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TATA INSTITUTE OF FUNDAMENTAL RESEARCH

A Tour of Space-Time Physics: Newton, Einstein, Hawking and Beyond

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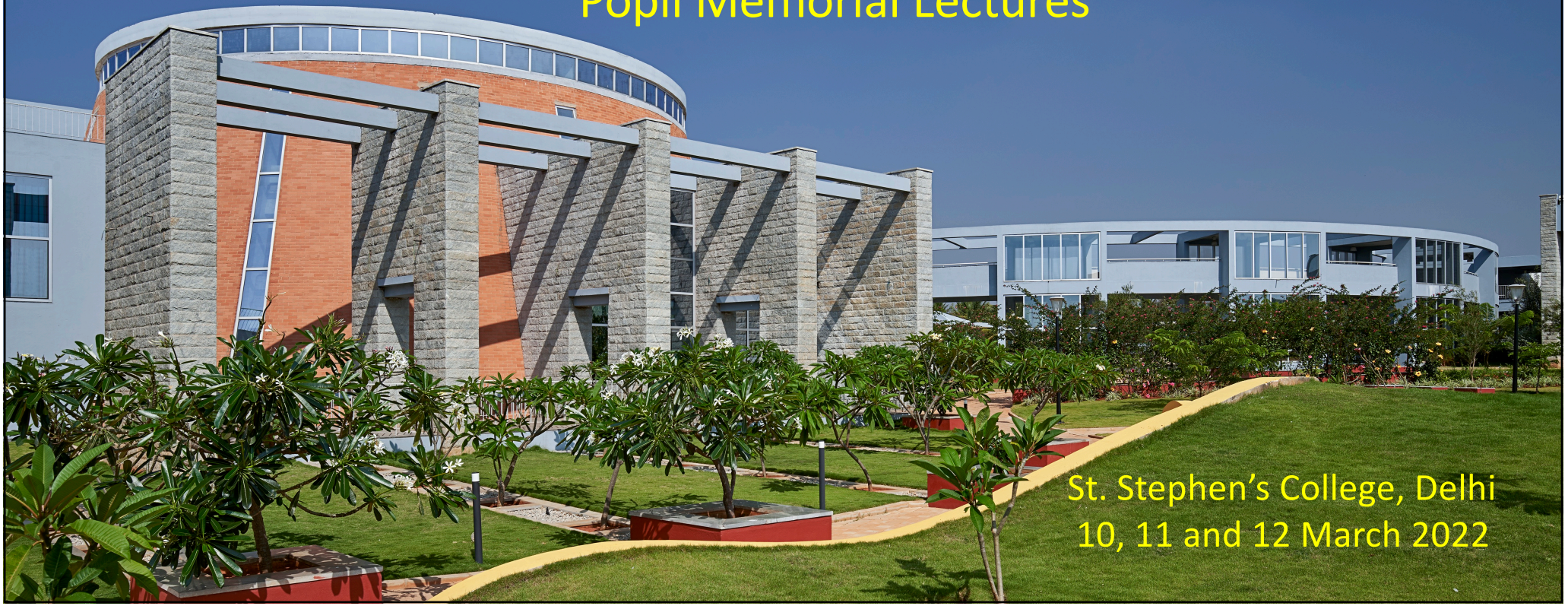
Popli Memorial Lectures



St. Stephen's College, Delhi
10, 11 and 12 March 2022

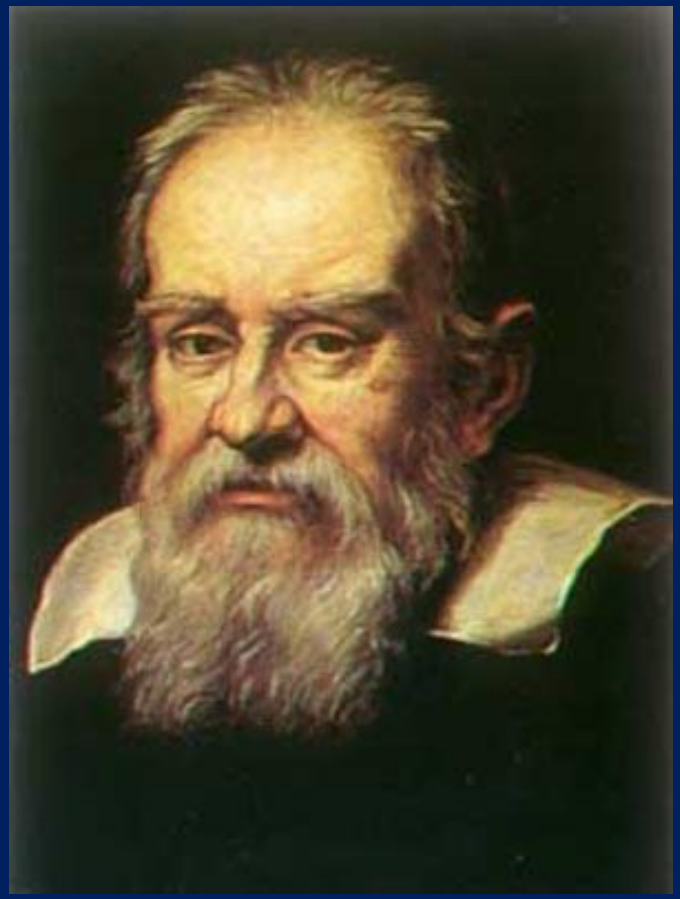
Newtonian Framework of Physics
and
the Law of Universal Gravitation

Galileo (1564-1642)

A pioneer of the modern scientific method

Discovered a new law of gravity

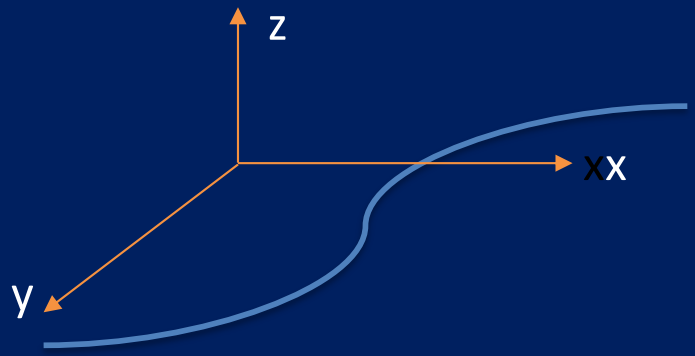
Gravity acts in the same way on all bodies: they all fall in the same way independent of their mass: $m_{inertial} = m_{gravitational}$ ($1/10^{13}$ precision, today) Plays a key role in Einstein's theory of General Relativity



Isaac Newton
(Principia Mathematica 1687)

Establishes a framework of mechanics

Newton formulated the laws of motion in terms of the flow in time of the position of a point particle in 3-dims.

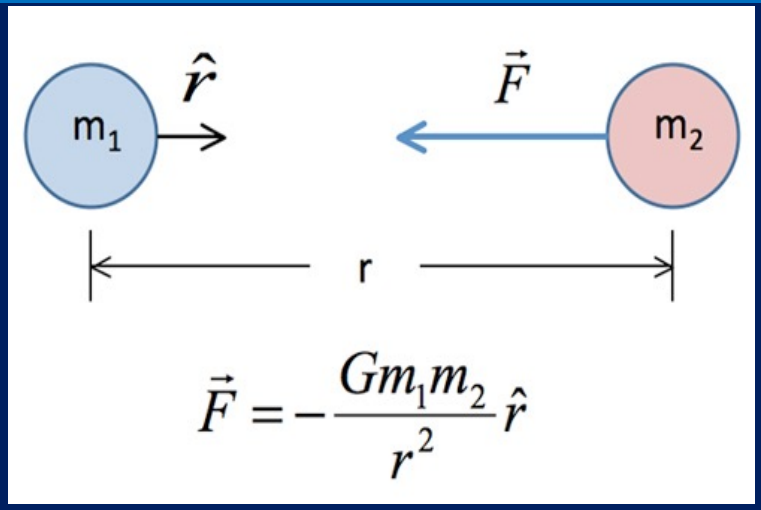


(x(t), y(t), z(t)) is a curve; velocity and acceleration are given by time derivatives of the curve. Geometry, Calculus and Mechanics provide a description of motion.

Time is absolute and the same for all observers. Coordinates may be rotated or moved with constant velocity.

Newton's law of motion:
Force = m_i a
Foundation of ALL classical physics

Newton's law of Universal Gravitation



- Gravitational force acts instantaneously at a distance
- Always attractive
- Force (on particle 1 due to particle 2) = $m_{1i}\, a_1$
- Inertial mass $m_{1\,i}$ = gravitational mass m_1
- Hence gravitational acceleration does not depend on the mass (Galileo)

In the simple case of 2-bodies leads to the famous elliptical orbits of planets described by Kepler, when the coordinate system is centered at the Sun; he found symmetry in big `data'.

Newton on the Law of Universal Gravitation

Newton (1692):

"That one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it."

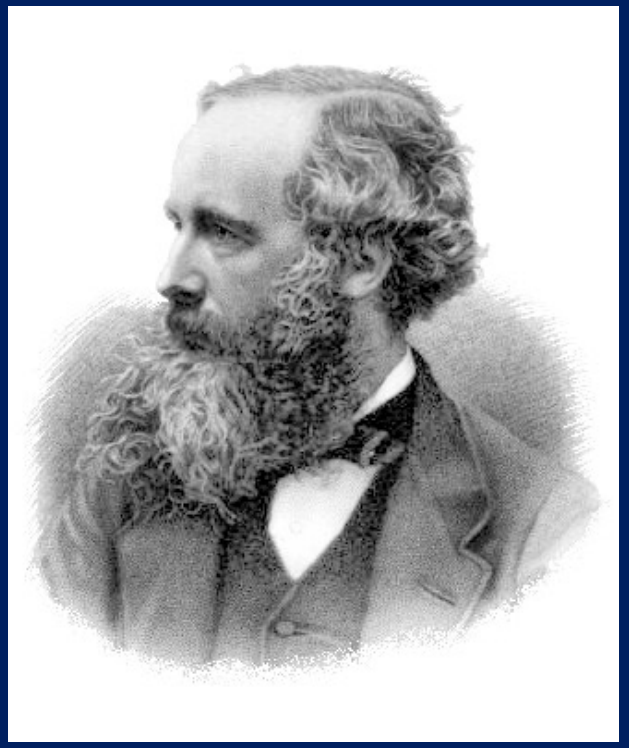
Newton (1713):

"I have not yet been able to discover the cause of these properties of gravity from phenomena and I feign no hypothesis. It is enough that gravity does really exist and acts according to the laws I have explained, and that it abundantly serves to account for all the motions of celestial bodies."

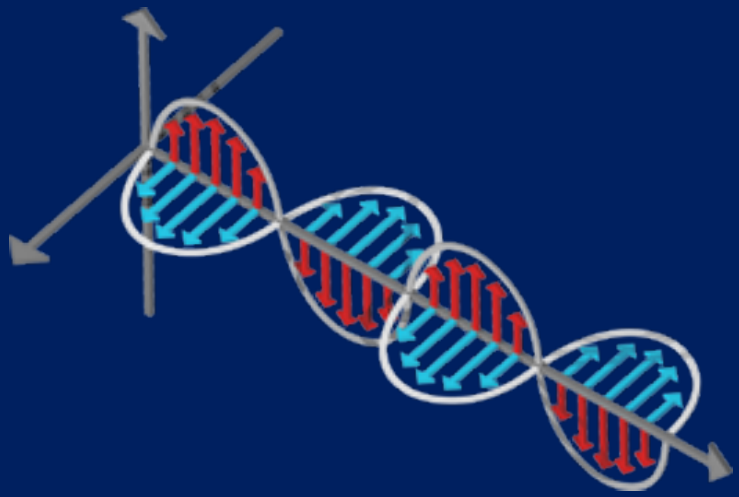
Electricity and Magnetism
and
Electromagnetic waves

Electric and Magnetic Fields and Waves

Michael Faraday had demonstrated that light is polarized by a magnetic field.



James Clerk Maxwell unified electricity and magnetism, predicted the existence of electromagnetic waves and identified light as an electromagnetic wave of oscillating electric and magnetic fields moving with a speed c (in vacuum)
$c = 3.1 \times 10^5$ kms/sec (1865).
Unifies electricity, magnetism and optics.

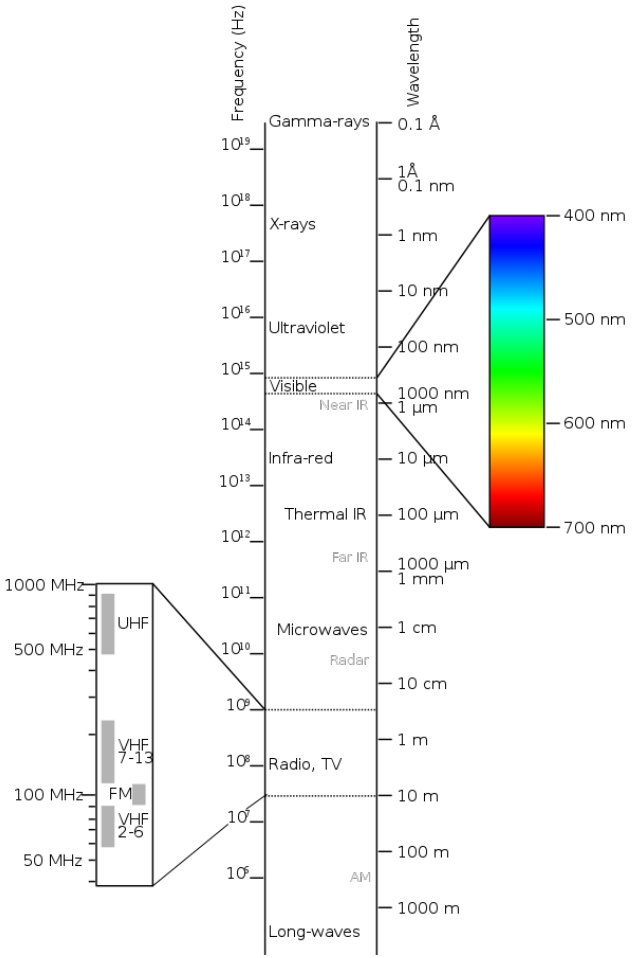


Force between charged particles is not instantaneous. It is transmitted by the electromagnetic field at the speed of light which is finite.

Experimental tests of Maxwell's theory

- In 1887 Heinrich Hertz demonstrated existence of radio waves that were predicted by Maxwell's theory with properties the same as visible light except the wavelength is 10^4 times longer.

- In 1895 Wilhelm Conrad Roentgen made the serendipitous discovery of X-Rays, which are electromagnetic waves of wavelength 10^{-2} times shorter than visible light.



Maxwell's Equations for $\vec{E}(\vec{x},t)$ + $\vec{B}(\vec{x},t)$

$$\nabla\cdot\vec{E} = \rho \qquad \nabla\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}$$

$$\nabla\cdot\vec{B} = 0 \qquad \nabla\times\vec{B} = \frac{1}{c}\frac{\partial}{\partial t}\vec{E} + \frac{1}{c}\vec{J}$$

If sources $\rho = 0$ and $\vec{J} = 0$, and space-time has no boundaries, we can write Maxwell eqns in Fourier modes:

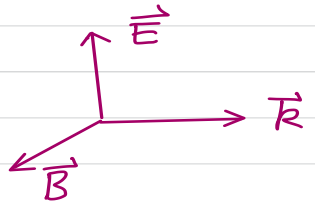
$$\vec{E}(\vec{x},t) = e^{i(\omega t - \vec{k}\cdot\vec{x})}\,\vec{E}(\vec{k},\omega)$$

$$\vec{B}(\vec{x},t) = e^{i(\omega t - \vec{k}\cdot\vec{x})}\,\vec{B}(\vec{k},\omega)$$

$$\vec{k}\cdot\vec{E}(\vec{k},\omega) = 0 = \vec{k}\cdot\vec{B}(\vec{k},\omega)$$

$$\vec{k}\times\vec{E}(\vec{k},\omega) = \frac{\omega}{c}\vec{B}(\vec{k},\omega)$$

$$\vec{k}\times\vec{B}(\vec{k},\omega) = -\frac{\omega}{c}\vec{E}(\vec{k},\omega)$$



Waves

$$\left(\vec{k}^2 - \frac{\omega^2}{c^2}\right)\vec{E}(\vec{k},\omega) = 0 = \left(\vec{k}^2 - \frac{\omega^2}{c^2}\right)\vec{B}(\vec{k},\omega)$$

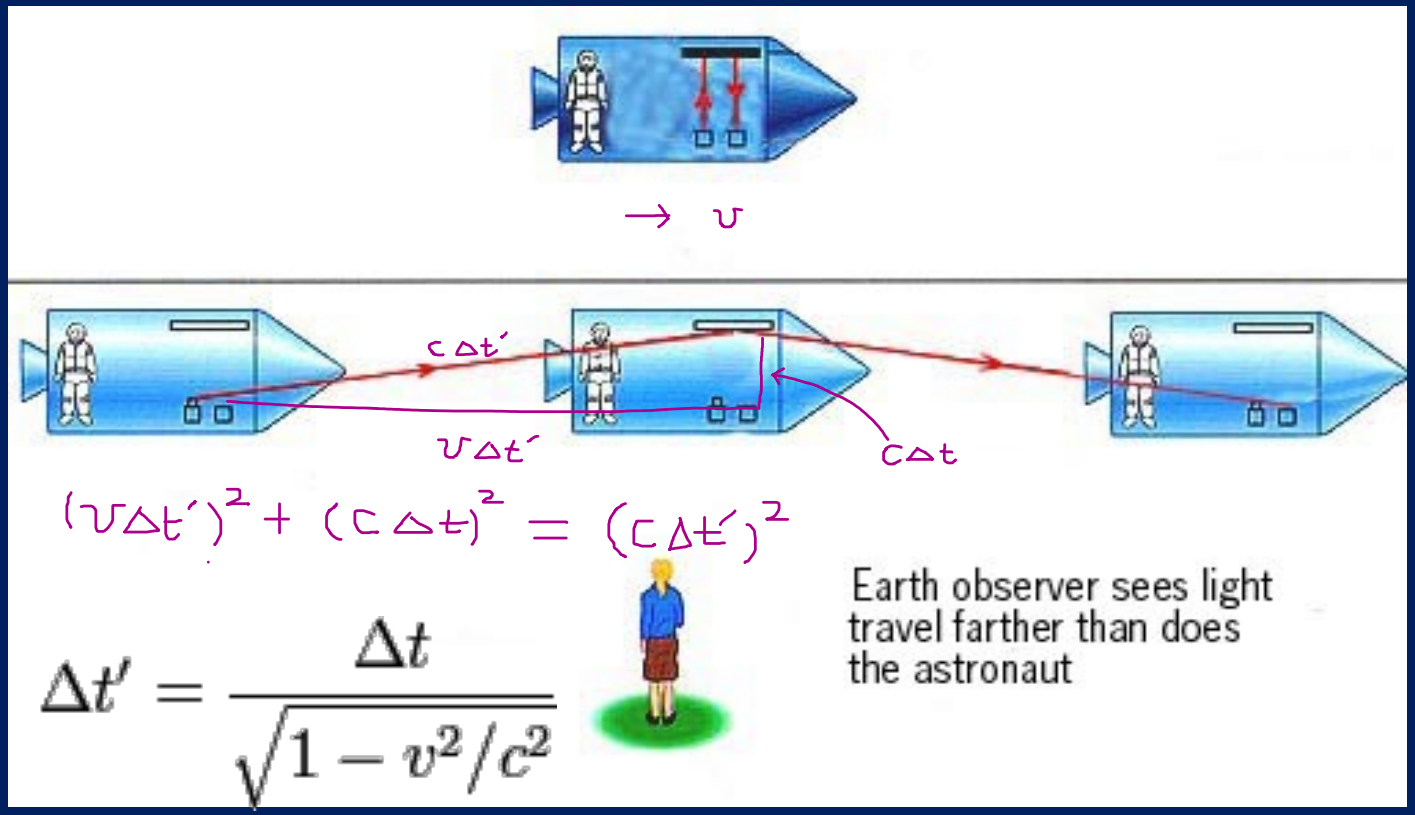
$$\vec{k}\cdot\vec{E} = \vec{k}\cdot\vec{B} = \vec{E}\cdot\vec{B} = 0\,, \quad \vec{k}^2 = \frac{\omega^2}{c^2}$$

Lorentz, Poincare, Einstein: Special Relativity (1905)

Implications of Maxwell's theory:

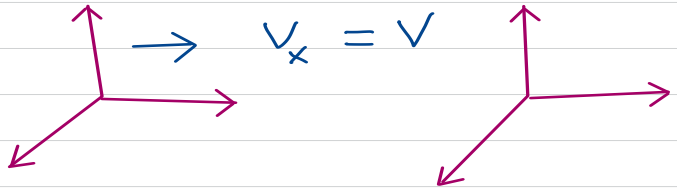
Speed of light is the same whether you run towards it or away from it. Space and time adjust themselves to ensure this!

Time intervals between events depend on your state of motion; things happen (according to us) more slowly for a moving observer than for us.



Lorentz invariance of Maxwells eqns

Lorentz transformation:



$$x' = \gamma\left(t - \frac{V}{c^2}x\right), \quad t' = \gamma(x - Vt)$$

$$\gamma = \left(1 - \frac{V^2}{c^2}\right)^{-1/2}$$

Define $\frac{V}{c} = \tanh\phi$

$$x' = -t\sinh\phi + \frac{x}{c}\cosh\phi$$
$$t' = t\cosh\phi - \frac{x}{c}\sinh\phi$$

} Mixing of space & time

$(x,t) \longmapsto (x',t')$ is a 'rotation'

$$(ct')^2 - x'^2 = (ct)^2 - x^2$$

The $\vec{E}$ and $\vec{B}$ fields also transform so that Maxwell's eqns are invariant.

4-dimensional Space-Time Geometry

Hermann Minkowski (1864-1909)

"Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

Introduced Geometry in Special Relativity

- Time is a 4th component of a 4-dim. space-time $x_4 = ct$; a new geometry described by an infinitesimal Lorentz invariant distance

  $ds^2 = (dx_4)^2 - (d\mathbf{x})^2$

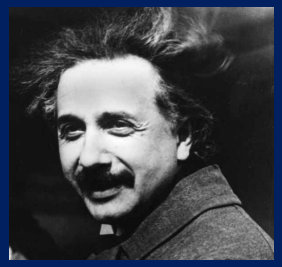
- Recall Euclid's geometry:

  $ds^2 = (dx_4)^2 + (d\mathbf{x})^2$

  Is the rotational invariant distance.

General Relativity

General Relativity-1



Einstein

Given that the speed of light is the same for observers in relative motion there can be no action at a distance which is simultaneous for all inertial observers in relative motion.

Hence Newton's law of gravitation needs to be modified.

Waging one of the great struggles in modern scientific history in 1915 Einstein completed the General Theory of Relativity, which is a theory of gravity in which the force of gravity is not instantaneous. It is communicated at a finite speed c (the same as the speed of light) by changes in the geometry of space-time.

Here the role of the electromagnetic field is played by the `metric' of spacetime.

Einstein's Two Puzzles/Questions

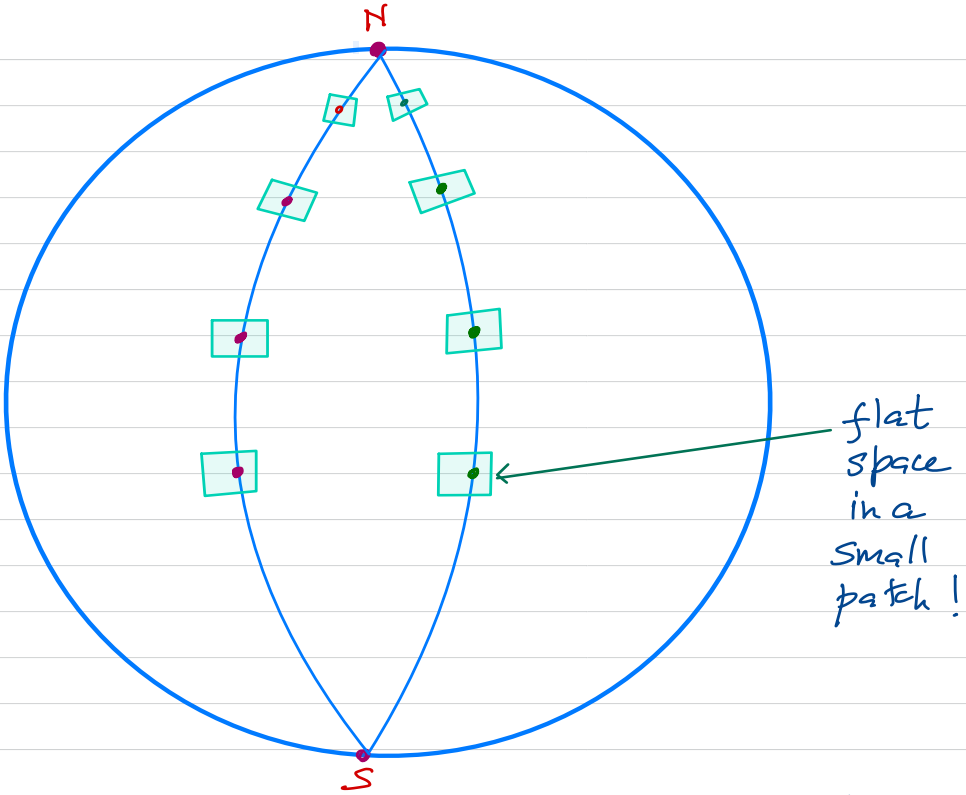
Newton's law of gravity in conflict with special relativity, and why only special relativity?

- That the force of gravity acts instantaneously is not consistent with Special Relativity!
- Einstein would like to have the force of gravity communicated at the speed of light by a field analogous to the electro-magnetic field of Faraday and Maxwell.
- Special Relativity is restricted to frames with relative constant velocity, but the laws of physics must be valid in any spacetime reference frame including those which are accelerating.

Special Relativity → General Relativity

- A possible resolution occurred in 1907.
- The Principle of Equivalence: ("the happiest/luckiest thought of my life"- Einstein)

  "The effect of a constant gravitational field is equivalent to a uniformly accelerated frame".
- In general, special relativity holds in sufficiently small regions of space-time in a non-uniform gravitational field.
- Patching up such small regions of space-time to construct the full space-time needed Riemannian geometry (curved space-time) and Marcel Grossmann (1912, Zurich).



2 bugs walking along the 'geodesics' on a sphere.... meet at the N pole.

Over 'small distances' $\ll$ (radius of sphere) the walk appears to be in 'flat space'.

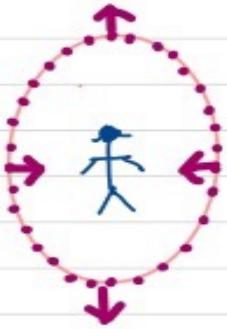
Only after walking for a while the bugs notice that they are walking on a curved space!

Free fall in the gravitational field of Earth

She is in a spherical dome of tiny beads

Key lesson:

Without looking down at Earth, she can infer that she is in the gravitational field of Earth.



After a while the beads sphere becomes an ellipsoid

'Visual Diff Geom & Forms' – T. Needham

The Riemann curvature tensor :

Riemann formulated geometry in terms of a metric tensor : $g_{\mu\nu}(x)$

$$ds^2 = \sum_{\mu,\nu} g_{\mu\nu}(x)\, dx^\mu dx^\nu$$

For flat Euclidean/Minkowski space $g_{\mu\nu}$ is diagonal:

$$g^{00} = \pm 1, \quad g^{ii} = +1$$

For a sphere of radius R : $ds^2 = R^2(d\theta^2 + \sin^2\theta\, d\phi^2)$

$$g_{\mu\nu} = R^2 \begin{bmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{bmatrix}$$

Deviations from a `flat' Euclidean or Minkowski space-time are given in terms of the Riemann tensor $R_{\mu\nu\rho\sigma}$:

$$ds^2 - ds^2_{flat} \cong \frac{1}{12} \sum_{\mu,\nu,\rho,\sigma} R_{\mu\nu\rho\sigma}(x)\, \delta^{\mu\nu}\, \delta^{\rho\sigma} + \ldots$$

$$\delta^{\mu\nu} = x^\mu dx^\nu - x^\nu dx^\mu$$

There is a formula for $R_{\mu\nu\rho\sigma}$ in terms of $g_{\mu\nu}$.

Einstein eqns (Quick derivation)

Recall Maxwell's eqns sourced by moving charges: $(\rho, \vec{J})$

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

Maxwell

$$\frac{\partial}{\partial x^\mu} F^{\mu\nu} = J^\nu$$

($\frac{\partial}{\partial x^\mu}$: fields; J^ν: source)

Note $\frac{\partial}{\partial x^\nu} J^\nu = 0$ because $F^{\mu\nu}$ is antisymmetric.

Gravity is sourced by energy-momentum

$T_{\mu\nu}(x)$ is a symmetric tensor that sources the gravitation field $g_{\mu\nu}$.

Form a symmetric tensor from $R_{\mu\nu\rho\sigma}$:

$R_{\mu\nu} = R_{\rho\mu\sigma\nu}\, g^{\rho\sigma}$ (Ricci tensor)

$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R$, $R = g^{\mu\nu} R_{\mu\nu}$

is conserved!

Einstein

$$\therefore \quad \boxed{R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = \frac{8\pi G_N}{c^4} T_{\mu\nu}}$$

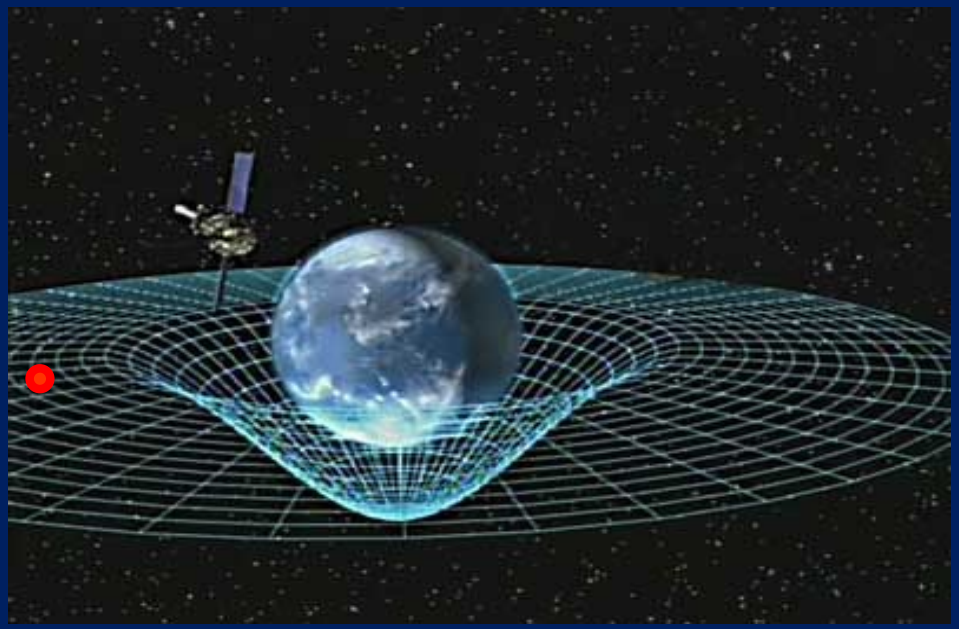
$G_N = 6.67 \times 10^{-8} \frac{cm^3}{gm.\, sec^2}$

General Relativity-2

GR describes the changes of the geometry of space-time caused by massive objects to which other objects respond. "matter tells space-time how to curve, and curved space-time tells matter how to move"(John Wheeler)

Space-time is like an 'elastic' grid, communicative and causal…but very very stiff...10^{20} times stiffer than steel!

In a curved space-time an object follows a path that maximizes the "proper time" in the frame of the object. These paths are geodesics.

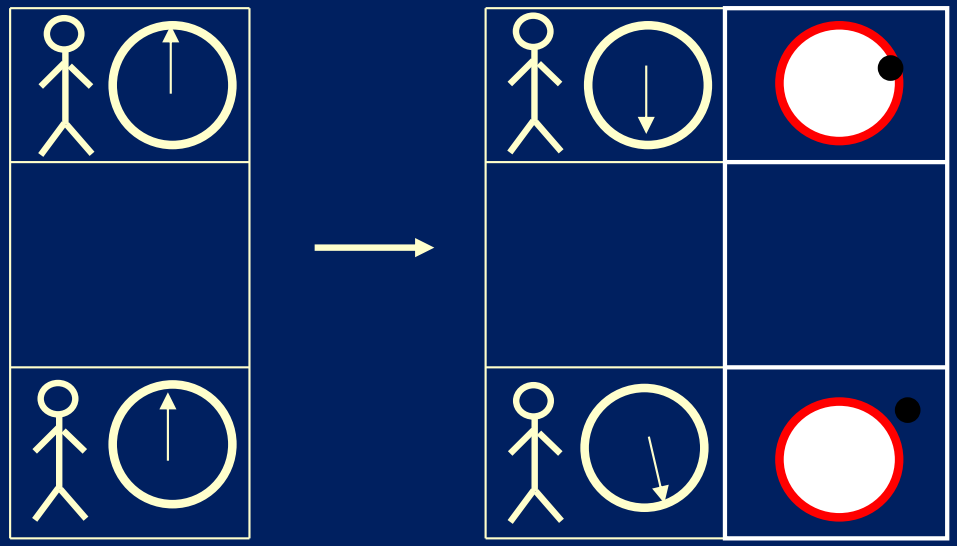


Time in General Relativity

Einstein's theory implies that time flows different for two observers in a gravitational field

Upper floor

Lower floor



Time flows more slowly

(A part in 10^{15}) for a height of 100 meters.

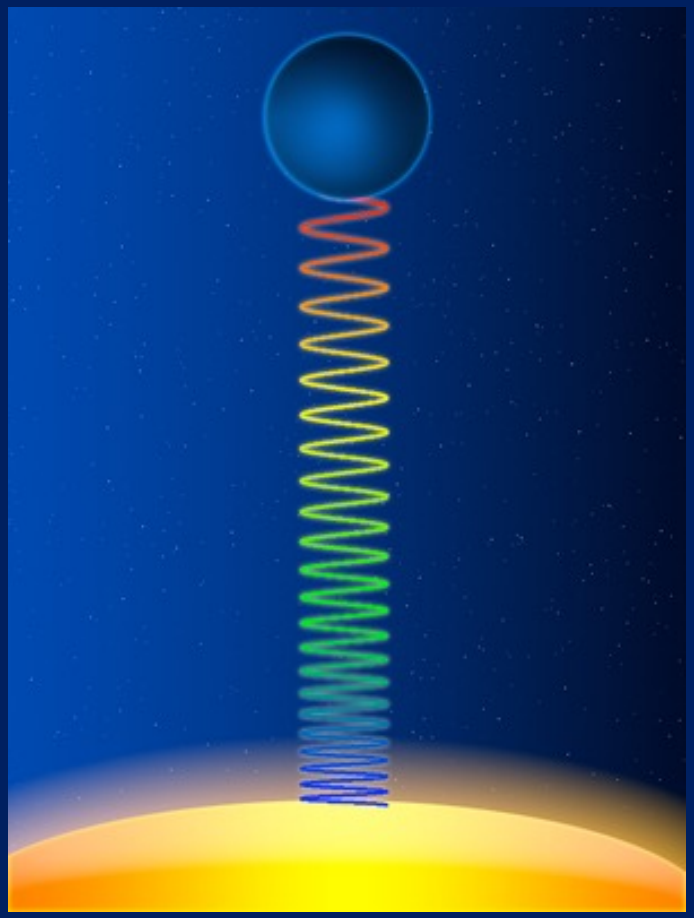
This effect is relevant for GPS!

J Maldacena

Experimental Tests of General Relativity

Tests of GR: 1. Gravitational Redshift

Wavelength of light shifts from blue (short) to red (long). Photons loose energy as they climb the against gravity.



- $(\lambda_{obs}-\lambda_e)/\lambda_e = \Delta\lambda, \quad \Phi_d-\Phi_u = \Delta\Phi$

$$\Delta\lambda = \Delta\Phi/c^2$$

λ is wavelength and Φ is the gravitational potential

- First verified by Pound and Rebka in 1959 at Harvard

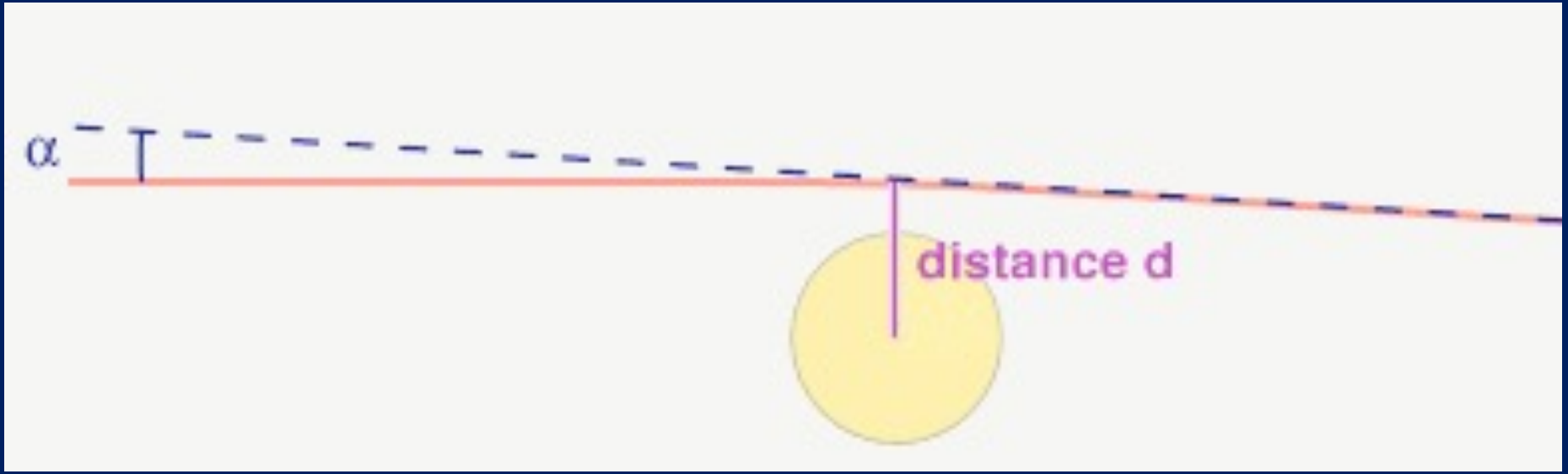
- Astrophysical tests:

  Sun: $\Delta\lambda \approx 2\times10^{-6}$

  White Dwarf: $\Delta\lambda \approx 0.001$

  Neutron Star: $\Delta\lambda \approx 0.1$

2. Bending of light by massive objects

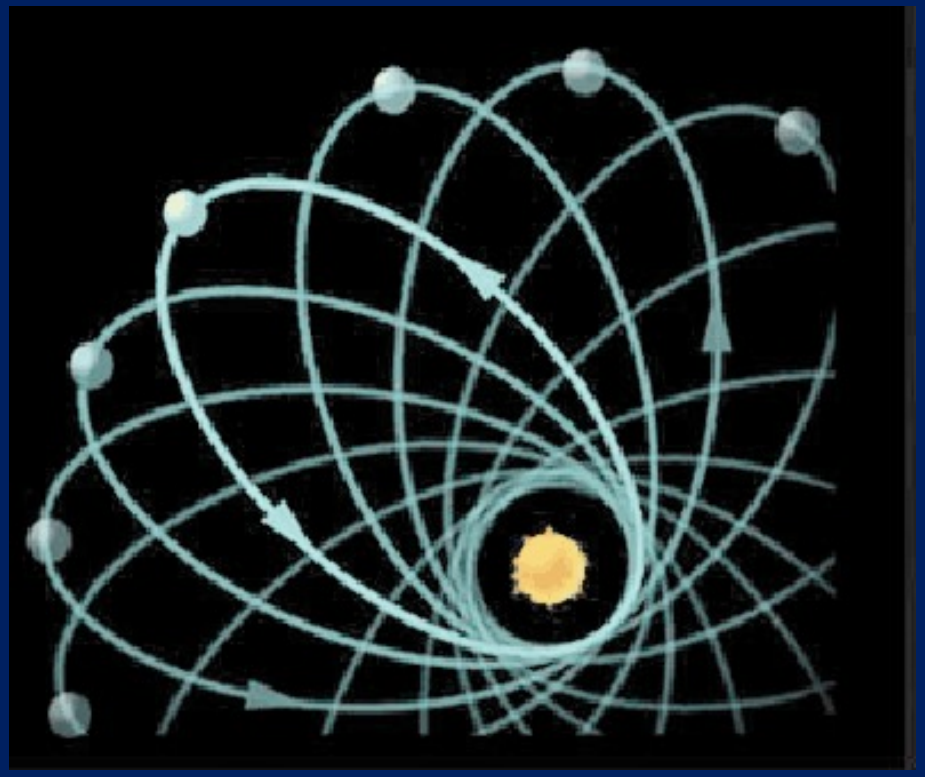


Newtonian theory $\alpha = 2GM/c^2d = 0.9$ arc seconds
Einstein theory $\alpha = 2x0.9$ arc seconds, twice the Newtonian value, agrees with experiment.

Experiments carried out during a solar eclipse: Eddington 1919, Lick Observatory 1922, Yerkes Observatory 1953, U Texas 1973.
Long baseline radio interferometry 1960s gave the most accurate verification of the GR prediction.

When asked by an assistant if general relativity had not been confirmed by Eddington and Dyson in 1919, Einstein famously made the quip: "Then I would feel sorry for the dear Lord. The theory is correct anyway."

3. Perihelion of Mercury



- 5600 seconds of arc per century (one second of arc=1/3600)
- Newton: predicts a precession of 5557 seconds of arc per century. There is a discrepancy of 43 seconds of arc per century.
- Einstein's theory accounts for this discrepancy, by taking into account that space-time is curved by the mass of the Sun.

GR as a framework for to describe gravitational phenomenon

Einstein's equations have surprising and remarkable solutions describing different space-times

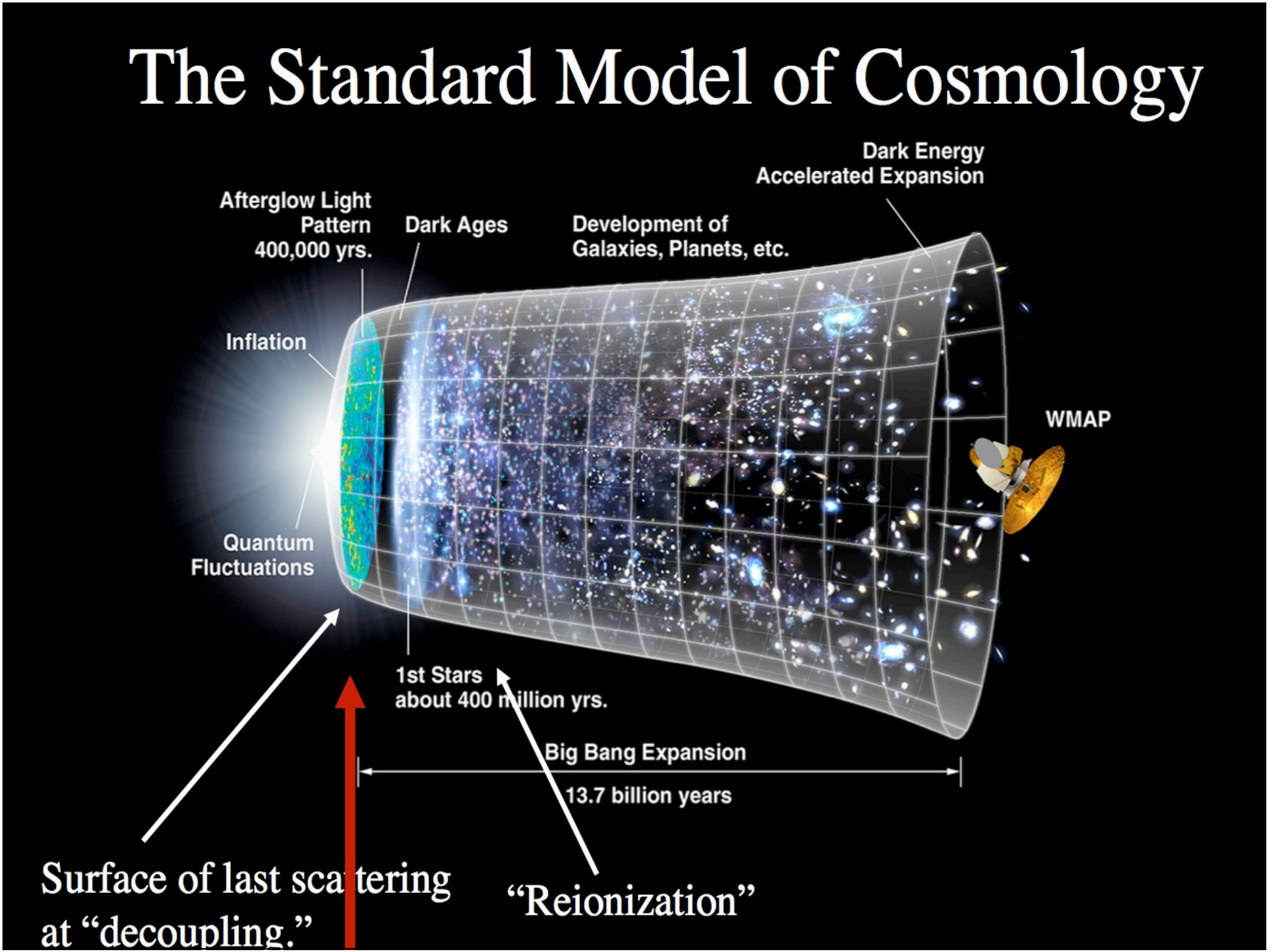
1. Expanding and accelerating space-time (+ve cosmological constant) (Friedman 1922, LeMaitre 1927)
2. Gravitational waves (Einstein 1916)
3. Black Holes (Schwarzschild 1916; Kerr 1963)

GR becomes a framework to discuss

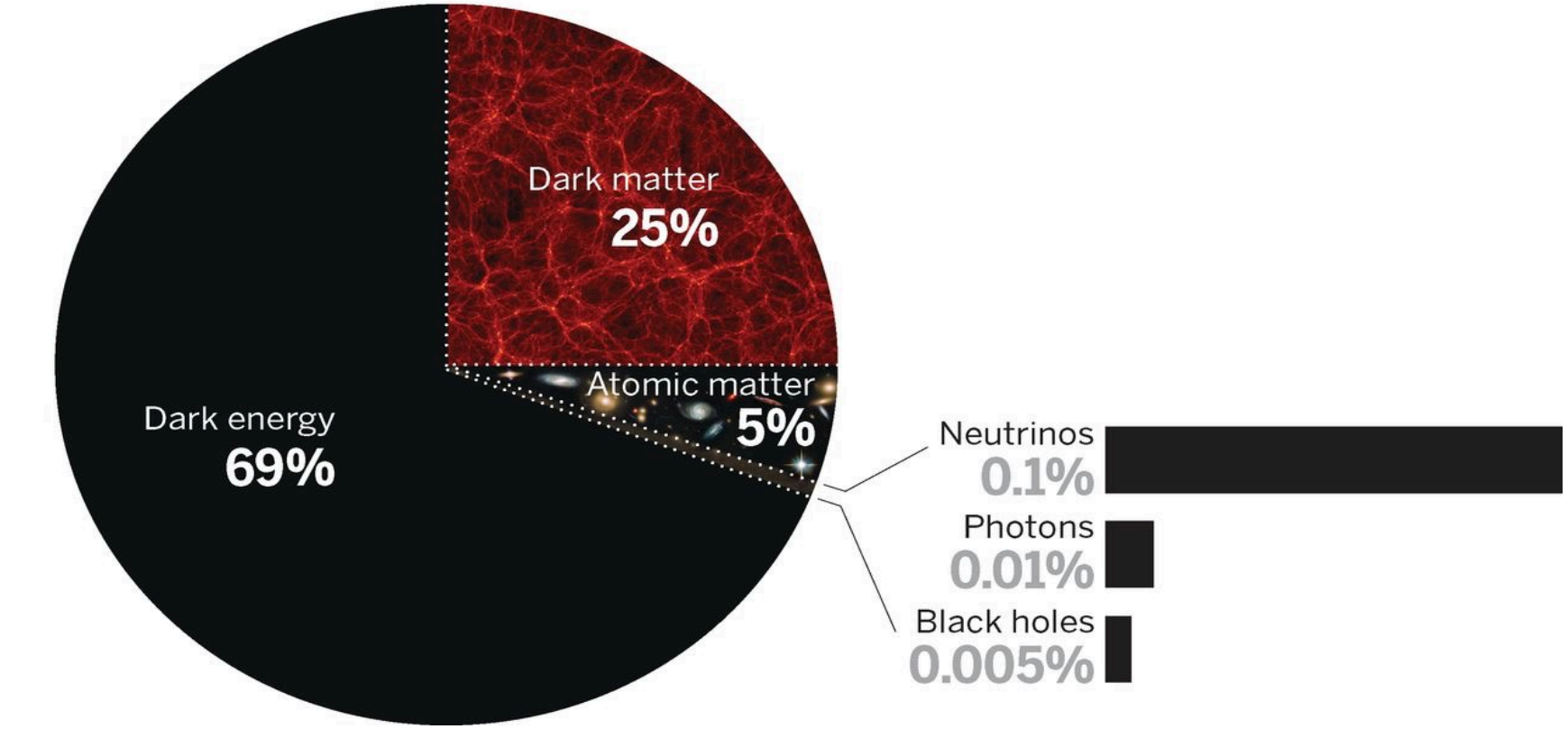
Cosmology, gravitational waves and black holes …

by now these applications are experimentally confirmed!

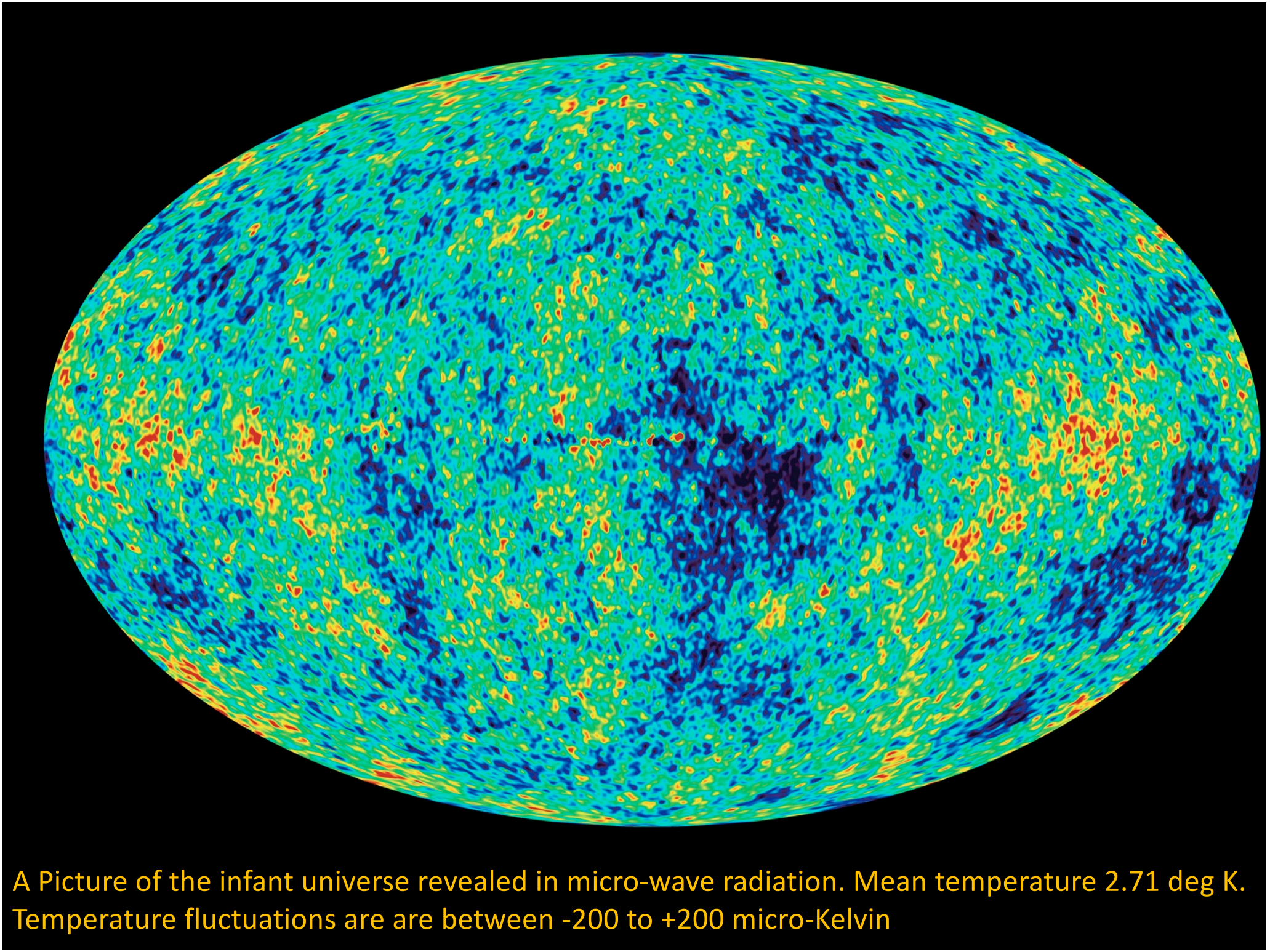
The Standard Model of Cosmology



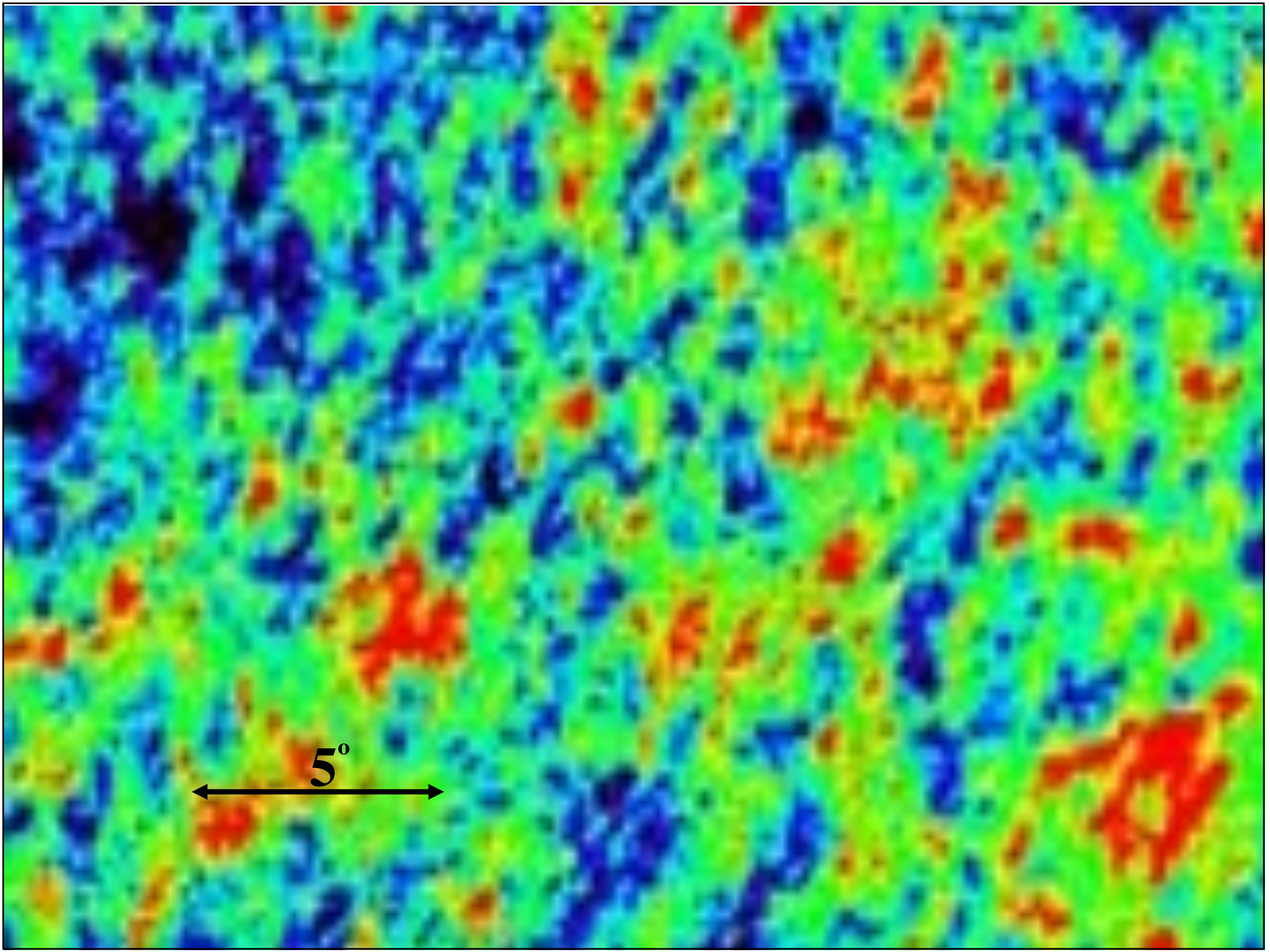
The multiple components that compose our universe. Dark energy comprises 69% of the mass energy density of the universe, dark matter comprises 25%, and "ordinary" atomic matter makes up 5%.



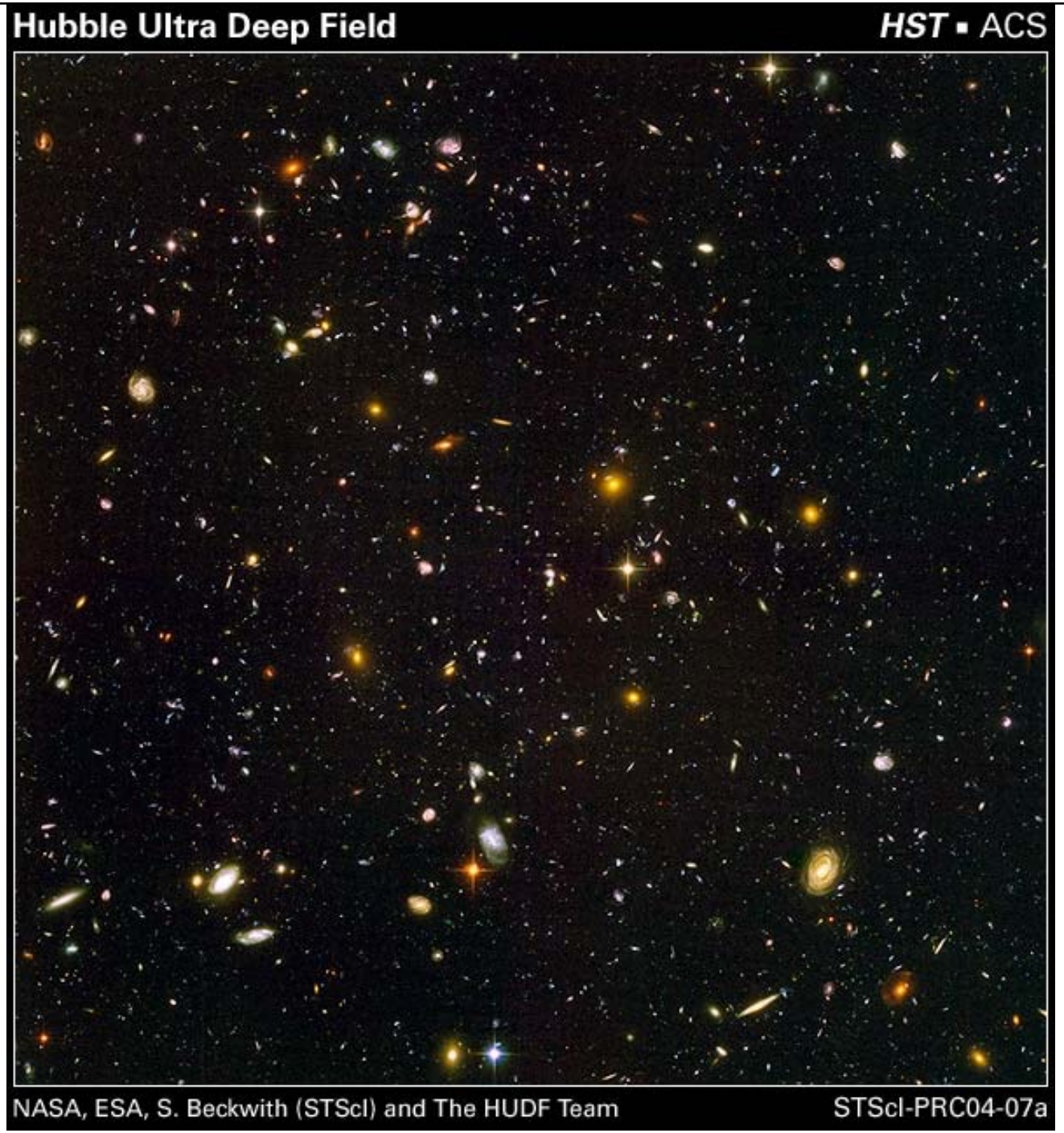
David N. Spergel Science 2015;347:1100-1102



A Picture of the infant universe revealed in micro-wave radiation. Mean temperature 2.71 deg K. Temperature fluctuations are are between -200 to +200 micro-Kelvin


5°

The the seeds of observed galaxies are tiny density fluctuations at the surface of last scattering which are in turn imprinted by fluctuations of quantum gravity during inflation.



~10^{11} Galaxies in Observable Universe

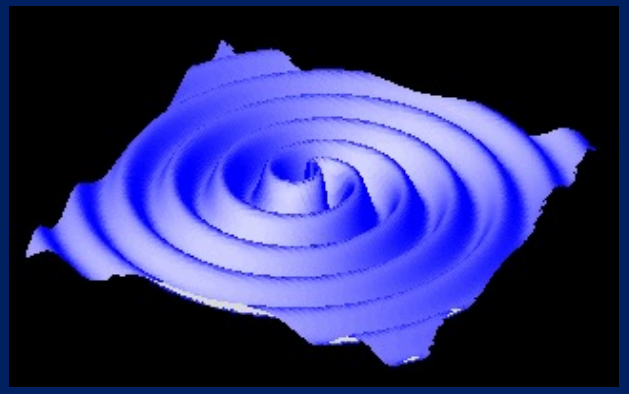
STScI/NASA Field & Levay

Gravitational Waves

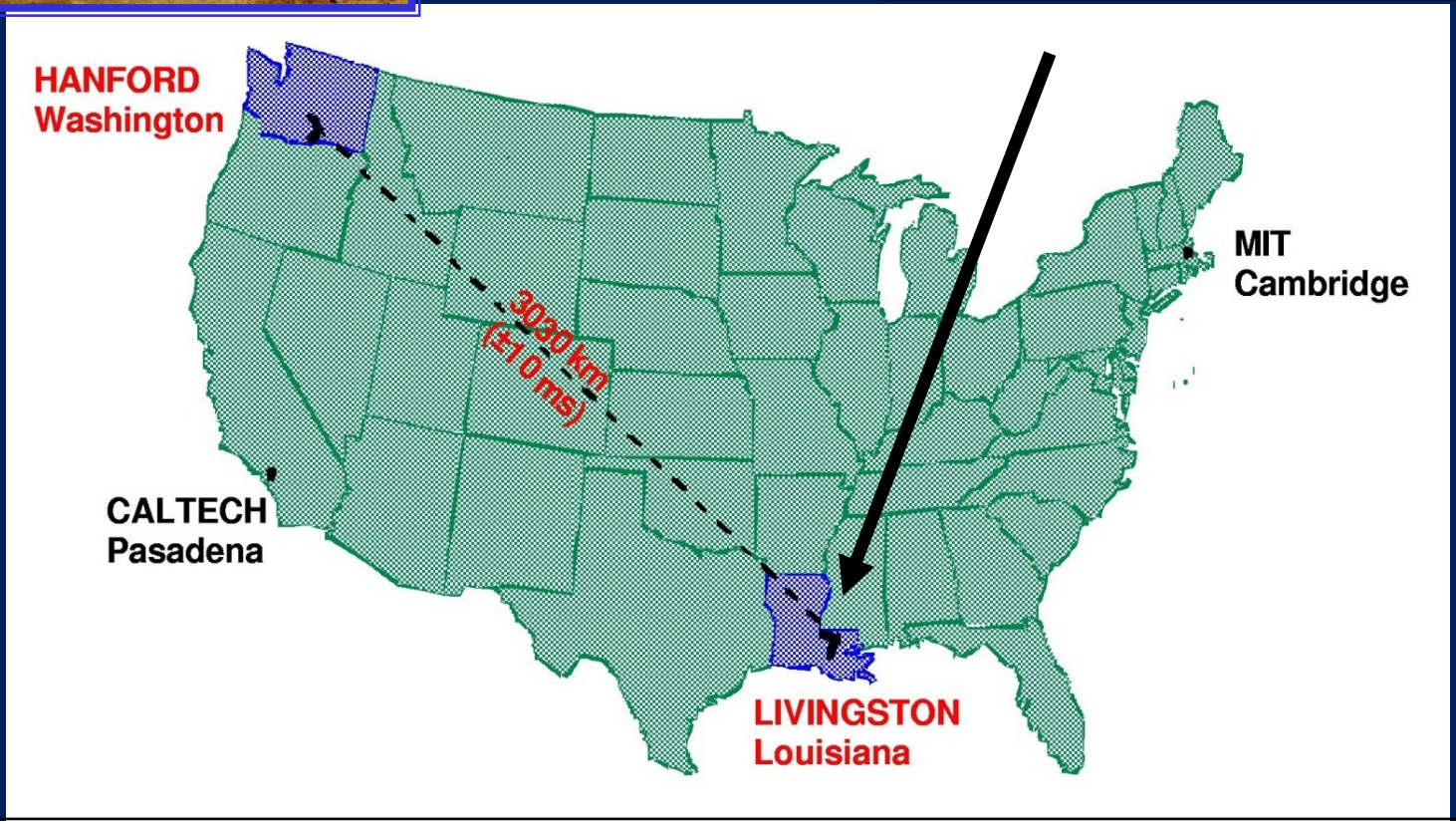
- Gravitational waves (GWs), ripples in the fabric of space-time, were predicted by Einstein in 1916 and detected in 2015.
- Indirect observation in binary pulsar (Hulse & Taylor, 1974)
- GWs will profoundly influence our knowledge of the universe and its past…it will see what `light' cannot.

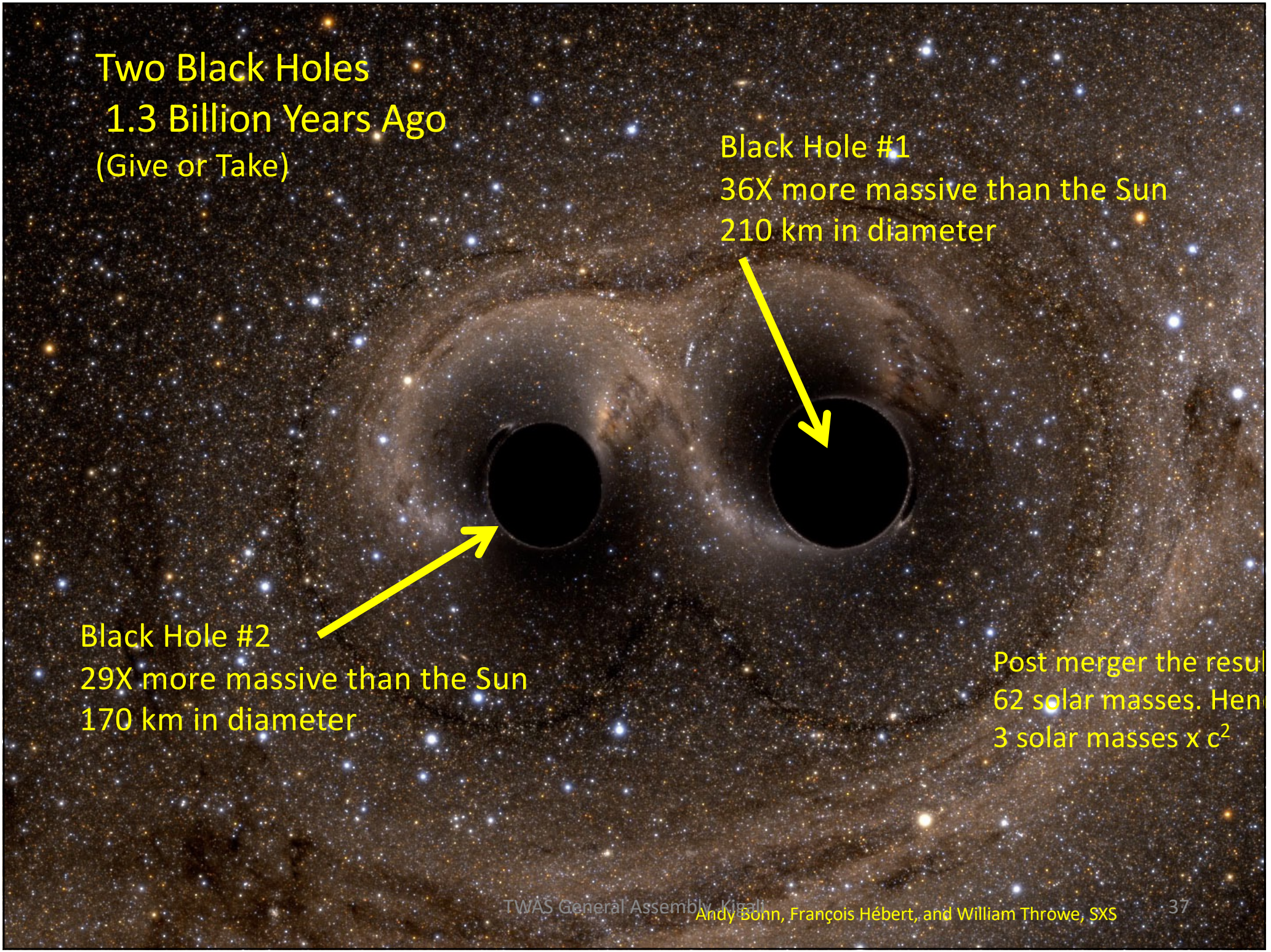
accelerating charges (time-varying dipole moment) → electromagetc waves -> photons

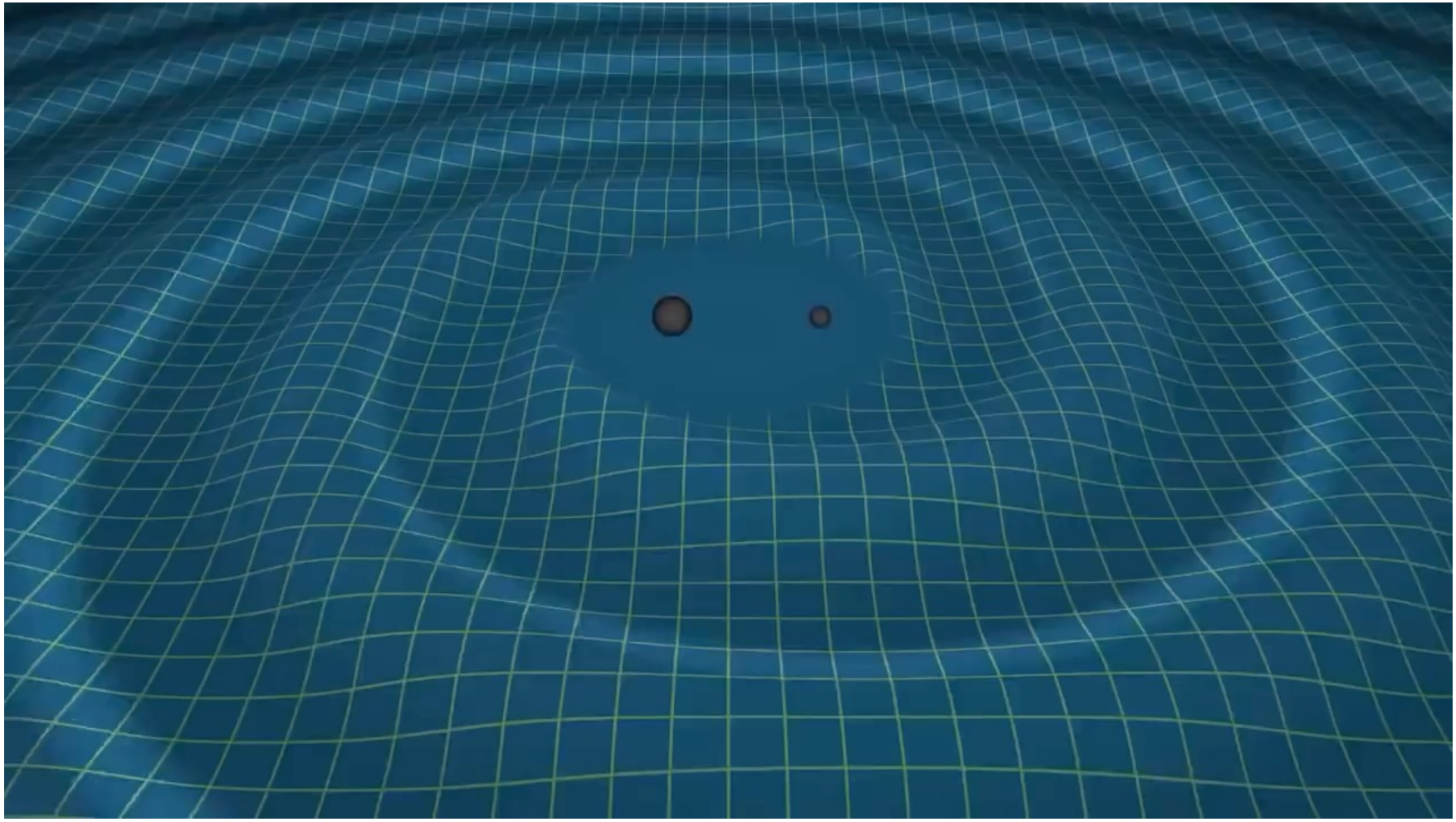
accelerating masses (time-varying quadrupole moment) → gravitational waves -> gravitons



On 14 September 2015, at the LIGO sites gravitational waves were detected



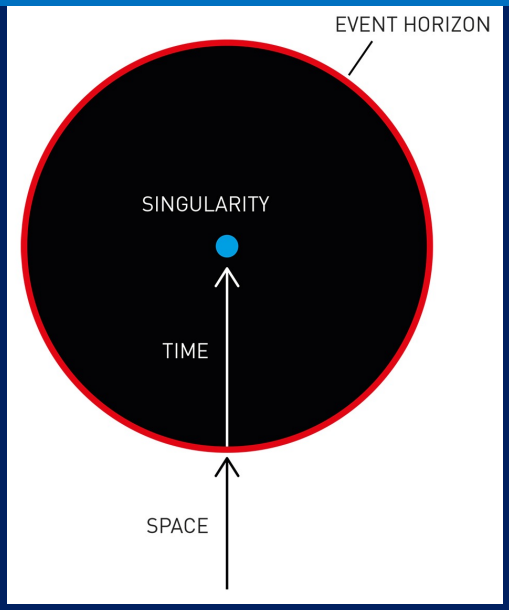

Two Black Holes
1.3 Billion Years Ago
(Give or Take)
Black Hole #1
36X more massive than the Sun
210 km in diameter
Black Hole #2
29X more massive than the Sun
170 km in diameter
Post merger the resul
62 solar masses. Hen
3 solar masses x c2
Andy Bonn, François Hébert, and William Throwe, SXS



Classical Black Holes

Black Holes-1

Karl Schwarzschild
1916



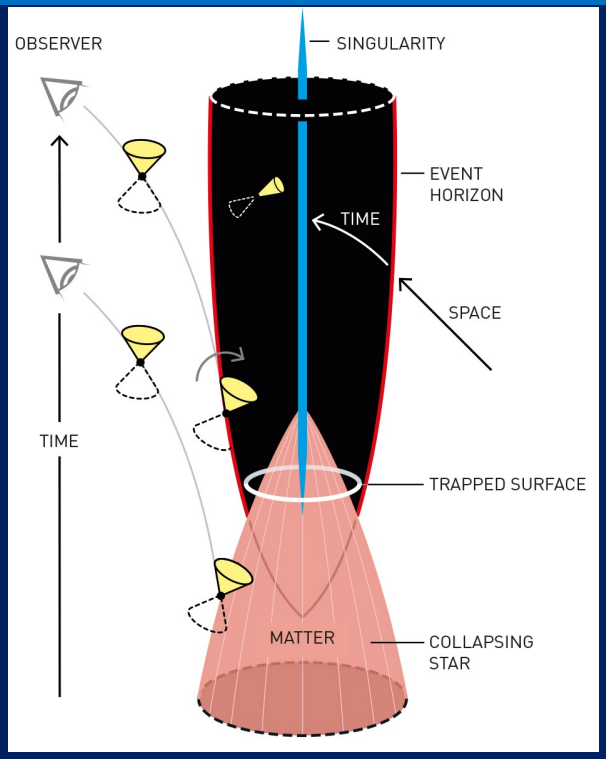
nobelprize.org

They are solutions of Einstein's equations which divide space-time into 2 parts. Interior and exterior separated by a surface called the horizon, which is a one-way gate. Even light entering it cannot get out. Hence the name BH.

The BH space-time has a curvature singularity in the interior. Inside the horizon the radial direction is time-like, and time ends at the singularity.

Black Holes-2

Roger Penrose



nobelprize.org

- Penrose (1973) developed topological arguments to show that BH formation is a robust prediction of the General Theory of Relativity.
- The horizon is a null surface of radius $r_s=2GM/c^2$ (Schwarzschild radius), G is Newton's constant. $r>r_s$ describes the exterior and $r<r_s$ the interior.

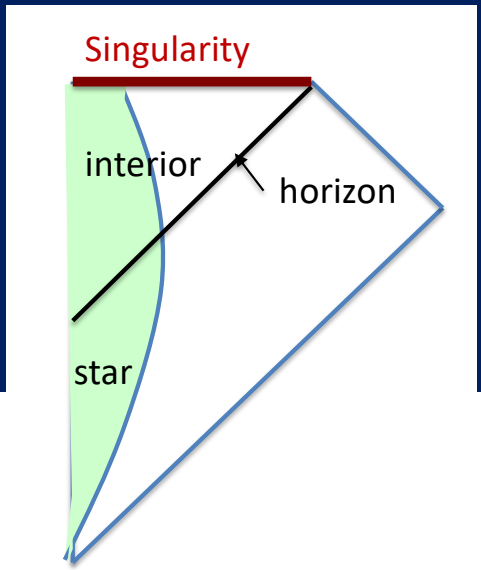
The horizon has area: $A_h = 4\pi r_s^2 = 16\pi G^2M^2/c^4$

Schwarzschild metric of space-time

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 d\Omega_2^2 \ .$$

$r_s=2GM/c^2$ is the Schwarzschild radius, that defines a null surface of codimension 2 called the horizon of the black hole. M is the mass of the BH and G is Newton's constant. For $r>r_s$ the metric describes a collapsing star. The apparent singularity at r_s can be removed by a coordinate transformation.

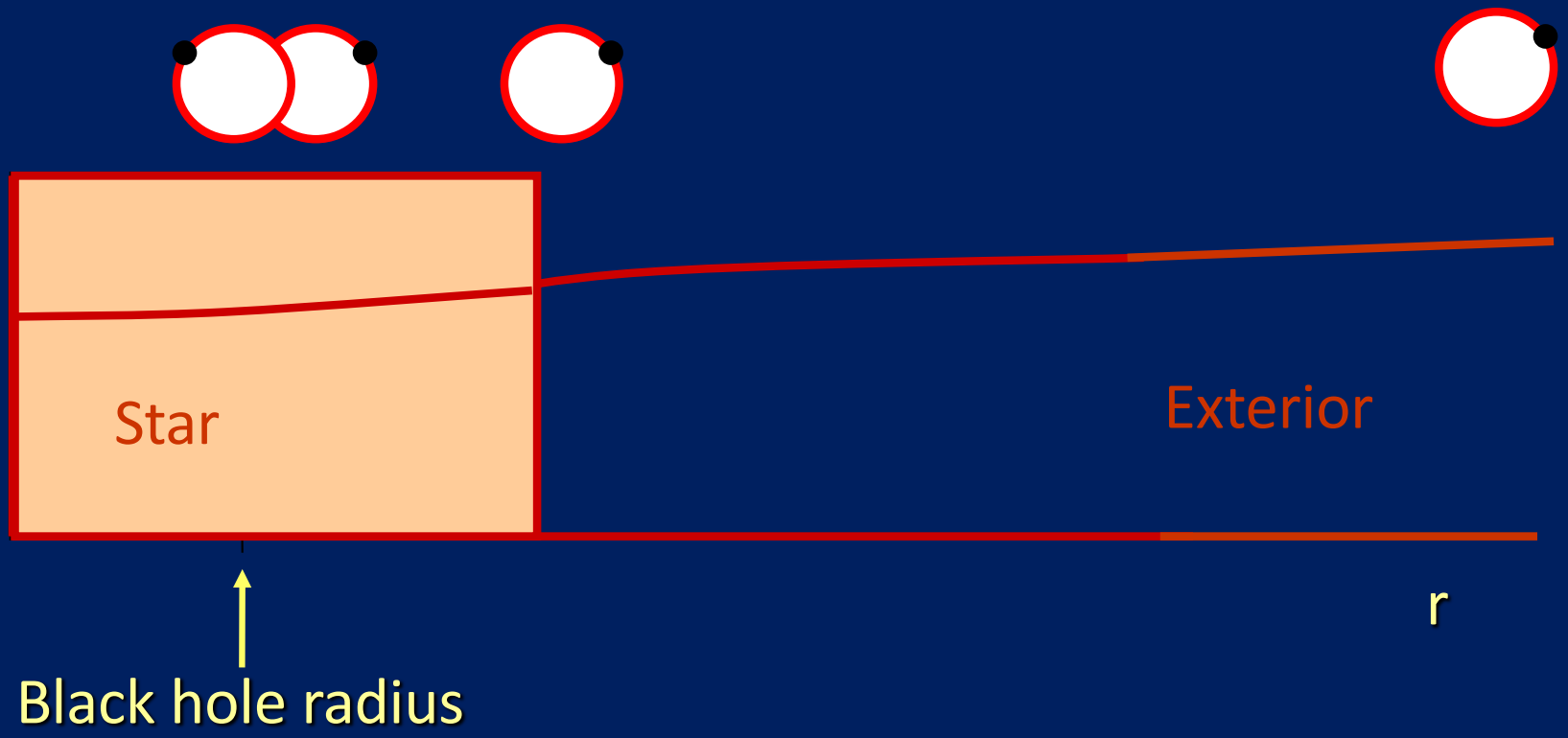
Penrose diagram of collapsing star



Flow of time in the presence of a massive Star and a Black Hole

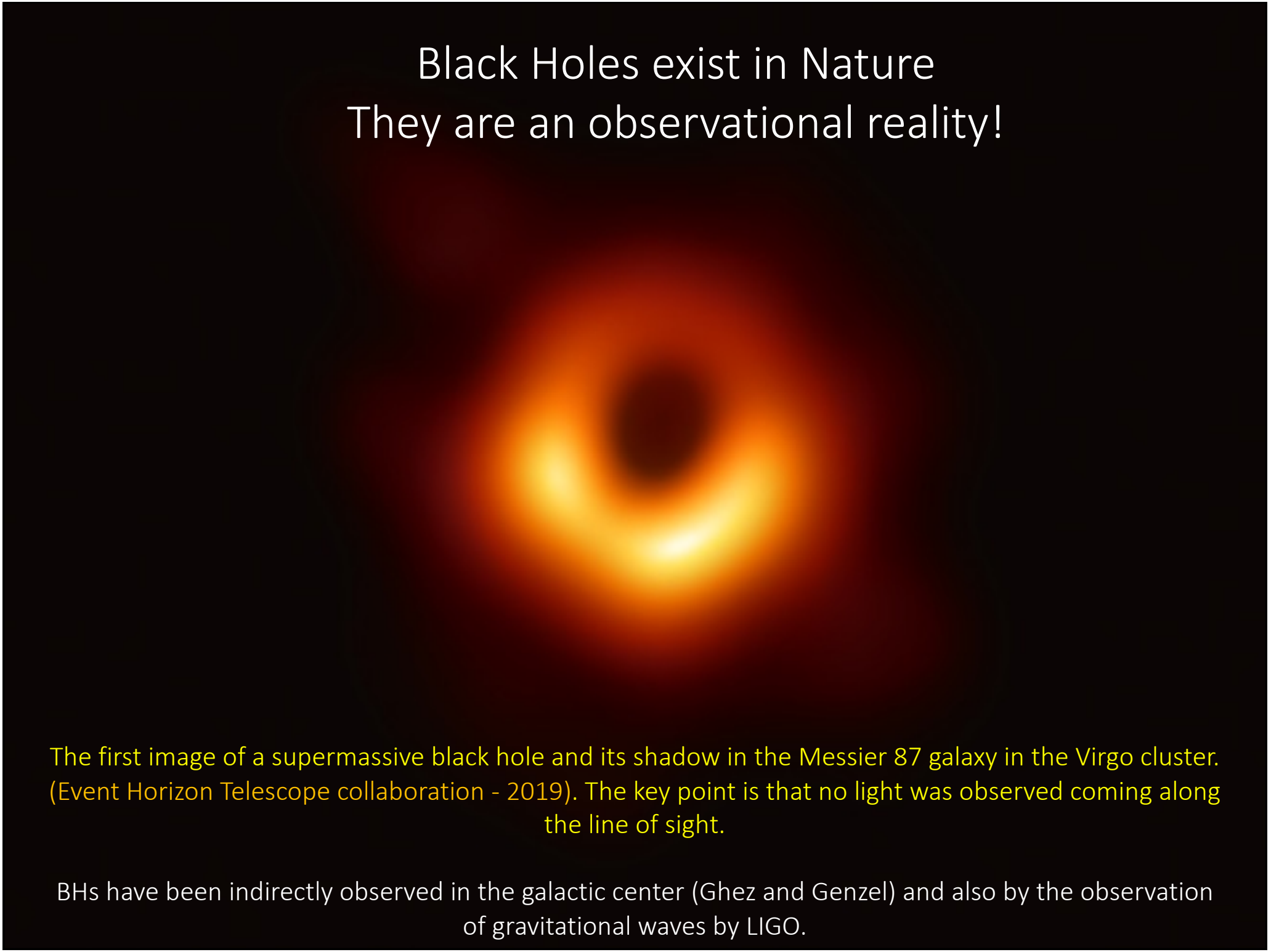
Flow of time

$$\sqrt{1-\frac{r_h}{r}} \qquad r_h = 2G\frac{M}{c^2}$$



J Maldacena

Black Holes exist in Nature
They are an observational reality!



The first image of a supermassive black hole and its shadow in the Messier 87 galaxy in the Virgo cluster. (Event Horizon Telescope collaboration - 2019). The key point is that no light was observed coming along the line of sight.

BHs have been indirectly observed in the galactic center (Ghez and Genzel) and also by the observation of gravitational waves by LIGO.

Black Hole Thermodynamics

Area theorems of GR (Hawking, 1971)

$Bh_1 + Bh_2 \rightarrow Bh_{12} +$ radiation

Horizon area always increases $A_{12} > A_1 + A_2$

A BH cannot be split apart and hence the reverse process is impossible.

Bekenstein (1973):

- Take a hot object and throw it into a black hole, it disappears decreasing the entropy of the universe, and that violates the 2nd law of thermodynamics.
- A contradiction is avoided if one attributes a thermodynamic entropy to a black hole proportional to the area of the horizon.

Black Hole Entropy - 2

$$S = b\text{Area}_h = b4\pi r_s^2 = b4\pi(2GM)^2$$

where b is an undetermined constant

The area theorems imply the 2nd law for the generalized coarse-grained entropy:

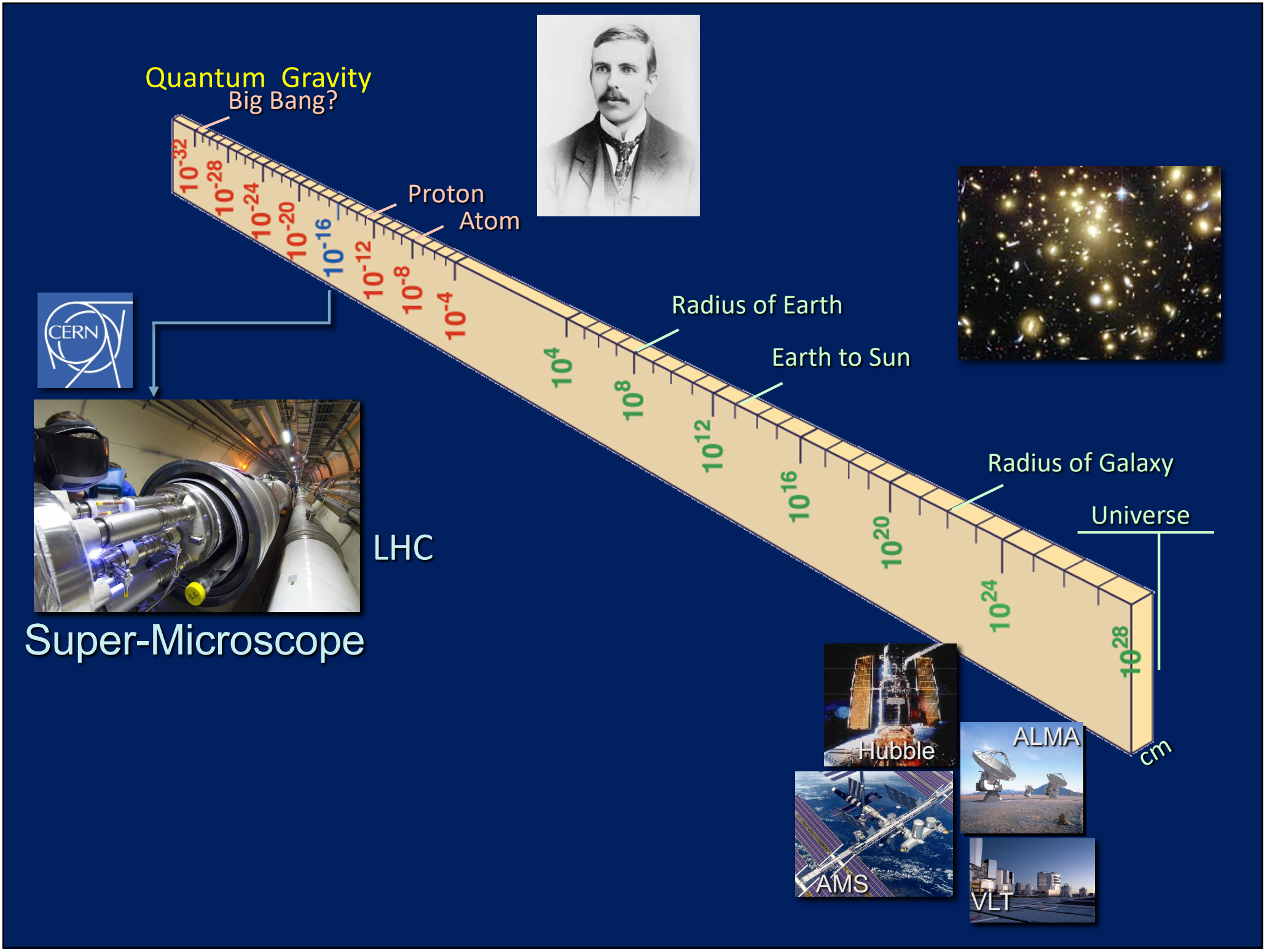
$$S_{gen} = S_{BH} + S_{matter}$$

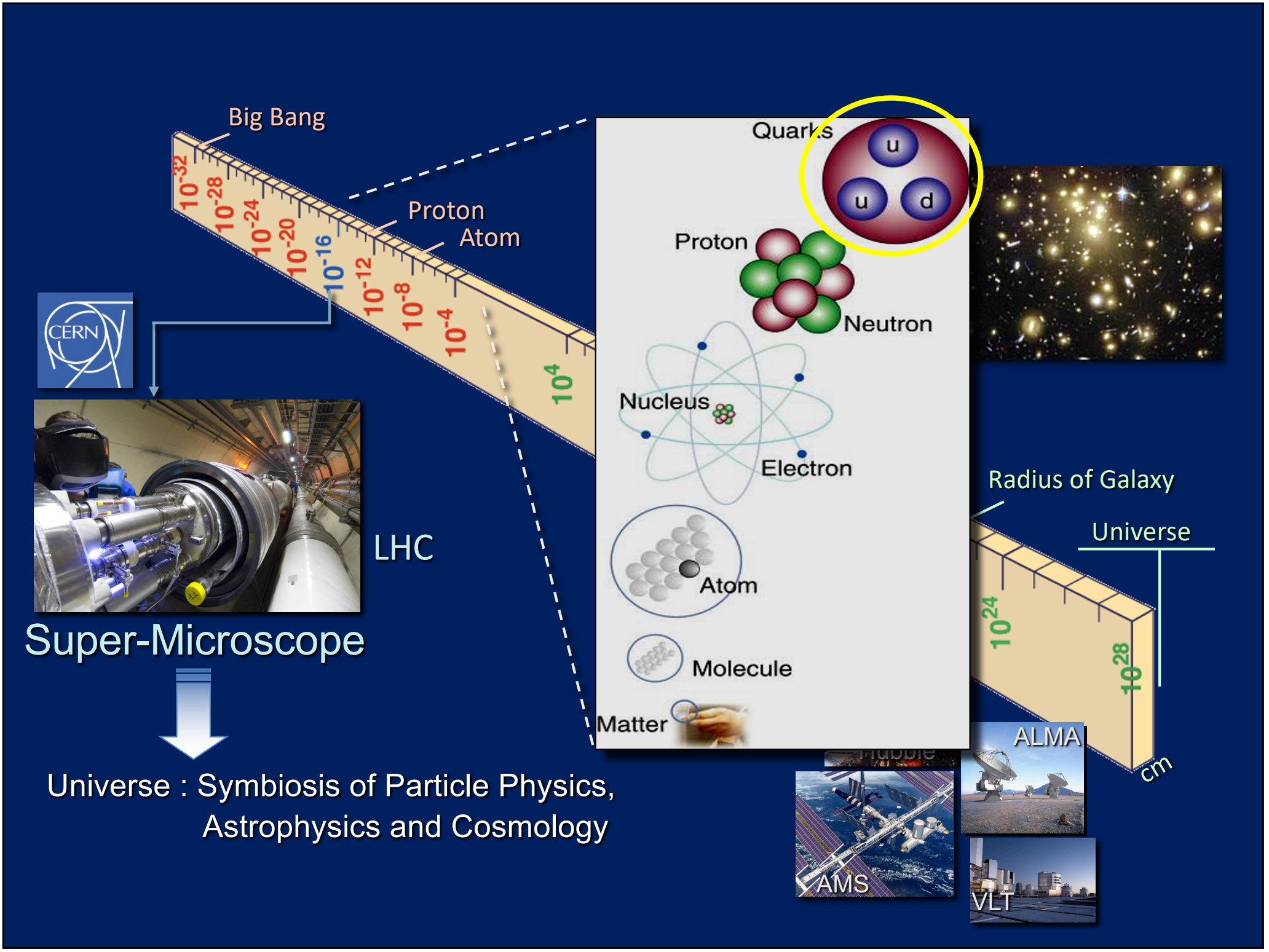
S_{matter} is the entropy of matter/radiation outside the BH horizon

(Bekenstein, A. Wall)

When the hot water bucket falls into the BH, S_{matter} decreases but S_{BH} increases by more.

Quantum Mechanics, Hawking Radiation and Information Puzzle


Quantum Gravity
Big Bang?
Proton
Atom
Radius of Earth
Earth to Sun
Radius of Galaxy
Universe
10^{-32}
10^{-28}
10^{-24}
10^{-20}
10^{-16}
10^{-12}
10^{-8}
10^{-4}
10^{4}
10^{8}
10^{12}
10^{16}
10^{20}
10^{24}
10^{28}
cm
CERN
LHC
Super-Microscope
Hubble
ALMA
AMS
VLT


Big Bang
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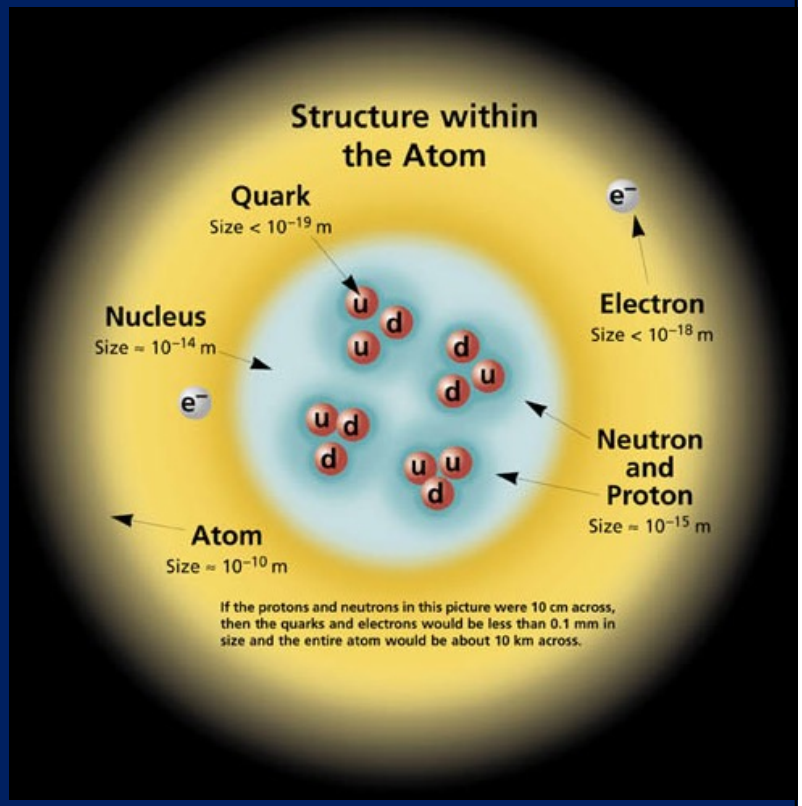
CERN
LHC
Super-Microscope
Quarks
u
u
d
Proton
Neutron
Nucleus
Electron
Atom
Molecule
Matter
Radius of Galaxy
Universe
10^{24}
10^{28}
cm
ALMA
AMS
VLT
Universe : Symbiosis of Particle Physics,
Astrophysics and Cosmology

Enter Quantum Mechanics

- A 20th century scientific revolution:

(Planck, Einstein, Bohr, Heisenberg, Schrodinger, Born, Dirac, Feynman and many many others)



- New laws for all particles
  especially below atomic scales.

- Electronic devices, computers, lasers, superconductors, superfluids, quantum computing…

- Colliding elementary particles in the LHC in Geneva all follow the laws of quantum mechanics …
  tested to 10^{-16} cms

Quantum Mechanics Rules

- QM assigns an amplitude (a complex number) for an event;

e.g., the amplitude for the motion of a particle from a point a to b is (Feynman)

- A(a , b) = $\sum_C$

- The sum is over all possible paths that connect a to b. Each path contributes a factor exp(iS/$\hbar$), S = $\int_{ta}^{tb} \mathbf{L}\,\mathbf{dt}$. L is the Lagrangian associated with the path C.

- A(b , a) = A(a , b)*

- The amplitude satisfies a Schrodinger type equation

- The PROBABILITY of the event is given by

  P(a , b) = A(b , a) A(a , b)*

Quantum Mechanics and Black Holes

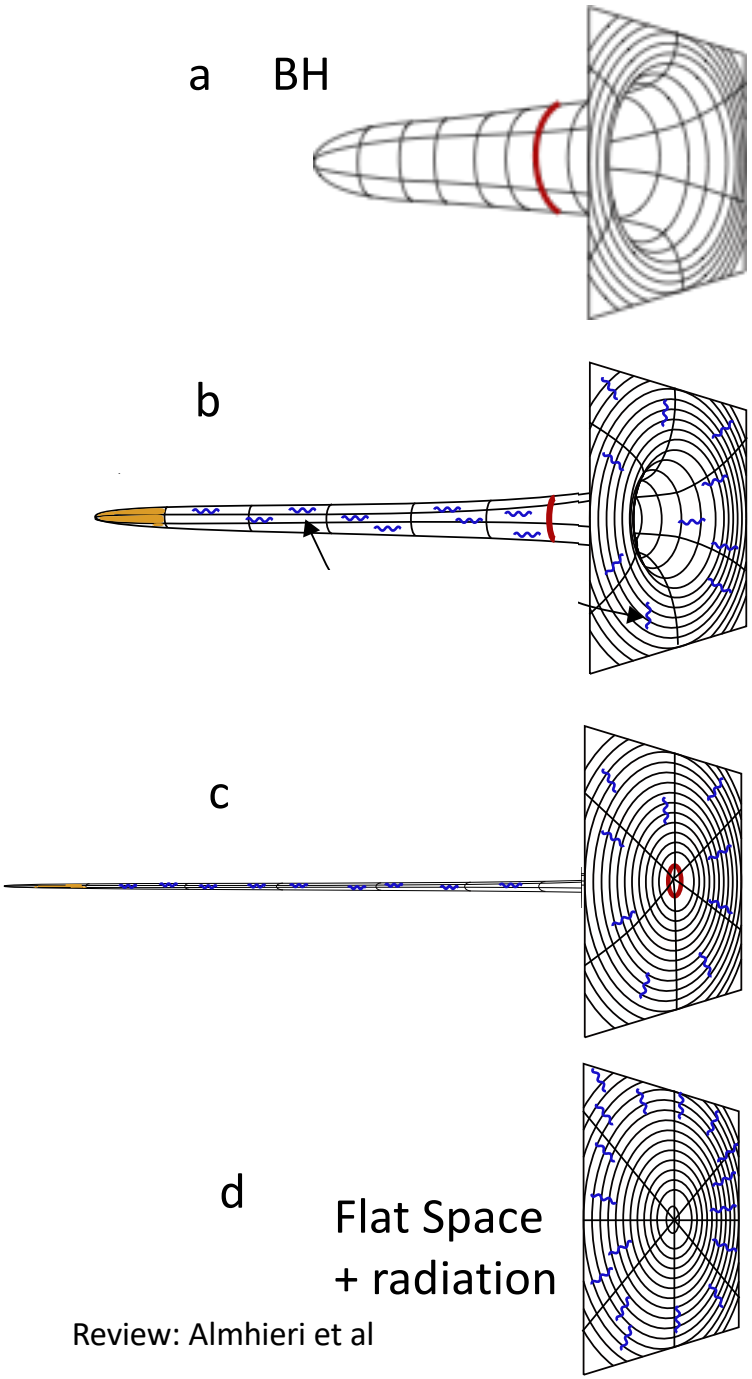
In QM if a particle to falls into a BH with a certain probability, then there is an equal probability for it to be emitted.

Hence quantum mechanics requires that

black holes must radiate! (Hawking 1974)

Hawking calculated the temperature of the black hole and argued that the end point of BH evaporation is purely thermal radiation.

Black Hole Evaporation



Review: Almhieri et al

Hawking Temperature and Entropy

In one of the celebrated calculations of the second half of the 20th century Hawking (1974) discovered that BHs radiate due to quantum effects. He calculated the temperature of the BH and found his famous formula which is on his tombstone:

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G k_{\mathrm{B}} M}$$

$T_{sun} = 3.6 \times 10^{-7}$ K

$T_{earth} = 0.1$ K

$T_{M=10^{18}kg} = 7000$ K

Using the first law of thermodynamics dM=TdS the BH entropy is (note b=1/4)

$l_p = 1.6 \times 10^{-35}$ m

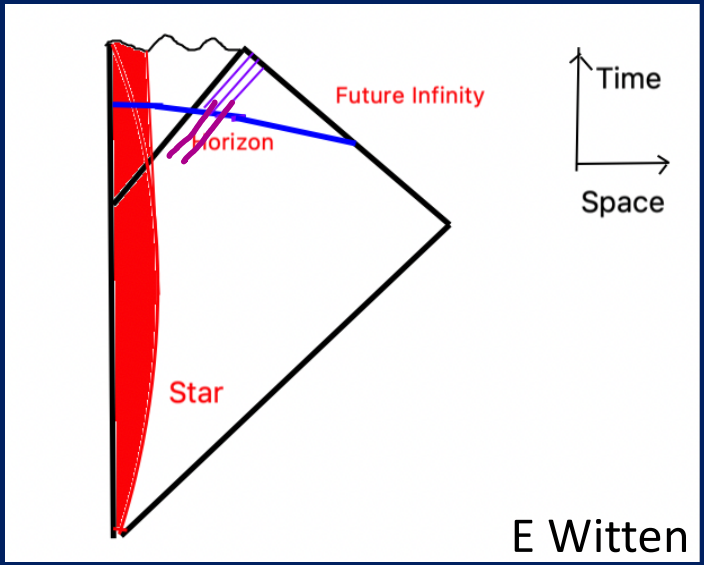
A BH has a very large entropy.

$$S_{BH} = \frac{k_B c^3}{4 \hbar G} A$$

$$\frac{c^3}{\hbar G} = \frac{1}{l_p^2}$$


Remember to look up at the stars
$S = \frac{k c^3 A}{4 \hbar G}$
STEPHEN HAWKING
FELLOW
1965-2018
and not down at your feet

Universal Black Hole Temperature & Entropy



Penrose diagram of a BH formed by collapsing matter.

The formula for BH temperature and entropy is universal and independent of how the BH was formed.

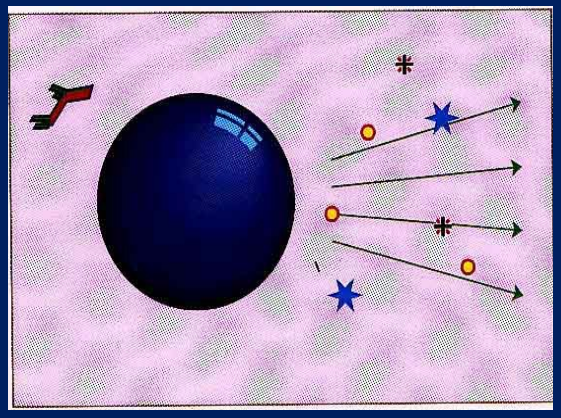
This is because the answer at late times depends only on very high energy Hawking quanta (or radiation) that emanate from near the horizon of the BH, after the collapsing matter has crossed the horizon.

The radiation appears to be thermal because we are NOT keeping track of the part that is on the other side of the horizon and is entangled with the radiation we are observing.

Information Loss is in conflict with Quantum Mechanics!

Like all hot bodies black holes radiate.

A black hole forms in various ways, but it always evaporates in the same way into thermal radiation leading to information loss as there is no memory of its initial state.



This violates a fundamental principle of unitarity in quantum mechanics:

A state cannot evolve to a mixed state in a closed system.

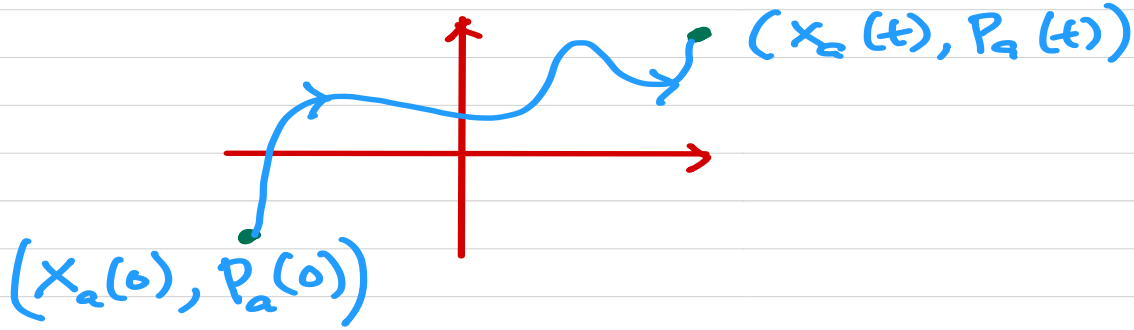
Hence, we have a serious problem

GR leads to information loss and QM cannot allow it!

Some people called it a crisis, but it presented an opportunity for progress

QM - 2

In Classical mechanics the 'state' of a system of N particles is specified at a given time by $(X_a(t), P_a(t))$, $X_a(t)$ & $P_a(t)$ are the position & momentum of the 'ath' particle.



The motion is generated by a version of Newton's laws formulated by Hamilton.

$$\frac{d}{dt} X_a(t) = + \frac{\partial H(X_a, P_a)}{\partial P_a}, \quad \frac{d}{dt} P_a(t) = - \frac{\partial H(X_a, P_a)}{\partial X_a}$$

$$H(X_a, P_a) = \frac{1}{2} \sum_a P_a^2 + V(X_a) \quad \text{(Hamiltonian)}$$

In Quantum mechanics the 'state' of a system is specified by a wave function $\Psi(x_a, t)$ which is a complex number, and an element of a linear Hilbert space.

Time evolution is given by Schrodinger's equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(x_a, t) = H\left(-i\hbar \frac{\partial}{\partial x_a}, x_a\right) \Psi(x_a, t)$$

or equivalently

$$\Psi(x_a, t) = e^{-\frac{i}{\hbar} H t} \Psi(x_a, 0)$$

Time evolution is generated by

$$U(t, 0) = e^{\frac{i}{\hbar} H t} \quad \text{(unitary)}.$$

Pure state $\nleftrightarrow$ Mixed state

Born rule: $\Psi^*(x_a,t)\,\Psi(x_a,t) = \rho(x_a,x_a,t)$

is the probability of finding the system in the state $\Psi(x_a,t)$.

$$\int \prod_a dx_a \; \rho(x_a,x_a,t) = 1$$

Density matrix:

$$\rho(x_a,y_a,t) = \Psi^*(x_a,t)\,\Psi(y_a,t)$$

$$\int \prod_a dz_a \; \rho(x_a,z_a,t)\rho(z_a,y_a,t) = \rho(x_a,y_a,t)$$

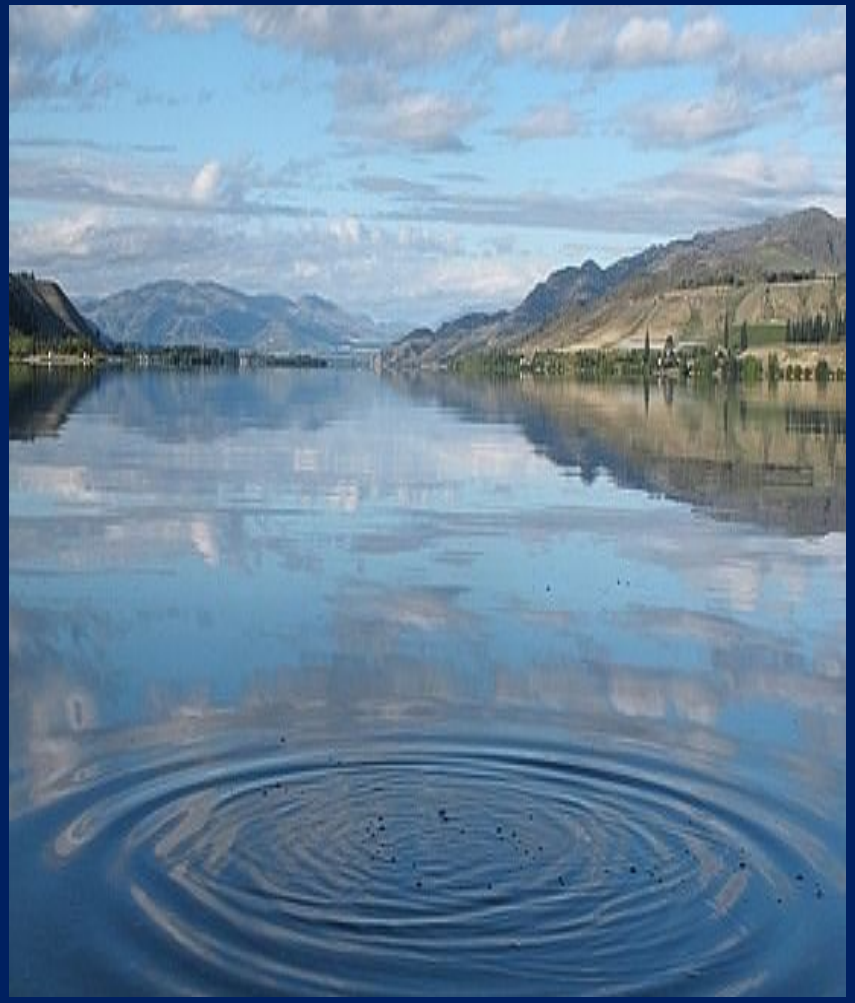
is a fundamental fact of QM. $\rho^2=\rho$

All states that satisfy the above equation are called "pure states".

The thermal density matrix $\rho = \frac{1}{Z} e^{-\beta H}$ represents a "mixed state." and $\rho^2 \neq \rho$

Resolving the Information Puzzle - Black Hole Microstates

An Analogy: Einstein's theory is an effective geometrical theory analogous to a fluid dynamics description of water



Drop a big pebble in a calm lake

It will cause a wave (distortion) to travel outwards from where it was dropped.

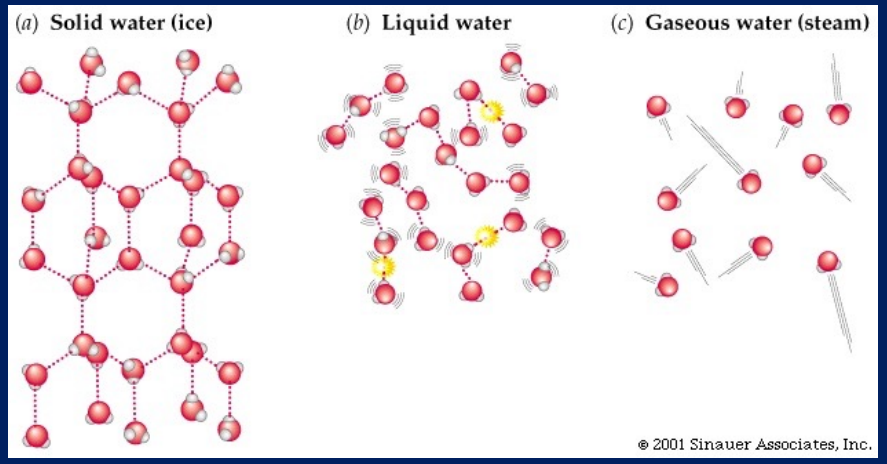
The wave will travel at the speed of sound in water and its effect will be felt a distance away.

The passing wave will wiggle a small sized object floating in the water!

There is a cause and an effect communicated by a wave traveling at a definitive speed…analogous to gravitational waves set off by colliding black holes.

The hidden structure of water/space-time?

- Water has a molecular structure underlying its smoothness… and various phases!



What is the hidden structure underlying the `smooth' geometry of space-time?

- What are the analogues of the molecules of water and their interactions in gravity?

- Clue in the study of black holes (which are predicted by Einstein's theory and exist in nature)… in String Theory

Attempts to resolve the information puzzle 1990 onwards: BH Microstates?

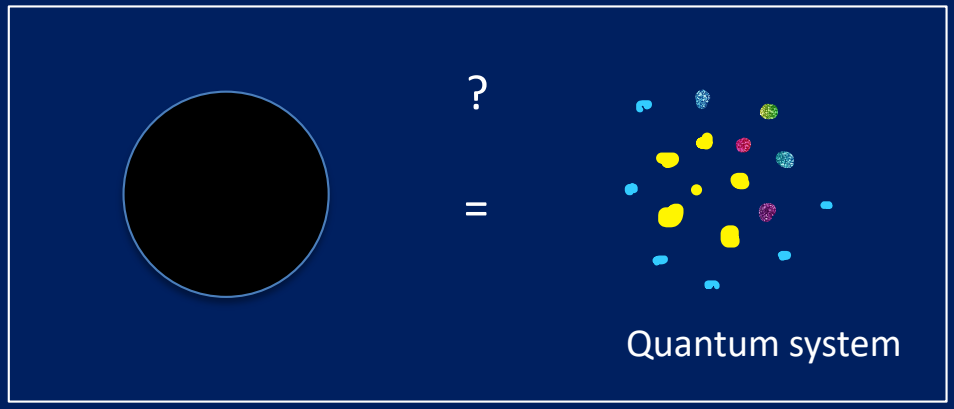
Since a BH is a thermodynamic system, can information loss for black holes be understood as due to an averaging process of many internal states, like in usual treatments of quantum statistical mechanics?

Like the burning of a piece of wood?

Is there a theory of quantum gravity in which black hole entropy is given by Boltzmann's formula ?

$$S_{BH} = k \log(N)$$

What are the internal states of the BH, that would account for BH entropy?



Black hole micro-states-D branes

In 1996 Strominger and Vafa provided the first concrete evidence in a calculable supersymmetric model that the black hole space-time is a sort of a hydrodynamic description of more basic underlying quantum system of D-branes in string theory.

D-branes which are domain walls of string theory were discovered by Polchinski. Quantum Gravity has more degrees of freedom than `gravitons'.

Hawking radiation and BH thermodynamics can be calculated in the framework of statistical mechanics in this constituent model of the black hole!

Microscopic model of Hawking radiation was developed post 1996 over many years:

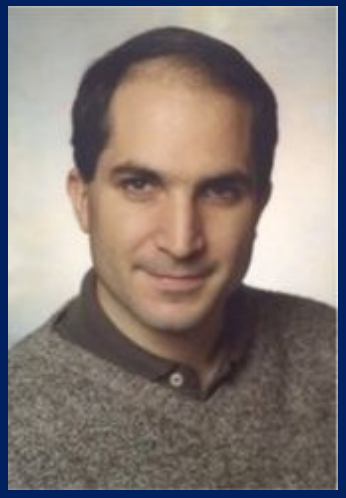
J. David, G. Mandal, S. R. Wadia - Physics Reports (2002)

Higher order corrections to BH entropy were computed by A. Dabholkar, A. Sen and others in the framework of string theory.

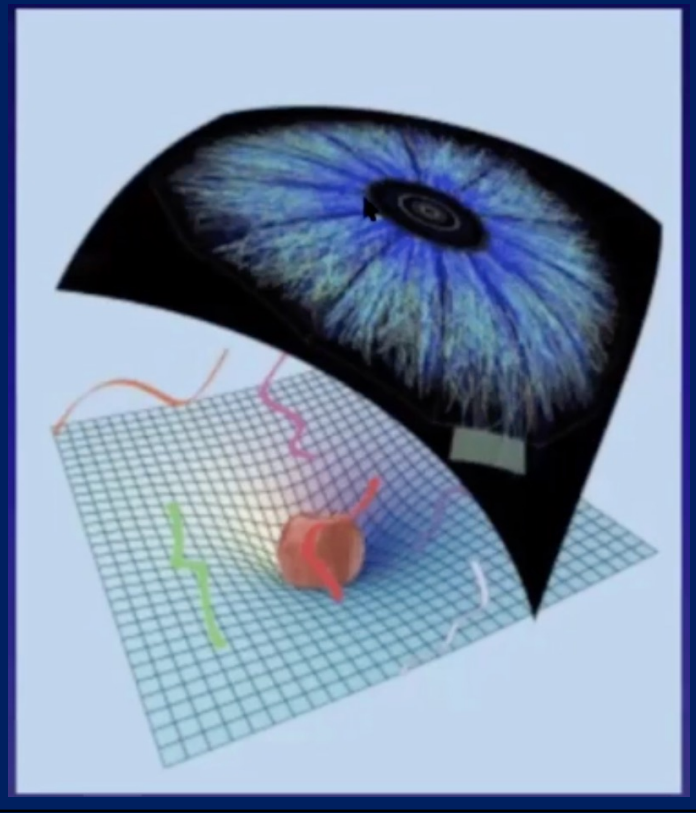
These development gave enormous evidence that string theory is the correct theory of quantum gravity and led to the holographic description of quantum gravity.

Holography and non-perturbative Quantum Gravity

Another approach to resolve the information puzzle is the AdS/CFT holography proposed by Maldacena (1997). It says that gravity and string theory in an asymptotically AdS (hyperbolic) space-time are holographically coded in a unitary QFT on its boundary.
Hence in principle one can track BH formation and evaporation in the QFT and that by the holographic correspondence would be a unitary process.

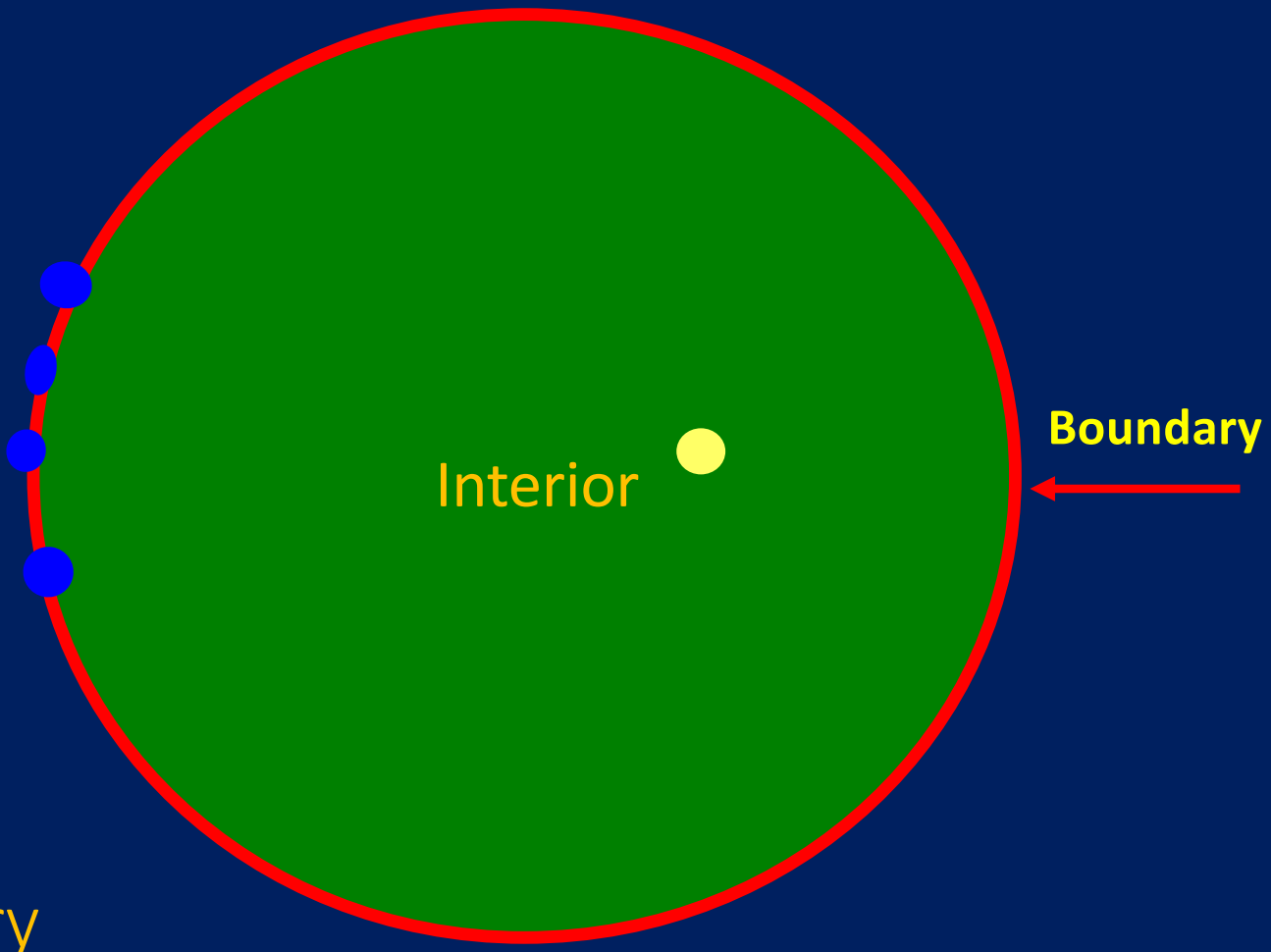


Juan Maldacena



Gravity in the interior → described by interacting particles on the boundary.

In the interior is a string theory (gravity) in anti-de Sitter space-time.

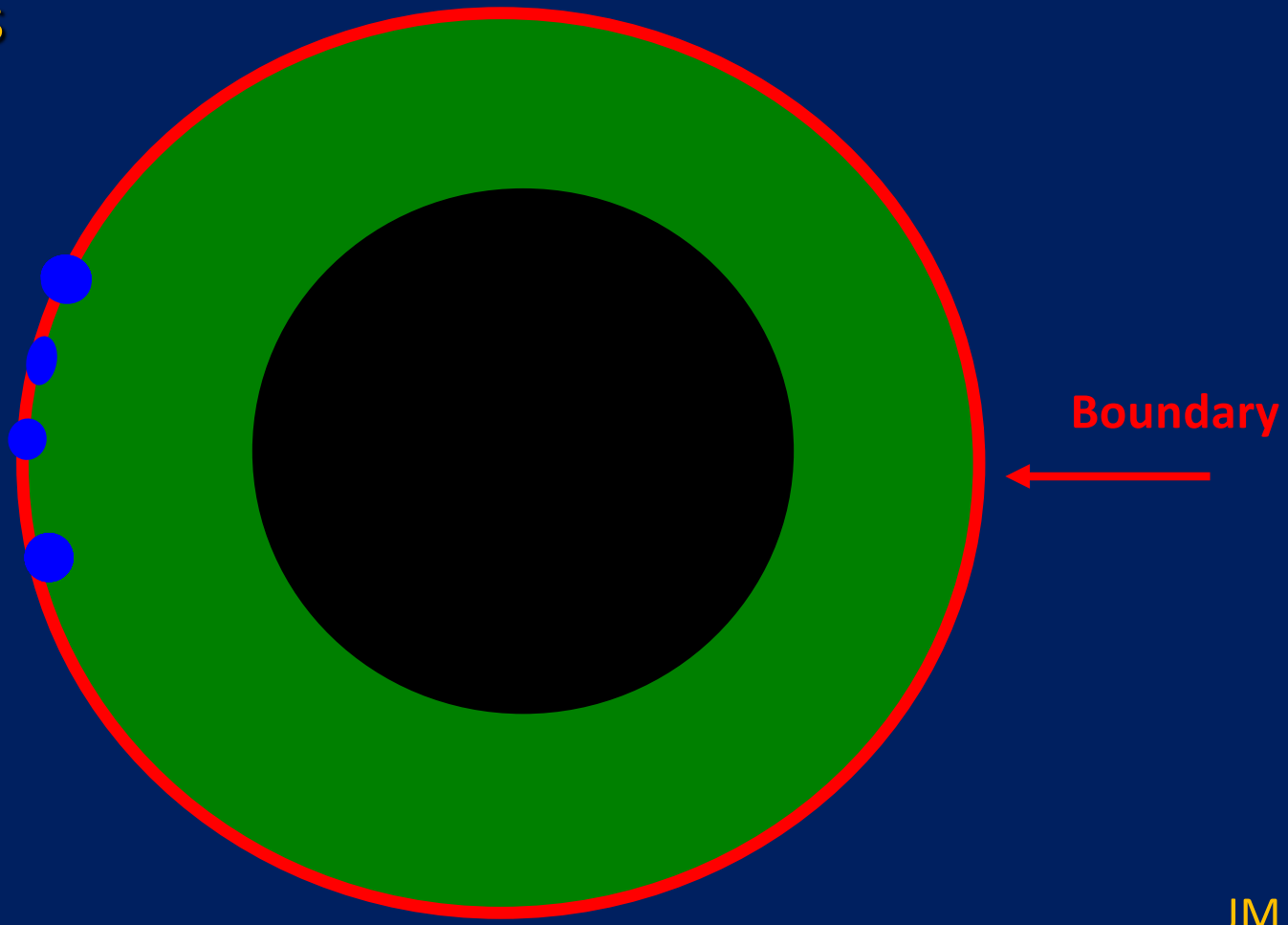


On the boundary of AdS lives a unitary QFT

JM

Black holes in AdS

Temperature and entropy
→ motion of particles
on the boundary
(Witten)



In principle
solves information
puzzle because

JM

BH formation is thermalization of the theory on the boundary.
Hawking conceded that there is in principle no information loss.

How does gravity solve the Information Puzzle?

There is recent progress in answering this question. It involves understanding `Information loss' as `quantum information loss' and uses concepts form quantum information theory. In particular, the concept of Quantum Entanglement and its measure by the fine grained or entanglement entropy (Von Neumann, 1932) and its geometrization.

Work along these lines was initiated by S. Ryu and T. Takayanagi (2006), Hayden and Preskill (2007), Maldacena and Susskind, S. Mathur, A. Almheiri, D. Marolf, J. Polchinski, J. Sully (AMPS), S. Raju, K. Papadodimas, Dong, D. Harlow, N. Engelhardt, A. Wall, T. Hartman and many others.

A good diagnostic is the Page curve (1993) of the time evolution of the entanglement entropy of an entangled composite quantum system. It is a direct consequence of the unitarity of quantum mechanics.

In what follows I will give a rough sketch of recent developments.

Entanglement Entropy in Gravity and Information Puzzle

Entangled states and density matrix

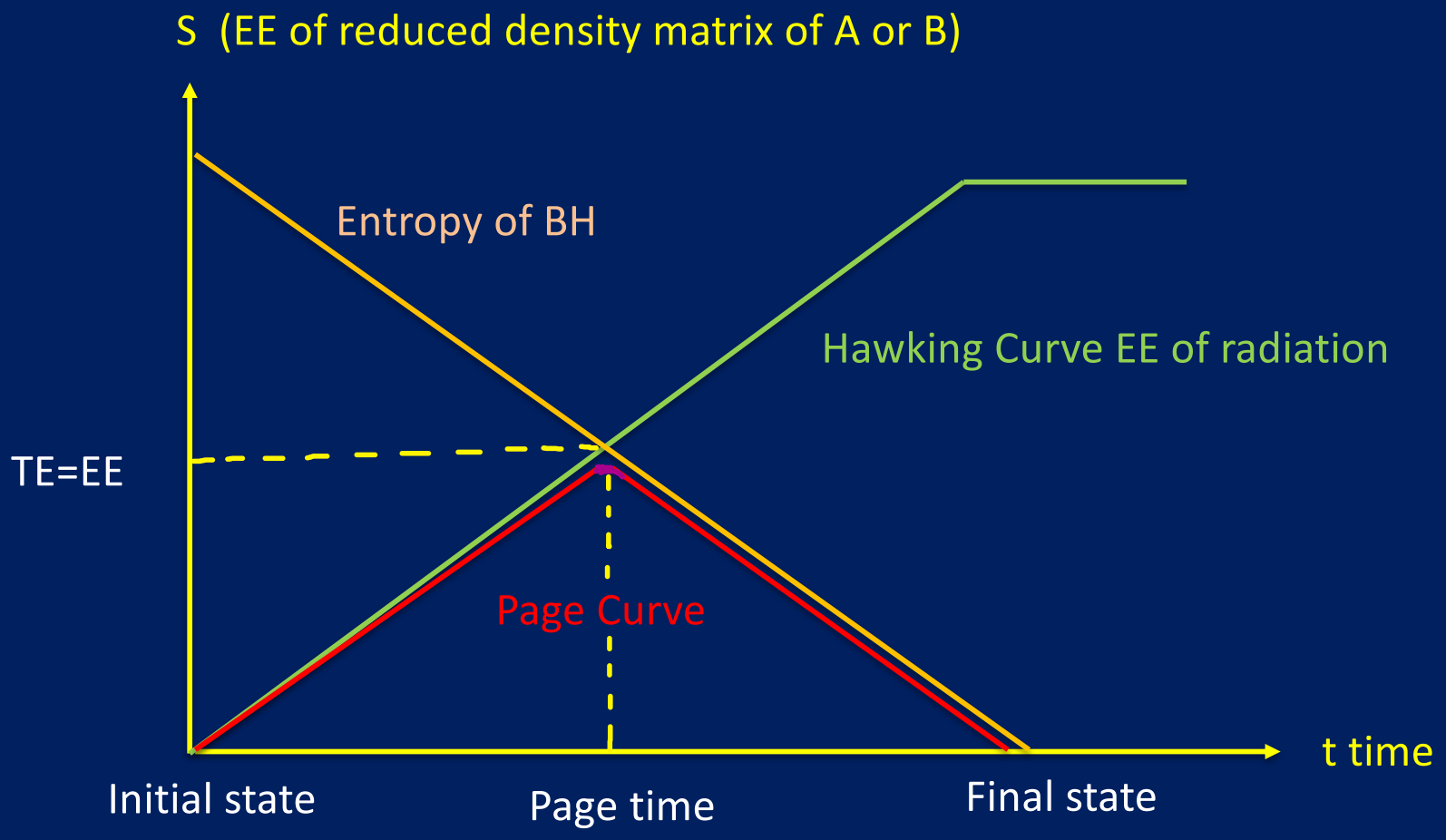
- Entanglement is a property of QM that distinguishes it from classical mechanics and accounts for its very non-intuitive consequences. Here we illustrate this very simple point in the case of two entangles two state systems. It is at the heart of quantum sensing, computing and cryptography.
- A and B are 2 state systems A: $\{|0\rangle_A, |1\rangle_A\}$, B: $\{|0\rangle_B, |1\rangle_B\}$, and C = AxB is the composite system
- We can consider separable product states like $|0\rangle_A \times |1\rangle_B$ in which both systems A and B are in a definite state.
- However, the state $|AB\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \times |1\rangle_B - |1\rangle_A \times |0\rangle_B)$ is not a product of two states. It is an `entangled' state of the system C. It does not belong to A or B.
- A state of a system in QM can be equivalently described by a `density matrix' e.g. $\rho(\mathrm{AxB}) = |AB\rangle\langle AB|$. Since $|AB\rangle$ is normalized we have $\rho(\mathrm{AxB})^2 = \rho(\mathrm{AxB})$, a projection operator.
- In case we do not want to observe system B we trace over all the states of system B. $\mathrm{tr}_B\, \rho(\mathrm{AxB}) = \rho(\mathrm{A})$ is the reduced density matrix of A and it turns out that $\rho(\mathrm{A})^2 \neq \rho(\mathrm{A})$

Entanglement Entropy (Von Neumann, 1932)

- Von Neumann entropy: $S = -tr\rho \ln \rho$ and $tr\rho = 1$
- $S = -tr\rho\ln\rho = 0$, iff ρ is a `pure state'. A non-zero S is a measure of deviation from a maximally entangled pure state. $S(\rho(AxB)) = 0$ but $S(\rho(A)) = \ln 2$
- $S(\rho(A)) = S(\rho(B))$, EE of the reduced density matrices are equal.
- $S(\rho) = S(U^{+}\rho U)$, where U is a unitary matrix. Hence $S(\rho)$ is constant under unitary time evolution.
- $S(\rho) \leqslant \log N$, where N is the dim of the Hilbert space.
  $\log N = S_{th}$ is the thermodynamic entropy, which has no information
- $S_{th} - S_{vn}(\rho) \geqslant 0$, is a measure of information loss.
- For a bipartite system AxB, $S(AUB) \leqslant S(A) + S(B)$, which is very different from the thermodynamic entropy, where the entropy of the combined system is always $\geqslant$ the entropy of the parts.

Page curve as a diagnostic for BH-Radiation entangled system

Imagine evaporating BH is system A and Hawking radiation is system B



Hawking curve violates unitarity Page curve is consistent with Unitarity

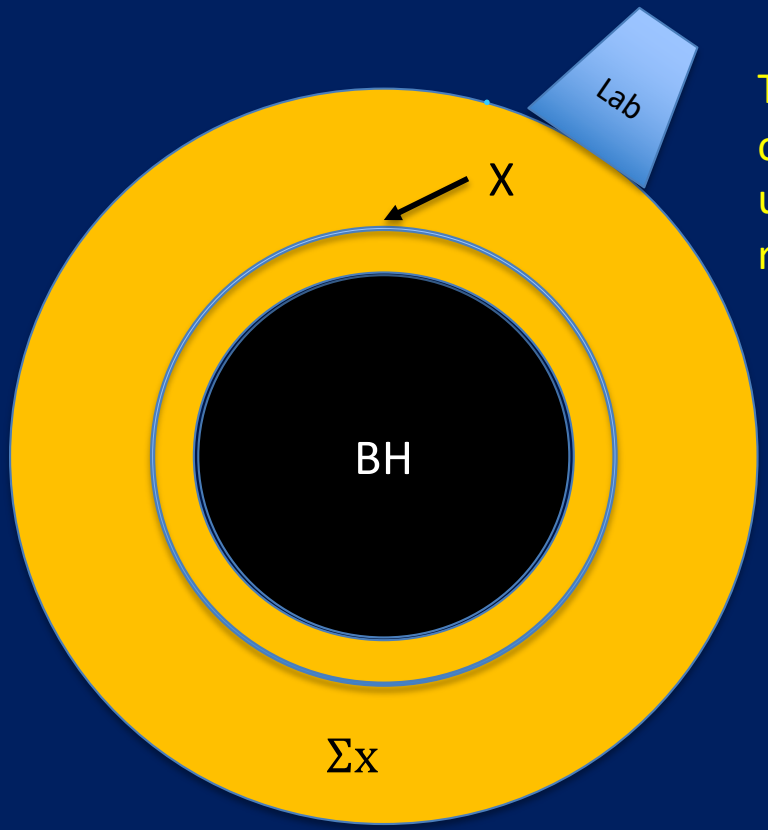
What is the correct EE formula formula in gravity?

It is the same formula as before except the `surface' under consideration is not the horizon!
The `surface' is obtained by looking at the extrema of

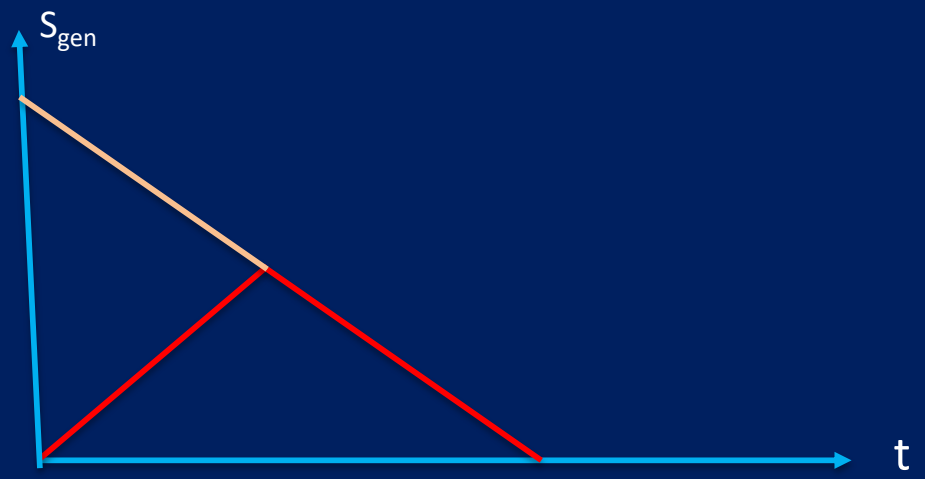
$$S(X(t))_{gen} = \left[\frac{A(X(t))}{4G} + S(\Sigma x(t))_{matter} \right]$$

Pennington (2019)
Almheiri, Engelhardt, Marolf, Maxfield (2019)

where X(t) is a surface or a union of surfaces, $\Sigma x(t)$ is the region bounded by X(t) and the cut-off surface. (If X is the horizon, then we get Hawking's result).



The key point is that X(t) can also be partly or fully inside the horizon and can be a union of surfaces...around islands, as required by the variational principle.

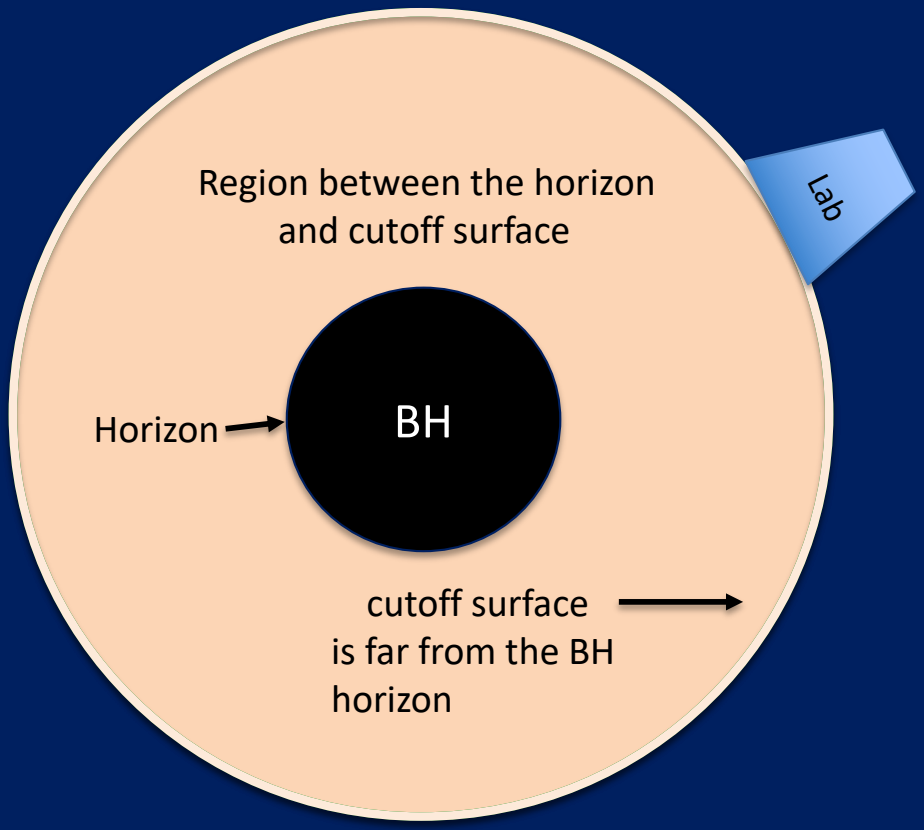


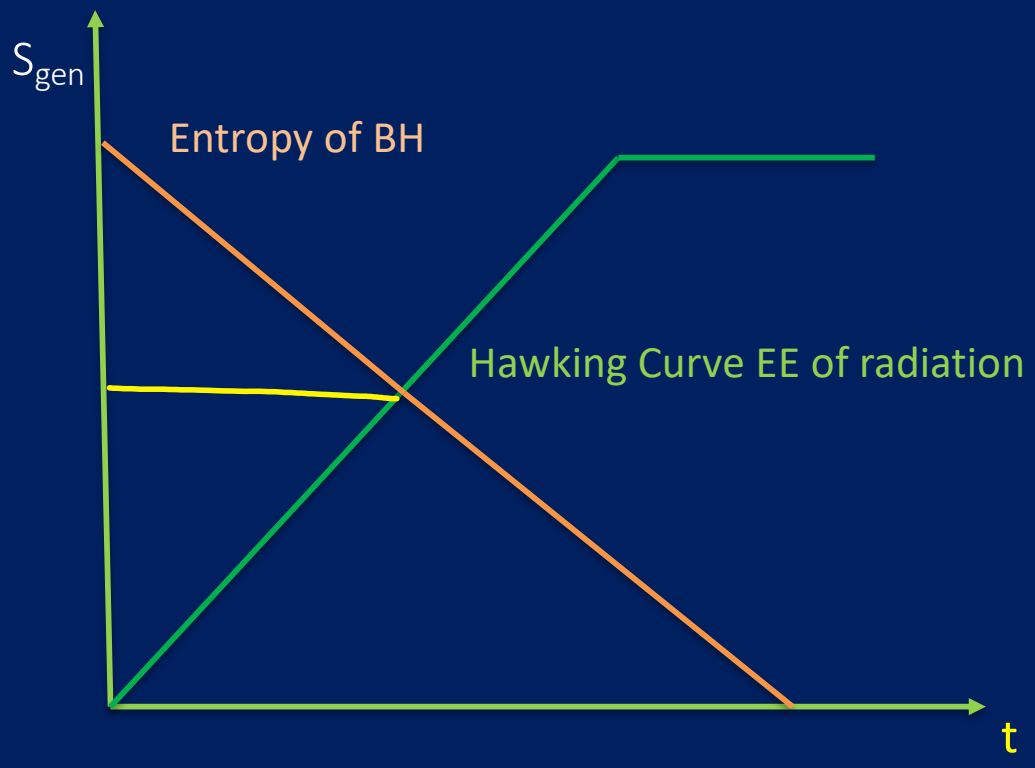
Entanglement Entropy formula that computes the Hawking curve of an evaporating BH

Recall the Bekenstein-Hawking formula for the generalized entropy. It can be shown that it is a formula for EE. The area term is the EE of the Hawking quanta near the inside and outside the horizon (Sorkin 1982) and S_{matter} is the EE of matter/radiation outside the horizon.

$$S_{gen} = \frac{A}{4G} + S_{matter}$$ (units $c=\hbar=1$) (N. Engelhardt and A. Wall, 2015)

(The UV divergences between the two terms cancel, and the Newton constant is finite).





The EE of radiation at the Page time exceeds the the TE of the BH and that is a violation of unitarity

Summary

We have reviewed a particular promising development in theoretical physics which brings us closer to a reconciliation of Quantum Mechanics and General Relativity within semi-classical gravity. This has been enabled by a new gravity formula for the entanglement entropy.

Explicit verification of the Page curve has been done in low dimensional systems.

In this talk I have not included the following topics:

1. Discussion of the interior of the BH and its imprint on the Hawking radiation.

2. Wormhole geometries and entanglement entropy.

3. Explicit dynamical evolution models of BH formation and evaporation in lower dimensional systems like the SYK model of random spins which is holographic to 2-dim gravity.

One of the reasons to study BHs is to understand the principles and degrees of freedom of quantum gravity and develop a framework to discuss the quantum nature of the cosmological event horizon which also has a characteristic temperature and Hawking radiation.

Thank you!