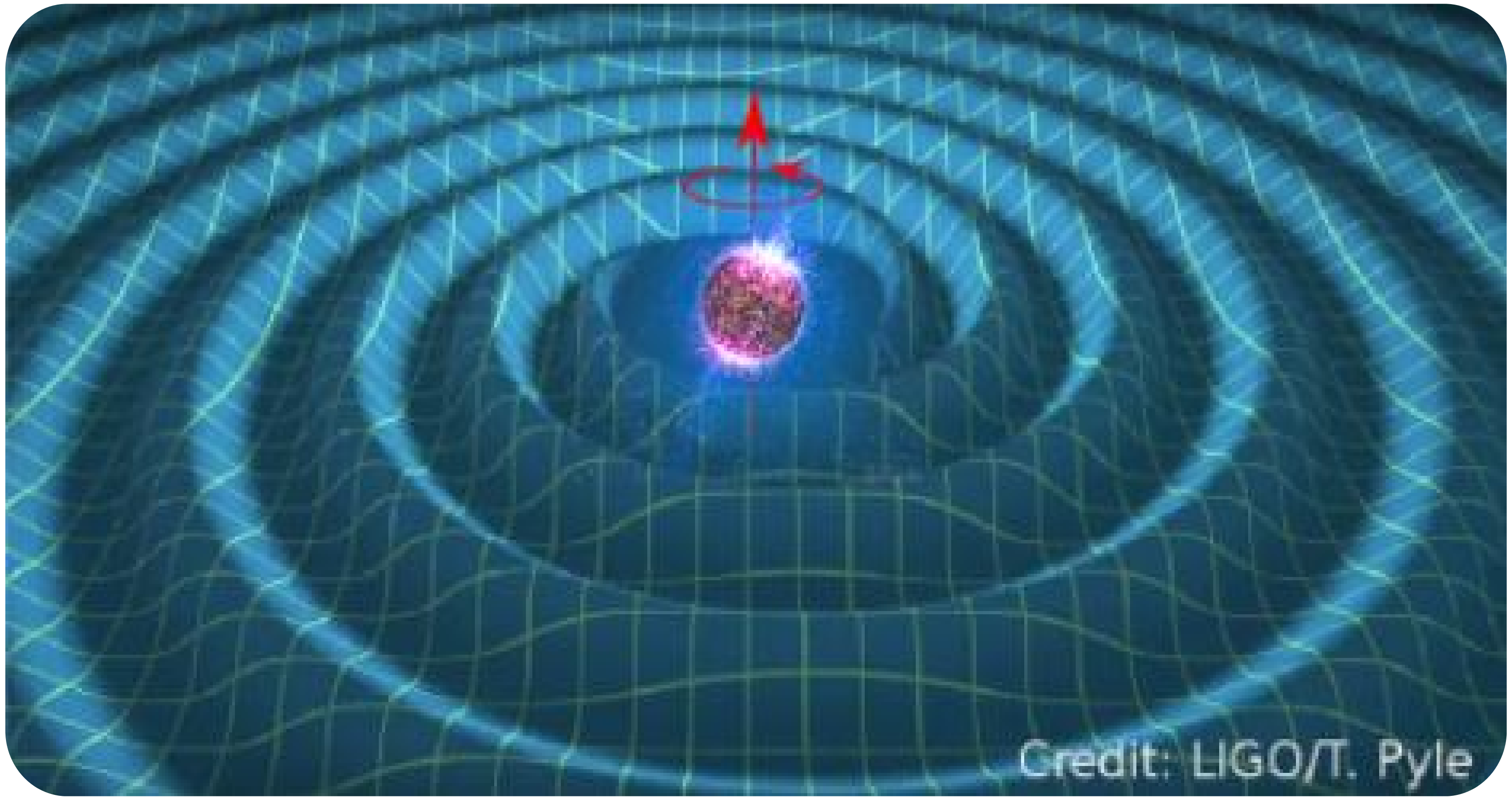
LIGO Science Workshop, ICTS, 2023-10-27

DCC: LIGO-G2302106-v2

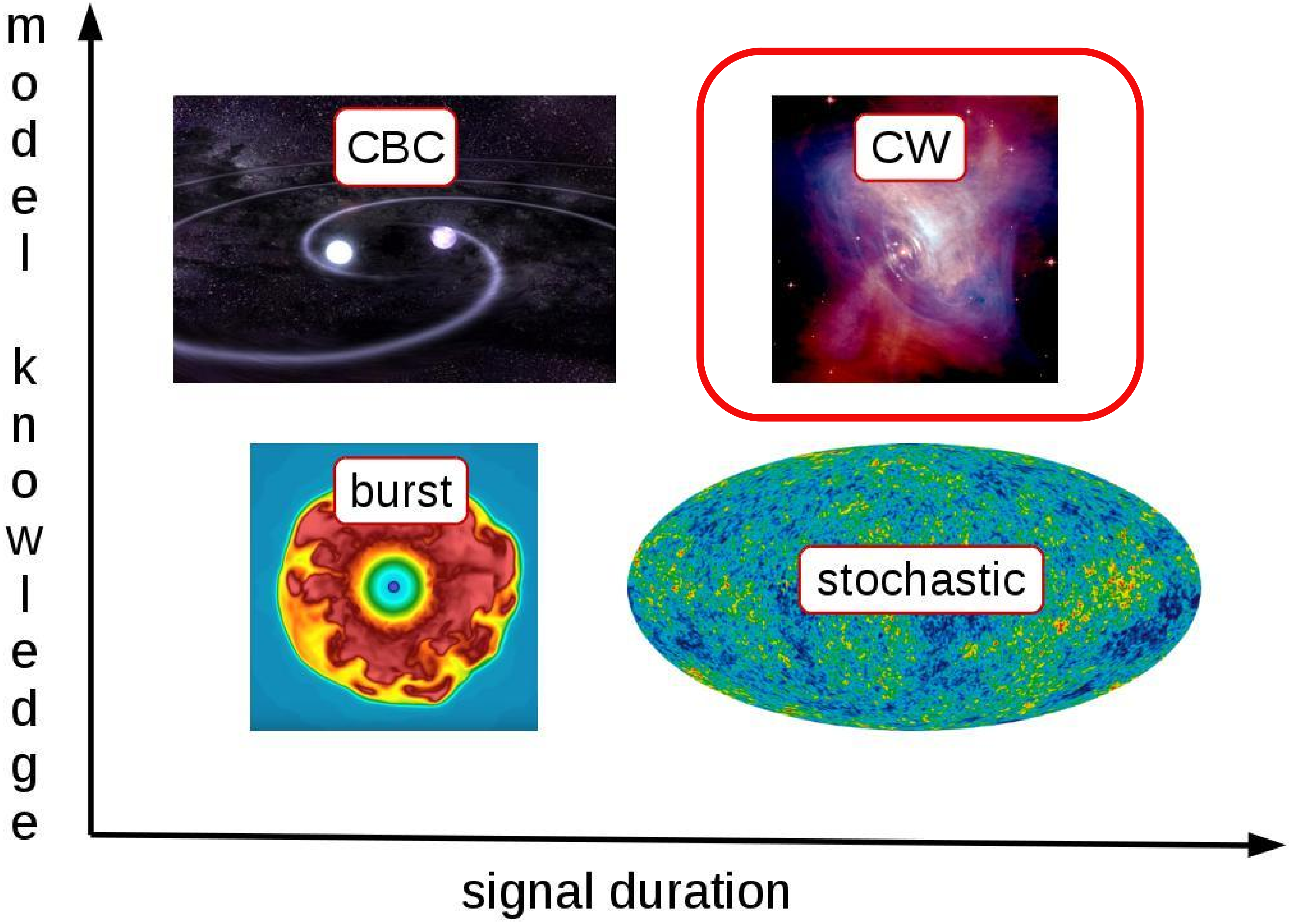
Continuous Waves – data analysis

David Keitel (IAC3 / UIB)


Credit: LIGO/T. Pyle

beyond CBCs: GW signal types

(for LVK network)



a brief review of (recent) reviews

[2019]

universe MDPI

Review

Continuous Gravitational Waves from Neutron Stars: Current Status and Prospects

Magdalena Sieniawska * and Michał Bejger

Chapter 1

Handbook of Gravitational Wave Astronomy: Isolated Neutron Stars

[2020]

Brynmor Haskell and Kai Schwenzer

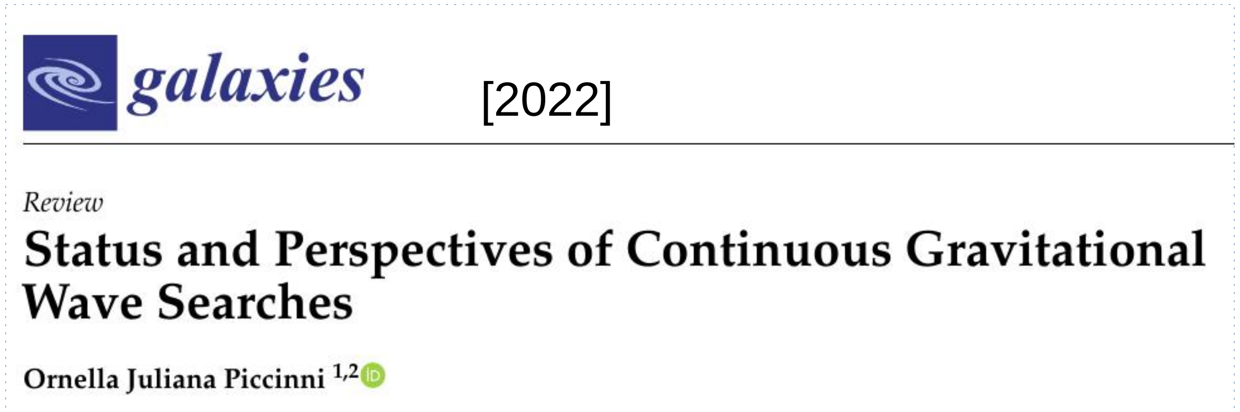
[2021]

universe MDPI

Review

Search Methods for Continuous Gravitational-Wave Signals from Unknown Sources in the Advanced-Detector Era

Rodrigo Tenorio *, David Keitel and Alicia M. Sintes



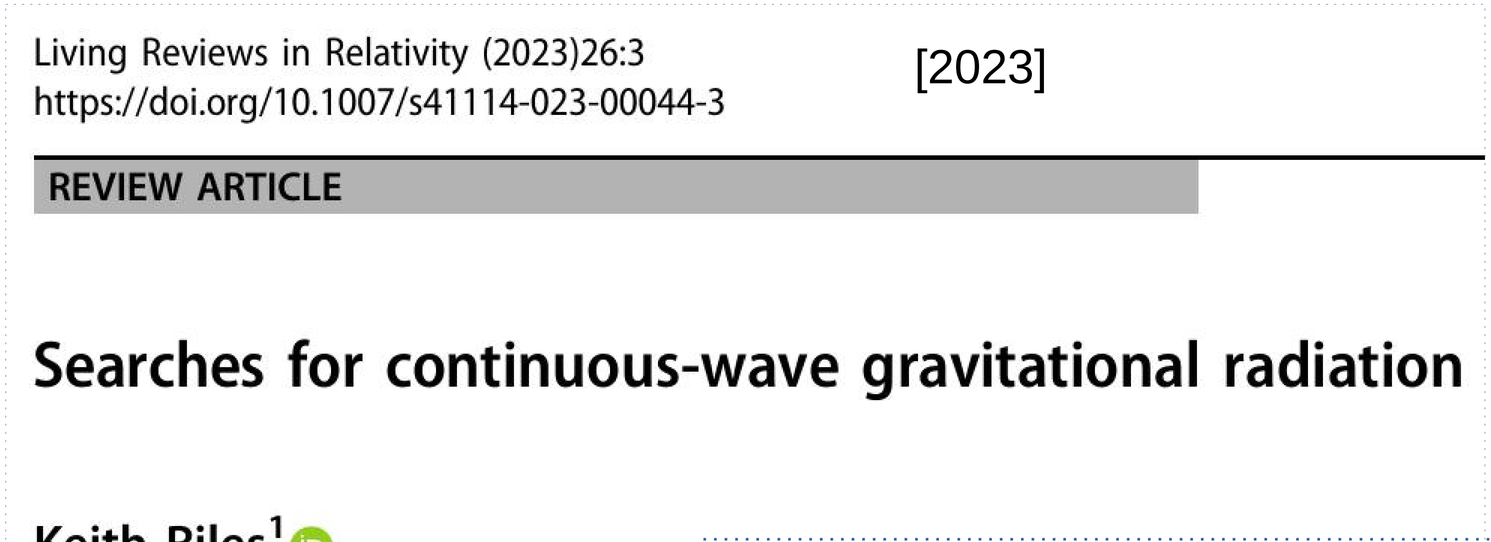
[2022]

galaxies

Review

Status and Perspectives of Continuous Gravitational Wave Searches

Ornella Juliana Piccinni [1,2]



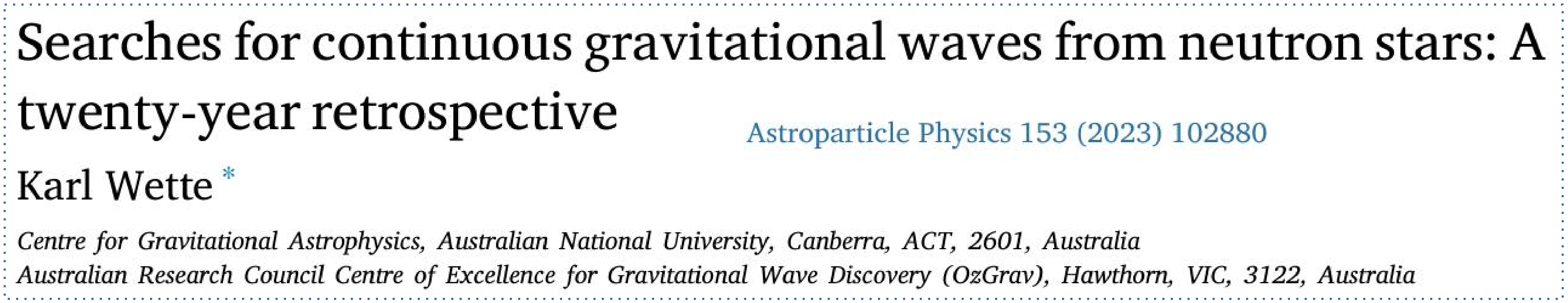
Living Reviews in Relativity (2023)26:3
https://doi.org/10.1007/s41114-023-00044-3

[2023]

REVIEW ARTICLE

Searches for continuous-wave gravitational radiation

Keith Riles[1]



Searches for continuous gravitational waves from neutron stars: A twenty-year retrospective

Astroparticle Physics 153 (2023) 102880

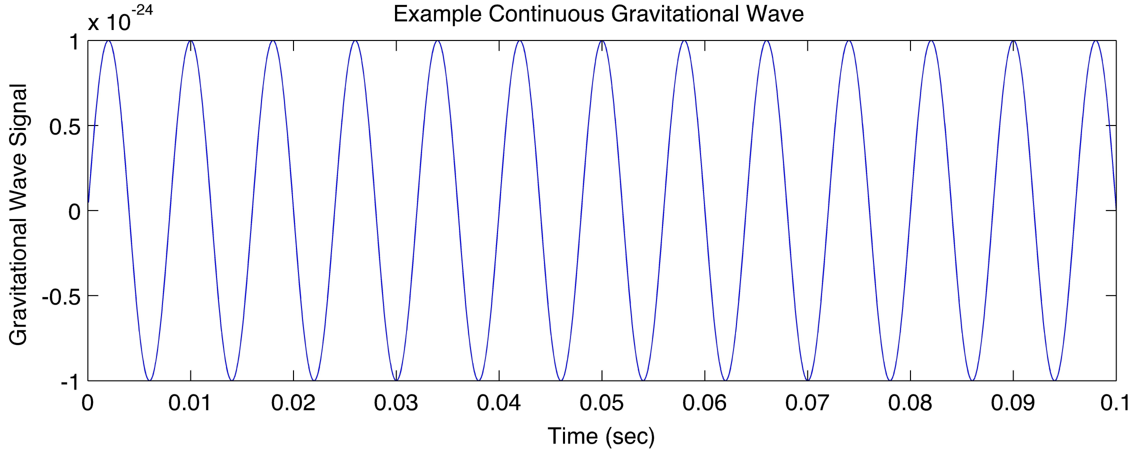
Karl Wette *

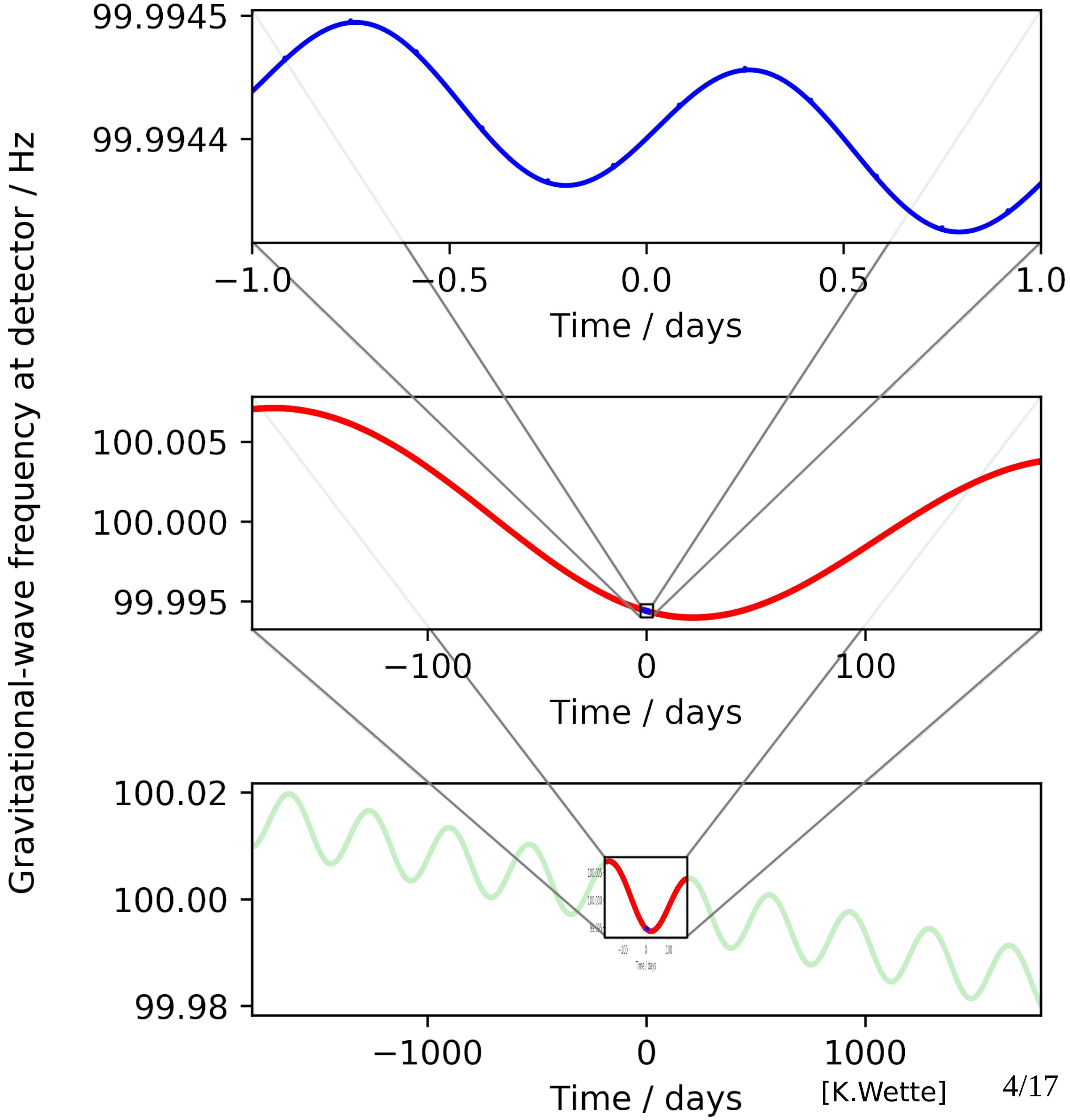
Centre for Gravitational Astrophysics, Australian National University, Canberra, ACT, 2601, Australia
Australian Research Council Centre of Excellence for Gravitational Wave Discovery (OzGrav), Hawthorn, VIC, 3122, Australia

clear signs of a mature but vibrant field

CW signals: basic concepts

- “quasi-stationary” and “quasi-monochromatic”
- Slow frequency & amplitude modulation:
  - intrinsic pulsar spin-down (energy loss)
  - daily rotation of Earth
  - yearly orbit of Earth around Sun
  - optional: source binary orbit
- Much simpler than CBC waveforms, usually no need for simulations-informed waveform models.
- Long-duration matched filter sensitive to tiny offsets in template parameters.
- → searches (for unknown sources) very computationally expensive: extremely dense template banks, up to 10^{17} in all-sky searches!





[K.Wette]

CW signals: Taylor series spindown model

- the usual two polarization components:

$$h_+(\tau) \equiv A_+ \cos\Phi(\tau)\,, \quad h_\times(\tau) \equiv A_\times \sin\Phi(\tau)$$

- phase evolution:

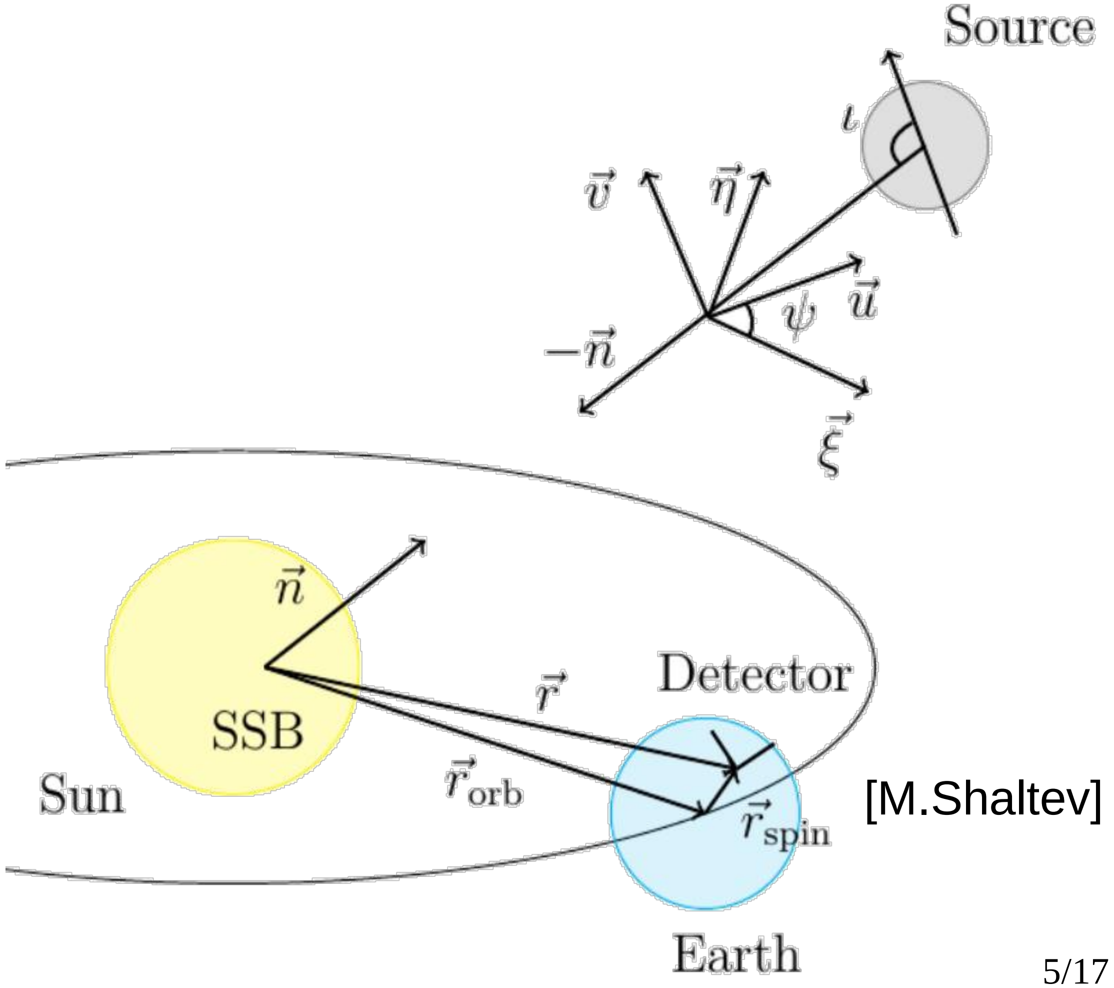
$$\Phi(\tau) = \phi_0 + 2\pi \sum_{s=0}^{s_{\max}} \frac{f^{(s)}(\tau_{\mathrm{ref}})}{(s+1)!}(\tau - \tau_{\mathrm{ref}})^{s+1} \qquad \text{with} \qquad f^{(s)}(\tau_{\mathrm{ref}}) \equiv \left.\frac{\mathrm{d}^s f(\tau)}{\mathrm{d}\tau^s}\right|_{\tau_{\mathrm{ref}}}$$

- Same as in radio timing, just at GW frequencies, e.g. $f_{\mathrm{GW}} = 2f_{\mathrm{rot}}$ for “mountains”

- If only..
  - pulsar has negligible proper motion
  - we had an ideal omnidirectional detector at the solar system barycentre

→ this would be all there is to it!

- *BUT* need to take into account:
  - actual detector response
  - timing corrections between SSB and detector frames.



[M.Shaltev]

CW signals: Doppler modulation and detector response

- real GW detectors on Earth: Doppler modulation from daily&yearly motion
- timing relation between wavefront arrivals in detector frame and in SSB:

$$\tau(t;\boldsymbol{n},\mathbf{b}) = t + \frac{\boldsymbol{r}(t)\cdot\boldsymbol{n}}{c}$$

- modulated signal waveform at detector:

$$\begin{aligned} h(t;\mathcal{A},\boldsymbol{\lambda}) =& F_+(t;\boldsymbol{n},\psi)\,A_+\cos\left[\phi_0+\phi(t;\boldsymbol{\lambda})\right] \\ &+ F_\times(t;\boldsymbol{n},\psi)\,A_\times\sin\left[\phi_0+\phi(t;\boldsymbol{\lambda})\right] \end{aligned}$$

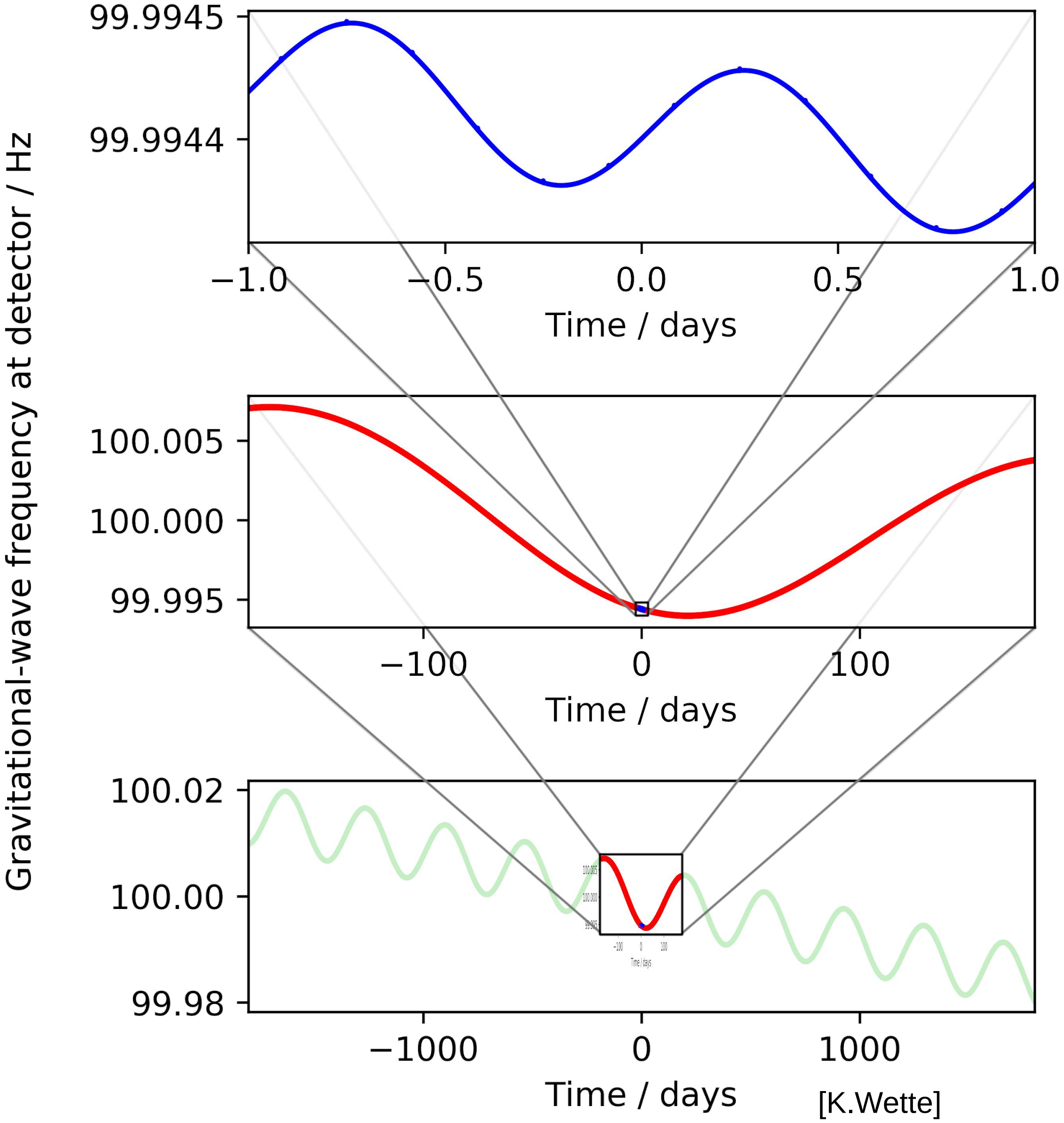
  (including detector response / antenna pattern)

- frequency evolution parameters ("Doppler parameters", λ):

  intrinsic spindown terms,
  sky position (alpha,delta)

- in data analysis also, also called "barycentring"

  → a main cost factor

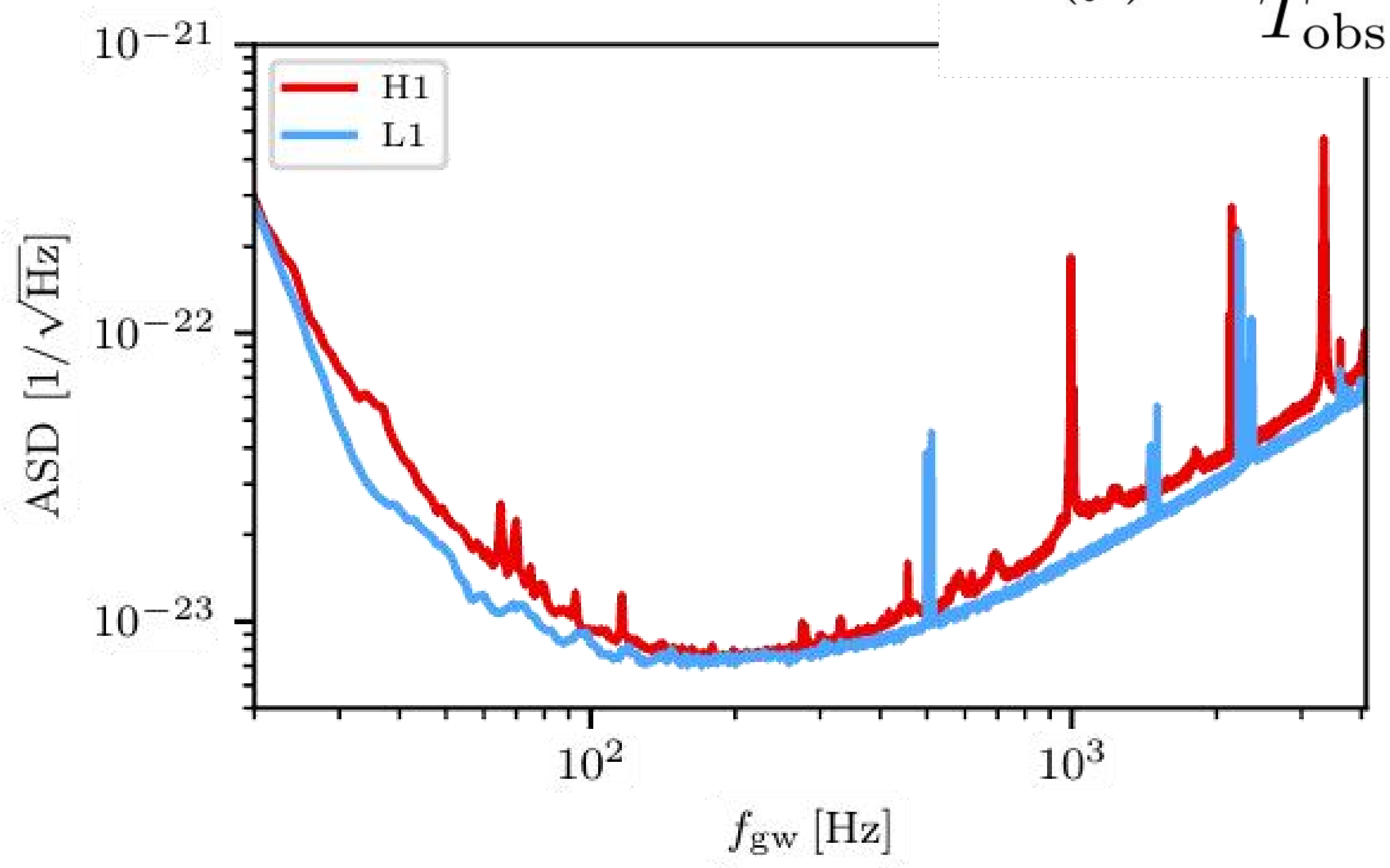


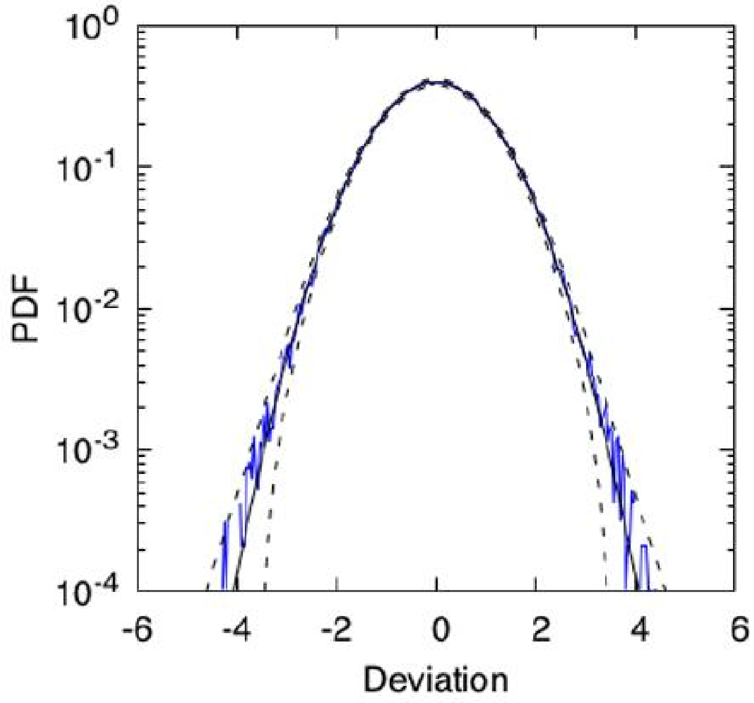
[K.Wette]

detector noise

- GW detector noise is ≈ ***Gaussian***
  (especially when averaging over long durations)***.***
- Fully described by Power Spectral Density (PSD).
- Frequency-dependent PSD (“coloured noise”):

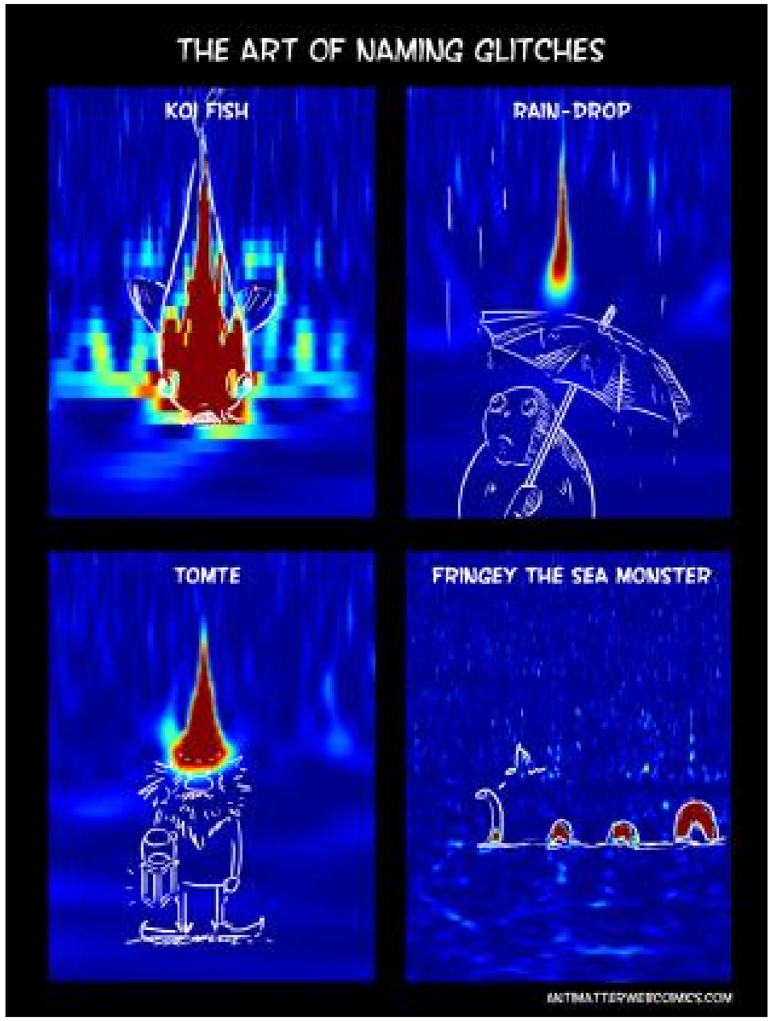
$$S_{\mathrm{n}}(f) \approx \frac{2}{T_{\mathrm{obs}}} E\left[|\widetilde{n}(f)|^2\right]$$





Real noise ***not*** perfectly Gaussian, contains artifacts like

- *glitches* (short duration, complex shapes)
- *lines* (fixed frequency, can be persistent, main CW headache)



detector noise & CW data analysis

- Quasimonochromatic signals → usually work in the Fourier domain
- Can extract “narrowband” data sets, noise PSD locally almost constant

$$(\boldsymbol{x}|\boldsymbol{y}) \approx 2 \sum_{X}^{N_{\mathrm{Det}}} S_X^{-1}(f_{\mathrm{s}}) \int_0^T x^X(t)\, y^X(t)\, dt$$

→ timeseries inner product
→ matched filter

- Usually split data into Short Fourier Transforms (SFTs) (T_{SFT}=1800s or similar).
  → only assume PSD is constant over each SFT:

$$\langle \boldsymbol{x} \,|\boldsymbol{y}\,\rangle \approx 2 \sum_{X=1}^{N_{\mathrm{det}}} \sum_{\alpha=1}^{N_{\mathrm{SFT}}^X} \frac{1}{S_\alpha^X} \int\limits_0^{T_{\mathrm{SFT}}} x_\alpha^X(t')\, y_\alpha^X(t')\, \mathrm{d}t'$$

(Virgo groups use different, but conceptionally similar formats)

- PSD estimates from per-SFT periodograms, time-averaged:

$$\widehat{S}^X(f') \equiv \frac{1}{N_{\mathrm{SFT}}^X} \sum_{\alpha=1}^{N_{\mathrm{SFT}}^X} \frac{2\left|\tilde{x}_\alpha^X(f')\right|^2}{T_{\mathrm{SFT}}}$$

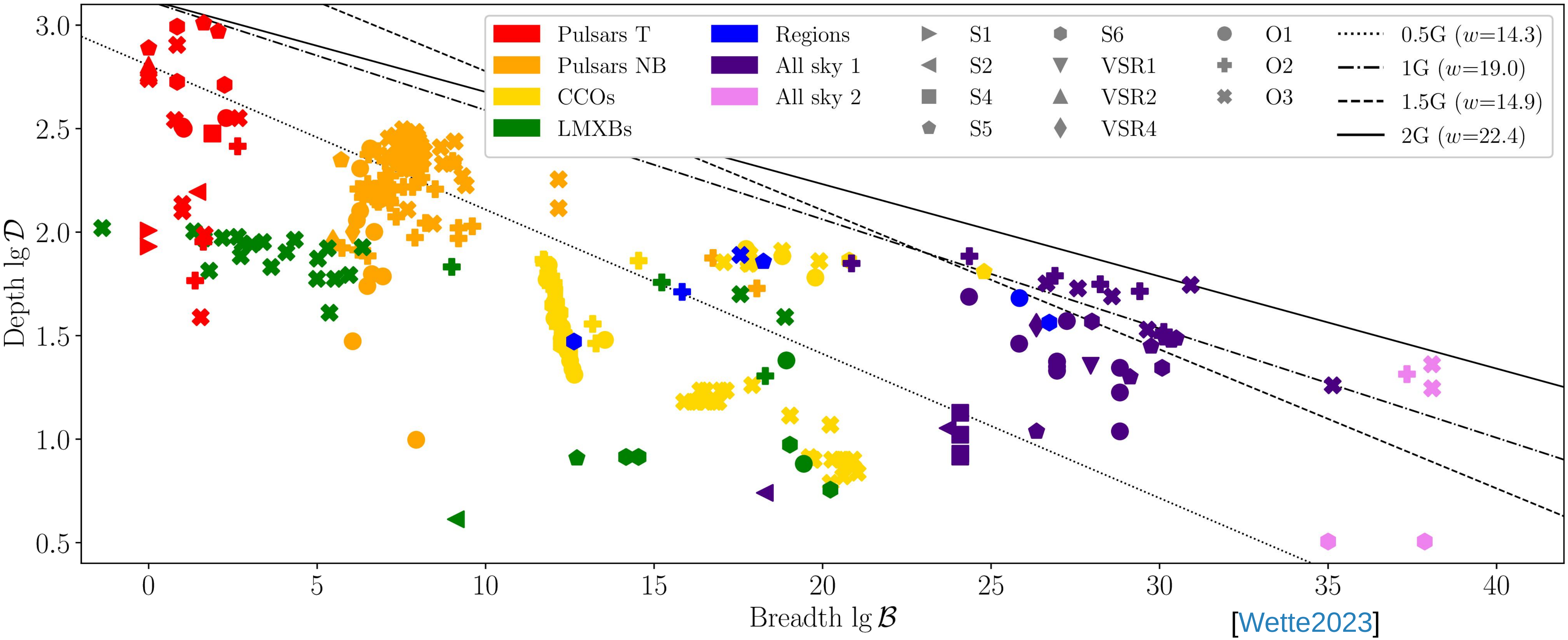
CWs: the key points

- Quasi-monochromatic: simple templates
- Incredibly weak.
- Long-duration:
  - gain SNR by integrating longer
  - data very close to Gaussian (except near narrow disturbances)
  - precise frequency resolution from long-term phase coherence
  - precise sky localisation, even with a single detector,
    because Earth moves during observation
  - computational cost for *unknown* targets grows steeply with observing time
    (or at least with *coherence time* – more later).
    - *Not* mainly from cost of a single long matched filter (~T).
    - *But* because *template banks* become so dense.
    - logical flip-side of the great resolution

CW searches

Categorisation by amount of prior information:

- targeted searches
- narrowband searches
- directed searches
- spotlight searches
- allsky searches
- dark matter: “no sky”



[Wette2023]

targeted searches

- Pulsar ephemerides (radio, X-ray, gamma-ray)
  → cheap and very sensitive ***fully-coherent analysis***:
  Taylor signal model, GWs coupled tightly to rotation frequency, no phase jumps
- Crucial milestone per target: ***spindown upper limit*** ↔ all energy loss into GWs:

$$h_0 \leq h_{\mathrm{sd}} = \frac{1}{d}\sqrt{\frac{5G\,I_{zz}}{2c^3}\,\frac{|\dot{\nu}|}{\nu}}$$

- no GW detection → ***observational upper limits*** on GW strain:
  if source emitting at h_0 would have detected a louder outlier (with e.g. 90% confidence)
- Crab and Vela pulsars: spindown limit first beaten with initial LIGO/Virgo in 2000s.

THE ASTROPHYSICAL JOURNAL, 683: L45–L49, 2008 August 10
© 2008. The American Astronomical Society. All rights reserved. Printed in U.S.A.

BEATING THE SPIN-DOWN LIMIT ON GRAVITATIONAL WAVE EMISSION FROM THE CRAB PULSAR

- As of O3: beaten for 23 pulsars [Abbott+ ApJ935:1]

THE ASTROPHYSICAL JOURNAL, 737:93 (16pp), 2011 August 20 doi:10.1088/0004-637X/737/2/93
© 2011. The American Astronomical Society. All rights reserved. Printed in the U.S.A.

BEATING THE SPIN-DOWN LIMIT ON GRAVITATIONAL WAVE EMISSION FROM THE VELA PULSAR

- *Narrowband* approach: allow EM–GW mismatch ≲ 1 Hz,
  still beat limit for 7 pulsars in O3 [Abbott+ ApJ922:133]

directed & all-sky searches

- Large parameter space (frequency, spindowns, possibly sky) at affordable cost.
- Simple signal model, dimensionality not too high;
  but parameter space is curved and highly structured.
- approximate metric as time-average of phase model derivatives:

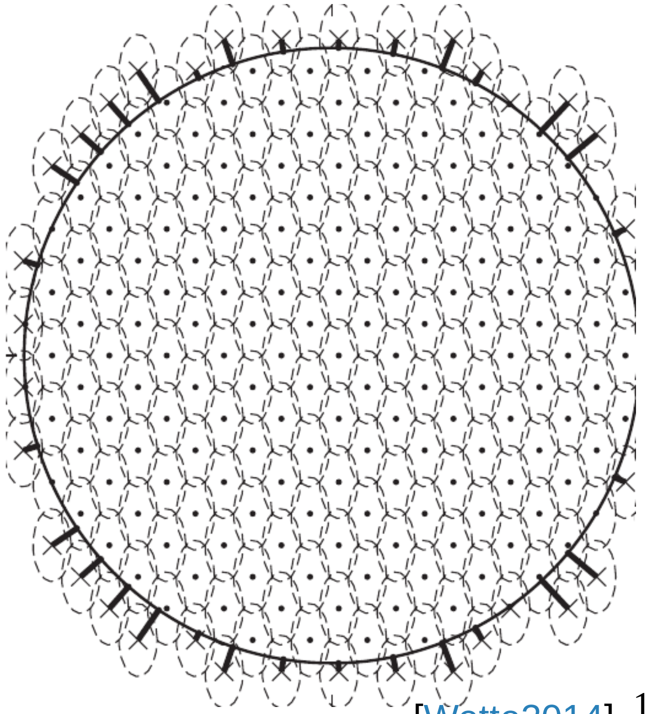
$$g_{ij} \sim \langle \partial_i\phi\,\partial_j\phi\rangle - \langle\partial_i\phi\rangle\langle\partial_j\phi\rangle \longrightarrow g_{\theta\theta} \propto f^2\, T_{\rm obs}^2\, (V/c)^2\,, \qquad g_{ff} \propto T_{\rm obs}^2\,, \qquad g_{\dot f\dot f} \propto T_{\rm obs}^4$$

where $V/c \sim 10^{-4}$ is the max Doppler shift from Earth's motion [Prix2007]

- For a $\underline{\text{4D parameter space } d^4\boldsymbol{\lambda} = d\Omega \times df \times d\dot{f}}$ → number of templates:

$$dN_p \propto \sqrt{|\det g_{ij}|}\, d^4\boldsymbol{\lambda} \propto T_{\rm obs}^5\, f^2\, d^4\boldsymbol{\lambda}$$ *(or even steeper!)*

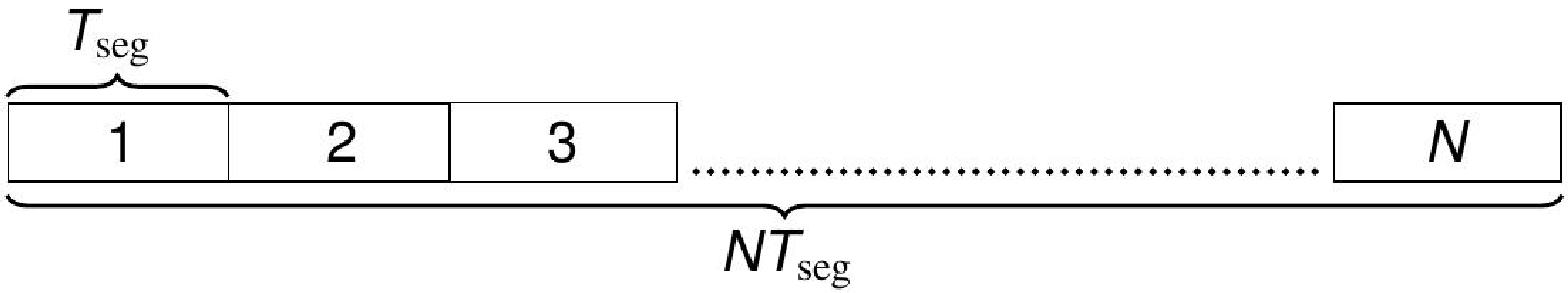
- $\boxed{\text{Computational cost } \propto T_{\rm obs}^6\, f^2\, d^4\boldsymbol{\lambda}}$ (per-template MF: cost $\propto T_{\rm obs}$)
- Intuitive reason for steep scaling of N_p:
  growing $T_{\rm obs}$ → small offsets give big dephasing
  → shrinking "mismatch ellipses" covered by each template
- Larger template banks → higher *trials factor*
  → less significance for the same signal!



[Wette2014]

semi-coherent searches

- Broad searches with T_{obs} > months → fully-coherent searches a bad choice
- Better sensitivity at fixed cost [Brady&Creighton1998]:



① split data $\boldsymbol{x}$ into N shorter segments $\boldsymbol{x}_k$

② compute coherent $\widetilde{\mathcal{F}}_k$ in each segment

③ incoherent combination, e.g. $\widehat{\mathcal{F}}(\boldsymbol{x}, \lambda) \equiv \sum_{k=1}^{N} \widetilde{\mathcal{F}}_k(\boldsymbol{x}_k, \lambda)$

- Template density only scales with T_{seg} instead of T_{obs}.
- No longer require phase-coherence across whole T_{obs}:
  → overall sensitivity reduced
  → more susceptible to spurious instrumental artifacts
  → more robust to astrophysical variations in the source (e.g. NS glitches)
- So much computational efficiency gain → better search depth at fixed budget!
- Spurious candidates can be taken out with *hierarchical follow-ups* [e.g. Tenorio+2021].

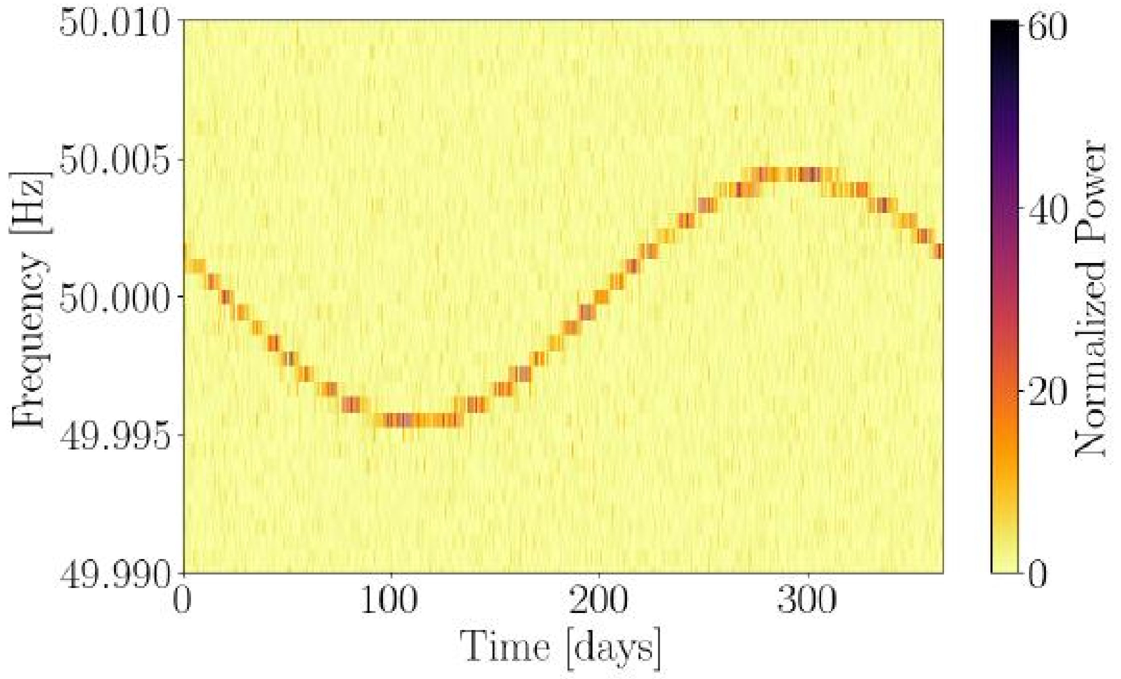
semi-coherent searches

- T_{seg} from single SFTs (e.g. 1800s for most “Hough” type searches) up to several days (for the most expensive Einstein@Home distributed computing searches)

- Instead of simple semi-coherent sum $\widehat{\mathcal{F}}(\boldsymbol{x}, \lambda) \equiv \sum_{k=1}^{N} \widetilde{\mathcal{F}}_k(\boldsymbol{x}_k, \lambda)$

  more sophisticated methods exist, e.g. using ***refinement***:

  - use “coarse grid” template banks $\{\lambda_k\}$ in each segment with resolution given by T_{seg}
  - evaluate final detection statistic on a “fine grid” $\{\lambda\}$ with resolution given by T_{obs}
  - Get that final $F(x,\lambda)$ from summing up $F_k(x_k,\lambda_k)$ along the λ time-frequency track
  - Optimal fine-grid construction and coarse/fine computational cost balancing is tricky and requires detailed understanding of parameter space structure (correlations/degeneracies between parameters).

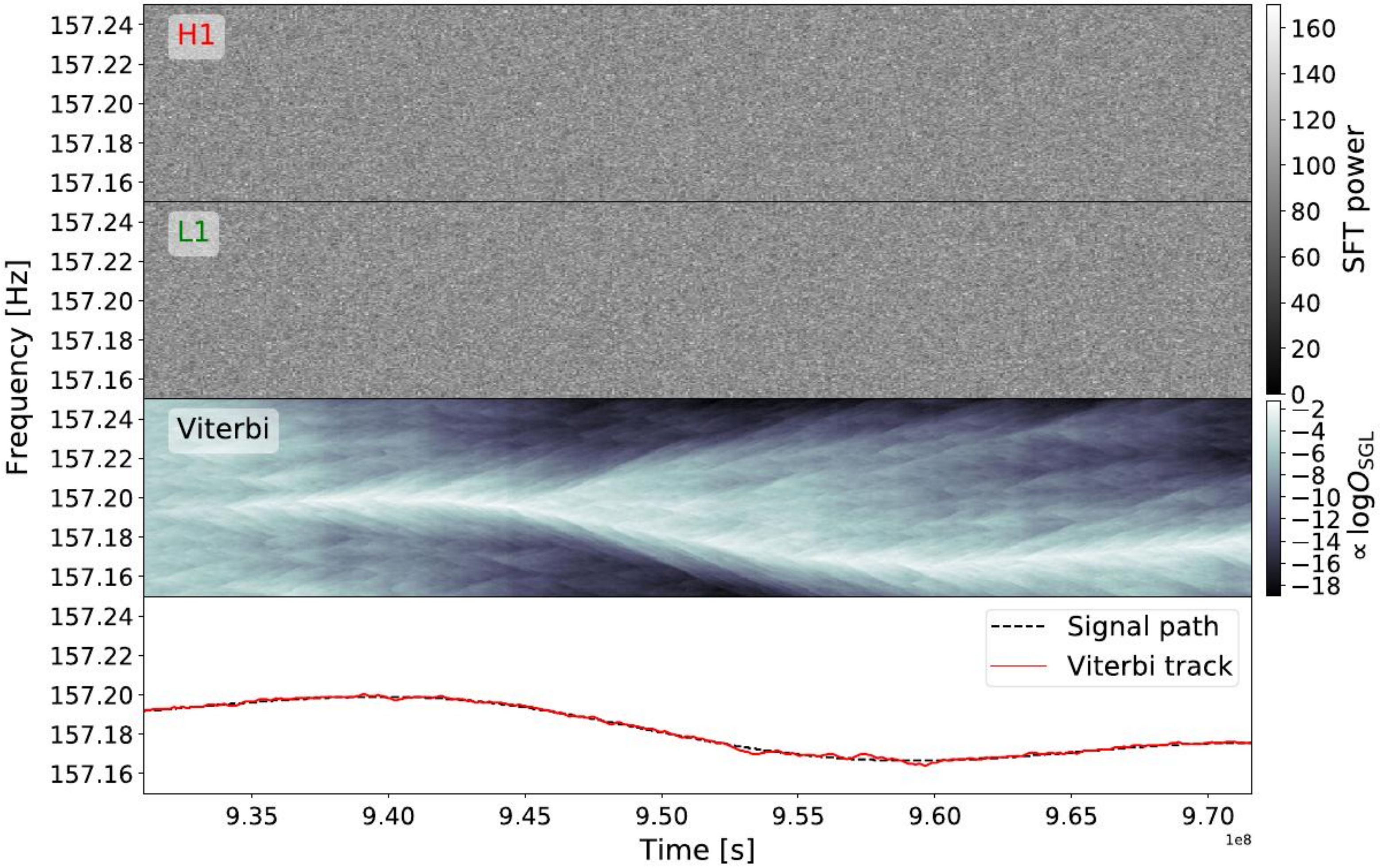


candidate post-processing

- Wide-parameter space searches (directed, all-sky) produce many outliers.
- Typical steps (in variable order):
  - vetos: use simple characteristics of noise or expected signals
    to “kill” candidates en masse
  - clustering: reduce number of candidates by identifying small volumes in parameter space with multiple outliers that could come from the same physical source (instrumental disturbance or real CW signal)
  - follow-up: run a new search around interesting candidates, with different methods or settings: switching to matched filter if not used in first stage, increasing the coherence times, MCMCs, etc
  - upper limits: if no detection ☹ : software injections of simulated signals to estimate the h_0 at which we’d detect 95% of signals (averaged over other parameters)
    - can then be astrophysically interpreted as max allowed ellipticity
      for a NS at a certain distance
    - or equivalently exclusion distance for NSs at given max ellipticity
    - or e.g. saturation amplitude under r-mode model

Viterbi methods

- Consider CW signal as a "hidden Markov model" and the GW model as the observable derived from it.
- points along a time-frequency track are the "states" of that model
- "Viterbi algorithm" is an efficient way to find the best track across [t,f] data range.
- → extremely cheap CW search



[Bayley+2019]

- robust against non-ideal signal evolution, e.g. NS glitchs, timing noise, spin wandering due to choppy accretion, …
- Suvorova+2016, Suvorova+2017, Sun+2017, Sun+2019, Bayley+2019
- Used in various directed and all-sky searches.

acknowledgments

D. Keitel is supported by the Spanish Ministerio de Ciencia, Innovación y Universidades (ref. BEAGAL 18/00148) and cofinanced by the Universitat de les Illes Balears, and would like to acknowledge additional support by the Spanish Agencia Estatal de Investigación grants CNS2022-135440, PID2022-138626NB-I00, PID2019-106416GB-I00, RED2022-134204-E, RED2022-134411-T, funded by MCIN/AEI/10.13039/501100011033; the MCIN with funding from the European Union NextGenerationEU/PRTR (PRTR-C17.I1); Comunitat Autonòma de les Illes Balears through the Direcció General de Recerca, Innovació I Transformació Digital with funds from the Tourist Stay Tax Law (PDR2020/11 - ITS2017-006), the Conselleria d’Economia, Hisenda i Innovació grant numbers SINCO2022/18146 and SINCO2022/6719, co-financed by the European Union and FEDER Operational Program 2021-2027 of the Balearic Islands; and the “ERDF A way of making Europe”.

Some of this material is based upon work supported by NSF’s LIGO Laboratory which is a major facility fully funded by the National Science Foundation. LIGO Laboratory and Advanced LIGO are funded by the United States National Science Foundation (NSF) as well as the Science and Technology Facilities Council(STFC) of the United Kingdom, the Max-Planck-Society (MPS), and the State of Niedersachsen/Germany for support of the construction of Advanced LIGO and construction and operation of the GEO600 detector. Additional support for Advanced LIGO was provided by the Australian Research Council. Virgo is funded, through the European Gravitational Observatory (EGO), by the French Centre National de Recherche Scientifique (CNRS), the Italian Istituto Nazionale di Fisica Nucleare (INFN) and the Dutch Nikhef, with contributions by institutions from Belgium, Germany, Greece, Hungary, Ireland, Japan, Monaco, Poland, Portugal, Spain. KAGRA is supported by Ministry of Education, Culture, Sports, Science and Technology (MEXT), Japan Society for the Promotion of Science (JSPS) in Japan; National Research Foundation (NRF) and Ministry of Science and ICT (MSIT) in Korea; Academia Sinica (AS) and National Science and Technology Council (NSTC) in Taiwan.