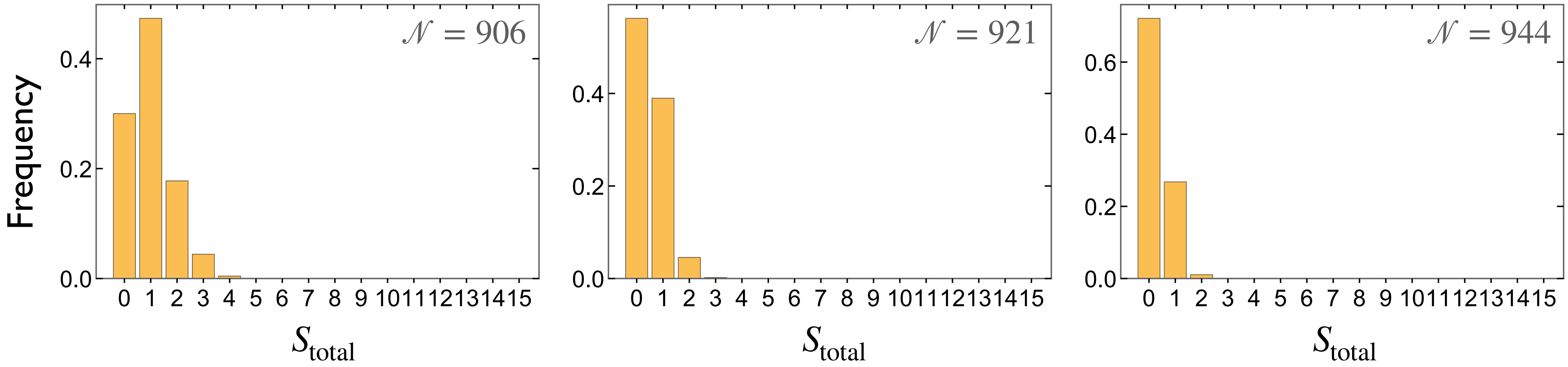
S_{total} small — falls with increasing $\bar{k}$:



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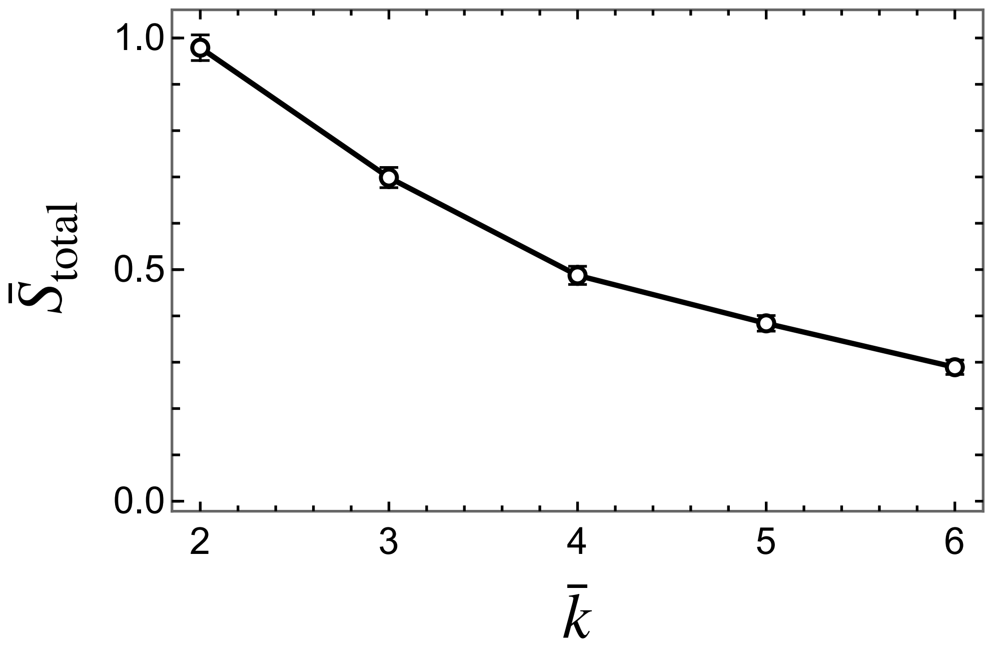

Magnetization falls with more neighbors

Random (connected) graphs

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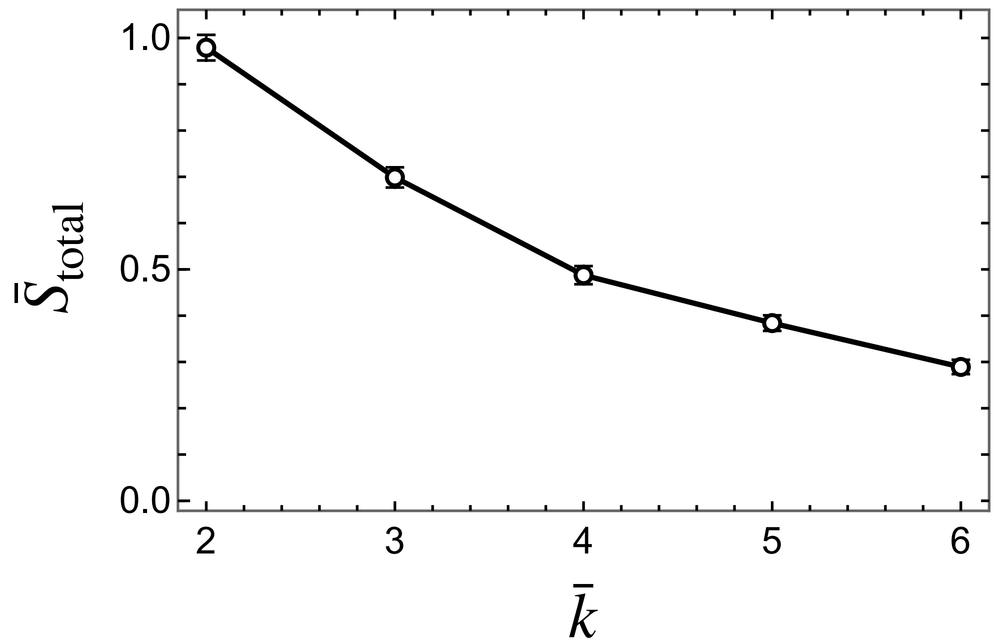
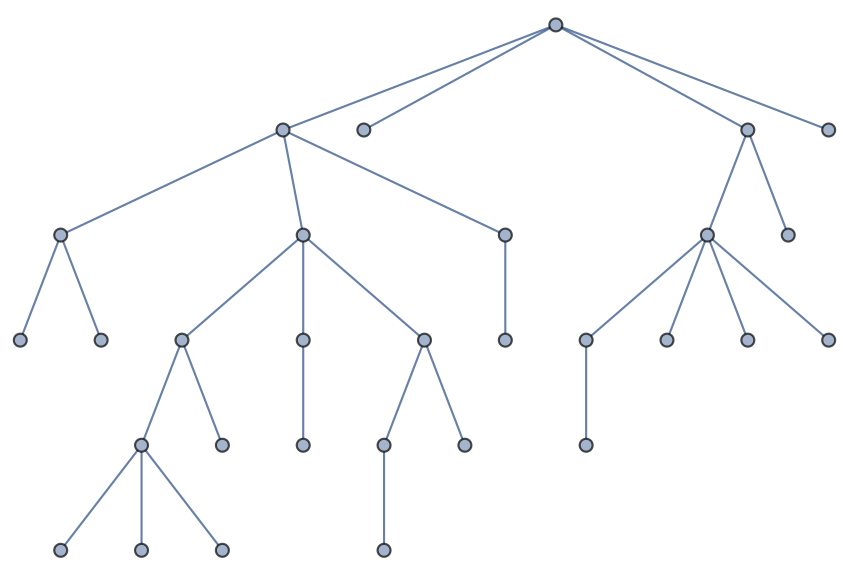


Magnetization falls with more neighbors

$N_e^{\text{min}} = N - 1$



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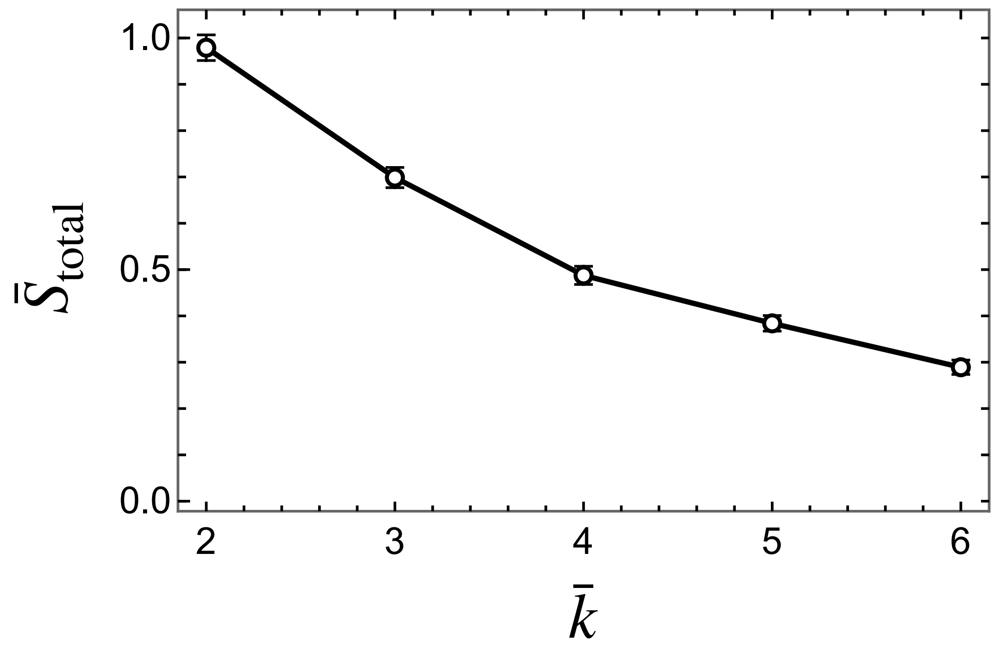

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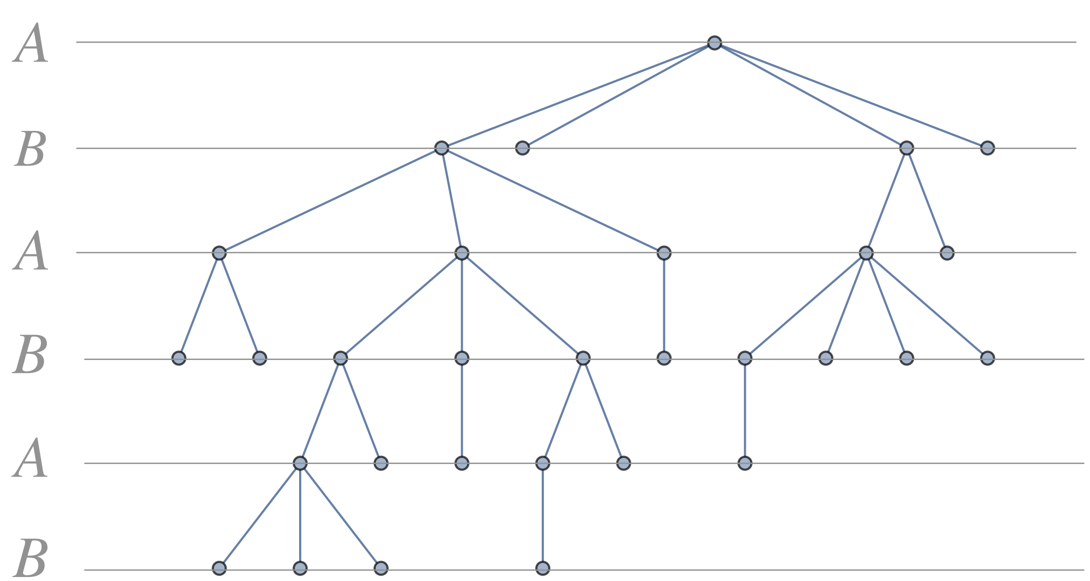

$S_{\text{total}} = |N_A - N_B|/2$

Random (connected) graphs

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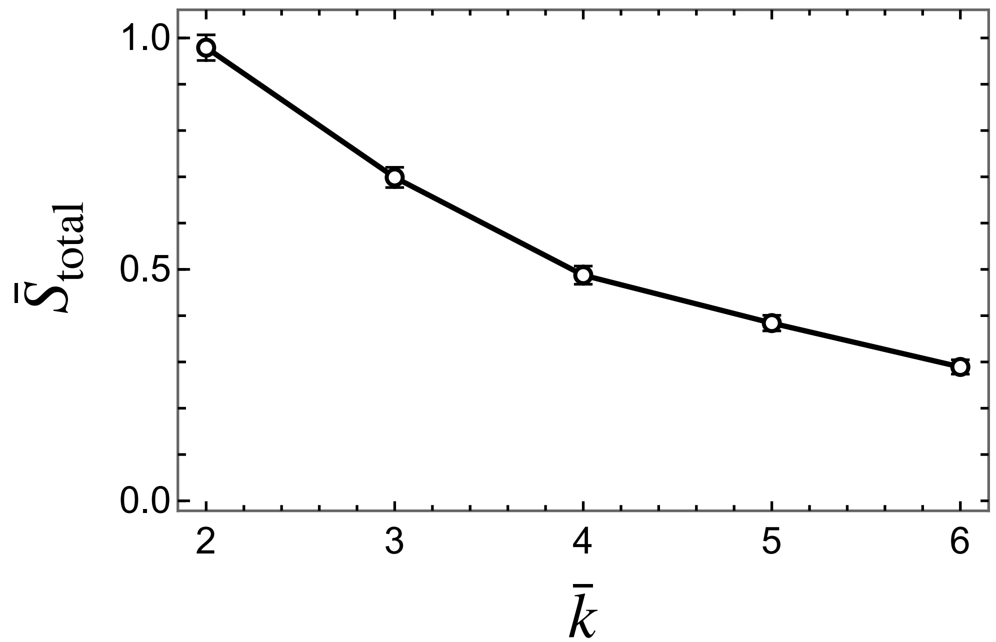

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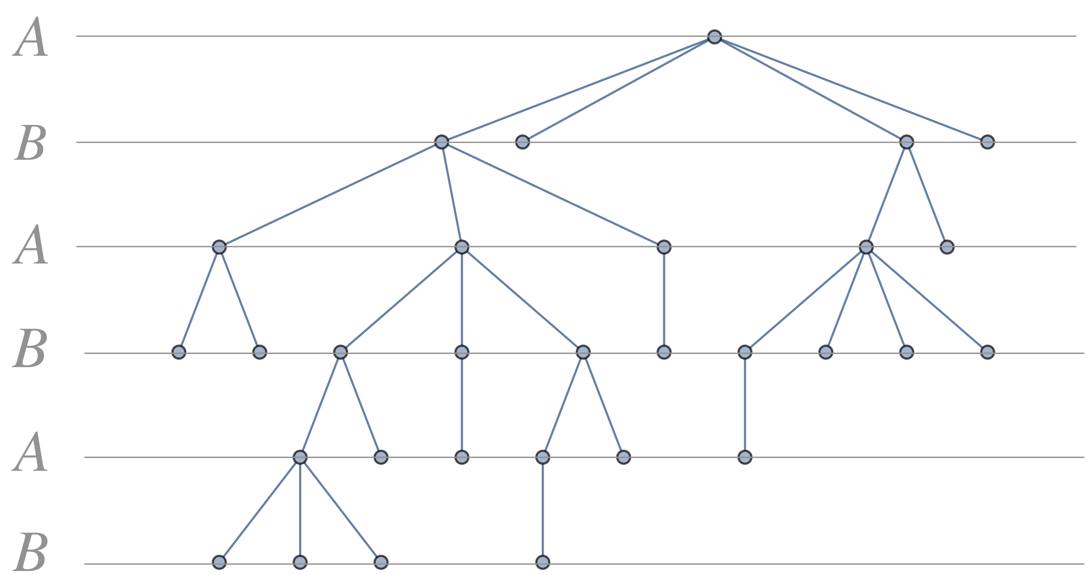
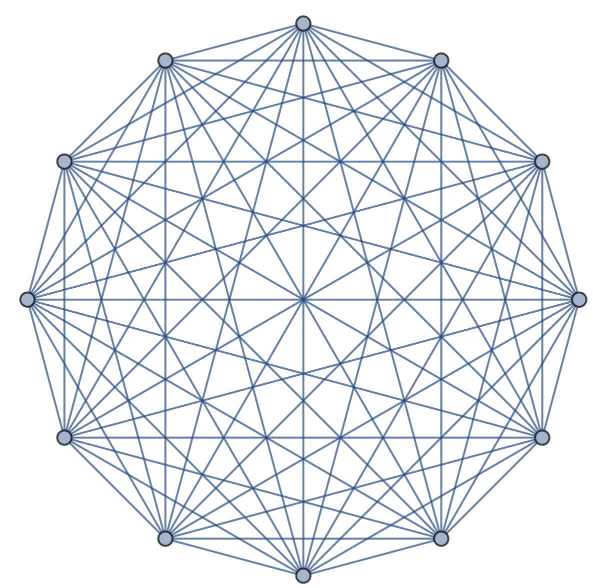


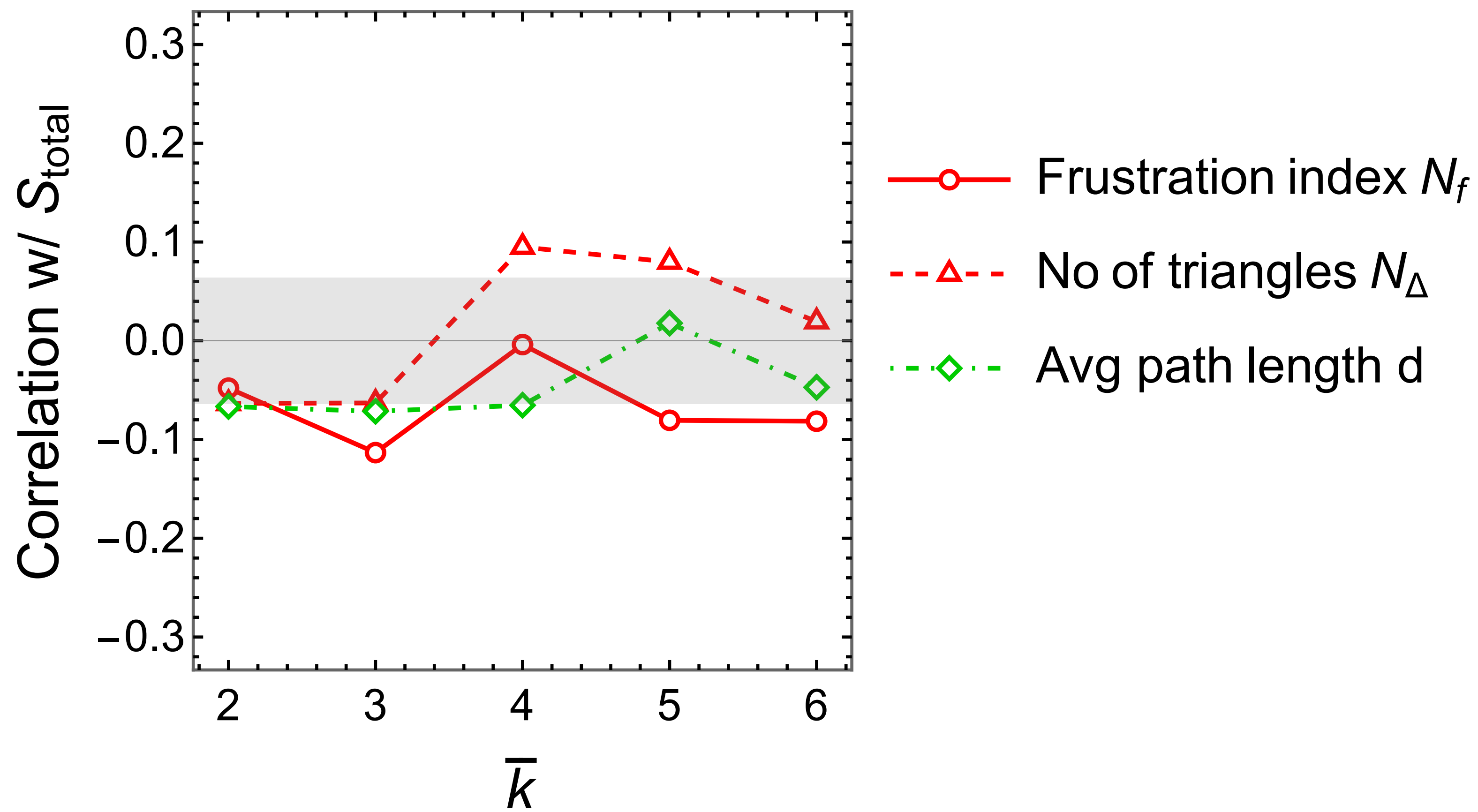
$S_{\text{total}} = |N_A - N_B|/2$

$N_e^{\max} = N(N-1)/2$



$S_{\text{total}} = 0$

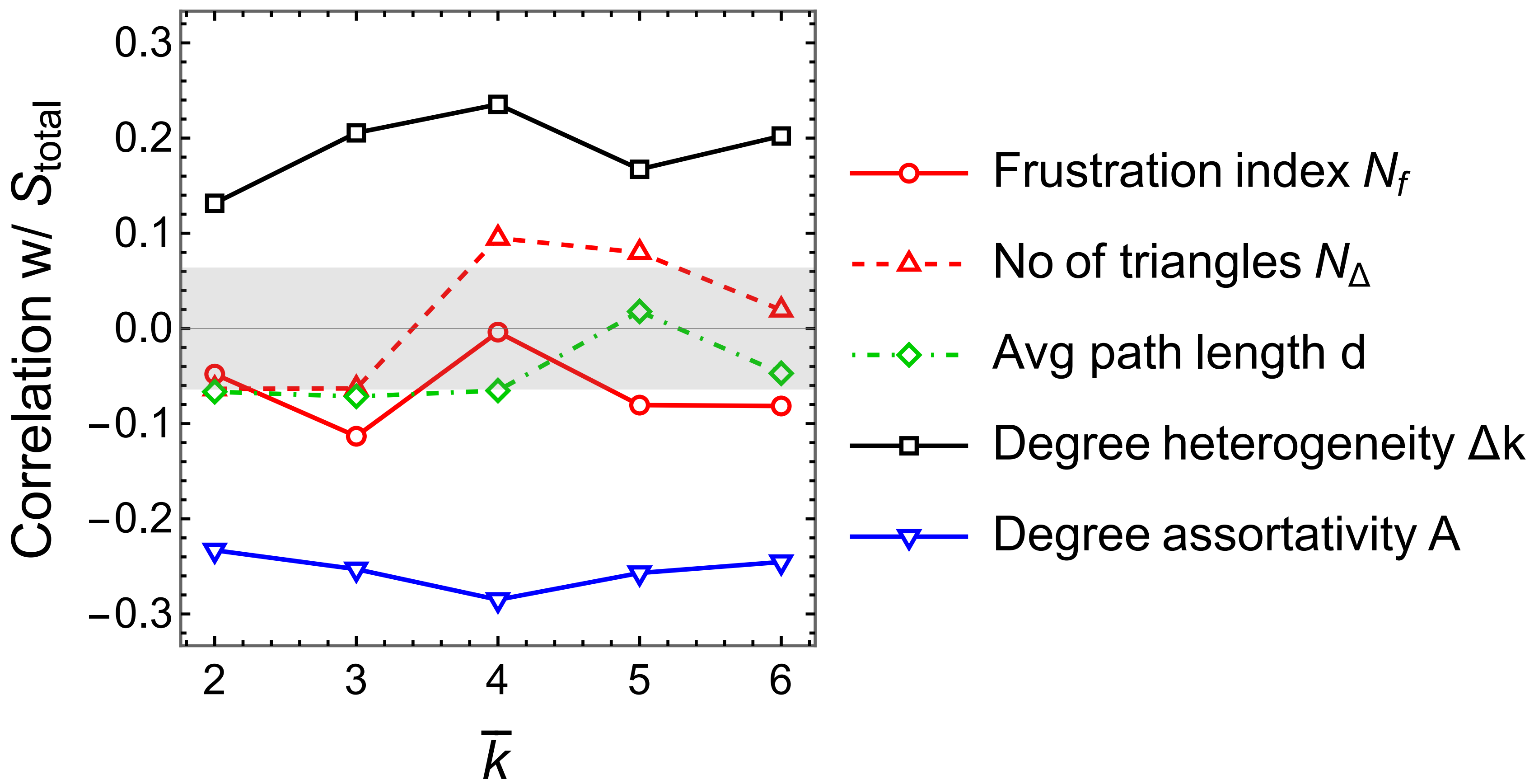
Random (connected) graphs — correlations



N_f : number of bonds to cut to make bipartite

Weak correlation w/ frustration

Random (connected) graphs — correlations



N_f : number of bonds to cut to make bipartite

$A \in [-1,1]$: +ve $\Longrightarrow$ high-degree nodes connect to high-degree nodes (& vice versa)

Newman, PRE 67, 026126 (2003)

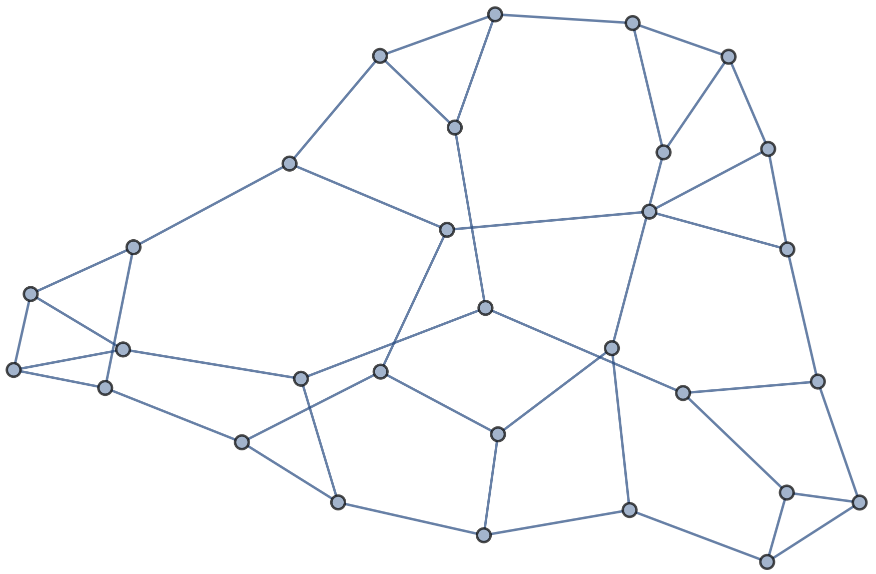
Weak correlation w/ frustration

Strong correlation w/ heterogeneity & assortativity

Heterogeneity

No heterogeneity: Random regular graphs

Every spin has k neighbors



$N = 30$

$k = 3$

No heterogeneity: Random regular graphs

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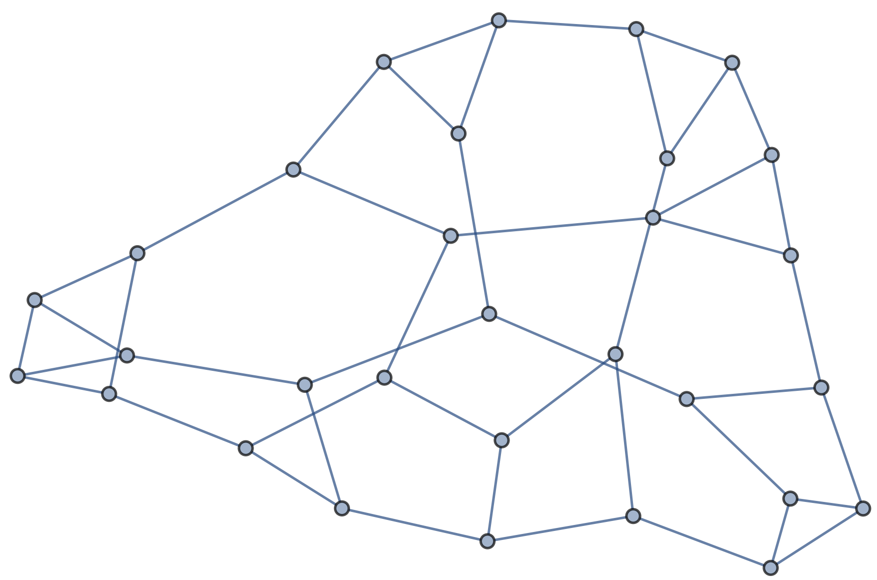

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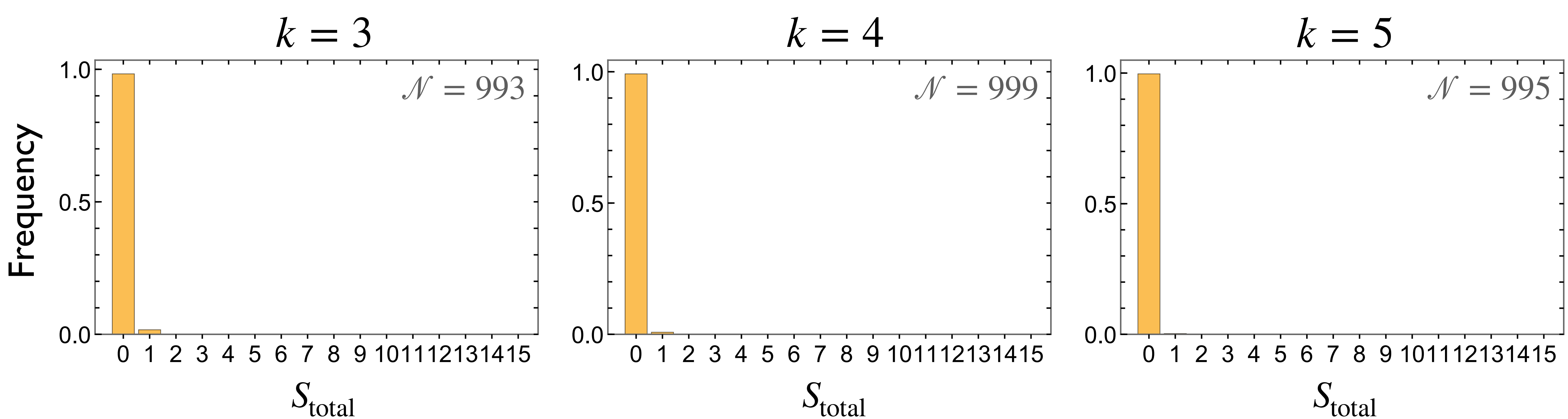

$\Longrightarrow$ **Nonzero S_{total} requires spread in degree (# of neighbors)**

Increased heterogeneity: Scale-free graphs

Power-law degree distribution

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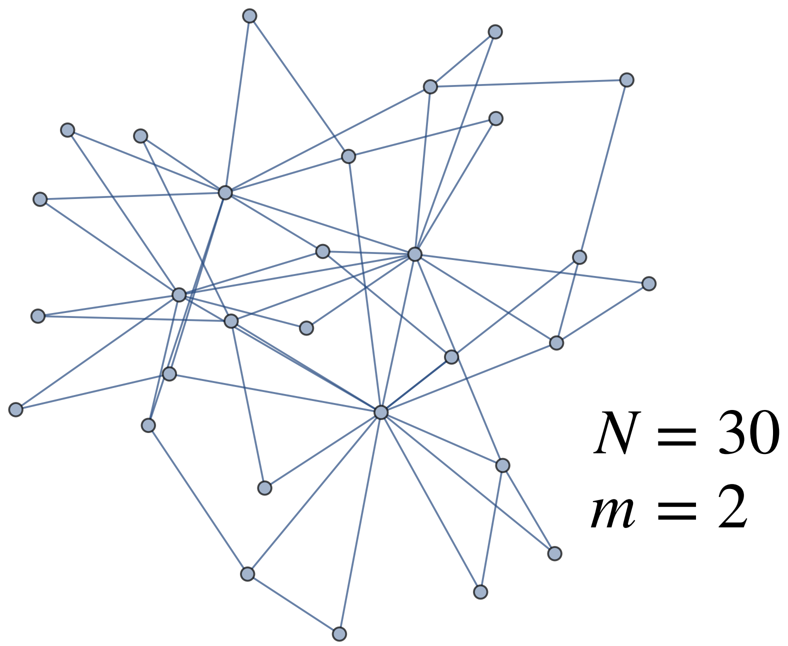
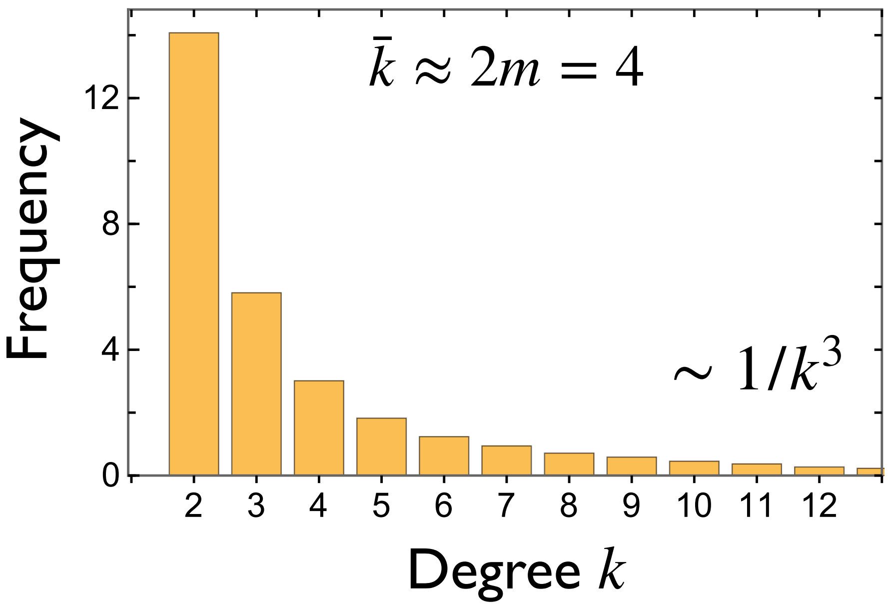
Power-law degree distribution

Barabasi-Albert: each new node connects to m existing nodes following preferential attachment — $p_i \propto k_i$

RMP 74, 47 (2002)





Increased heterogeneity: Scale-free graphs

Power-law degree distribution $\Longrightarrow$ Hubs

Barabasi-Albert: each new node connects to m existing nodes following preferential attachment — $p_i \propto k_i$

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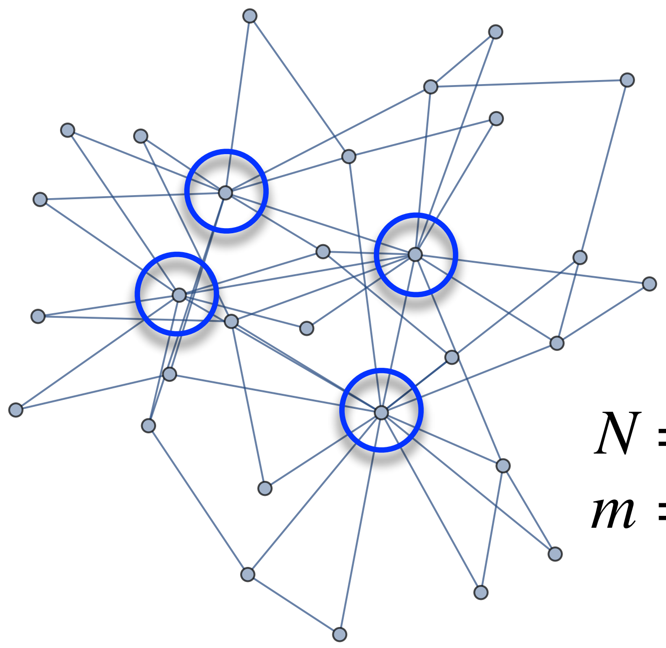
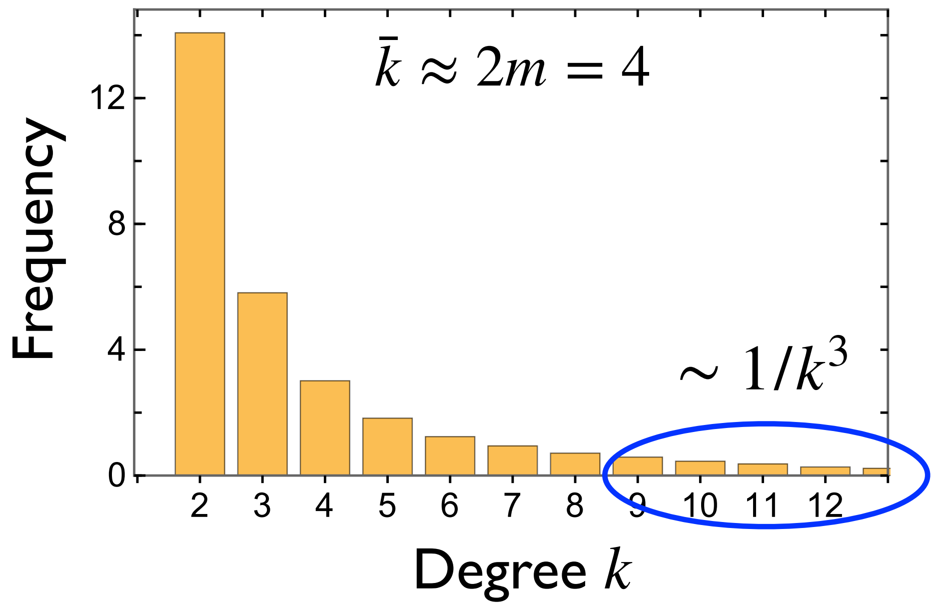


$N = 30$
$m = 2$



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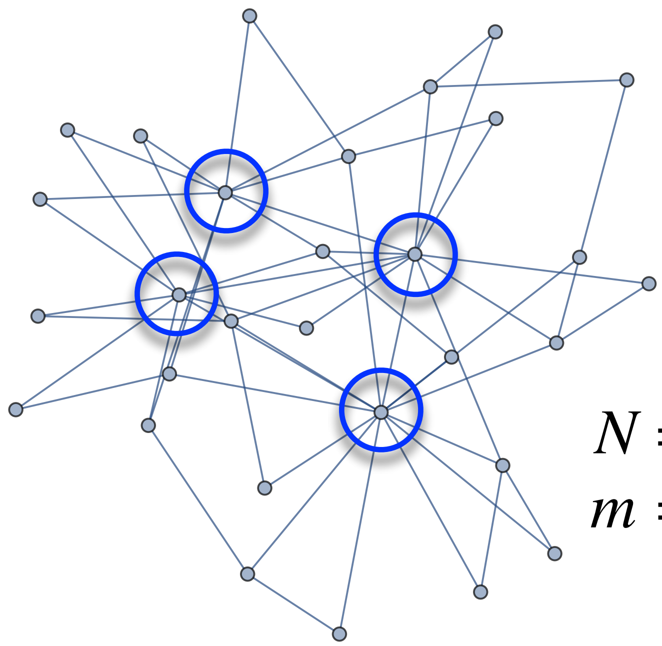

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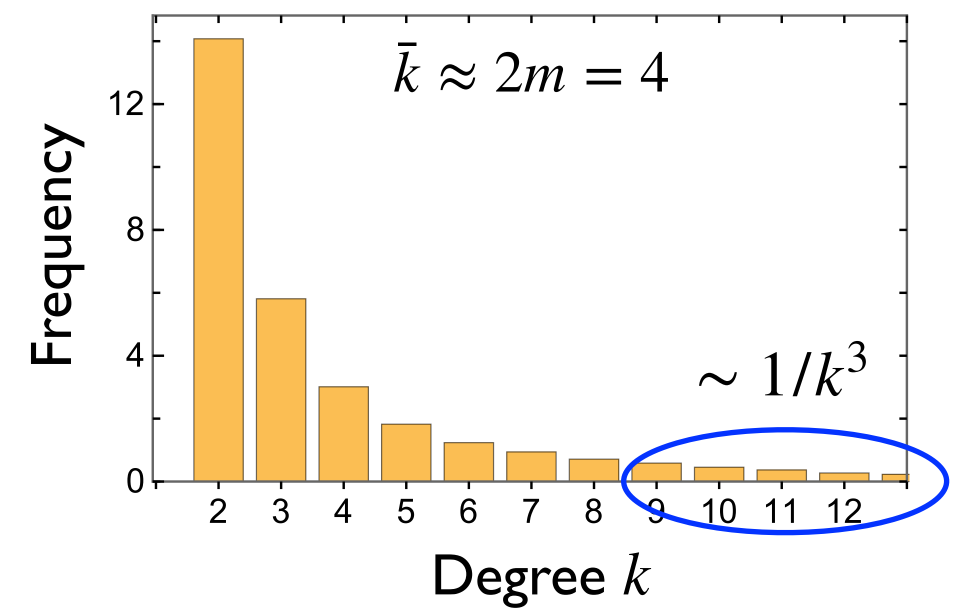
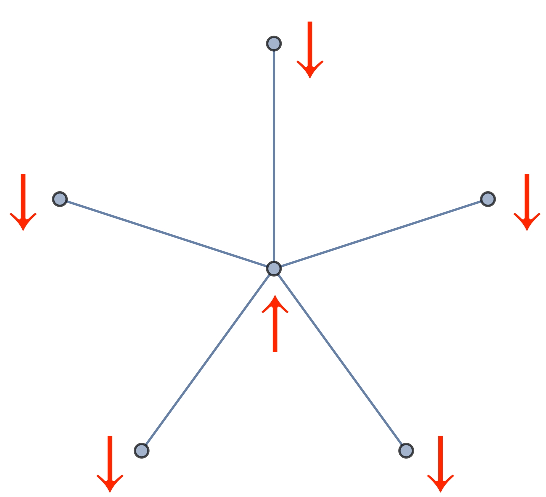
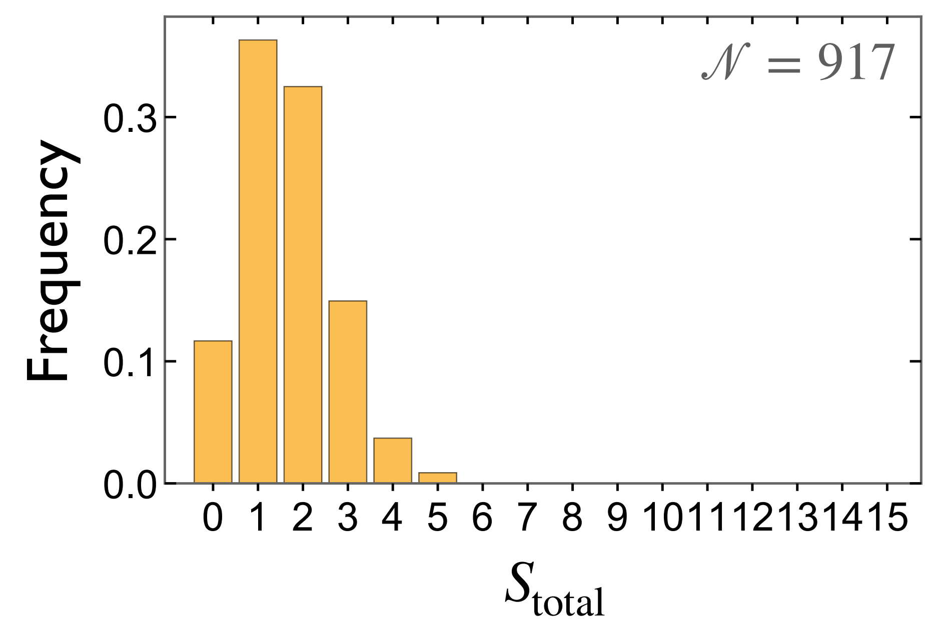
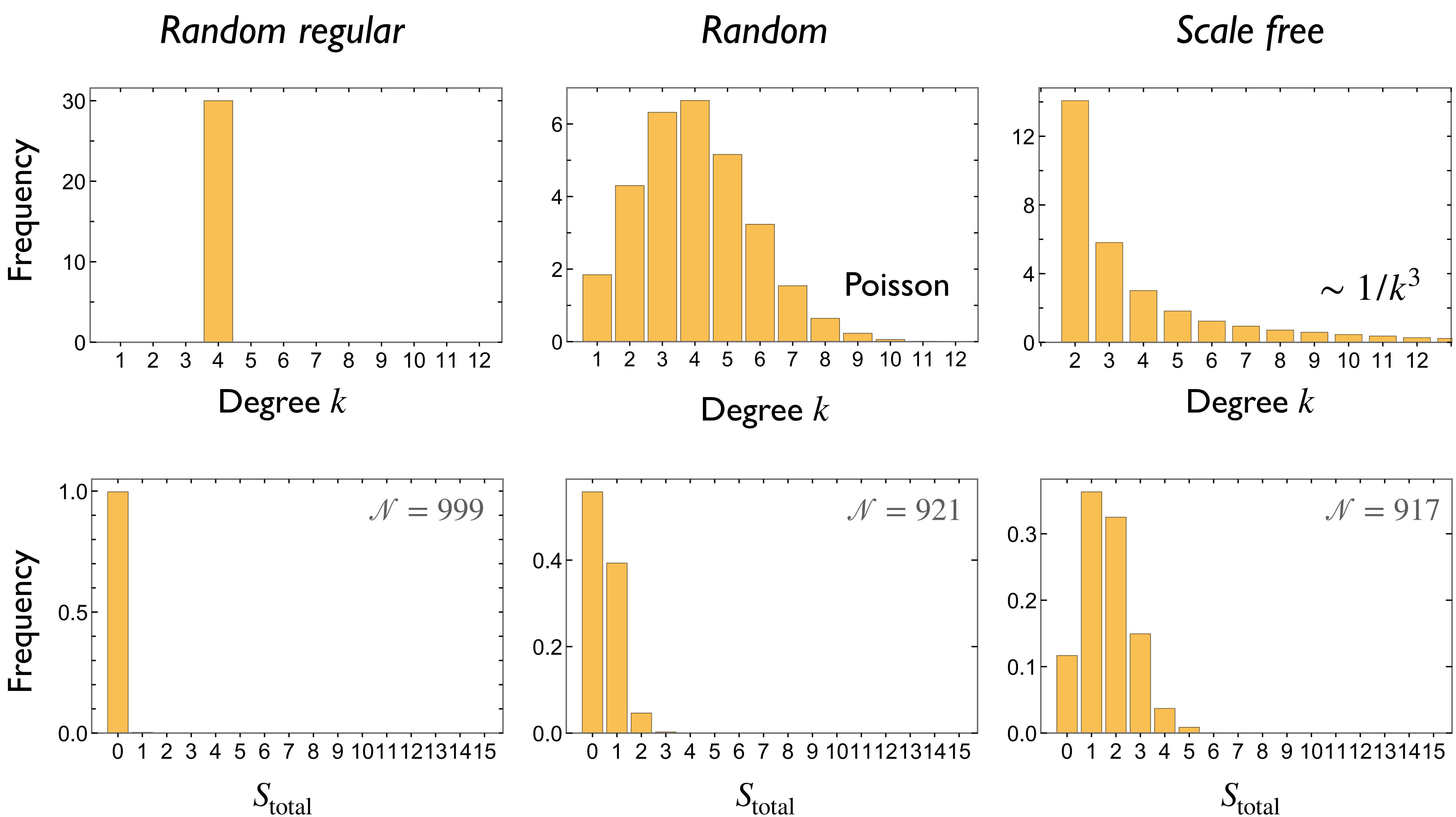




Large S_{total}



Enhanced magnetization



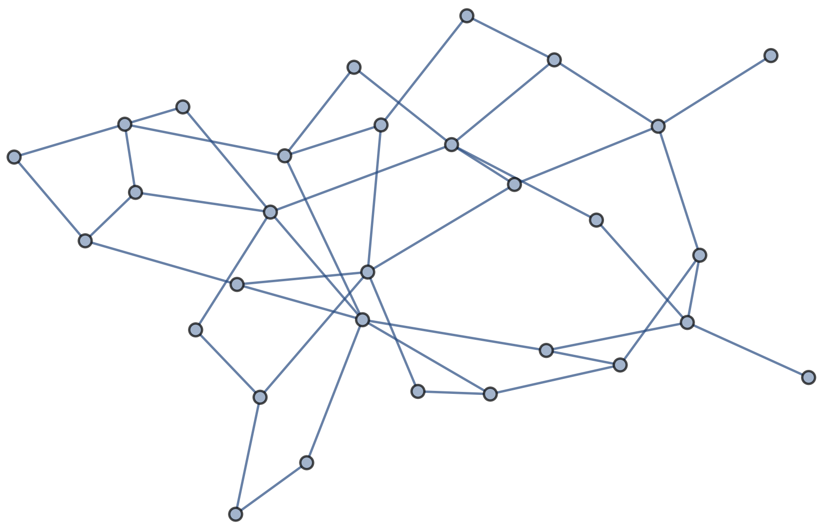
Results for: $N = 30$, $\bar{k} = 4$

Frustration level

Insensitivity to frustration level

Remove all triangles

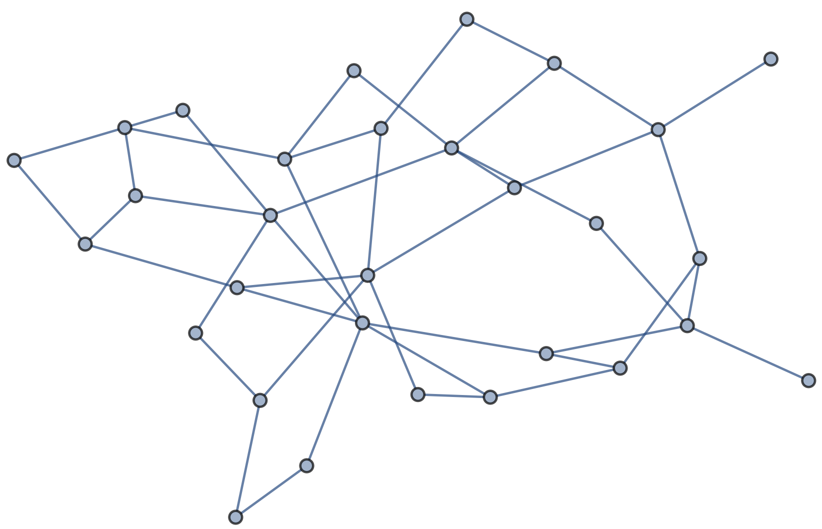
Bayati, Montanari, Saberi, arXiv:0811.2853



$N = 30$, $N_e = 45$, $N_\Delta = 0$

Remove all triangles

Bayati, Montanari, Saberi, arXiv:0811.2853



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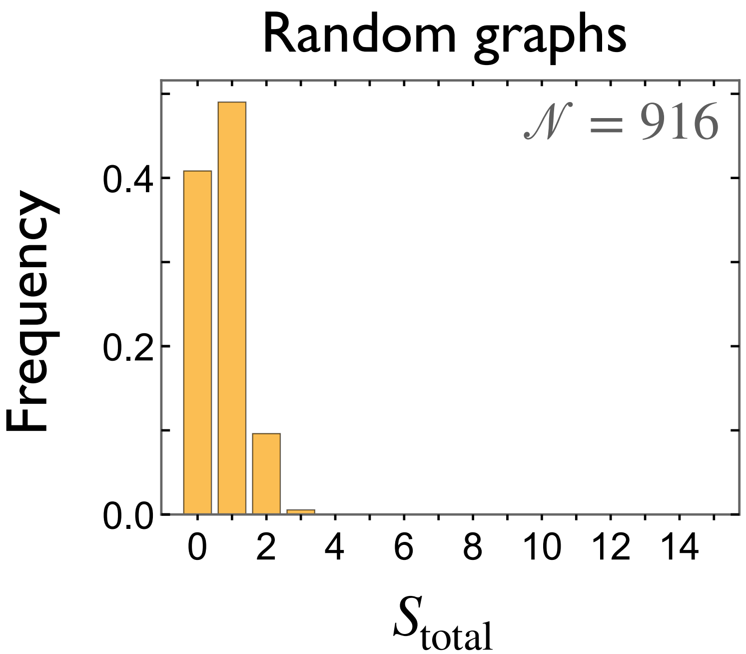
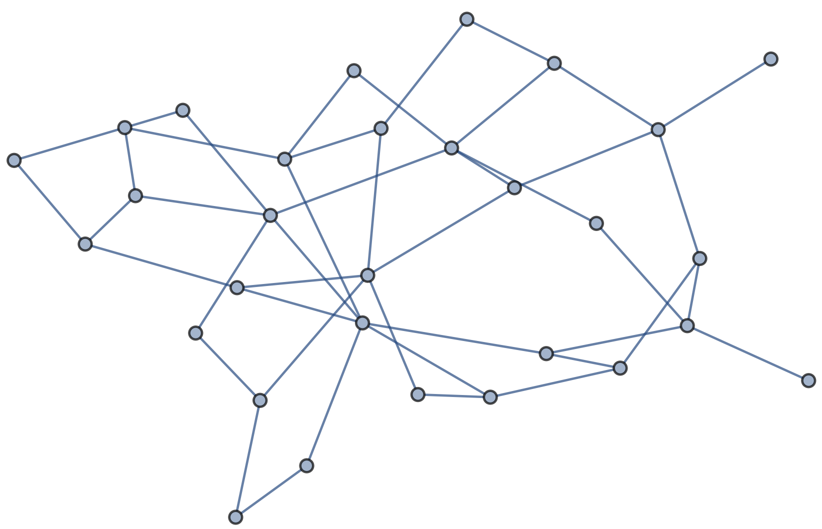


Insensitivity to frustration level

Remove all triangles $\Longrightarrow$ Spin distribution unaffected

Bayati, Montanari, Saberi, arXiv:0811.2853



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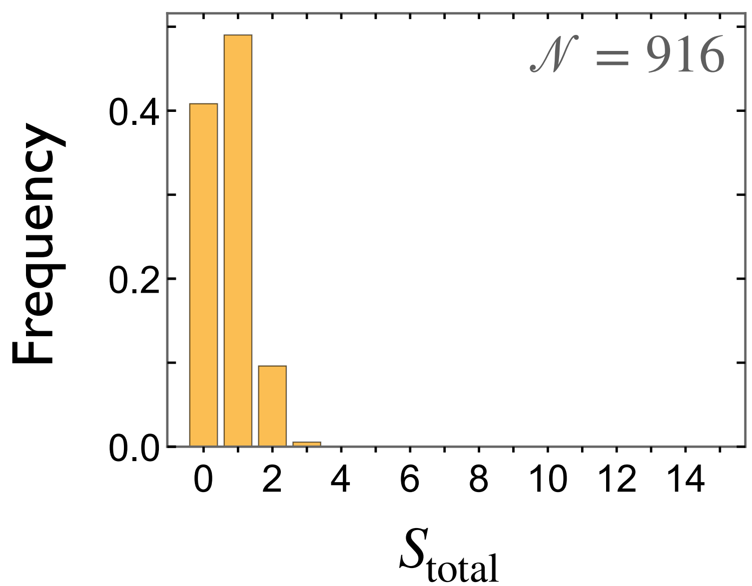
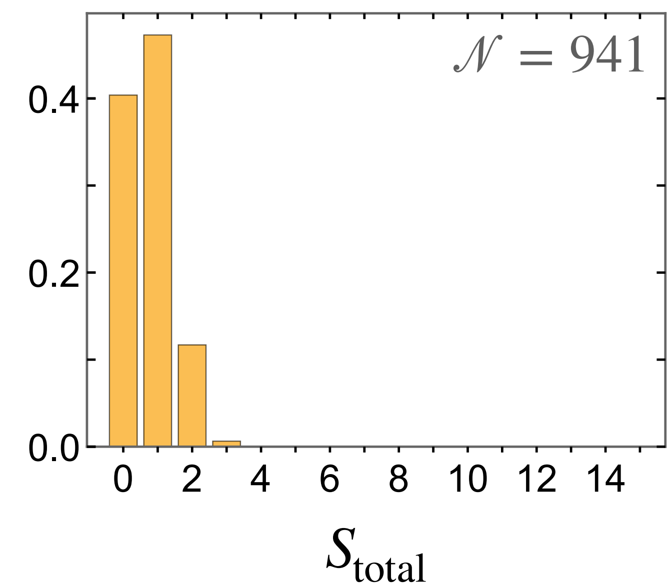


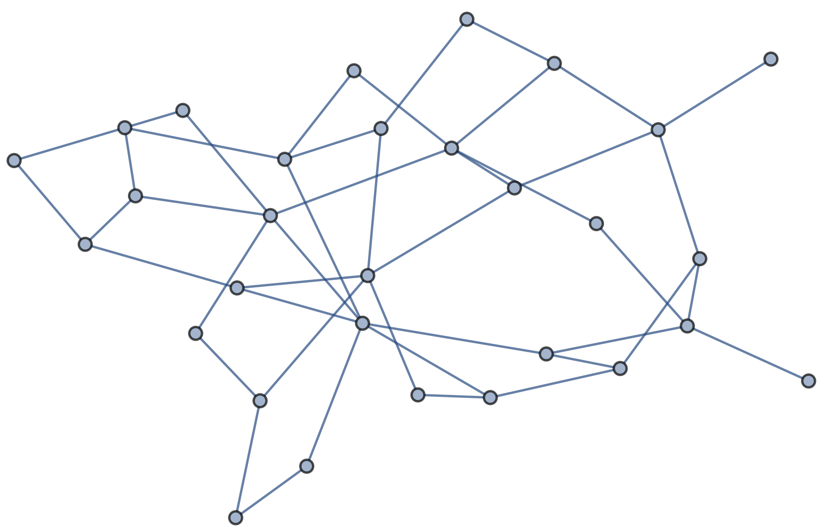
Random graphs, $N_\Delta = 0$



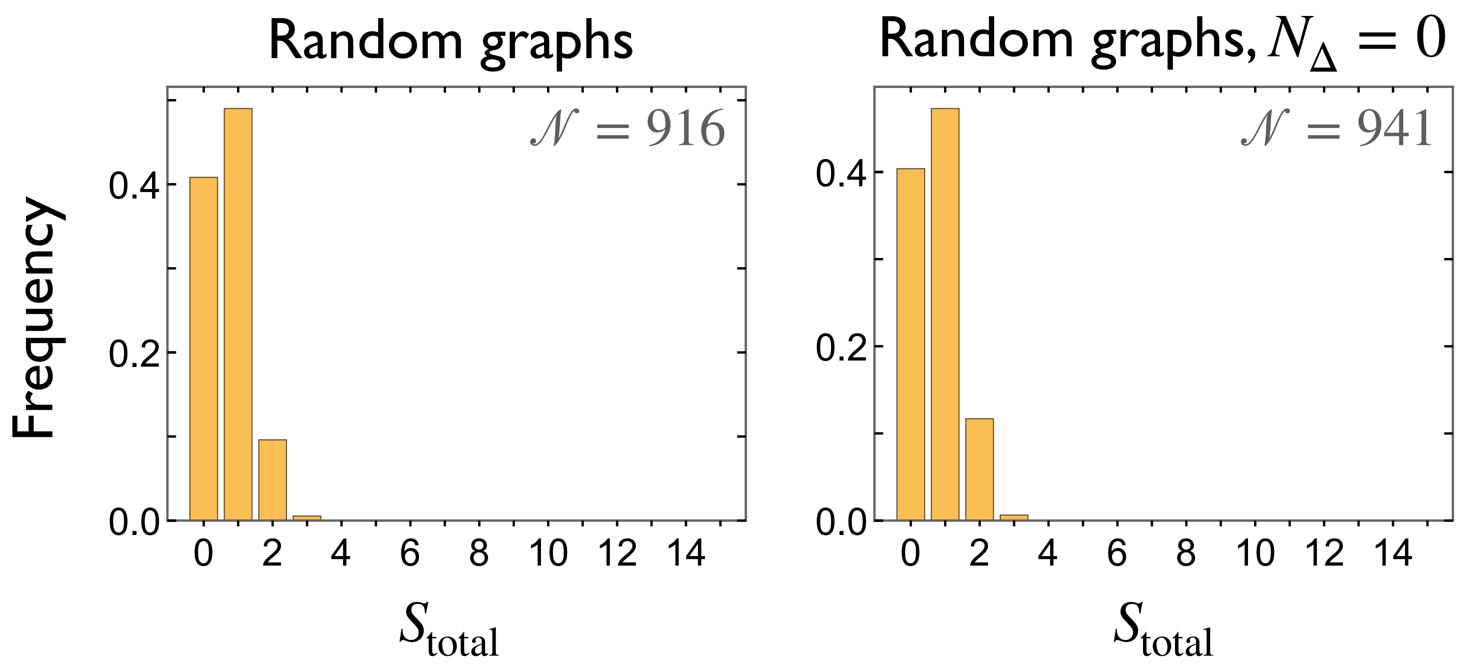
Insensitivity to frustration level

Remove all triangles $\Longrightarrow$ Spin distribution unaffected

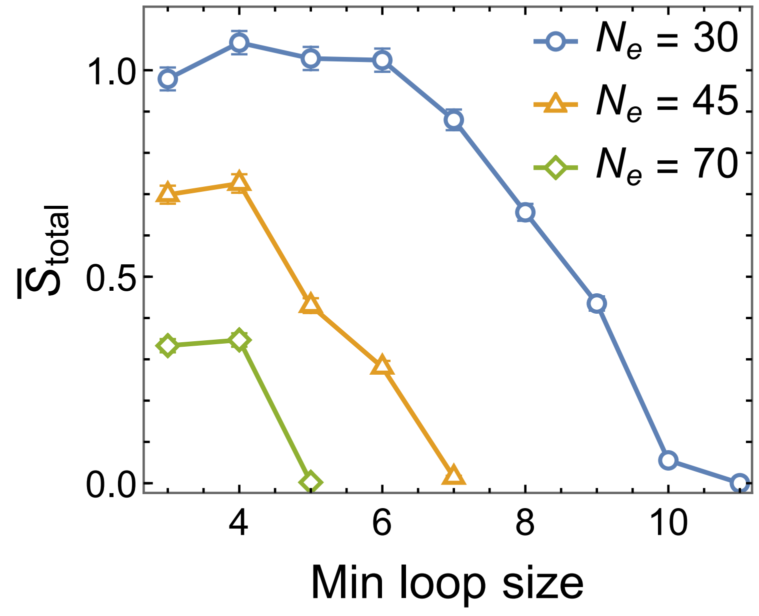
Bayati, Montanari, Saberi, arXiv:0811.2853



$N = 30$, $N_e = 45$, $N_\Delta = 0$



Remove short loops



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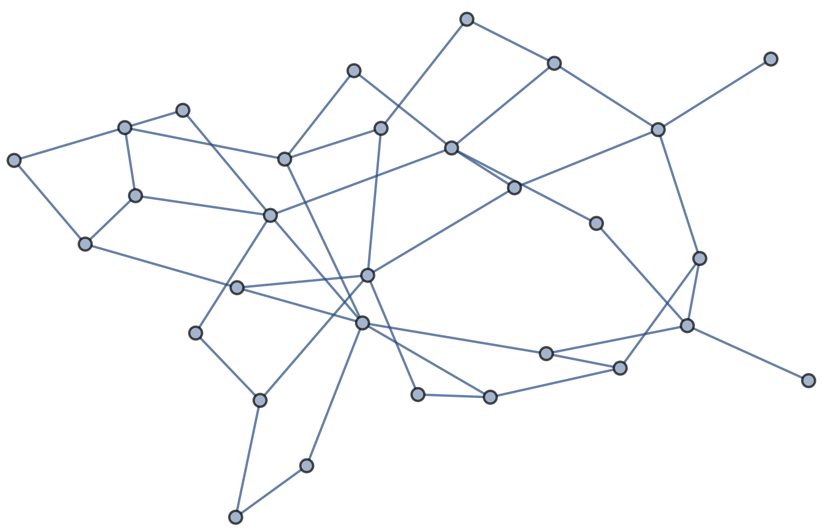

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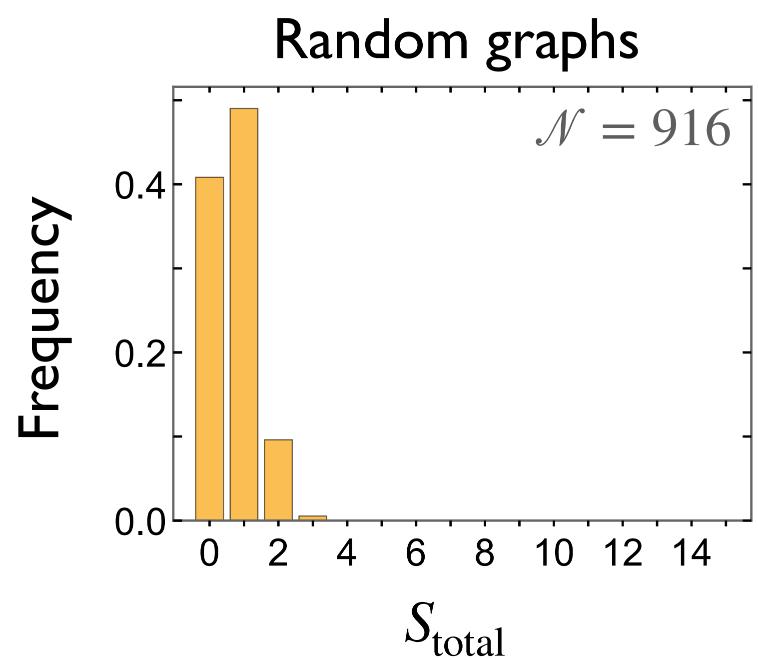
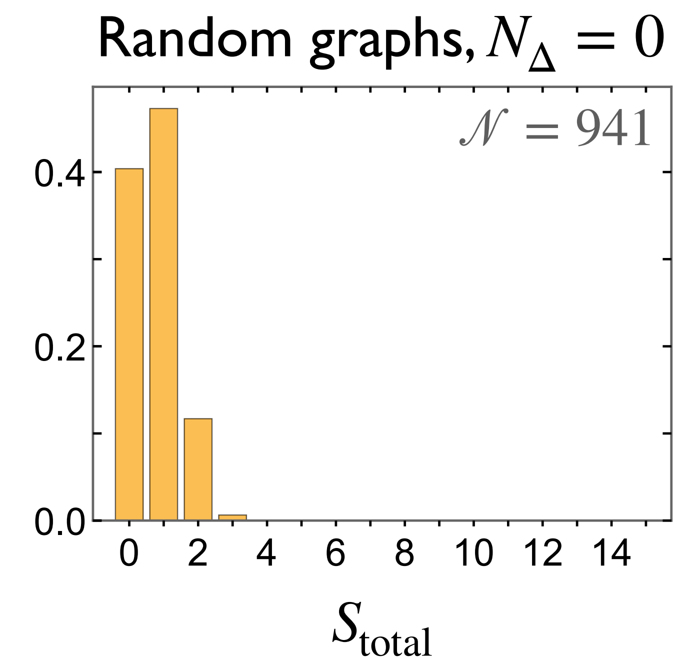
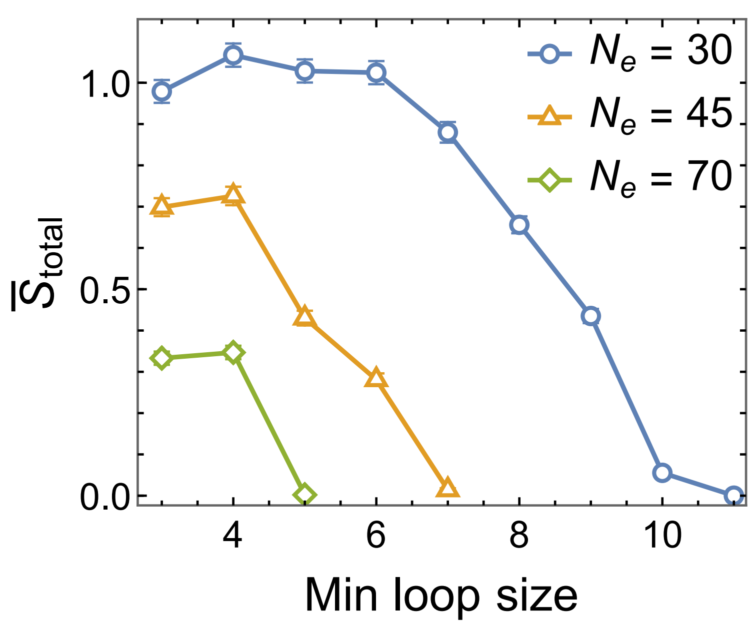
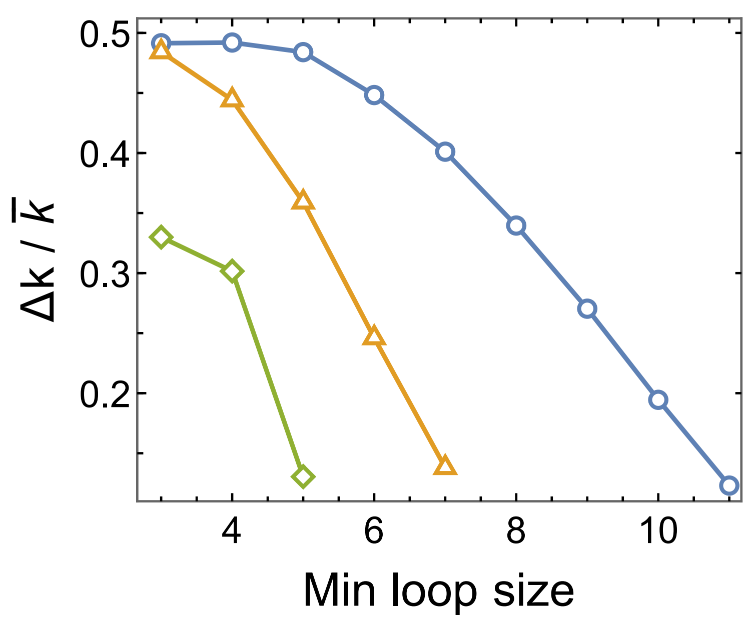




Remove short loops $\Longrightarrow$ Magnetization follows heterogeneity





Insensitivity to frustration level

Remove all triangles $\Longrightarrow$ Spin distribution unaffected

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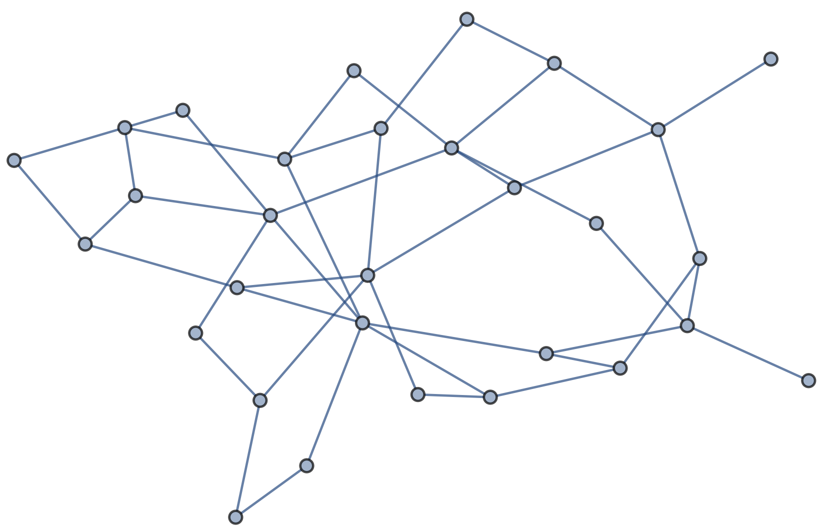
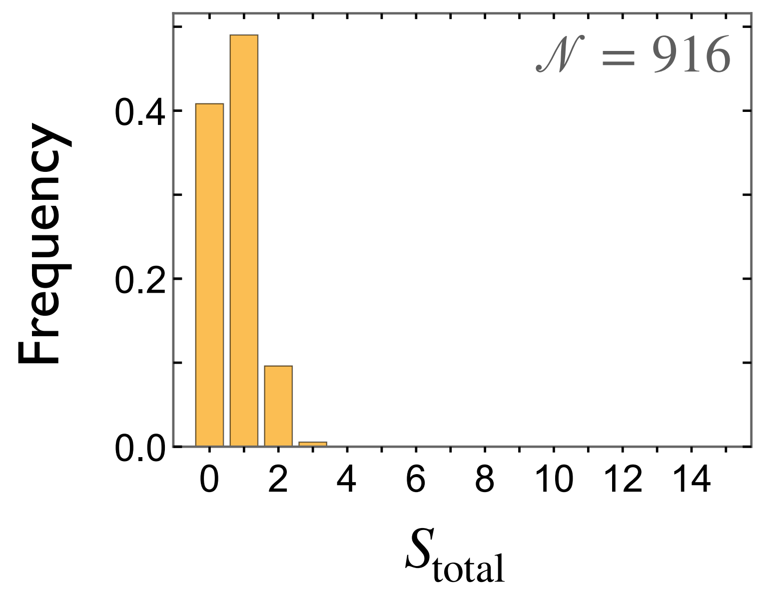
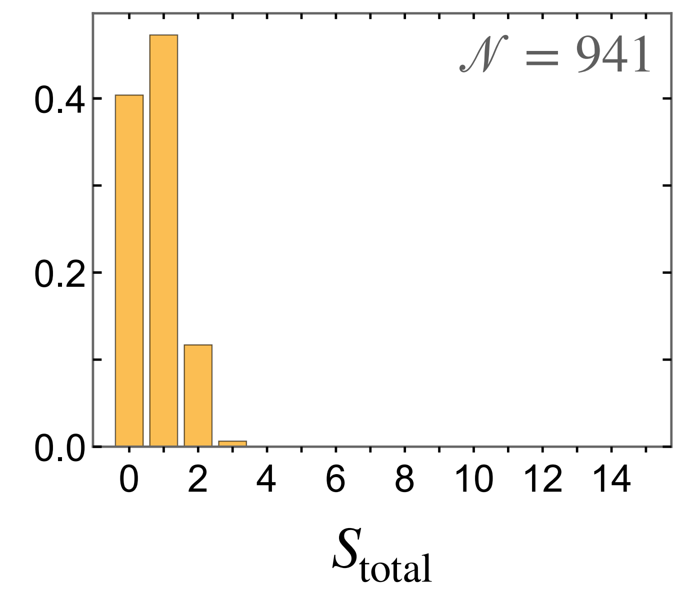


$N = 30,\ N_e = 45,\ N_\Delta = 0$

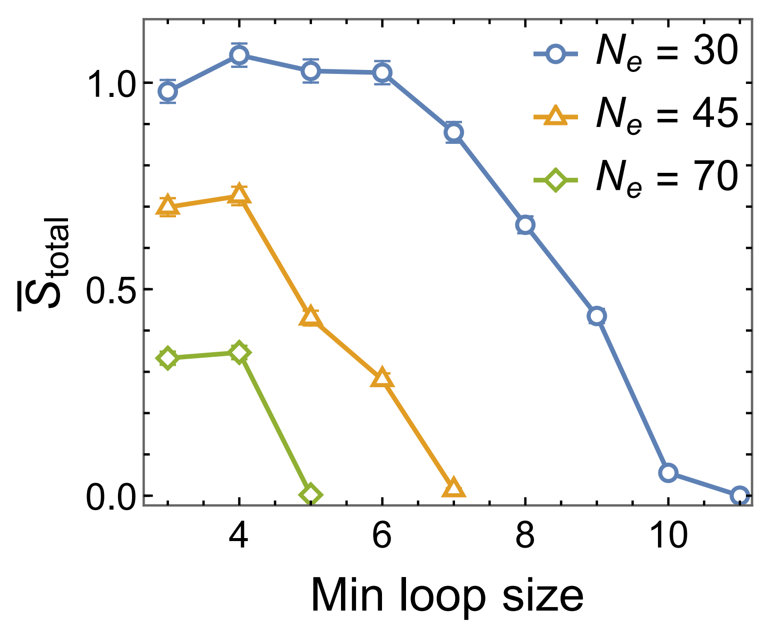
Random graphs

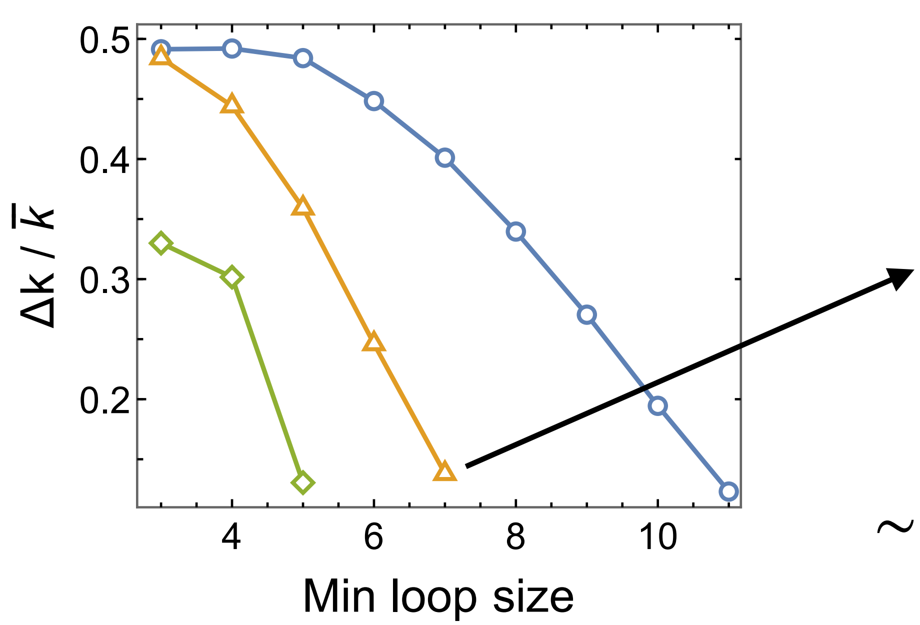


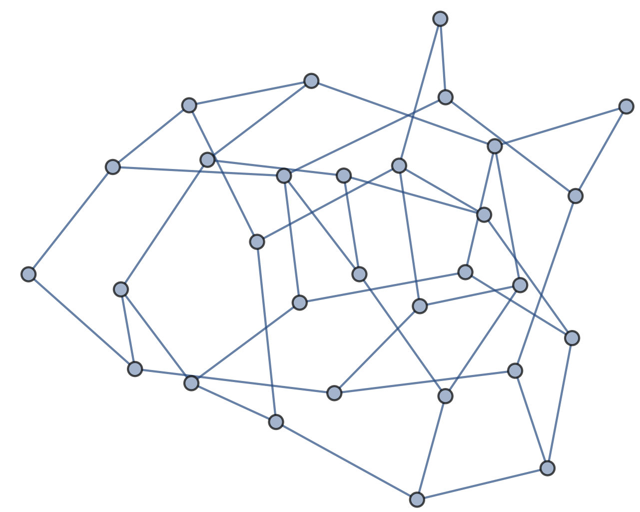
Random graphs, $N_\Delta = 0$



Remove short loops $\Longrightarrow$ Magnetization follows heterogeneity







~ random regular

Insensitivity to frustration level

Tune N_Δ without changing degree distribution $\left(\sim 1/k^3\right)$

Holme and Kim,
PRE 65, 026107 (2002)

Insensitivity to frustration level

Tune N_Δ without changing degree distribution $\left(\sim 1/k^3\right)$

Holme and Kim,
PRE 65, 026107 (2002)

For every new node:

(1) Connect to existing node i with prob $p_i \propto k_i$

(2) With prob p connect to a neighbor of i, else repeat (1)

Insensitivity to frustration level

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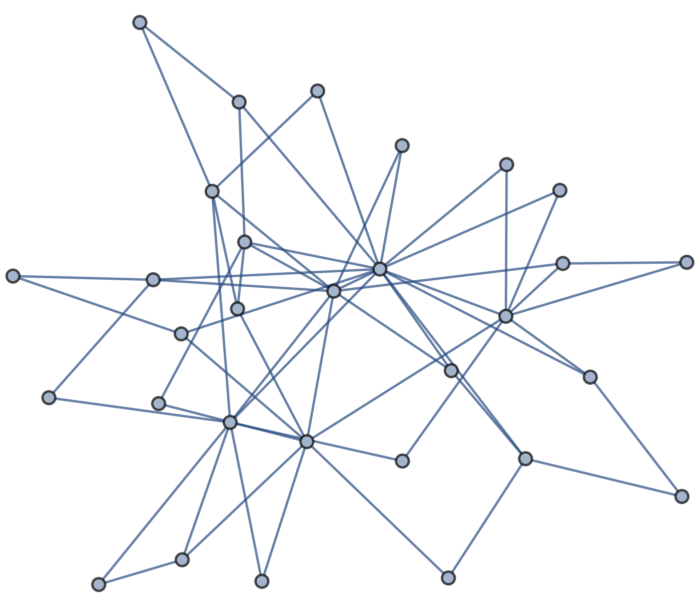
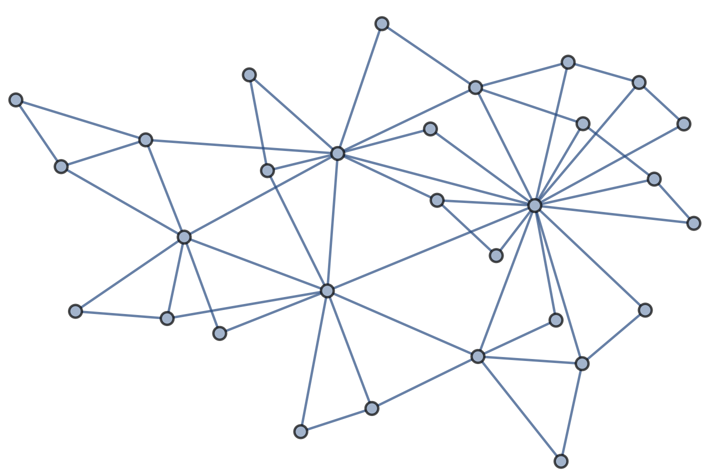


Fewer triangles



More triangles

Tune N_Δ without changing degree distribution $\left(\sim 1/k^3\right)$

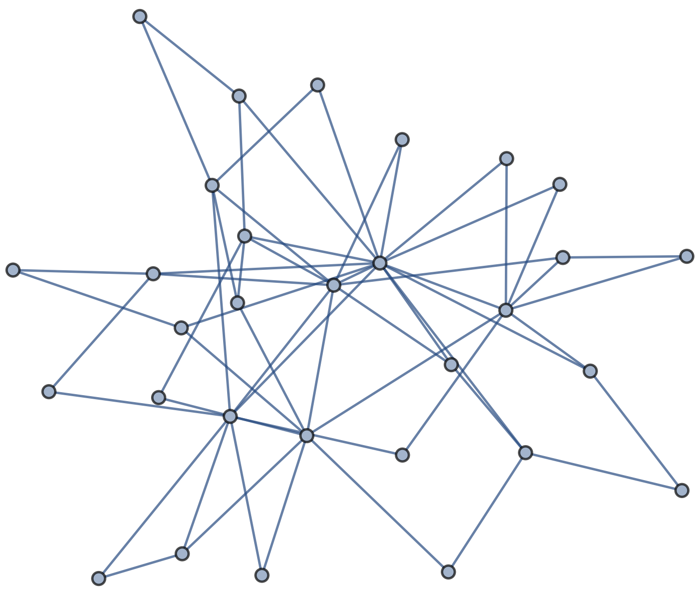
Holme and Kim,
PRE 65, 026107 (2002)

$\Longrightarrow$ **Weak variation of** S_{total}

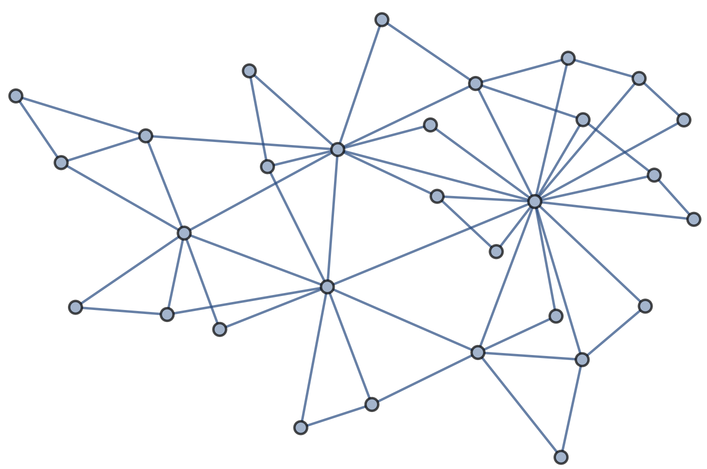
For every new node:

(1) Connect to existing node i with prob $p_i \propto k_i$

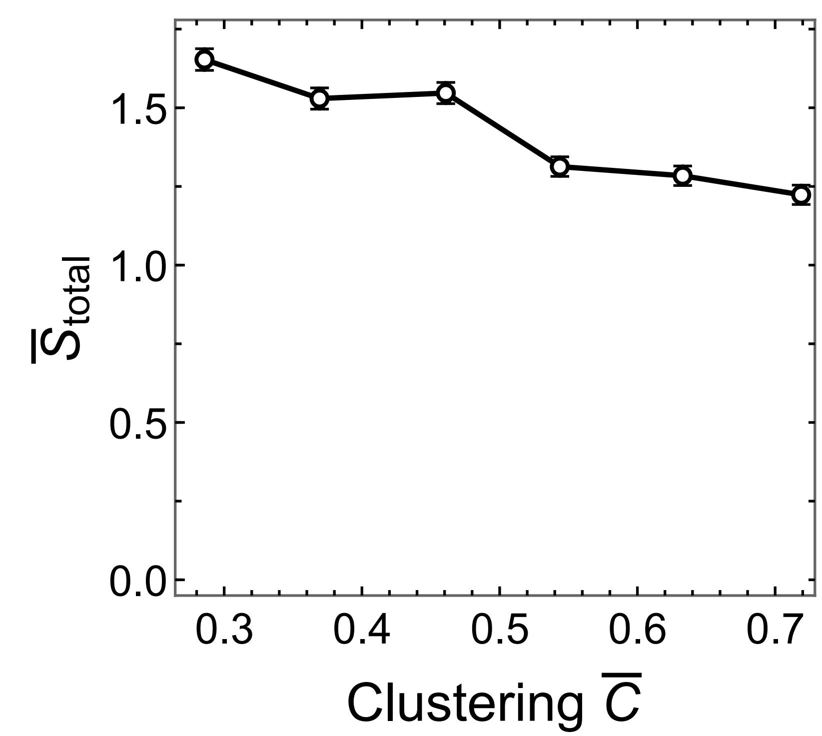
(2) With prob p connect to a neighbor of i, else repeat (1)



Fewer triangles



More triangles

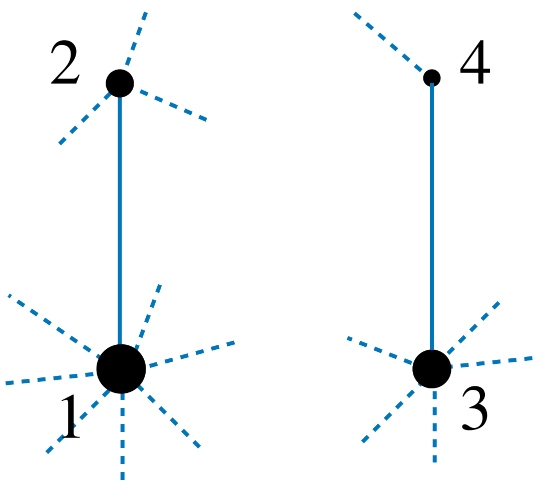


Assortativity

Tuning assortativity

Van Mieghem *et al*,
EPJ-B 76, 643 (2010)

Degree-preserving rewiring



$k_1 > k_3 > k_2 > k_4$

Tuning assortativity

Van Mieghem *et al*,
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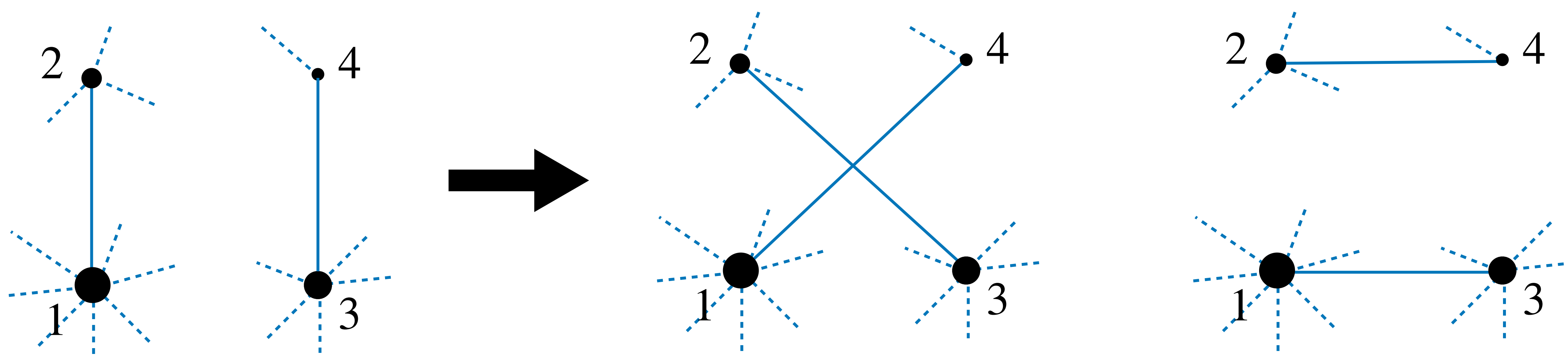
Degree-preserving rewiring



$k_1 > k_3 > k_2 > k_4$

Disassortative

Assortative

Tuning assortativity

Van Mieghem *et al*,
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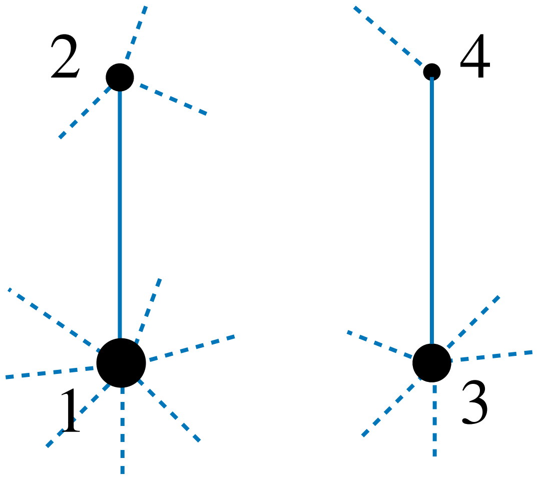
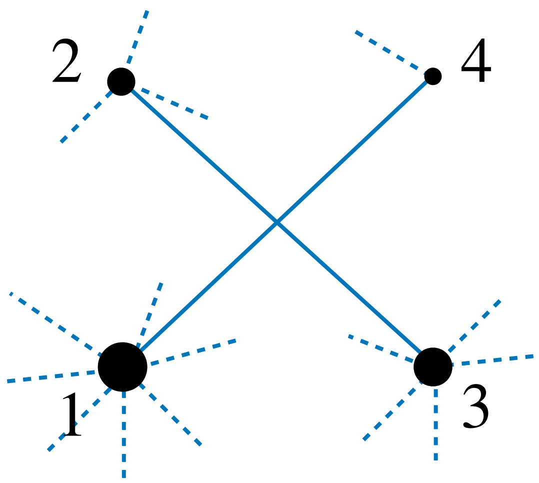
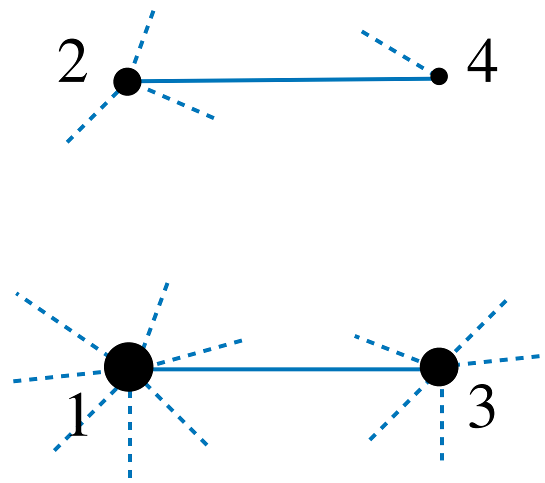
Degree-preserving rewiring



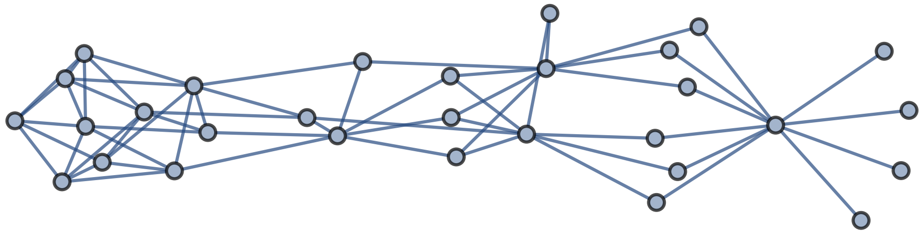
$k_1 > k_3 > k_2 > k_4$



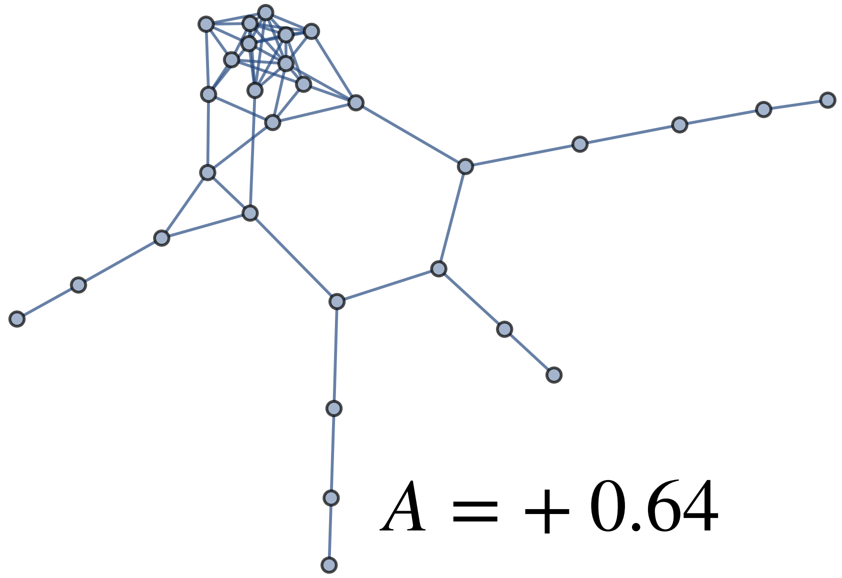
Disassortative



Assortative



$A = -0.95$

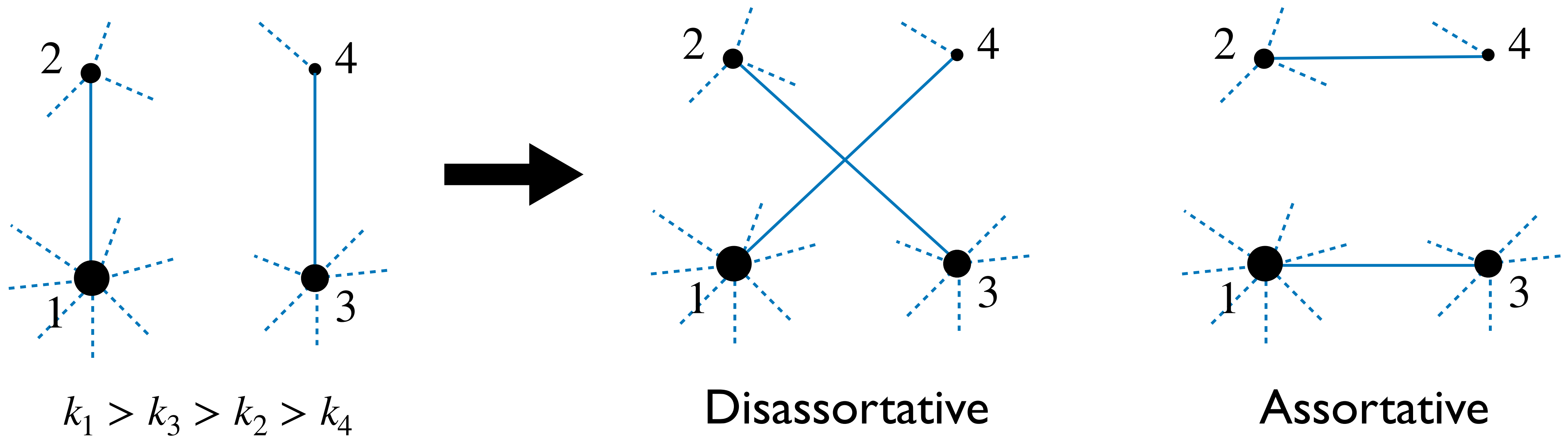


$A = +0.64$

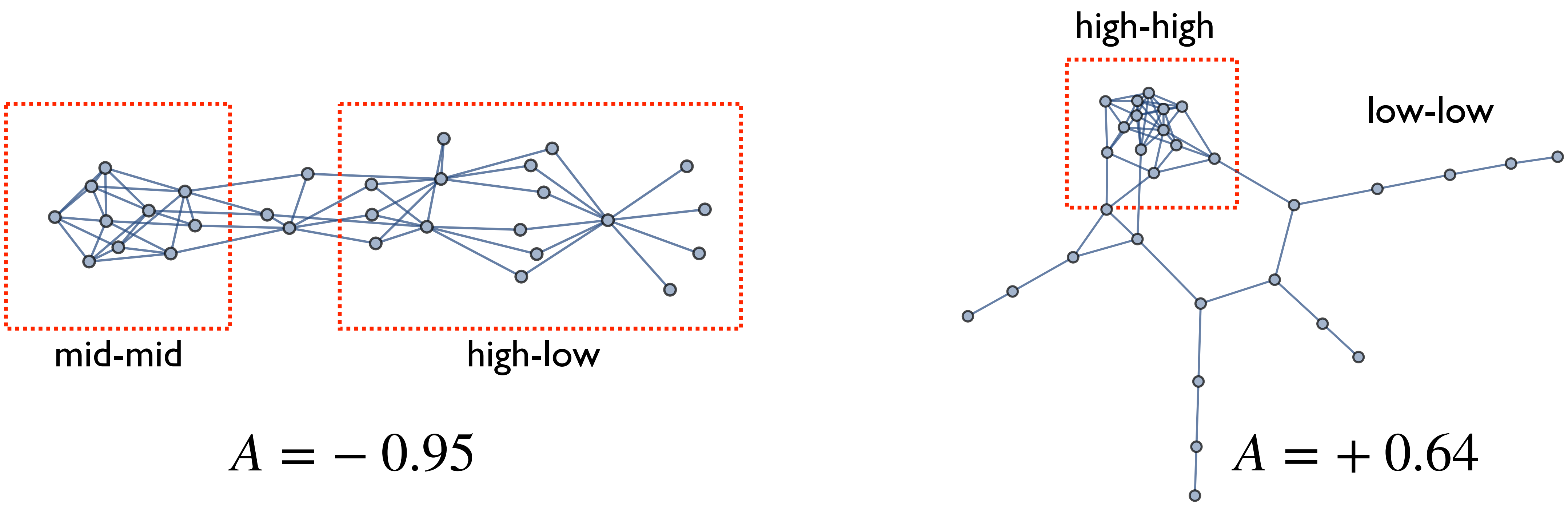
Tuning assortativity

Van Mieghem *et al*, EPJ-B 76, 643 (2010)

Degree-preserving rewiring



$k_1 > k_3 > k_2 > k_4$

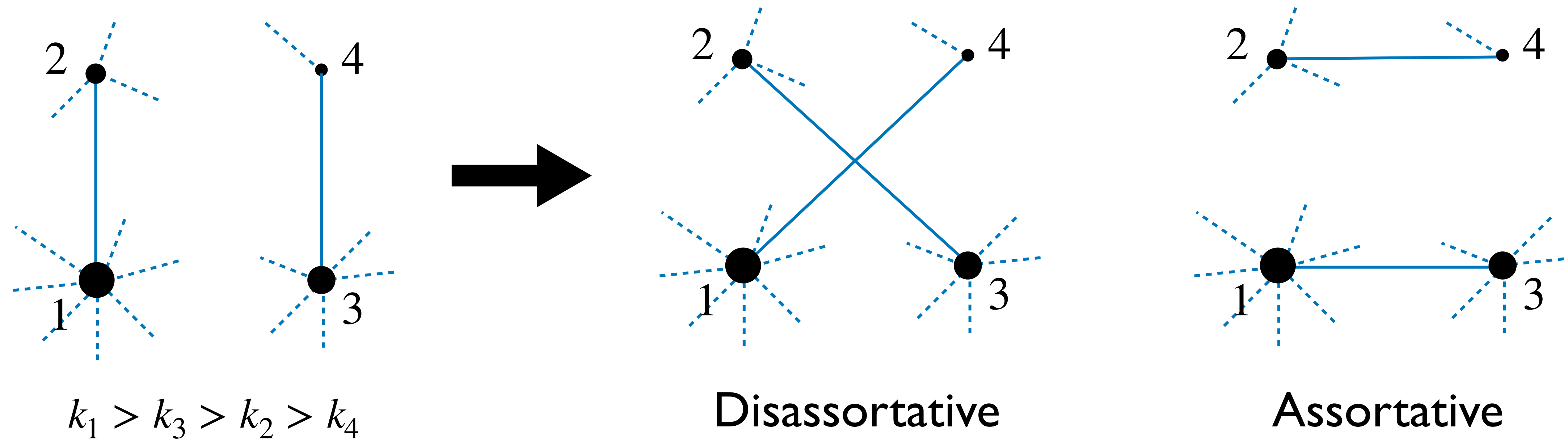


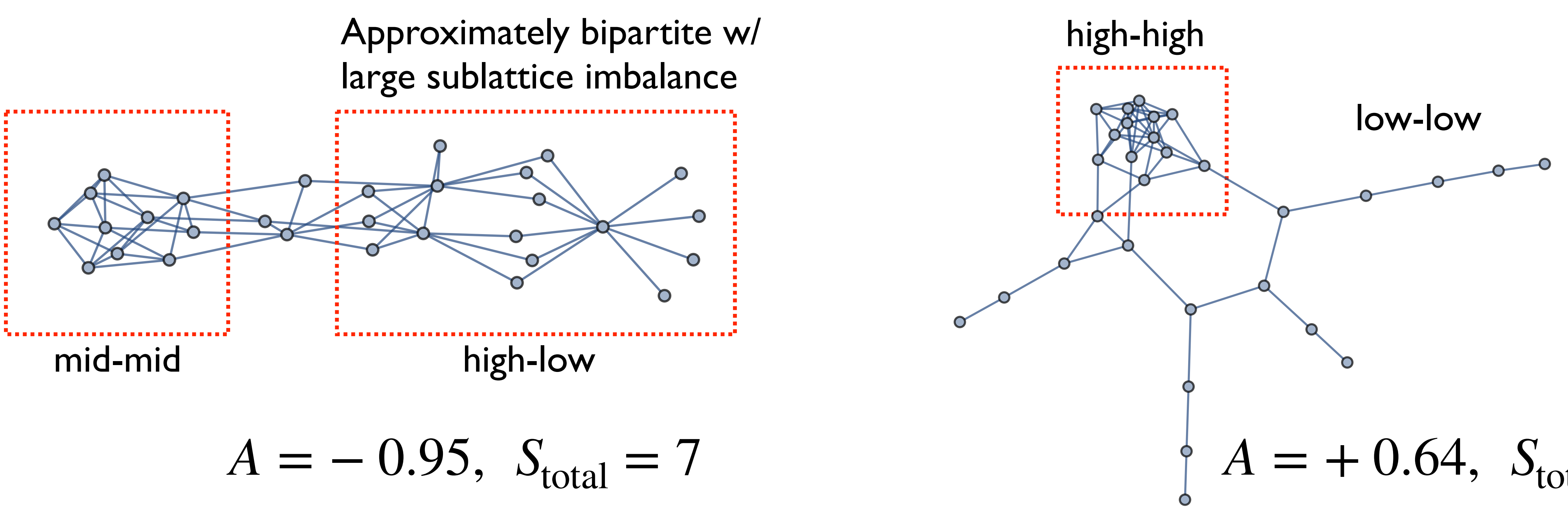
$A = -0.95$ $A = +0.64$

Tuning assortativity

Van Mieghem *et al*, EPJ-B 76, 643 (2010)

Degree-preserving rewiring



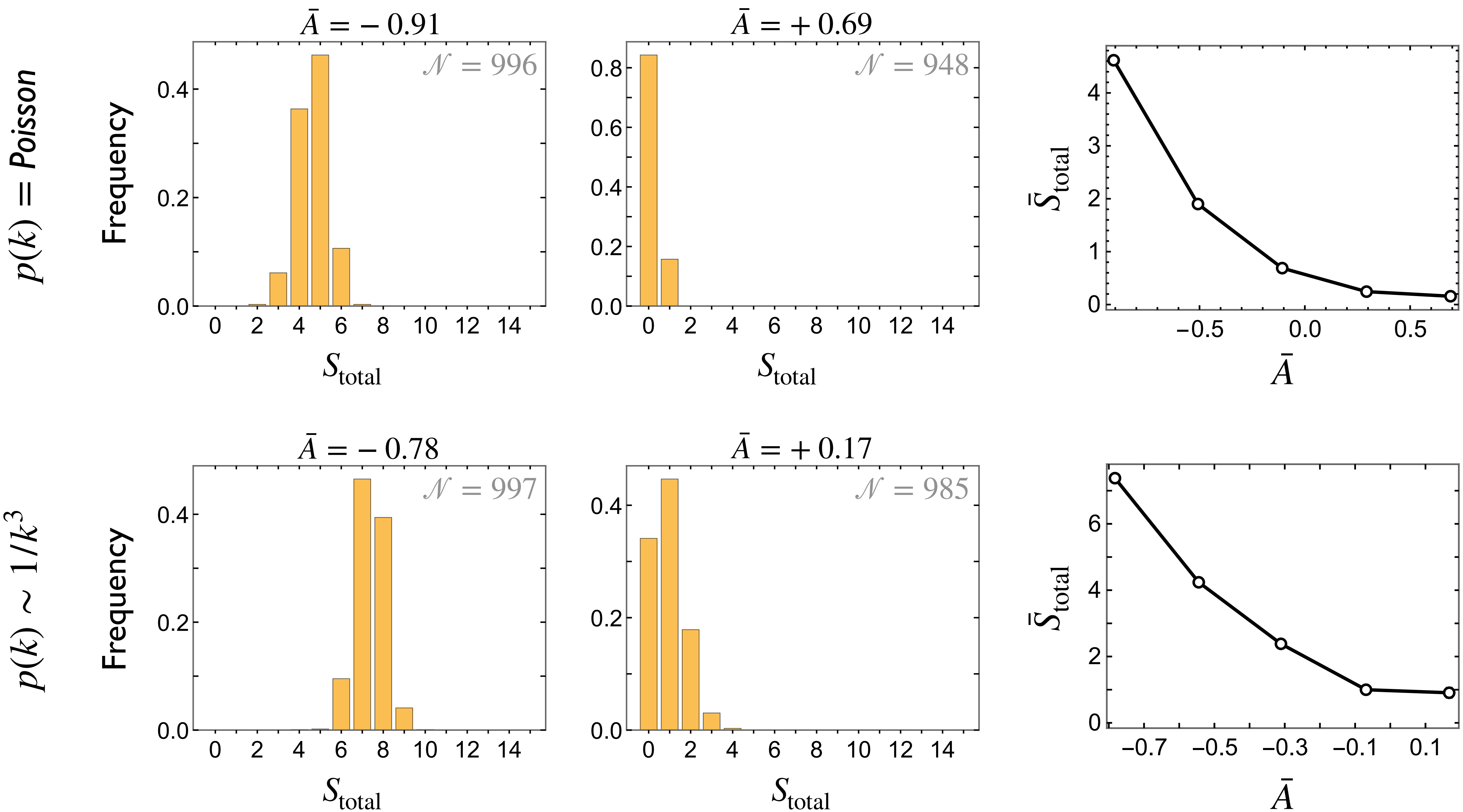


$A = -0.95, \ \ S_{\text{total}} = 7$

$A = +0.64, \ \ S_{\text{total}} = 0$

Magnetization falls w/ assortativity

Most disassortative | *Most assortative*



Results for: $N = 30$, $\bar{k} = 4$

Putting together:
Tunable spin distribution

Embedding disassortative hubs: Copy model

Parameters: N (no of spins), m ($\approx \bar{k}/2$), p (probability)

Alam, Perumalla, and Sanders
Data Sci. Eng. 4, 61 (2019)

Embedding disassortative hubs: Copy model

Parameters: N (no of spins), m ($\approx \bar{k}/2$), p (probability)

Alam, Perumalla, and Sanders
Data Sci. Eng. 4, 61 (2019)

For every new node j :

- Randomly pick an existing node i
- With prob p connect (i, j)
- With prob $1 - p$ connect to a neighbor of i w/ $p_{i'} \propto k_{i'}$
- Repeat m times

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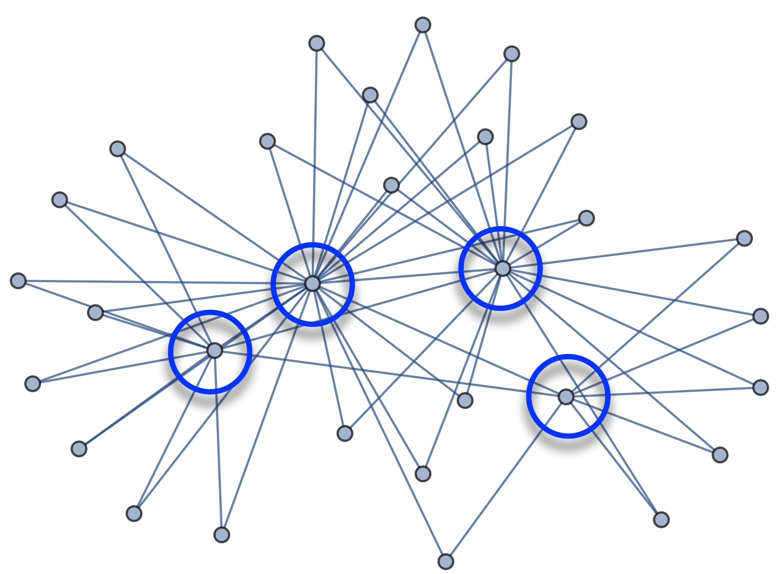
$p = 0$: embedded hubs



$N = 30, m = 2$

Embedding disassortative hubs: Copy model

Parameters: N (no of spins), m ($\approx \bar{k}/2$), p (probability)

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Data Sci. Eng. 4, 61 (2019)

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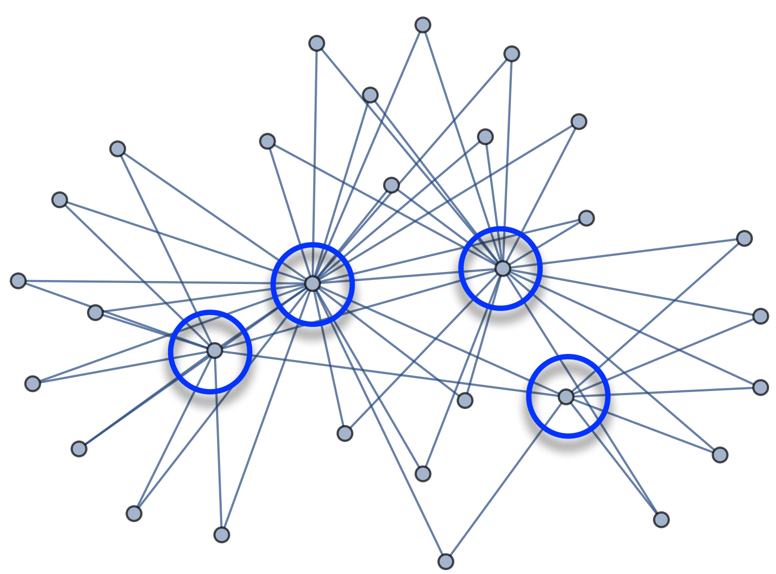
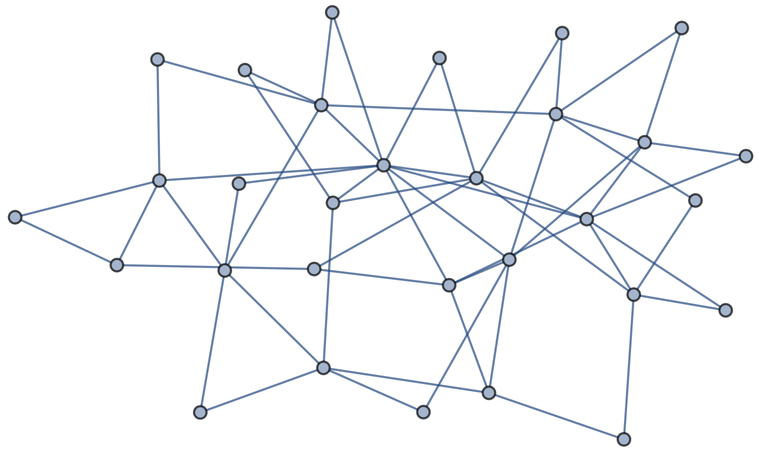
$p = 0$: embedded hubs



$p = 1$: random



$N = 30,\ m = 2$

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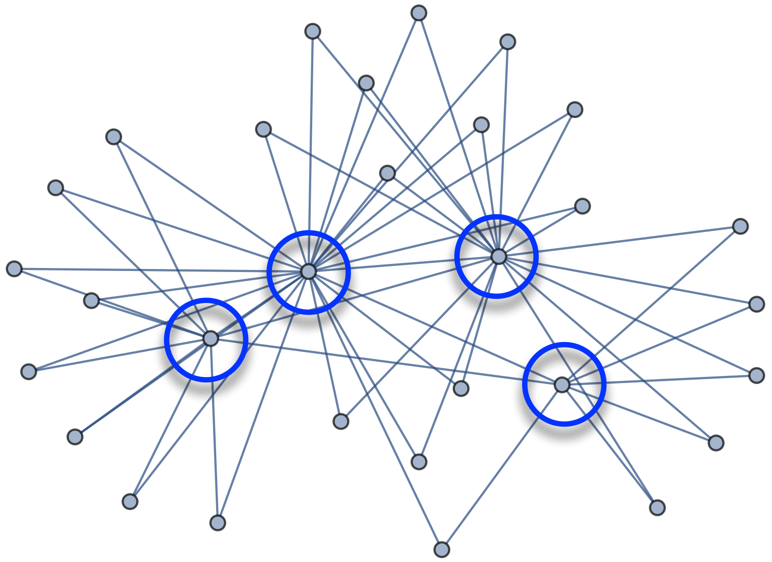

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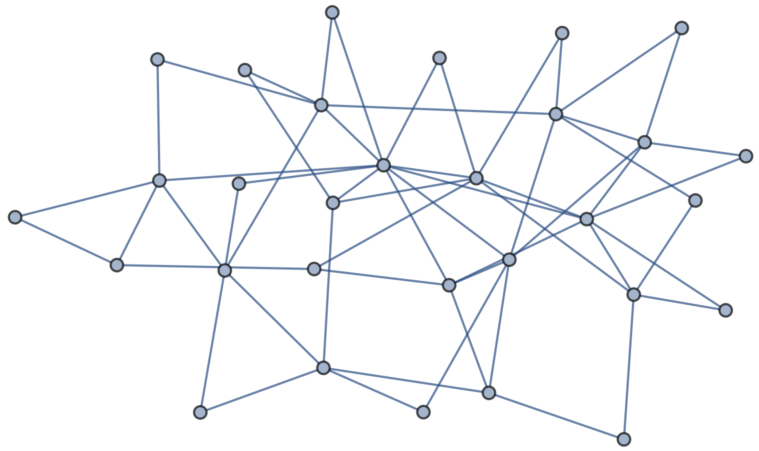
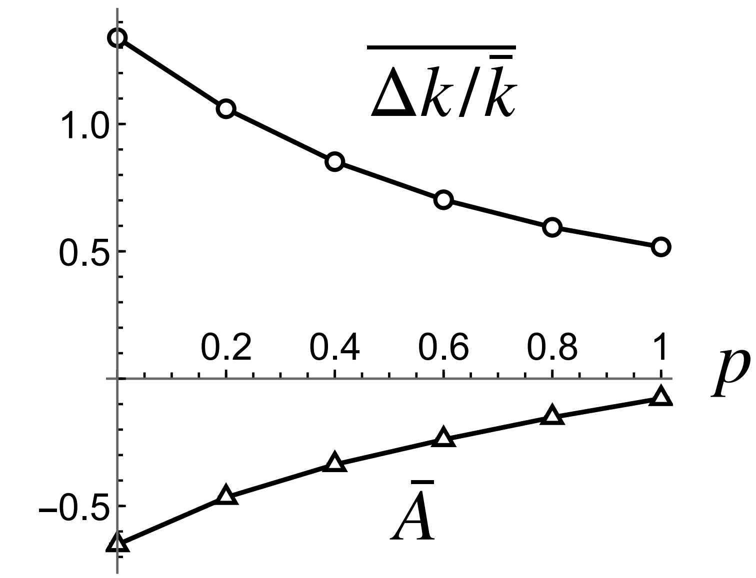
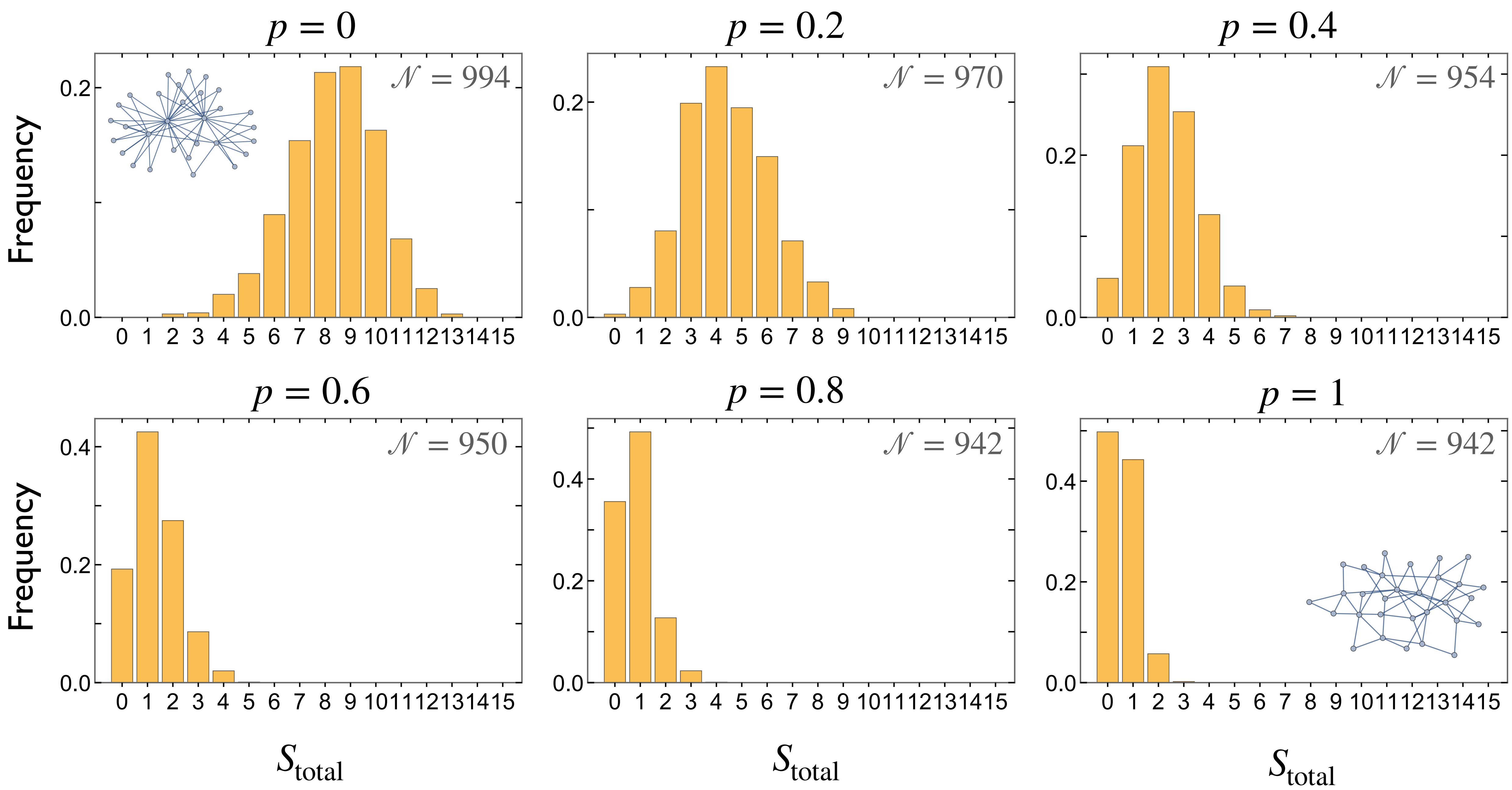


Heterogeneity + Assortativity



$N = 30,\ m = 2$

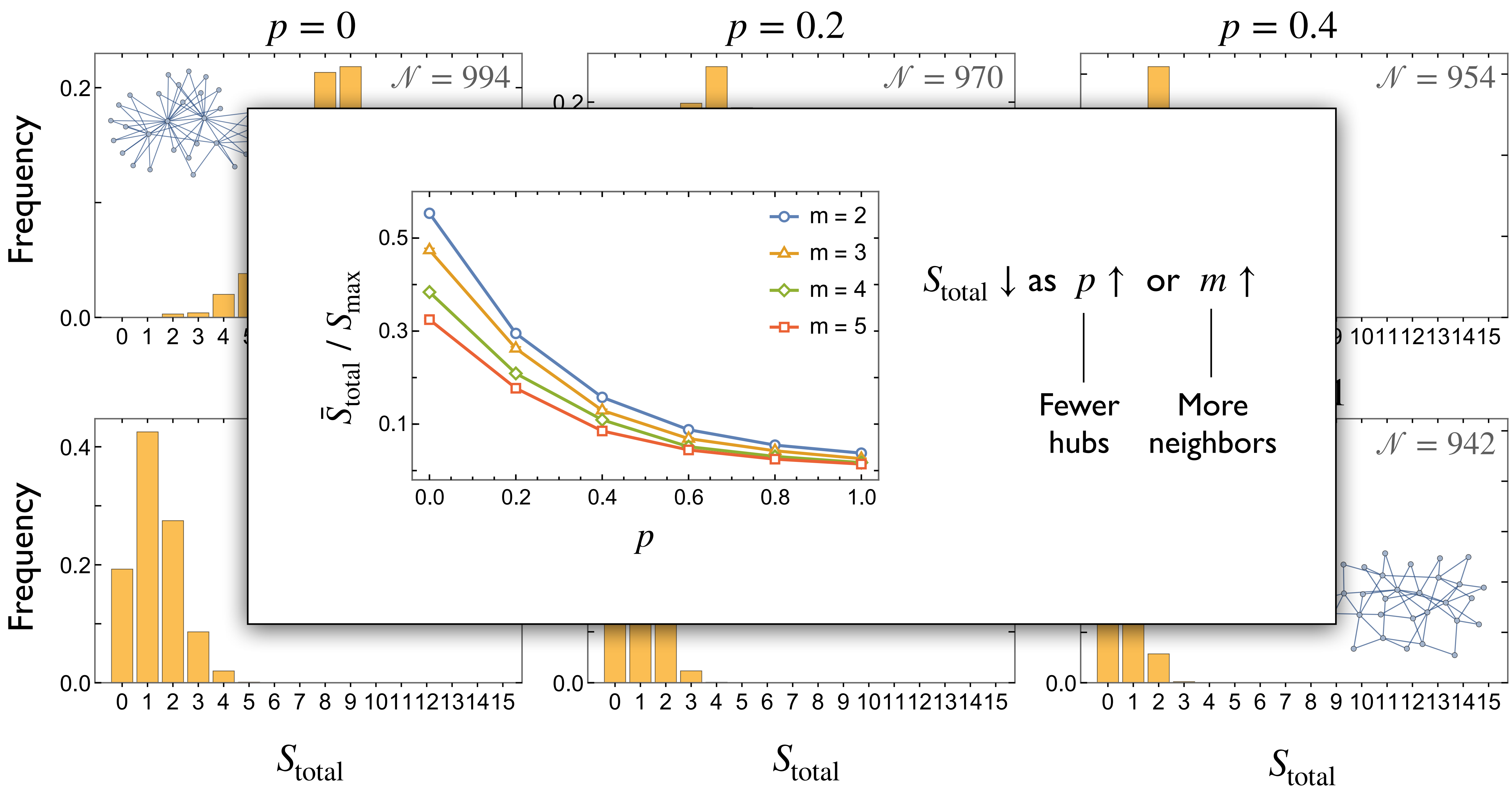
Tunable spin distribution:



$N = 30$, $m = 2$

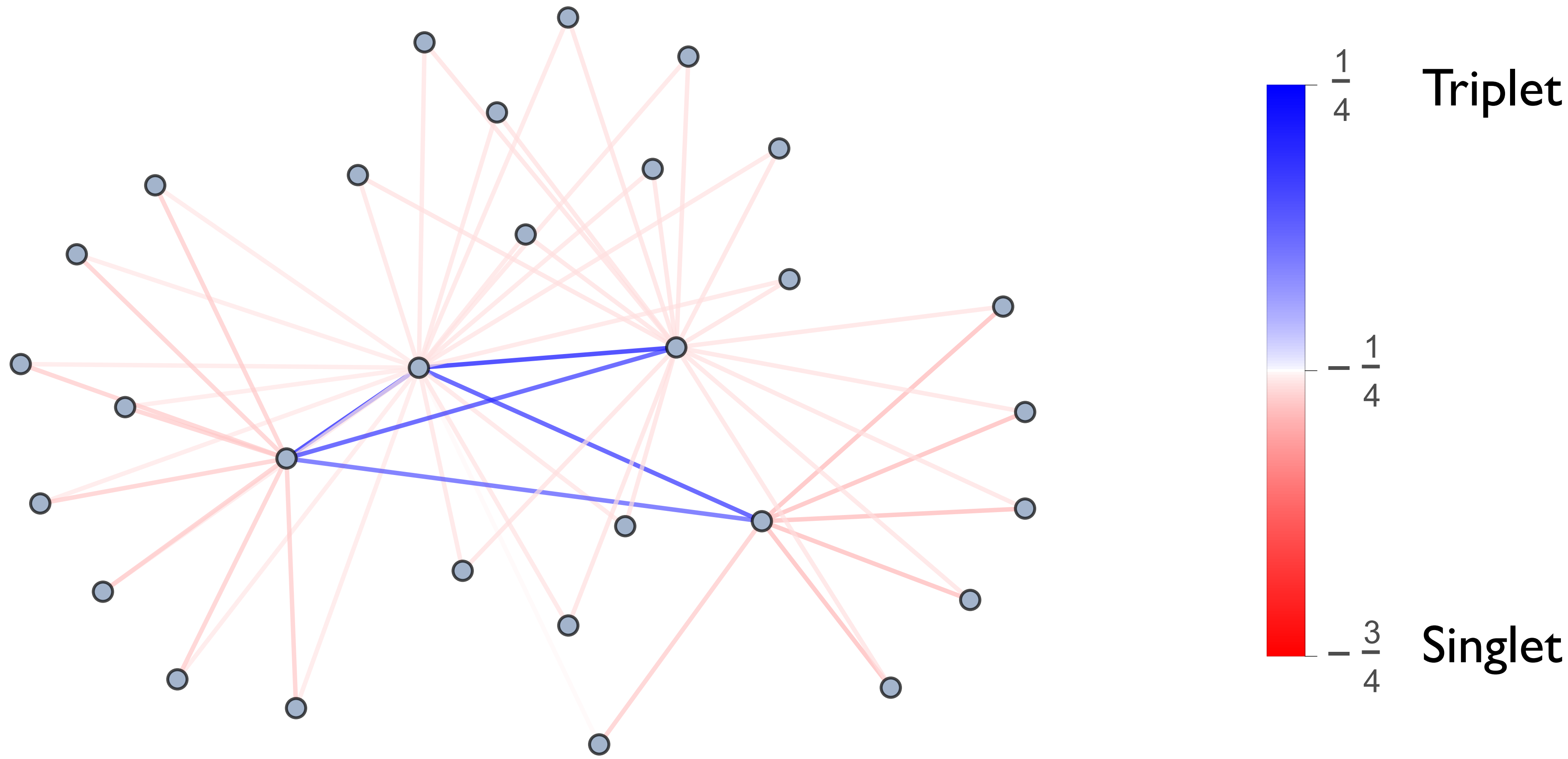
Embedding disassortative hubs: Copy model

Tunable spin distribution:



Embedding disassortative hubs: Copy model

Pairwise alignment: $\langle \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j \rangle$



Hubs aligned opposite to other nodes

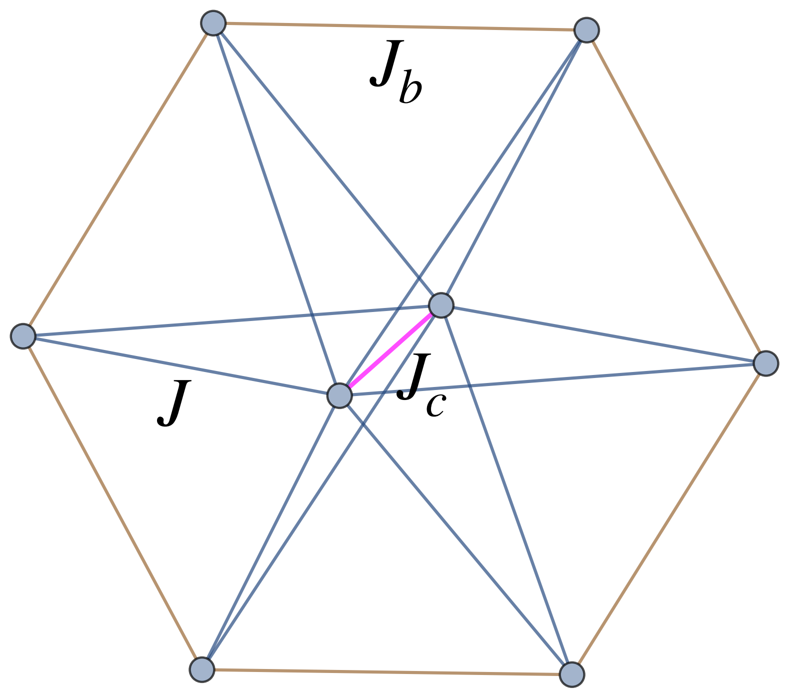
Can we tune S_{total} in a

non-random (frustrated) graph?

Frustrated hub: Wheel

N_c central spins (fully connected) + N_b outer spins

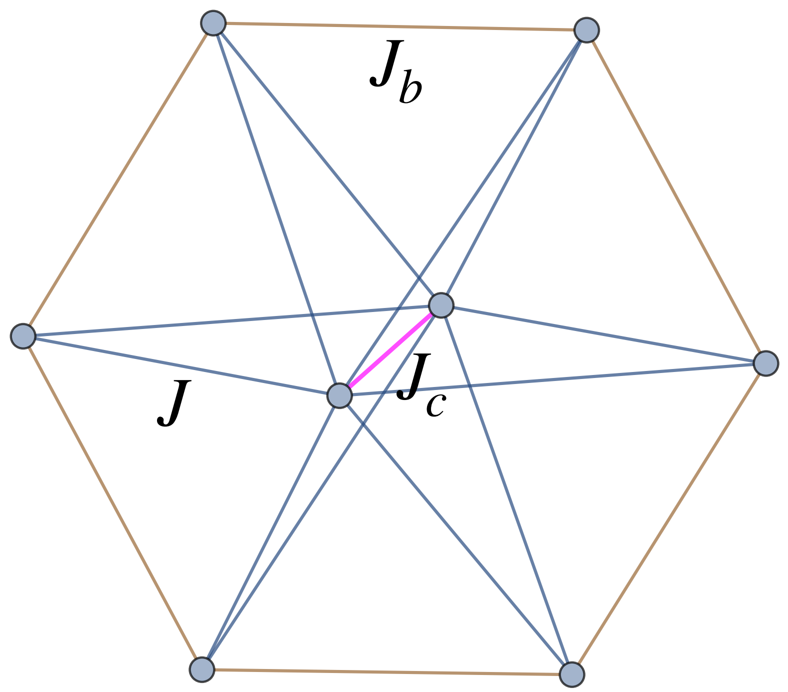
$J, J_b, J_c > 0$



$$\hat{H} = (J_c/2)\ \hat{S}_c^2 + J\ \hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + \ J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$

Frustrated hub: Wheel

N_c central spins (fully connected) + N_b outer spins $J, J_b, J_c > 0$



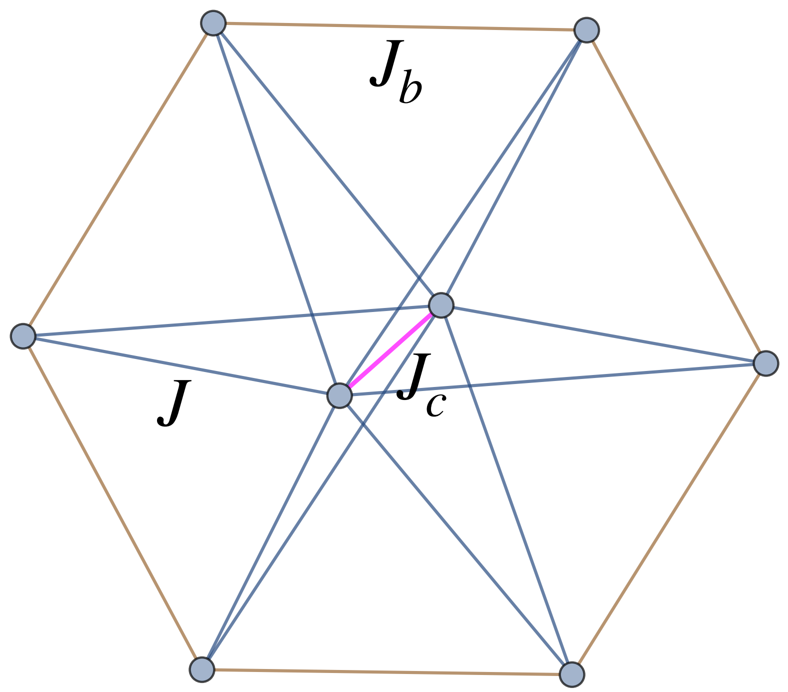
$$\hat{H} = (J_c/2)\,\hat{S}_c^2 + J\,\hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$

- $J_c \gg J$: $S_c = 0$

Frustrated hub: Wheel

N_c central spins (fully connected) + N_b outer spins

$J, J_b, J_c > 0$



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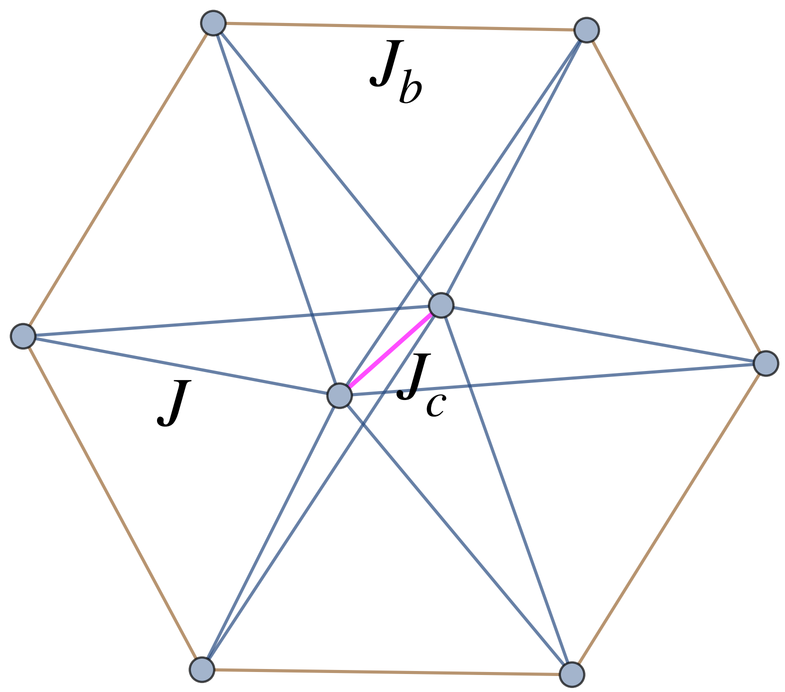
- $J_c \gg J: \; S_c = 0 \implies S_b = 0 \implies S_{\text{total}} = 0$

Frustrated hub: Wheel

N_c central spins (fully connected) + N_b outer spins

$J, J_b, J_c > 0$

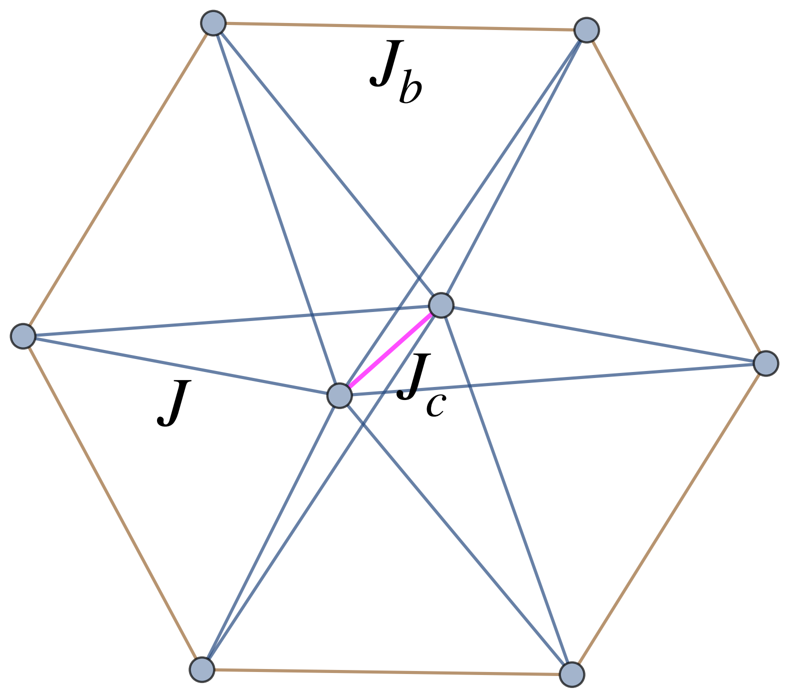


$$\hat{H} = (J_c/2)\,\hat{S}_c^2 + J\,\hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$

- $J_c \gg J: \; S_c = 0 \implies S_b = 0 \implies S_{\text{total}} = 0$
- Lower $J_c: \; S_c \sim 1$

Frustrated hub: Wheel

N_c central spins (fully connected) + N_b outer spins $\quad J, J_b, J_c > 0$

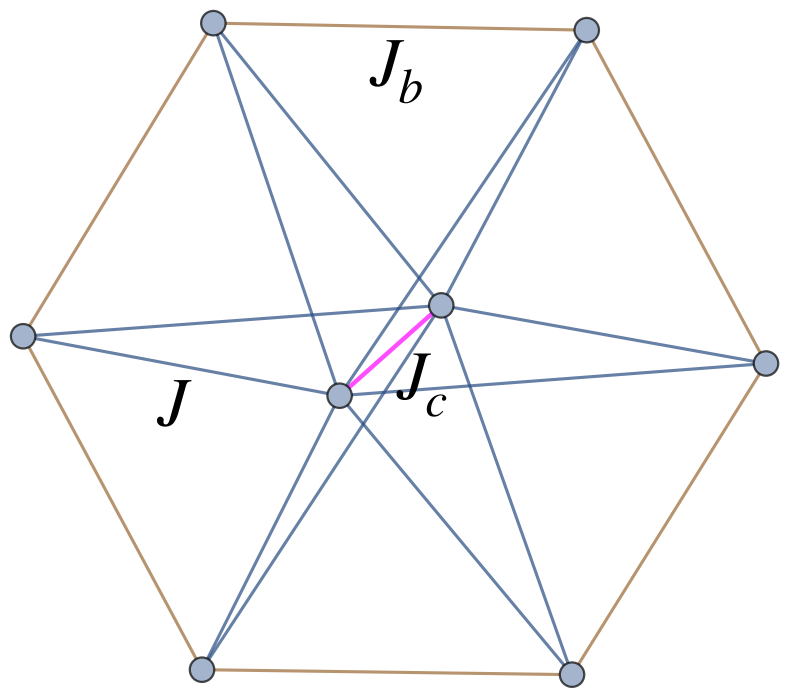


$$\hat{H} = (J_c/2)\, \hat{S}_c^2 + J\, \hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$

- $J_c \gg J$: $S_c = 0 \implies S_b = 0 \implies S_{\text{total}} = 0$
- Lower J_c : $S_c \sim 1 \implies$ if $J_b \ll J$: $S_b = S_b^{\max} = N_b/2$
  $S_{\text{total}} \sim (N_b - 1)/2$

Frustrated hub: Wheel

N_c central spins (fully connected) + N_b outer spins $\quad J, J_b, J_c > 0$



$$\hat{H} = (J_c/2)\,\hat{S}_c^2 + J\,\hat{\mathbf{S}}_b \cdot \hat{\mathbf{S}}_c + J_b \sum_{n=1}^{N_b} \hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}$$

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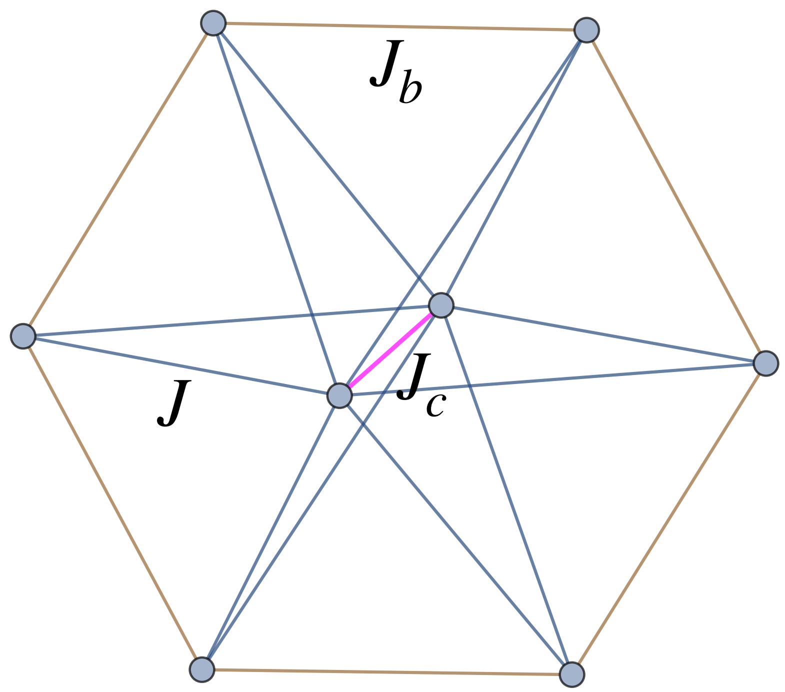
  $S_{\text{total}} \sim (N_b - 1)/2$
- $J_c/J \downarrow \implies S_c \uparrow$, $J_b/J \uparrow \implies S_b \downarrow$ —Variable S_{total}

Frustrated hub: Wheel

N_c central spins (fully connected) + N_b outer spins $\quad J, J_b, J_c > 0$



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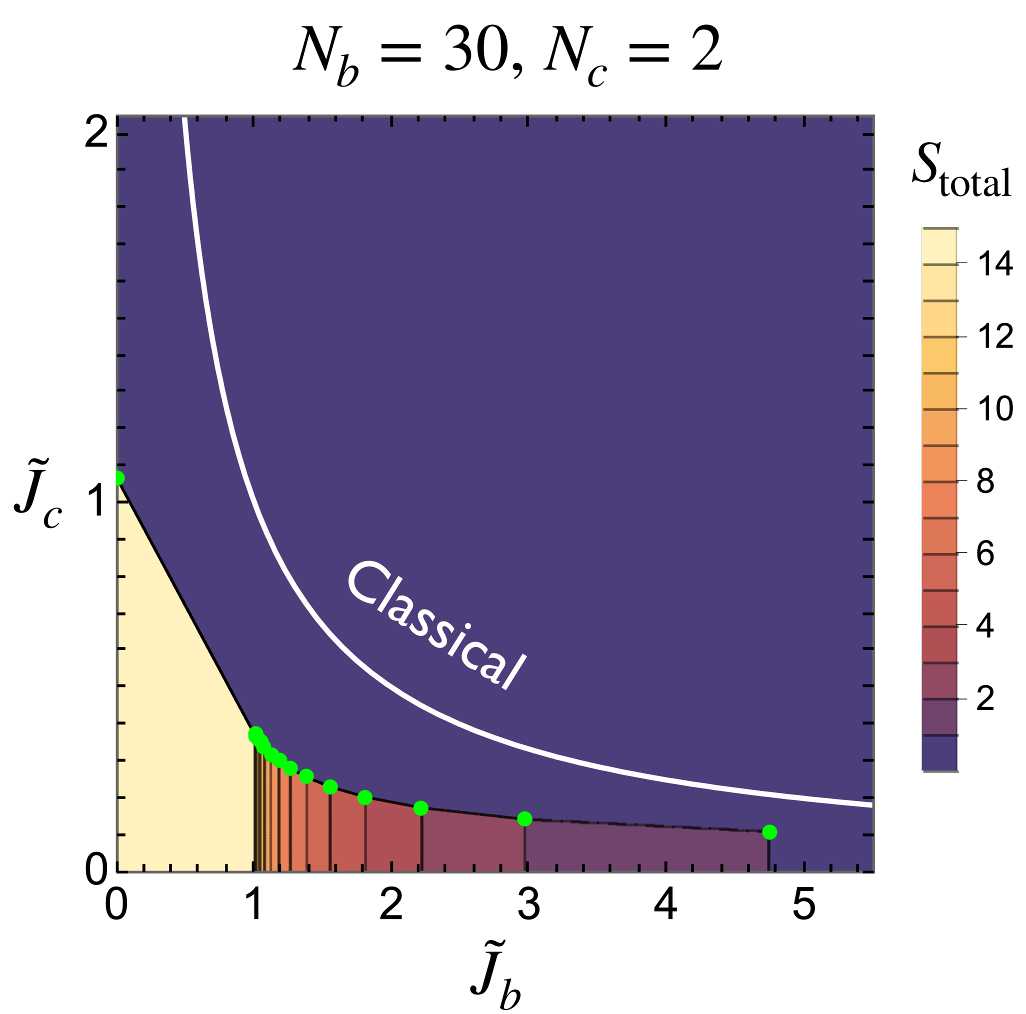
- $J_c \gg J: \; S_c = 0 \implies S_b = 0 \implies S_{\text{total}} = 0$
- Lower $J_c: \; S_c \sim 1 \implies$ if $J_b \ll J: \; S_b = S_b^{\max} = N_b/2$
  $S_{\text{total}} \sim (N_b - 1)/2$
- $J_c/J \downarrow \implies S_c \uparrow, \; J_b/J \uparrow \implies S_b \downarrow$ — Variable S_{total}

Exactly solvable:

$S_b, S_c, S_{\text{total}}$ good quantum numbers — Energy minimized for $S_{\text{total}} = S_{bc} := |S_b - S_c|$

$$\implies E(S_b, S_c) = \frac{J}{2} S_{bc}(S_{bc}+1) + \frac{J_c - J}{2} S_c(S_c+1) - \frac{J}{2} S_b(S_b+1) + J_b \underbrace{E_{\min}^{XXX}(N_b, S_b)}_{\text{Bethe Ansatz}}$$

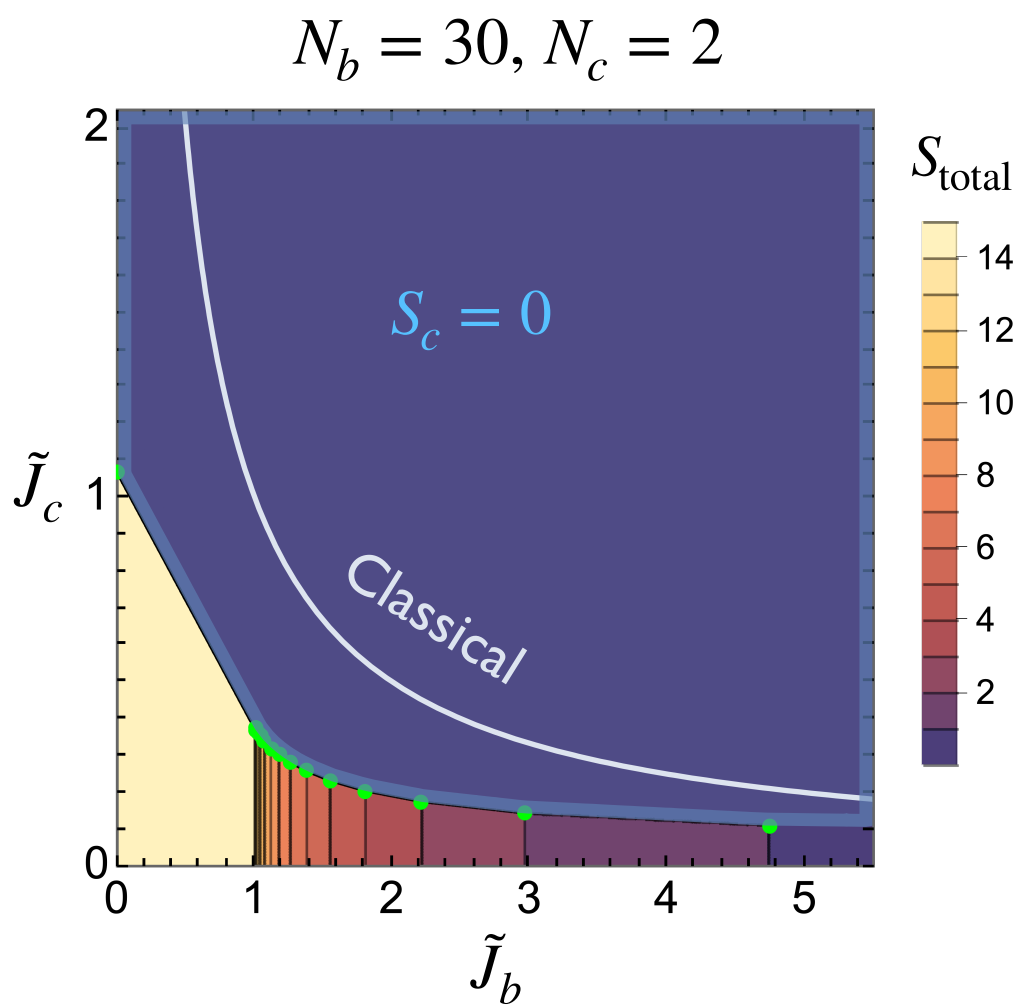
Frustrated hub: Wheel



$$\tilde{J}_b := \frac{4J_b}{JN_c}$$

$$\tilde{J}_c := \frac{J_c N_c}{JN_b}$$

Frustrated hub: Wheel



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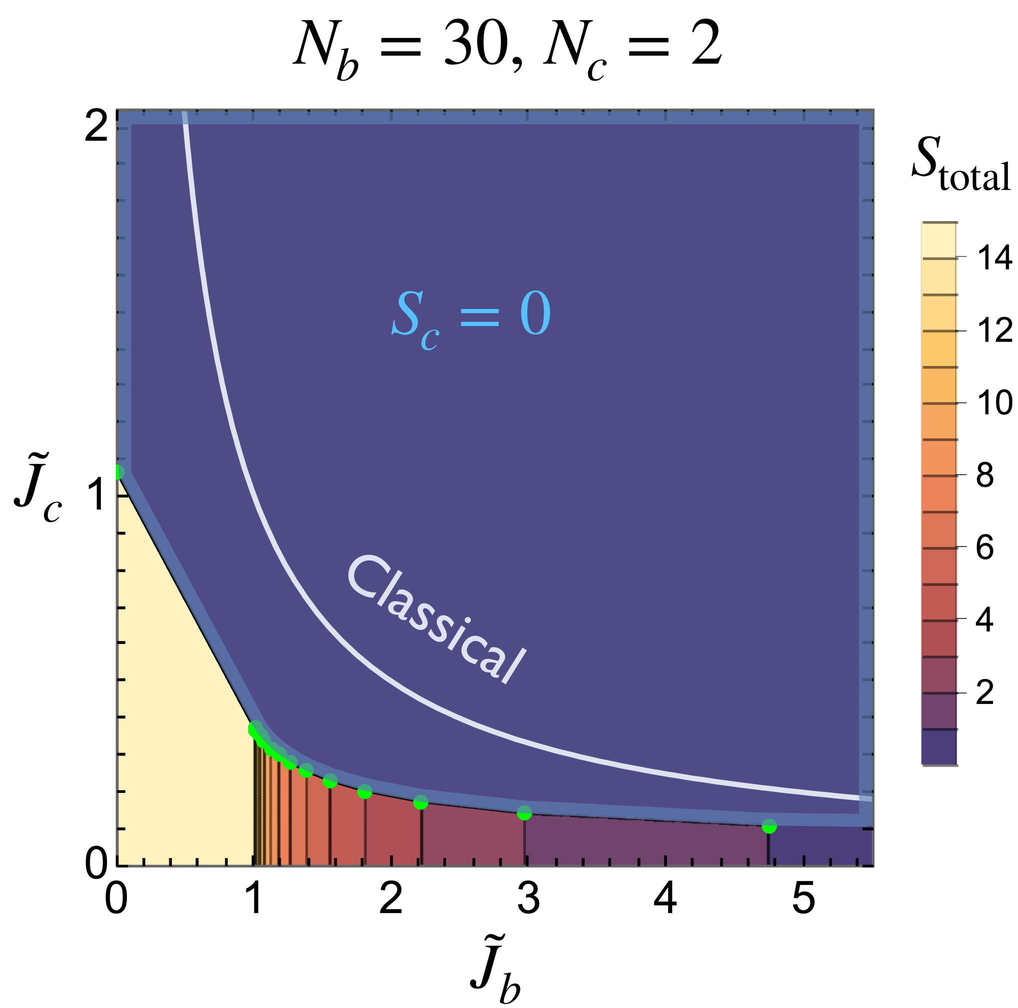


$$\tilde{J}_b := \frac{4J_b}{JN_c}$$

$$\tilde{J}_c := \frac{J_c N_c}{JN_b}$$

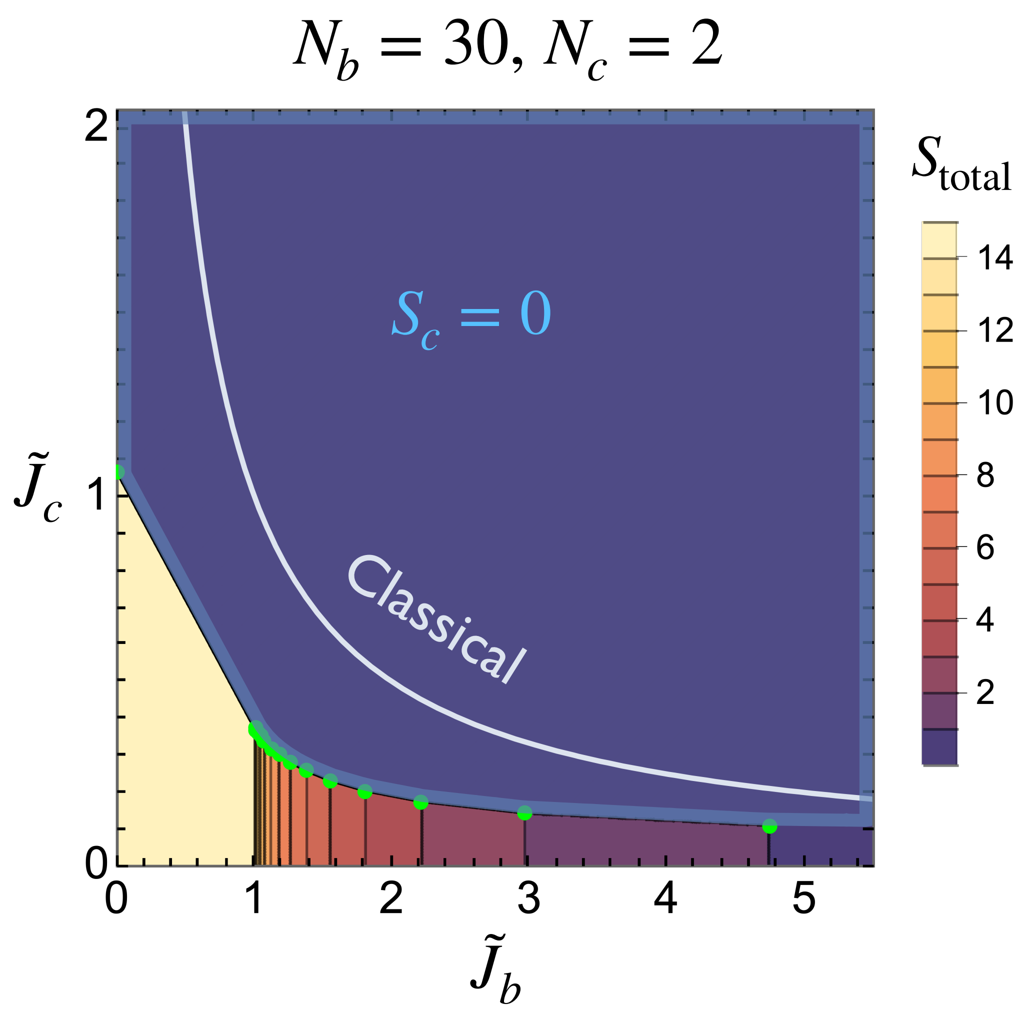
J_c small: $S_c = 1$

$$\implies E = J_b \underbrace{E^{XXX}_{\min}(N_b, S_b)}_{\sim S_b^2} - JS_b$$

Frustrated hub: Wheel

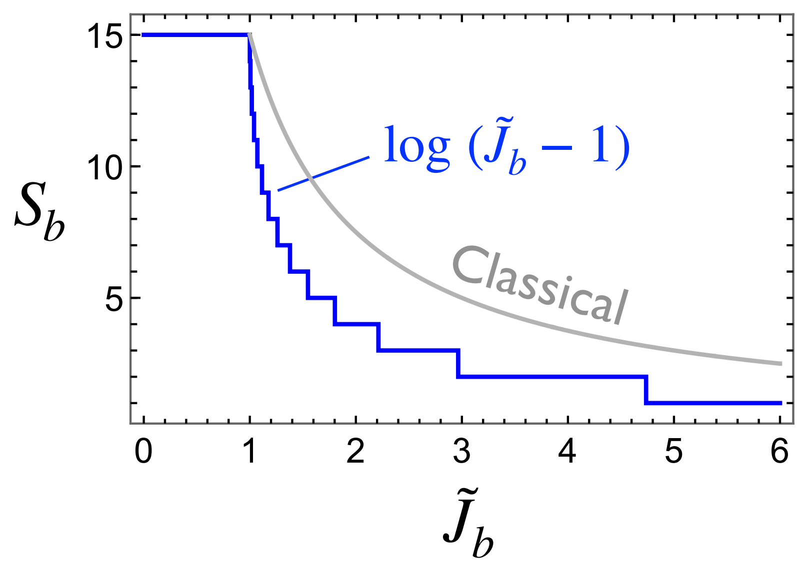
$\tilde{J}_b := \frac{4J_b}{JN_c}$

$\tilde{J}_c := \frac{J_c N_c}{JN_b}$

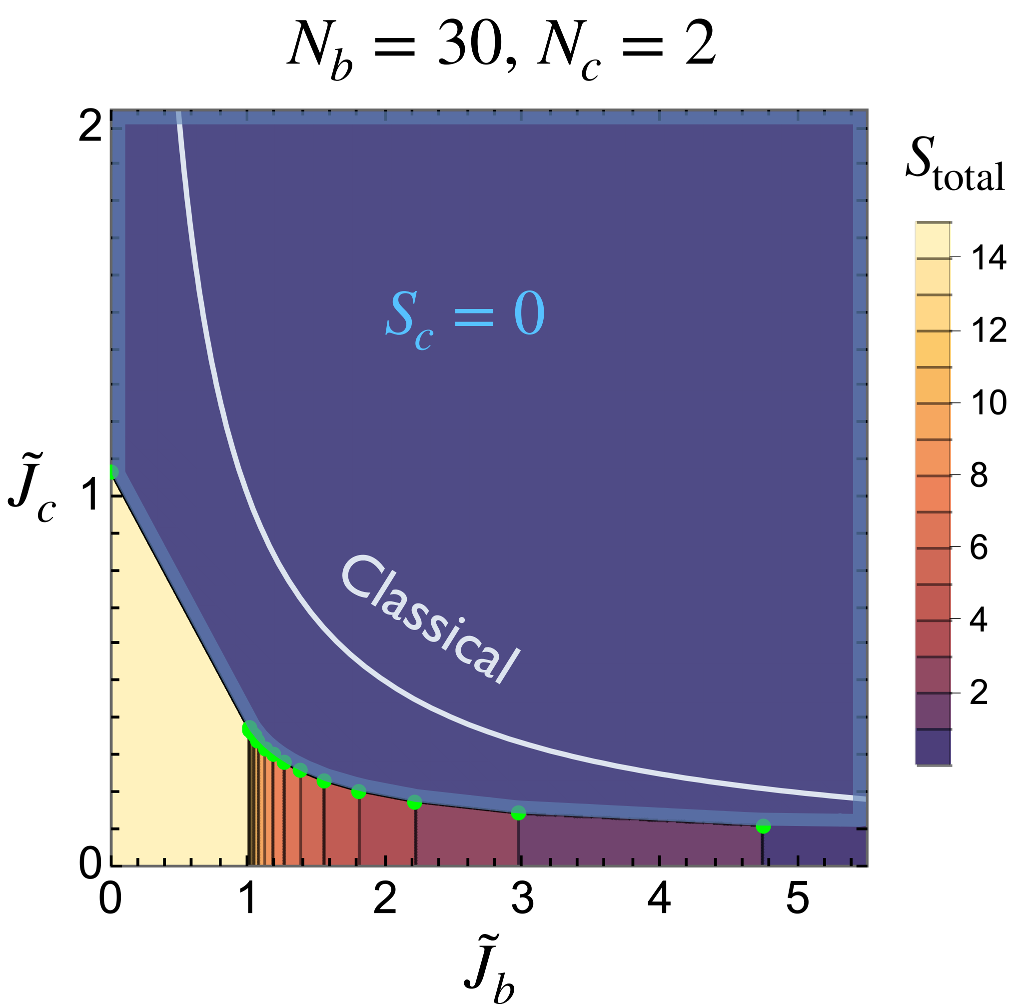


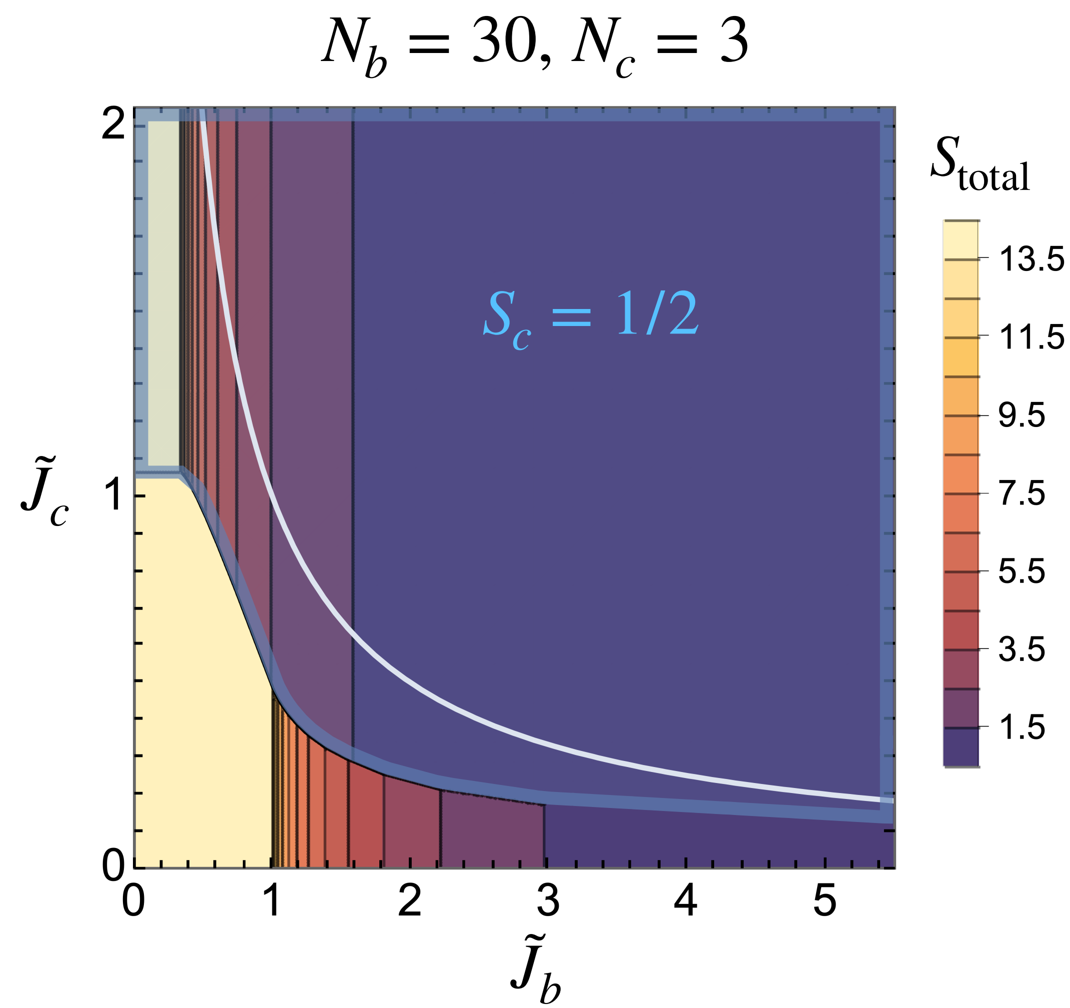
J_c small: $S_c = 1$

$$\Longrightarrow \; E = J_b \underbrace{E_{\rm min}^{XXX}(N_b, S_b)}_{\sim S_b^2} - JS_b$$



Frustrated hub: Wheel





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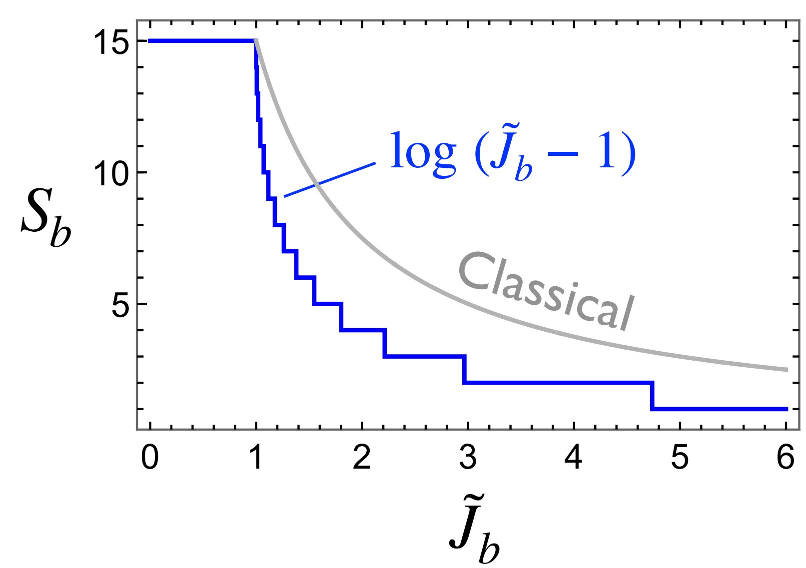
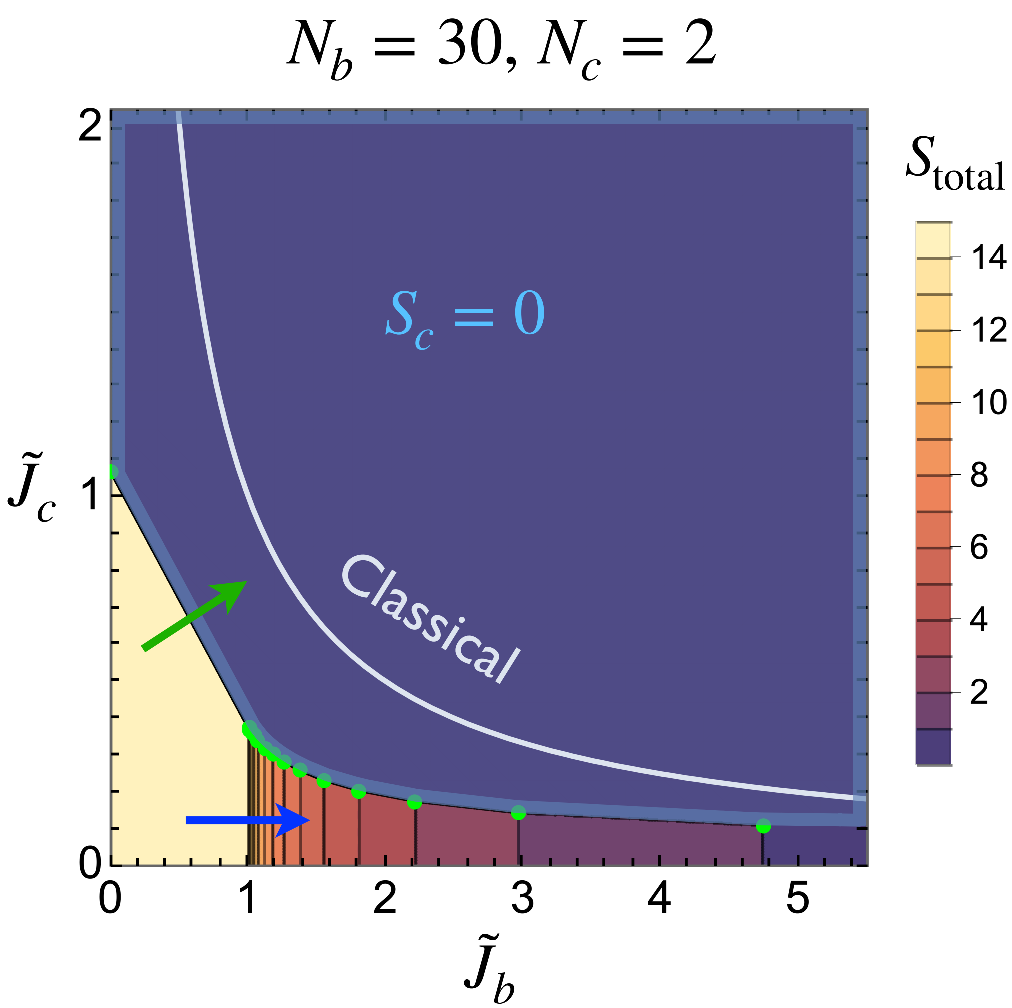
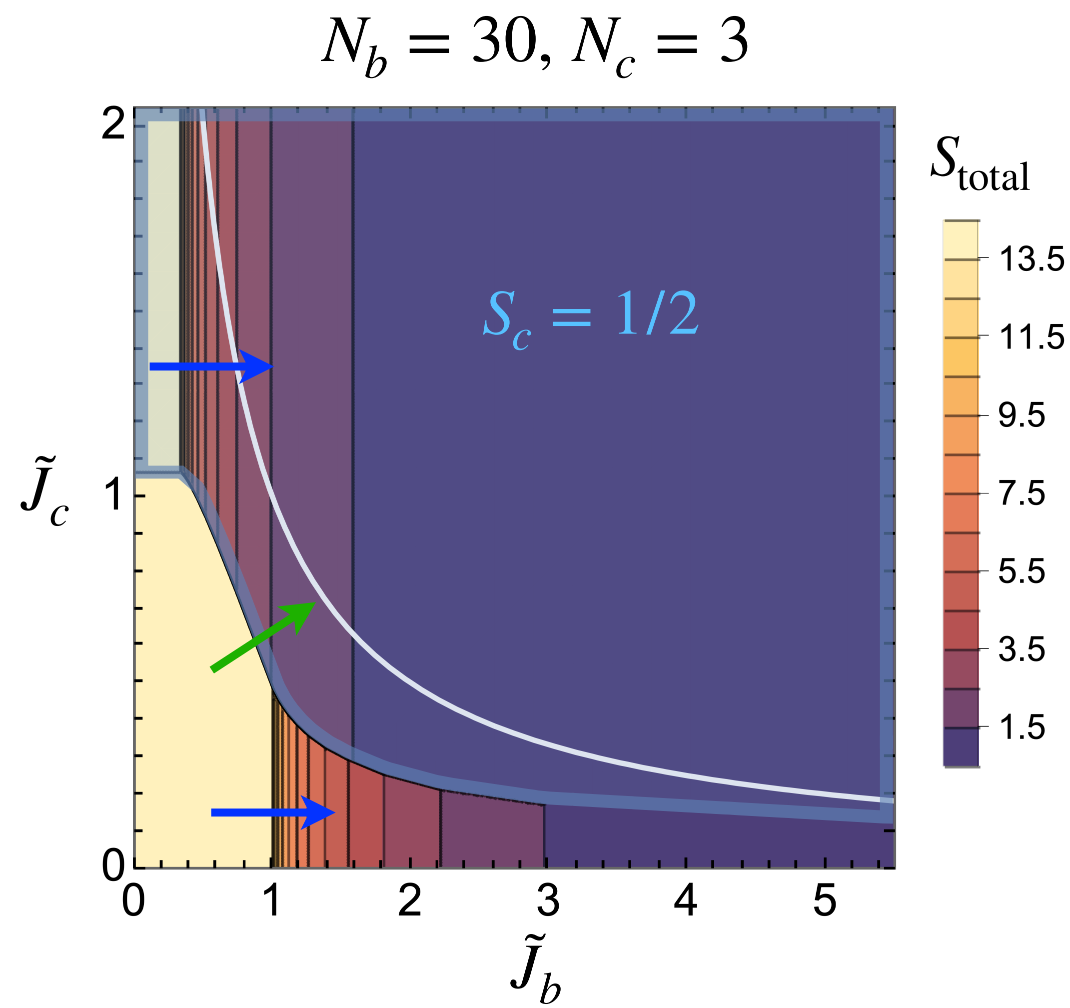


Frustrated hub: Wheel





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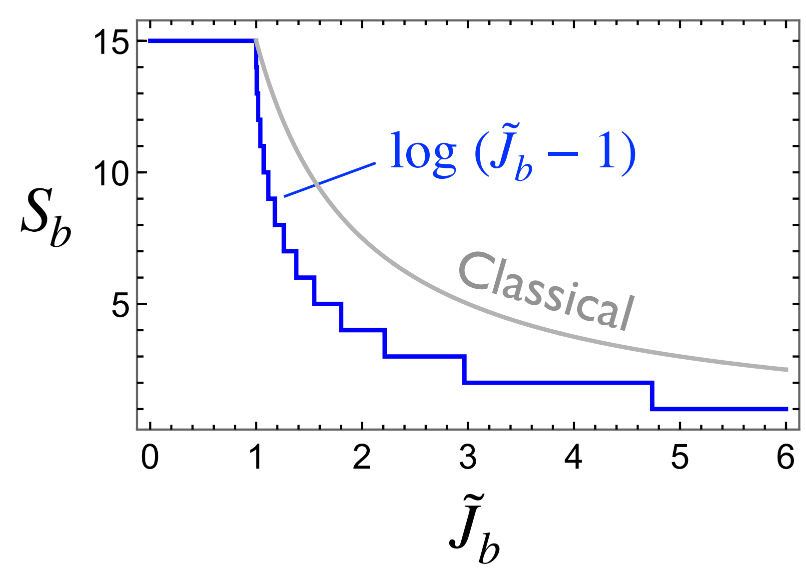
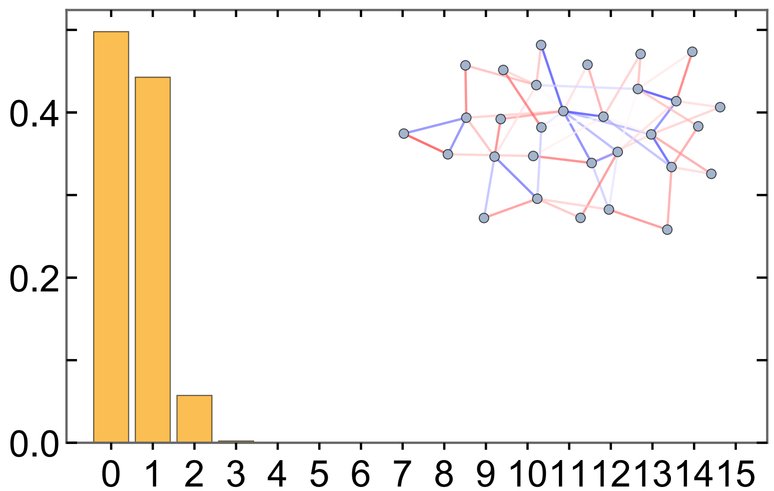
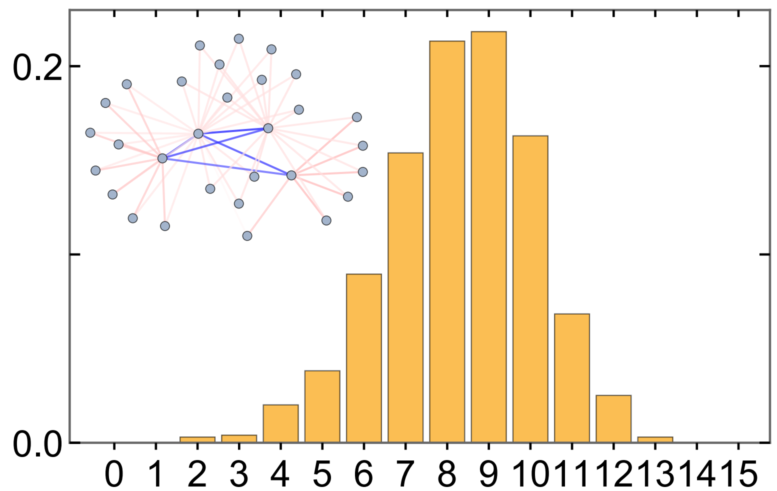
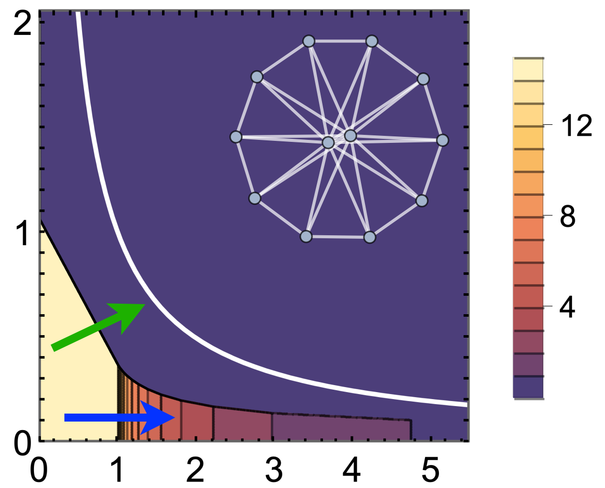


S_{total} tunable over entire range across discontinuous & continuous transitions

- Degree mismatch — disassortative hubs — essential for nonzero S_{total}
- S_{total} not sensitive to frustration level & falls w/ more neighbors
- S_{total} tunable over full range in nonbipartite graphs







Open questions:

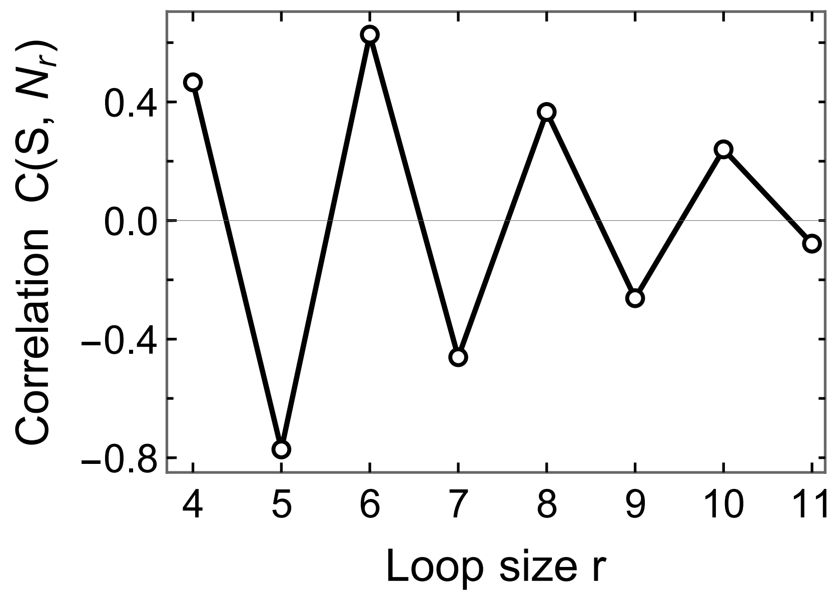
- Structure of ground state — spin liquids?
- Analytic understanding — importance of embedded hubs
- Contrast w/ kinetic magnetism

Preethi G and SD, in prep

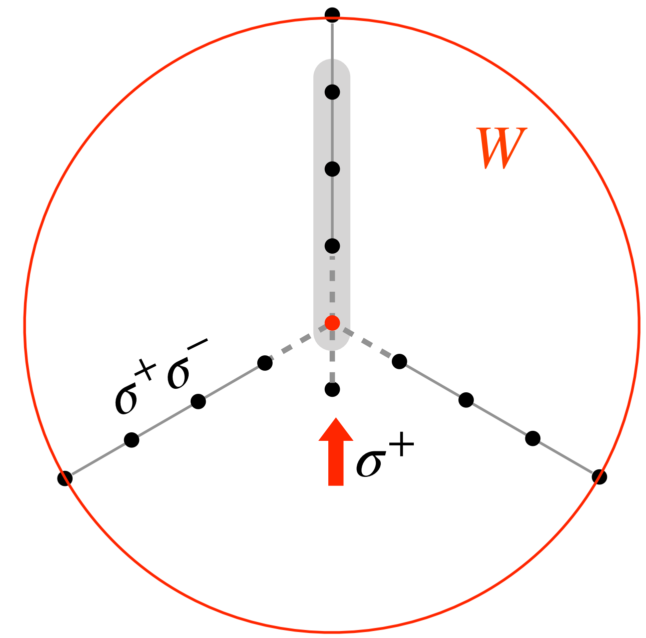
Other recent work

Nagaoka physics on general graph
Frustration level key!

Revathy BS

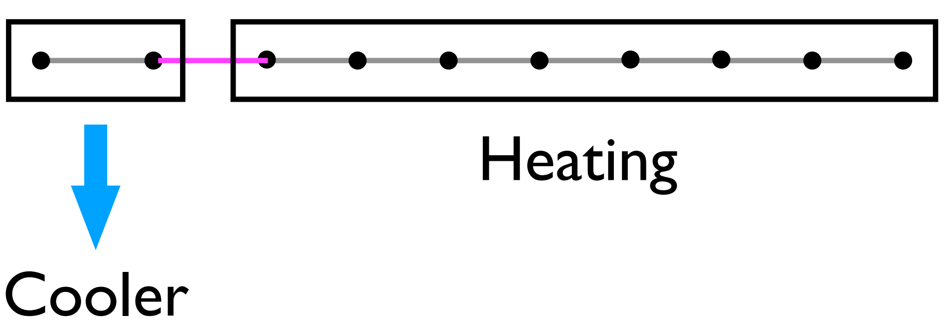


Long-range multipartite entanglement from local pump & static coupling

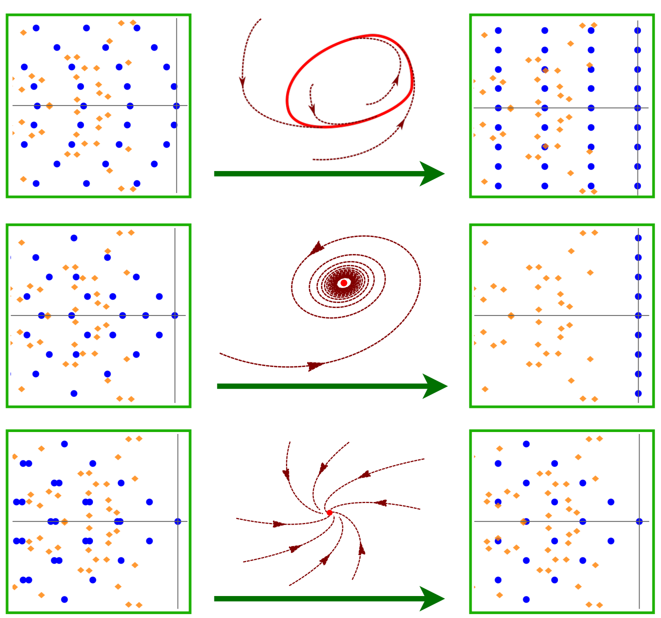


Global heating from local cooling (or vice versa)

Jaswanth Verma
Masud Haque
Paul McClarty



Emergence of classical nonlinear phenomena



Shu Zhang
Masud Haque

Nigel Cooper
Stefan Kuhr
R Moessner
Sanjukta Roy
Apoorv S

Summary

Thank you :)

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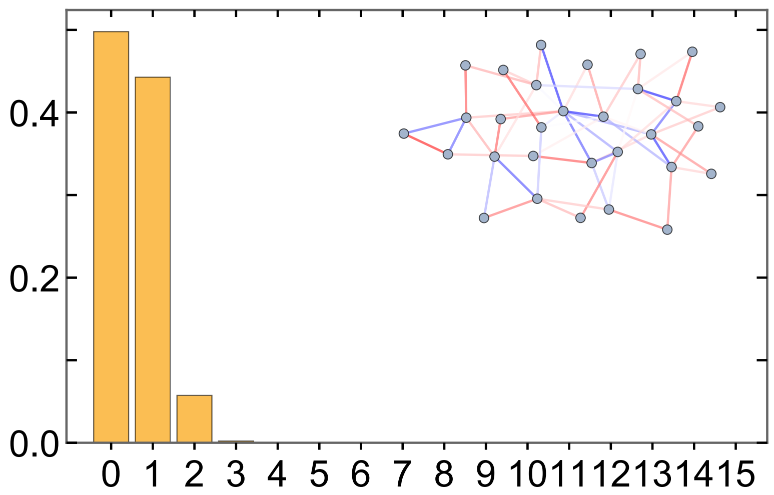

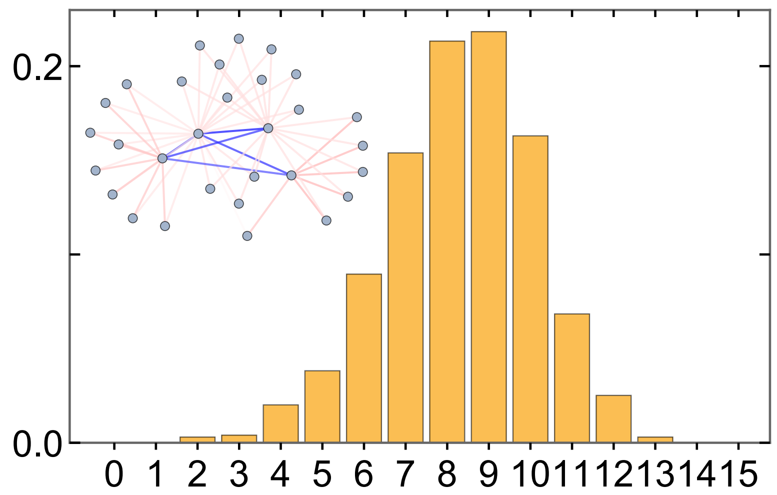

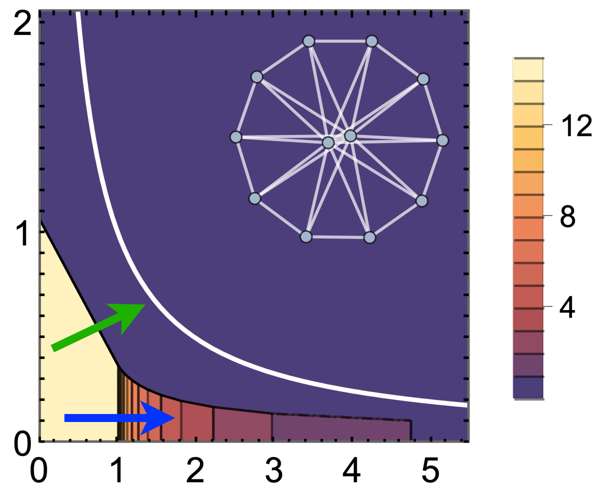

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Preethi G and SD, in prep