Dynamically emergent correlations in bosons via quantum resetting

Sanjib Sabhapandit
Raman Research Institute, Bangalore

+ Satya N. Majumdar, Manas Kulkarni

J. Phys. A: Math. Theor. (2025).

10th Statistical Physics Community Meeting
23-25 April 2025
ICTS, Bangalore

Quantum gases

- A gas of interacting bosons can be realized in cold atom systems.
- Analytically tractable models for correlated bosonic gases are very rare.
- **Lieb-Liniger model** (1963): $H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g\sum_{1\le i<j\le N}\delta(x_i - x_j).$
  - The solution can be written in the form of a Bethe ansatz.
  - However, the presence of an external trap breaks integrability.
- **Gross-Pitaevskii equation**: $i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}) + g\,|\,\Psi(\vec{r},t)\,|^2\right)\Psi(\vec{r},t).$

  Often used to describe the collective behavior of a weakly interacting Bose gas.
  - Exact solutions are hard to come by except for $V(\vec{r}) = 0$ (free particles).

Motivation

- There is a paucity of analytically tractable models of strongly correlated quantum gases.
- Even if a solution can be found in some cases (e.g., in the Lieb-Liniger model without external potential), analytical calculations of various observables, such as the extreme/order statistics, the full counting statistics, etc., are very difficult.
- Thus, there is a growing need to engineer analytically tractable models of correlated quantum gases, which are also experimentally feasible.

Noninteracting bosons in a harmonic trap subjected to stochastic resetting

[Kulkarni, Majumdar, SS (2025)]

(1) Prepare the system in the ground state $|\Psi_0\rangle$ of the Hamiltonian $H_0 = \sum_{j=1}^{N} \left[\frac{p_j^2}{2m} + \frac{1}{2} m\,\omega^2 (x_j - a)^2 \right]$

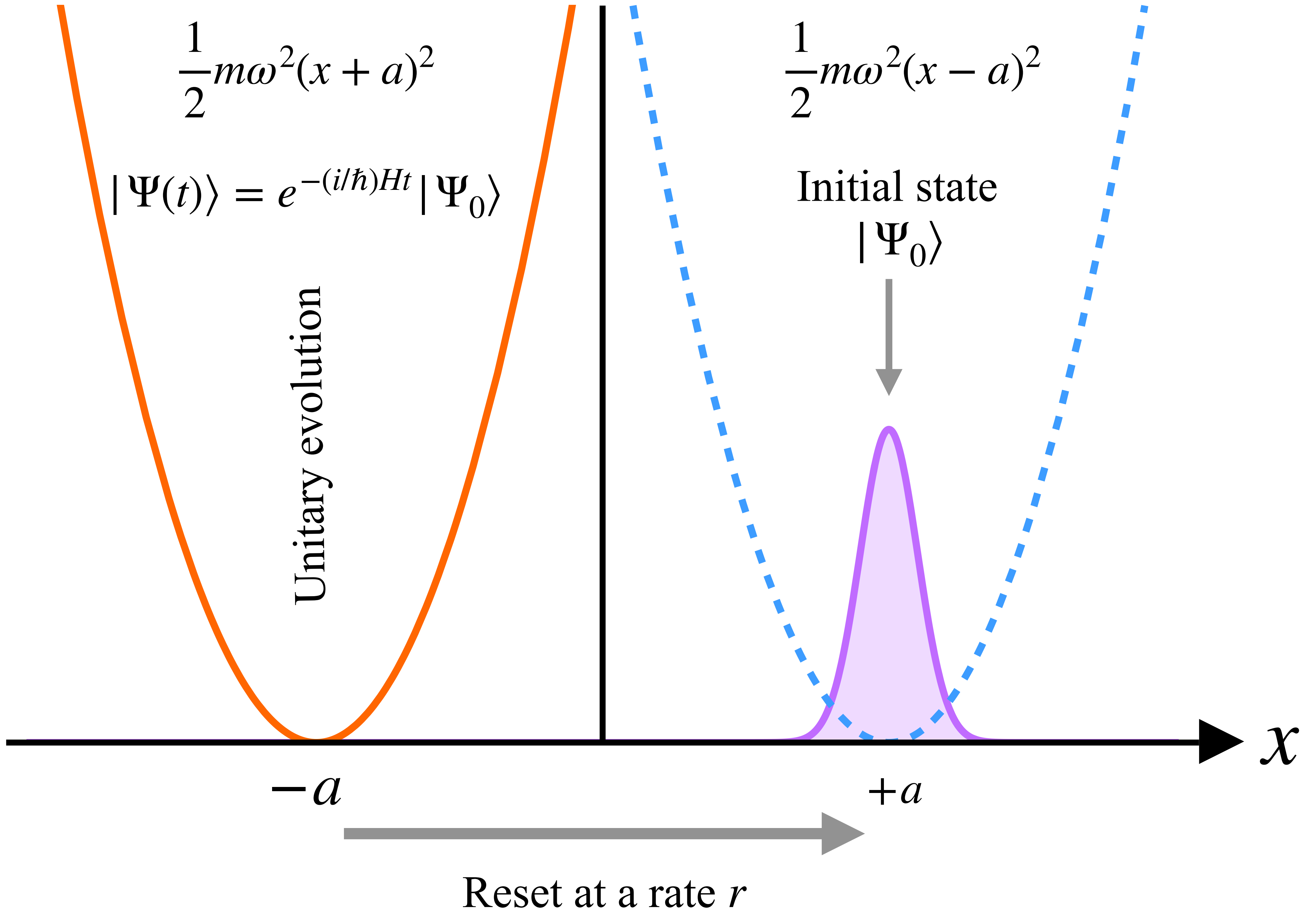
(2) Instantaneously quench to a new Hamiltonian

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2m} + \frac{1}{2} m\,\omega^2 (x_j + a)^2 \right]$$

(3) Evolve unitarily $|\Psi(\tau)\rangle = e^{-\frac{i}{\hbar}H\tau} |\Psi_0\rangle$ for a random time τ drawn from $p_r(\tau) = r\,e^{-r\tau}$

(4) Reset: $|\Psi(\tau)\rangle \to |\Psi_0\rangle$ instantaneously.

(5) Repeat steps (2)—(4)



The system evolving by this protocol approaches a NESS at long times.

The quantum JPDF with resetting

- During unitary evolution: $|\Psi(\tau)\rangle = e^{-\frac{i}{\hbar}H\tau}|\Psi_0\rangle$
- The density matrix: $\varrho(t) = |\Psi(t)\rangle\langle\Psi(t)|$
- In the presence of resetting: [Mukherjee, Sengupta, Majumdar (2018)]

$$\varrho_r(t) = e^{-rt}\varrho(t) + r\int_0^t d\tau e^{-r\tau}\varrho(\tau)$$

- The quantum joint probability density function (JPDF) is the matrix element: $P_r(x_1, x_2, \ldots, x_N, t) = \langle x_1, x_2, \ldots, x_N|\varrho_r(t)|x_1, x_2, \ldots, x_N\rangle$

Stationary joint probability density function

In the limit $t \to \infty$: $P_r(x_1, x_2, \ldots, x_N, t) \to P_{\text{st}}(x_1, x_2, \ldots, x_N)$

$$P_{\text{st}}(x_1, x_2, \cdots, x_N) = \int_{-3a}^{a} du\, h(u) \prod_{j=1}^{N} \frac{1}{\sqrt{\pi\sigma^2}} e^{-\frac{1}{\sigma^2}(x_j - u)^2} \quad \text{with } \sigma = \sqrt{\frac{\hbar}{m\,\omega}}$$

$$h(u) = \frac{r/\omega}{\sinh(\pi r/\omega)} \frac{1}{\sqrt{4a^2 - (a+u)^2}} \cosh\left[\frac{r}{\omega}\left(\pi - \cos^{-1}\left(\frac{1}{2}(1 + u/a)\right)\right)\right] \quad u \in [-3a, a]$$

CIID structure —Biroli, Larralde, Majumdar, Schehr (2023, 2024); Biroli, Kulkarni, Majumdar, Schehr (2024); Sabhapandit, Majumdar (2024) — makes the models analytically tractable.

Micro and Macro observables

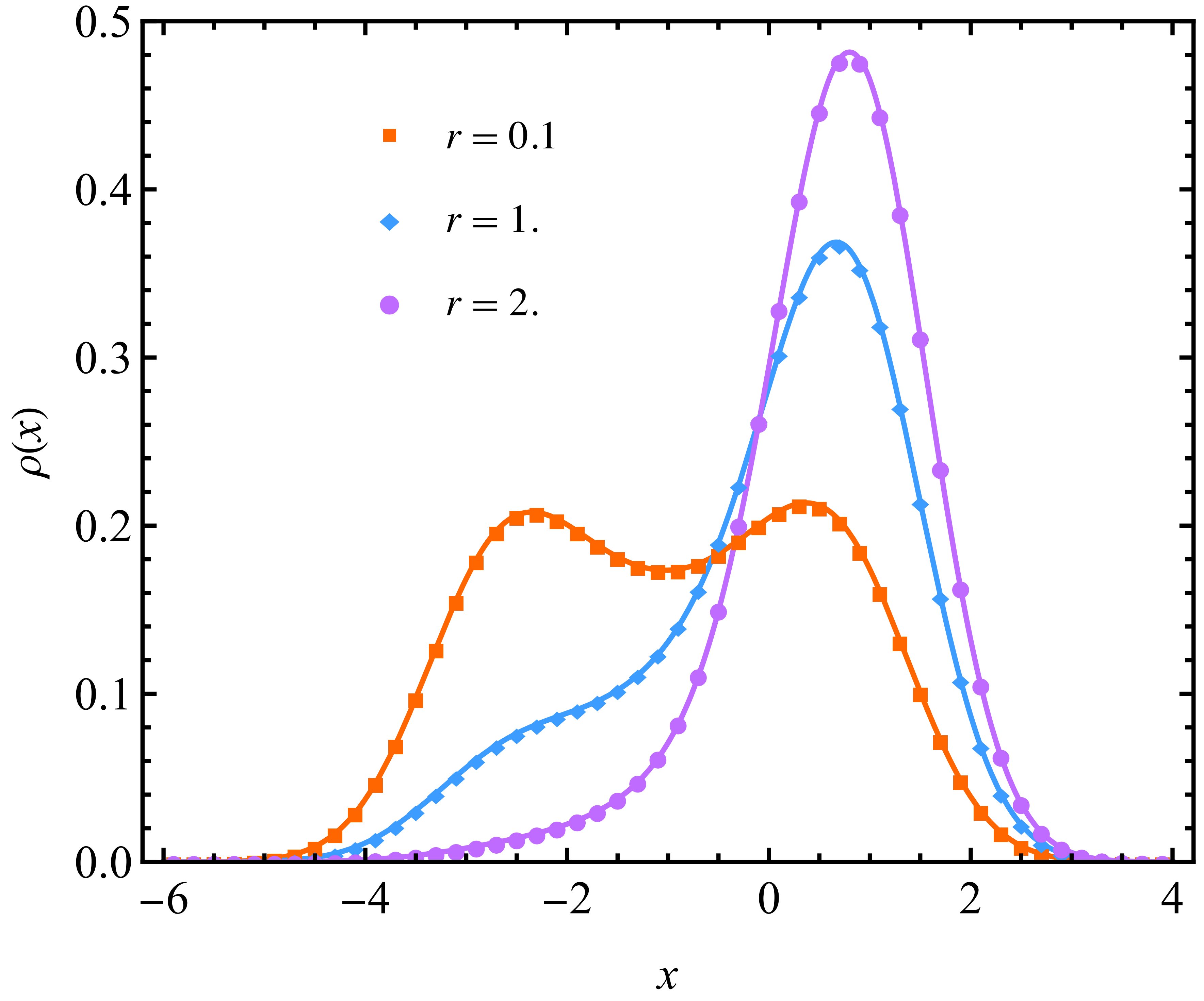
- Average density profile: $\rho(x) = \left\langle \frac{1}{N} \sum_{i=1}^{N} \delta(x - x_i) \right\rangle$
- Correlation function: $C_{i,j} = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$
- Extreme value statistics and order statistics: e.g., $M_1 = \max(x_1, x_2, \ldots, x_N)$
- Spacing/gap distribution (between successive positions)
- Full counting statistics: # particles in a given interval, e.g., in $[-L, L]$

Correlation function

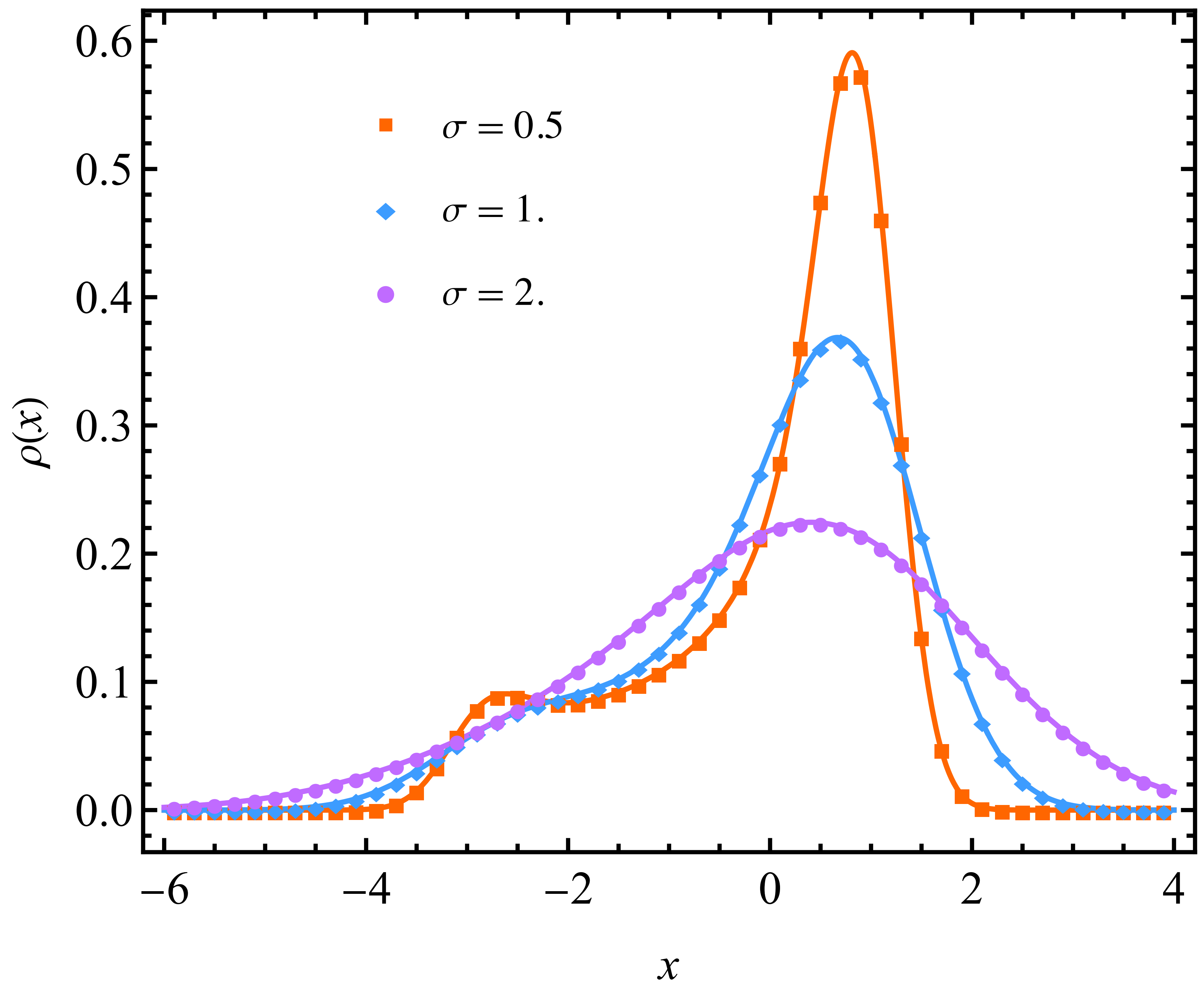
$$C_{i,j} = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle = \frac{4a^2 \left[5(r/\omega)^2 + 2\right]}{\left[(r/\omega)^2 + 1\right]^2 \left[(r/\omega)^2 + 4\right]} + \delta_{i,j} \frac{\hbar}{2\, m\, \omega}$$

Average density profile

$$\rho(x) = \frac{1}{\sqrt{\pi\sigma^2}} \int_{-\infty}^{\infty} du\, h(u)\, \exp\left(-\frac{(x-u)^2}{\sigma^2}\right)$$



$\sigma = 1$



$r = 1$

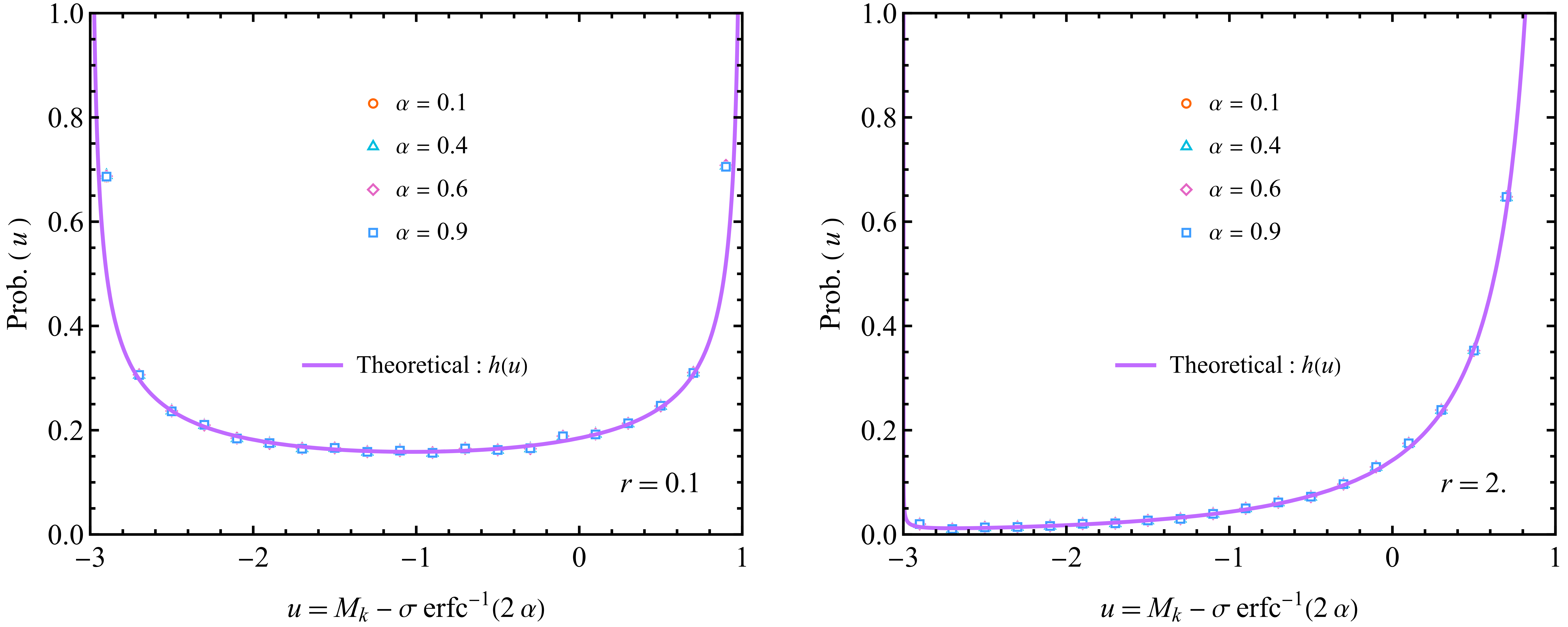
Order statistics

We first arrange the positions $\{x_1, x_2, \ldots, x_N\}$ in descending order $\{M_1 > M_2 > \cdots > M_N\}$ such that

$M_1 = \max\{x_1, x_2, \ldots, x_N\}, \;\; M_N = \min\{x_1, x_2, \ldots, x_N\},$ and
M_k represents the position of the k-th particle from the right.

$$\text{Prob}\,.\,[M_k = w] \simeq h(w - l_k) \quad \text{where} \;\; l_k \simeq \begin{cases} \sigma\,\text{erfc}^{-1}(2\alpha) & \text{when } \dfrac{k}{N} = \alpha \sim O(1) \\ \sigma\sqrt{\ln N} & \text{when } k \sim O(1) \end{cases}$$

Order statistics for the bosons



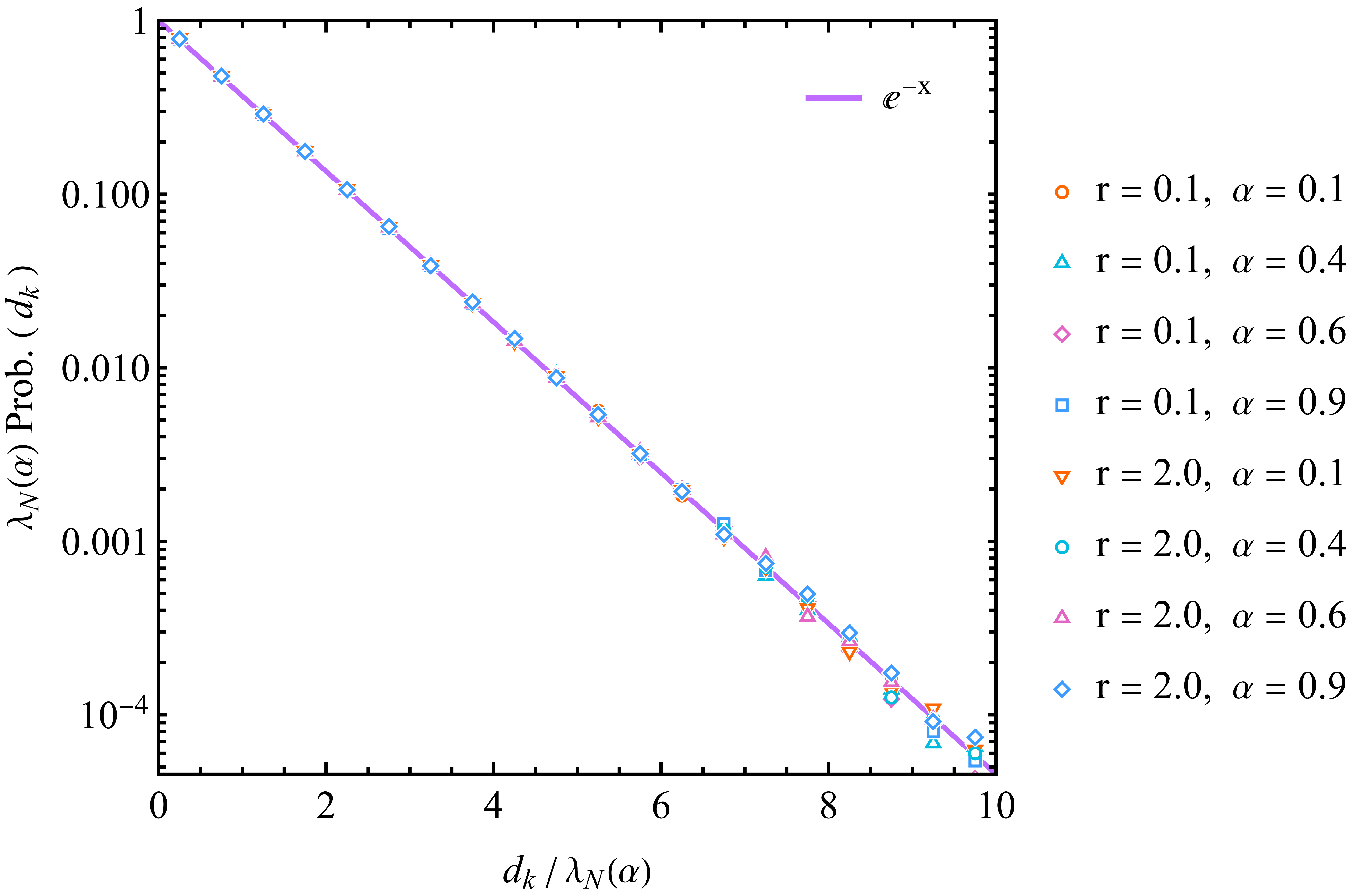
Gap statistics

$$\text{Prob}\,.\,(d_k = g) = \int_{-\infty}^{\infty} du\, h(u)\, \text{Prob}\,.\,\big(M_k(u) - M_{k+1}(u) = g\big)$$

$$\text{Prob}\,.\,(d_k = g) \simeq \frac{1}{\lambda_N} \exp\left(-\frac{g}{\lambda_N}\right)$$

$$\lambda_N \simeq \begin{cases} \left[\dfrac{N}{\sigma\sqrt{\pi}} \exp\big(-[\text{erfc}^{-1}(2\alpha)]^2\big)\right]^{-1} & \text{when } \dfrac{k}{N} = \alpha \sim O(1) \\ \dfrac{\sigma}{2k} \dfrac{1}{\sqrt{\ln N}} & \text{when } k \sim O(1) \end{cases}$$

Gap statistics



Full Counting Statistics (FCS)

- $P(N_L, N) =$ probability distribution of the number of particles N_L in the region $[-L, L]$

- For large N and N_L , it has the scaling form

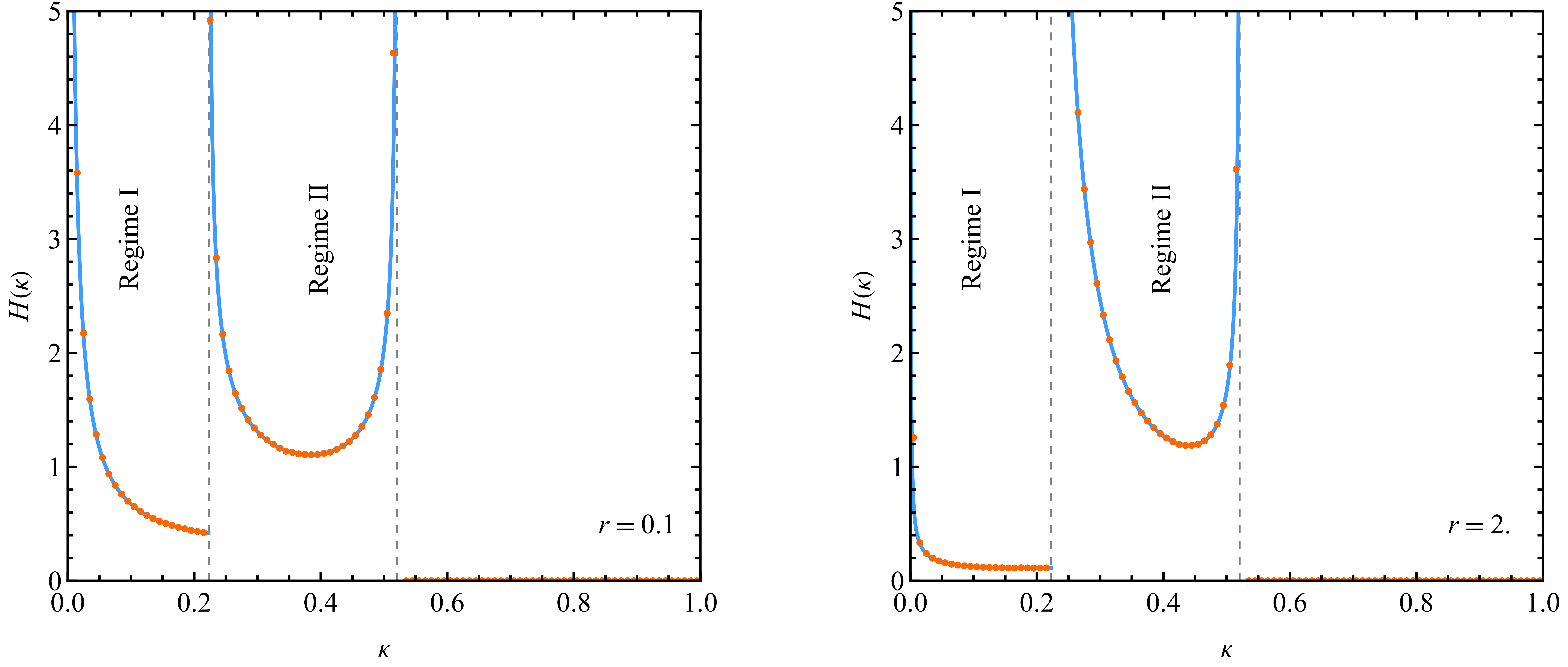
$$P(N_L, N) \simeq \frac{1}{N} H\left(\frac{N_L}{N}\right) \quad \text{where} \quad \frac{N_L}{N} = \kappa \in [\kappa_{\min}, \kappa_{\max}]$$

with $k_{\min} > 0$ and $\kappa_{\max} < 1.$

- The scaling function $H(\kappa)$ depends on the system via $h(u)$.

$$
H(\kappa) \simeq \begin{cases} \dfrac{A_1}{\sqrt{\kappa - \kappa_{\mathrm{min}}}} & \text{as } \kappa \to \kappa_{\mathrm{min}} \\ A_2 & \text{as } \kappa \to \kappa^* \text{ from below/left} \\ \dfrac{A_3}{\sqrt{\kappa - \kappa^*}} & \text{as } \kappa \to \kappa^* \text{ from above/right} \\ \dfrac{A_4}{\sqrt{\kappa_{\mathrm{max}} - \kappa}} & \text{as } \kappa \to \kappa_{\mathrm{max}} \end{cases}
$$

FCS for bosons



Conclusions

- We have engineered a strongly correlated quantum gas, where the correlations between noninteracting particles emerge dynamically.

- The joint probability density function has a special form

$$P_{\mathrm{st}}(x_1, x_2, \ldots, x_N) = \int_{-\infty}^{\infty} du\, h(u) \prod_{j=1}^{N} p(x_j \mid u)$$

- The conditional IID structure of the JPDF allows the analytical computation of several observables in a strongly correlated system.