Estimating parameters by fitting correlation functions of continuous quantum measurement

Pierre Guilmin, Pierre Rouchon, Antoine Tilloy

January 23, 2025

1. Introduction
Experimental QEC with superconducting circuits

2. Today’s solution
Typical workflow for characterising superconducting circuits

3. A simple method
Fitting correlation functions of continuous measurement

4. Three numerical examples
Does the method actually work?

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Building a fault-tolerant quantum computer

- Qubits suffer from **errors**: bit-flips and phase-flips.
- The solution: **quantum error correction** (QEC).
- Use a **larger Hilbert space** to protect from **uncorrelated local errors** → **delocalize** the information.

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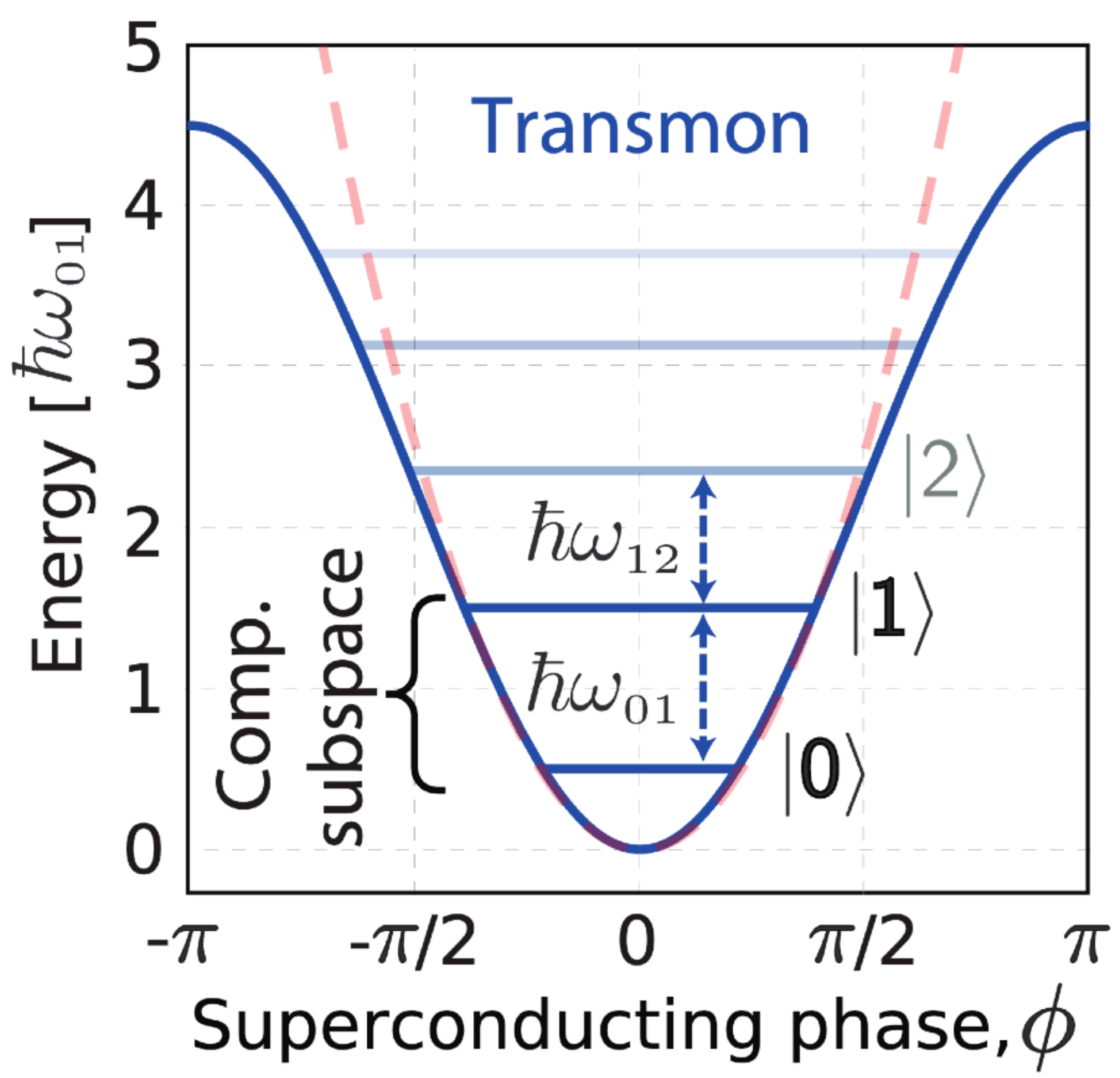
- The solution: **quantum error correction** (QEC).
- Use a **larger Hilbert space** to protect from **uncorrelated local errors** → **delocalize** the information.
- Two examples using superconducting circuits.
  - Example 1: qubit + surface code.
  - Example 2: bosonic code (choose an encoding) + simpler code.

    ↪ Errors are local in phase-space.

Example 1: surface code with transmon qubits

Google
IBM

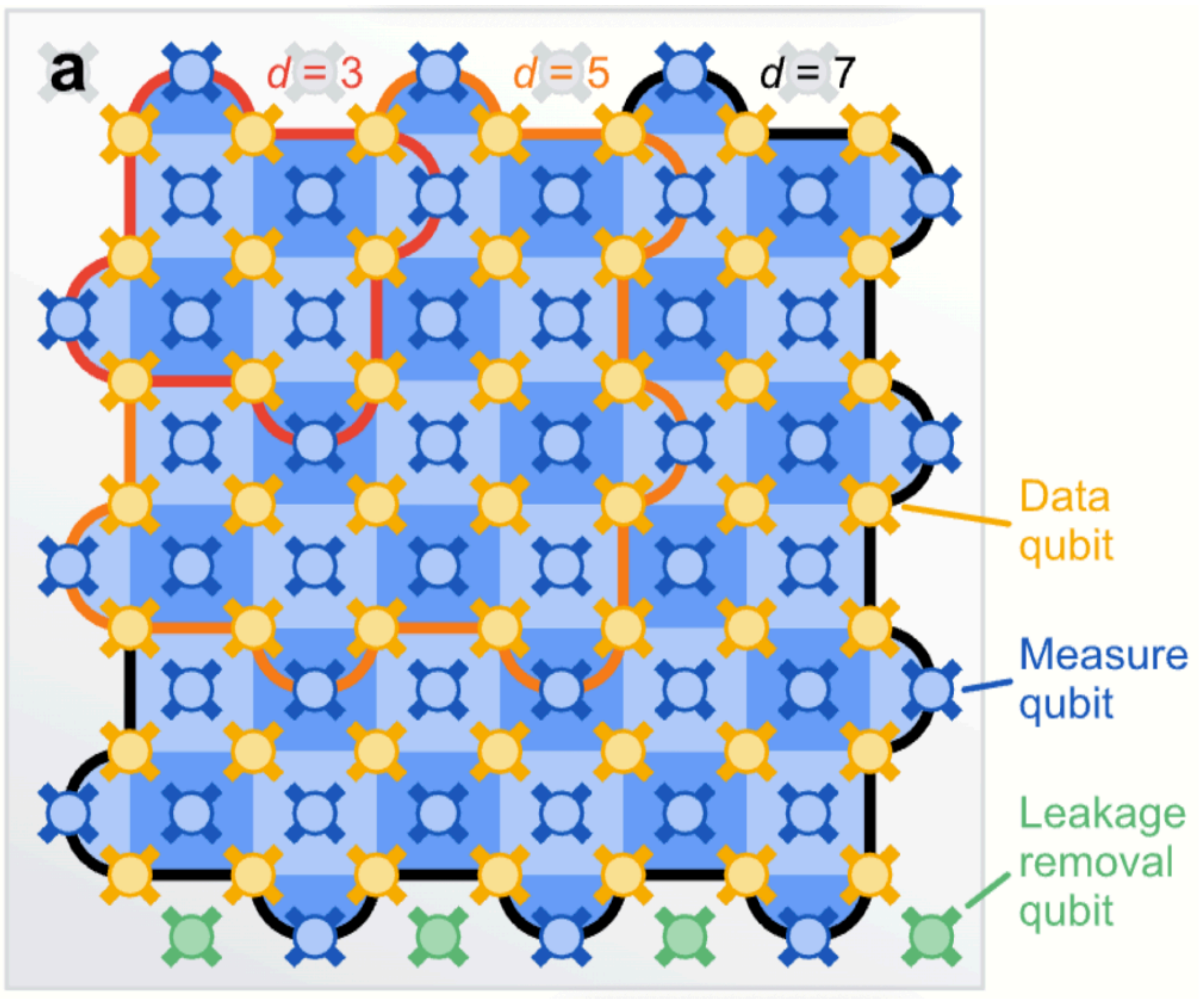
The qubit



Qubit encoded in the $\{|0\rangle, |1\rangle\}$ energy subspace

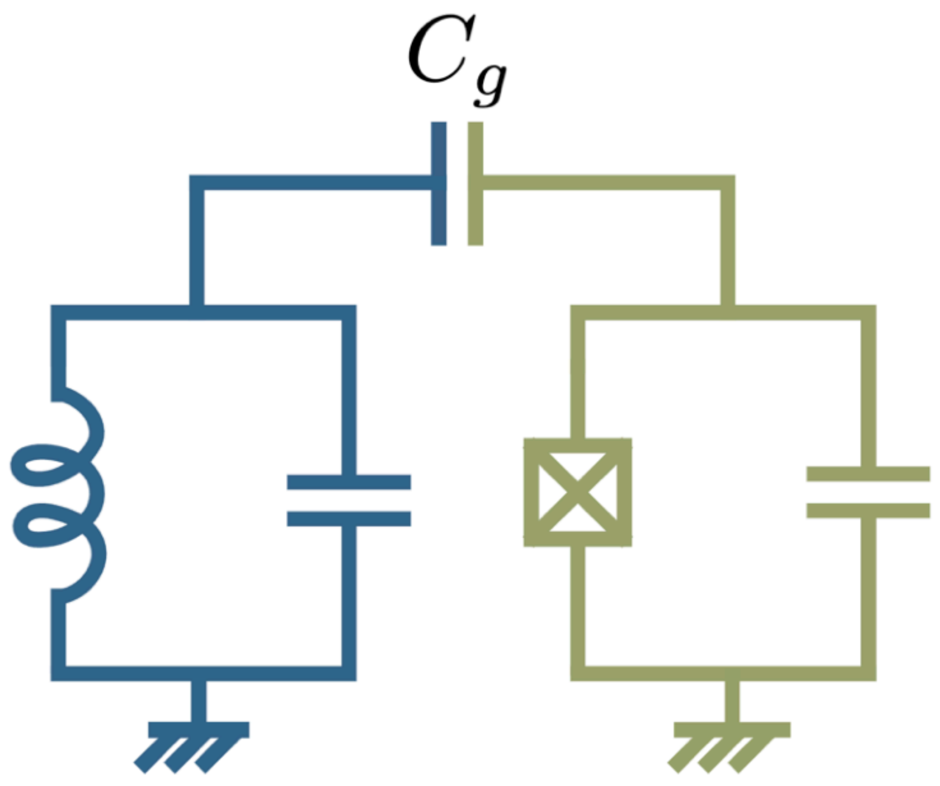
The code

(correct bit and phase flips)



$n \approx 1000$ for one logical qubit

In practice



$$\hat{H} = 4E_C(\hat{n} + \hat{n}_r)^2 - E_J \cos\hat{\varphi} - \sum_m \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m$$

approximations

$$\hat{H}_{\text{disp}} \approx \hbar\omega_r' \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_q'}{2}\hat{\sigma}_z + \hbar\chi \hat{a}^\dagger \hat{a} \hat{\sigma}_z$$

$$L = \sqrt{\gamma}\sigma_- \quad L_\varphi = \sqrt{\gamma_\varphi}\sigma_z$$

Krantz, Philip, et al. "A quantum engineer's guide to superconducting qubits." Applied physics reviews 6.2 (2019).
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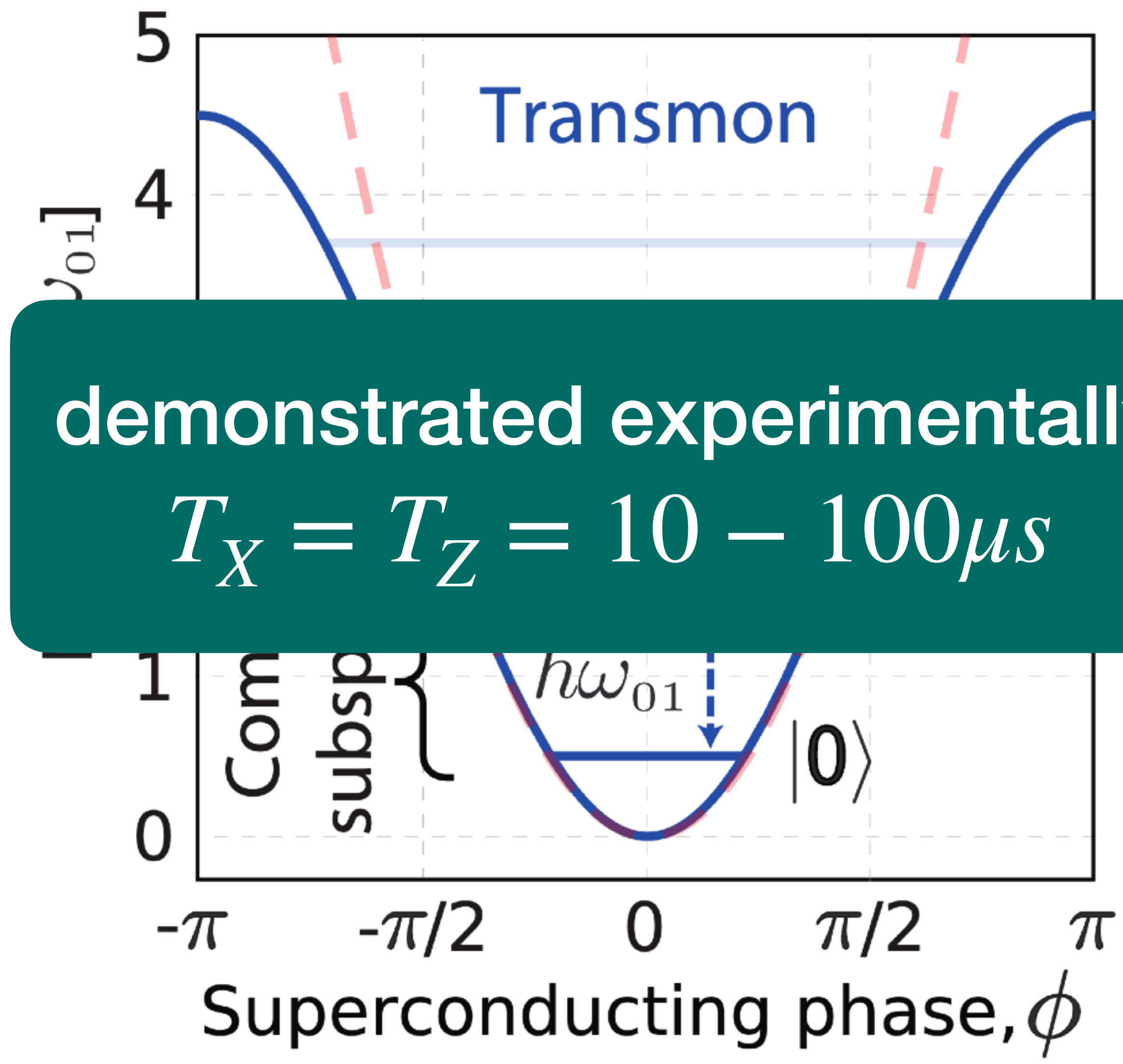

demonstrated experimentally
$T_X = T_Z = 10 - 100\mu s$

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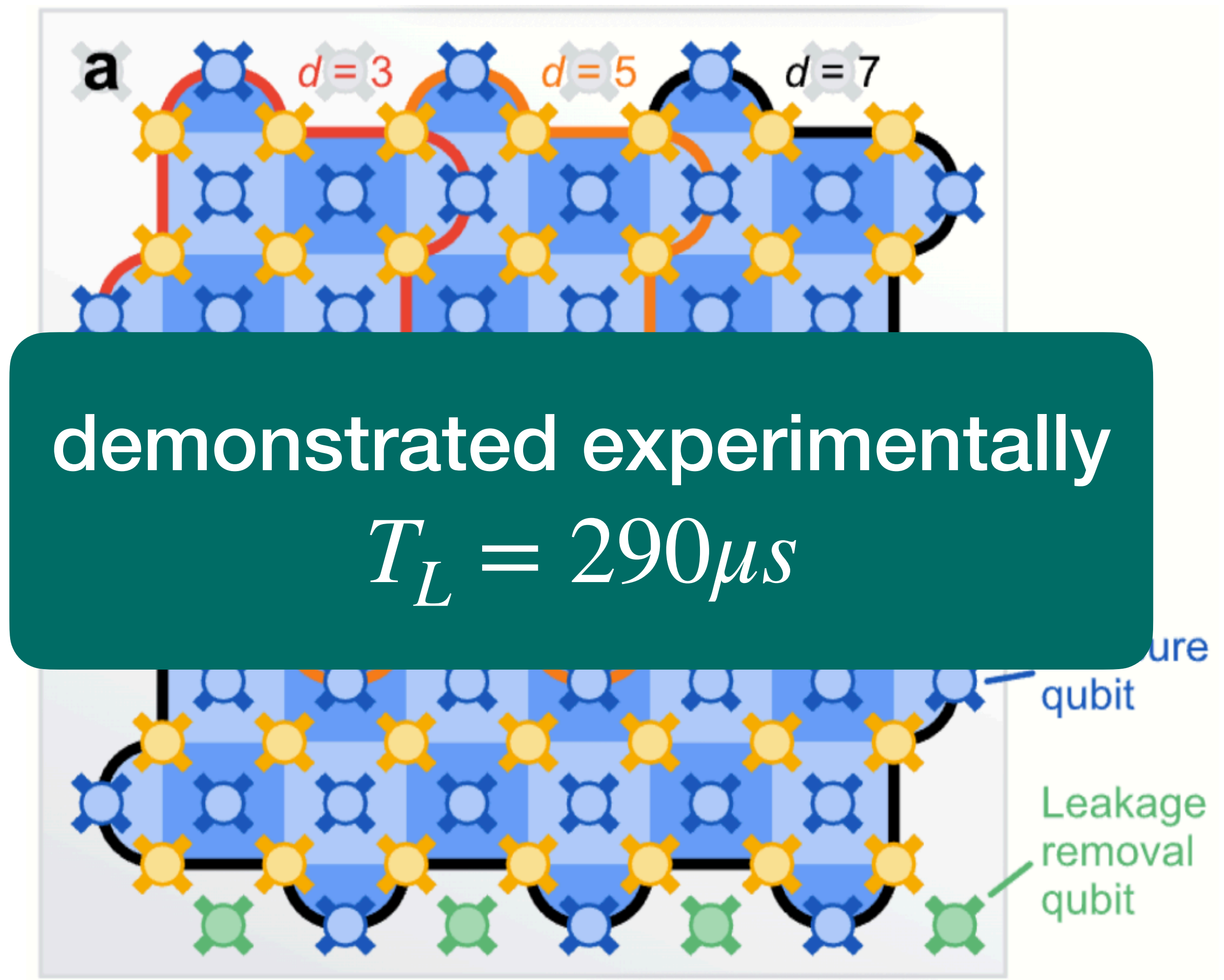
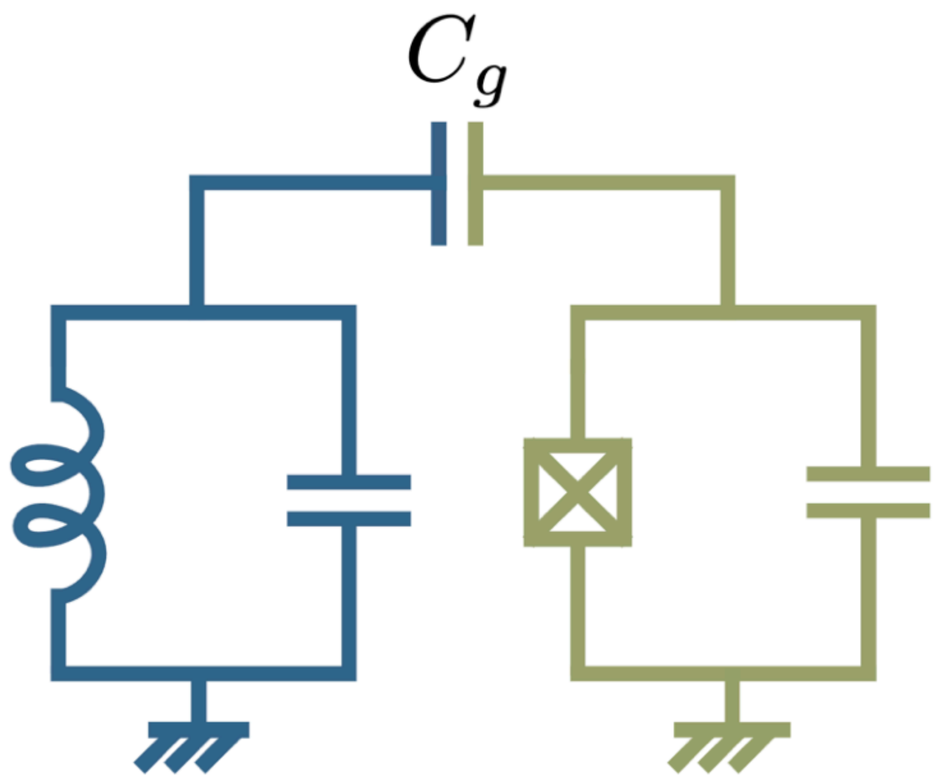


demonstrated experimentally
$T_L = 290\mu s$

$n \approx 1000$ for one logical qubit

In practice



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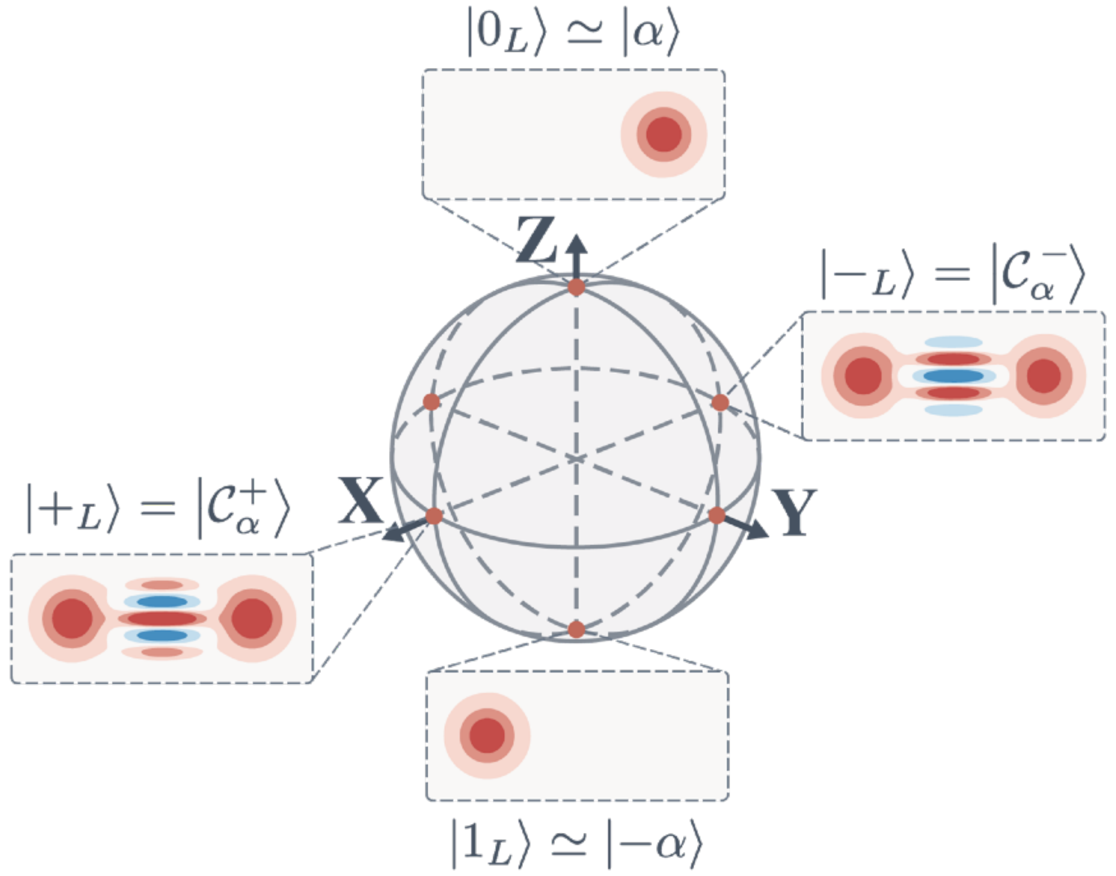
Example 2: repetition code with cat qubits

ALICE & BOB

aws

The qubit

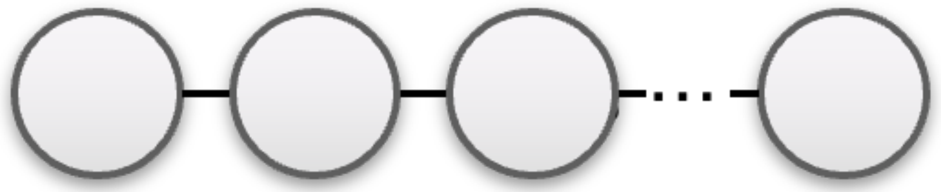
(correct bit flips)



Qubit encoded in the
$\{|-\alpha\rangle, |+\alpha\rangle\}$
subspace

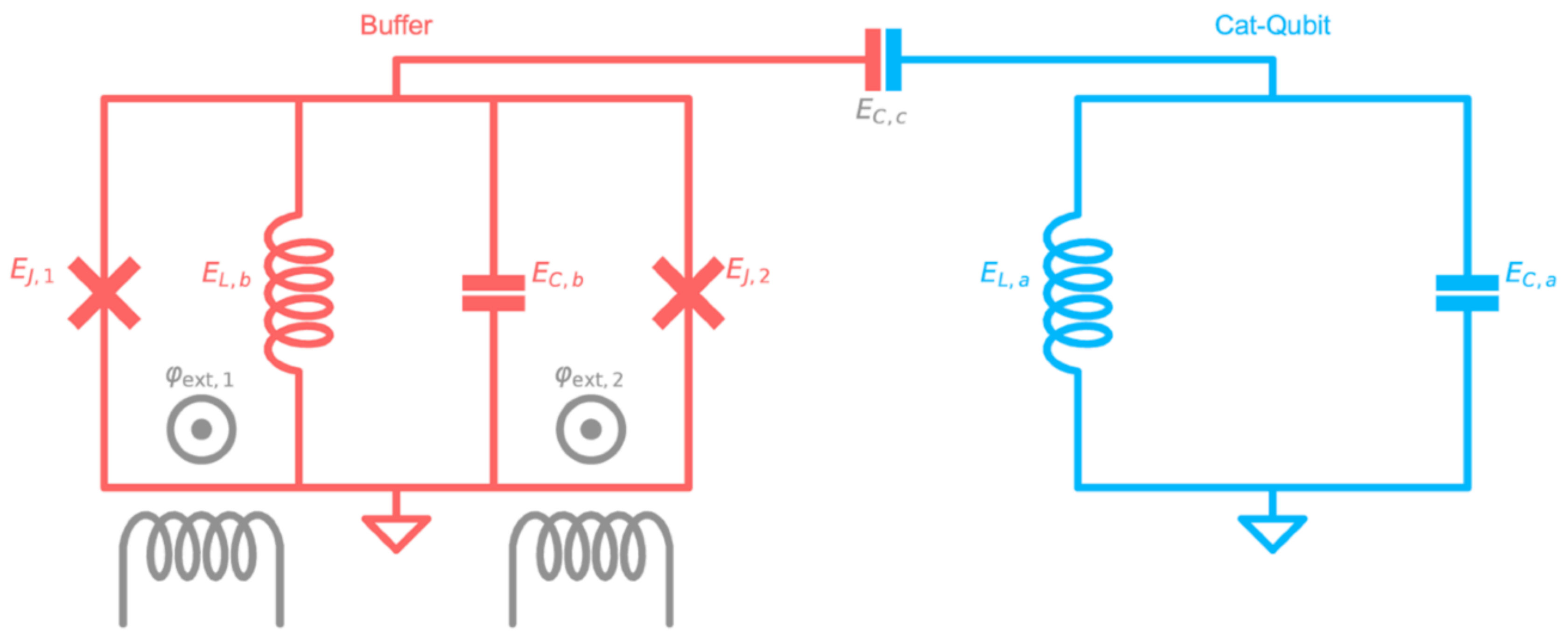
The code

(correct phase flips)



$n \approx 15$ for one logical qubit
(with LDPC code)

In practice



$$U(\varphi) = \frac{1}{2}E_{L,b}\varphi^2 - E_{J,1}\cos(\varphi + \varphi_{\text{ext},1}) - E_{J,2}\cos(\varphi - \varphi_{\text{ext},2})$$

↓ approximations

$$H = 0$$
$$L_1 = \sqrt{\kappa_1}a$$
$$L_2 = \sqrt{\kappa_2}(a^2 - \alpha^2)$$

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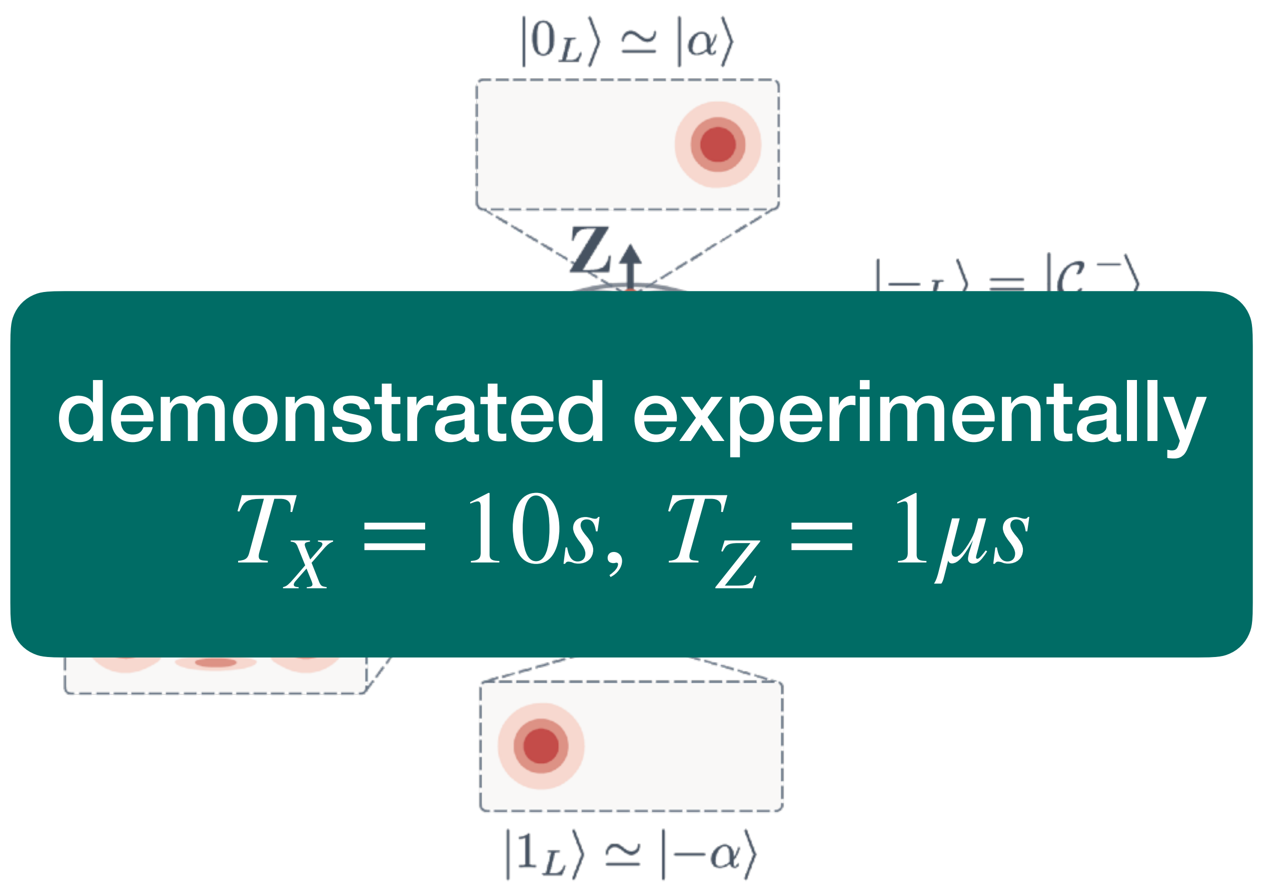

demonstrated experimentally
$T_X = 10s,\ T_Z = 1\mu s$

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(correct phase flips)

work in progress

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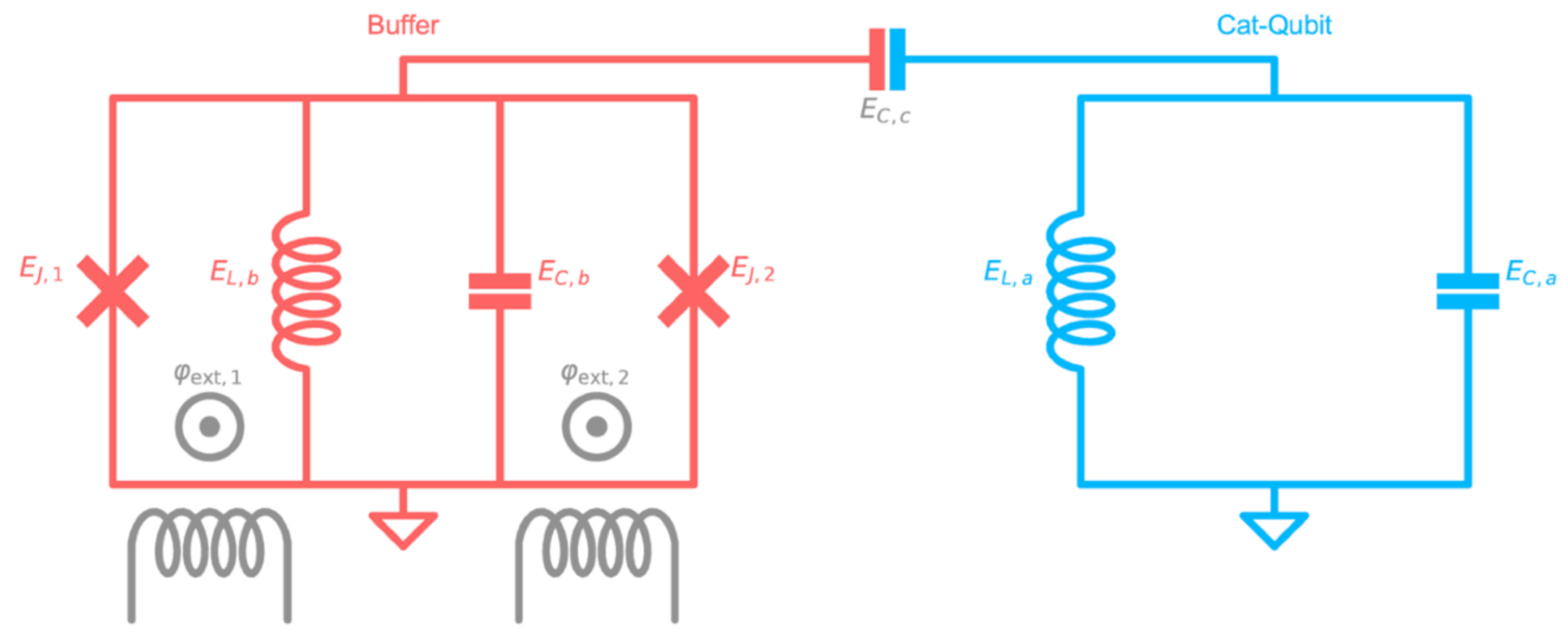

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The recipe for building a quantum computer
*with superconducting circuits

1. Choose a **target dynamics** for your qubit (a specific Liouvillian $\mathscr{L}$).
2. Find **a circuit** that implement this dynamics.
3. **Build** the circuit.
4. **Measure** all the system parameters.
5. Implement **physical gates** (e.g. X gate, $CNOT$ gate).
6. Implement **QEC** and **logical gates**.
7. (optional) Break bitcoin, become rich and famous.

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parameterised Liouvillian $\mathscr{L}(\theta)$
with parameters $\theta = (\theta_1, \ldots, \theta_p) \in \mathbb{R}^p$

Goal of the characterisation: estimate θ

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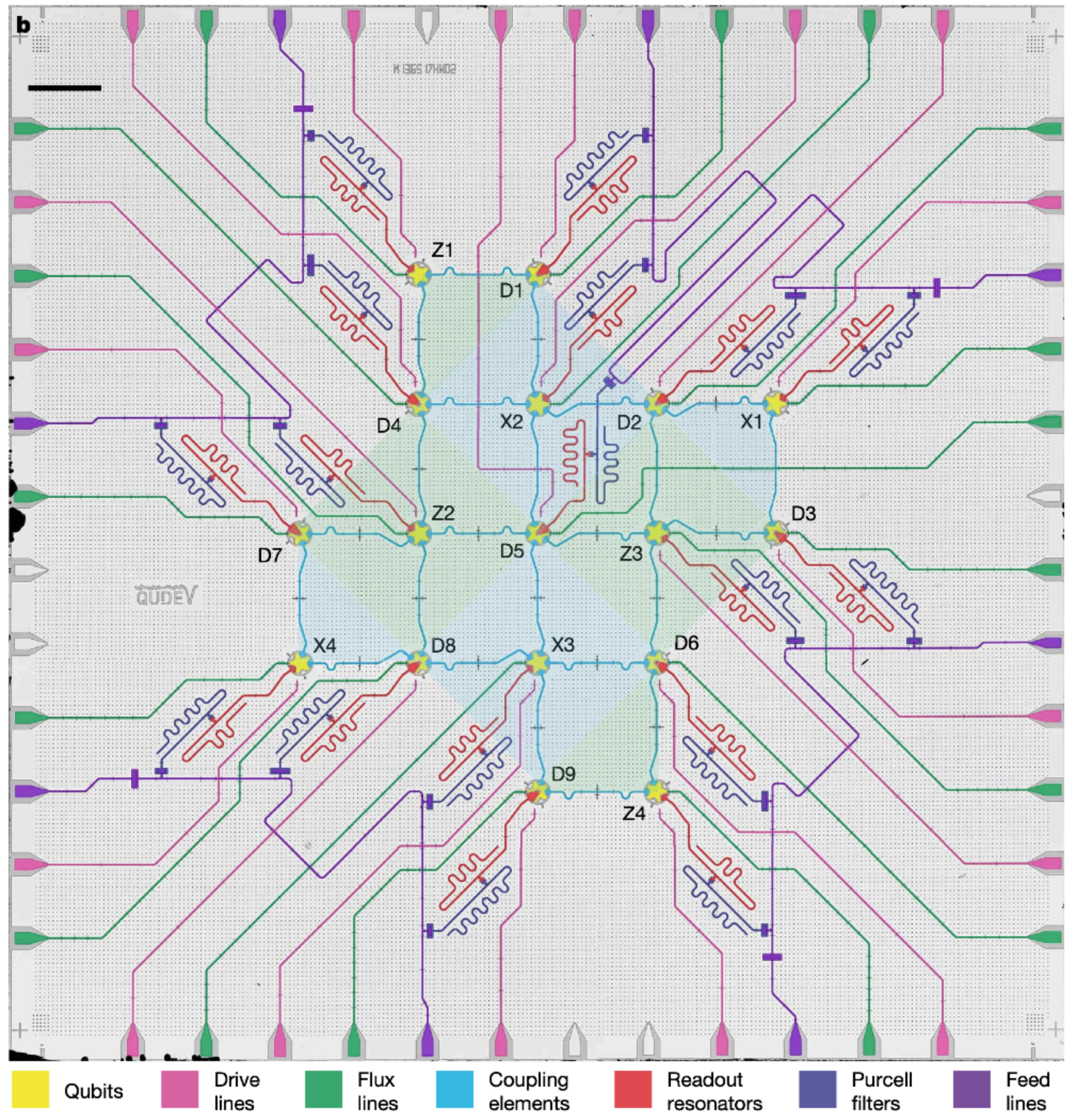
Transmon example:

- Fit the simplified qubit-resonator model $\theta = (\omega_r, \omega_q, \chi, \gamma, \gamma_\varphi, \ldots)$
- Fit the full circuit model $\theta = (E_J, E_C, \omega_m, \ldots)$

Characterising superconducting circuits

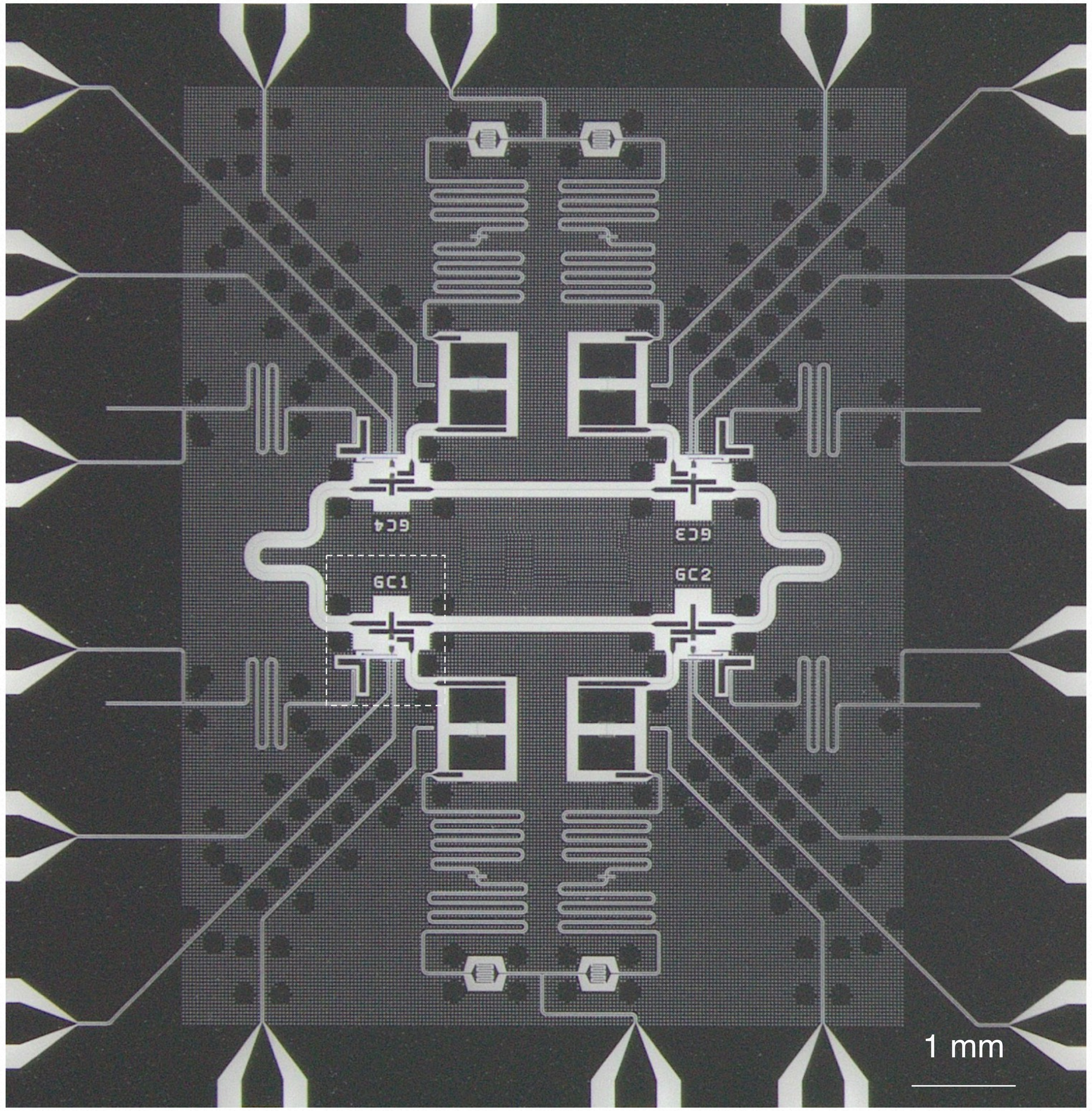


$\longrightarrow$ $\mathscr{L}(\theta)$ $\longrightarrow$ θ ?

David DiVincenzo's lectures

Why do we need to **characterise** anything if we designed the circuit?

Krinner, Sebastian, et al. "Realizing repeated quantum error correction in a distance-three surface code." *Nature* 605.7911 (2022): 669-674.



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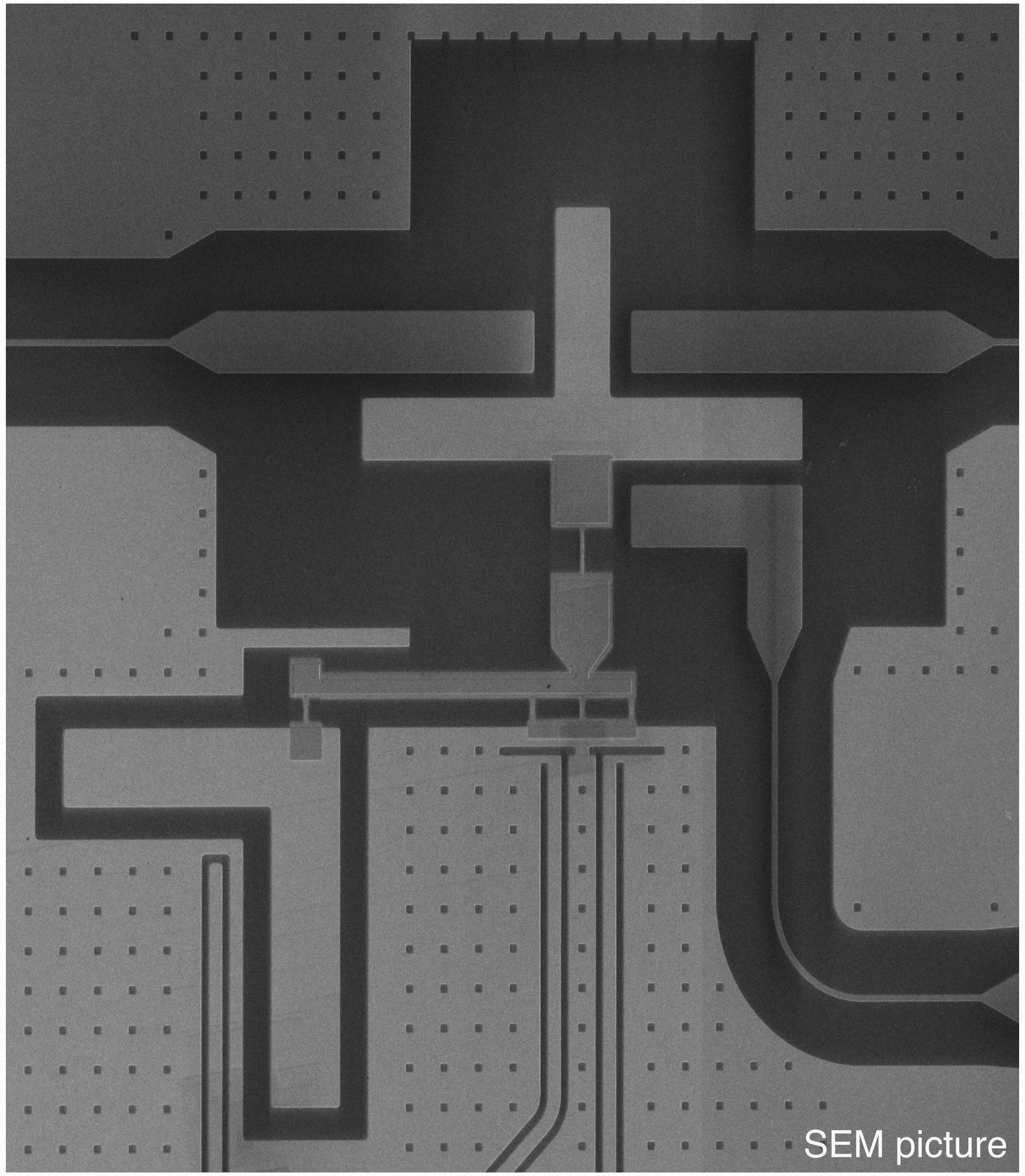
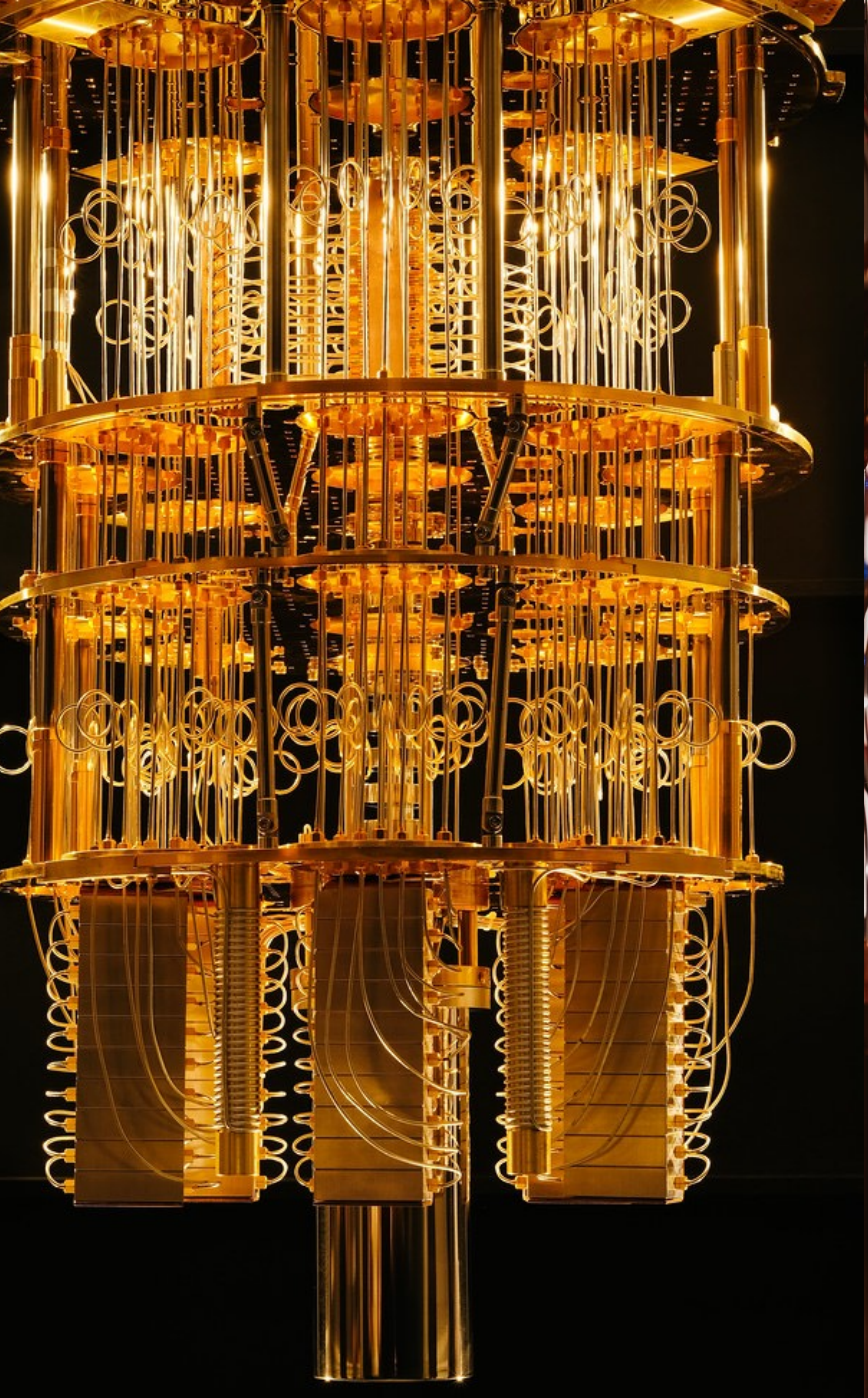
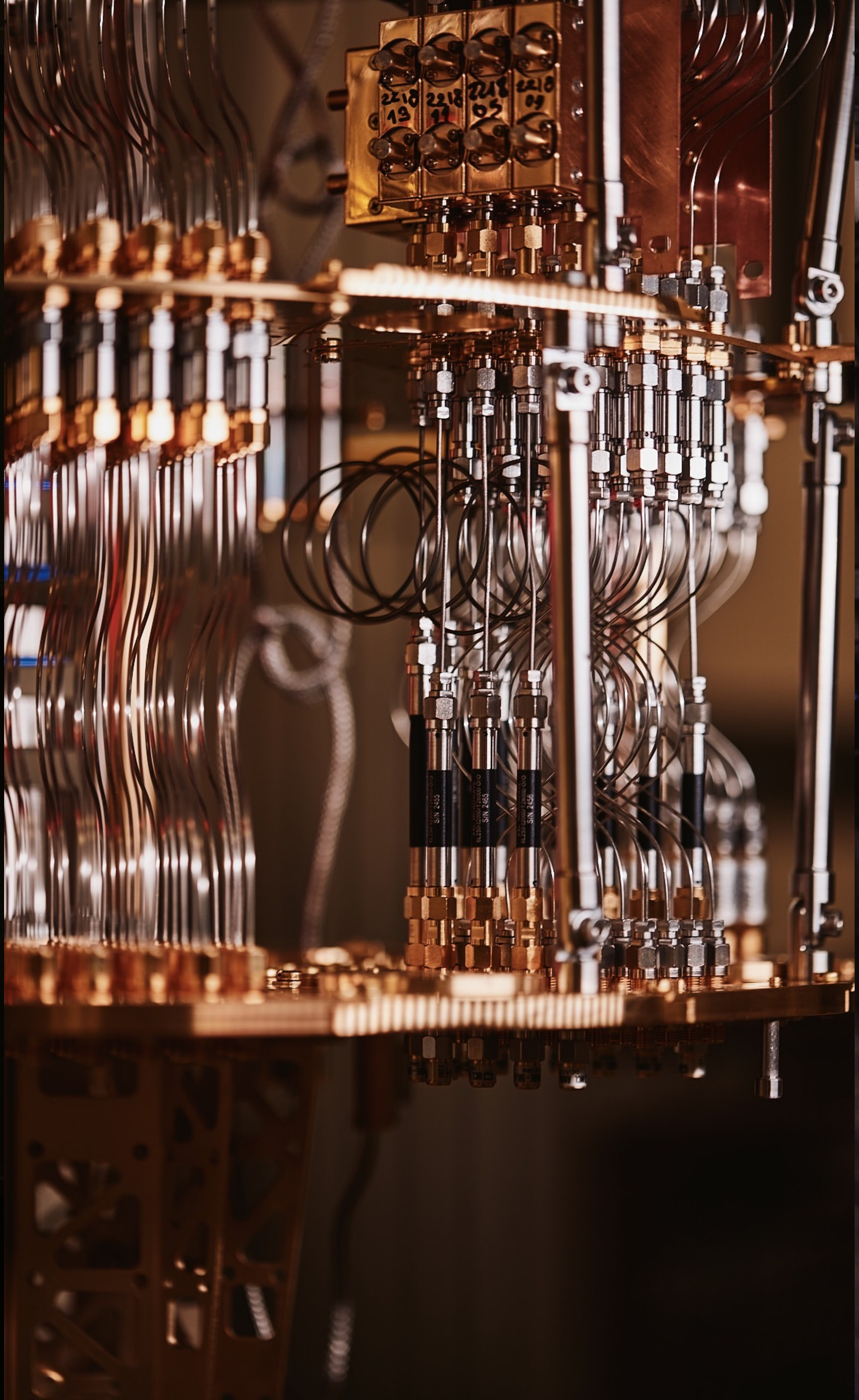
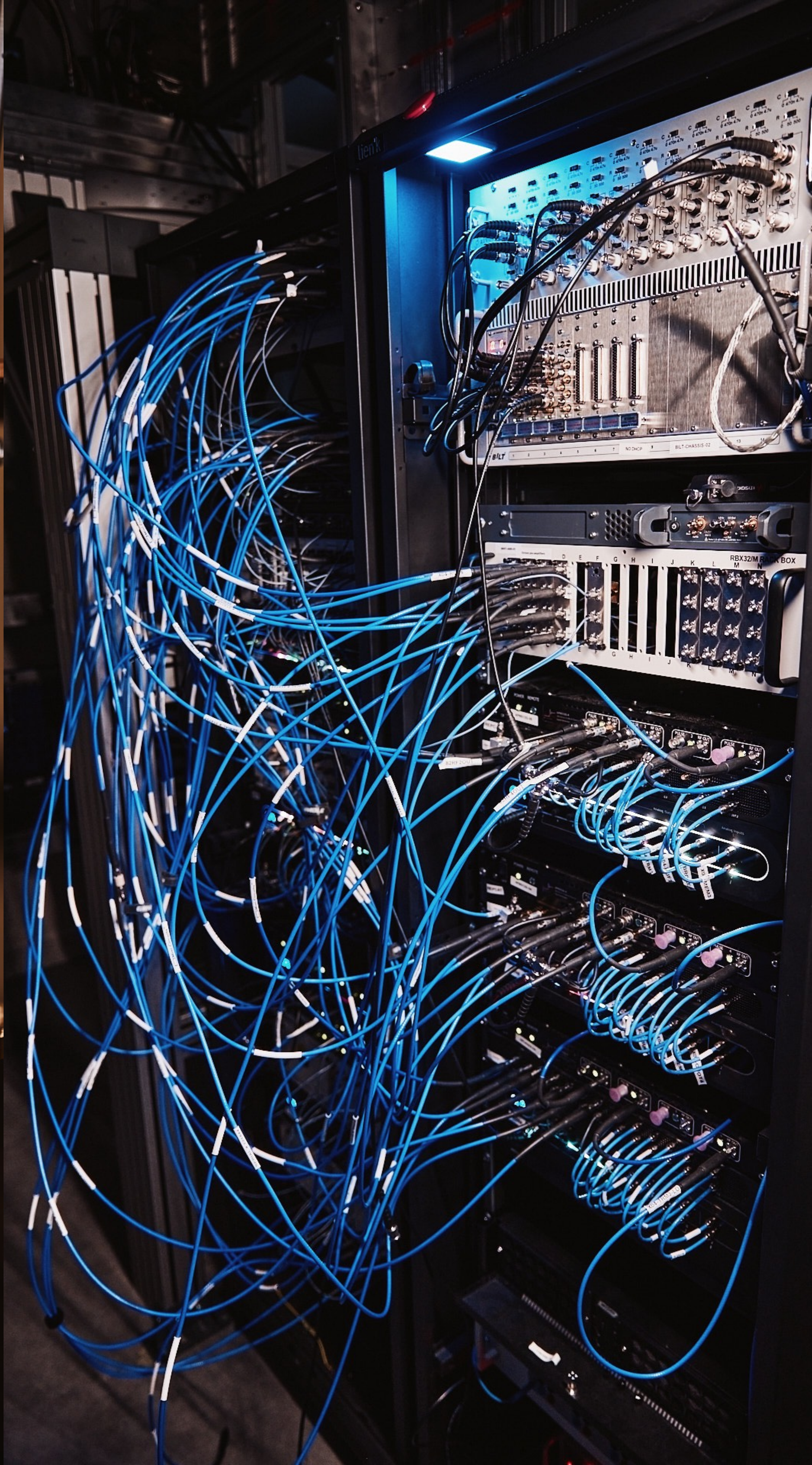


SEM picture


1 cm

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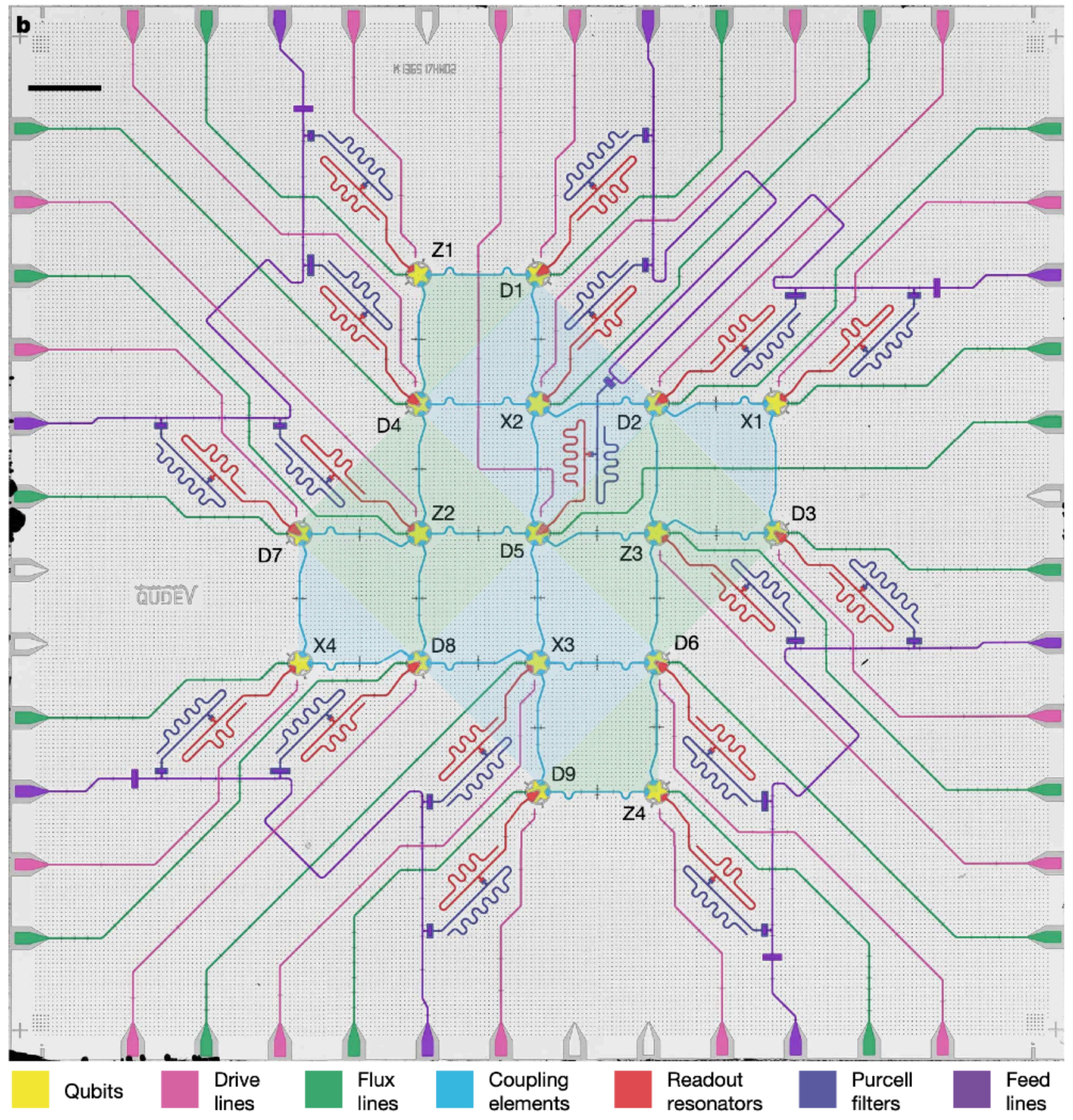

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→ **fabrication**

→ **cooldown**

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The challenge

- A double-edged sword ⚔️: you can **choose** the dynamics you want, but…
  - **Many parameters** to estimate (~10-20 for a single qubit).
  - **Large** Hilbert space dimensions (not qubits).
  - New parameters for **every new fabricated chip** (not like an atom energy levels).
  - Parameters **drift** over time (need frequent recalibration).
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- We are moving towards **larger** and **more complex** systems.
  - First applications start at ~100 logical qubits (~10^5 transmons, ~1500 cats).
  - New modeling issues, unexpected effects and more parameters to estimate.

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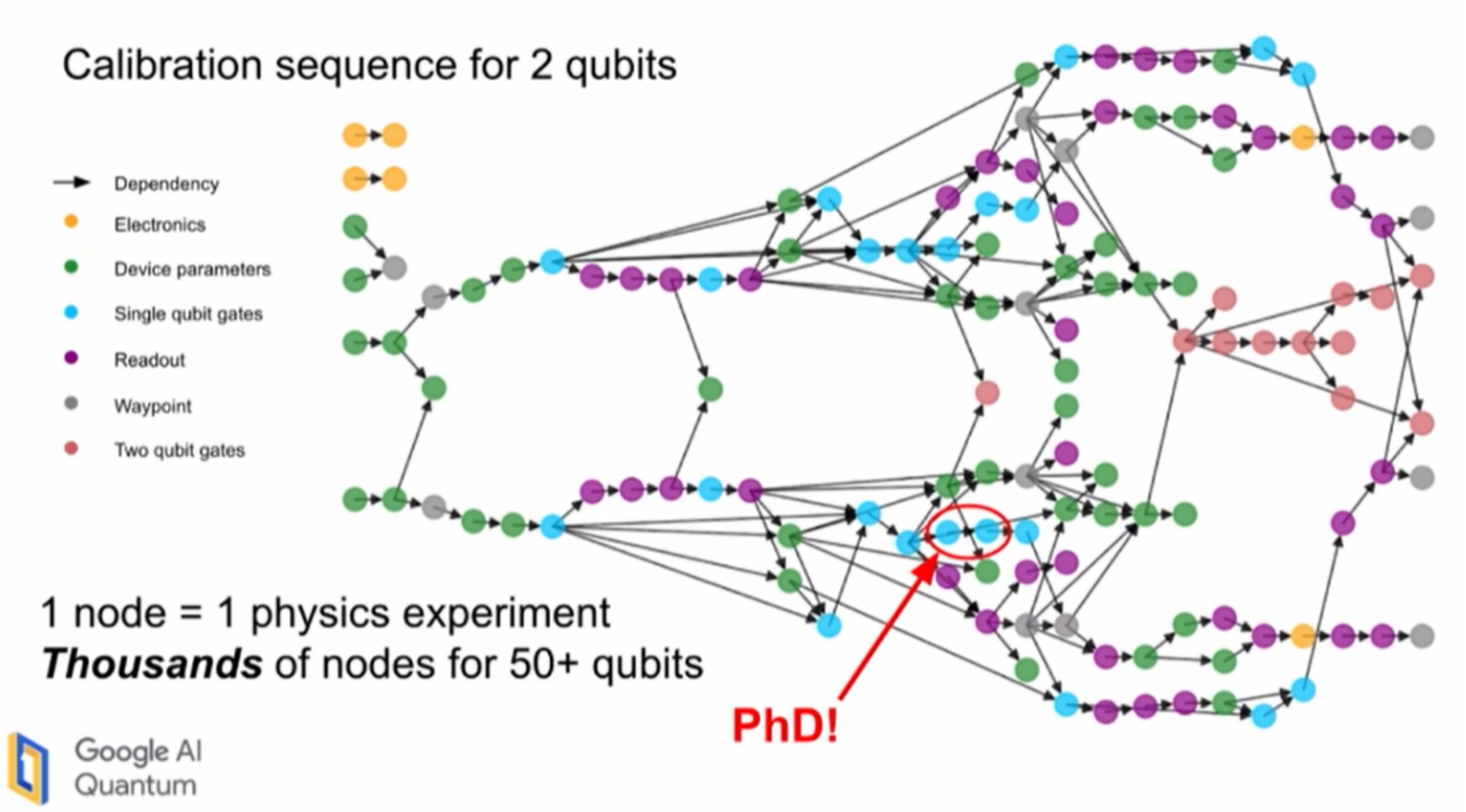
The measurement graph

- One measurement **handcrafted** for each parameter to estimate
- Based on **physical intuition**.
- Data = observables **expectation value**: $\langle O_k \rangle = \mathrm{Tr}[O_k \rho]$.



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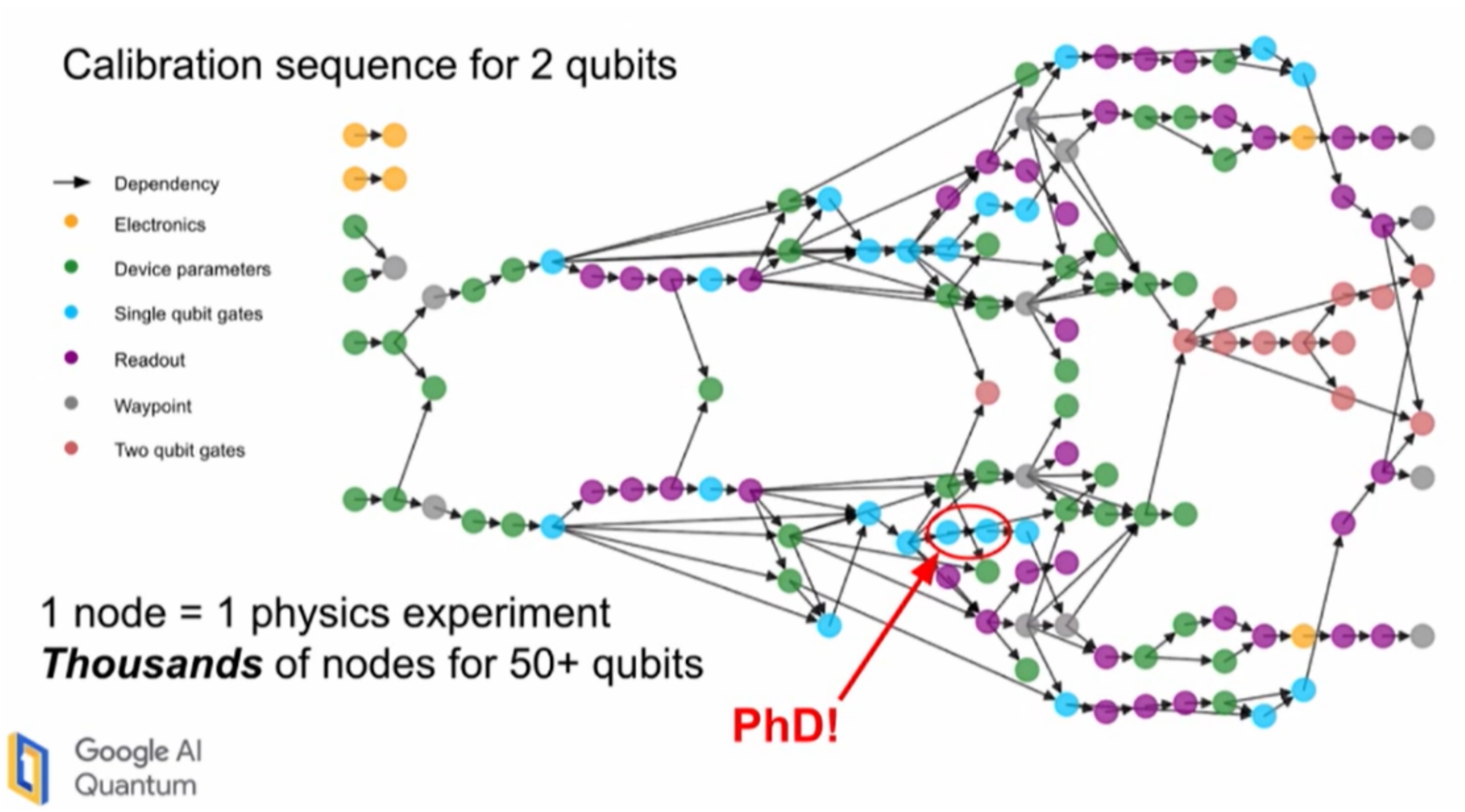

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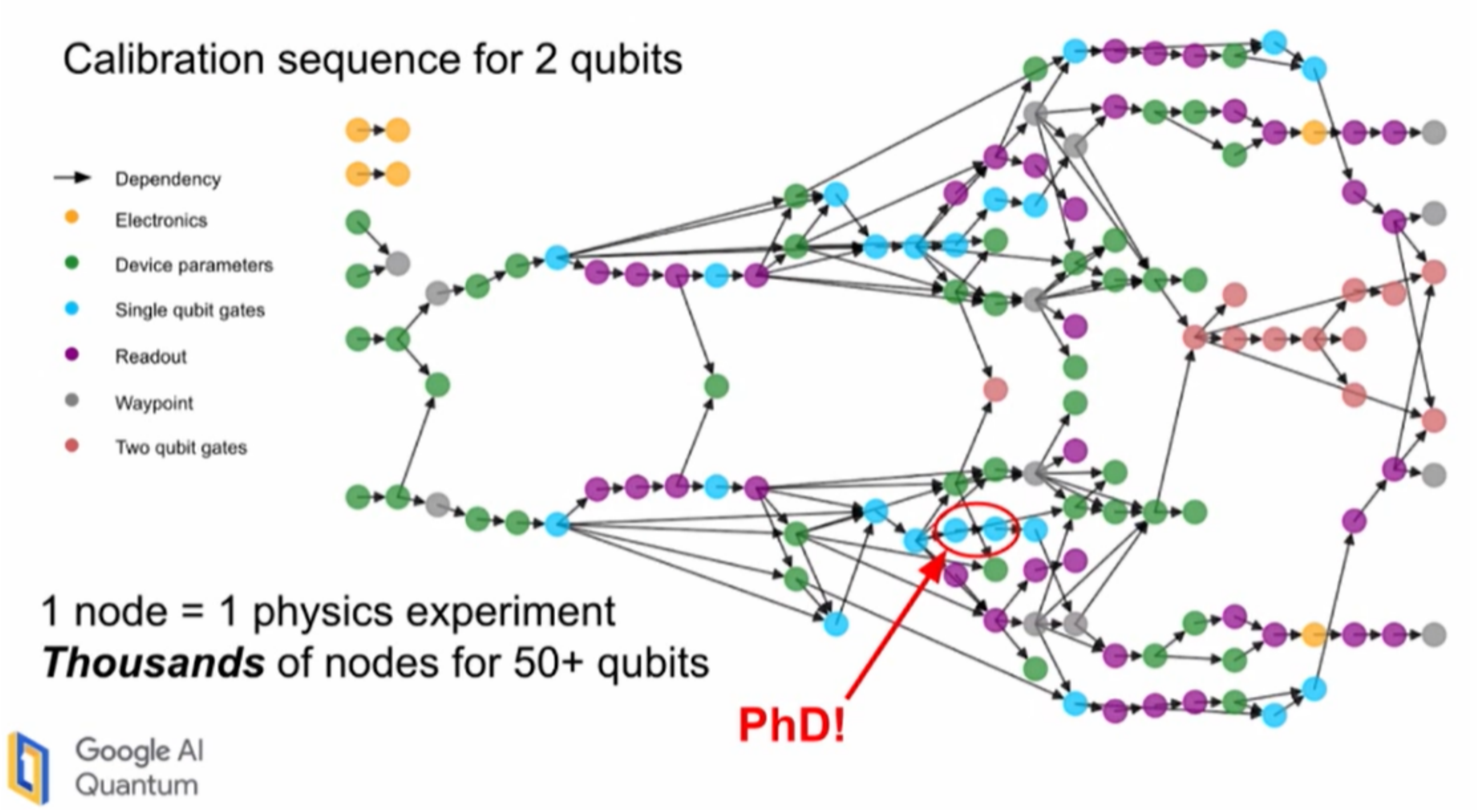

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We need more automated methods to scale to larger devices.

And for this, we need more expressive data.

Projective measurements in superconducting circuits

=

time-averaged continuous measurements

Measurements in superconducting circuits

- Typically: **heterodyne detection** of a field averaged over long times.
- Readout devices = **amazing probes**, but only used to measure σ_z projectively.
- Can sample **lots** of trajectories (e.g. $1\mu s$ measurement → 10^6 trajectories in $1s$).
- SME formalism: how the state ρ_t depends on the **signal** $I_t = \mathrm{d}Y_t/\mathrm{d}t$:

average Lindblad evolution + measurement back-action

$$\mathrm{d}\rho_t = \mathscr{L}(\rho_t)\,\mathrm{d}t + \mathscr{M}(\rho_t, \mathrm{d}Y_t)$$

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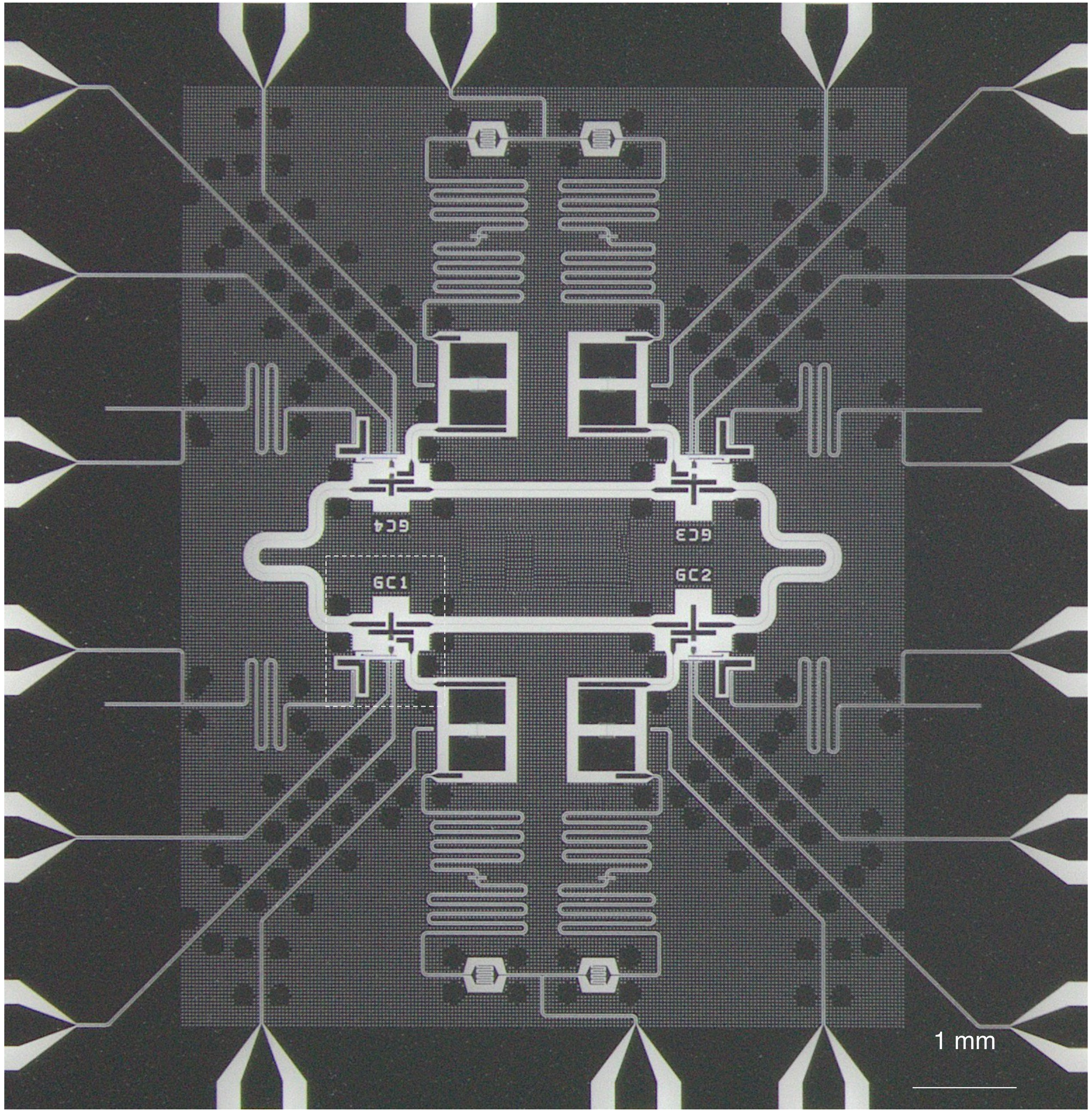
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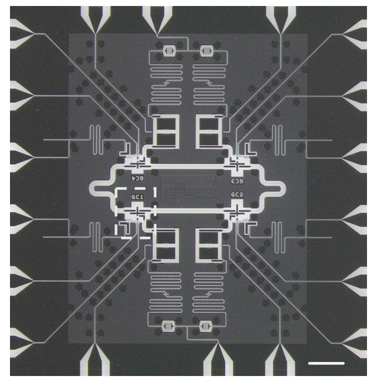

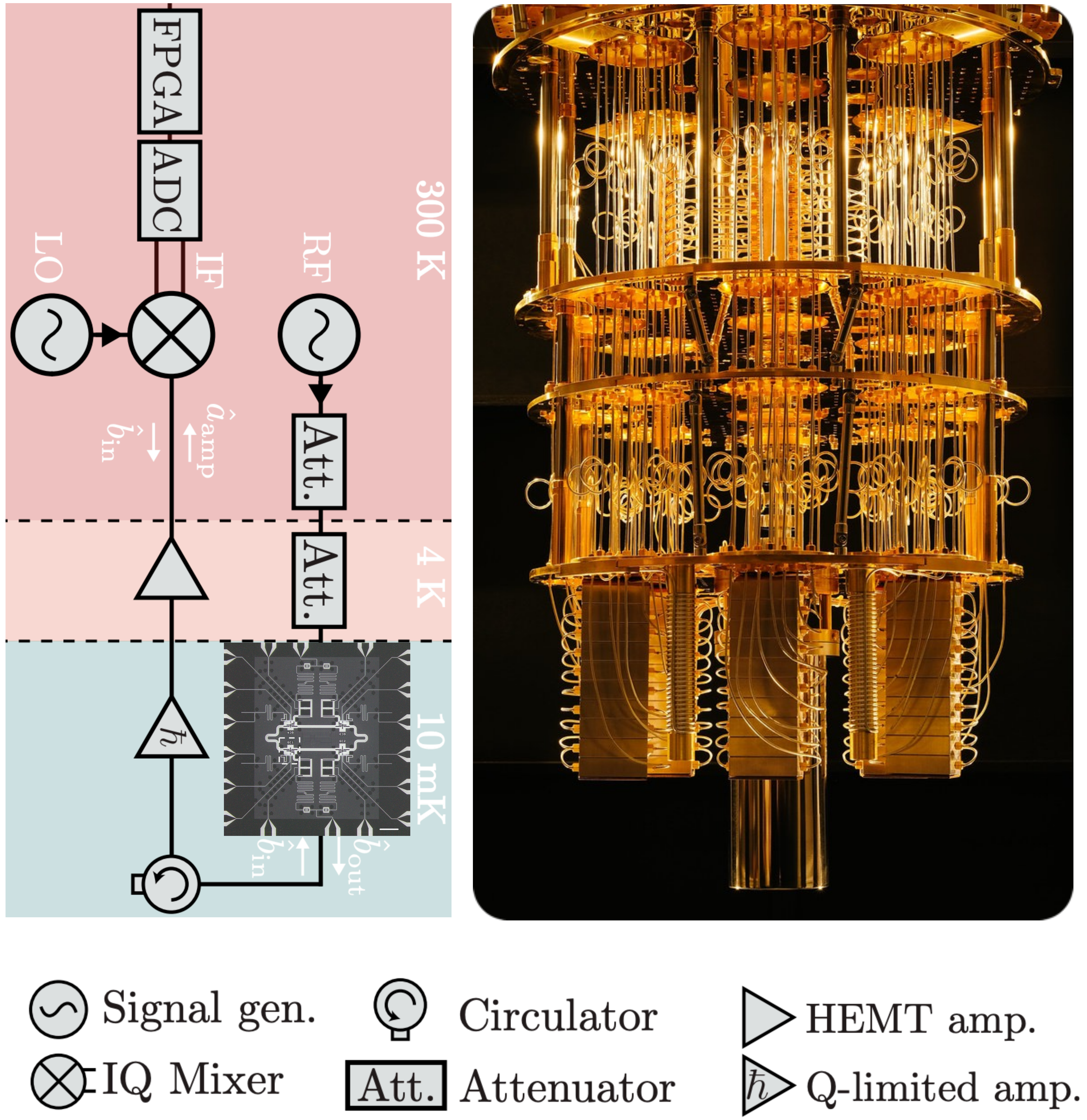

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The *actual* experimental data

- I_t is **not a measurable quantity**.
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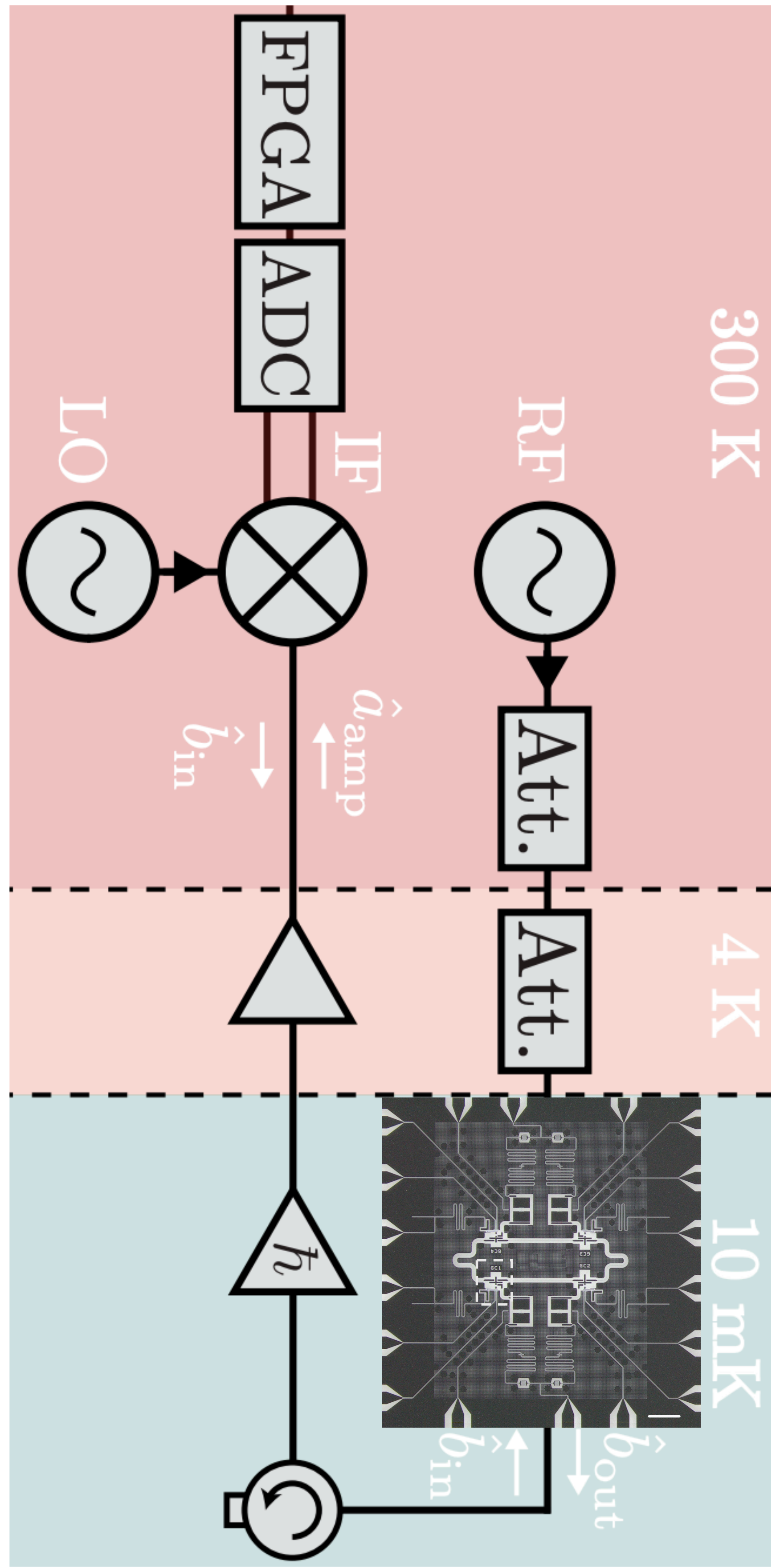
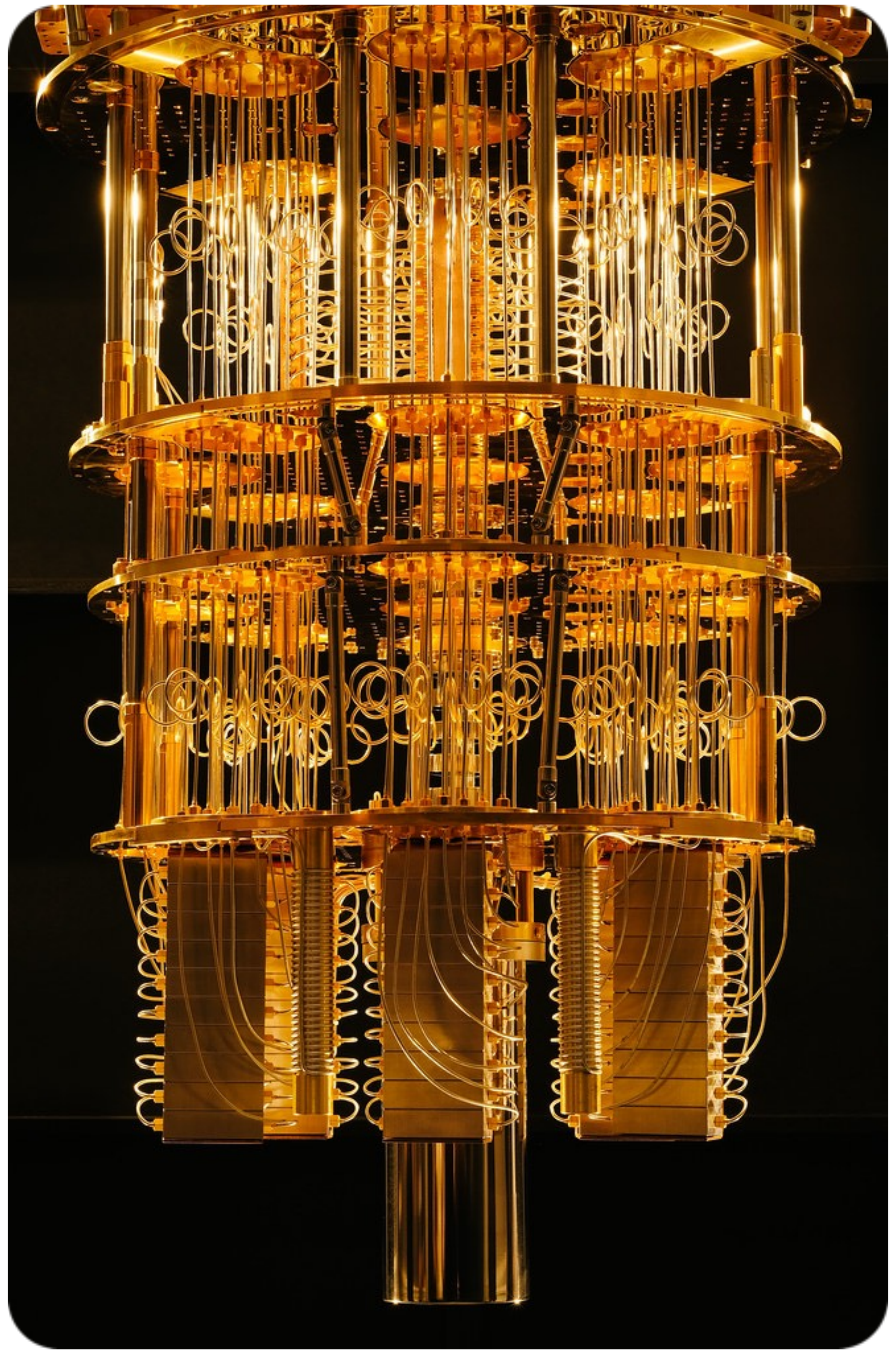
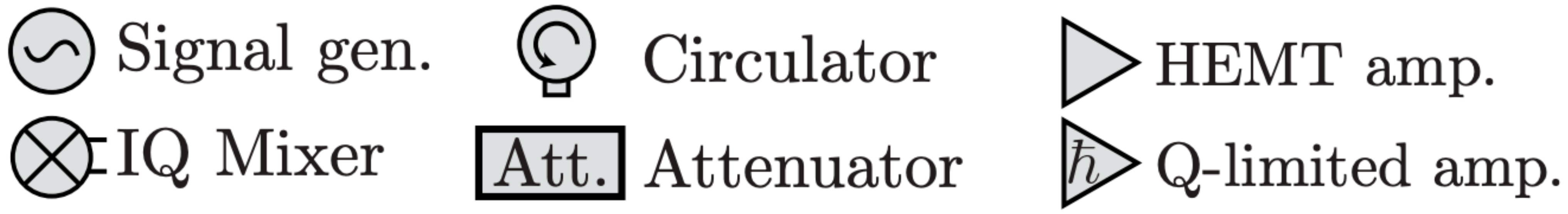
impulse response of the acquisition chain







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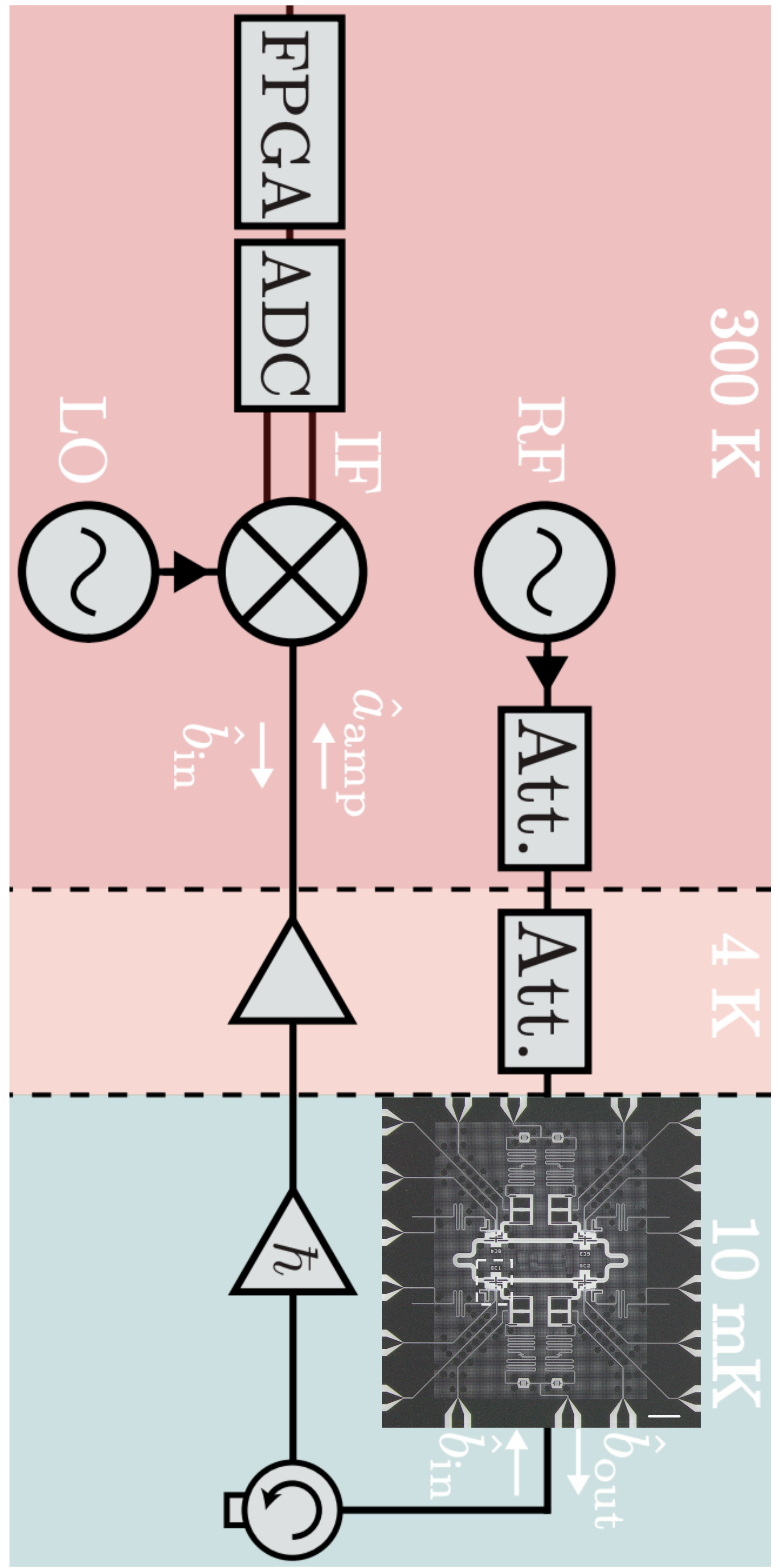
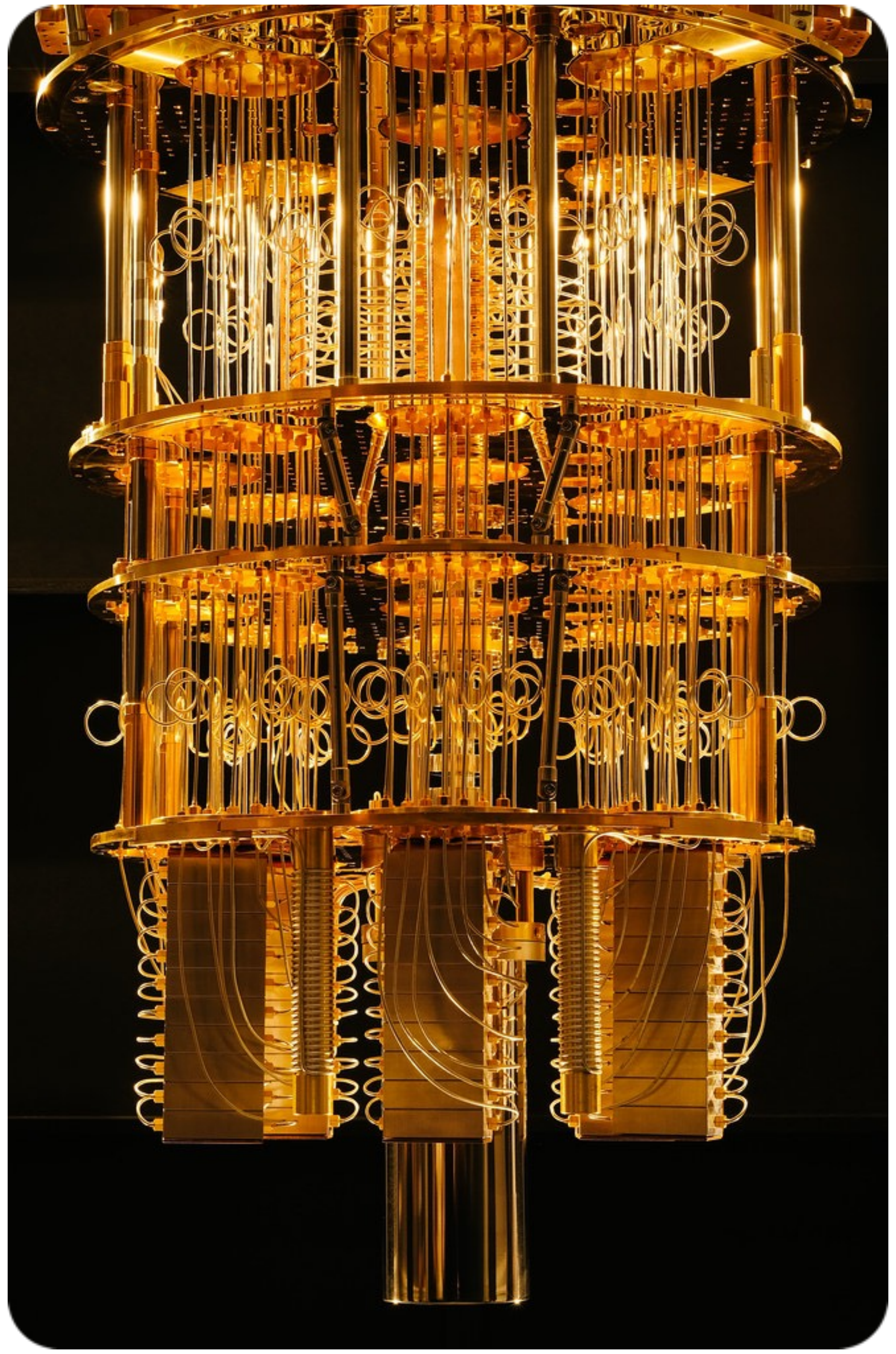
- ADC → **digitised signal** I_k.
- The *actual* data is a **discrete-time record**:

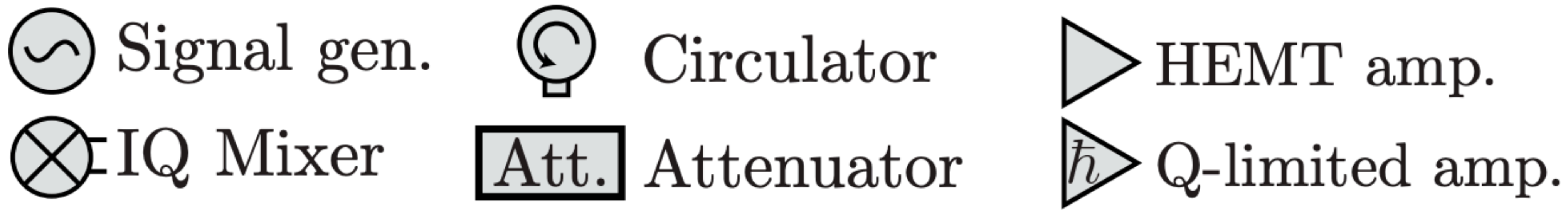
$$Y = \left\{ (I_0^{(j)}, I_1^{(j)}, \ldots, I_n^{(j)}) \right\}_{1 \leq j \leq n_{exp}}$$

Jump: $\{(0,0,0,0,0,1,0,0,1,0,\ldots), \ldots\}$

Diffusive: $\{(0.2, -0.44, 0.3, 1.2, \ldots), \ldots\}$







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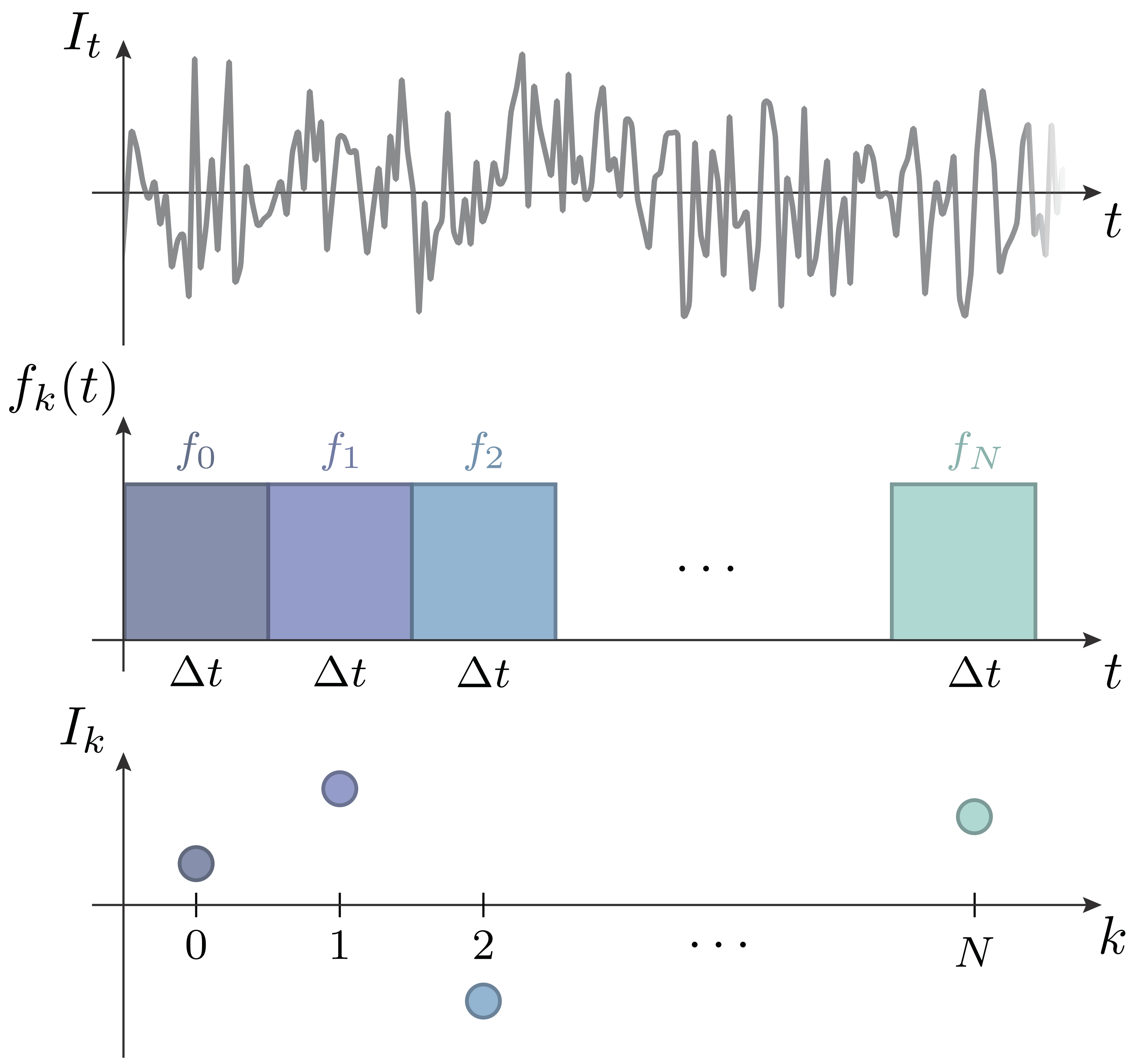
$$\{I_t\}_{t \in [0,T]} \to \{I_0, I_1, \ldots, I_N\}$$



$$I_k = \int_{k\Delta t}^{(k+1)\Delta t} f_k(t) I_t \, \mathrm{d}t \longrightarrow \{I_0, I_1, I_2, \ldots, I_N\}$$

How to estimate the system parameters from this data?

$$Y = \left\{ (I_1^{(j)}, I_2^{(j)}, \ldots, I_n^{(j)}) \right\}_{1 \leq j \leq n_{exp}} \xrightarrow{?} \theta$$

Two existing methods

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Bayesian inference

Find the parameters that maximise the likelihood of the data Y:

$$\theta = \text{argmax}_\theta \ \mathbb{P}[Y|\theta]$$

✅ Optimal use of the information.
❌ Exponential cost in the number of parameters.
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Machine Learning

Supervised
Learn to solve the inverse problem $Y \to \theta$.

Self-supervised
Learn to solve the direct problem $Y \to \rho_T$.

✅ Quite general + handle non-Markovianity.
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Shared drawback: lack of interpretability

⇒ difficult to debug and hard to trust

Let's do something simpler.

1. Introduction

Experimental QEC with superconducting circuits

2. Today's solution

Typical workflow for characterising superconducting circuits

3. A simple method

Fitting correlation functions of continuous measurement

4. Three numerical examples

Does the method actually work?

A new method

- What is hard? A practical gap between what is **predicted by the theory** and what is **observed experimentally**.

A new method

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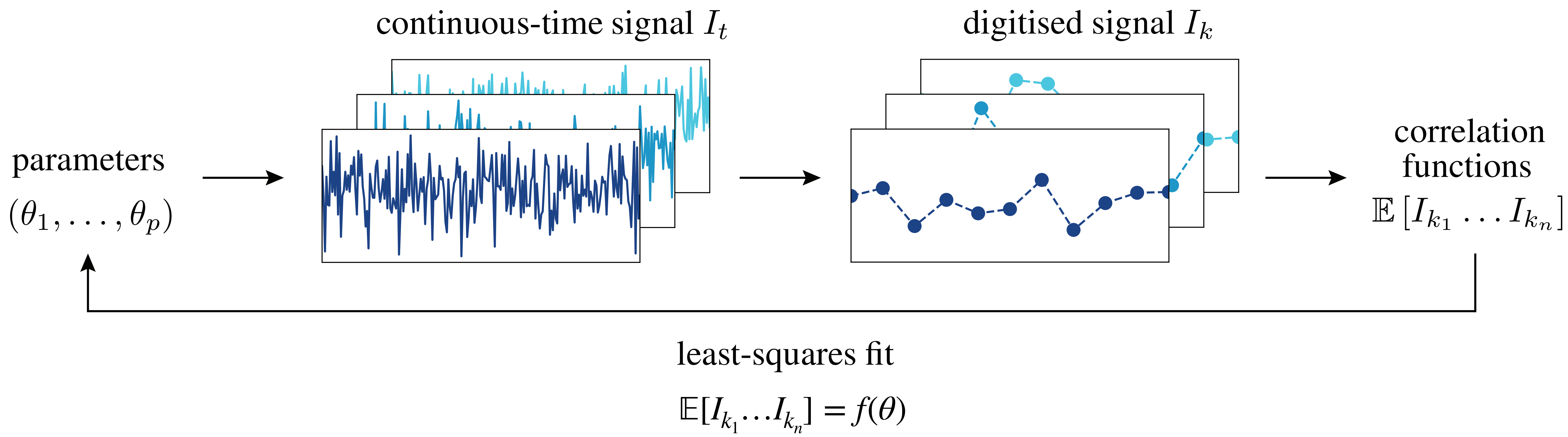
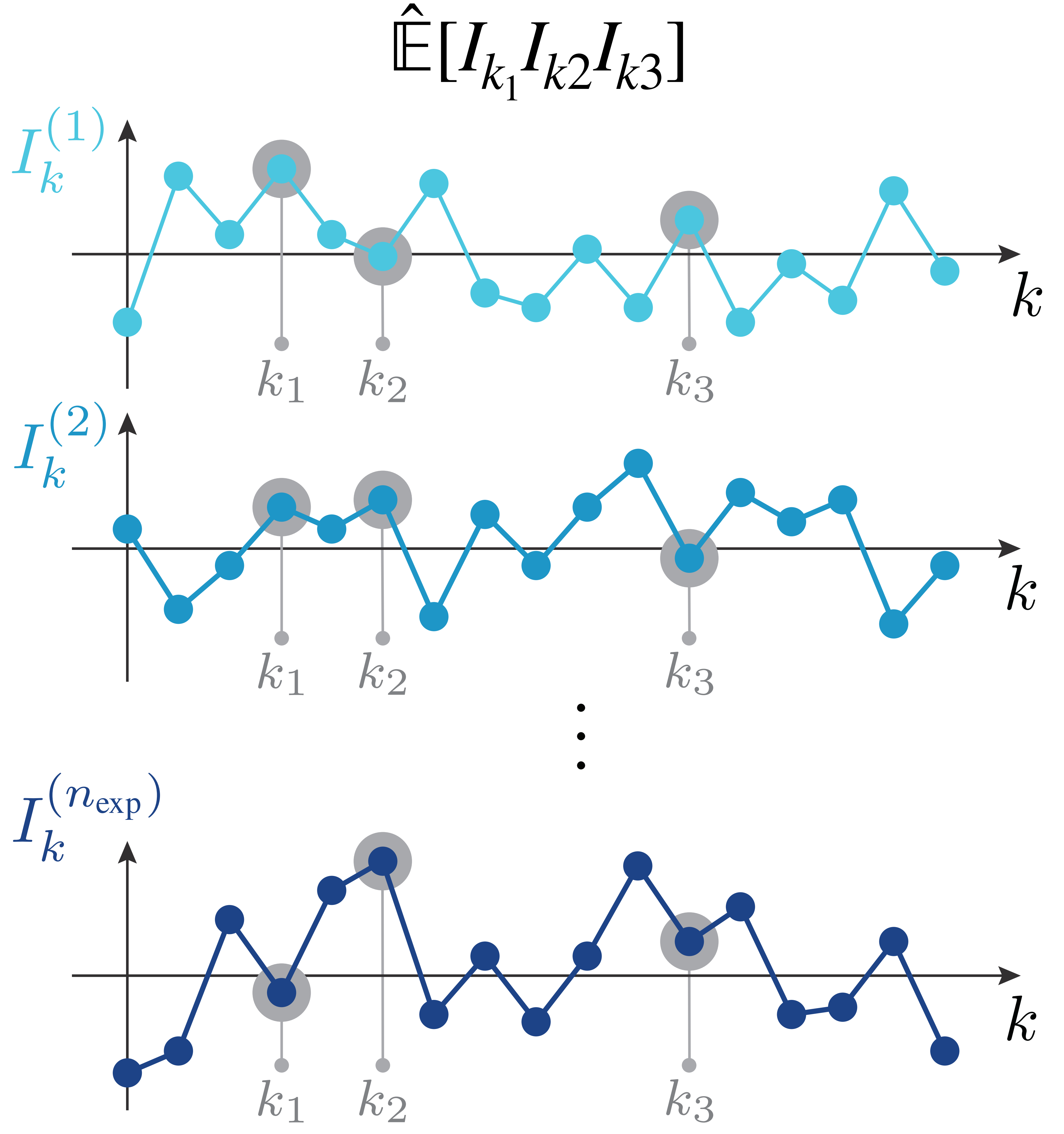
- Proposition: **fit correlation functions** $\mathbb{E}[I_{k_1} \dots I_{k_n}]$ of the signal with an **exact formula**.



What is a correlation function?

- A random variable X is fully characterized by its **moments** $\mathbb{E}[X^n]$.
- A stochastic process I_t is fully characterized by its **correlation functions** $\mathbb{E}[I_{t_1} \dots I_{t_n}]$.

$\mathbb{E}$ is the average over all quantum trajectories (⇔ over all measurement records)



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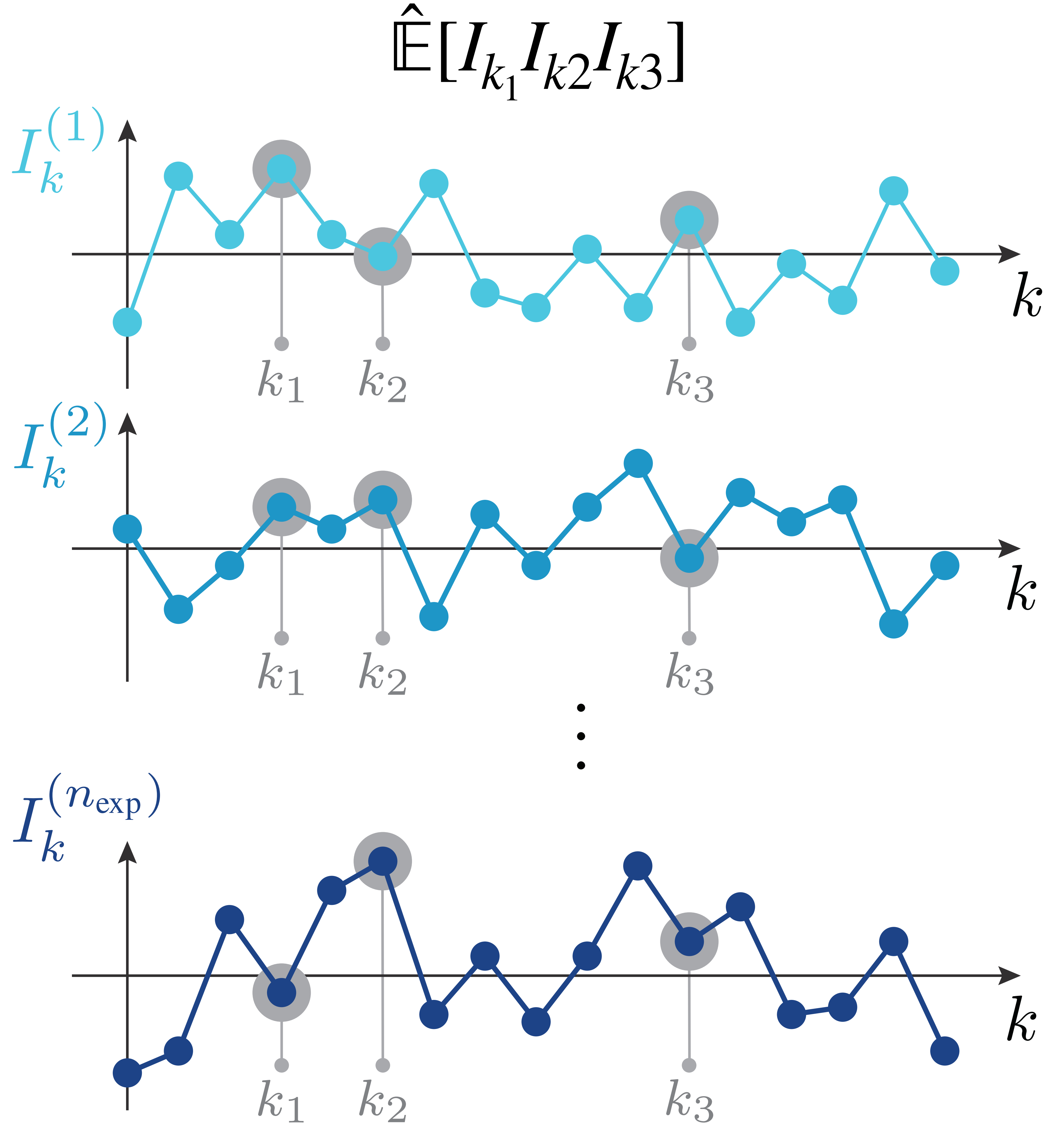

- Reduced data:

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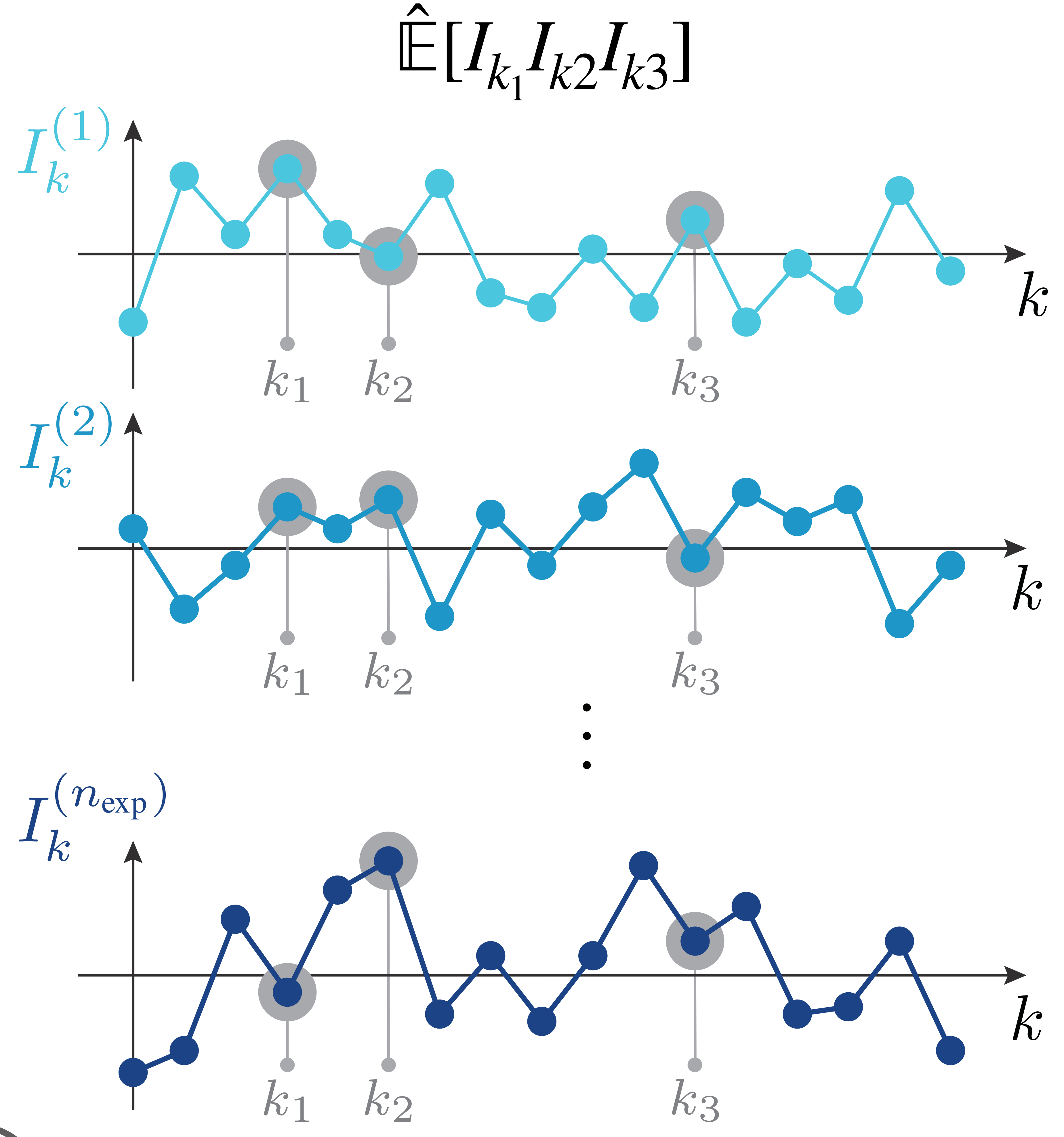

- Reduced data:
  - multiple correlators
  - estimates: $\mathbb{E} \to \hat{\mathbb{E}}$

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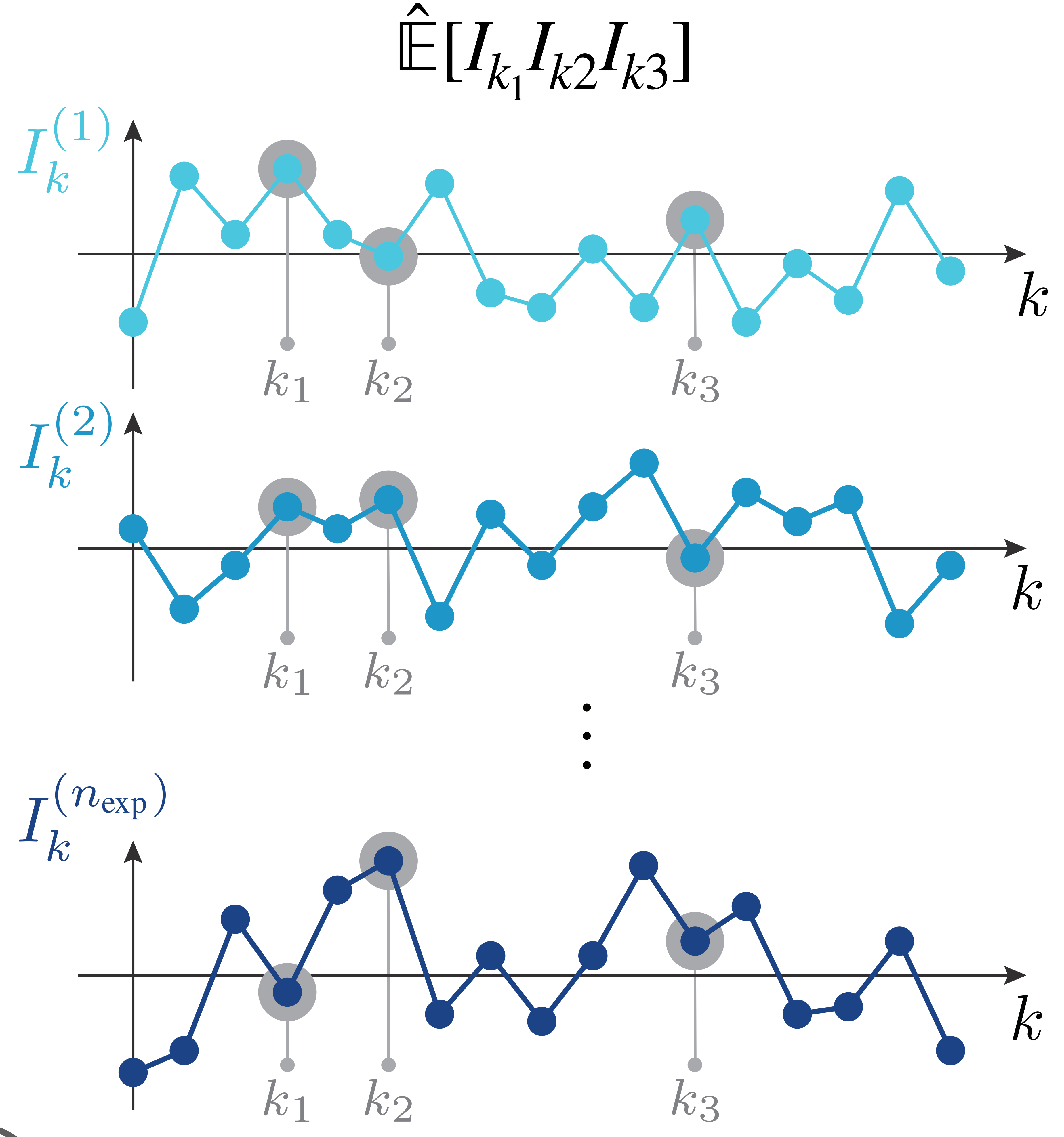

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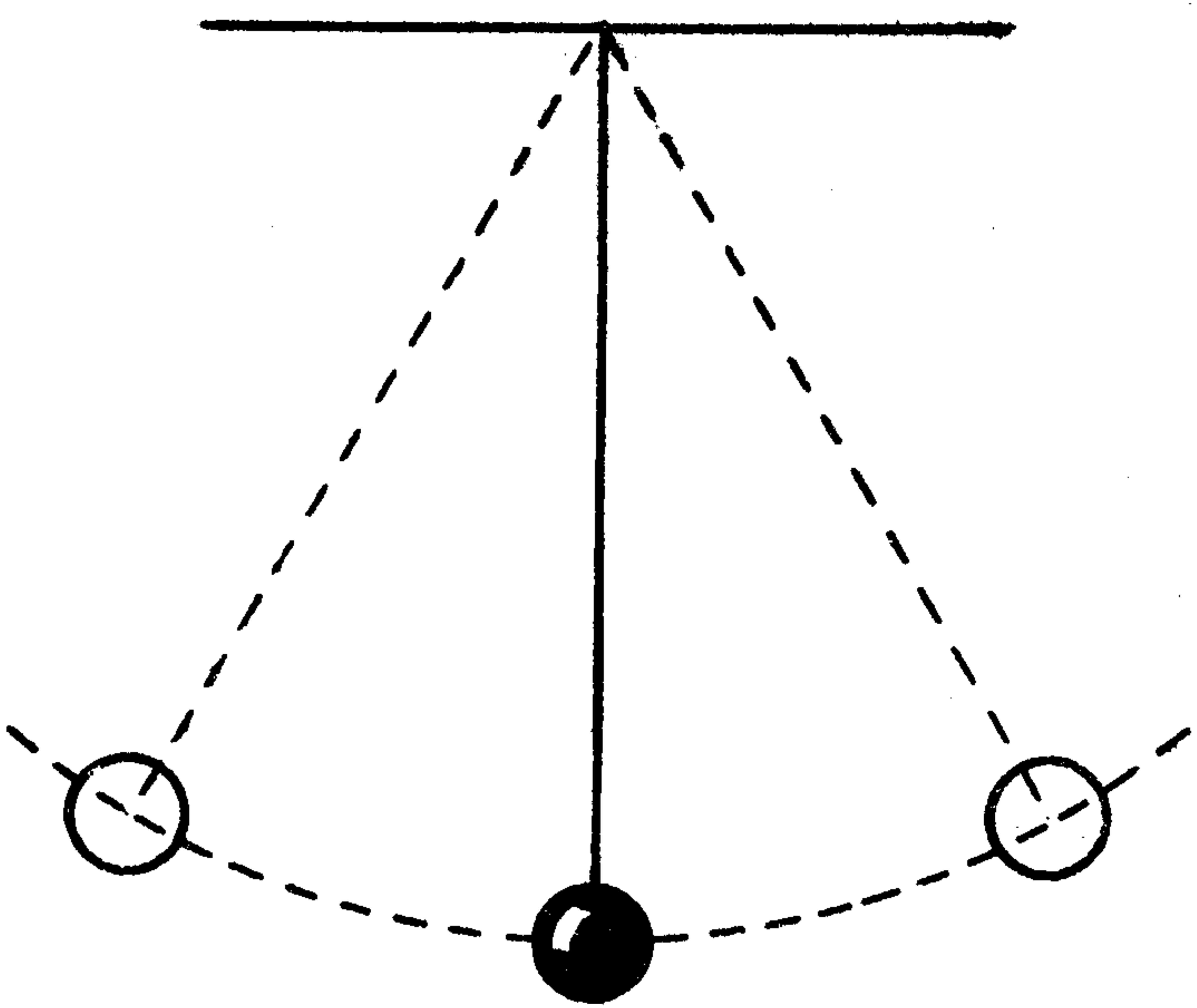
$N = 250,\ n_{exp} = 10^6$
⇒ 250×10^6 data points

1-point + 2-point ($k_1 = 0$)
⇒ 500 data points

The pendulum analogy

The pendulum analogy



Theory 1/3 (history)

- **Naïve way** to compute correlation functions: simulate the SME → expensive and imprecise.
- There is **an exact formula** for $\mathbb{E}[I_{k_1}\dots I_{k_n}]$ that only depends on the SME and the initial state!

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- History:
  - **2018**: paper by Antoine Tilloy

    ↪ First proof in the diffusive case (linear SME + Girsanov's theorem).
  - **2023**: paper by Pierre^2 and Antoine

    ↪ Simpler proof (in discrete-time) for both jump and diffusive + efficient numerics.
  - **2024**: paper by Pierre^2 and Antoine

    ↪ Shorter proof (using Itô's lemma) + demonstration on numerical examples.

Tilloy, Antoine. "Exact signal correlators in continuous quantum measurements." Physical Review A 98.1 (2018): 010104.
Guilmin, Pierre, Pierre Rouchon, and Antoine Tilloy. "Correlation functions for realistic continuous quantum measurement." IFAC-PapersOnLine 56.2 (2023): 5164-5170.
Guilmin, Pierre, Pierre Rouchon, and Antoine Tilloy. "Parameters estimation by fitting correlation functions of continuous quantum measurement." arXiv preprint arXiv:2410.11955 (2024).

$$\mathcal{C}_L(\bullet) = \theta \bullet + \eta L \bullet L^\dagger \qquad \text{for the jump SME,}$$
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Theory 2/3 (formulas)

Sharp signal I_t

- Lindblad evolution interspersed with the **correlation superoperator** $\mathcal{C}_L$.

$$\mathbb{E}[I_{t_1} \dots I_{t_n}] = \mathrm{Tr}\Big[\mathcal{C}_L e^{(t_n - t_{n-1})\mathcal{L}} \dots \mathcal{C}_L e^{t_1 \mathcal{L}}(\rho_0)\Big]$$

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Filtered signal I_f

- Solve a system of coupled linear ODEs with an augmented state.
- Lindblad with additional **source terms**.
- Numerical cost = **solving Lindblad**! And cheap gradient $\nabla_\theta \mathbb{E}[I_{k_1} \dots I_{k_n}]$! (bwd. diff. in $\mathcal{O}(1)$ w.r.t p)

$$\mathbb{E}[I_{f_1} I_{f_2}] = \mathrm{Tr}[\rho_T^{12}]$$

$$\begin{bmatrix} \dot\rho \\ \dot\rho^1 \\ \dot\rho^2 \\ \dot\rho^{12} \end{bmatrix} = \begin{bmatrix} \mathcal{L} & 0 & 0 & 0 \\ f_1\, \mathcal{C}_L & \mathcal{L} & 0 & 0 \\ f_2\, \mathcal{C}_L & 0 & \mathcal{L} & 0 \\ f_1 f_2 & f_2\, \mathcal{C}_L & f_1\, \mathcal{C}_L & \mathcal{L} \end{bmatrix} \begin{bmatrix} \rho \\ \rho^1 \\ \rho^2 \\ \rho^{12} \end{bmatrix}$$

for T s.t. $\forall t \geq T, f_1(t) = f_2(t) = 0$

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Bridges the gap!

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Theory 3/3 (proof bits)

- **Moment-generating function** for X. $\mathbb{E}[X^n] = \frac{\mathrm{d}^n}{\mathrm{d}t^n} Z(t)\Big|_{t=0}$ $\quad Z(t) = \mathbb{E}[e^{tX}]$

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- Observe that $\mathcal{Z}(j) = \mathrm{Tr}[\rho^j_\infty]$ and apply **Itô's lemma** to find that ρ_t^j obeys a simple linear differential equation!

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$$\implies \quad \mathcal{Z}(j) = \mathrm{Tr}\left[\mathscr{T}\exp\left(\int_0^\infty \mathcal{L}^j_t \,\mathrm{d}t\right)(\rho_0)\right]$$

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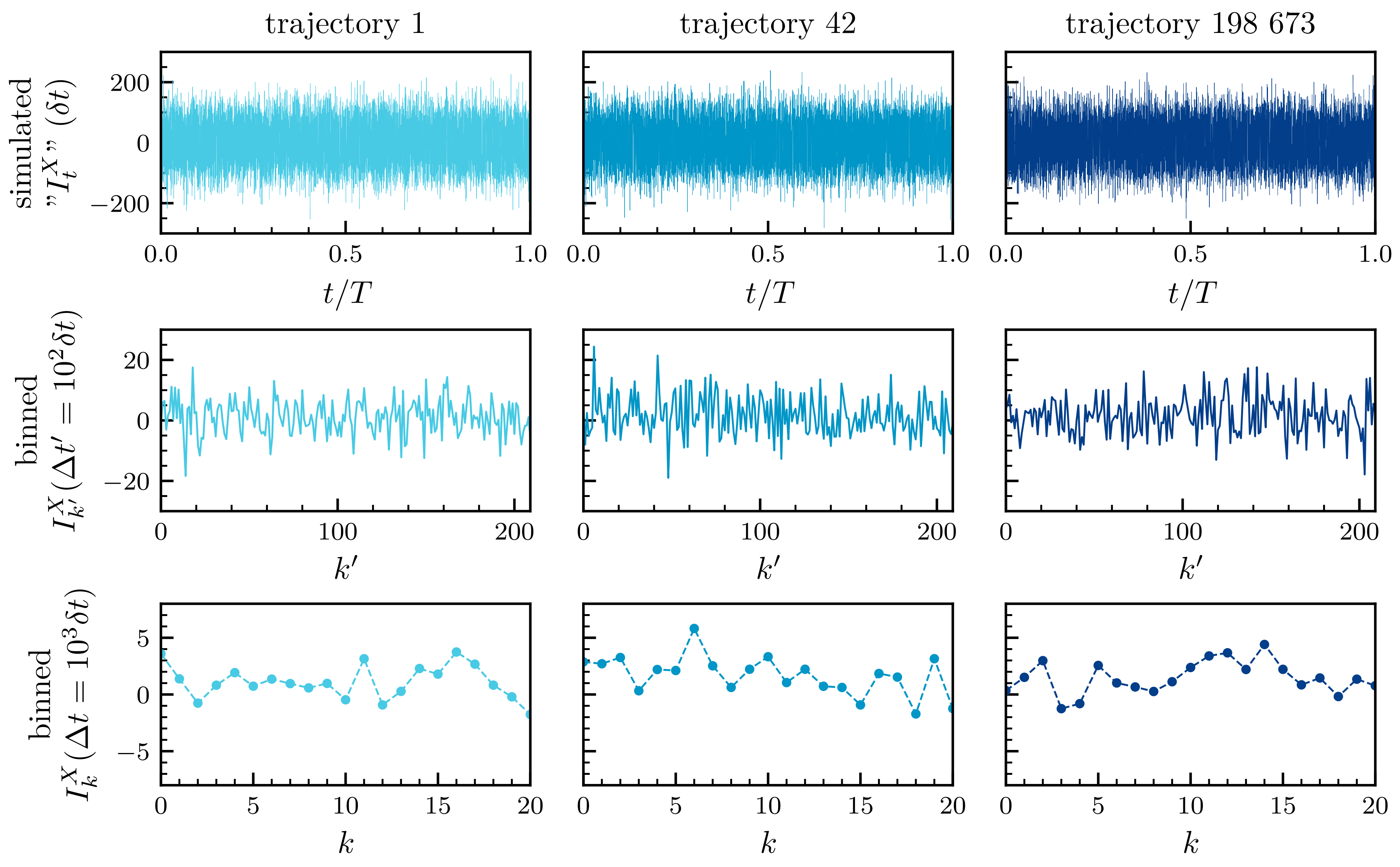
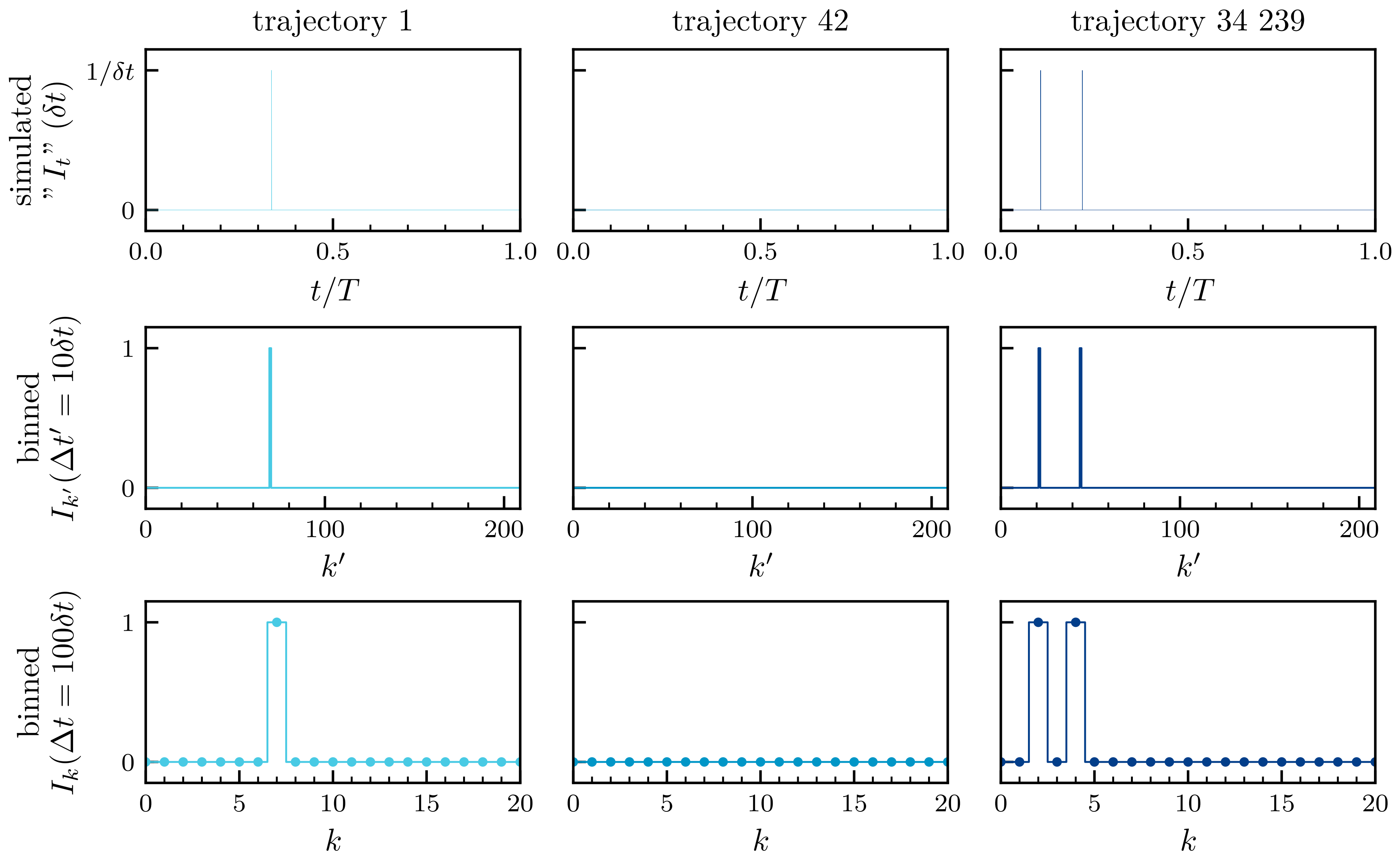
Does it really work?

Numerical results

- Three examples.
- Generate **fake experimental data** by **simulating** the SME for 10^5 or 10^6 trajectories.
- Simulate with a small δt and **average** the signal on $\Delta t = 10^3 \delta t$.
- **Least-squares fit** (with SciPy).

  ↪ Less than a minute on a regular CPU.





Numerical results — example 1/3

- Driven anharmonic oscillator:

$$H = -K/2\, a^{\dagger 2} a^2 + \epsilon^* a + \epsilon a^\dagger,$$
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$$\mathrm{d}Y_t^X = \sqrt{\eta\kappa/2}\,\mathrm{Tr}\big[(a + a^\dagger)\rho_t\big]\mathrm{d}t + \mathrm{d}W_t^X,$$
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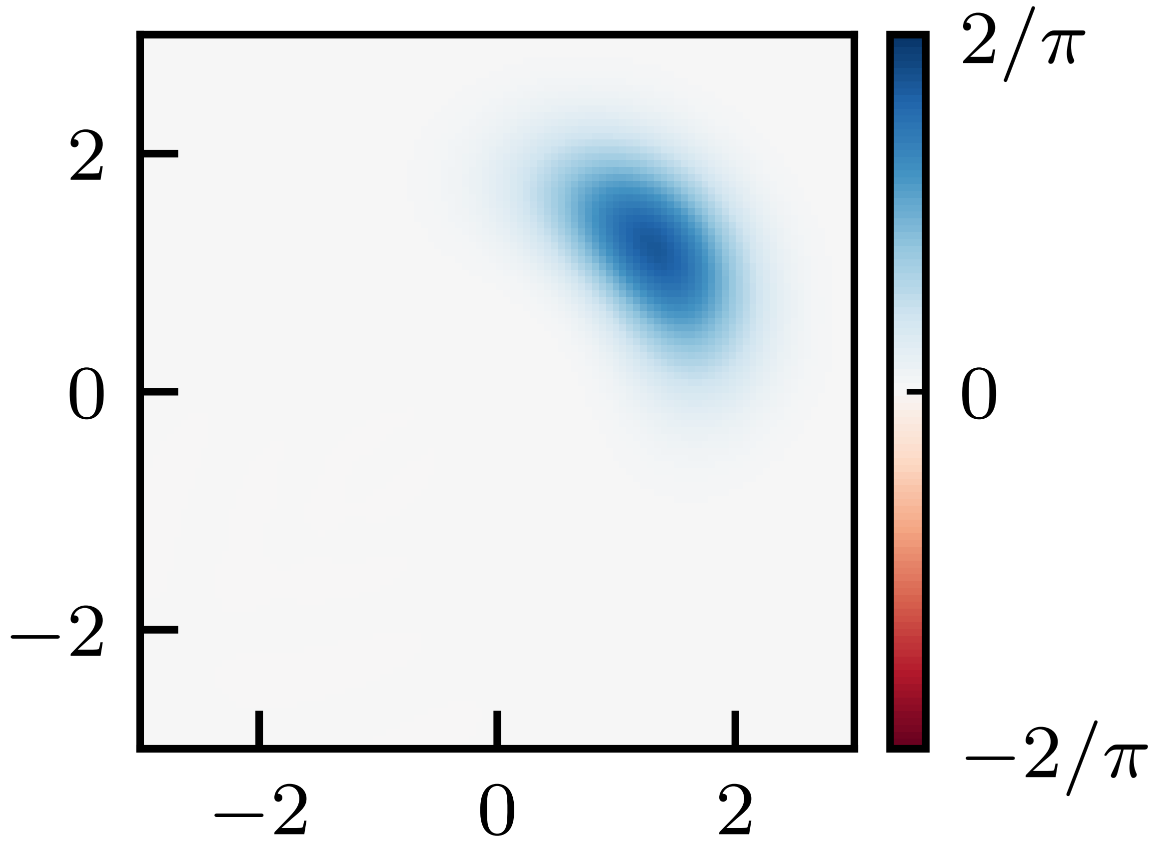

- In the steady state ρ_∞ (fitted as well).
- **Which experiments** would *you* do to estimate $\theta = (K, \epsilon_x, \epsilon_y, \eta)$?

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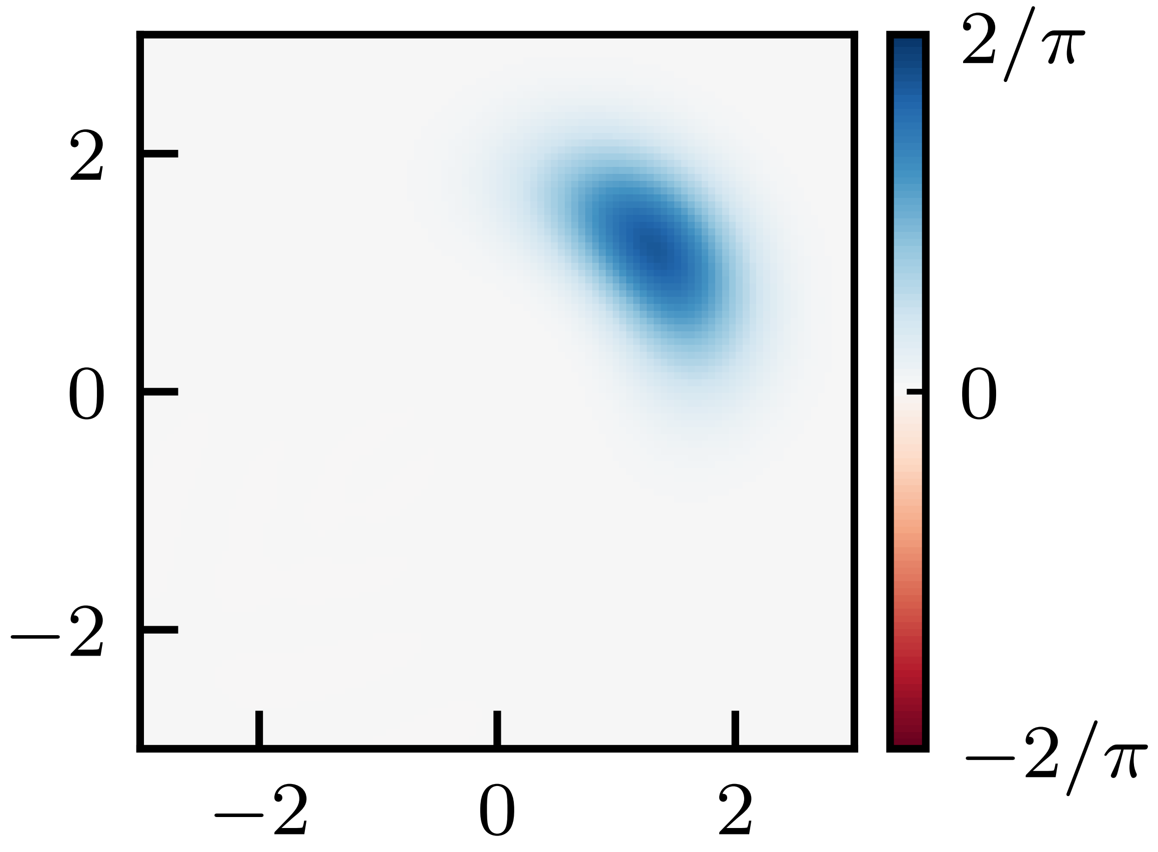

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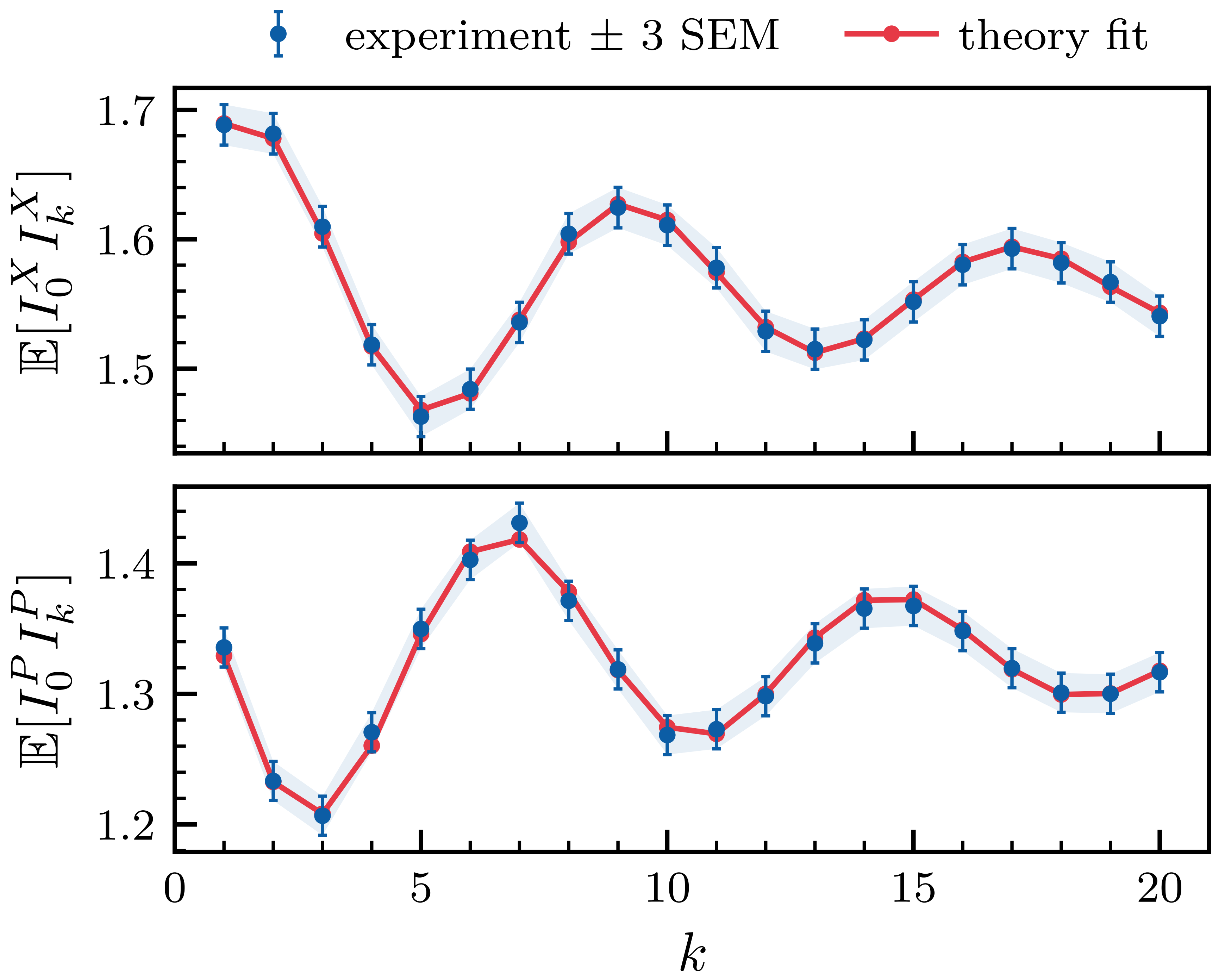
1-point = a single value in ρ_∞

- Just one by fitting the 2-point!



	Parameter	Value	Estimated
$K/(2\pi)$	self-Kerr	100 kHz	100 ± 1 kHz
$\epsilon_x/(2\pi)$	drive (real)	300 kHz	299 ± 2 kHz
$\epsilon_y/(2\pi)$	drive (imaginary)	400 kHz	398 ± 2 kHz
$\kappa/(2\pi)$	photon loss rate	100 kHz	–
η	efficiency	0.8	0.80 ± 0.01

Numerical results — example 2/3

- Driven two-level system:

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$$L = \sqrt{\gamma}\,\sigma_-.$$

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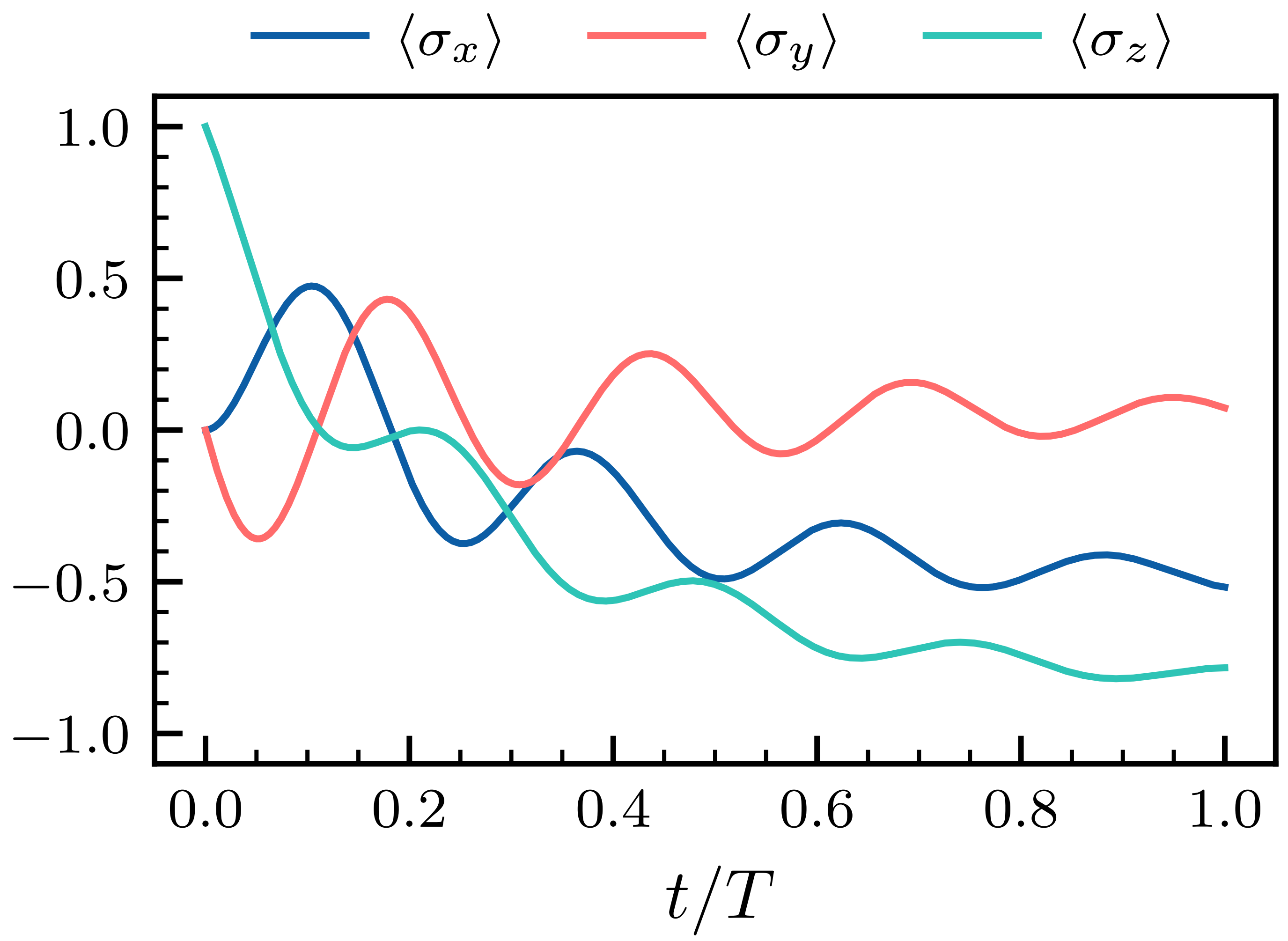
- Starting from $\rho = |e\rangle\langle e|$.

$$I_k = \int_{k\Delta t}^{(k+1)\Delta t} \mathrm{d}N_t$$



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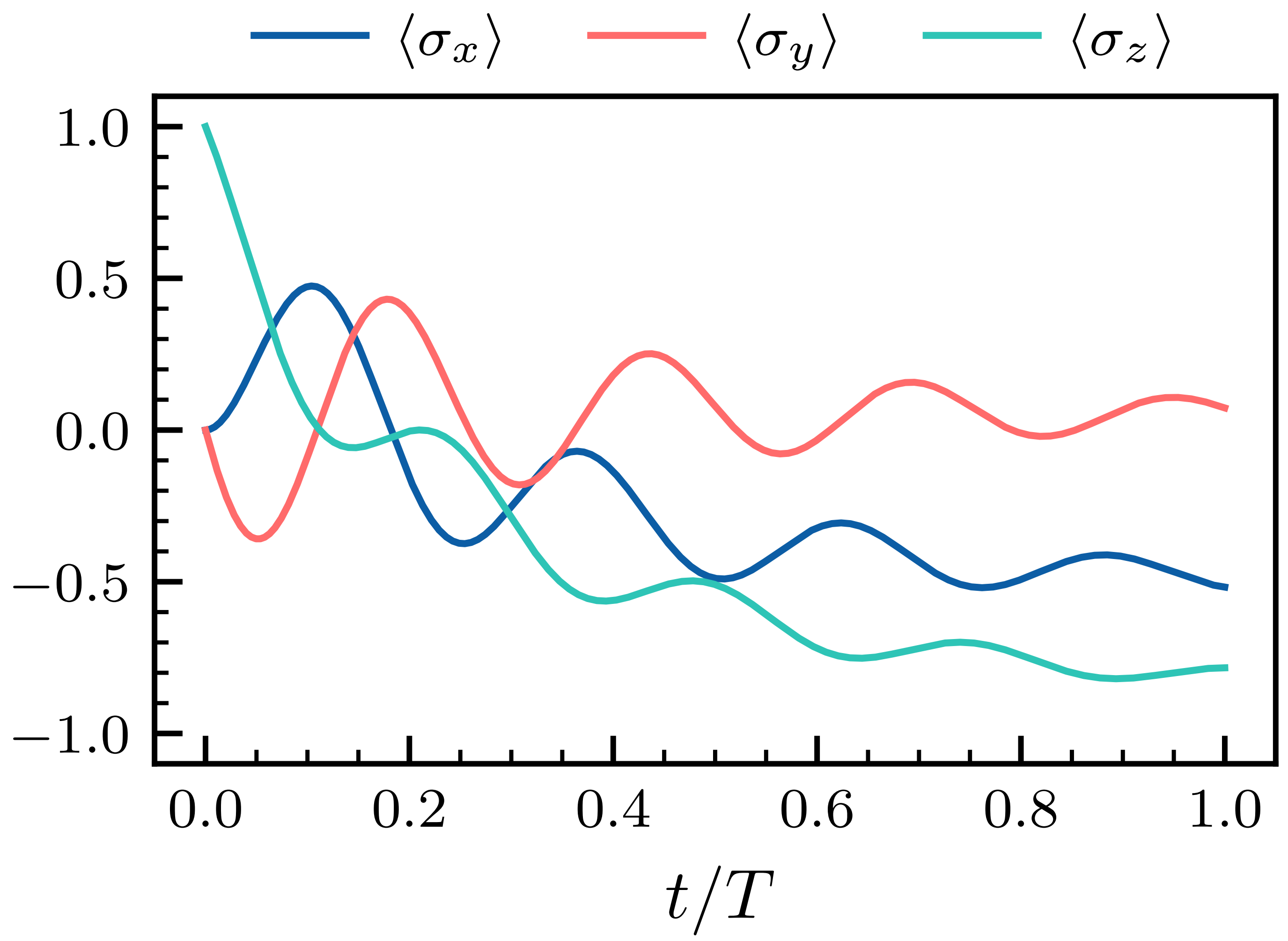
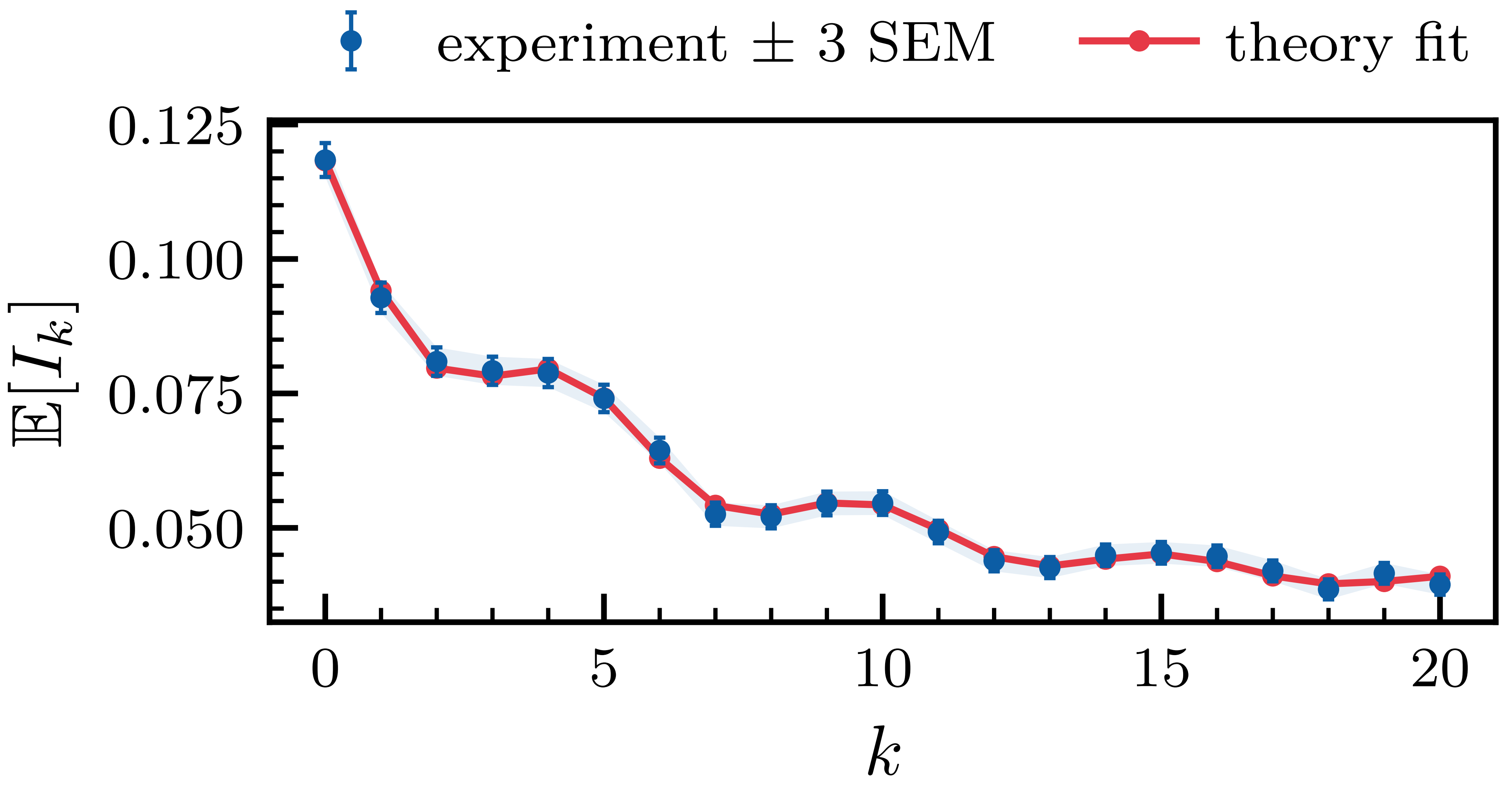


- **Which experiments** would *you* do to estimate $\theta = (\Delta, \Omega, \gamma, \theta, \eta)$?

- Just one by fitting the 1-point!



	Parameter	Value	Estimated
$\Delta/(2\pi)$	detuning	5 kHz	5.04 ± 0.03 kHz
$\Omega/(2\pi)$	drive	3 kHz	2.99 ± 0.05 kHz
$\gamma/(2\pi)$	loss rate	2 kHz	2.03 ± 0.06 kHz
$\theta/(2\pi)$	dark count rate	300 Hz	307 ± 7 Hz
η	efficiency	0.5	0.49 ± 0.01

Error bars by subsampling.

Numerical results — example 3/3

- Inspired by *C. Berdou et al*, ***One Hundred Second Bit-Flip Time in a Two-Photon Dissipative Oscillator*** (PRXQ, 2023).
- Dissipative cat qubit:

$$
\begin{aligned}
H &= 0, \\
L_1 &= \sqrt{\kappa_1}\, a, \\
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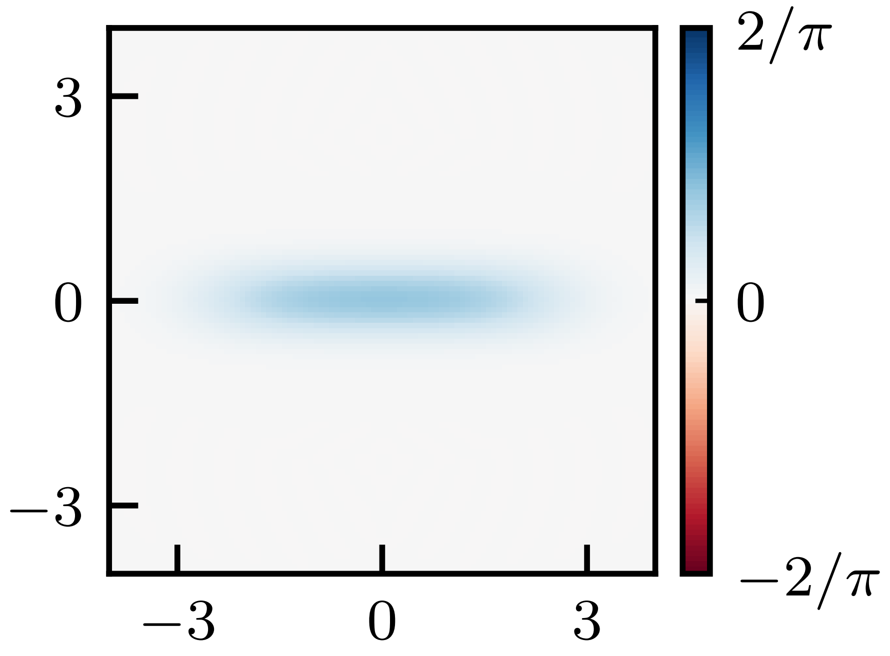
  - Homodyne measurement gives a **binned signal** I_k:

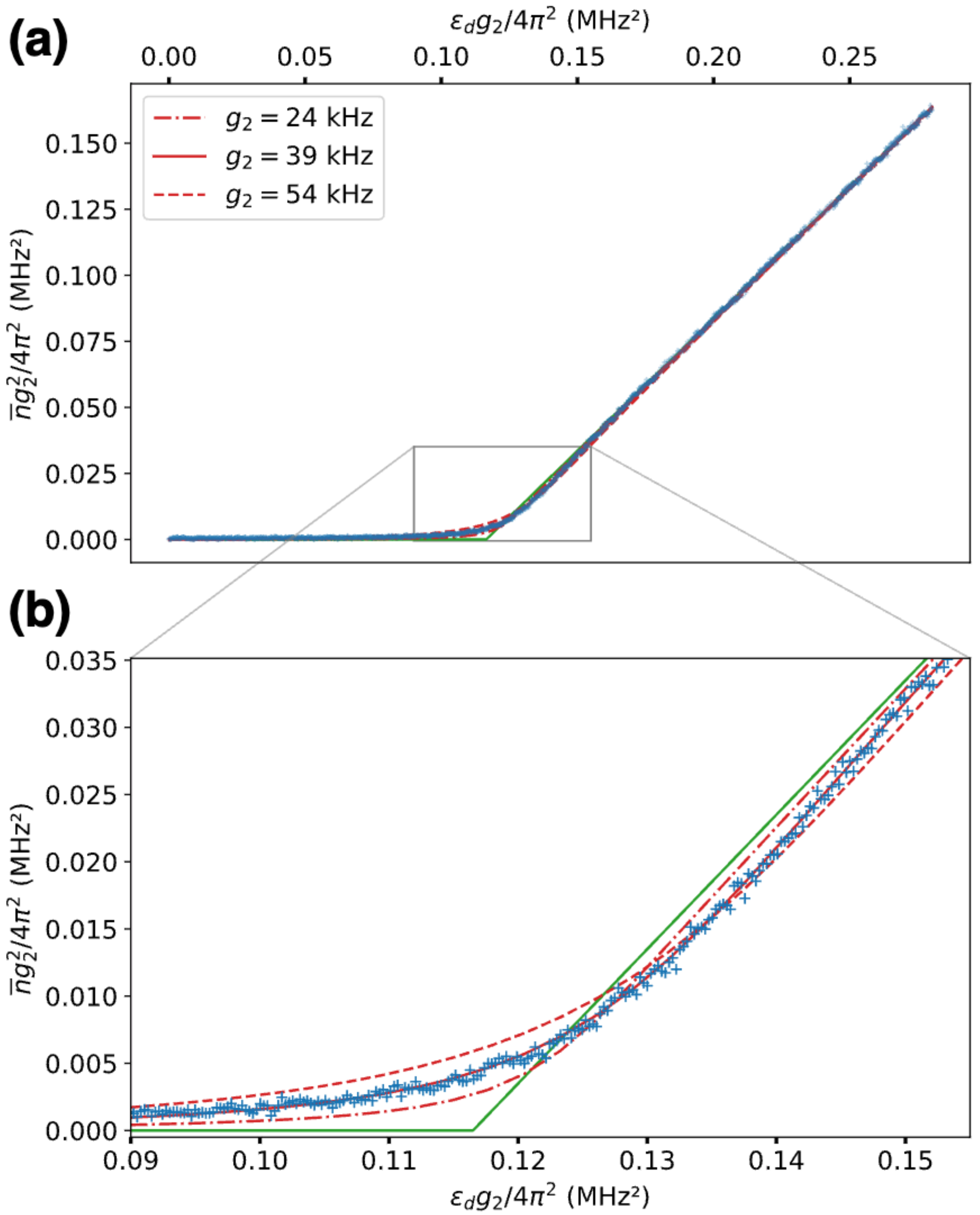
$$\mathrm{d}Y_t = \sqrt{\eta\kappa_1/2}\,\mathrm{Tr}\big[(a + a^\dagger)\rho_t\big]\mathrm{d}t + \mathrm{d}W_t,$$
$$I_k = \frac{G}{\Delta t}\int_{k\Delta t}^{(k+1)\Delta t} \mathrm{d}Y_t.$$

  - In the steady state ρ_∞.
  - Estimated $\theta = (\kappa_2, \alpha_2, \eta)$ with **>10% uncertainty**.

↪ Multiple experiments including fitting $\mathbb{E}[I_k^2]$ for various α_2.

↪ Gain confidence by fabricating, cooling down, and measuring another superconducting chip.





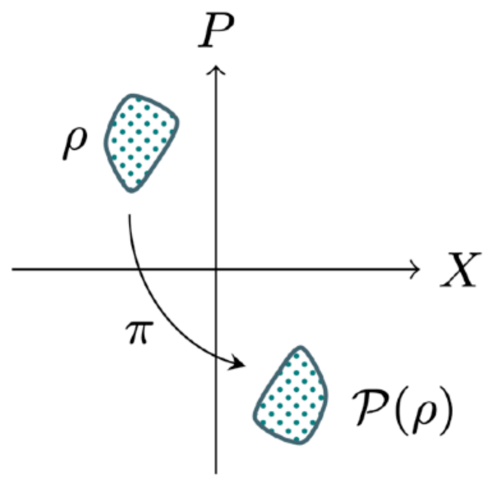
Numerical results 3/3

- Difficult case:

↪ **System symmetry** ⇒ all odd-order correlation functions are null.

Numerical results 3/3

General result: if $[\mathscr{L}, \mathcal{S}] = 0$ and $\rho_0 = \mathcal{S}(\rho_0)$ and $\{\mathscr{C}_L, \mathcal{S}\} = 0$ then $\mathbb{P}[I_t] = \mathbb{P}[-I_t]$.



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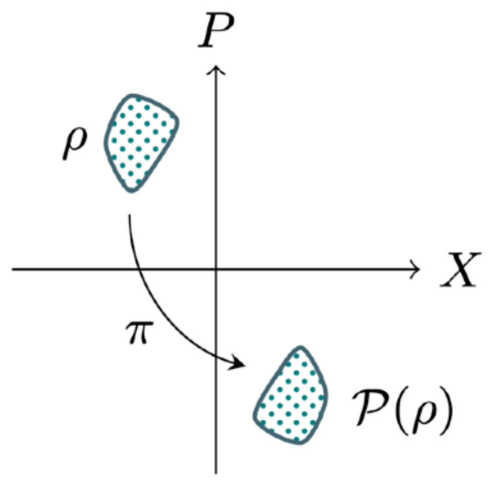

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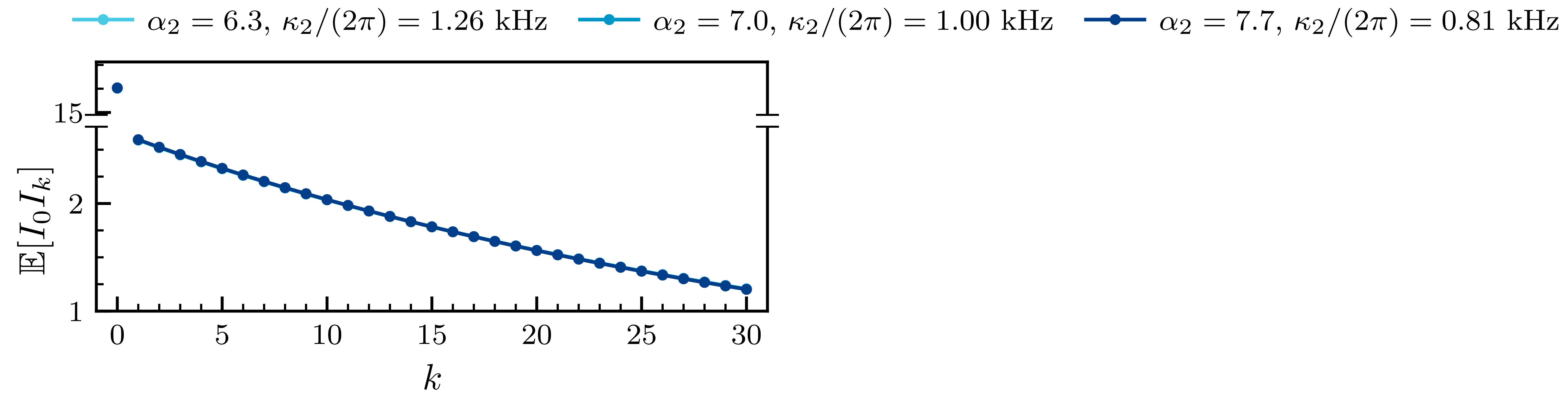

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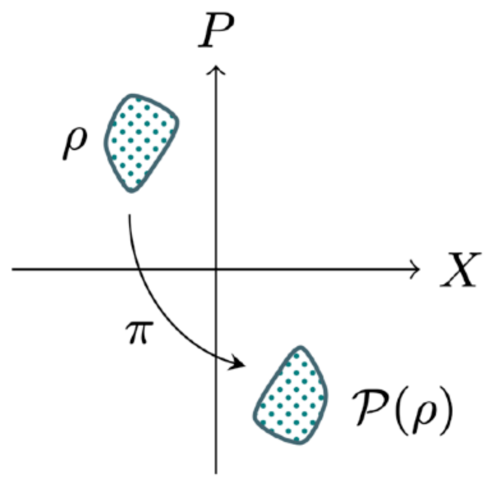

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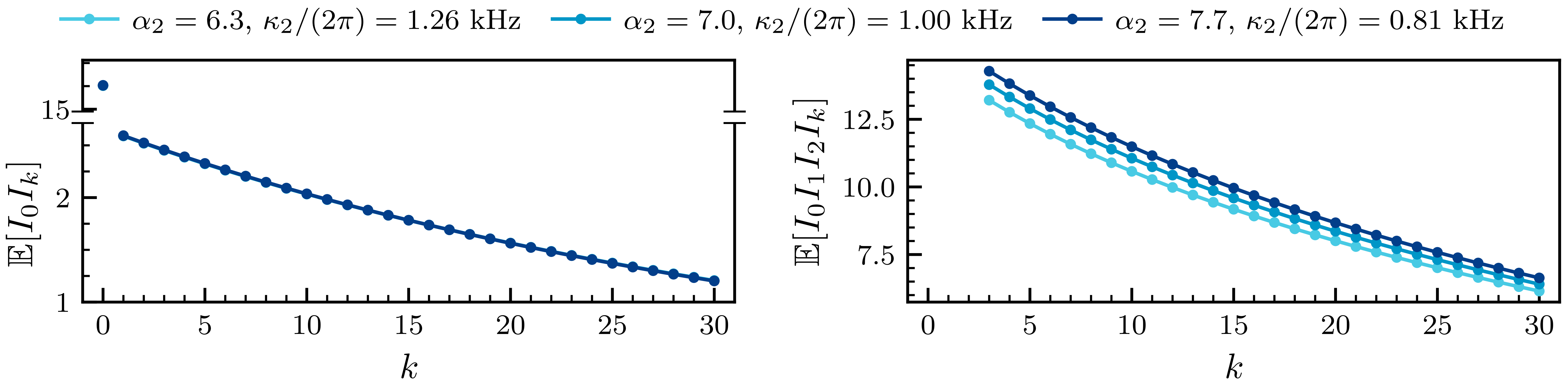

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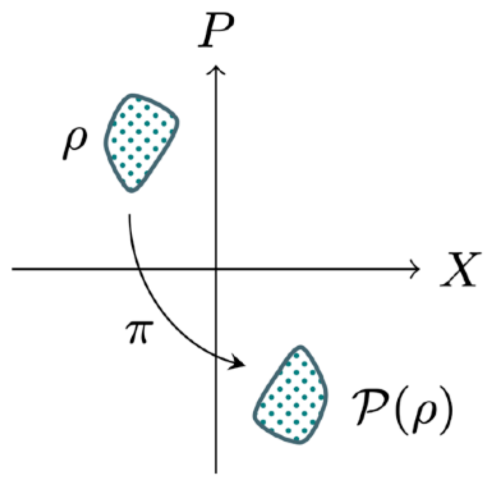

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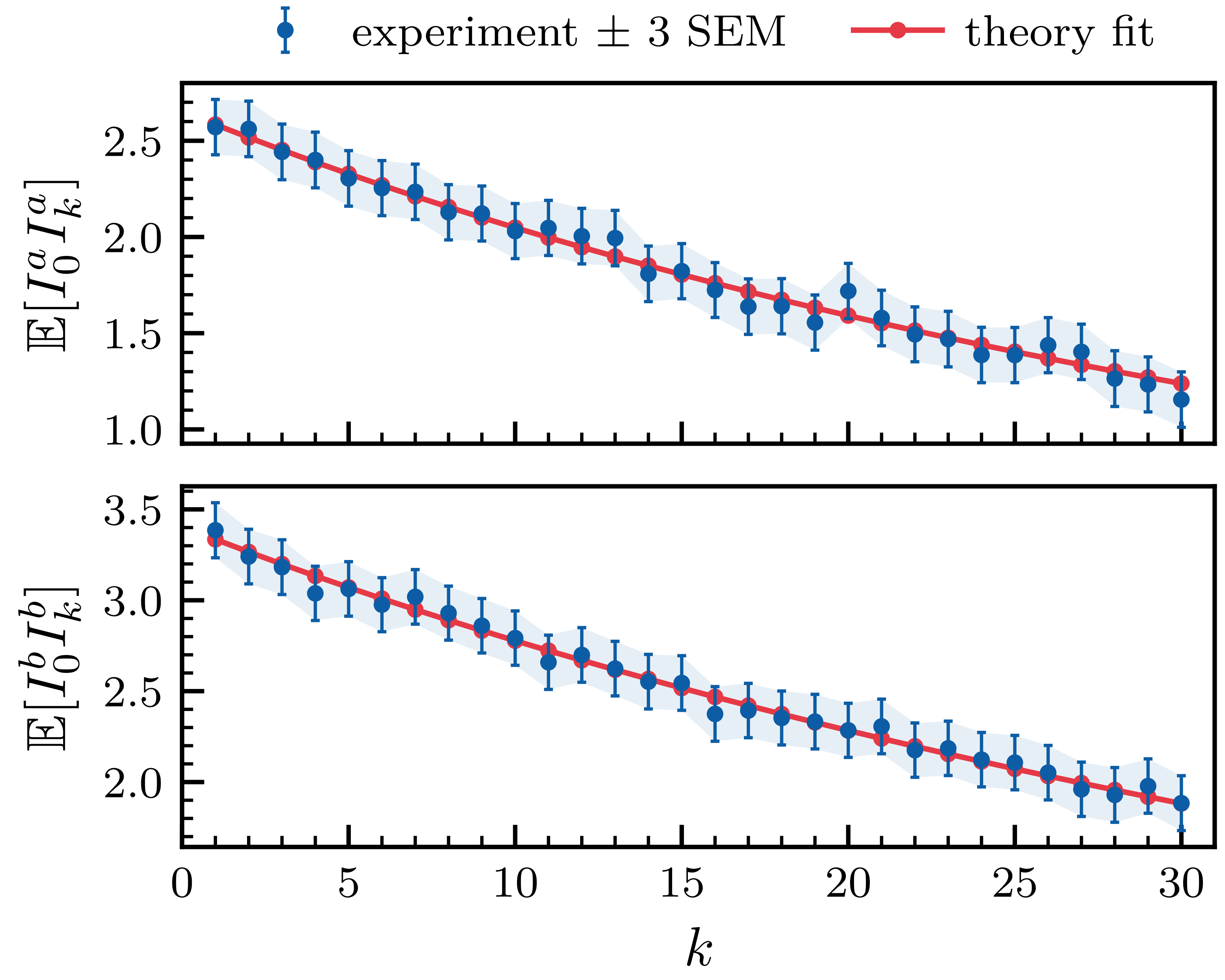
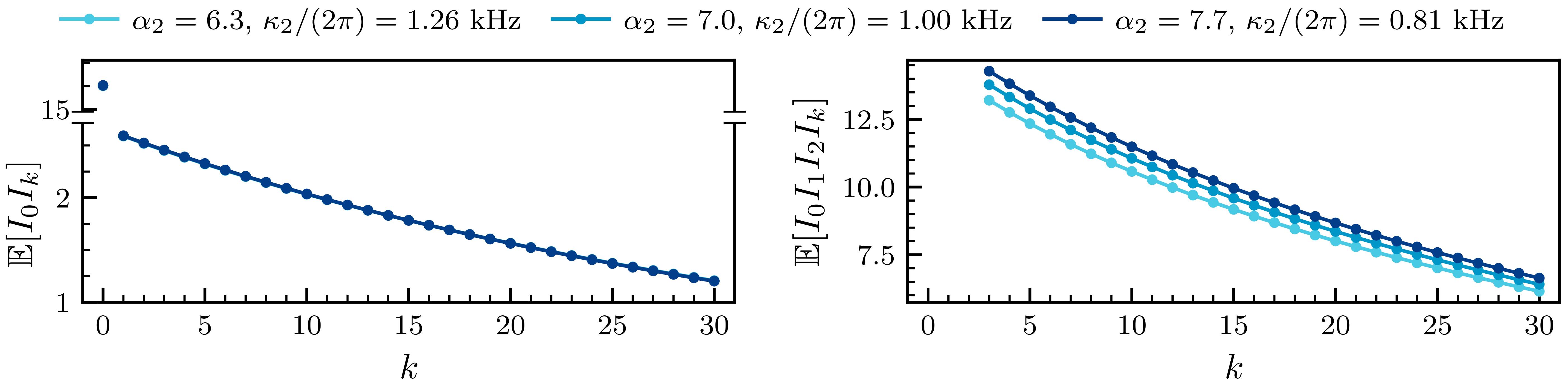
  ↪ **Vary** one of the system parameters (here α_2).





	Parameter	Value	Estimated
$\kappa_1/(2\pi)$	1-photon loss rate	100 kHz	–
$\kappa_2/(2\pi)$	2-photon loss rate	1 kHz	1.007 ± 0.004 kHz
α_2	amplitude	7	7.002 ± 0.005
η	efficiency	0.1	0.095 ± 0.002

Limitations and advantages

❌ Limitations

- Need a parameterised model → OK if missing an important term: **incorrect fit rather than biased fit**.
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- **General**: any SME, detector imperfections, arbitrary filtering.
- **Simple**: theoretically (directly models experimental data), experimentally, numerically.
- **Efficient**: numerical cost = solving Lindblad, doesn't depend on the nb. of parameters.
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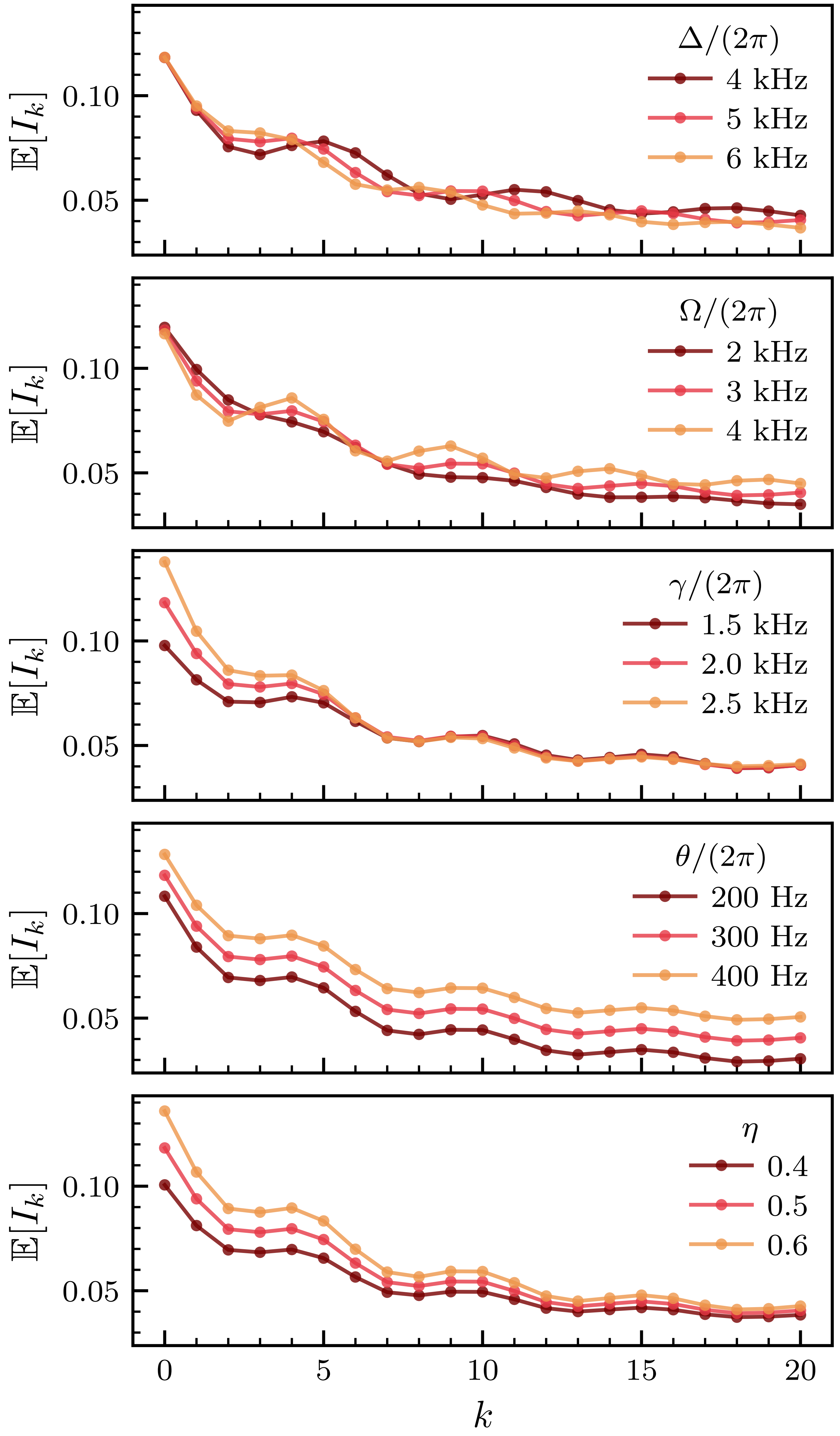
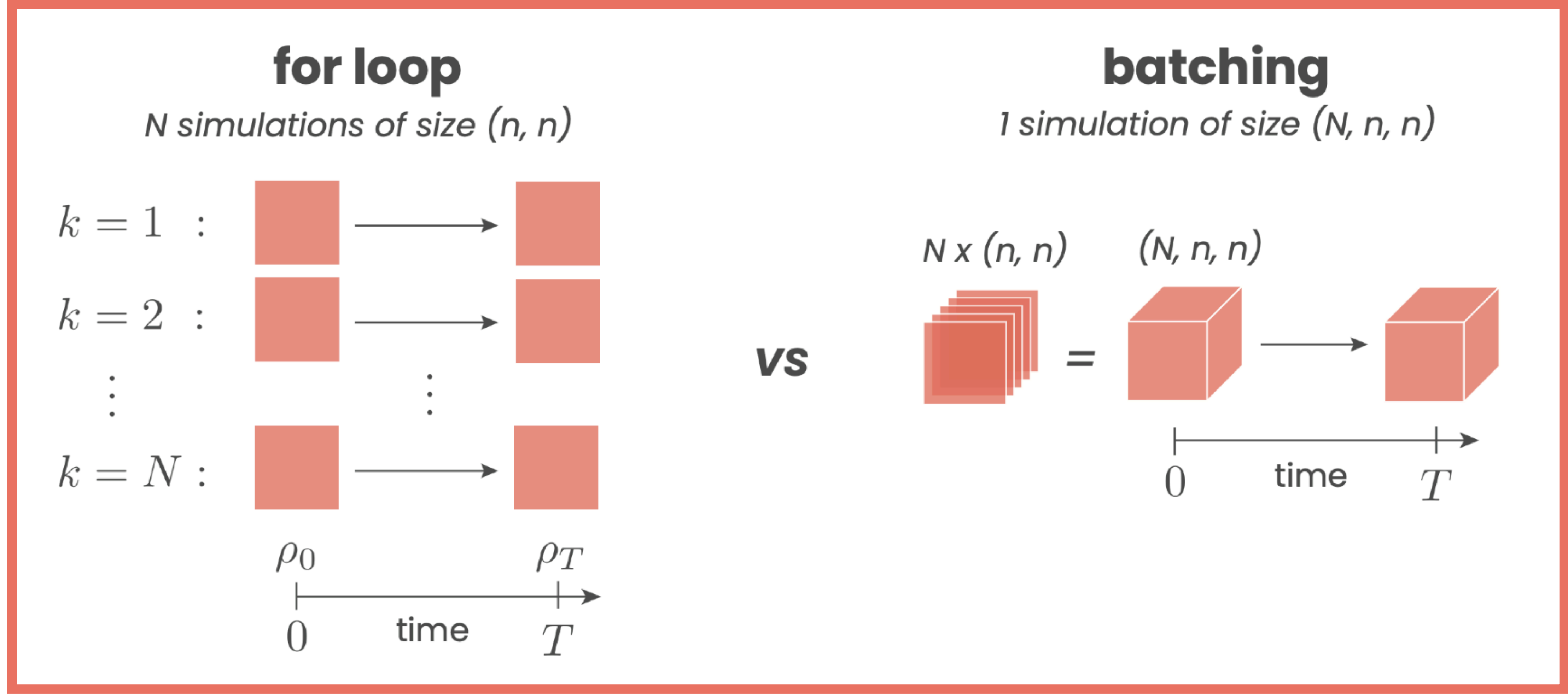


Teasers and conclusion

Three teasers

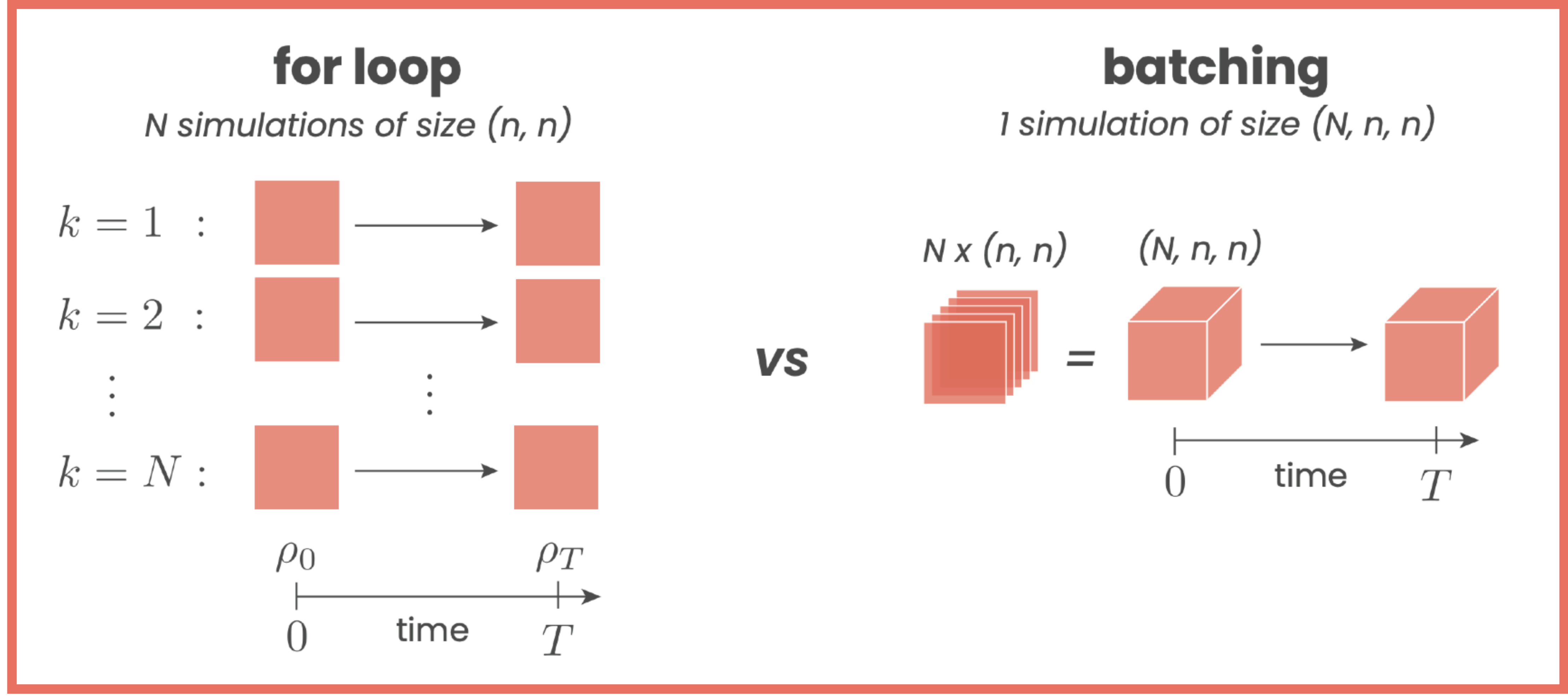


- [poster] The Dynamiqs library.

↪ 10^6 trajectories in a few seconds for a qubit, a few minutes for a bosonic mode.

↪ How? → GPU + vectorized-array computation + sparse DIA format + CPTP solvers.

Three teasers



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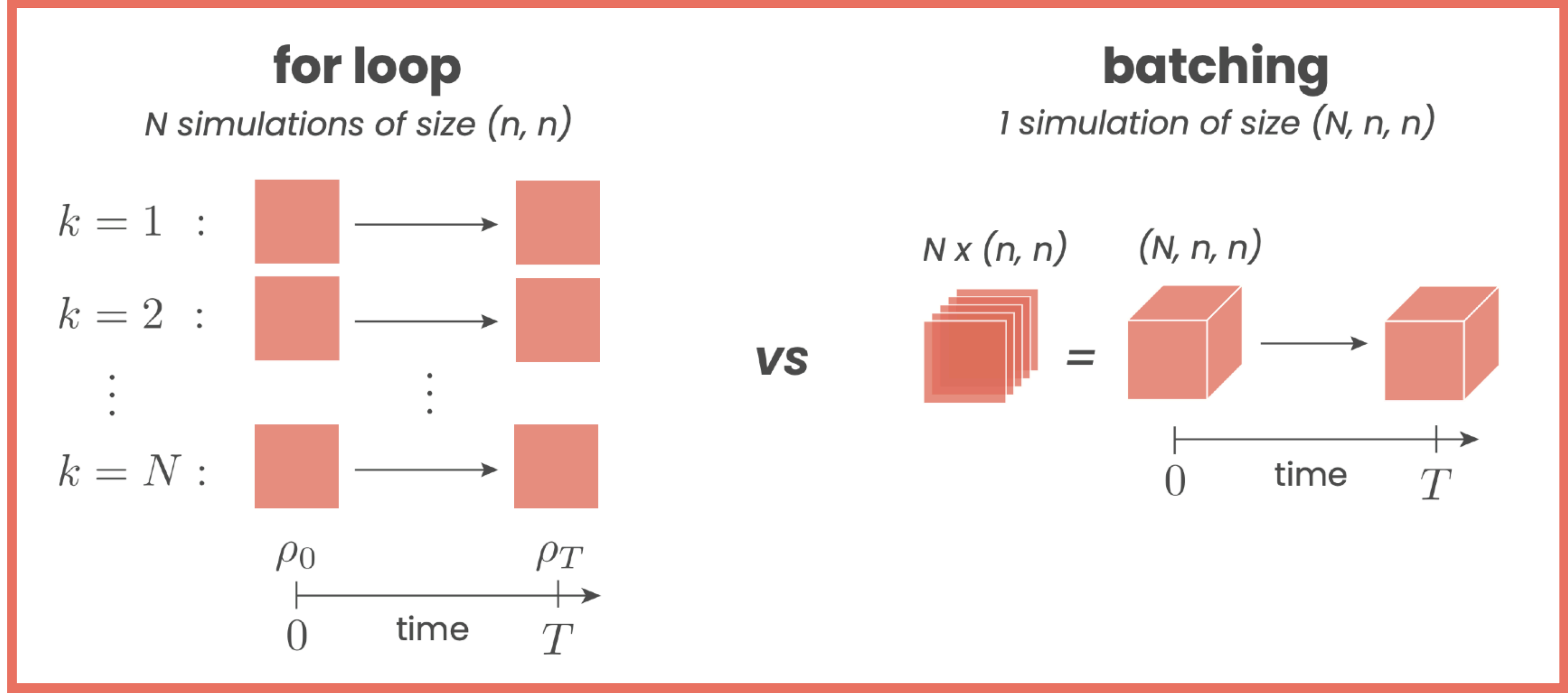

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- [Antoine Tilloy talk] We **partially** bridged the gap → what about **the state**?

↪ State reconstruction, Bayesian estimation, state-based feedback (etc.) with digitised signal.

Complete information	Time-averaged information	No information
$\{I_t\}_{t\in[0,T]} \to \rho_t$	$\{I_k\}_{0\leq k\leq N} \to \bar{\rho}_k = ?$	$\varnothing \to \bar{\rho}_t = \mathbb{E}[\rho_t]$
SME	?	Lindblad

We need new characterisation methods to build a FTQC with superconducting circuits.

Let’s fit correlation functions!

Pierre Guilmin, Pierre Rouchon, Antoine Tilloy, Parameters estimation by fitting correlation functions of continuous quantum measurement, arXiv:2410.11955 (2024).