Abelian and non-Abelian gravitating vortices

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The bottom-line at a high level..

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  where m is the maximum multiplicity of the zeroes of ϕ. Method of proof: Continuity method in α but the uniform estimate from the lower boundedness of an energy functional.

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- Bradlow proved that that a stability condition holds iff existence holds.

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- In the usual trivialisation of V on $\mathbb{C}$

$$\frac{\partial}{\partial\bar{z}}\left(H^{-1}\frac{\partial H}{\partial z}\right) = 4^{1/N}\lambda_s(\phi \otimes \phi^{*H} - \tau)e^{-2\alpha\|\phi\|_H^2 - \eta/N}, \tag{5}$$

where $e^{-\eta} = \det(H)$.

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$$\text{near } w = 0, \left\| H^{-1}\frac{\partial H}{\partial z} \right\|_{L^\infty(B(0,1))} \leq C, \left\| H^{-1}\frac{\partial H}{\partial w} - \frac{N}{w} \right\|_{L^\infty(B(0,1))} \leq C.$$

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Thank you