Celebrating the Science of Giorgio Parisi

Stochastic Resonance in Climate Change

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Giorgio Parisi

Alfonso Sutera
1950-2013

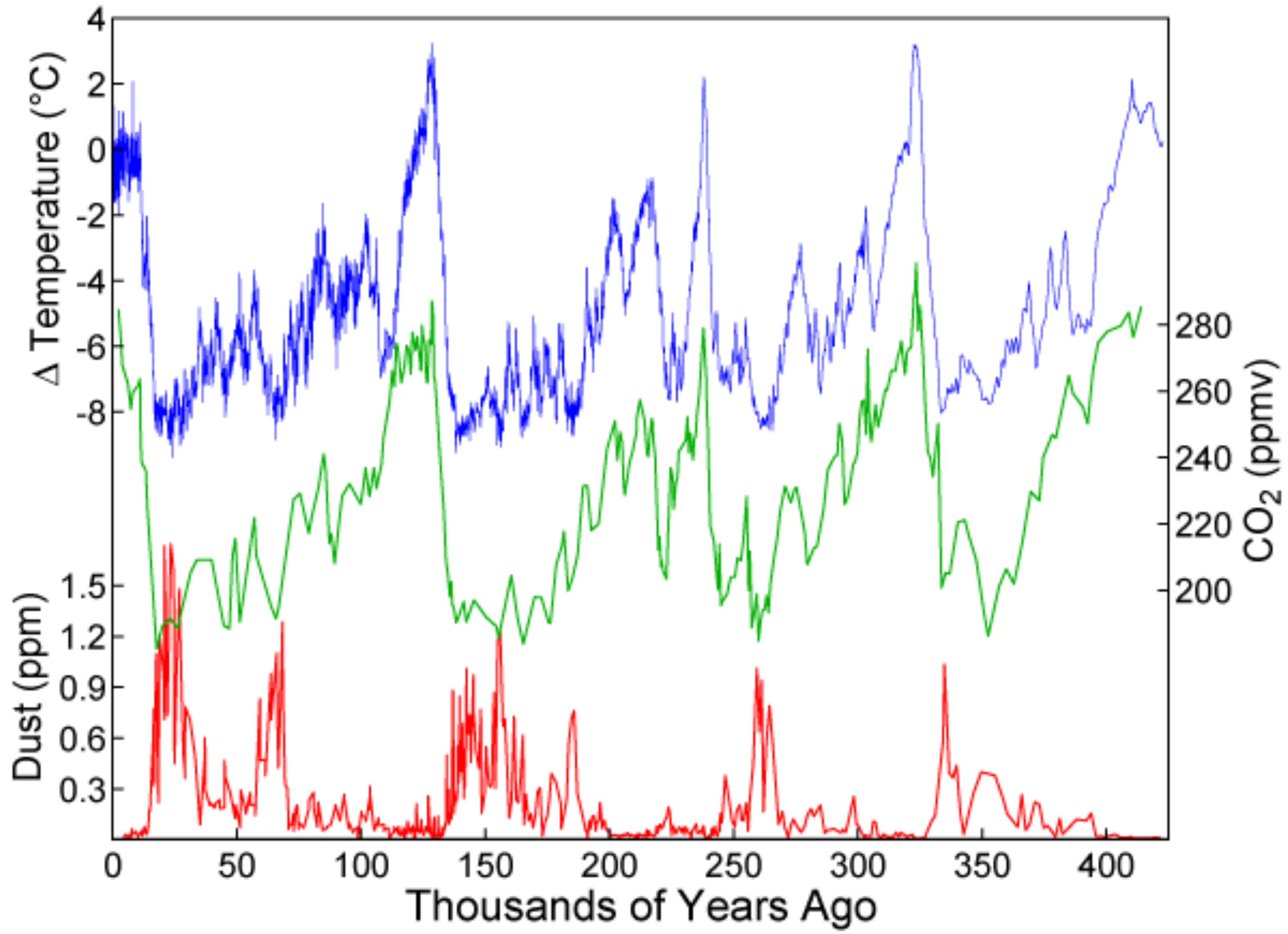
Angelo Vulpiani

Benzi, R., Sutera, A., and Vulpiani, A.: The mechanism of stochastic resonance, J. Phys. A, 1981.

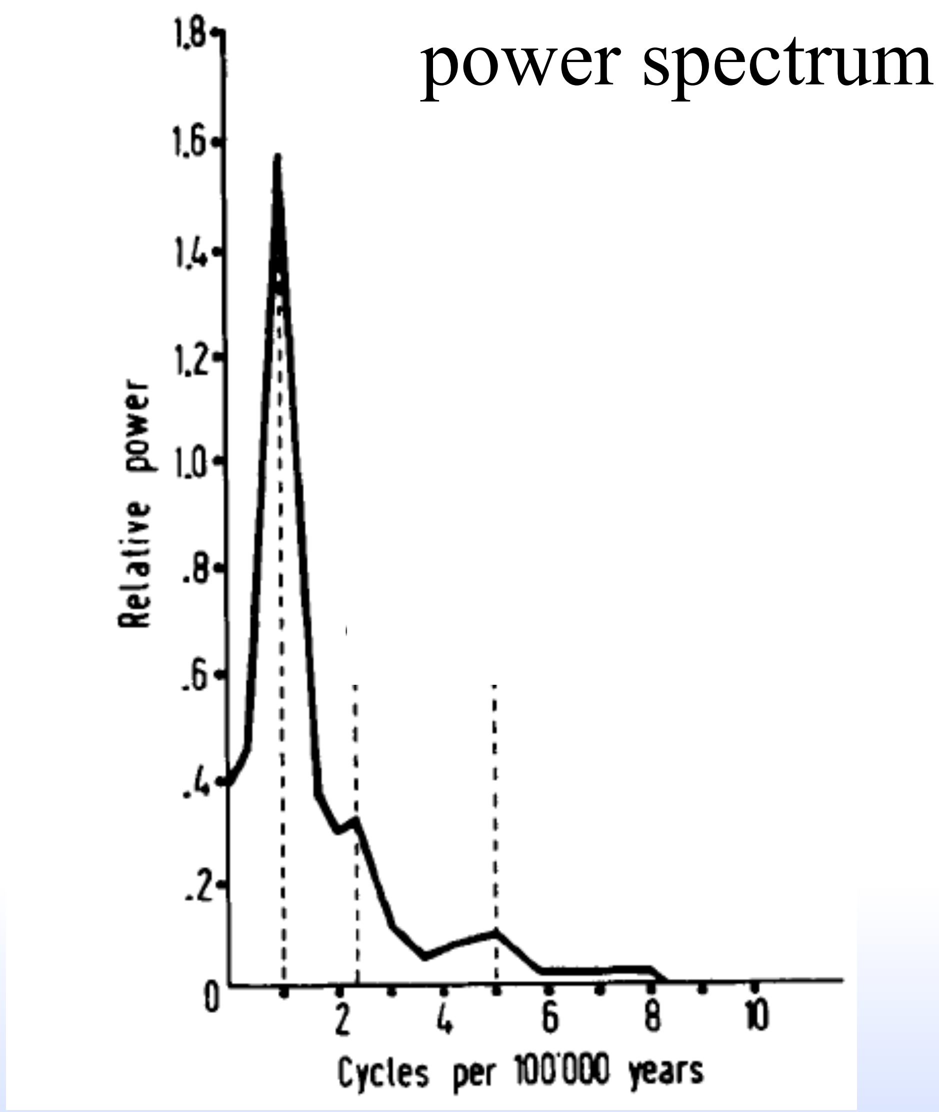
Benzi, R., Parisi, G., Sutera, A., and Vulpiani, A.: Stochastic resonance in climatic change, Tellus, 34, 10–16, 1982

Paleo Climate Records over the last one million year

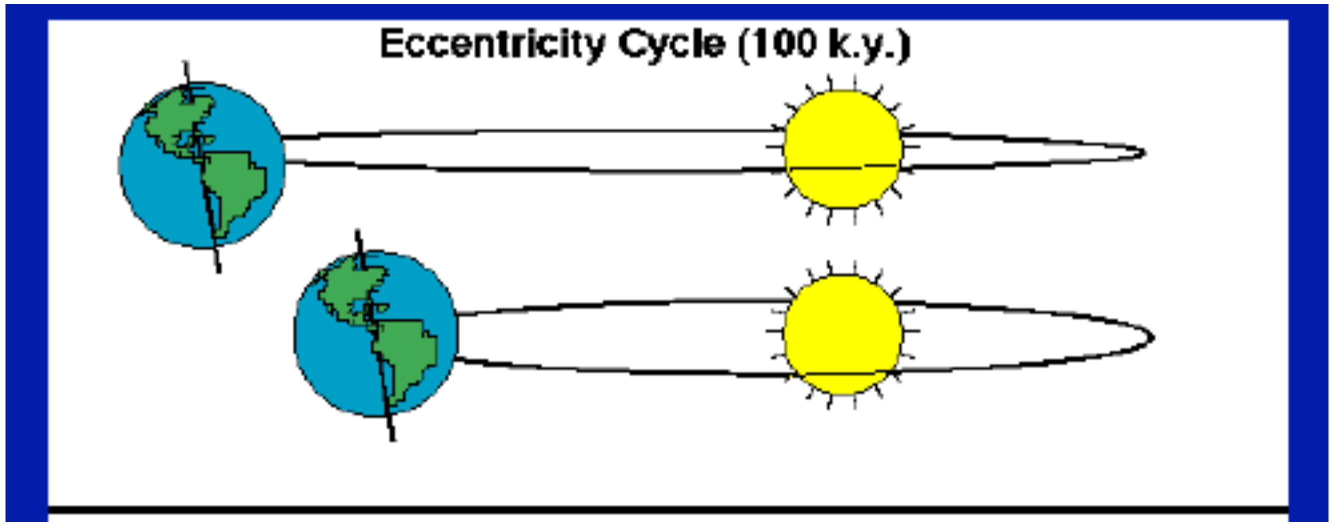
During the last 1million years climate changes almost periodically with a period of 100,000 years. **Why**?



Last 400,000 years



Variation in Orbital Eccentricity (~100,000 year cycle)

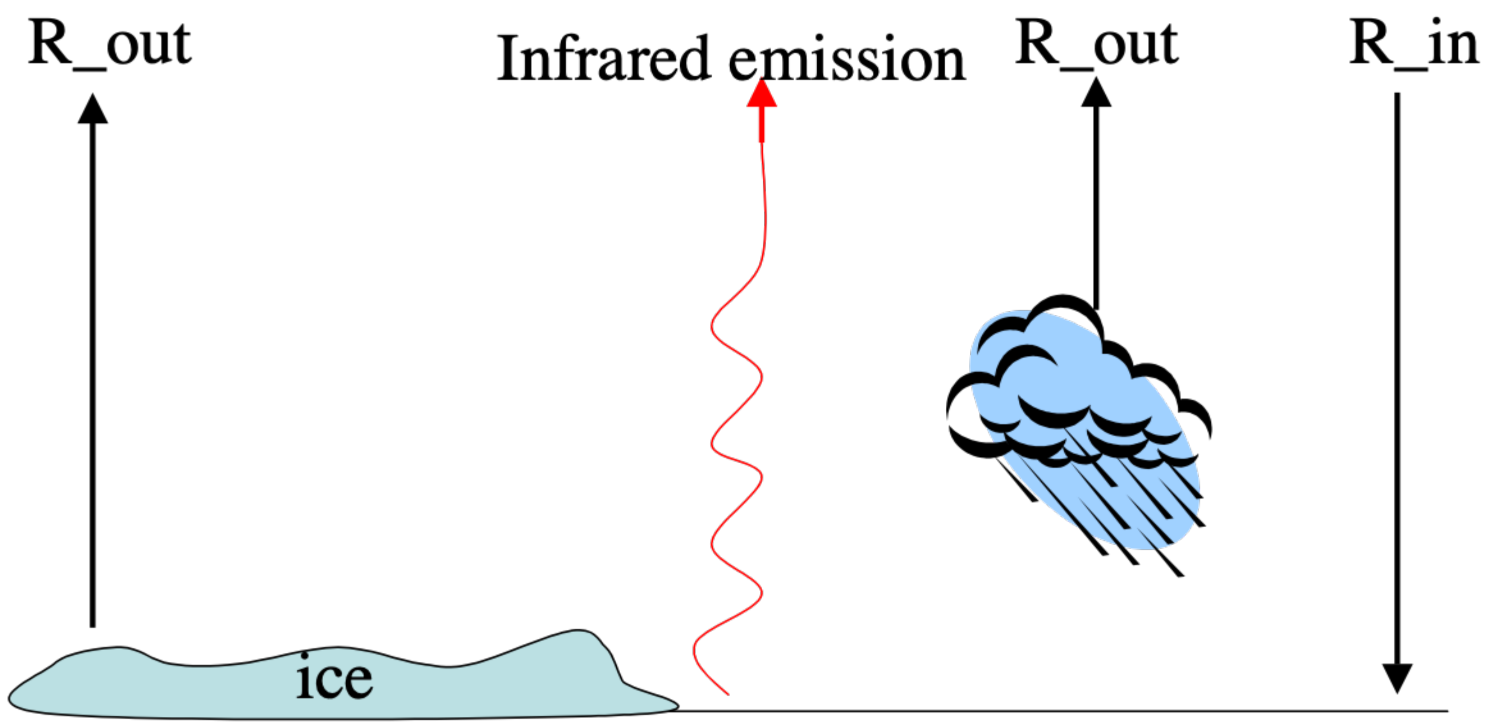


The astronomer Milutin Milanković formulated the astronomical theory of climate in 1920 under the title *Mathematical Theory of Heat Phenomena Produced by Solar Radiation*.

The change in the eccentricity produces a change in the global solar radiation of 0.25 W/m^2, which is about 0.1% of net incoming radiation.

Unfortunately, on the Earth mean temperature, the Milanković forcing produces a change of 0.2 degree and not 10 as observed in the records

The simplest non trivial model of climate (Budyko-Sellers)



$$C\frac{dT}{dt} = R_{in}(1-\alpha(T)) - \epsilon(T)$$

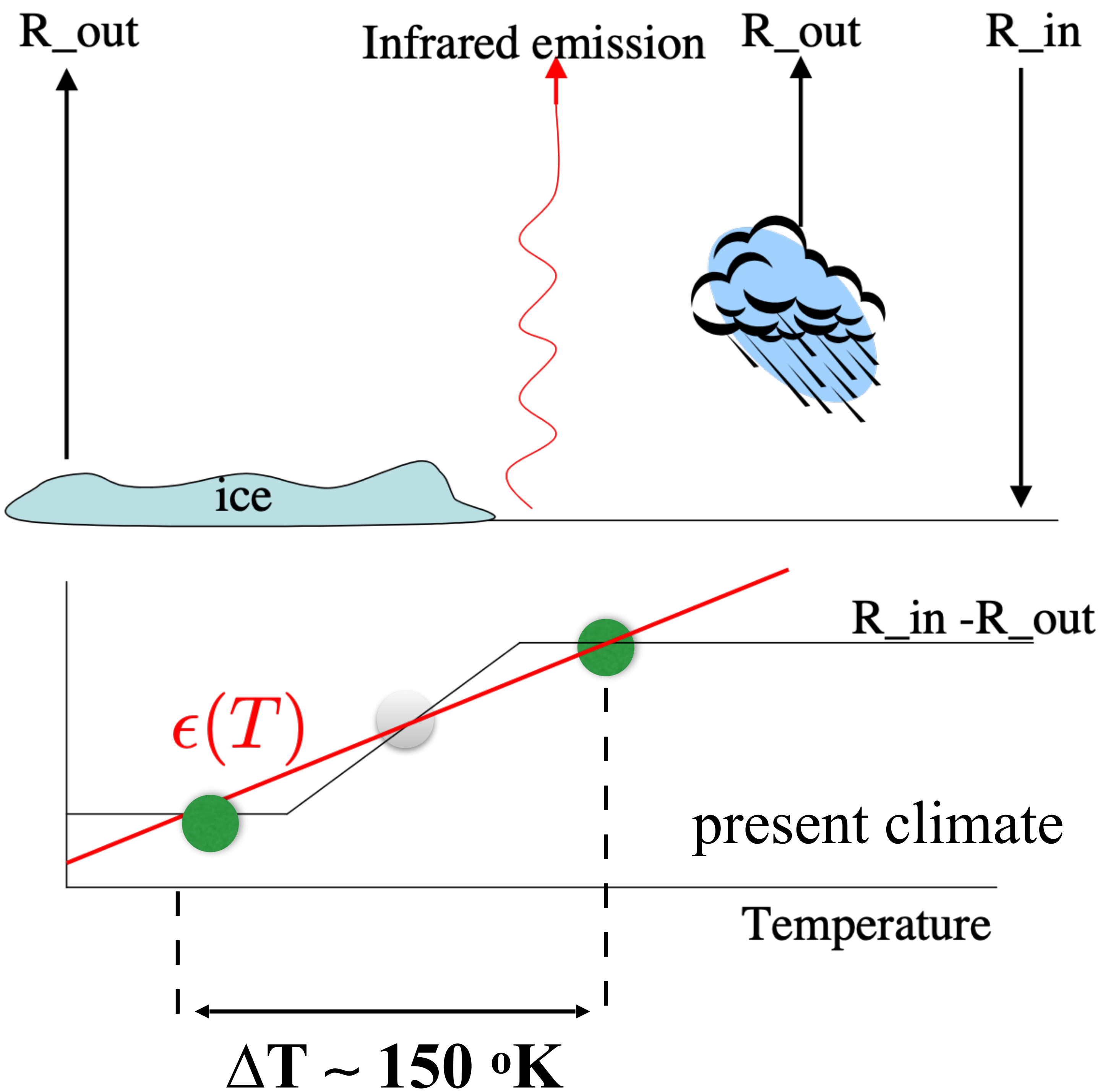
ALBEDO

$$\alpha(T) \equiv \frac{R_{out}}{R_{in}}$$

INFRARED EMISSION

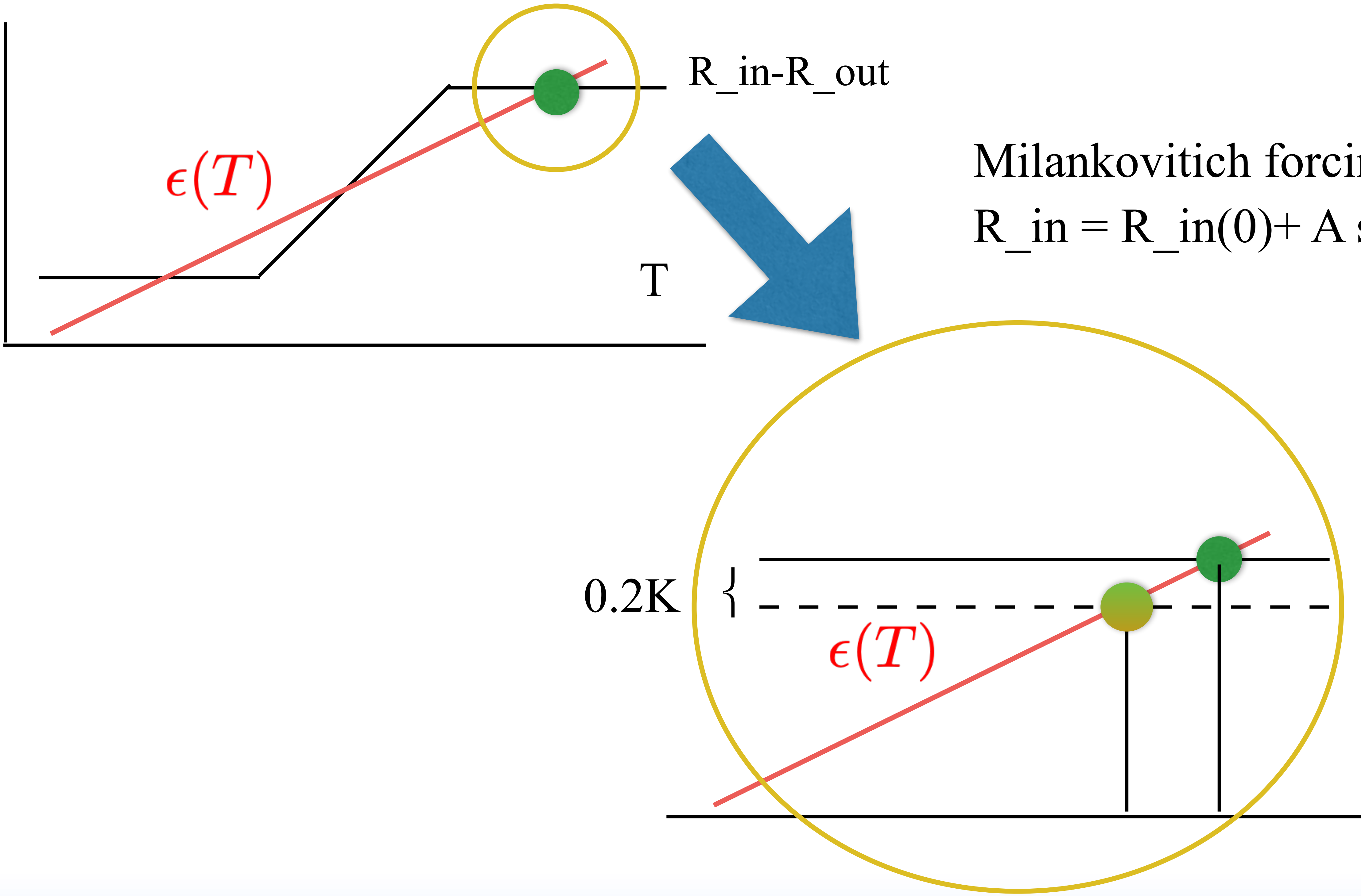
$$\epsilon(T)$$

The simplest non trivial model of climate



$$R_{in}(1-\alpha(T))$$

The effect of Milankovitch Forcing



Milankovitich forcing is

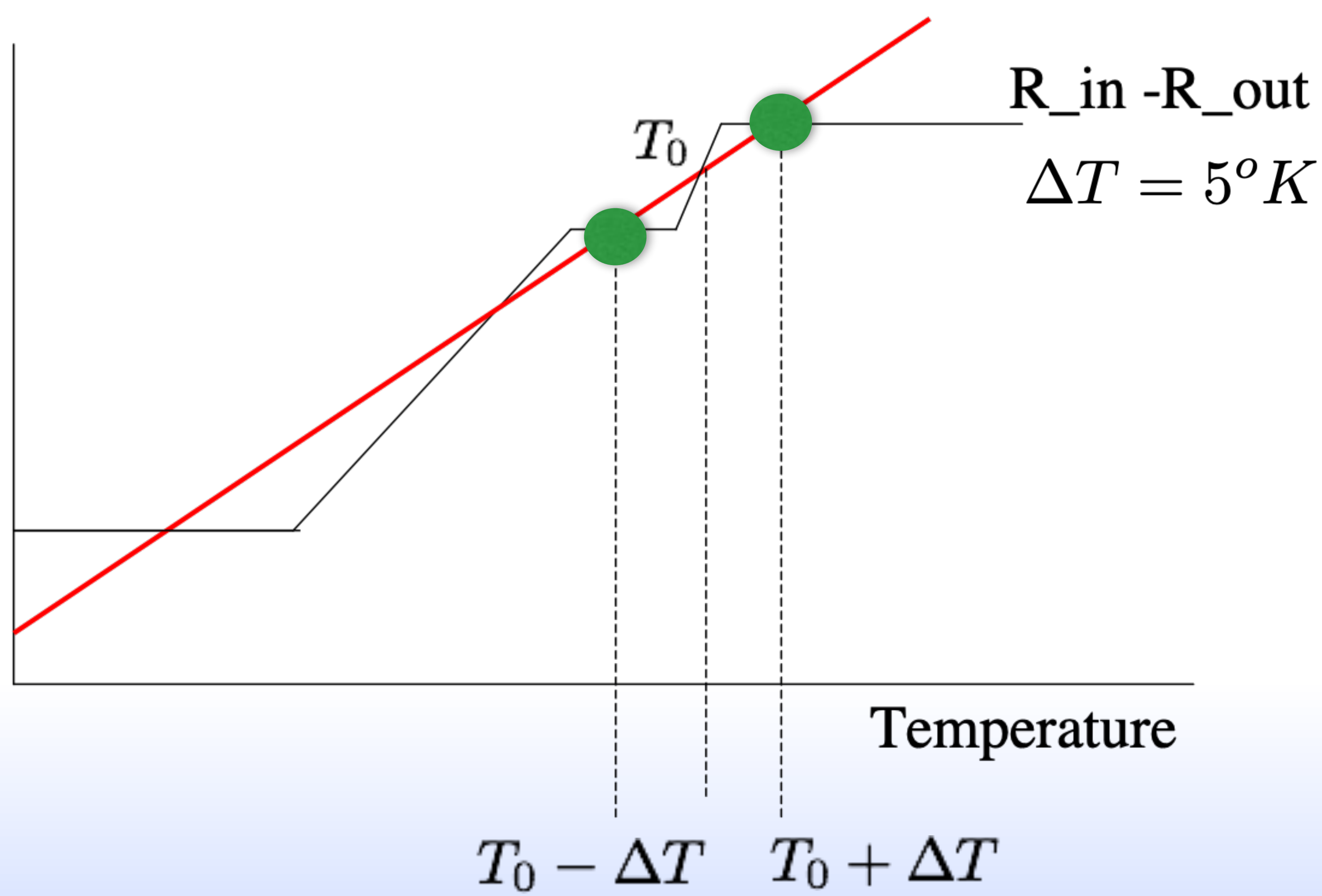
R_in = R_in(0)+ A sin(ωt)

How to explain 10K variation?

First step 1979: R.B., Parisi, Sutera, Vulpiani

$$C\frac{dT}{dt} = R_{in}(1-\alpha(T)) - \epsilon(T) \longrightarrow \frac{dT}{dt} = F(T)$$

Starting assumption: there exists two stable climate states corresponding to glacial and interglacial climates with temperatures $\mathbf{T_g}$ and $\mathbf{T_p}$ respectively.



$$T_p = T_0 + \Delta T$$
$$T_g = T_0 - \Delta T$$

The existence of the two states is supported by the paleo climate records and our present knowledge on ice albedo feedback as a function of the temperature.

Second step

Transitions between the glacial and interglacial climate due to the "climate noise"

Sutera(1981), RB, Pandolfo, Sutera (1981)

The idea of "climate noise" was introduced in 1976 by Klaus Hasselmann to investigate the Sea Surface Temperature (SST) variations in the ocean

$$d\delta T = -a\delta T dt + \underbrace{\sqrt{D}dw}$$

Fluctuations of the wind speed which are white on timescales of weeks to a year

δT = SST anomaly

a^{-1} = 2-5 months

From the present climatic fluctuations we can compute the **effective noise acting in the system**

$\boldsymbol{\delta}$T = fluctuations around present climate

$\boldsymbol{\tau}_s$ = 6-10 years

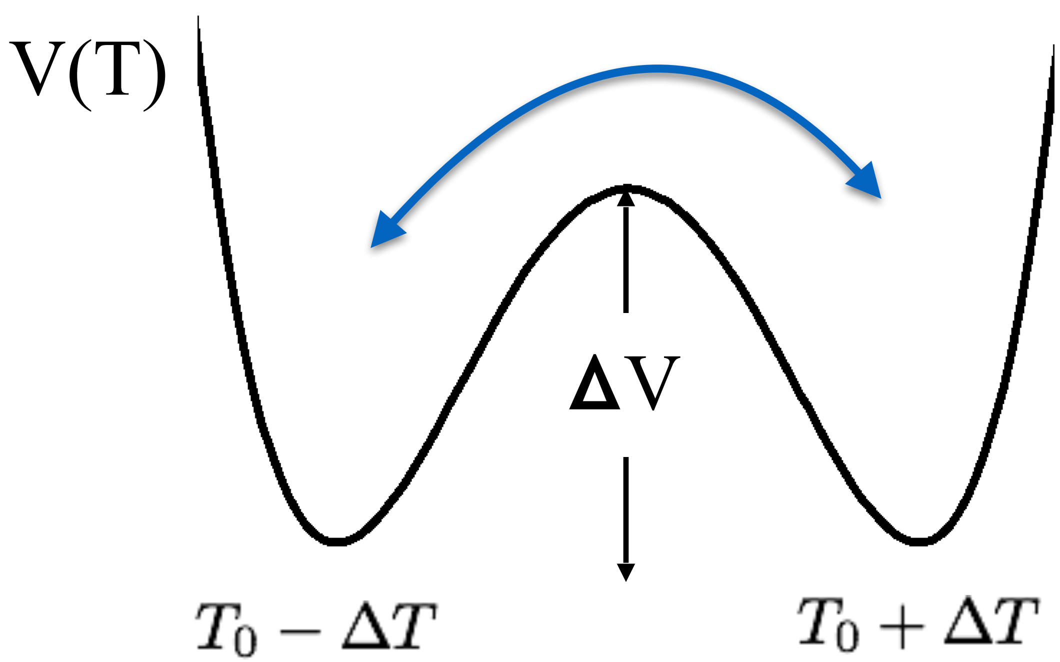
$$\frac{dT}{dt} = F(T) \quad \Longrightarrow \quad d\delta T = \left[-\frac{1}{\tau_s}\delta T\right] dt + \sigma dw$$

$$\langle(\delta T)^2\rangle = \frac{\sigma^2}{2\tau_s} \longrightarrow \sigma^2 \sim 0.2\ \mathrm{K}^2/\mathrm{year}$$

Average time of noise induced climate transition

$$dT = F(T)dt + \sigma dW \qquad V(T) = -\int dT F(T)$$

Close to T_0, V(T) can be approximated as a symmetric double well "potential".
All the parameters of V(T) can be computed knowing ΔT and $\boldsymbol{\tau}_s$



$\langle \tau \rangle$ = mean time to perform a transition

$$\langle \tau \rangle \sim \tau_s \exp\left[\frac{2\Delta V}{\sigma^2}\right]$$

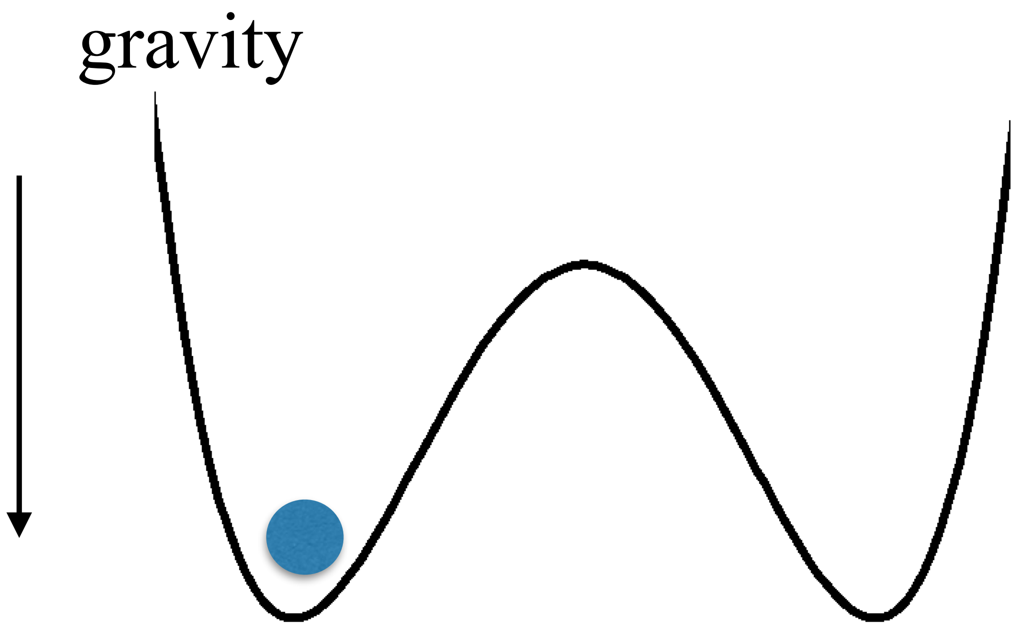
$\langle \tau \rangle \sim 50,000$ years !!

The "present" level of climate noise predicts random time transitions between the two climatic states with a characteristic time 10^5 years

What do we mean by “noise induced transition”

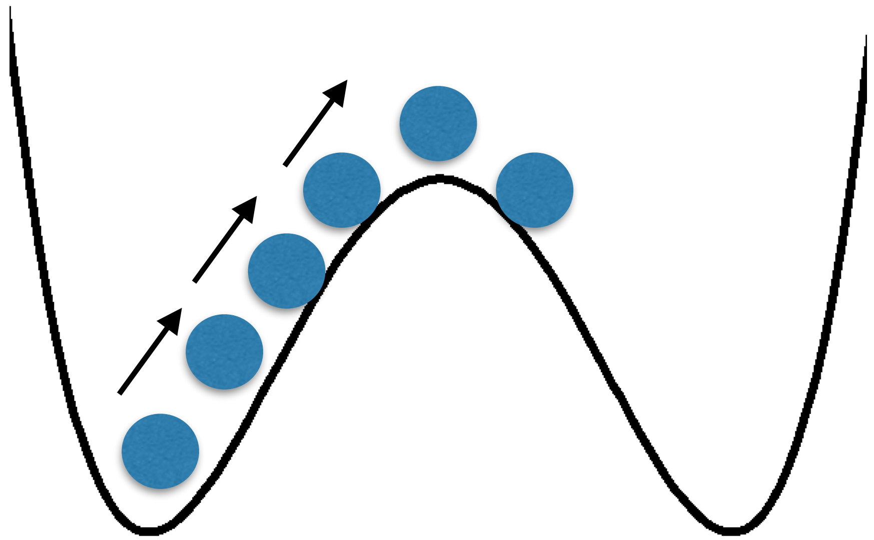
Consider a small ball with **large friction** on this surface

The ball is forced by small random kicks



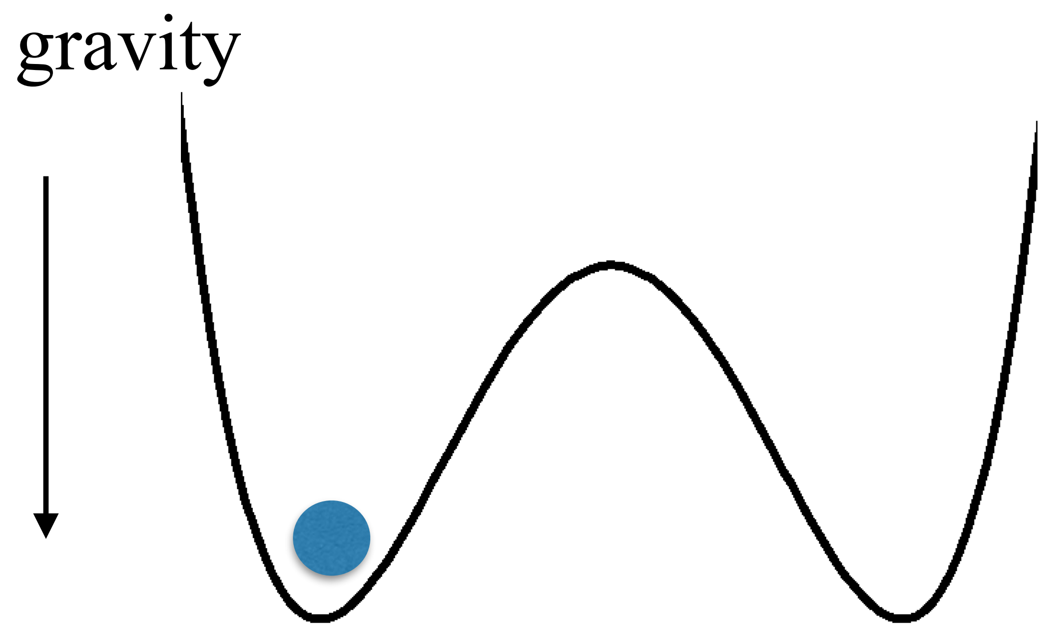
There exist very rare events which drives the ball from one stable equilibrium to the other

If the probability to go in one direction is P, we look for cases where we have N kicks in the same direction. This happens with probability P^N

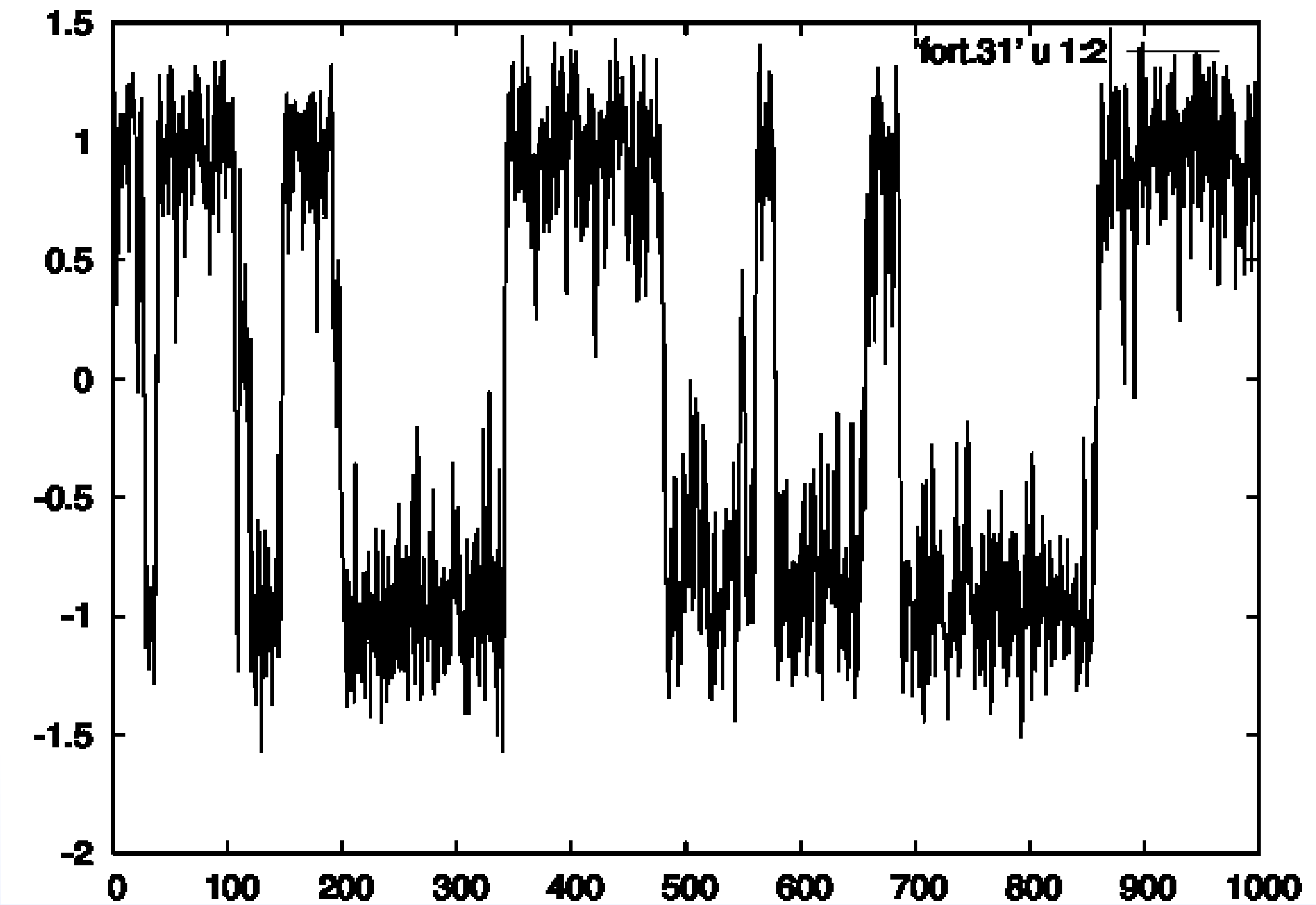


The concept of “**large deviations**” in the probabilistic sense was yet unknown

Many people in climate theory (and not only in this community!) thought that transitions between states are due to large fluctuations in the noise.



This is what we obtain from a numerical simulations

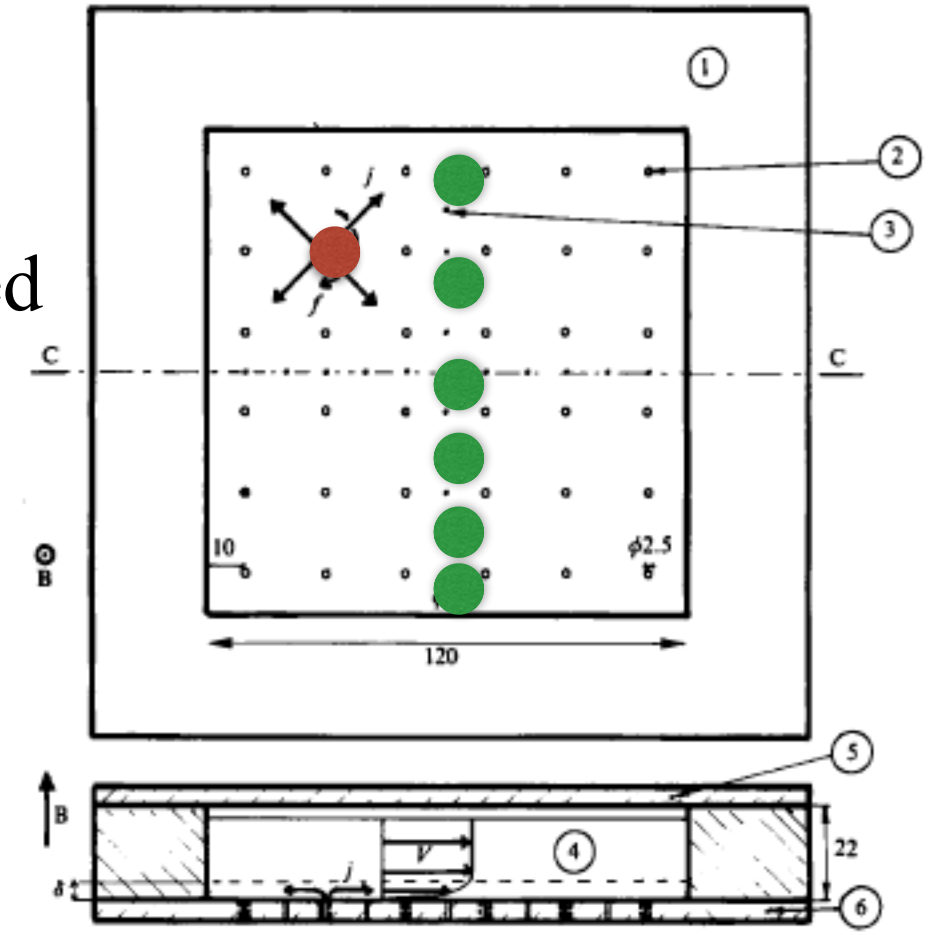


A more general view on the concept of “noise”

Let consider a fluid in a two dimensional box with small scale forcing.

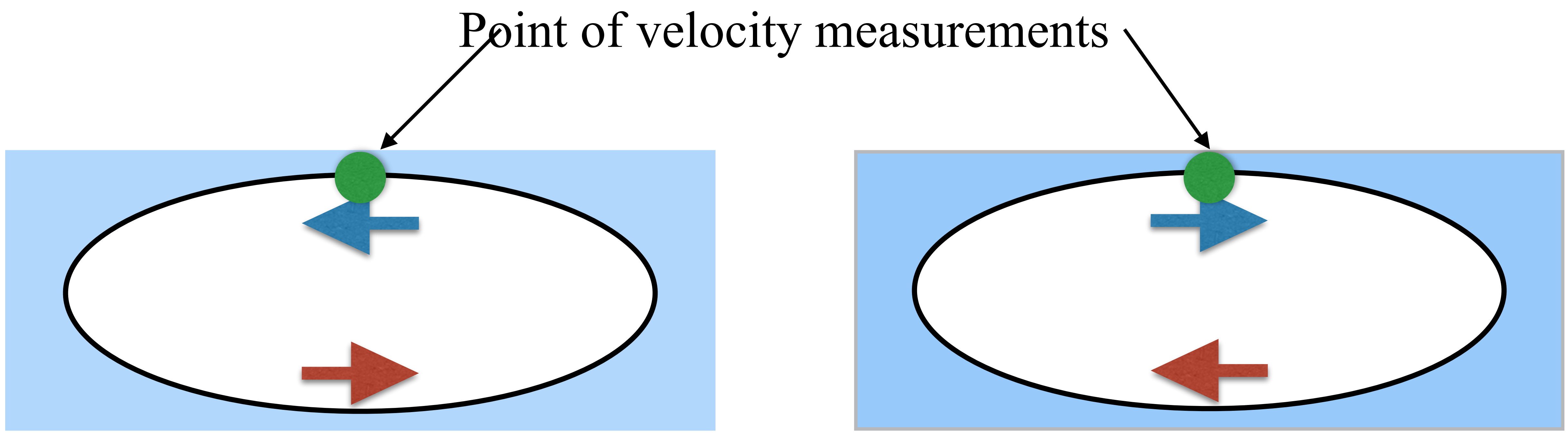
- Forcing of the fluid
- Point where velocity is measured

Top view of the experiment

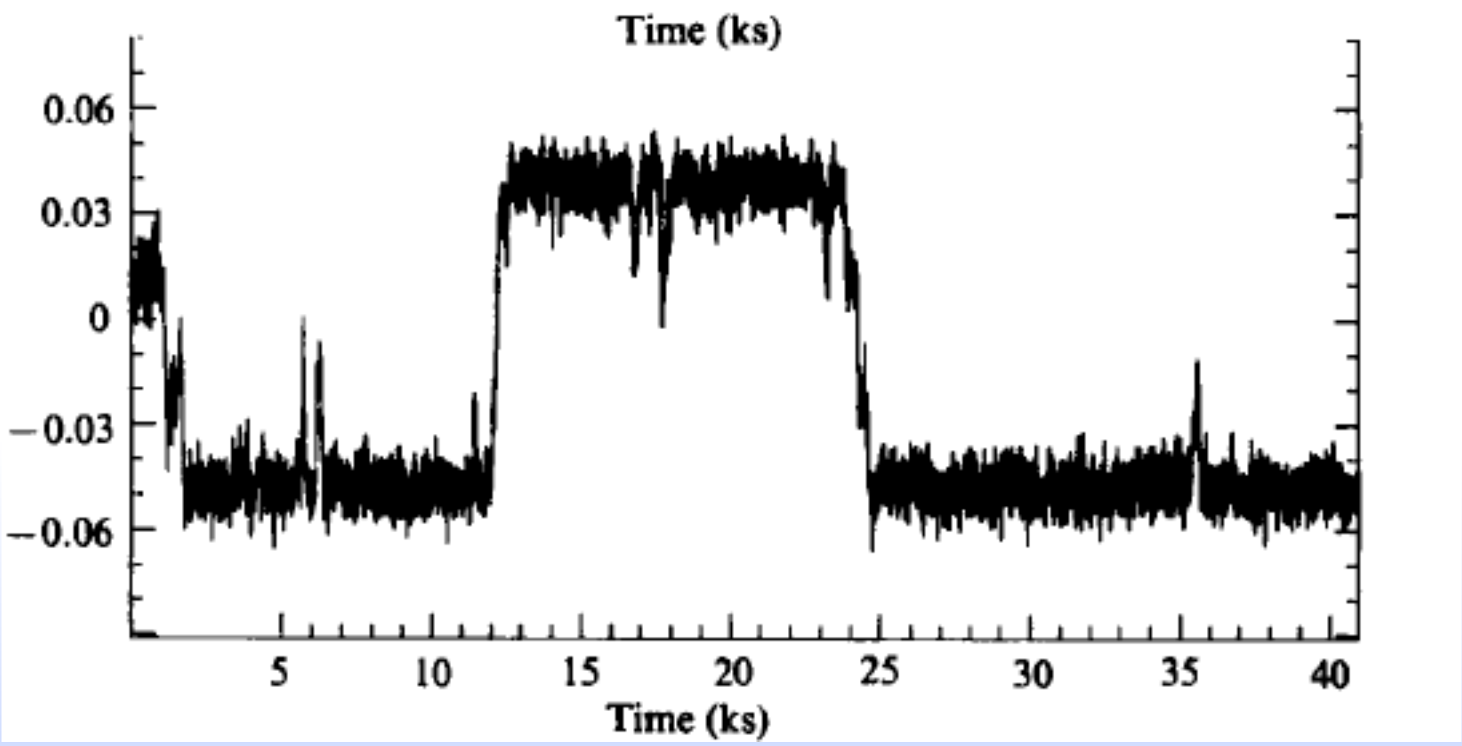


J. Sommeria 1986

The large scale "mean flow" has two possible states



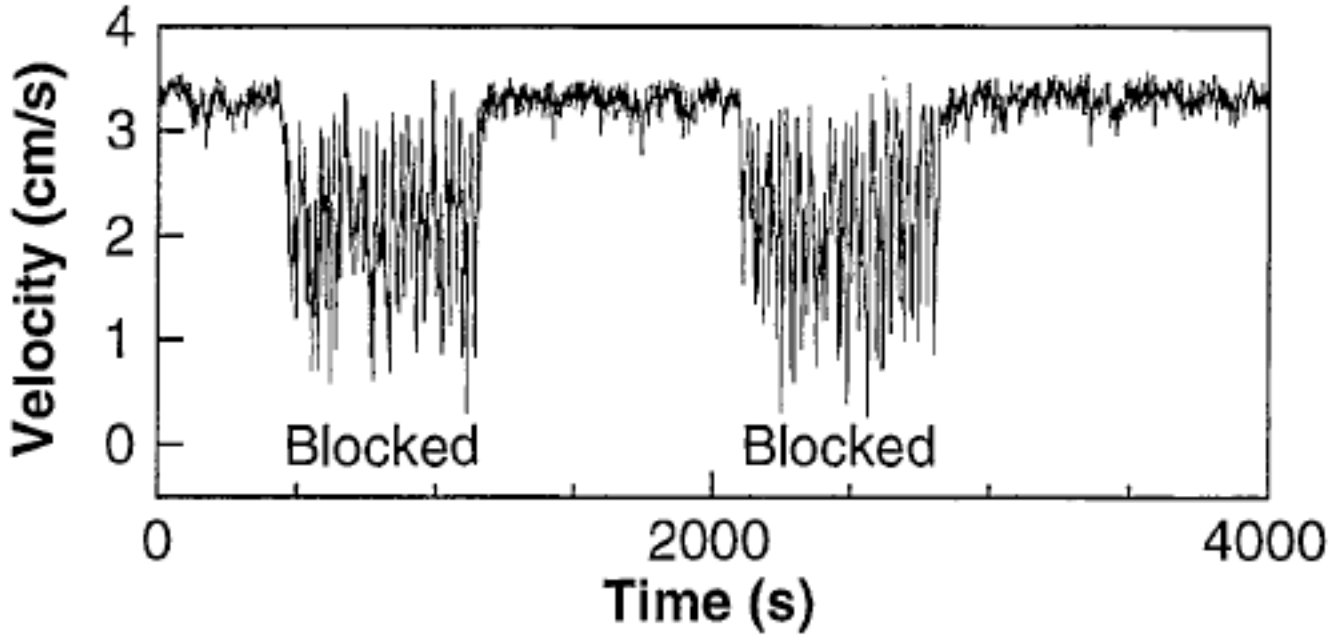
Observed records of velocity: similar to the effect of the noise



Another example

Transitions Between Blocked and Zonal Flows in a Rotating Annulus with Topography

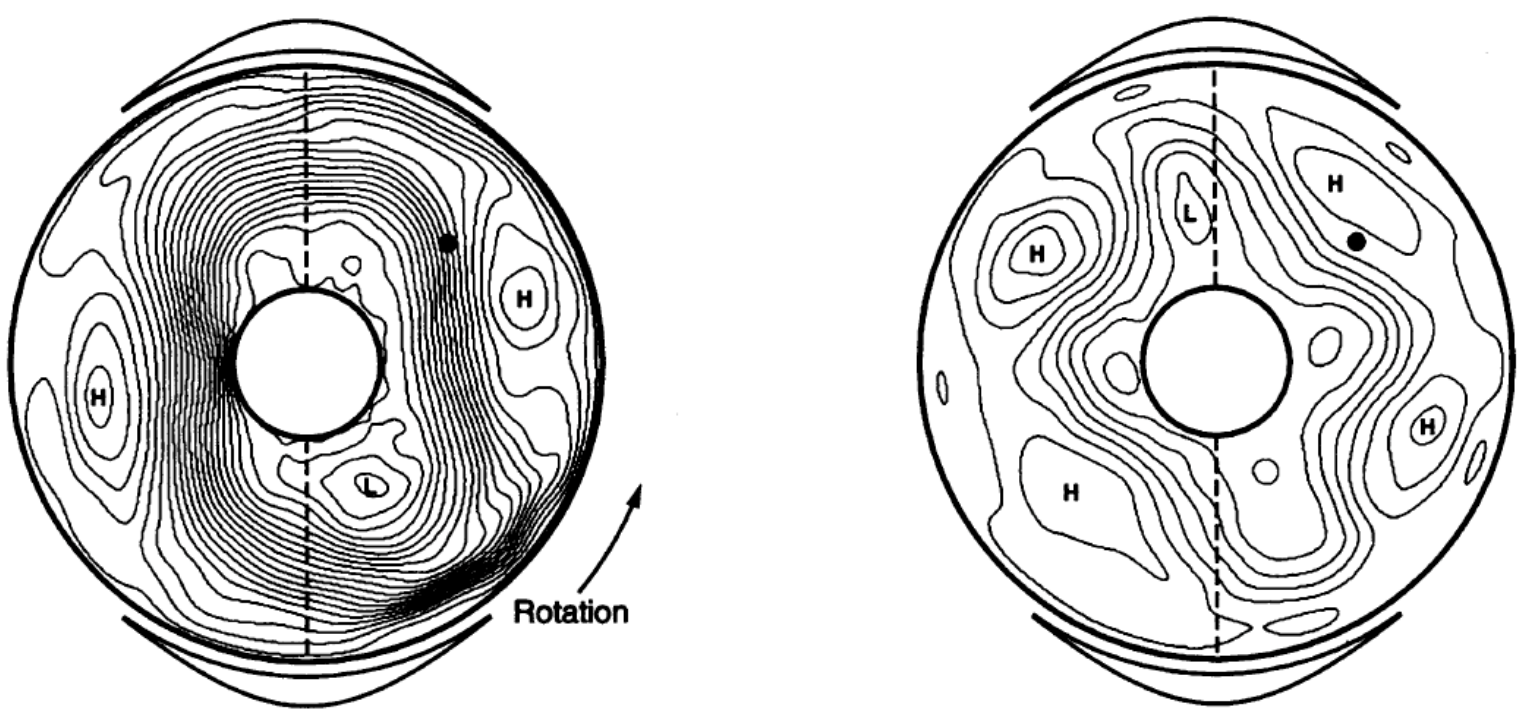
Eric R. Weeks, Yudong Tian, J. S. Urbach,* Kayo Ide, Harry L. Swinney,"richael Ghil, Science 1997



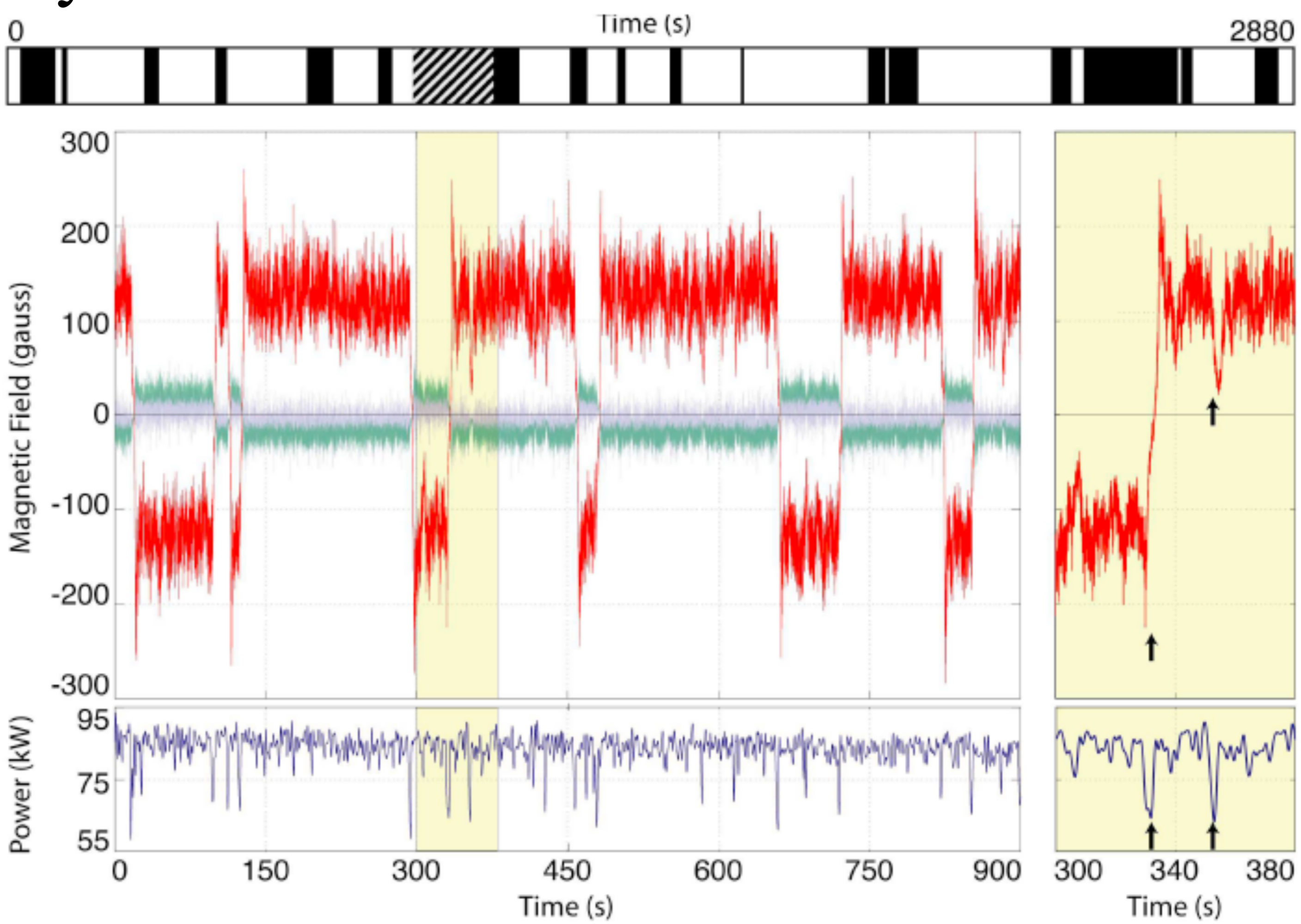
Velocity time series showng intermittent transitions between zonal and blocked flow

Zonal

Blocked

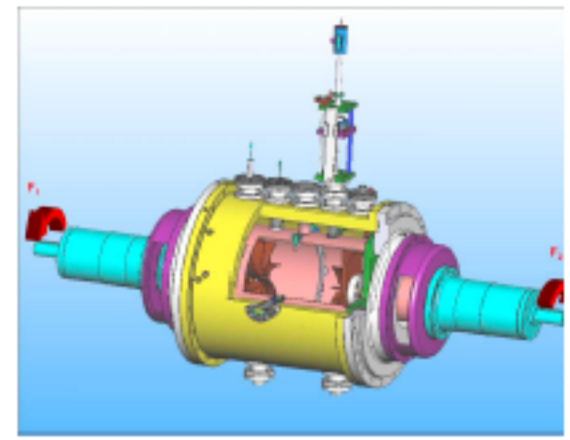


Dynamo reversals



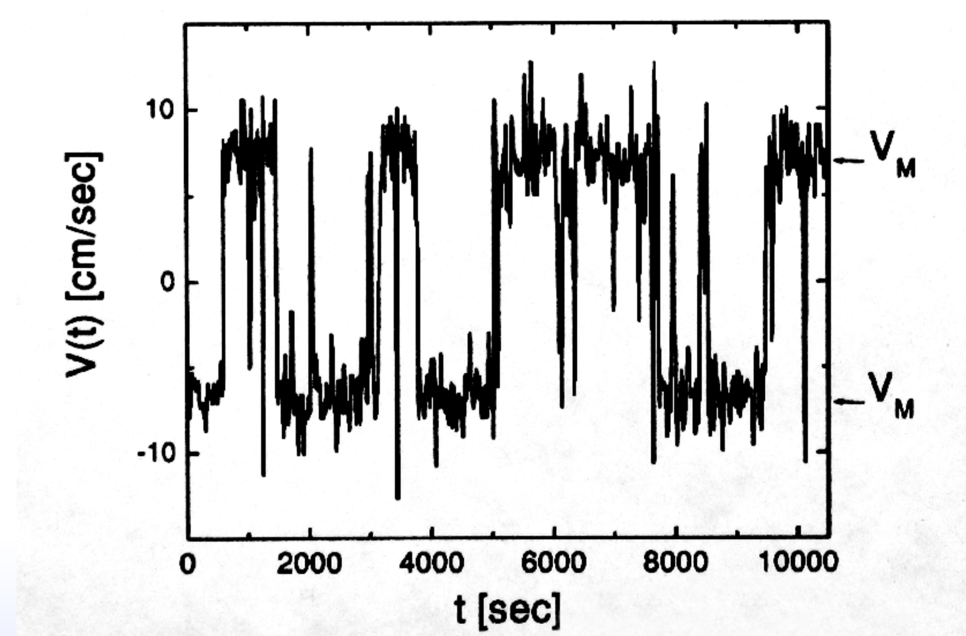
Magnetic field reversals in an experimental turbulent dynamo

M. Berhanu[1], R. Monchaux[2], S. Fauve[1], N. Mordant[1], F. Pétrélis[1], A. Chiffaudel[2], F. Daviaud[2]
B. Dubrulle[2], L. Marié[2,+], F. Ravelet[2,*], M. Bourgoin[3,#], Ph. Odier[3], J.-F. Pinton[3], R. Volk[3]



Sreenivasan, Bereshadskii, Niemela 2002

PHYSICAL REVIEW E **65** 056306

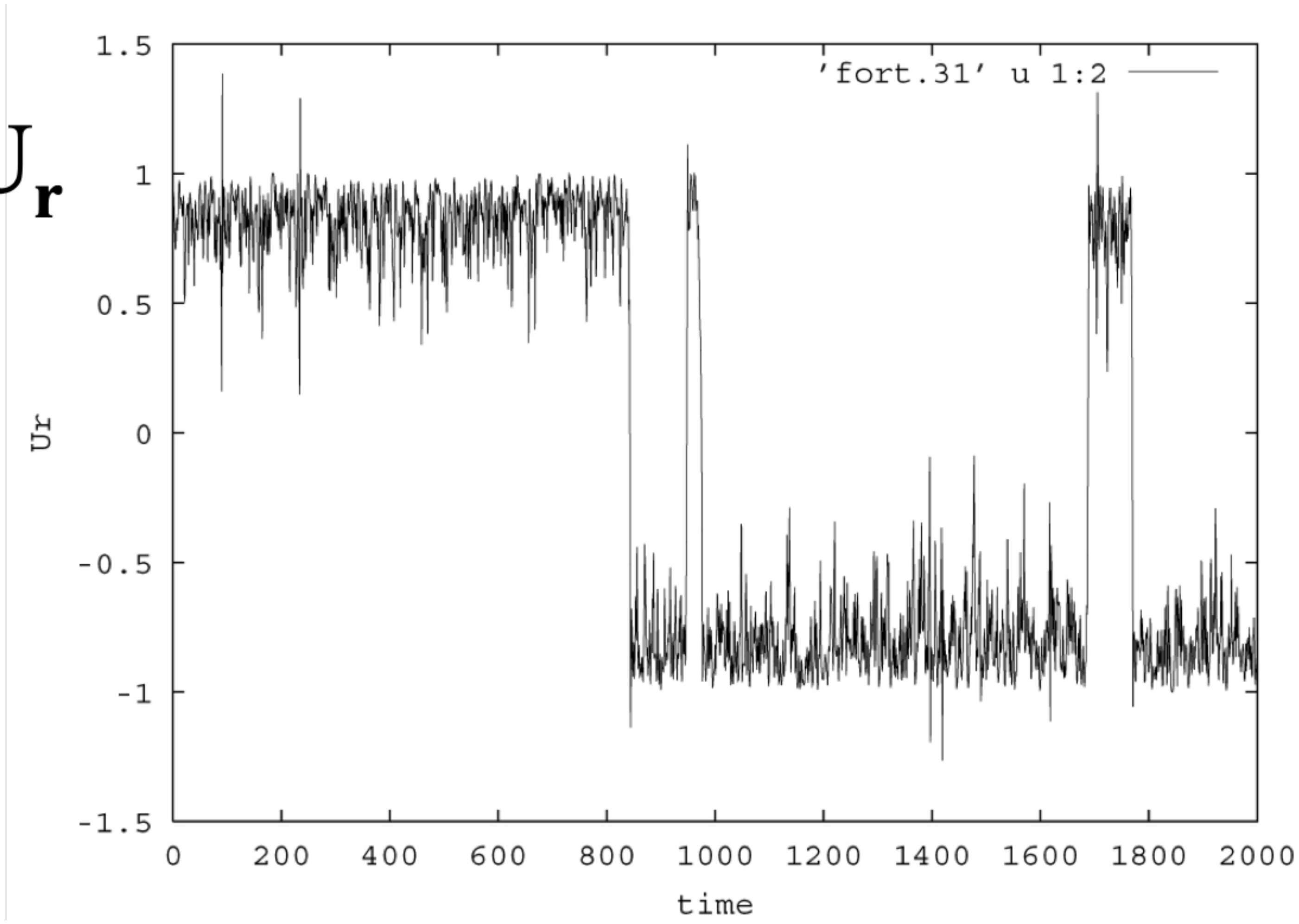


Rayleigh Bénard convection

Numerical simulations of homogenous turbulence and **no noise**

$$\partial_t U_r = U_r(1 - U_r^2) + Navier - Stokes\ eq.s$$

U_r



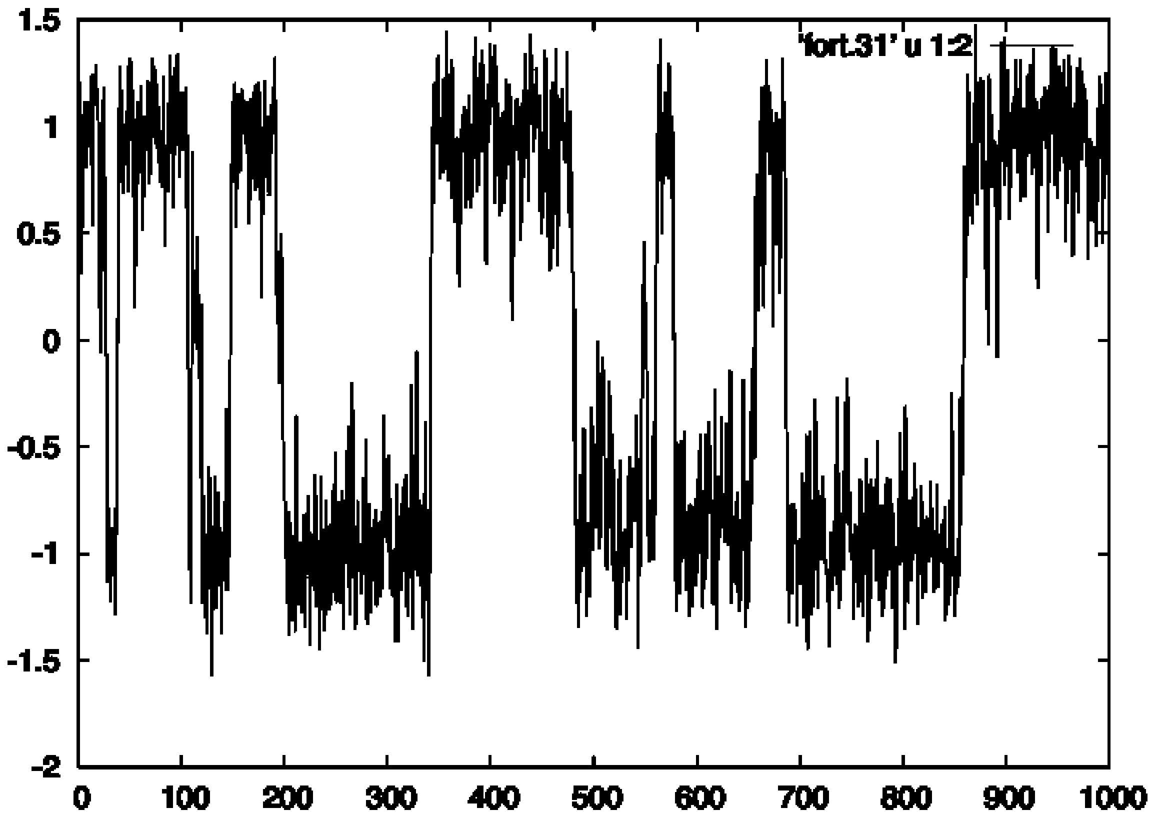
Random reversal in time

R. B. PRL 2005

Curiosity: some people refers to this paper saying that "transition are due to stochastic perturbations" !!!

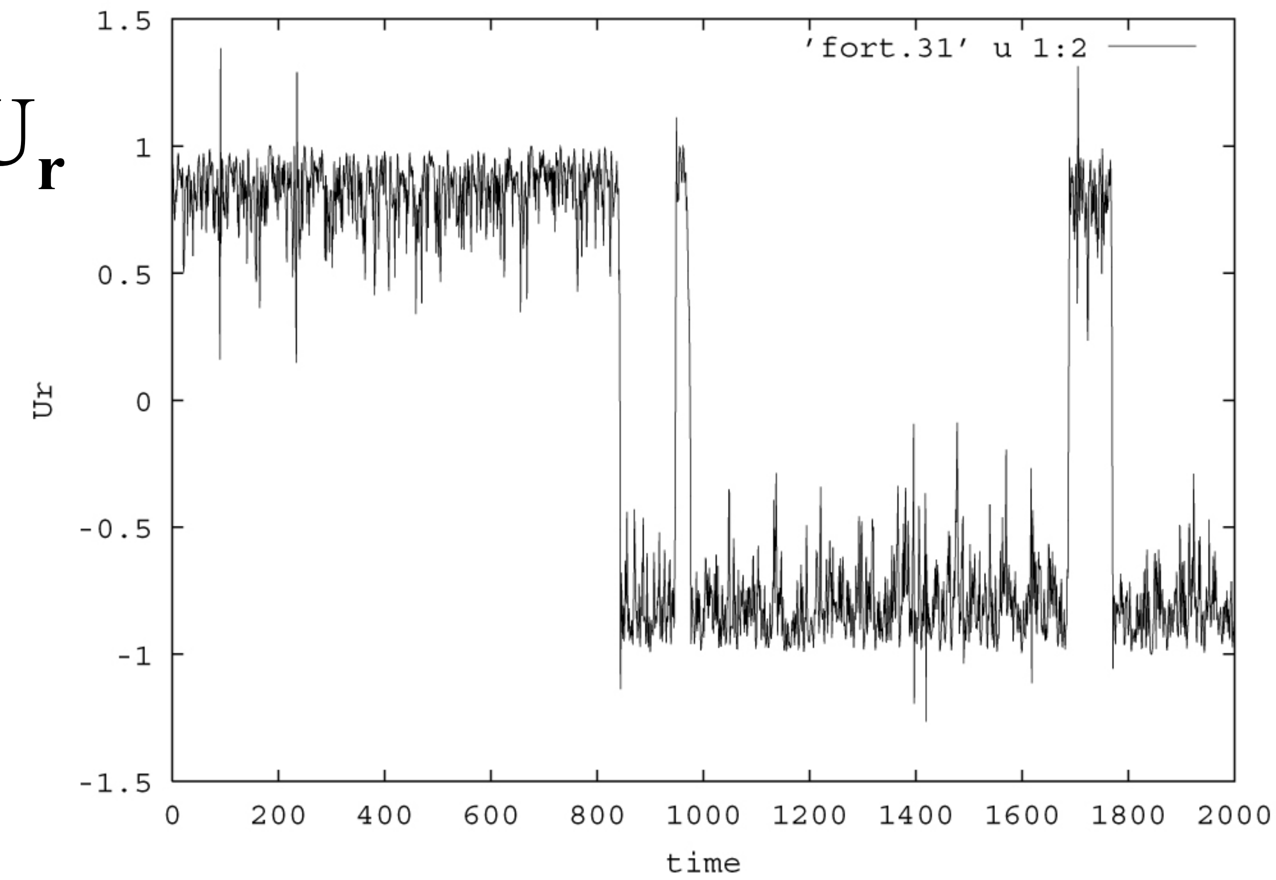
Noise and dynamics face to face

noise 1981



Navier Stokes dynamics 2005

U_r



When the fluid becomes turbulent the dynamics on the flow is equivalent (on very long times) to the effect of a noise!

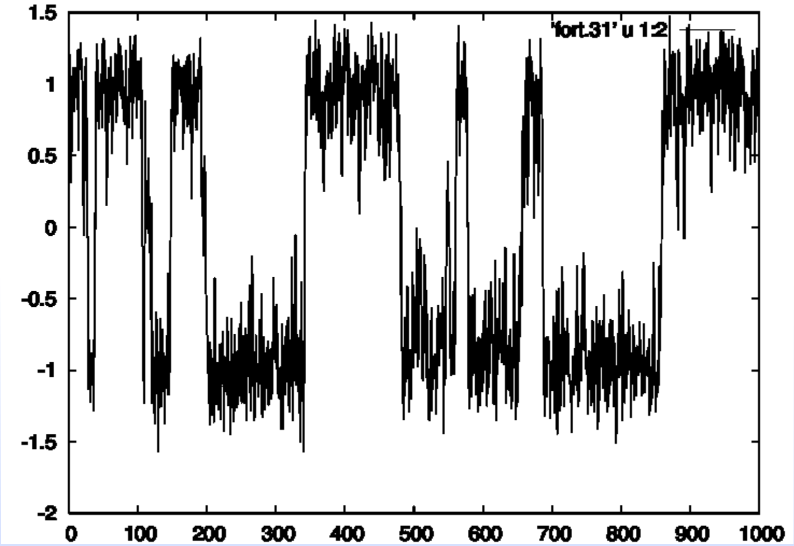
What do we **physically** mean by “noise induced transition”?

Climate “noise” is a way to take into account the internal variability of climate dynamics and, in particular, its effect on Earth global temperature.

At the time of our theoretical investigations, this idea was strongly criticised.

We now understand that we should interpret the concept of noise in a more general way.

However, the effect of the noise is not enough: the transitions between climate states occur randomly and not periodically.



November 1979: Giorgio Parisi hint on the problem

Let us now consider the effect of Milankovitch forcing

$$dT = [F(T) + A(t)]dt + \sigma dW$$

We consider the two cases $A(t)=\pm A_0$

$$\langle\tau\rangle_{A=+A_0} = \tau_s \exp\left[\frac{2\Delta V + 2A_0\Delta T}{\sigma^2}\right] = \langle\tau\rangle_{A=0} \exp\left[+\frac{2A_0\Delta T}{\sigma^2}\right]$$

$$\langle\tau\rangle_{A=-A_0} = \tau_s \exp\left[\frac{2\Delta V - 2A_0\Delta T}{\sigma^2}\right] = \langle\tau\rangle_{A=0} \exp\left[-\frac{2A_0\Delta T}{\sigma^2}\right]$$

The effect of the Milankovitch forcing becomes exponentially large !!

The internal noise and the external forcing can "cooperate" to provide an almost periodical behaviour.

February - March 1980

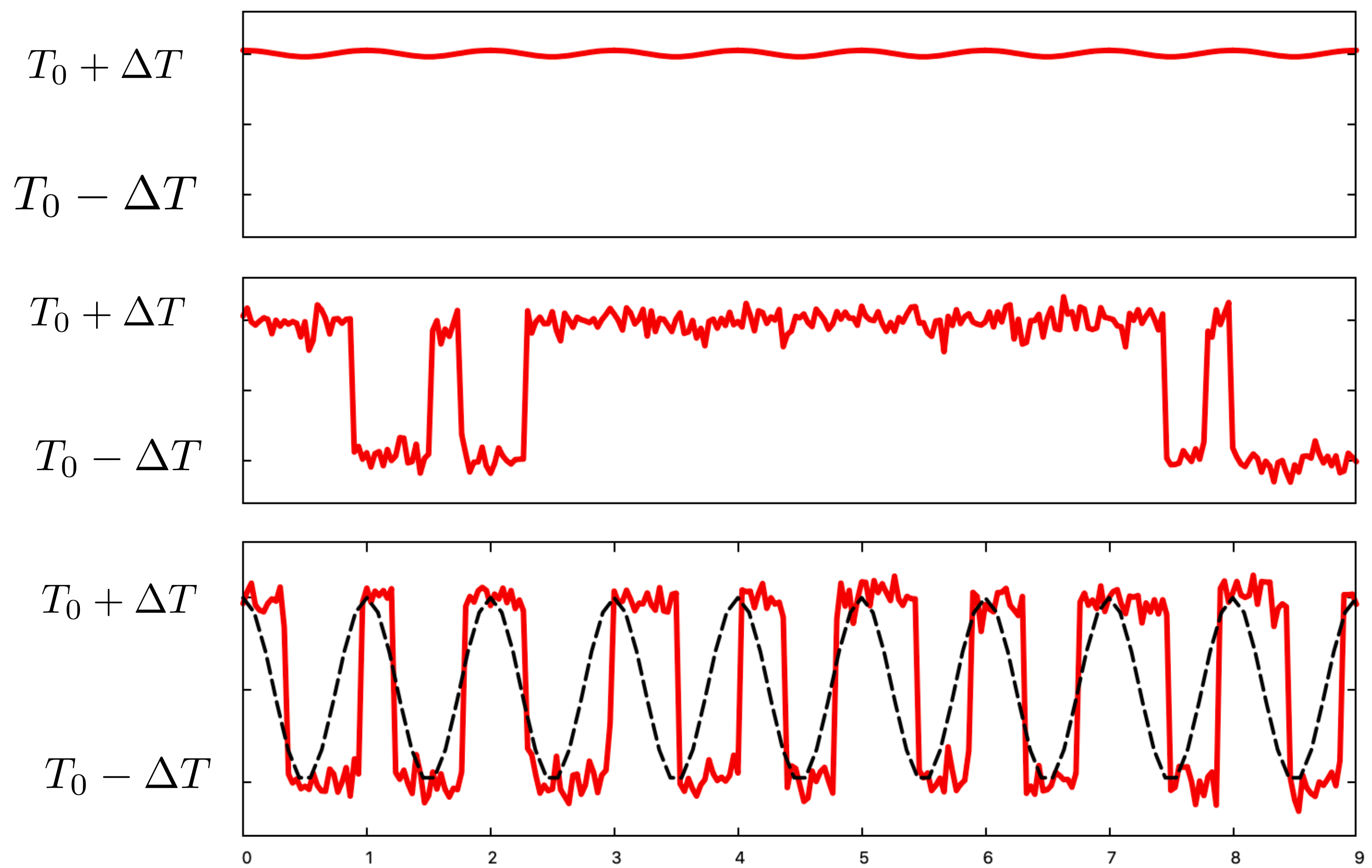
We knew how to compute the average $<\boldsymbol{\tau}>$ but not the $<\boldsymbol{\tau}^2>$ (we were "ignorant" together with a very large number of so called "experts" in stochastic differential equations !)

With Angelo Vulpiani we decided to perform numerical simulations. The naive intuitive approach was that if $<\boldsymbol{\tau}>$ is small also $<\boldsymbol{\tau}^2>$ should be small.

Numerical simulations performed during one night in February 1980: we got the correct result immediately. Phone to Alfonso (Yale Univ.) at 3 a.m.

Curiosity: we were stopped by the police questioning what we were doing in the basement of an empty building! We answered we were doing numerical simulations in the night (!) and the police probably thought we were too "stupid" to be dangerous!

A direct check using numerical simulations

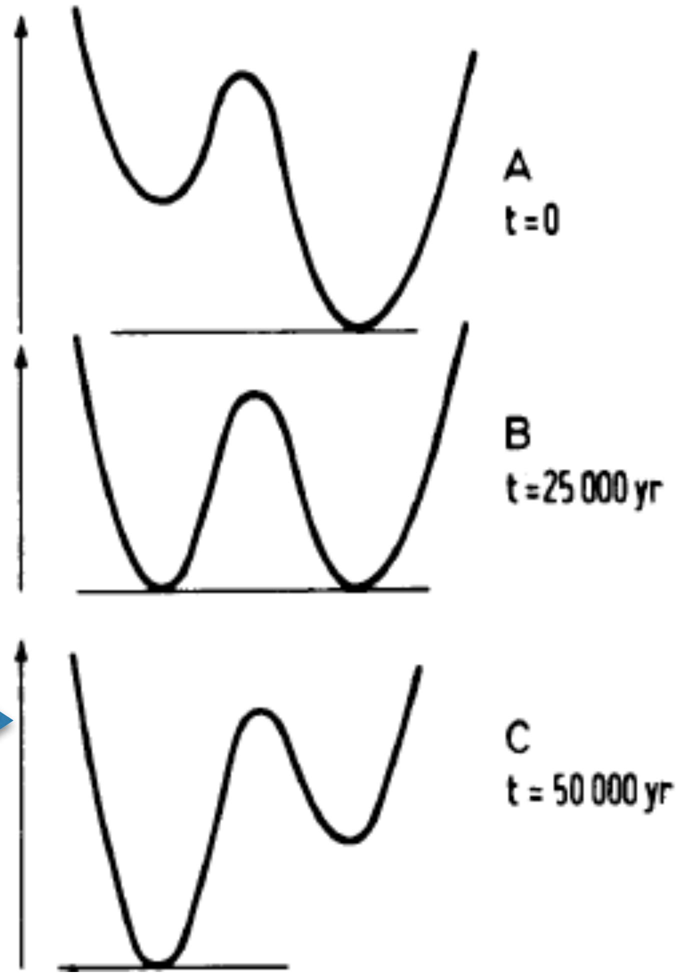


σ=0

A(t)=0

Both terms

V(T)



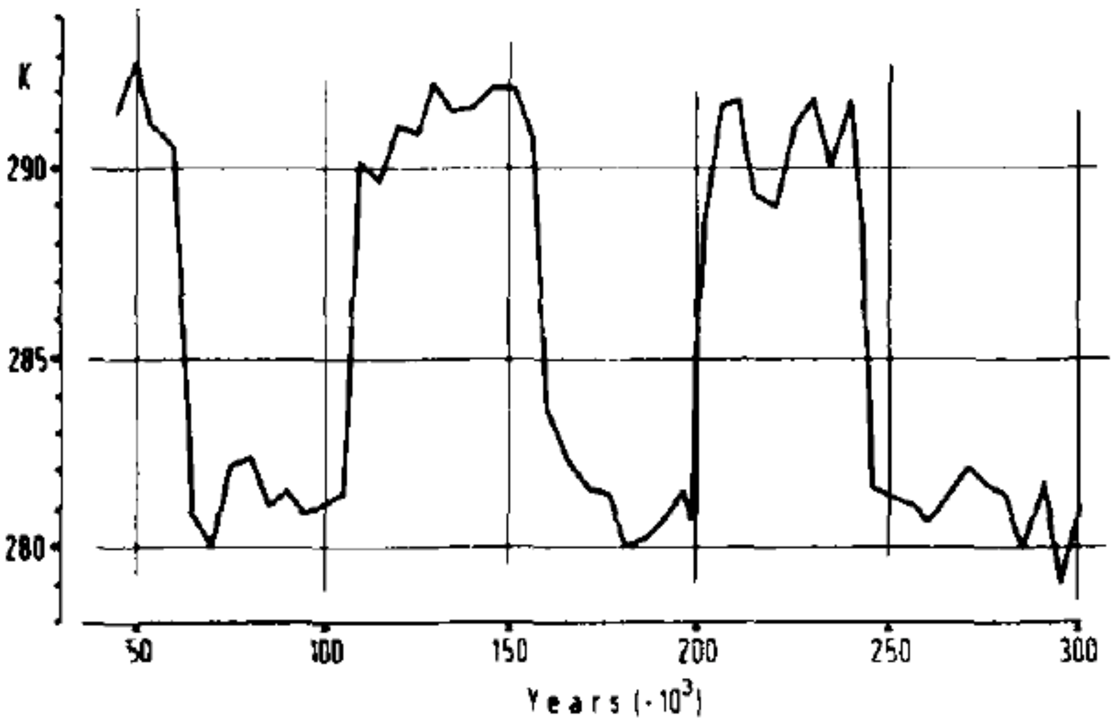
We explained the 100,000 years cycle in the climate change!

Tellus (1982) *34*, 10–16

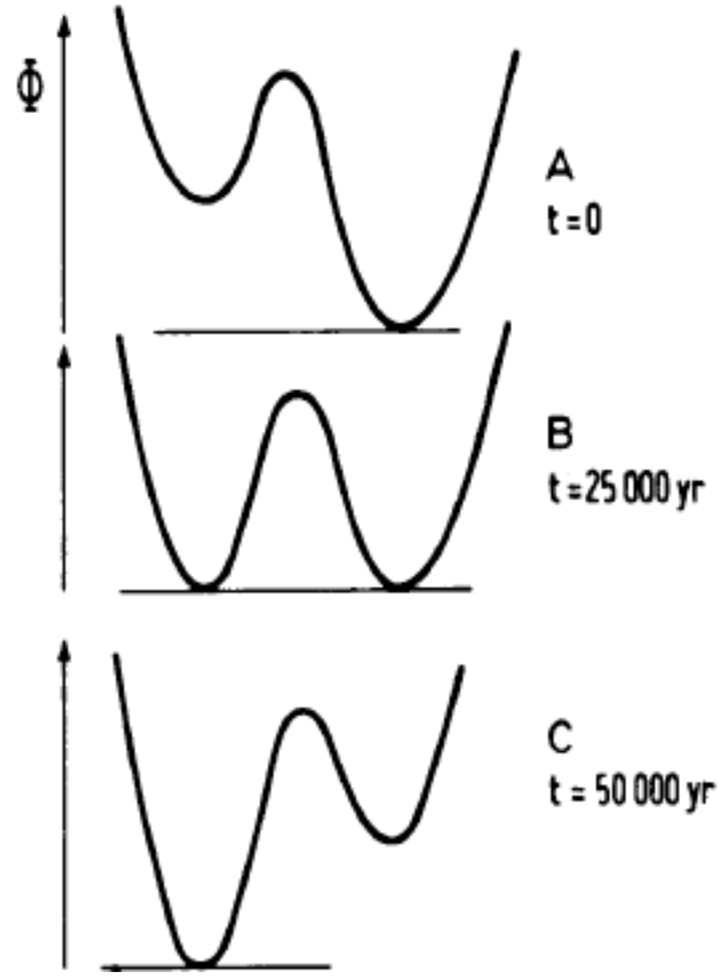
Stochastic resonance in climatic change

By ROBERTO BENZI, *Istituto di Fisica dell'Atmosfera, C.N.R., Piazza Luigi Sturzo 31, 00144, Roma, Italy,*
GIORGIO PARISI, *I.N.F.N., Laboratori Nazionali di Frascati, Frascati, Roma, Italy,*
ALFONSO SUTERA, *The Center for the Environment and Man, Hartford, Connecticut 06120, U.S.A.*
and ANGELO VULPIANI, *Istituto di Fisica "G. Marconi", Università di Roma, Italy*

(Manuscript received November 12, 1980; in final form March 13, 1981)



V(ϕ)



Erice Meeting on Climate: March 1980

RB gave a talk and introduced the term Stochastic Resonance after a discussion with J. Imbrie who asked “is this a resonance?”

Non trivial point: the phase shift of the signal respect to the forcing is small contrary to standard resonance!

Very enthusiastic comments on the presentation.

So we wrote the paper and sent it to **SCIENCE** and…

it was refused

We wrote another version of the paper sent it to **Atmospheric Science** and…

it was refused

Why it is called stochastic resonance ?

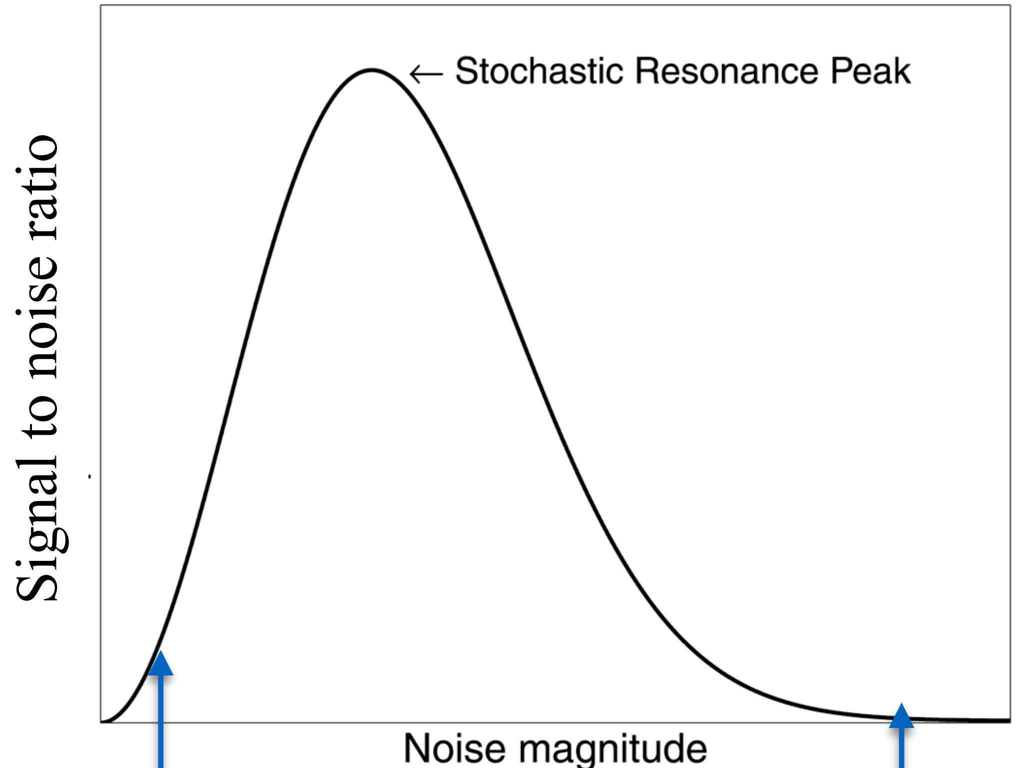
Resonance is a kind of magic word in physics and there it is not surprising that a new resonance mechanism can excite the scientific community.

In our case, the name stochastic resonance was introduce because of a short discussion with John Imbrie in March 1980. Just a few days after our first numerical simulation, I participated in a climate meeting in Erice where I gave a short talk on our results.

John Imbrie, one of the most famous scientist working on paleoclimate, asked whether what we found was somehow similar to a resonance and my answer was: “not exactly! It is a kind of stochastic resonance!”.

This how the name was introduced

At any rate the name is justified if one looks at the signal to noise ratio



For small noise, rare events, not in phase with the external forcing

For large noise, too many events to be in phase with the forcing

Some historical remark

At the time we worked on stochastic resonance, there was no clear hint about the probability distribution of the random transition time. This is the reason why we perform numerical simulations to check our idea.

There was no previous simulation done to study the statistical properties of the random transition times.

There was some controversy on how to perform a correct numerical simulations.

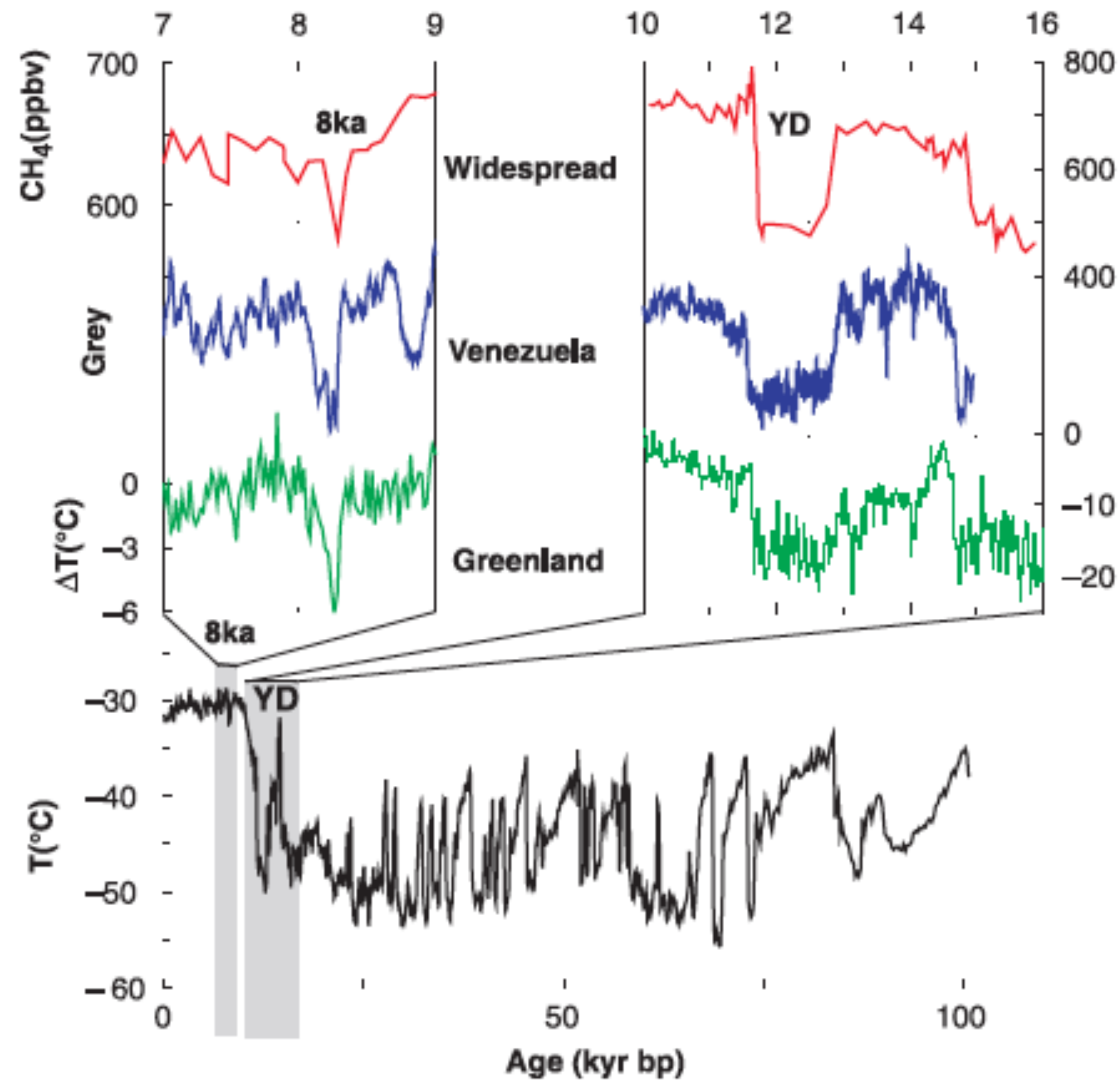
Some mathematicians were questioning our results until an independent experimental verification (with a simplified electronic system by Fauve and Heslot) showed that our original idea was correct.

The theory of chaos was at its infancy and was considered a mathematical problem. We showed that stochastic resonance applies also in chaotic systems

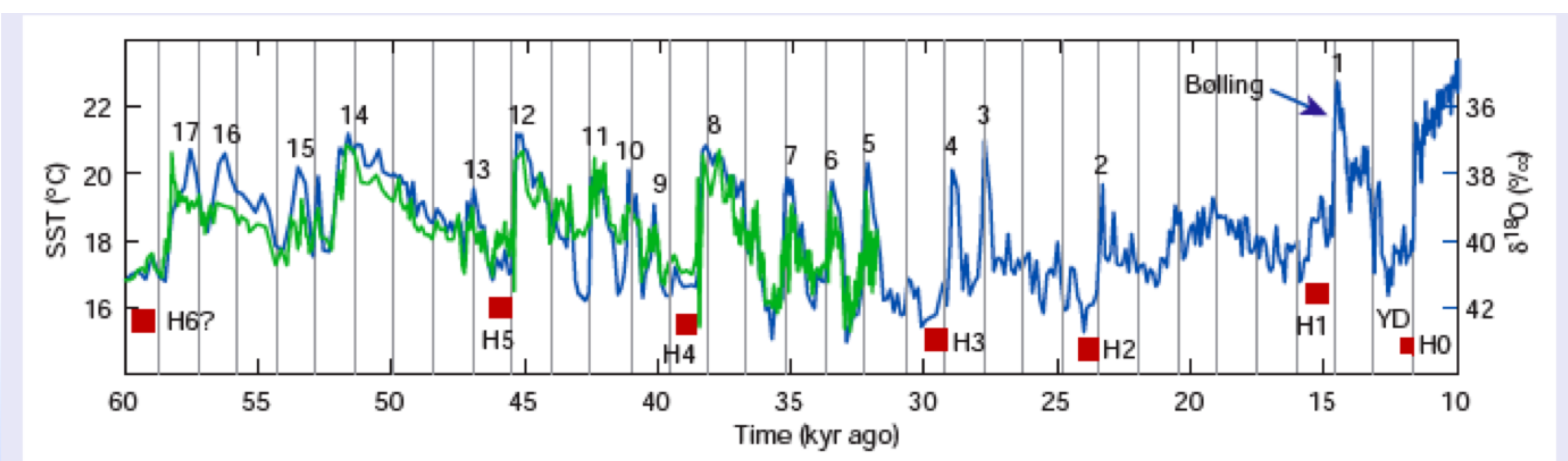
Criticisms concerning sharp in time change of climate state

However it was later discovered that abrupt climate change did occur in the past!

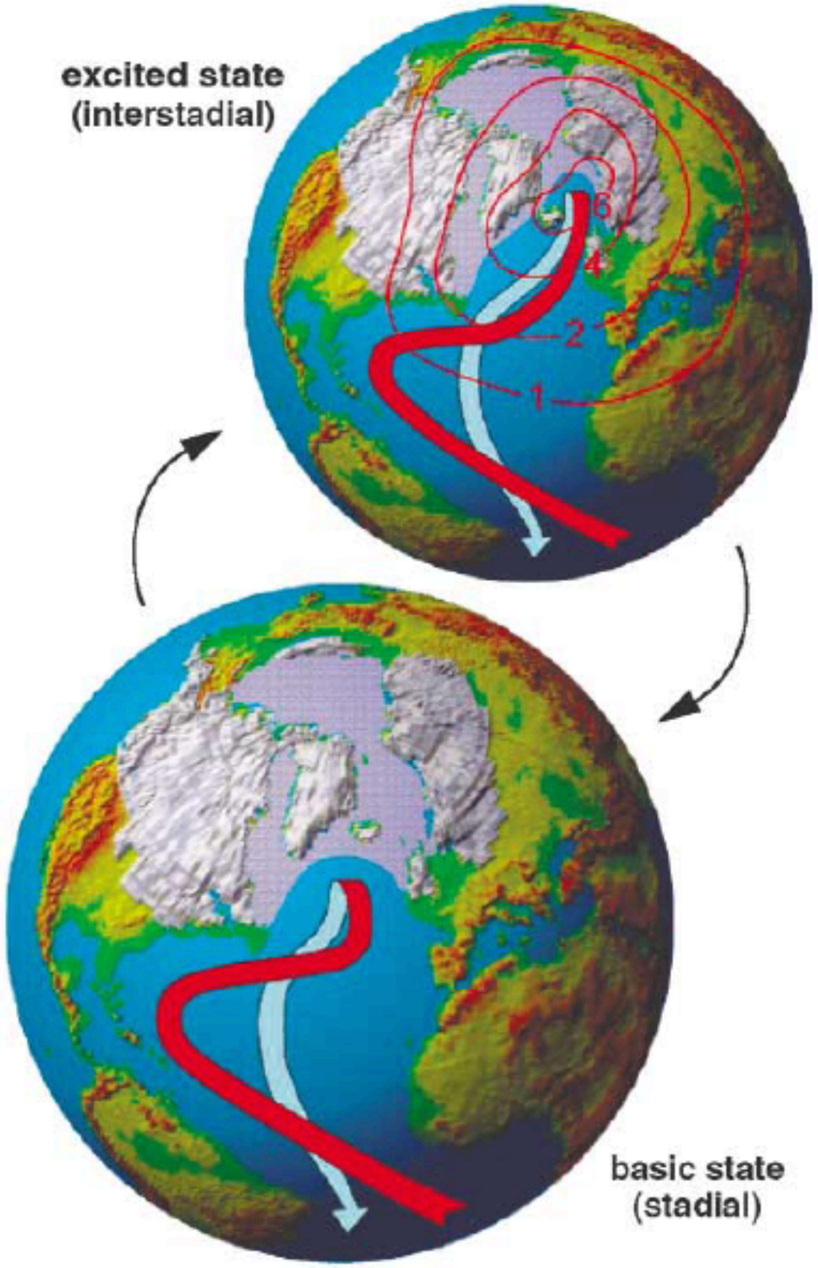
Alley 2000



Abrupt climate change

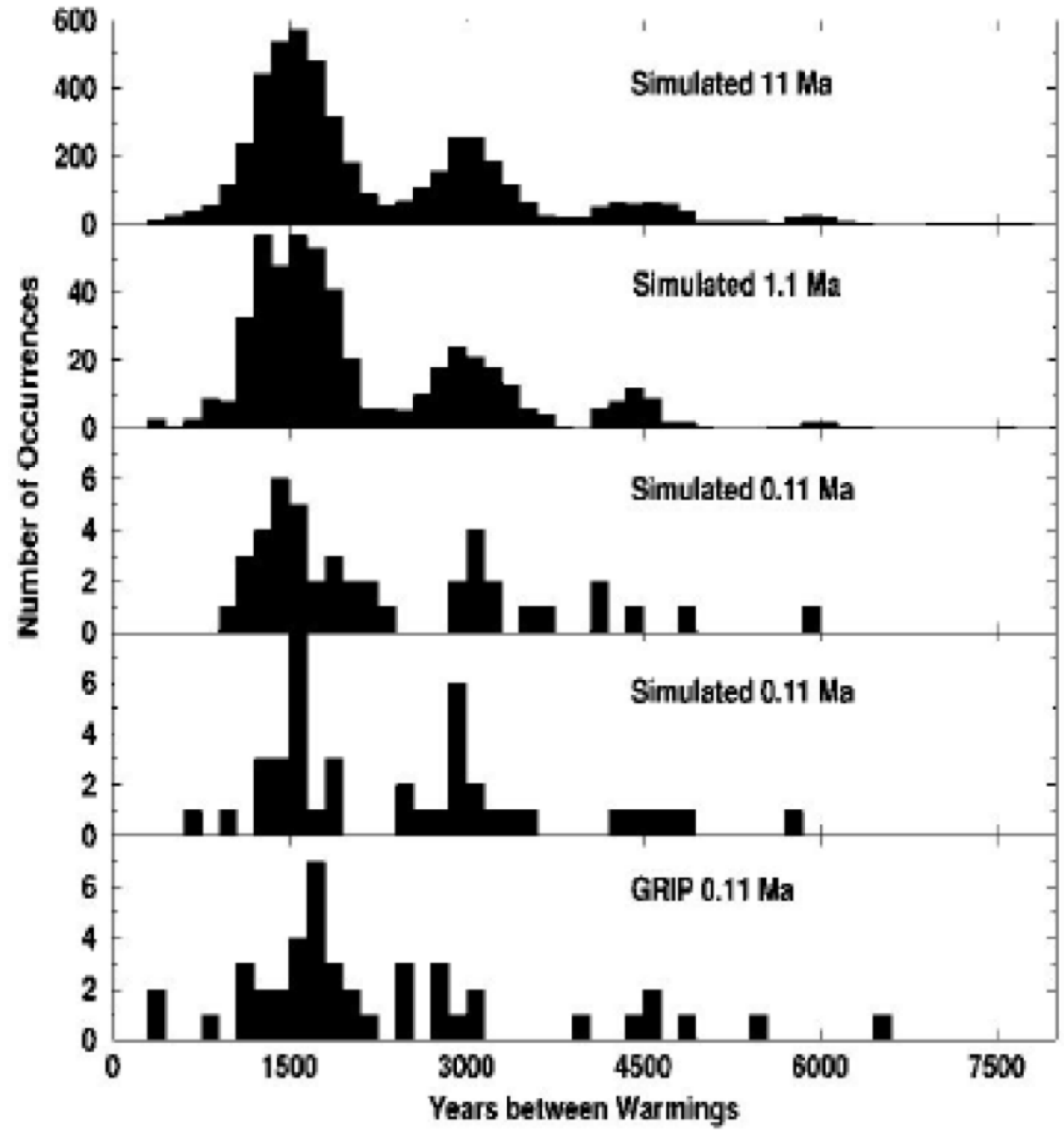


Further developments



Alley et. al. 2000

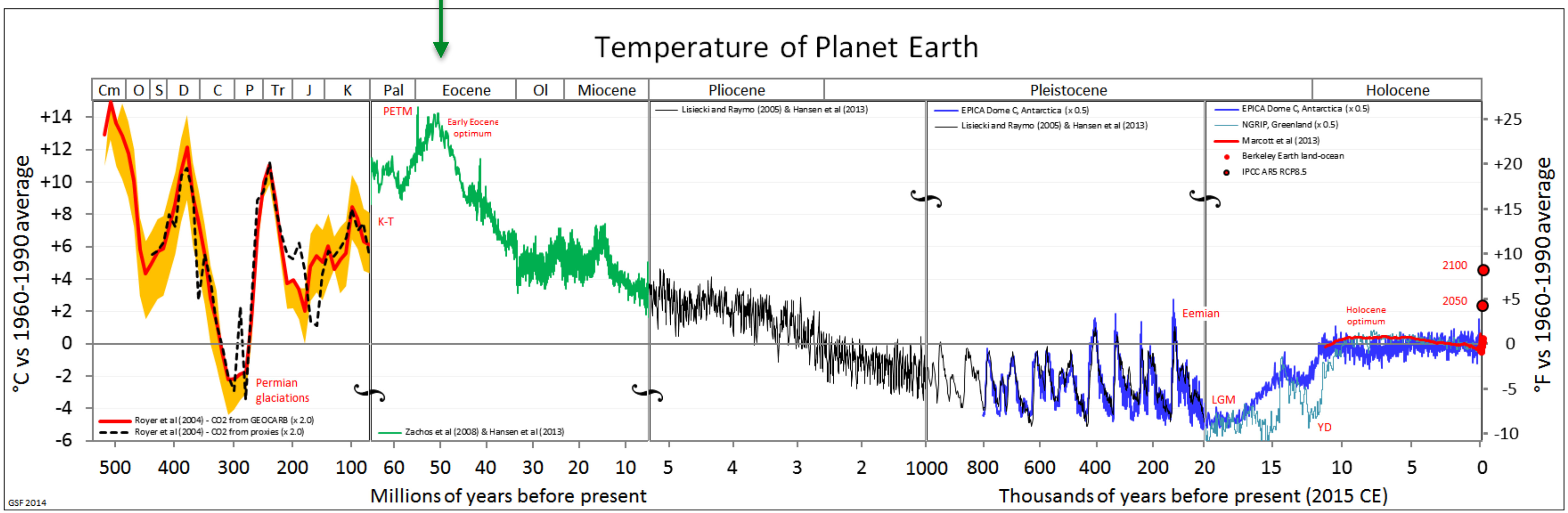
Resident time distribution



Abrupt Glacial Climate Changes due to Stochastic Resonance

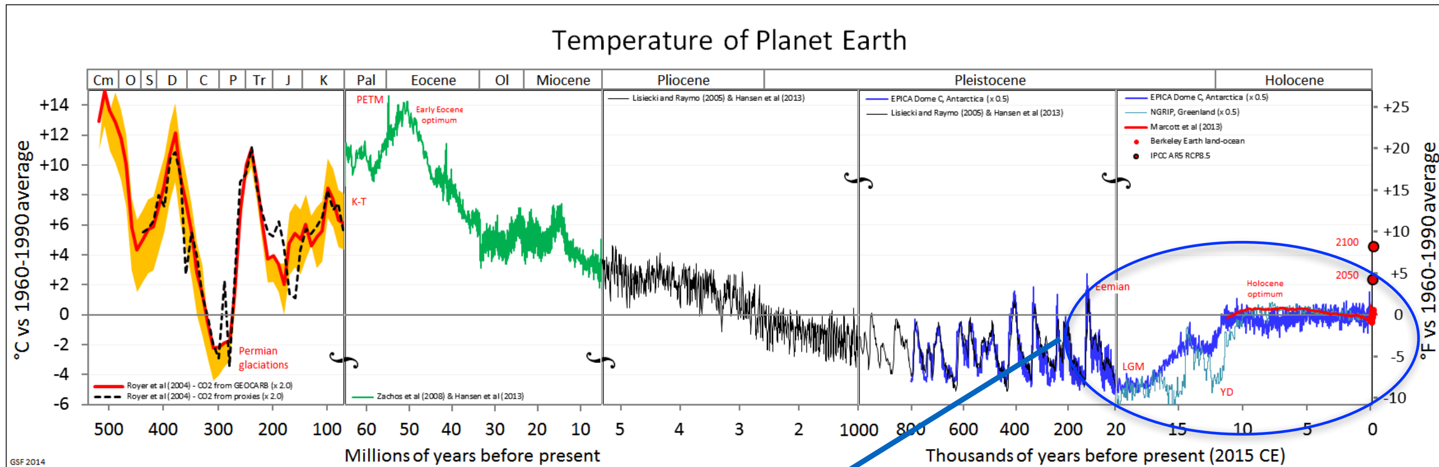
Andrey Ganopolski and Stefan Rahmstorf* 2002

Climate records over the last 64 million years



log(time)

What are climate changes?

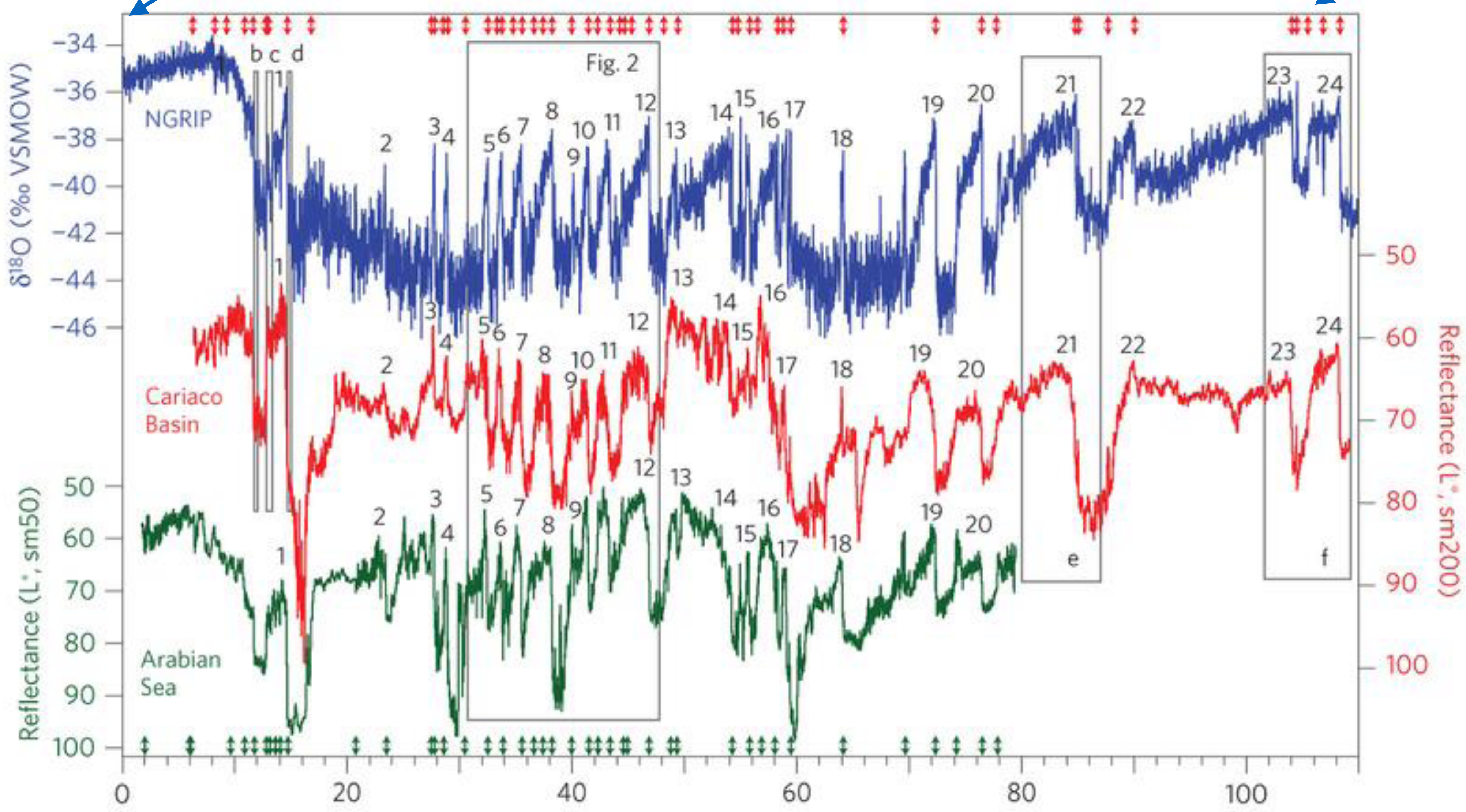


Look only at the **blue line** representing the Greenland temperature

abrupt climate changes in the past

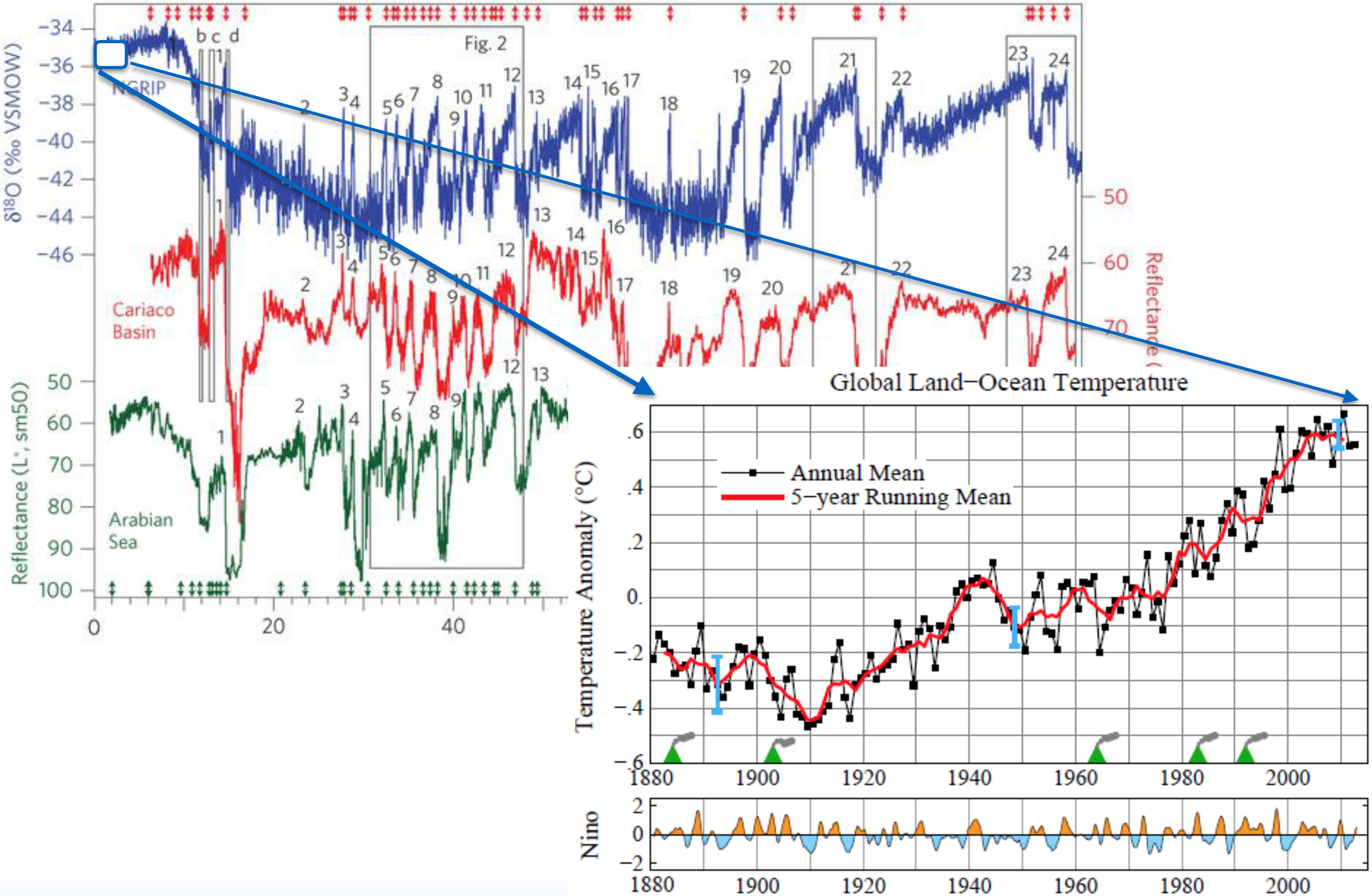
warmer

colder



Alley 2000

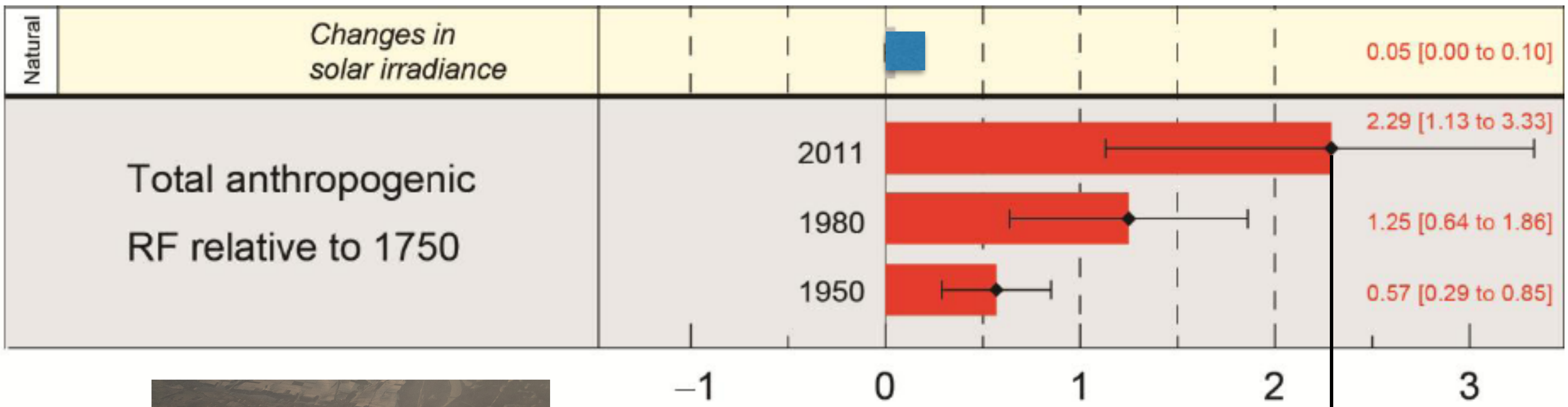
Last 60,000 years

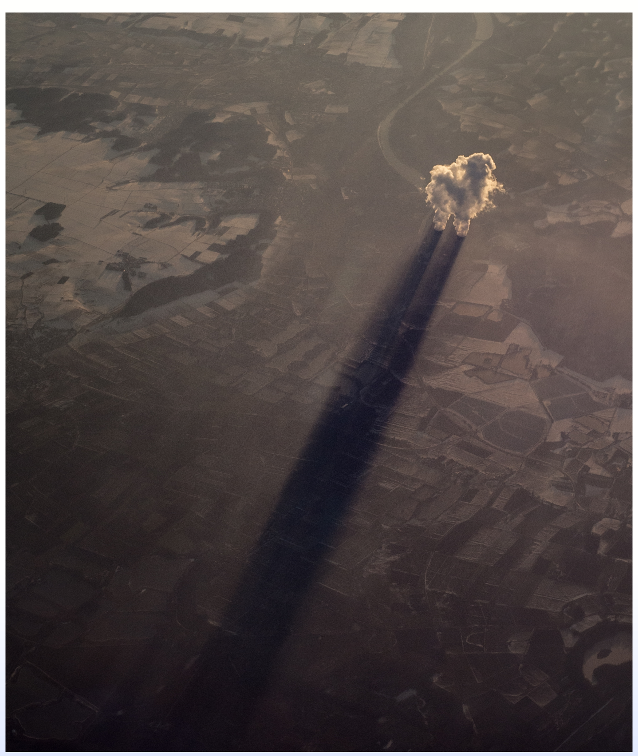


Last 150 years

How we “force” climate.

size of the solar forcing producing glacial interglacial climate changes





Naive estimate of temp. increase $\sim 0.2 \times 9 \sim 2\text{K}$

In agreement with IPCC estimate of temperature increase !

Thank for your attention