Topics in Discrete Harmonic Analysis

A Highlight of the series of Lectures

<u>Thm</u> (3 Prime Thm in sectors in $\mathbb{Z}[i]$)

$\forall\ \omega \subset \Pi \quad \exists\ N_\omega \quad \forall$ odd $n \in \mathbb{Z}[i]$

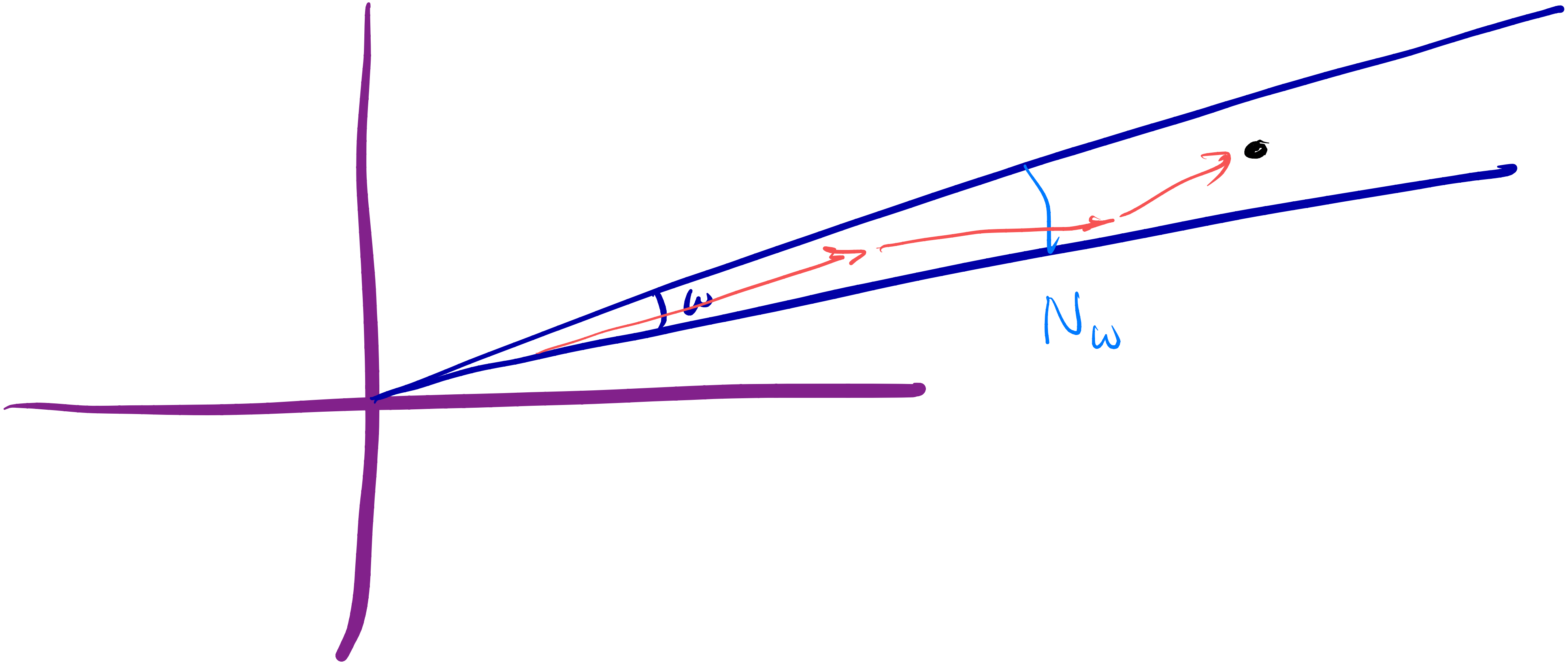
interval

w/ $\arg n \in \omega$, $\exists\ p_1, p_2, p_3$

primes in $\mathbb{Z}[i]$

w/ $\arg p_i \in \omega$ $\quad n = p_1 + p_2 + p_3$

$$\mathbb{Z}[i] = \mathbb{Z} + i\mathbb{Z}$$



Prior result: $\omega = \pi$, Mitsui (1960), Gen'l number fields

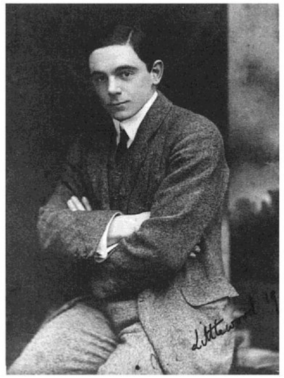
Towards this direction:

- Averages on $\mathbb{R}^d$, oscillatory estimates
- Their periodic versions
- Their versions on $\mathbb{Z}$, which require both.
- A glimpse at same topics on $\mathbb{Z}[i]$.

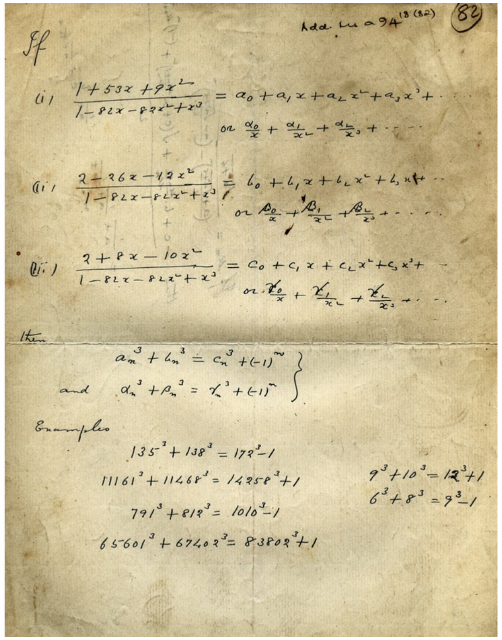
We will focus on

- Improving estimates on $\mathbb{Z}$, $\mathbb{Z}^d$,
- Utilize methods of analytic number theory

Hardy, Littlewood, Ramanujan, Vinogradov



Littlewood in 1907



Add. ms a94 13 (82) (82)

If

(i) $\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$

or $\frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \cdots$

(ii) $\frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \cdots$

or $\frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \cdots$

(iii) $\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots$

or $\frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \cdots$

then

$$\left.\begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and} \quad \alpha_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned}\right\}$$

Examples

$135^3 + 138^3 = 172^3 - 1$

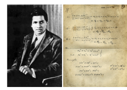
$11161^3 + 11468^3 = 14258^3 + 1$

$791^3 + 812^3 = 1010^3 - 1$

$65601^3 + 67402^3 = 83802^3 + 1$

$9^3 + 10^3 = 12^3 + 1$

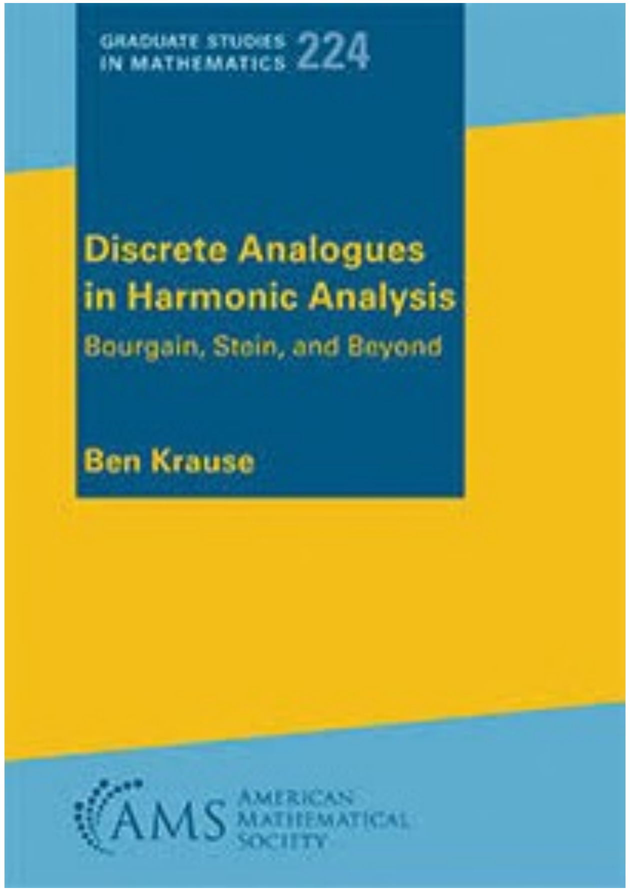
$6^3 + 8^3 = 9^3 - 1$



Harmonic Analysis, use ideas of

Stein, Bourgain, and

other modern masters


GRADUATE STUDIES IN MATHEMATICS 224
Discrete Analogues in Harmonic Analysis
Bourgain, Stein, and Beyond
Ben Krause
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