Quantum States under Floquet heating

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2023.6 — 2024.5

Based on

[1] TNI, A. Polkovnikov, Phys. Rev. B 104, 134308 (2021).

[2] TNI, S. Sugiura, A. Polkovnikov, arXiv:2311.16217

Jan. 22, 2024 Stability of Quantum Matter in and out of Equilibrium @ ICTS, India

Floquet theory and experiment in many-body systems

Reviews: Bukov et al,. Adv. Phys. (2015); Oka et al., Ann. Rev. Condens. Matter Phys. (2019)

Floquet quantum systems

$$\frac{d\psi}{dt} = -\frac{i}{\hbar}H(t)\psi$$

Time-Periodic Hamiltonian

$$H(t+T) = H(t)$$

e.g. Effective topological effect by periodic drives

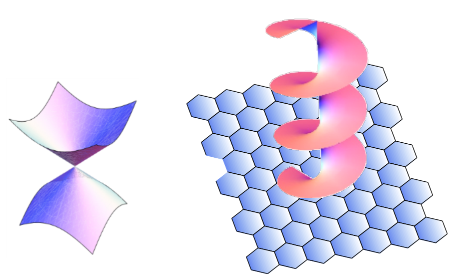
$H(t)$

periodic Hamiltonian

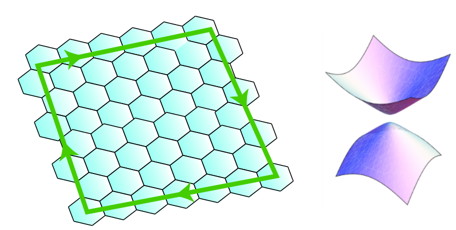
Floquet theory

H_F

effective Hamiltonian



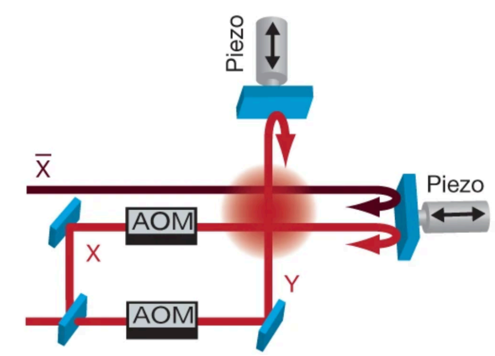
graphene + circ. pol. laser



topological insulator

Oka and Aoki, PRB (2009); Kitagawa, Oka et al. PRB (2011)

Cold-atom realization



Jotzu et al. Nature (2014)

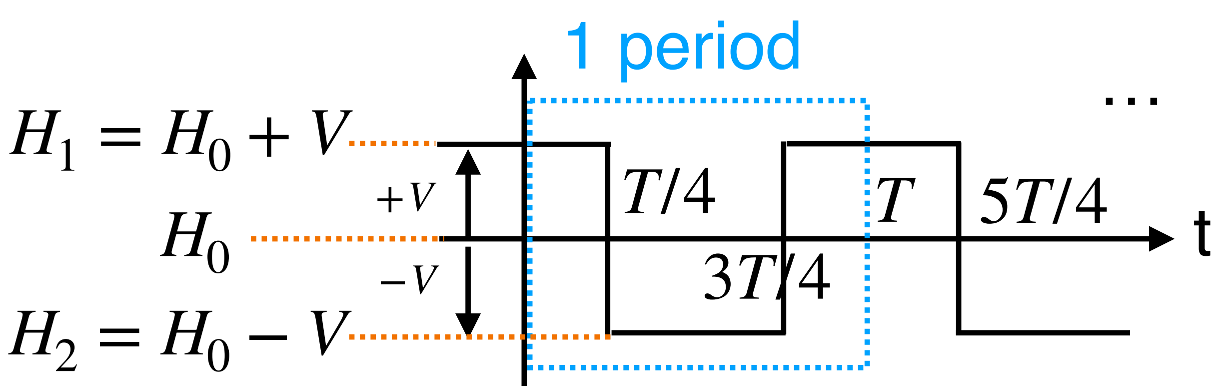
Effective Hamiltonian H_F in isolated many-body systems

For simplicity, we consider stroboscopic evolution (i.e., discrete times $t = kT$)

Exact effective Hamiltonian H_F^{ex}

Let U denote the one-cycle unitary: $|\Psi_k\rangle = U^k |\Psi_0\rangle$

e.g.



$$U = e^{-iH_1T/4}e^{-iH_2T/2}e^{-iH_1T/4}$$
$$=: e^{-iH_F^{\mathrm{ex}}T}$$

H_F^{ex} is exact but complex (non-local)

(high-frequency) effective Hamiltonian H_F ($\neq H_F^{\mathrm{ex}}$)

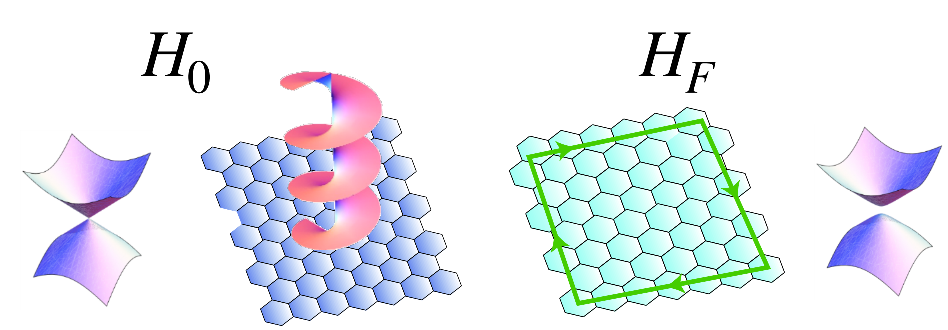
(high frequency = small T)

$$U = e^{-iH_1T/4}e^{-iH_2T/2}e^{-iH_1T/4}$$

BCH series terminated at some order:

$$U \approx U_F = e^{-iH_FT}$$

$$H_F = H_0 - \frac{T^2}{24 \cdot 2^3}[[H_2, H_1], H_1 + 2H_2] + CT^4 + DT^6 + \cdots$$



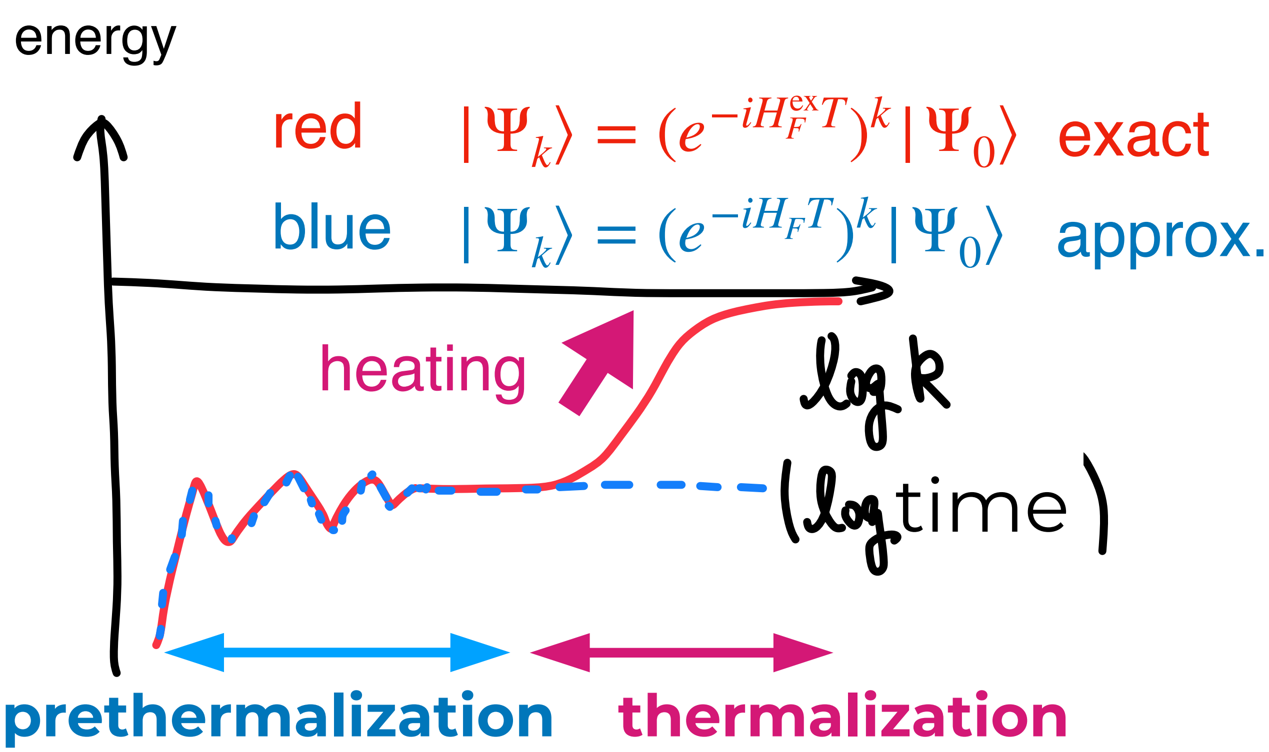
BCH (Magnus) expansion is an asymptotic series.

Kuwahara, Mori, Saito, Ann. Phys. (2016)

H_F is approximate but local

Floquet heating | Consequence of $H_F \neq H_F^{\mathrm{ex}}$

Floquet Prethermalization & Thermalization

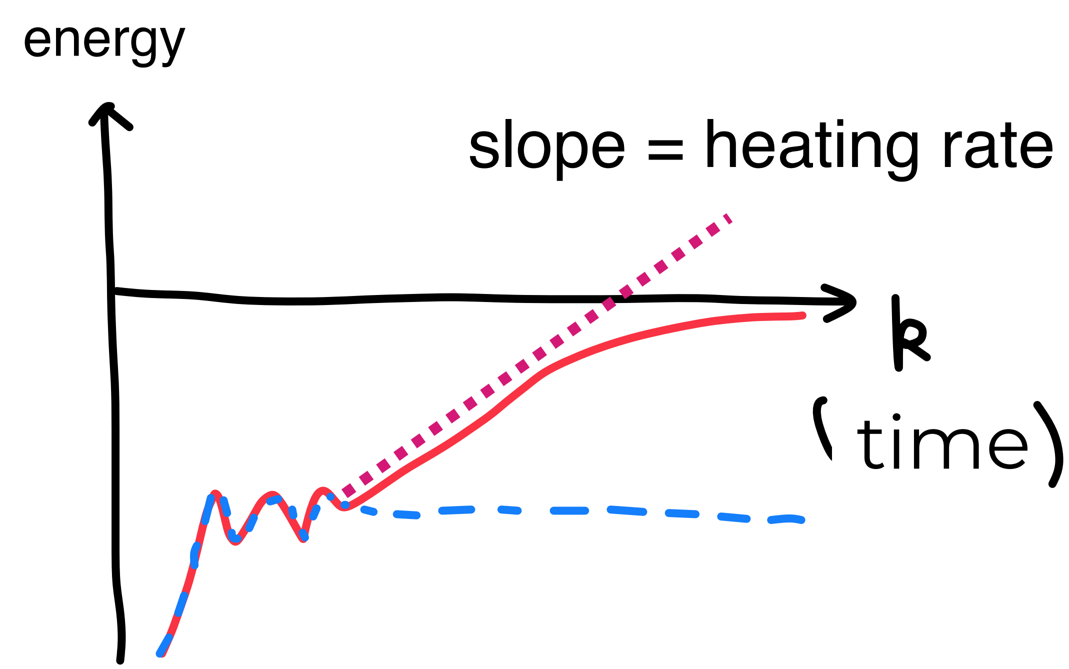


Kuwahara et al., Ann. Phys. (2016)
Abanin et al., PRL (2015)

Lazarides et al. PRE (2014)
D'Alessio et al. PRX (2014)
Kim et al., PRE (2014)

FE survives

FE broken?



Kuwahara et al., Ann. Phys. (2016)
Abanin et al., PRL (2015)
WW Ho et al., Ann Phys (2023)

Upper bound heating rate $< e^{-O(\omega)}$

Questions and our answers

States during Floquet heating? ➡ $e^{-\beta(t)H_F}/Z_t$ **FE survives at finite temperature**

Can we obtain heating rates instead of their upper bounds?

➡ **Yes by Fermi's golden rule (FGR)**

Review | (bare) FGR for small-amplitude drives

Mallayya and Rigol, PRL 123, 240603 (2019)

$$H(t) = H_0 + g(t)V$$

Assumptions

Amplitude $g(t+T) = g(t)$ is small.

H_0 is nonintegrable

Energy increase rate based on time-dependent perturbation theory and Fermi's golden rule (FGR)

$$\dot{E}(t) = 2\pi \sum_{m>0} g_m^2 \sum_{i,j} |\langle E_f^0 | V | E_i^0 \rangle|^2 (E_f^0 - E_i^0) P_i^0(t)$$

where $g(t) = \sum_{m>0} 2g_m \sin(m\omega t)$

State ansatz: Thermal state w.r.t. H_0 with a single parameter (time-dependent temperature)

$$P_i^0(t) = \langle E_i^0 | \rho_{\mathrm{GE}}(t) | E_i^0 \rangle \qquad \rho_{\mathrm{GE}}(t) = e^{-\beta(t)H_0}/Z_t^0$$

Self-consistent evolutions for energy and temperature increases

=> well reproduce the direct simulation of Floquet heating dynamics

Remaining issues

What about large amplitudes, for which $H_F \neq H_0$?

This case is relevant in the Floquet-engineering viewpoint.

How to deal with two effects (dressing $H_0 \to H_F$ and heating)?

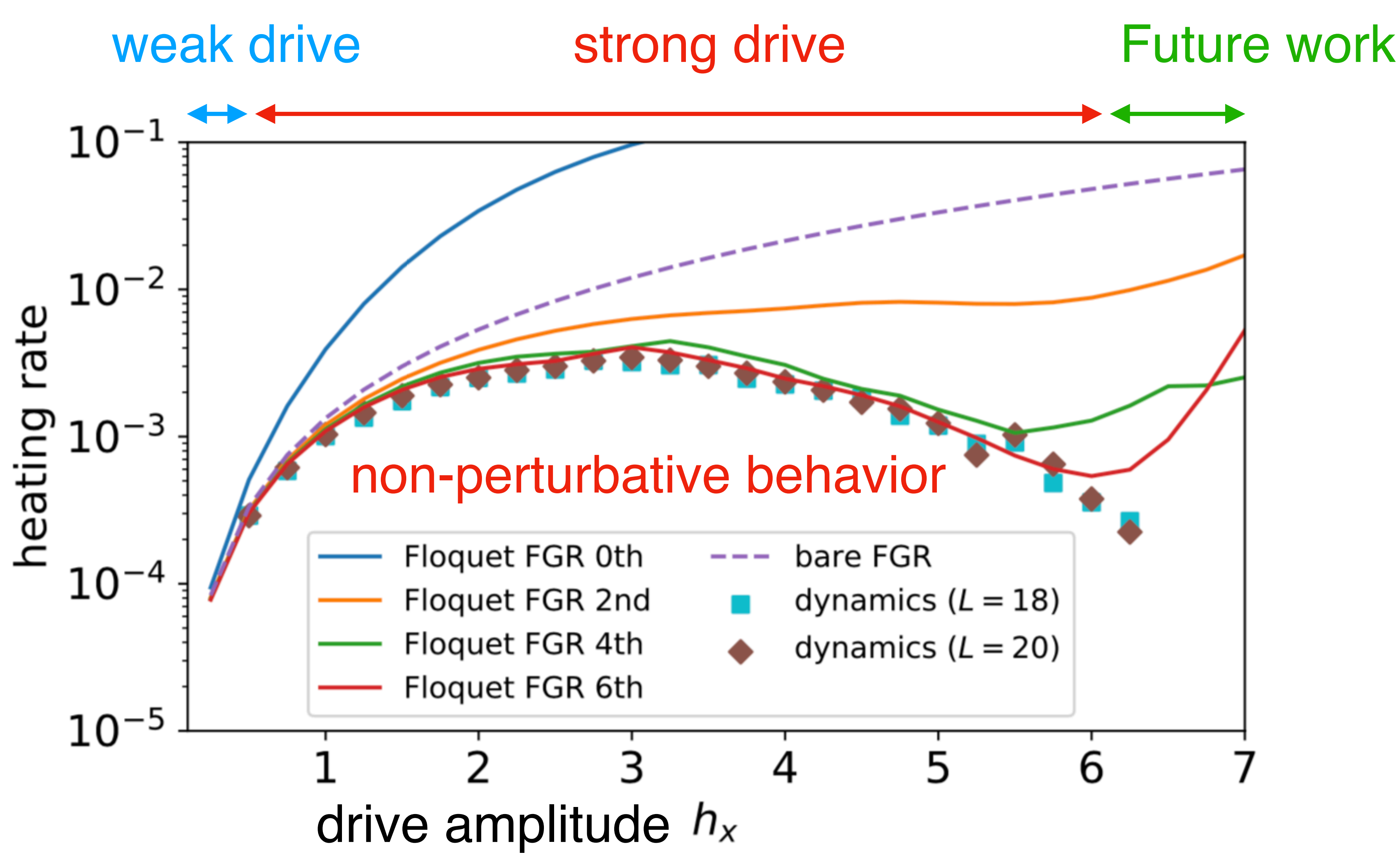
Outline of our results

TNI, A. Polkovnikov, Phys. Rev. B 104, 134308 (2021).

FGR description generalized to strong (high-frequency) Floquet drives

Fermi's Golden Rule = Floquet FGR

(Floquet FGR reduces to the previous work in the weak amplitude limit)



Floquet Fermi's golden rule (FGR)

H_F : obtained by high-frequency expansion terminated at some order

State ansatz: diagonal in eigenbasis of H_F $\quad \rho(t) = \sum_n P_n(t) \left| n \right\rangle \left\langle n \right|$

$$H_F \left| n \right\rangle = E_n \left| n \right\rangle$$
$$U_F \left| n \right\rangle = e^{-i\theta_n} \left| n \right\rangle$$

We assume H_F is non-integrable and further assume an effective temperature $\beta(t)$

$$P_n(t) = e^{-\beta(t)E_n}/Z_t$$

Master equation among eigenstate populations（see our paper for derivation）

$$\frac{dP_n(t)}{dt} = \sum_m \left[w_{m\to n} P_m(t) - w_{n\to m} P_n(t) \right],$$

$$\delta U \equiv U_F^\dagger U = e^{iH_F^{\mathrm{ex}}T} e^{-iH_F T} \neq I$$

$$w_{m\to n} = 2\pi \sum_{l\in\mathbb{Z}} \delta(E_n - E_m - l\omega) \left| \langle n | \delta U | m \rangle \right|^2$$

(this reduces to the previous work [Mallayya and Rigol 2019] when amplitude is weak.)

Self-consistent equations for the increments for $\beta(t)$ and $\langle H_F \rangle_t$

Floquet heating in spin chains | weak drive

L-site spin chain, PBC, initial state = thermal pure state, Krylov evolution method

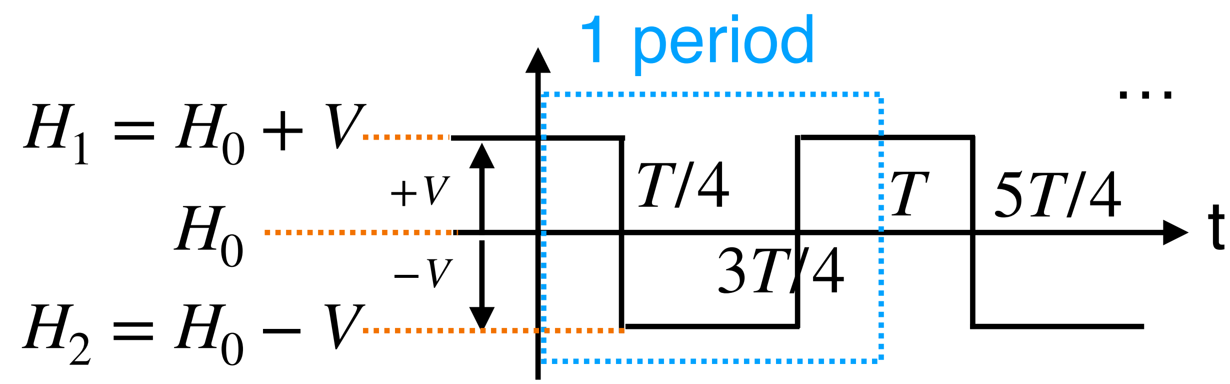
Sugiura & Shimizu (2012)

Machado et al. PRResearch (2020)

$$H_0 = J\sum_i \sigma_i^z\sigma_{i+1}^z + J'\sum_i \sigma_i^z\sigma_{i+2}^z + h_z\sum_i \sigma_i^z + J_x\sum_i \sigma_i^x\sigma_{i+1}^x$$

$V = h_x\sum_i \sigma_i^x$ h_x =driving amplitude

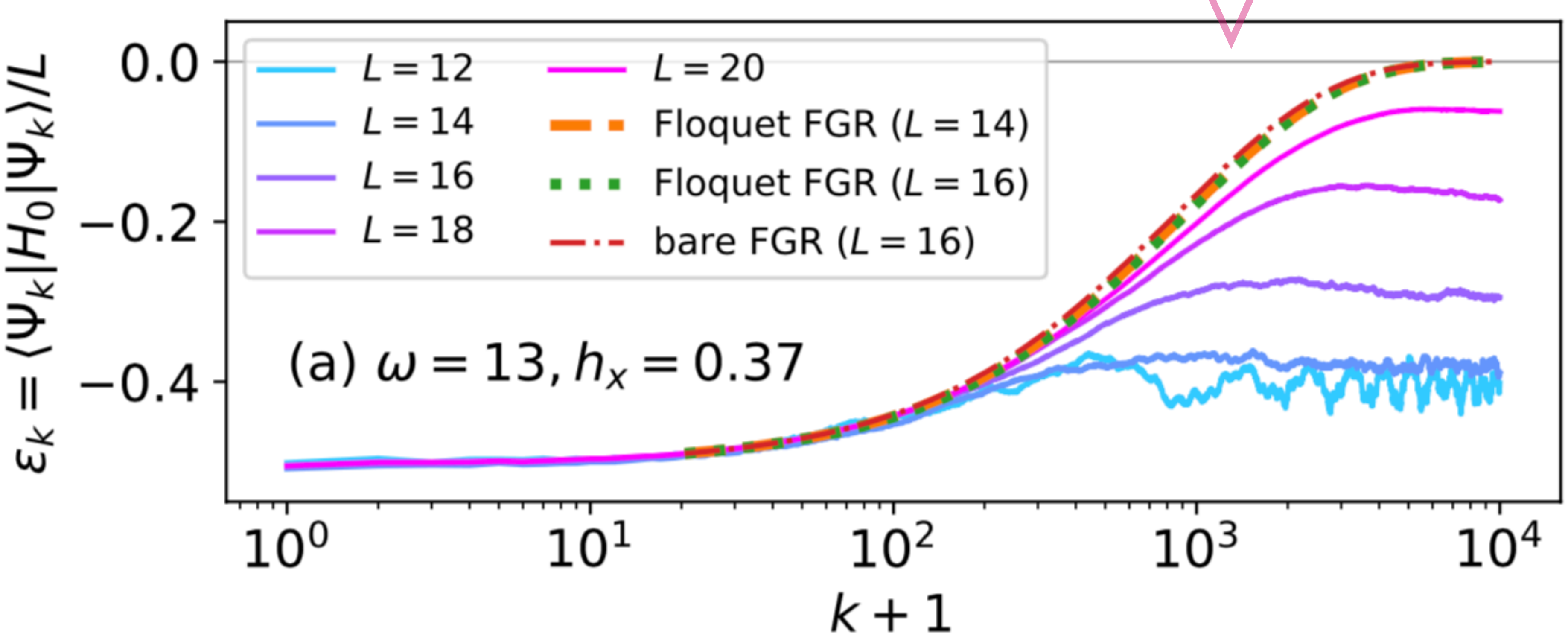
$(J = -1,\ J' = -0.4,\ h_z = 0.6,\ J_x = 0.75)$



Floquet heating simulation requires large L. $L = 20 \Leftrightarrow D = 2^L \approx 10^6$

Bare/Floquet FGR give same result for weak drive (hx=0.37).

FGR at small size (L=14) captures the thermodynamic limit.



H_F is 6th order in T

Floquet heating in spin chains | strong drive

L-site spin chain, PBC, initial state = thermal pure state, Krylov evolution method

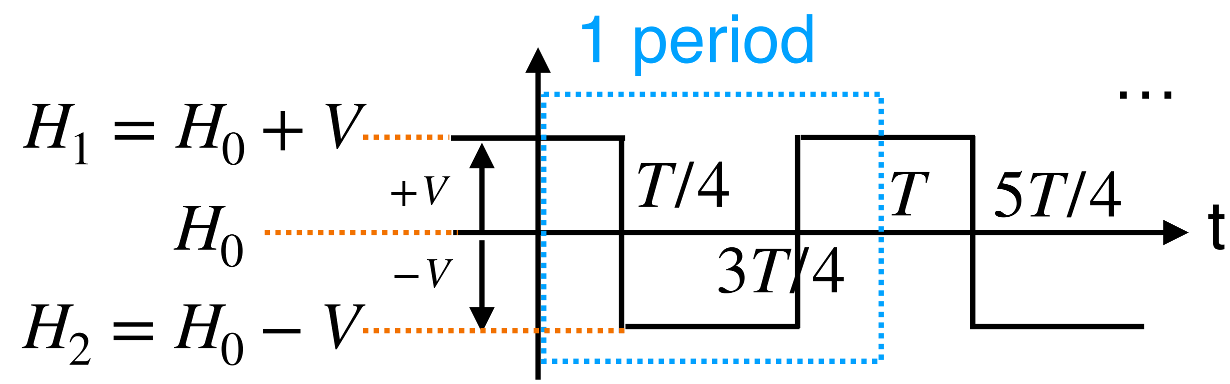
Sugiura & Shimizu (2012)

Machado et al. PRResearch (2020)

$$H_0 = J\sum_i \sigma_i^z\sigma_{i+1}^z + J'\sum_i \sigma_i^z\sigma_{i+2}^z + h_z\sum_i \sigma_i^z + J_x\sum_i \sigma_i^x\sigma_{i+1}^x$$

$V = h_x\sum_i \sigma_i^x$ h_x =driving amplitude

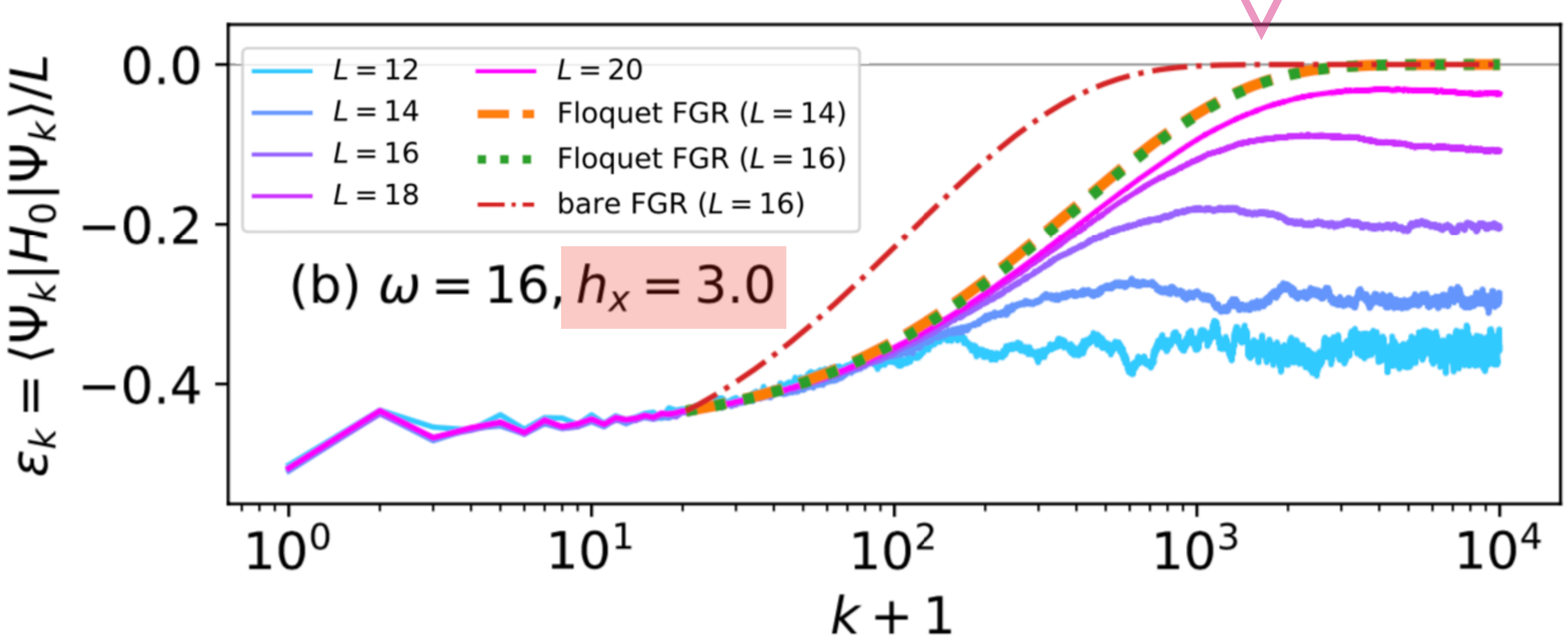
$(J = -1,\ J' = -0.4,\ h_z = 0.6,\ J_x = 0.75)$



Floquet FGR works unlike bare FGR for strong drive (hx=3.0).

Floquet FGR allows us to know thermodynamic limit only at L=14 calculations.

Slight overestimate could be improved by using higher-order approx. for H_F



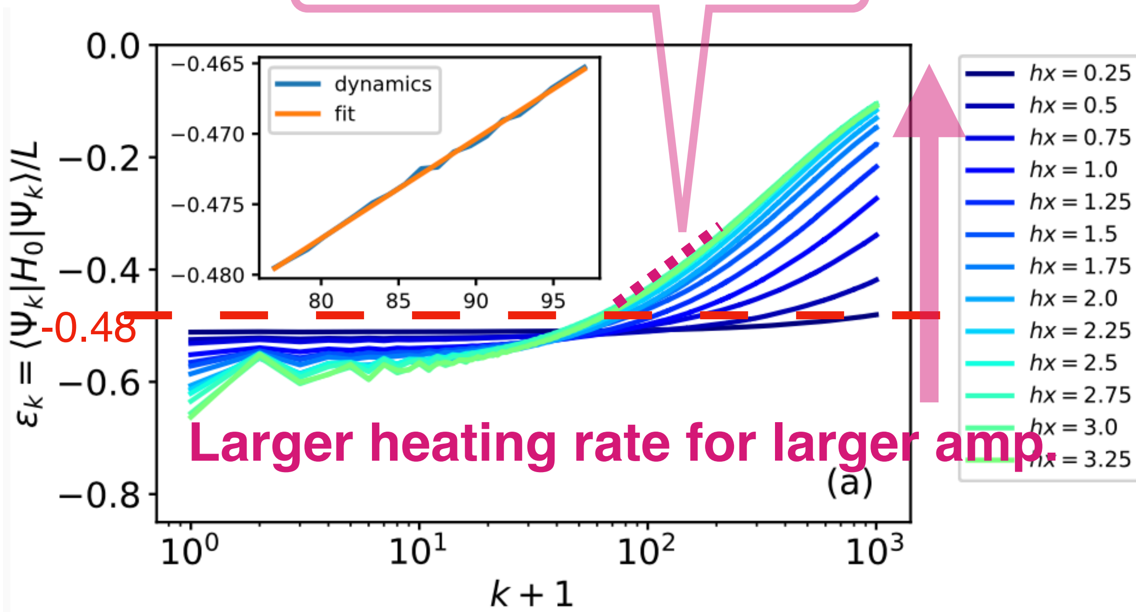
H_F is 6th order in T

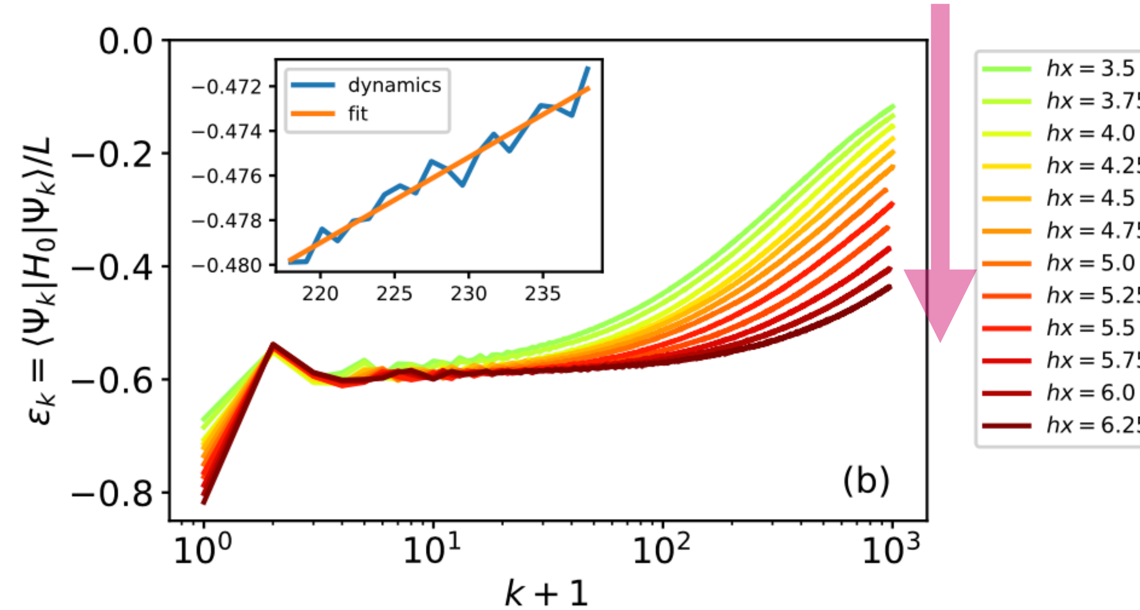
Importance of $H_F \neq H_0$ | non-monotonic heating rate

$\omega = 2\pi/T = 16$

slope = heating rate

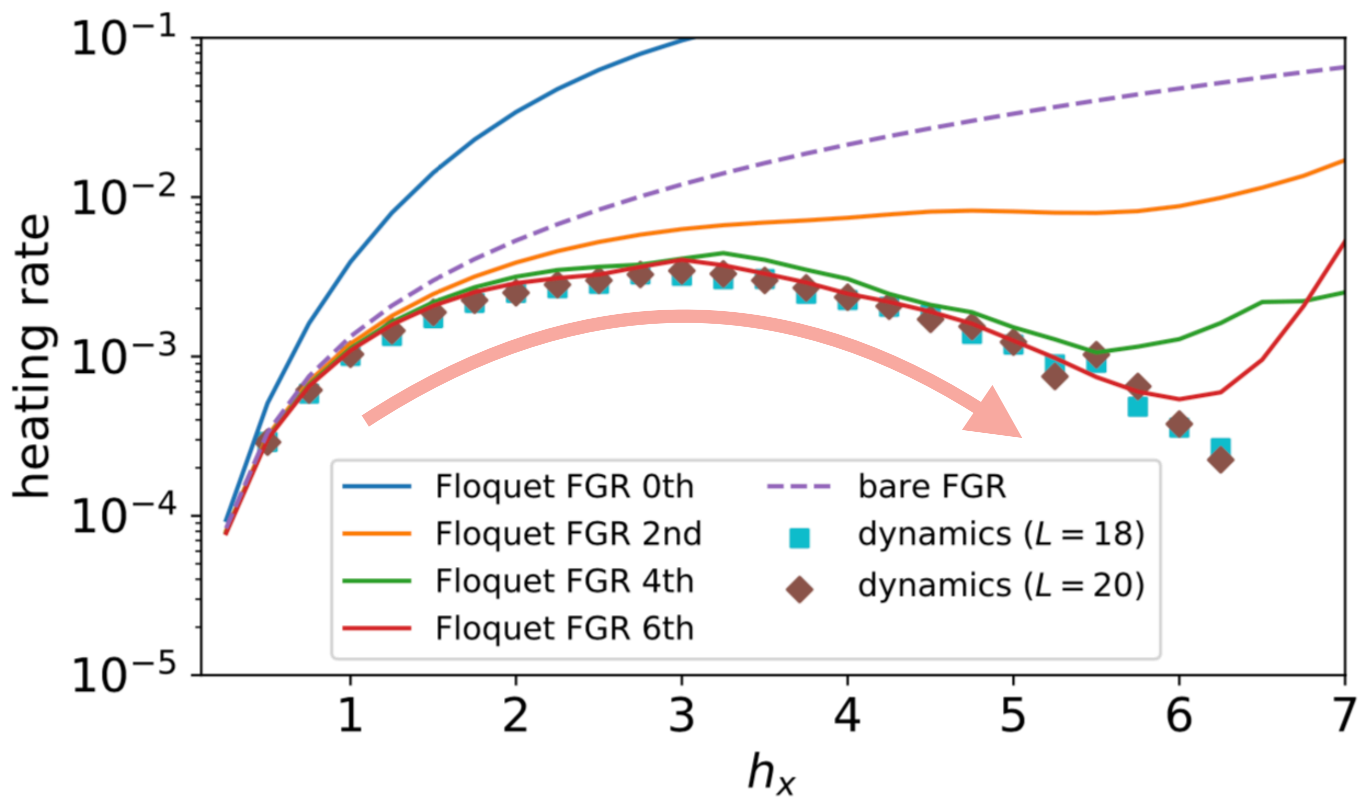
Smaller heating for larger amp.





cf. Das, PRB (2010); Haldar et al. PRB (2018)

Data points = heating rate extracted from least square fits



bare FGR $\propto h_x^2$

Floquet FGR reproduces non-monotonic behavior when H_F is calculated up to T^4, T^6

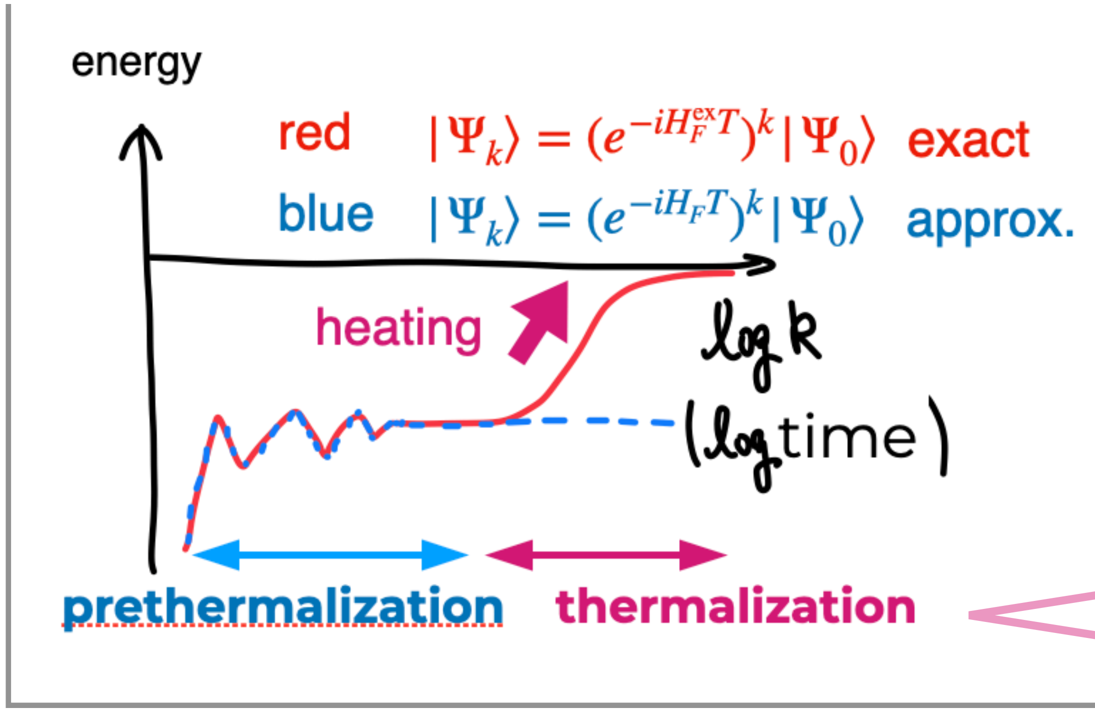
BCH series for H_F does not work for $h_x \gtrsim 6$ (at $\omega = 16$). Need other approaches.

see also Mori, PRL (2021)

Summary of Part 1

TNI, A. Polkovnikov, Phys. Rev. B 104, 134308 (2021).

Floquet heating in many-body systems



Floquet FGR description

Simple view of states during heating

Instantaneous thermal state w.r.t. H_F

$$\rho(t) = e^{-\beta(t)H_F}/Z_t$$

$$\frac{dP_n(t)}{dt} = \sum_m \left[w_{m\to n}P_m(t) - w_{n\to m}P_n(t)\right],$$

$$w_{m\to n} = 2\pi \sum_{l\in\mathbb{Z}} \delta(E_n - E_m - l\omega)\,|\langle n|\delta U|m\rangle|^2$$

- Floquet FGR tells us the TD limit at small-size calculations

- Floquet engineering (FE) makes sense even after heating sets in (finite-temp. FE).

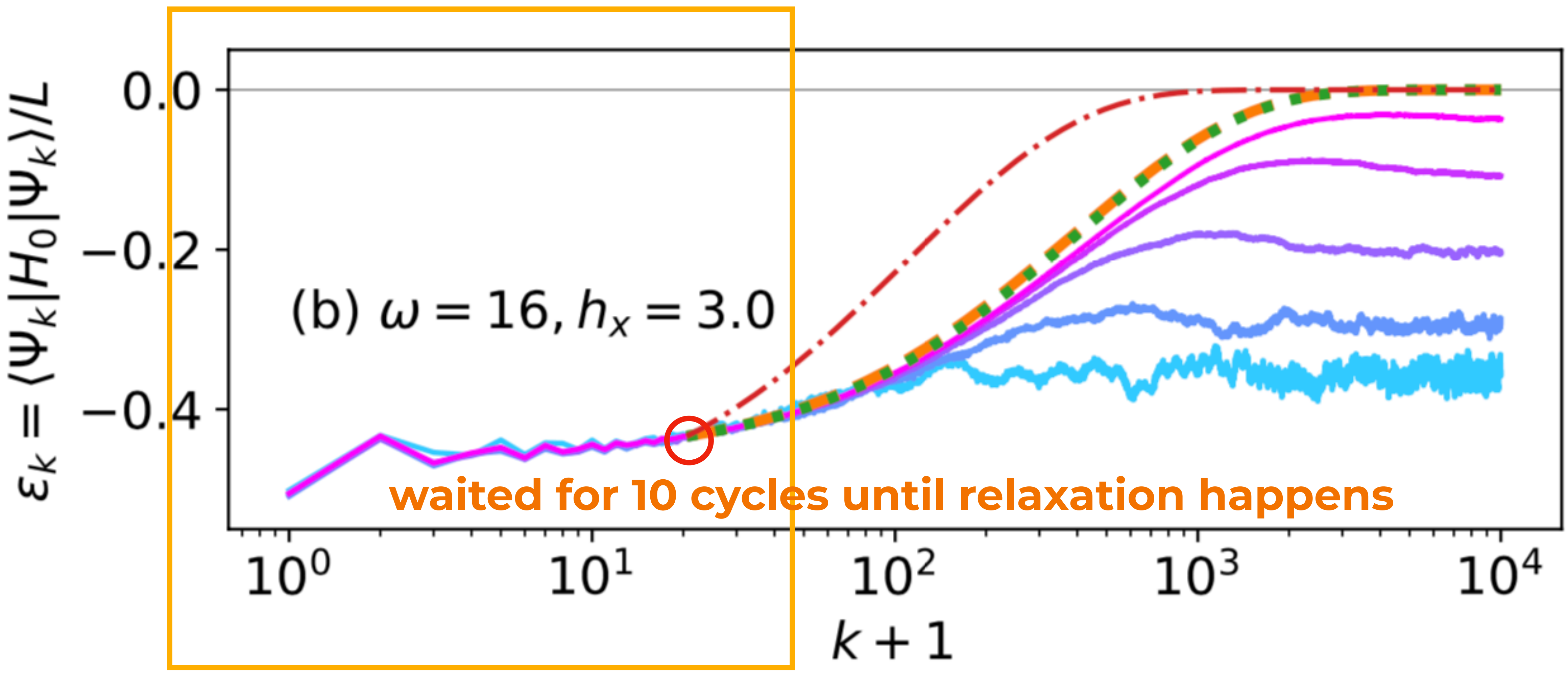
Outlook

Other ways to compute H_F used in Floquet FGR.

H_F has been assumed to be non-integrable. Generalizations to integrable ones?

Prerequisite for the Floquet FGR description

Another observation:
The initial state must be relaxed at high-enough temperature.



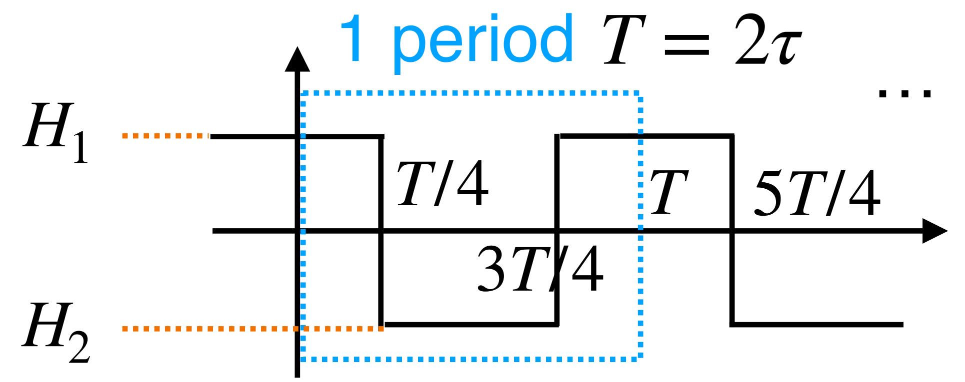
The ground state of the effective Hamiltonian could be much more robust against heating than expected from FGR.

Possible Trotter transition in a nonintegrable model

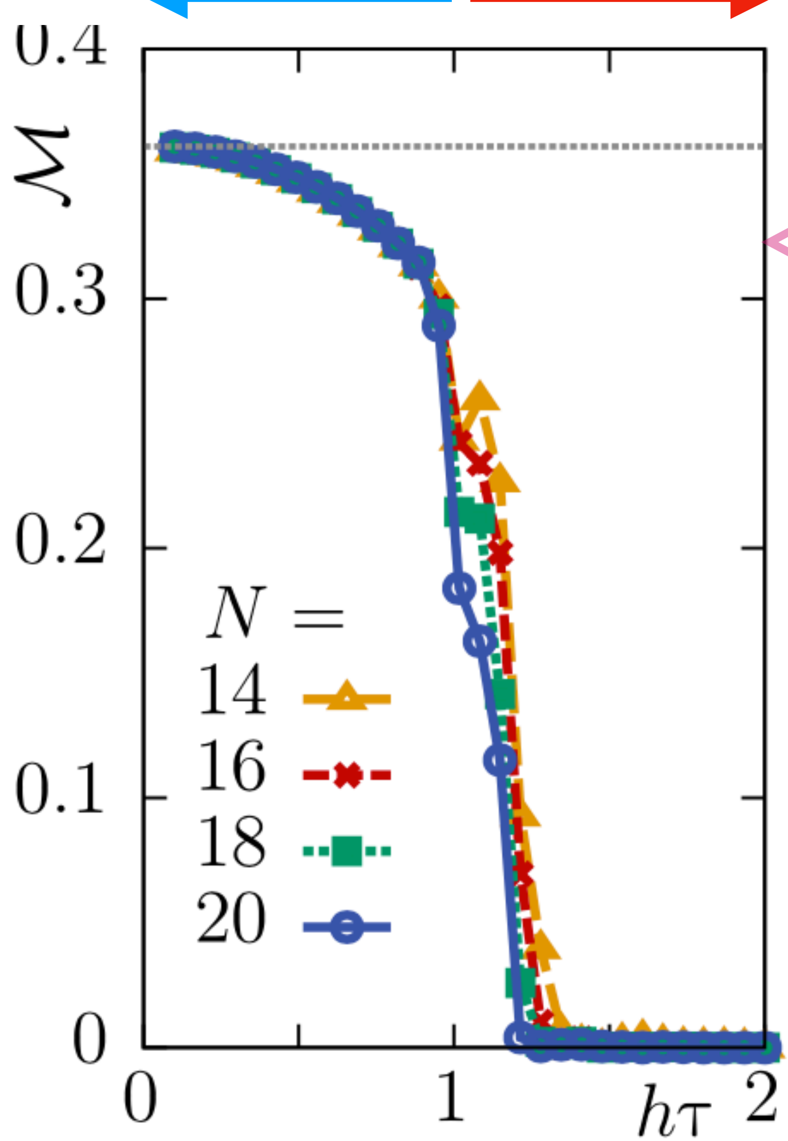
Heyl et al. Sci. Adv. (2019)

$$H_1 = -\sum_{j=1}^{L}\left(\frac{J}{4}Z_jZ_{j+1} + \frac{h}{2}Z_j\right),\ H_2 = -\frac{g}{2}\sum_{j=1}^{L}X_j$$

$$(J = g = h = 1)$$



non-heating **heating**



driving period ~ 1/(driving frequency)

$|\psi_0\rangle = |\uparrow\uparrow\cdots\uparrow\rangle$

Time(cycle)-averaged magnetization

Questions

Consistency with Floquet ETH?

Initial-state dependence of heating?

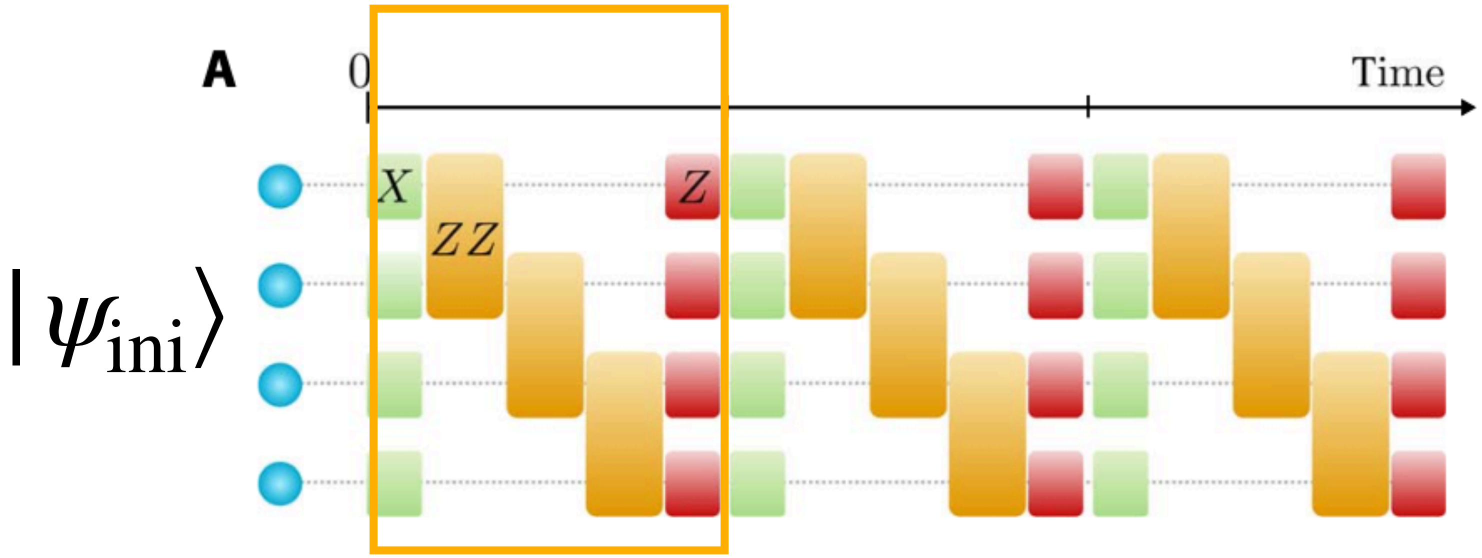
Implying robustness of effective ground state?

$$H_F = \frac{H_1 + H_2}{2} + O(\tau^2) \qquad \langle\psi_0|H_F|\psi_0\rangle = \text{small}$$

See also Prosen JPhysA 2007, D'Alessio et al. Ann Phys 2013 for non-Trotterized models

Method | Real-time evolution on quantum circuit simulator

$T(\tau) = \prod \text{gates}(\tau)$ One-cycle Floquet unitary = Trotter unit



Heyl et al. Sci. Adv. (2019)

$T(\tau)^n |\psi_{\text{ini}}\rangle$ obtained by

Efficient circuit simulator (Qulacs): Suzuki et al. Quantum (2021).

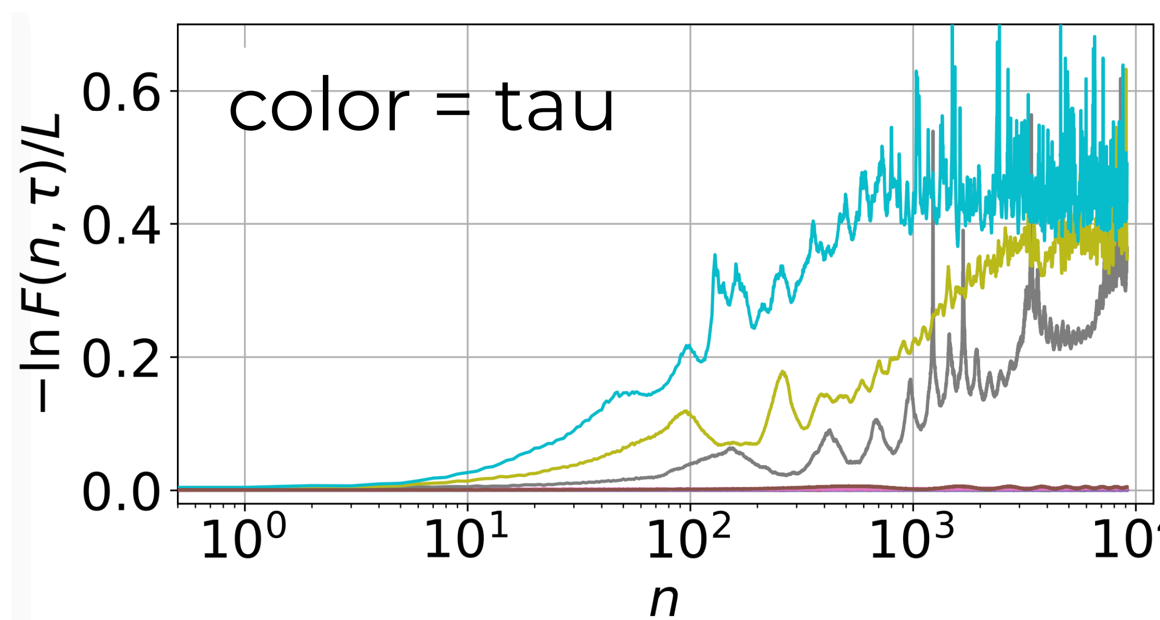
Faster than generic Krylov evolution.

Our robustness measure

Fidelity $F(n, \tau) = |\langle\psi_{\text{ini}}|T(\tau)^n|\psi_{\text{ini}}\rangle|^2$

Normalized ver. $s = -\dfrac{\ln F(n, \tau)}{L}$

Example data: L=30

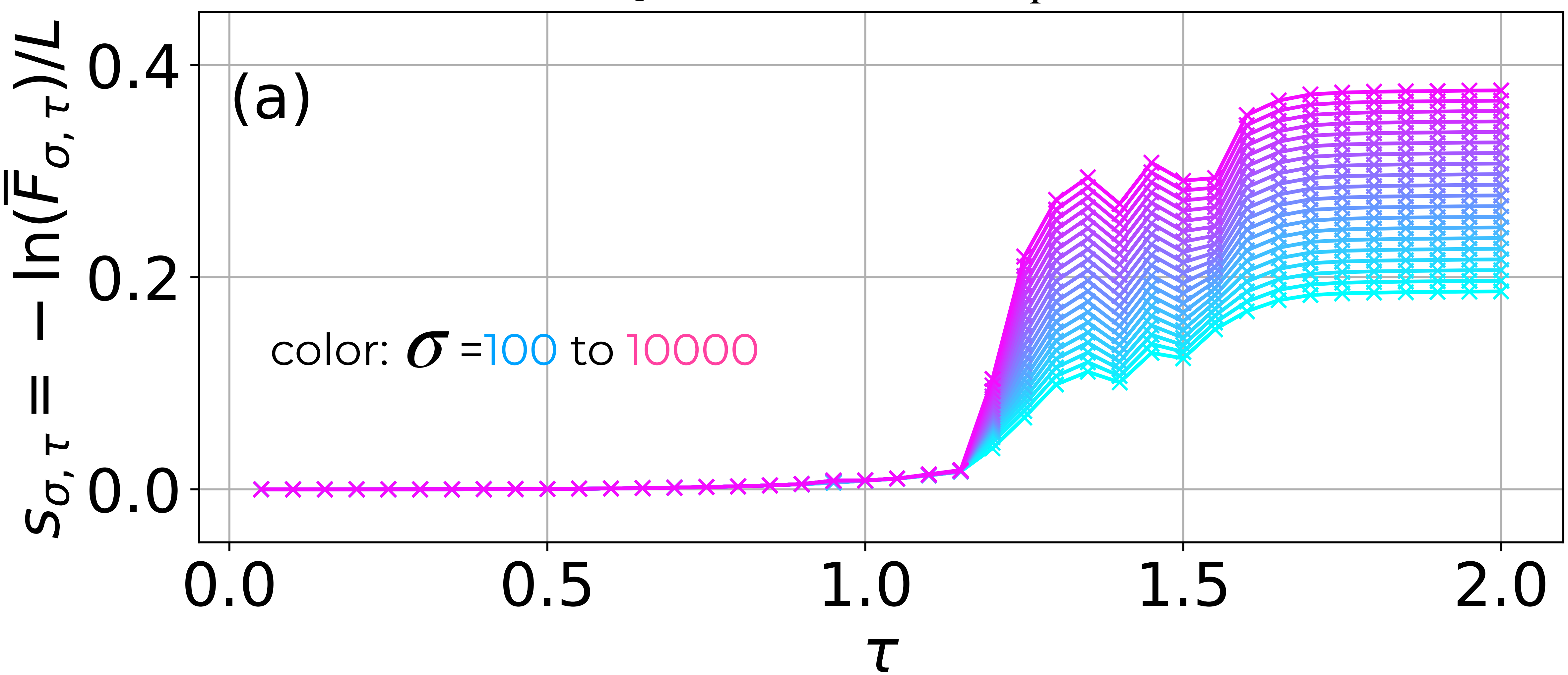


Result | (Possible) Trotter transition for the ground state

Average over $\sim\boldsymbol{\sigma}$ steps $$\overline{F}_{\sigma,\tau} = \frac{1}{\mathcal{N}_\sigma}\sum_{n=0}^{\infty} F(n,\tau)e^{-(n/\sigma)^2}$$

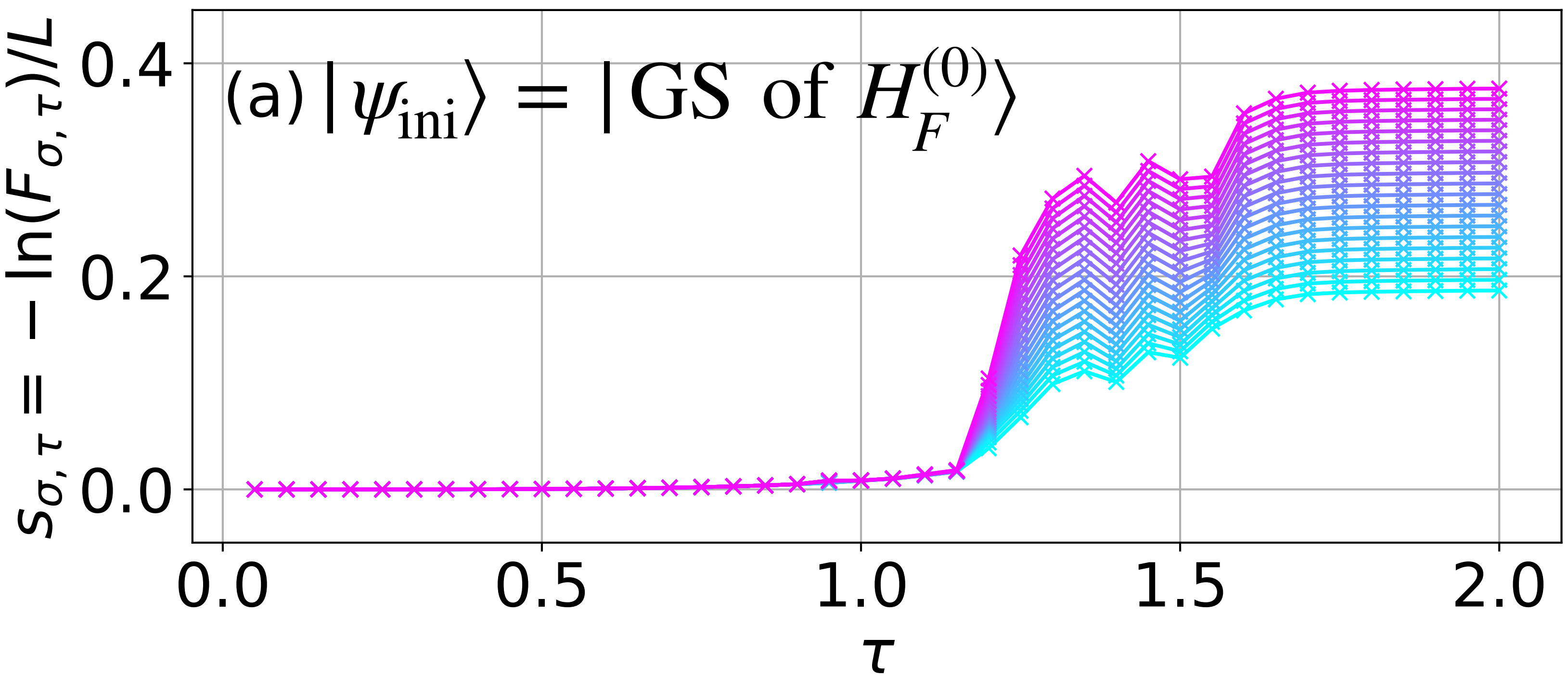
normalized $$s_{\sigma,\tau} = -L^{-1}\ln(\overline{F}_{\sigma,\tau})$$

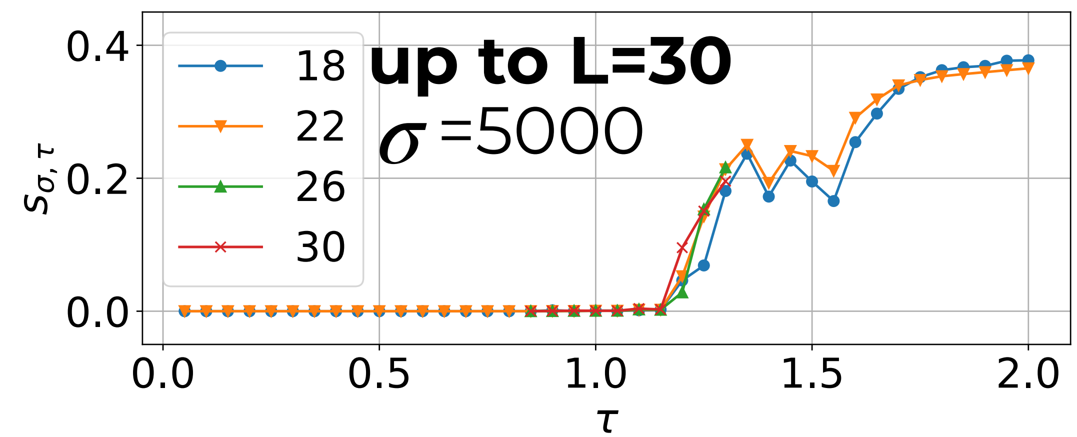
L=24, initial state = ground state of $H_F^{(0)} = (H_1 + H_2)/2$

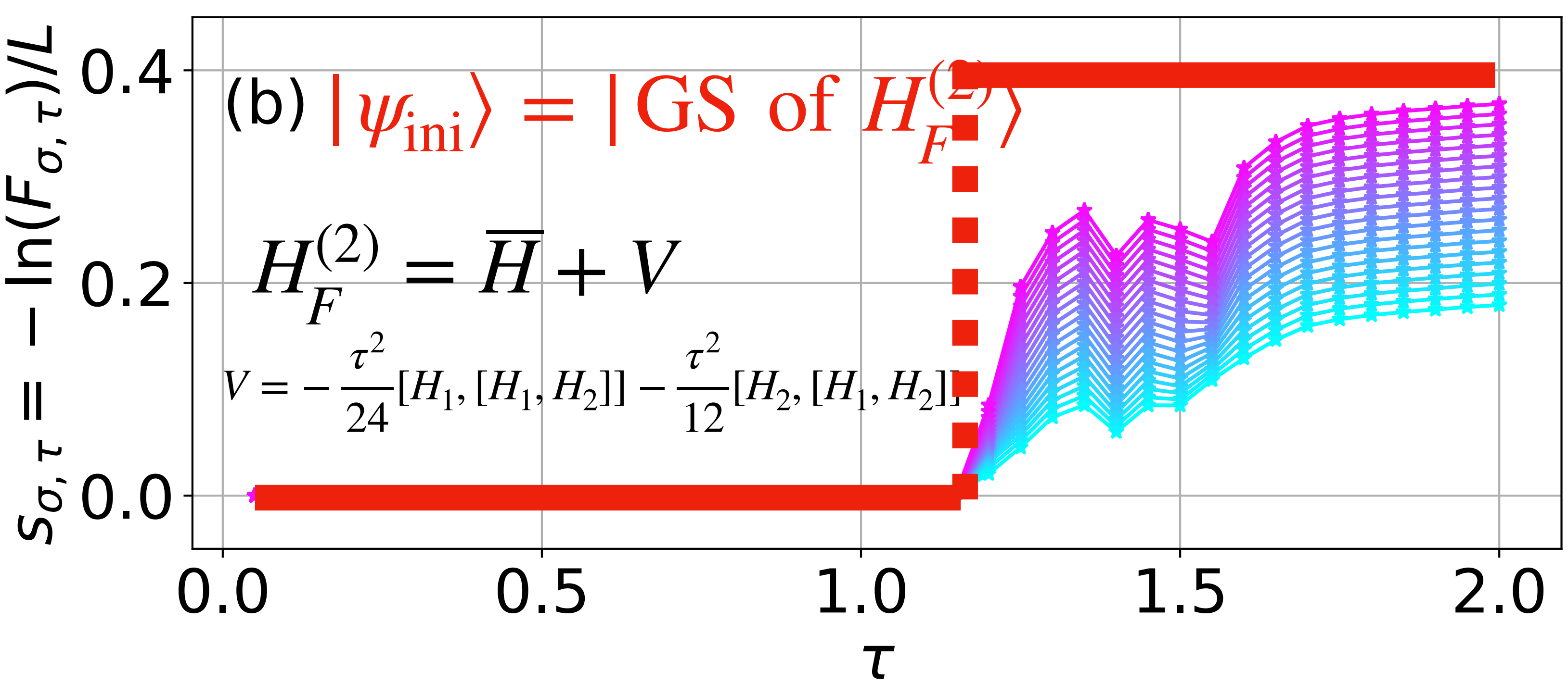


Clearer Trotter transition for the effective ground state

L=24 color: σ =100 to 10000







Conjecture

Trotter transition exists for the GS in the sense of

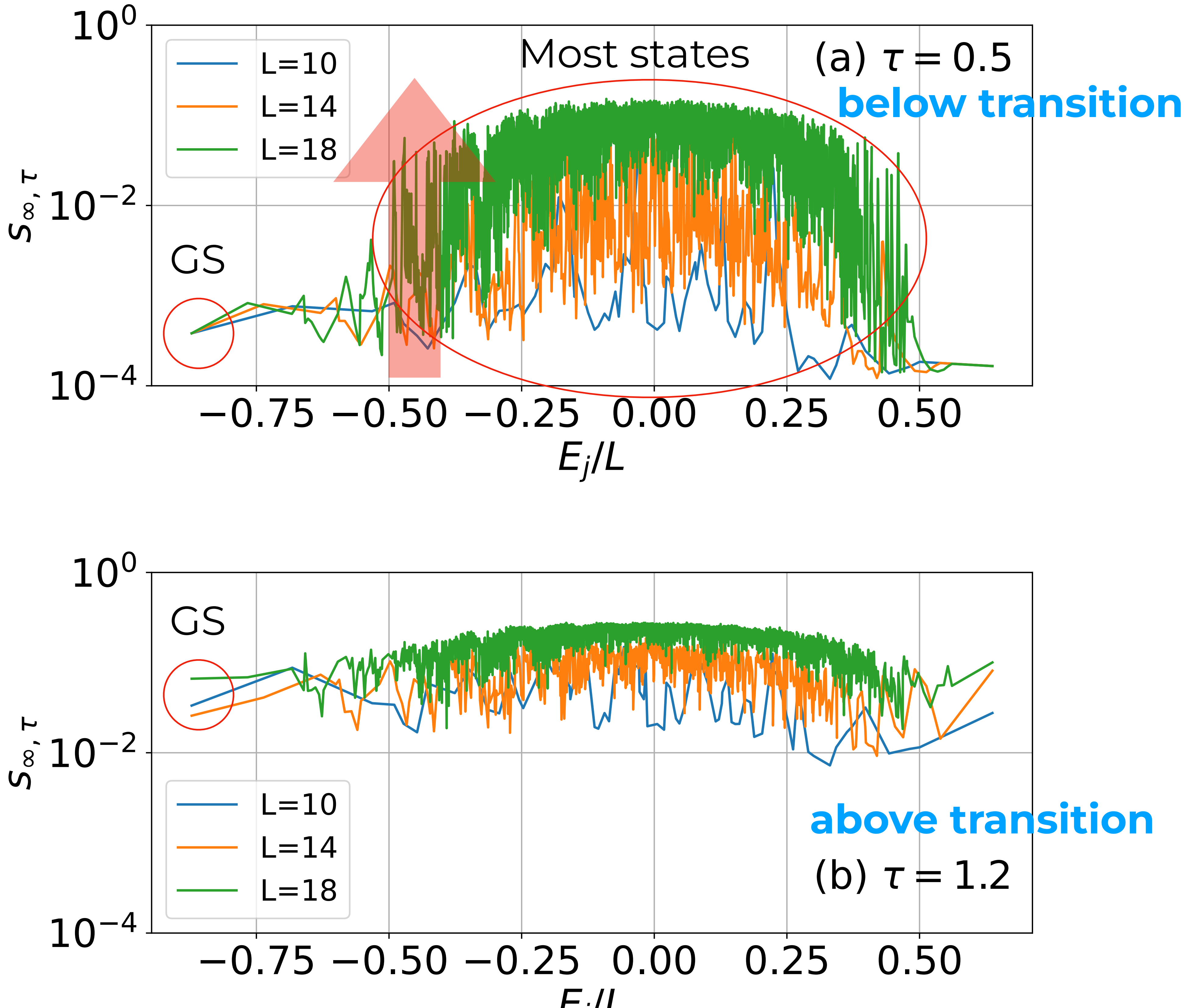
$$\lim_{\sigma\to\infty}\lim_{L\to\infty}$$

Specialty of GS | Most of other states do not show a transition

$$|\psi_{\rm ini}\rangle = |E_j\rangle$$

for all j

$$H_F^{(0)}|E_j\rangle = E_j|E_j\rangle$$



Scar-like Floquet eigenstates beyond convergence radius of HE

$$T(\tau)\,|\,\theta_\alpha(\tau)\rangle = e^{-i\theta_\alpha}\,|\,\theta_\alpha(\tau)\rangle$$

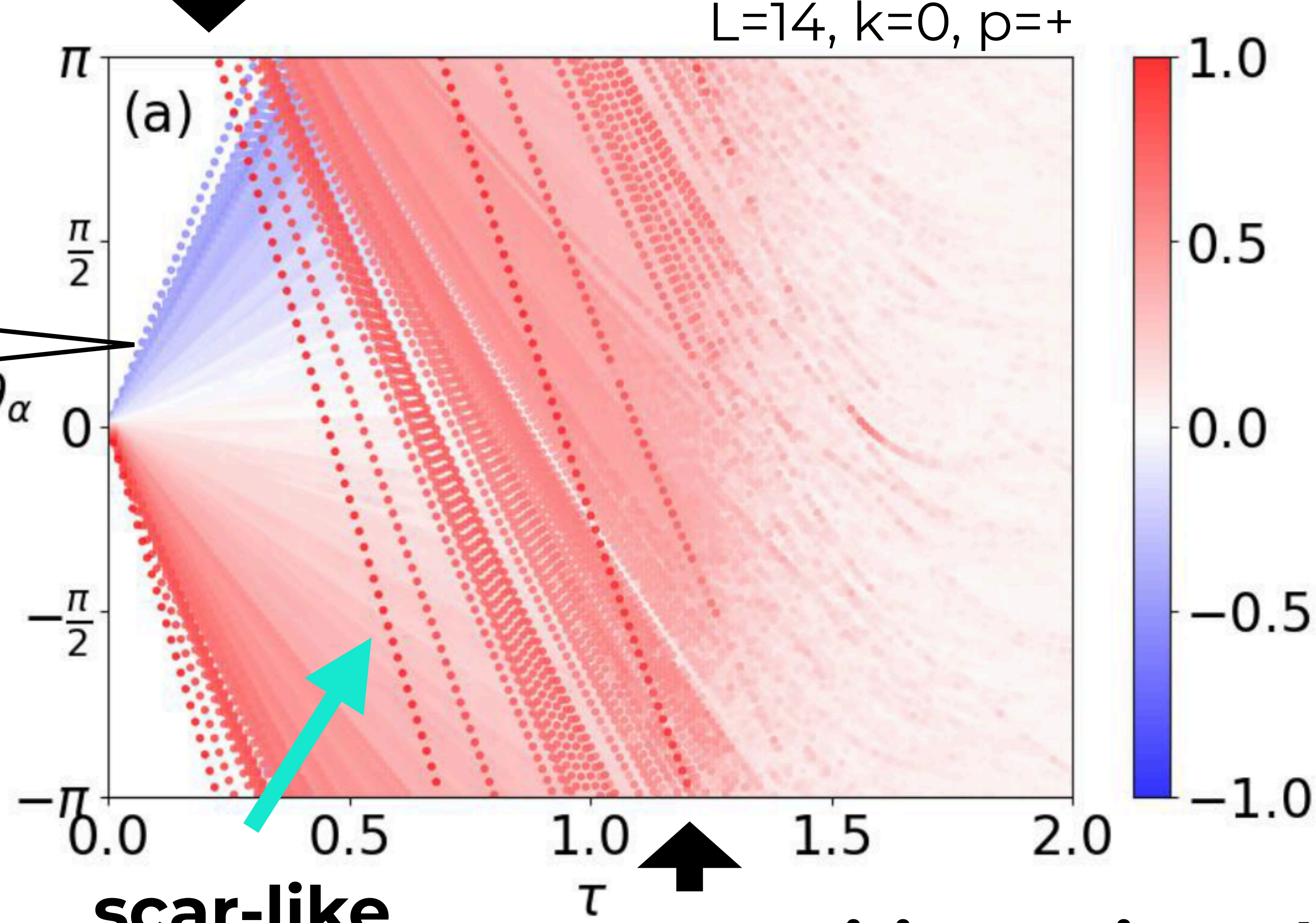
$O(L^{-1})$ convergence radius of high-frequency expansion (HE)

Short period (high frequency)

$$T \approx e^{-iH_F\cdot 2\tau}$$

$$\theta_n \approx E_n\tau \quad \mathrm{mod}\ 2\pi$$

$$|\,\theta_n\rangle \approx |\,E_n\rangle$$



GS of HF has large overlap with these

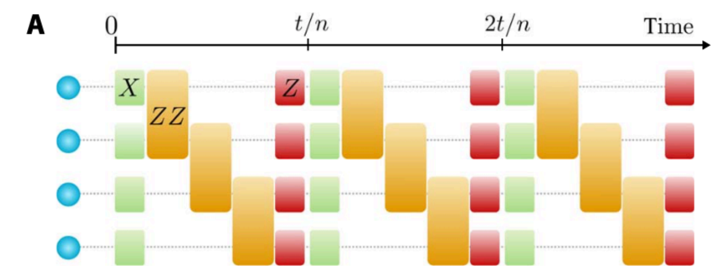
scar-like eigenstates

Transition point O(1)

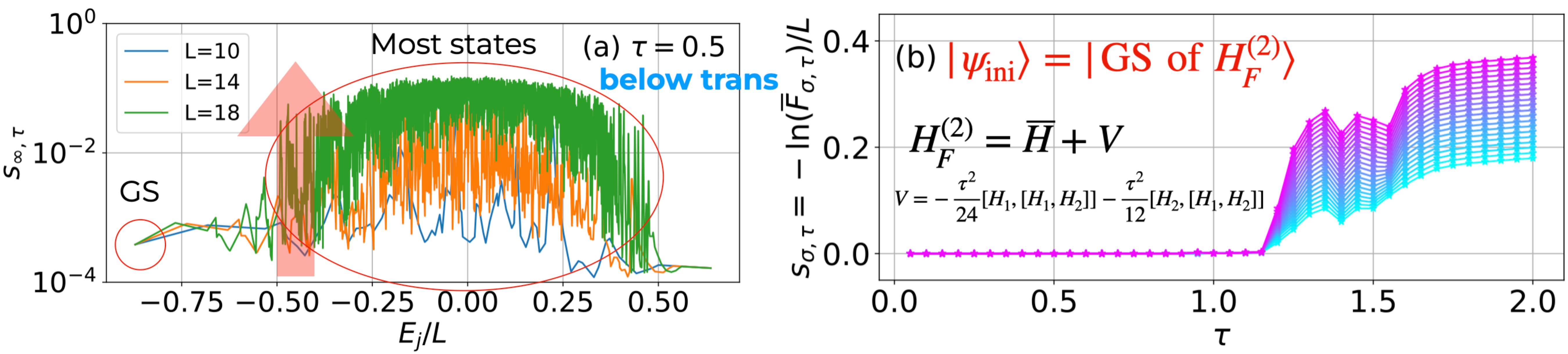
Summary of Part 2

TNI, S. Sugiura, A. Polkovnikov, arXiv:2311.16217

-Using a circuit model and simulator, we numerically studied Floquet dynamics up to L=30.



-Unlike most states, the effective GS is robust against heating and exhibits possible transition between heating/non-heating.



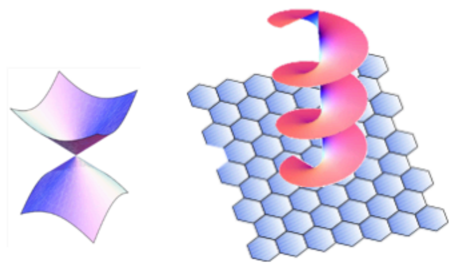
- a possible counter example to “Every state will eventually heat up in generic nonintegrable Floquet models (Floquet ETH)”

Remaining Questions: (1) Its Mechanism (2) Other models (3) how many

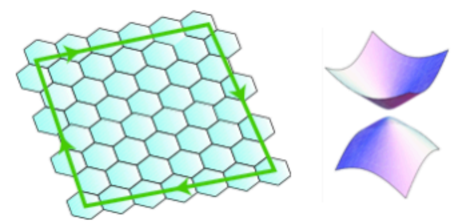
Grand Summary | Quantum States under Floquet heating

Floquet engineering

$H(t)$ periodic Hamiltonian — Floquet theory → H_F effective Hamiltonian



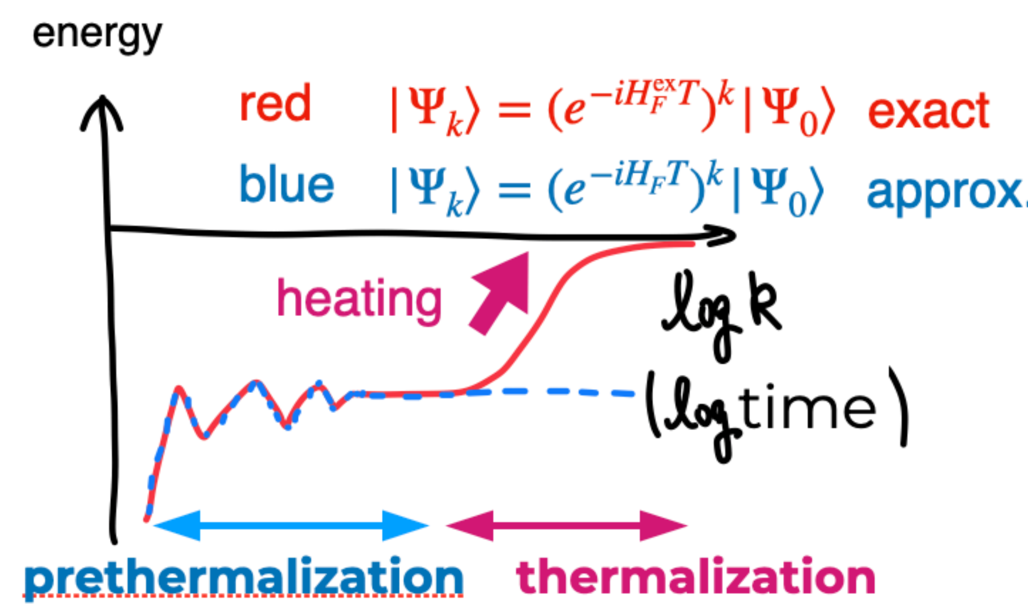
graphene + circ. pol. laser



topological insulator

Oka and Aoki, PRB (2009); Kitagawa, Oka et al. PRB (2011)

Floquet heating in many-body systems



Floquet FGR description for finite temperature states

Simple view of states during heating

Instantaneous thermal state w.r.t. H_F

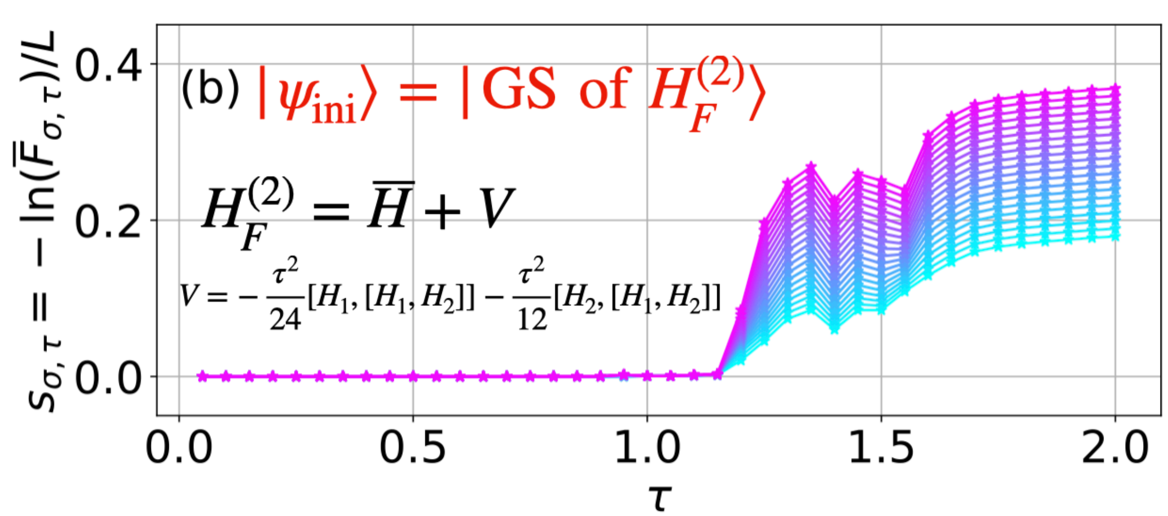
$$\rho(t) = e^{-\beta(t)H_F}/Z_t$$

$$\frac{dP_n(t)}{dt} = \sum_m \left[w_{m\to n}P_m(t) - w_{n\to m}P_n(t)\right],$$

$$w_{m\to n} = 2\pi\sum_{l\in\mathbb{Z}}\delta(E_n - E_m - l\omega)\,|\langle n|\delta U|m\rangle|^2$$

PRB 104, 134308 (2021)

Robust effective ground state



arXiv:2311.16217

More roles of effective Hamiltonian H_F

Appendix

Derivation of Floquet FGR

transition prob. in N >> 1 cycles

$$p_{m\to n} \equiv \left|\langle n|U^N|m\rangle\right|^2 \approx \left|\sum_{k=1}^{N} \langle n|U_F^k \delta K U_F^{N-k}|m\rangle\right|^2$$

$$\approx \left|\sum_{k=1}^{N} e^{i(\theta_n-\theta_m)k}\right|^2 |\langle n|\delta U|m\rangle|^2.$$

U = U_F + (small)

$$U = U_F U_F^\dagger U = U_F \delta U \approx U_F(I + i\delta K)$$

sum of phases

$$\left|\sum_{k=1}^{N} \mathrm{e}^{i(\theta_n-\theta_m)k}\right|^2 \approx 2\pi N \sum_{l\in\mathbb{Z}} \delta(\theta_n - \theta_m - 2\pi l),$$

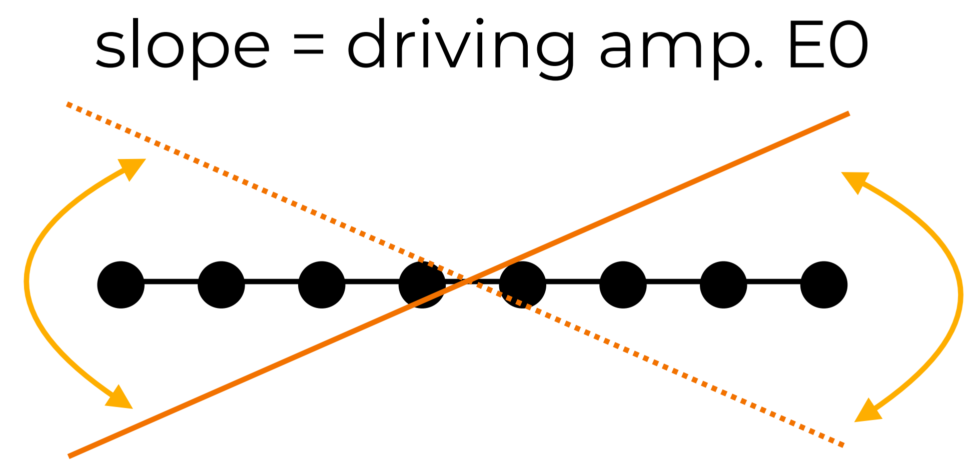
Floquet FGR for 1d Hubbard chain

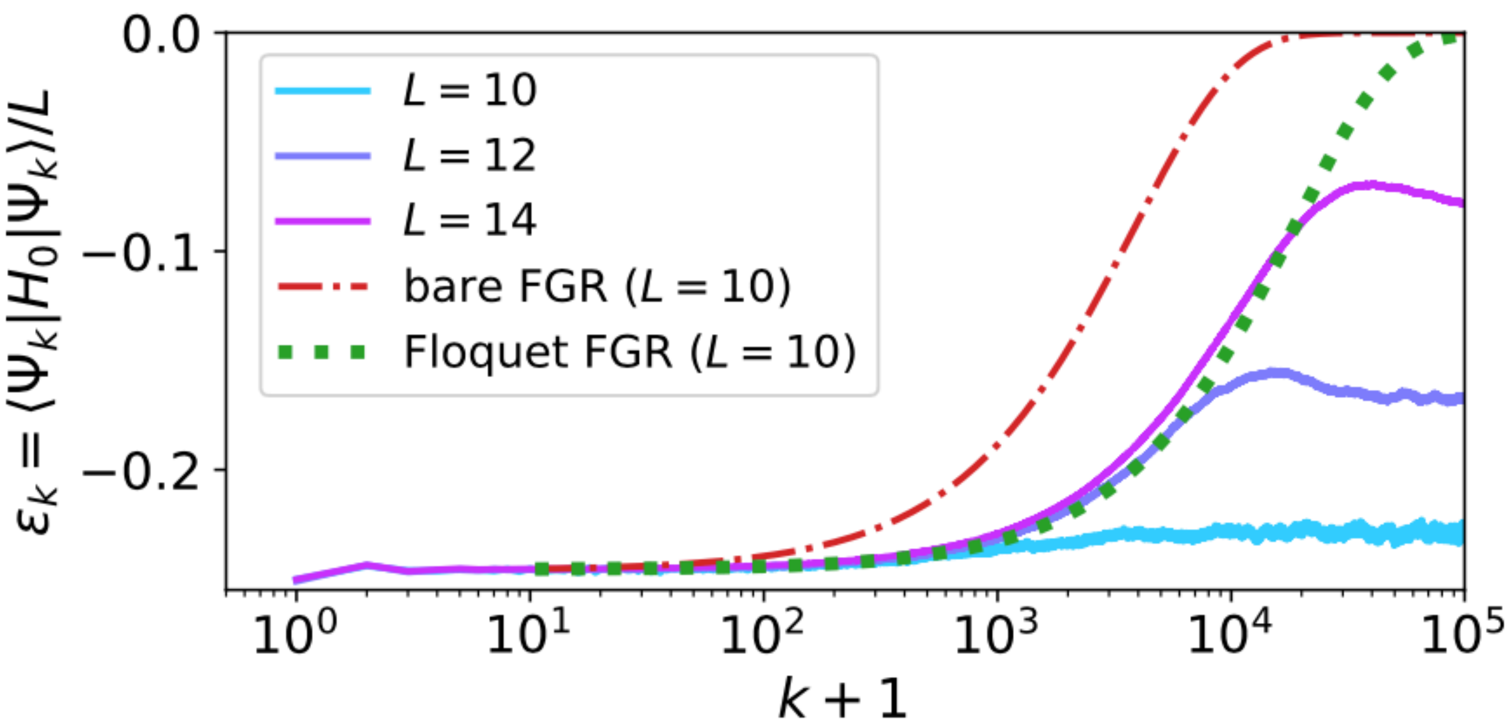
$$H_0 = H_{\text{hop}} + H_{\text{int}} + H_{\text{sb}},$$

$$H_{\text{hop}} = -t_1 \sum_{i=1}^{L-1} \sum_{s=\uparrow,\downarrow} (c^\dagger_{i,s} c_{i+1,s} + \text{H.c.}) - t_2 \sum_{i=1}^{L-2} \sum_{s=\uparrow,\downarrow} (c^\dagger_{i,s} c_{i+2,s} + \text{H.c.}),$$

$$H_{\text{int}} = U \sum_{i=1}^{L} n_{i,\uparrow} n_{i,\downarrow}$$

$$H_{\text{sb}} = h_b(n_{1,\uparrow} - n_{1,\downarrow}) + \mu_b(n_{L,\uparrow} + n_{L,\downarrow}).$$





Perturbation theory reproduces numerics

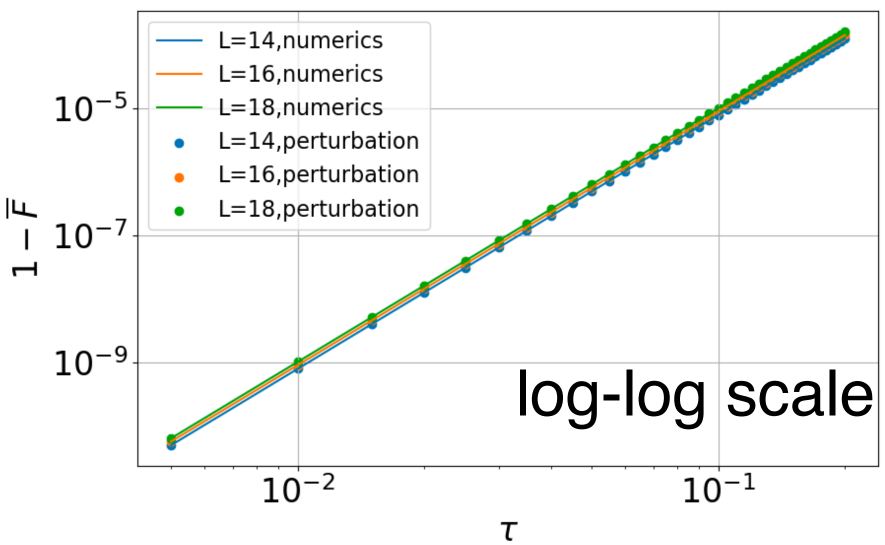
$$|\theta_\alpha\rangle = |E_{k_\alpha}\rangle + \sum_{l \neq k_\alpha} c_{k_\alpha, l} |E_l\rangle \qquad c_{kl} = -\frac{V_{kl}}{E_k - E_l} \qquad V = O(\tau^2)$$

negligible $O(\tau^8)$

Time-averaged fidelity for the GS of H0: $\overline{F}_0 = \sum_\alpha |\langle \theta_\alpha | E_0 \rangle|^4 = |\langle \theta_0 | E_0 \rangle|^4 + \sum_{\alpha \neq 0} |\langle \theta_\alpha | E_0 \rangle|^4$

According to perturbation theory, $|\langle \theta_0 | E_0 \rangle|^2 = 1 - \sum_{l \neq 0} \frac{|V_{0l}|^2}{(E_0 - E_l)^2} + O(V^4)$

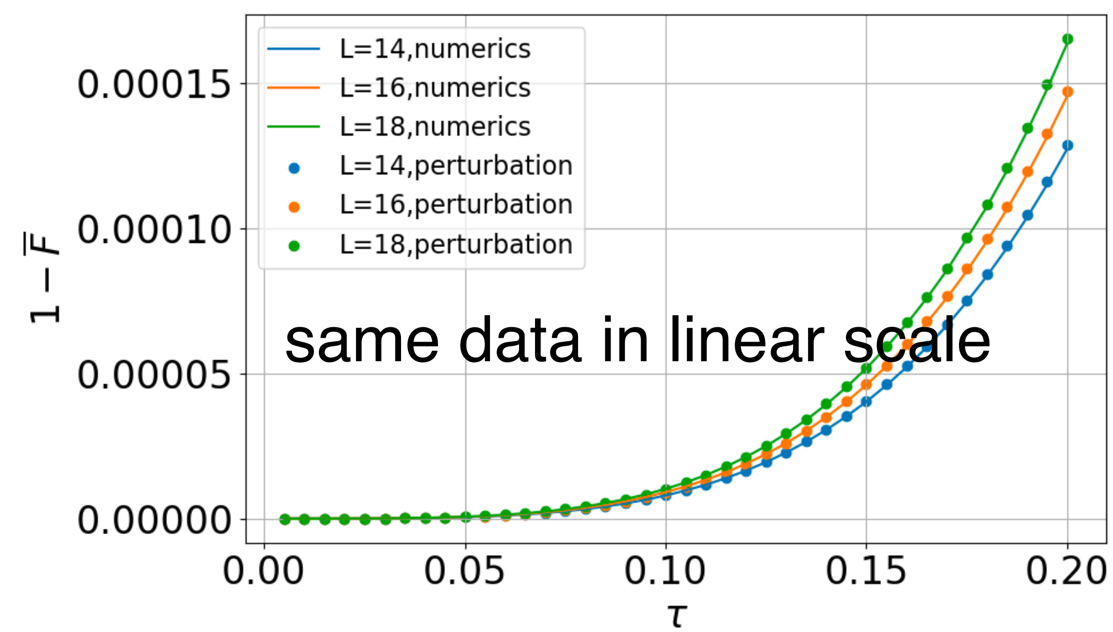
$$\overline{F}_0 = 1 - 2 \sum_{l \neq 0} \frac{|V_{0l}|^2}{(E_0 - E_l)^2} + O(\tau^8)$$



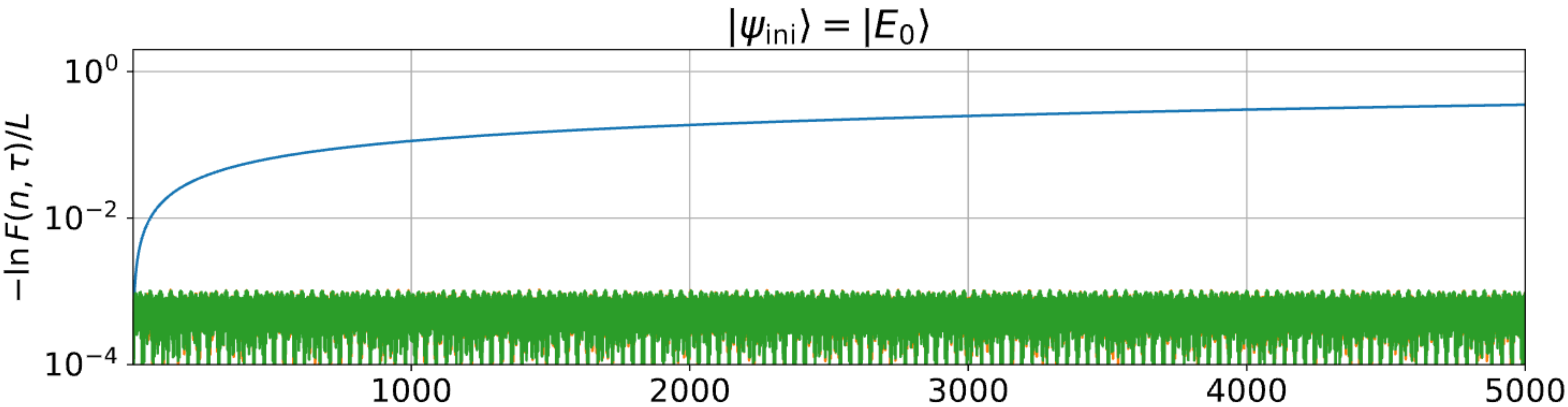
prop. to tau^4

but L-dependence?

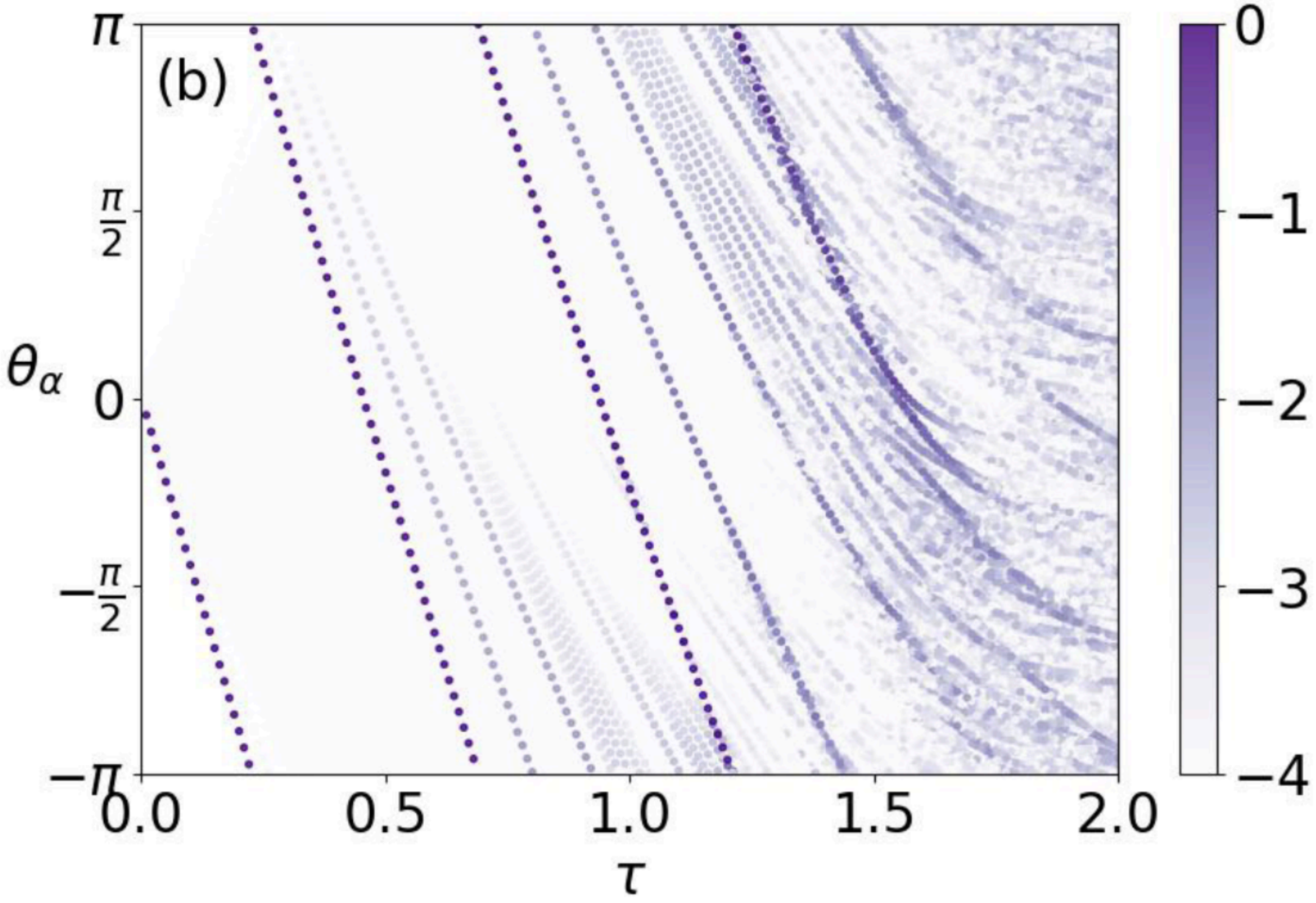
cf. $1 - \overline{F} = c\tau^4 e^{\gamma L}$



breakdown of FGR for GS



Weights on Floquet eigenstes



Tiny repulsions