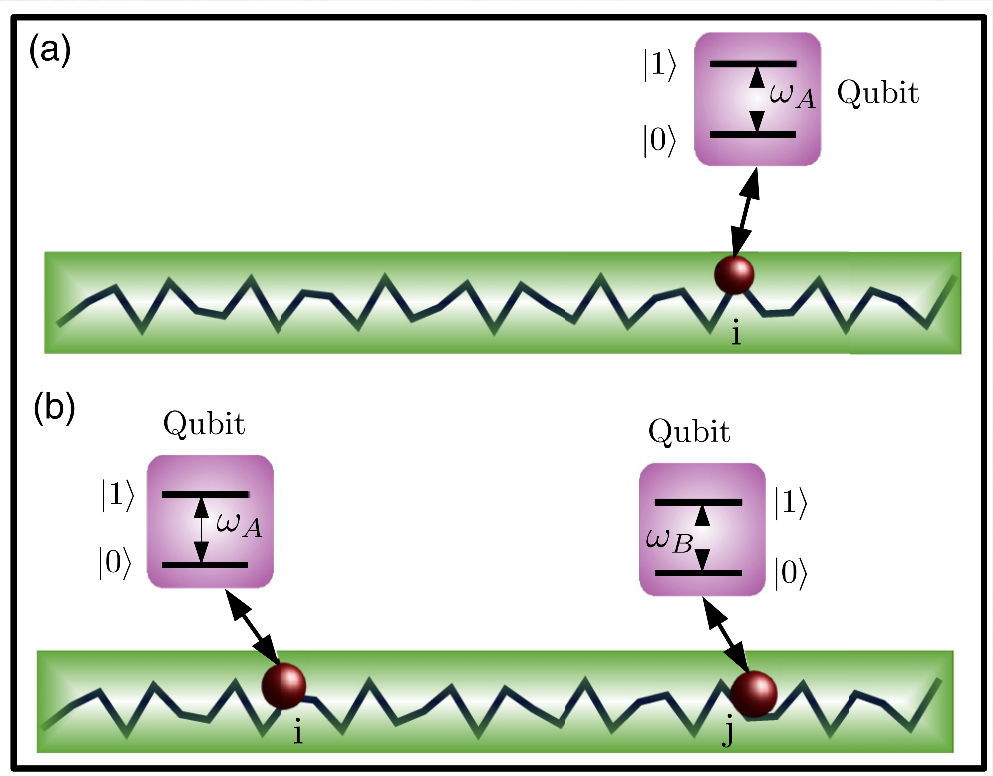

Readout of Quasi-periodic Systems using Qubits

B. Prasanna Venkatesh, IIT Gandhinagar

Recent Developments in Non-Hermitian Physics 2021, ICTS

In collaboration with …

Madhumita Saha
IISER Pune

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Based on: PHYSICAL REVIEW A **103**, 023330 (2021)

Talk Outline

- Introduction & Motivation
- Qubit Readout of GAAH Chain - Set-up and Dynamics
- Results

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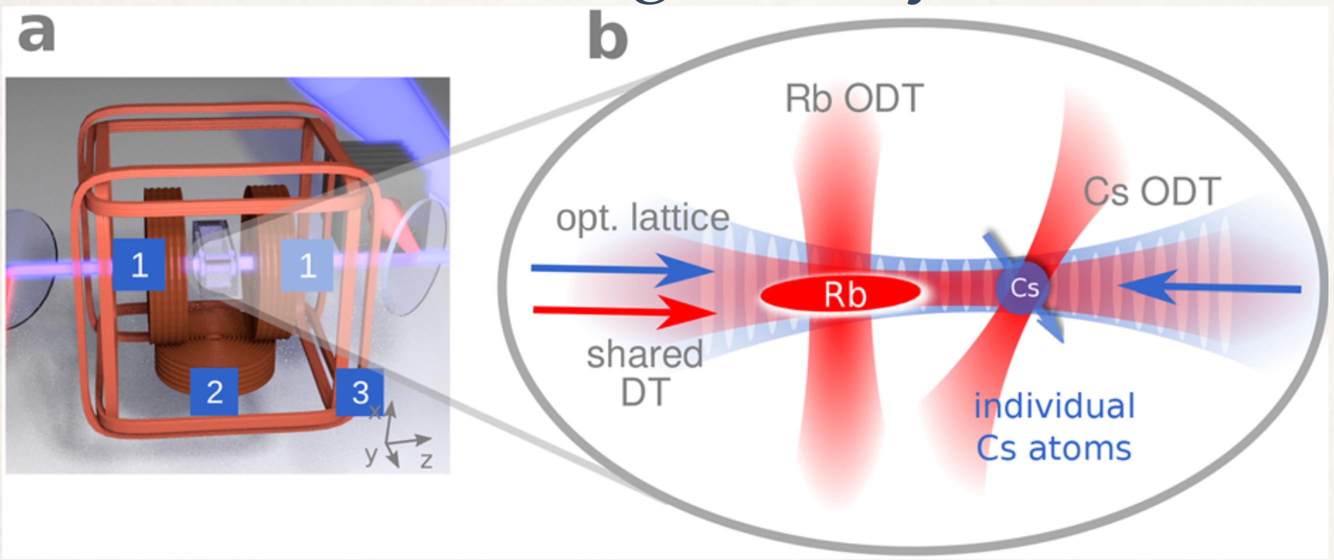
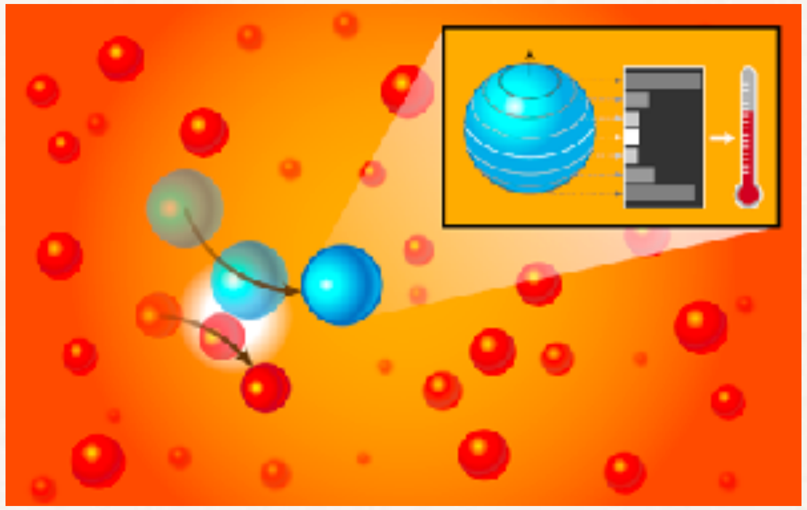
Quantum Systems - In Situ Readout

- Engineered Quantum Systems: Quantum Simulation to Quantum Control

- In-situ quantum measurement: Precise tool of quantum control

e.g: Cavity QED, circuit QED





Artur Widera Group, Mainz, Phys. Status Solidi B 2019, 256, 1800710; Phys. Rev. X 10, 011018 (2020); Stephen Clark, Physics Viewpoint13, 7

- In-situ quantum measurement: Controlled opening of system

Quasi-Periodic Systems

- Lattice with disordered on-site potential (uncorrelated)

Anderson Localization, Mobility Edge in 3-d

P.W. Anderson Phys. Rev. 109, 1492

- Quasi-Periodic Systems

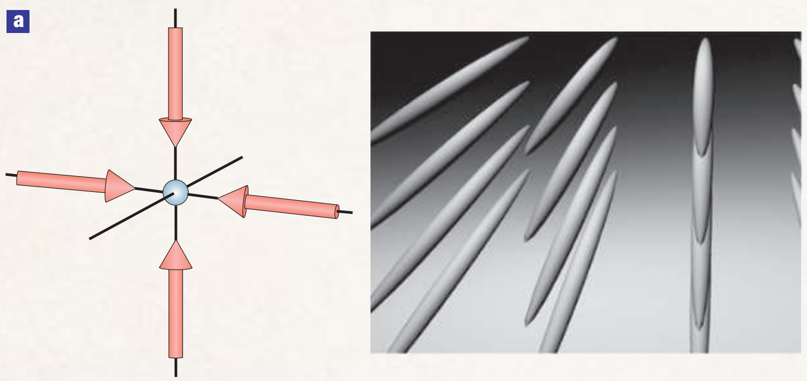
Neither periodic nor disordered systems: e.g. Aubre-André-Harper

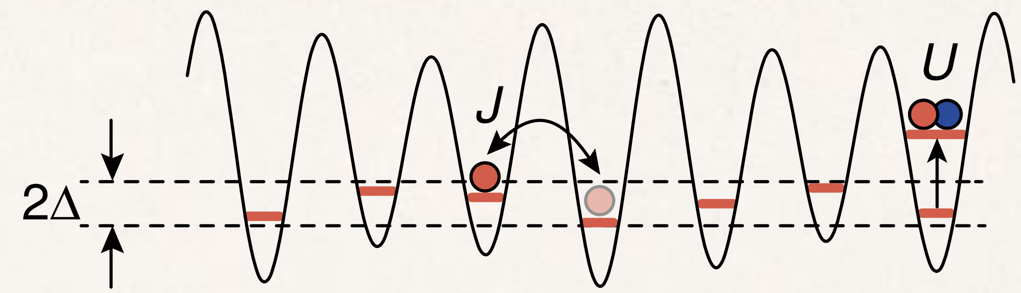
Mobility Edge in 1d

S. Aubry and G. André, Ann. Isr. Phys. Soc. 3, 18 (1980);
S. Ganeshan, K. Sun, and S. Das Sarma, Phys. Rev. Lett. 110, 180403 (2013).
S. Ganeshan, J. H. Pixley, and S. Das Sarma, Phys. Rev. Lett. 114, 146601 (2015).

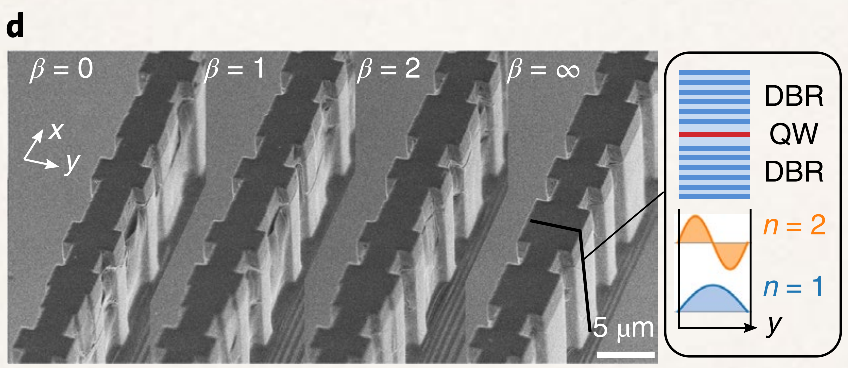
Quasi-Periodic Systems - Resurgence

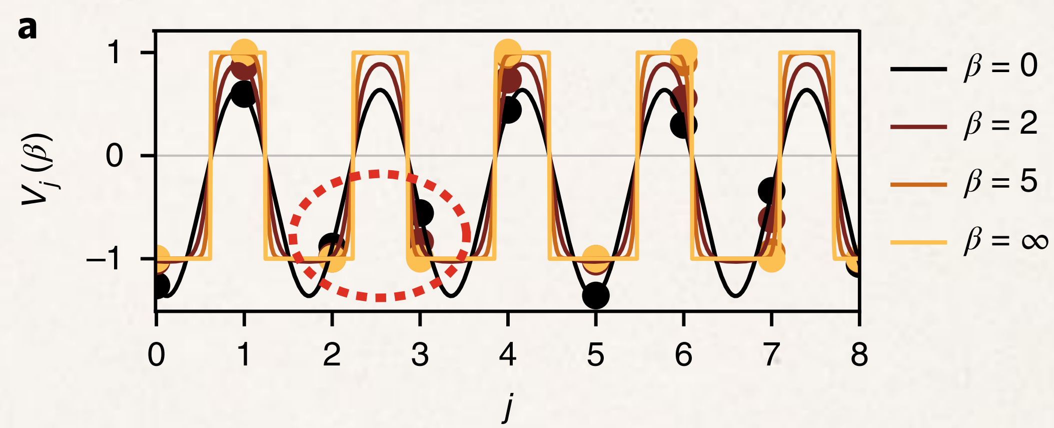
- Multiple Experimental Realizations





I. Bloch, Nature Physics **1**, 23(2005); G. Roati et.al., Nature **453**, 895(2008);
H.P. Lüschen et.al., Phys. Rev. Lett. **120**,160404 (2018)





V. Goblot et.al., Nature Physics volume **16**, 832 (2020)

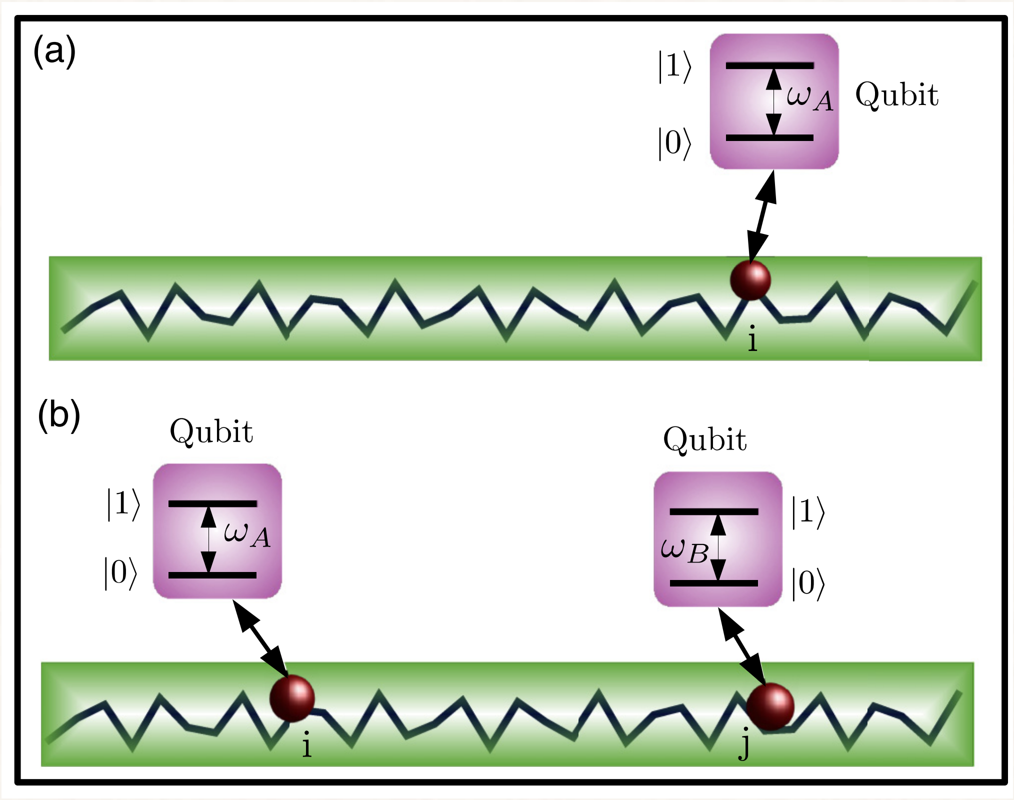
- Connections to Many Body Localization

M. Schreiber et.al, Science **49**, 842 (2015); R. Modak and S. Mukerjee, Phys. Rev. Lett. **115**, 230401 (2015).

- Open version of quasi-periodic systems

H.P. Lüschen et.al., Phys. Rev. X **7**, 011034 (2017); J. Sutradhar et.al., Phys. Rev. B **99**, 224204 (2019);
A. Purkayastha et.al., Phys. Rev. B 97, 174206 (2018); Phys. Rev. B 96, 180204(R) (2017)

This Talk



- Couple Qubit(s) to quasi-periodic GAAH lattice
- Read out nature of SPEs
- Read out transport properties

Qubits as Quantum Probes: Maniscalco, Palma etc.. see Phys. Rev. A **103**, 023330 (2021)

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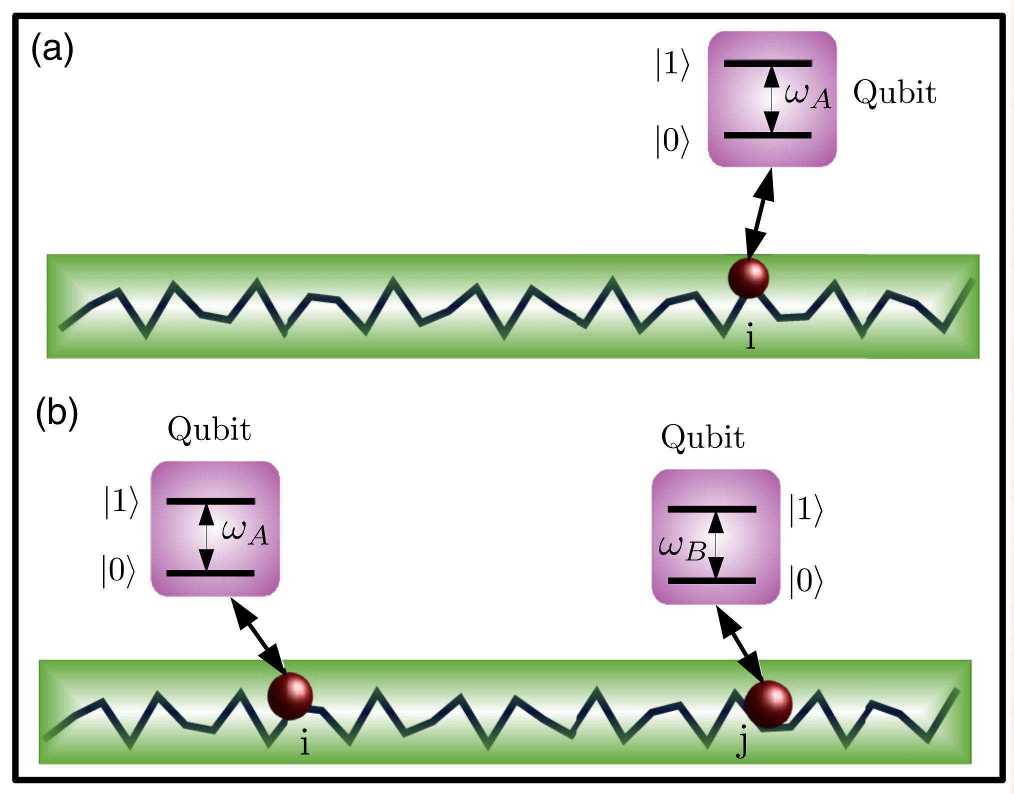
Set-up



(a)
$|1\rangle$
ω_A
Qubit
$|0\rangle$
i
(b)
Qubit
$|1\rangle$
ω_A
$|0\rangle$
Qubit
$|1\rangle$
ω_B
$|0\rangle$
i
j

Properties of AAH & GAAH

$$\hat{H} = \sum_{n=1}^{N} \left(\mu + \frac{2\lambda \cos[2\pi bn + \phi]}{1 + \alpha \cos[2\pi bn + \phi]} \right) \hat{c}_n^\dagger \hat{c}_n + \sum_{n=1}^{N-1} (\hat{c}_n^\dagger c_{n+1} + \hat{c}_{n+1}^\dagger \hat{c}_n).$$

b-irrational

Hopping Strength

$J = 1$

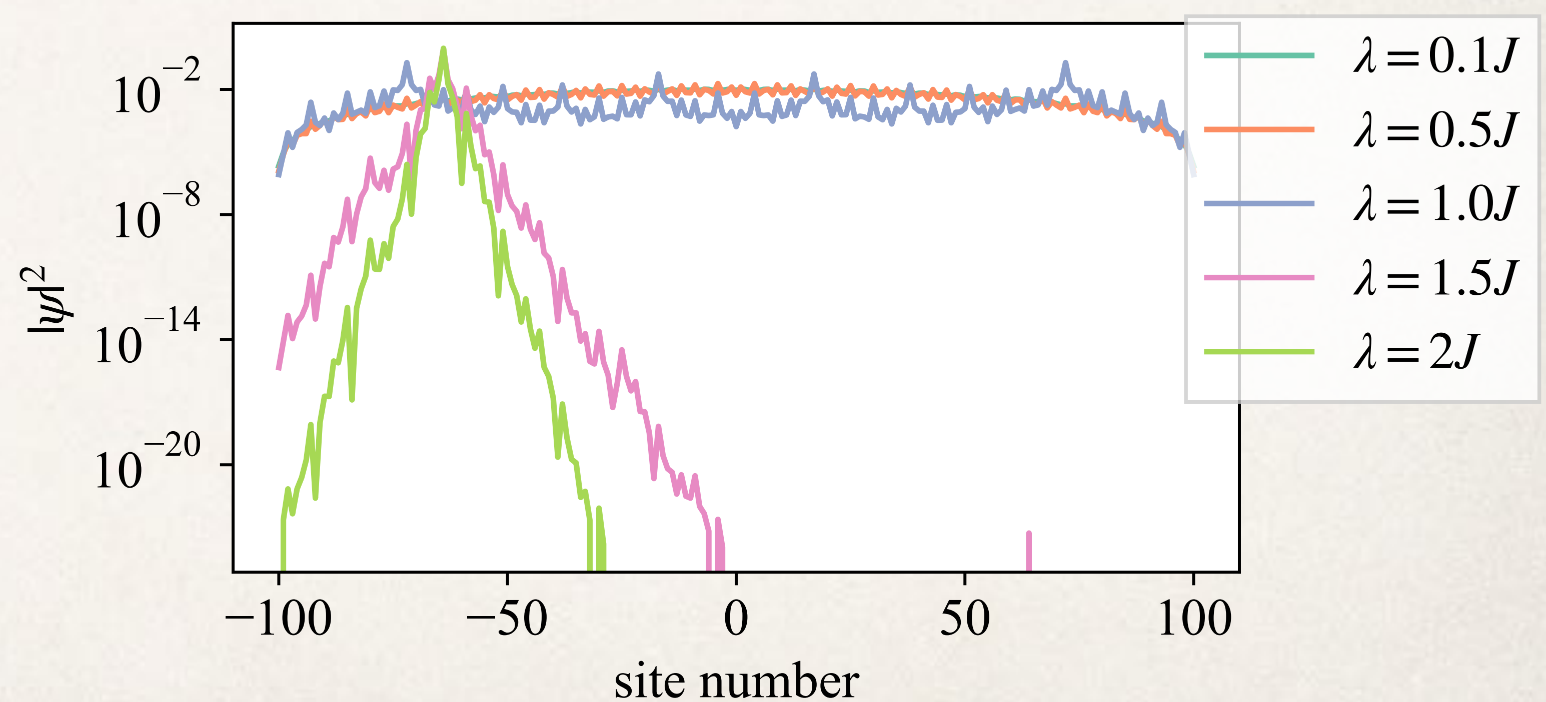
$\hat{H}\psi_k = \omega_k \psi_k$ SPEs

AAH Model $\alpha = 0$

$\lambda < J$ All SPEs Extended

$\lambda = J$ All SPEs Critical

$\lambda > J$ All SPEs Localized



Properties of AAH & GAAH

$$\hat{H} = \sum_{n=1}^{N} \left(\mu + \frac{2\lambda \cos[2\pi bn + \phi]}{1 + \alpha \cos[2\pi bn + \phi]} \right) \hat{c}_n^\dagger \hat{c}_n + \sum_{n=1}^{N-1} (\hat{c}_n^\dagger c_{n+1} + \hat{c}_{n+1}^\dagger \hat{c}_n).$$

Hopping Strength

$J = 1$

AAH Model $\alpha = 0$

IPR(N) = $\frac{\sum_{n=1}^{N} |\psi_{ni}|^4}{\left(\sum_{n=1}^{N} |\psi_{ni}|^2\right)^2}$

$\lambda < J$	All SPEs Extended	N^0
$\lambda = J$	All SPEs Critical	N^{-b}, with $0 < b < 1$
$\lambda > J$	All SPEs Localized	N^{-1}

Properties of AAH & GAAH

$$\hat{H} = \sum_{n=1}^{N} \left(\mu + \frac{2\lambda \cos[2\pi bn + \phi]}{1 + \alpha \cos[2\pi bn + \phi]} \right) \hat{c}_n^\dagger \hat{c}_n + \sum_{n=1}^{N-1} (\hat{c}_n^\dagger c_{n+1} + \hat{c}_{n+1}^\dagger \hat{c}_n).$$

Hopping Strength

$J = 1$

GAAH Model $\alpha > 0,\ \lambda > 0$

$\hat{H}\psi_k = \omega_k \psi_k$ SPEs

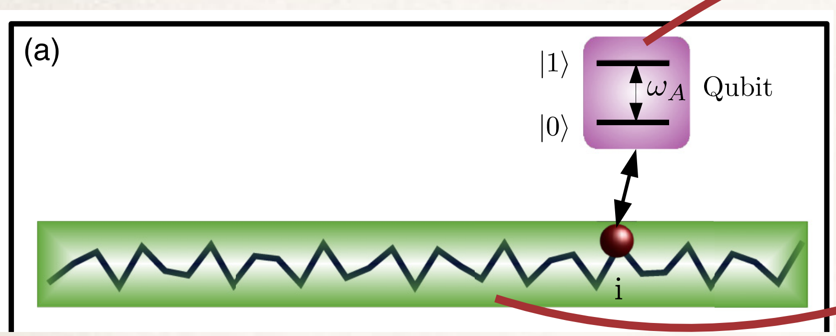
Mobility Edge

$$E = \mu + 2(J - \lambda)/\alpha$$

$\omega_k < E$ Extended

$\omega_k > E$ Localized

Single Qubit Coupled to GAAH Chain



$$\hat{H}_{1q} = \hat{H} + \frac{\omega_A}{2}\hat{\sigma}_z^i + g\hat{\sigma}_z^i(\hat{c}_i^\dagger + \hat{c}_i)$$

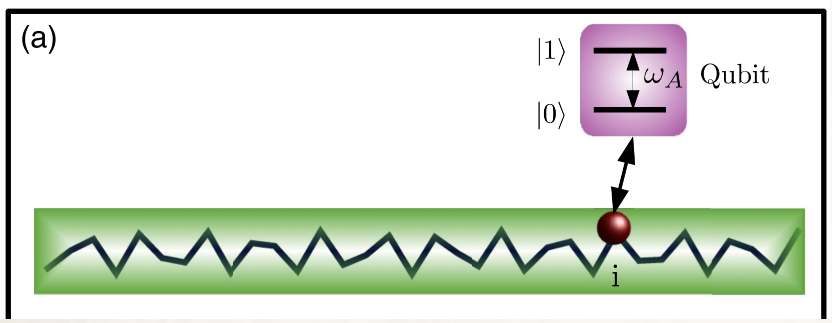
$$\hat{H} = \sum_{k=1}^{N} \omega_k \hat{\eta}_k^\dagger \hat{\eta}_k \qquad \hat{c}_i = \sum_{k=1}^{N} S_{i,k}\hat{\eta}_k \qquad g_k^i = gS_{i,k}$$

$$\hat{H}_{1q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \sum_{k=1}^{N}\omega_k \hat{\eta}_k^\dagger \hat{\eta}_k + \sum_{k=1}^{N}\hat{\sigma}_z^i\left(g_k^i \hat{\eta}_k^\dagger + g_k^{i*}\hat{\eta}_k\right)$$

Spin-Boson Dephasing Coupling - can be exactly solved

Dephasing Spin-Boson Solution

$$\hat{c}_i = \sum_{k=1}^{N} S_{i,k}\hat{\eta}_k$$



$$g_k^i = gS_{i,k}$$

$$\hat{H}_{1q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \sum_{k=1}^{N}\omega_k\hat{\eta}_k^\dagger\hat{\eta}_k + \sum_{k=1}^{N}\hat{\sigma}_z^i\left(g_k^i\hat{\eta}_k^\dagger + g_k^{i*}\hat{\eta}_k\right)$$

$$\hat{\rho}(0) = \hat{\rho}_s(0) \otimes (e^{-\beta\hat{H}}/Z_\beta) \quad Z_\beta = \mathrm{Tr}_B[e^{-\beta\hat{H}}]$$

Solution:

$$\hat{\sigma}_-^i(t) = e^{-i\omega_A t}\hat{\sigma}_-^i(0) \otimes \prod_{k=1}^{N}\hat{D}_k\left(\alpha_k^i\right)$$

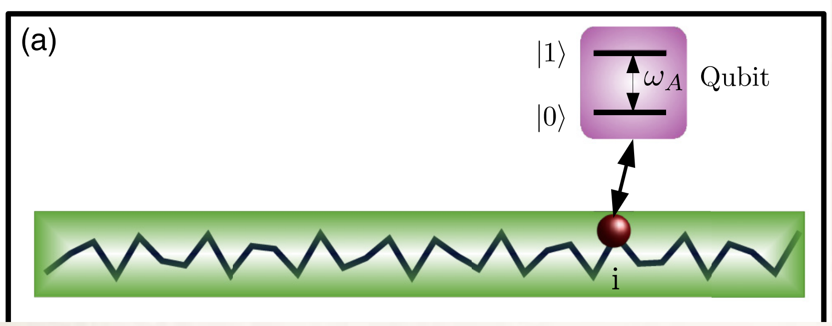
$$\hat{\sigma}_z(t) = \hat{\sigma}_z(0)$$

$$\alpha_k^i = \frac{2g_k^i(1-e^{i\omega_k t})}{\omega_k}$$

$$\hat{D}_k(\alpha) = \exp[\alpha\hat{\eta}_k^\dagger(0) - \alpha^*\hat{\eta}_k(0)]$$

Dephasing Spin-Boson Solution

$$\hat{c}_i = \sum_{k=1}^{N} S_{i,k}\hat{\eta}_k$$



$$g_k^i = gS_{i,k}$$

$$\hat{H}_{1q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \sum_{k=1}^{N}\omega_k\hat{\eta}_k^\dagger\hat{\eta}_k + \sum_{k=1}^{N}\hat{\sigma}_z^i\left(g_k^i\hat{\eta}_k^\dagger + g_k^{i*}\hat{\eta}_k\right)$$

$$\langle\hat{\sigma}_-^i(t)\rangle = \langle\hat{\sigma}_-^i(0)\rangle e^{-i\omega_A t - \Gamma_i(t)},$$

$$\Gamma_i(t) = 4\sum_{k=1}^{N}\left|g_k^i\right|^2 \coth\left(\frac{\beta\omega_k}{2}\right)\frac{1-\cos(\omega_k t)}{\omega_k^2}.$$

$$\Gamma_i(t) = \Gamma_{i,\mathrm{vac}}(t) + \Gamma_{i,\mathrm{th}}(t)$$

$$\Gamma_{i,\mathrm{vac}}(t) = \sum_{k=1}^{N}\frac{|\alpha_k^i|^2}{2}$$

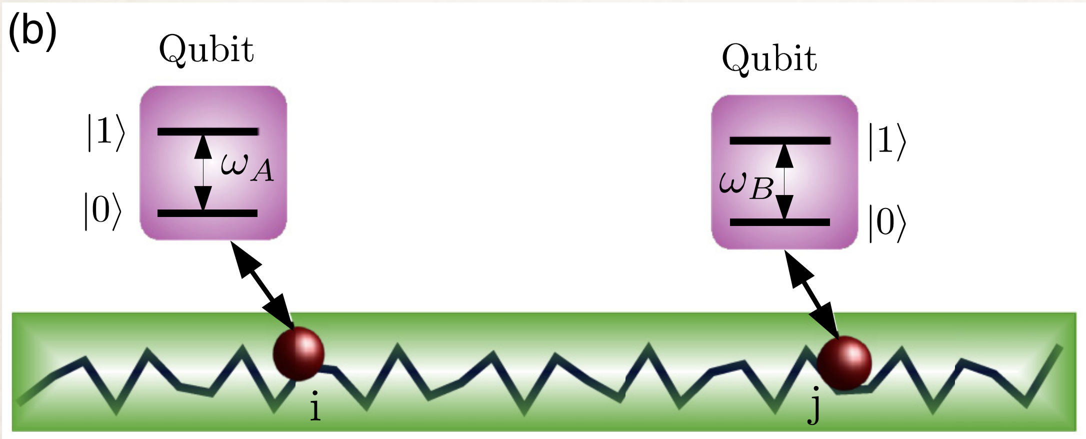
$$\Gamma_{i,\mathrm{th}}(t) = \sum_{k=1}^{N}\bar{n}(\beta,\omega_k)|\alpha_k^i|^2$$

Two Qubits Coupled to GAAH

Motivation: non-local probe for transport

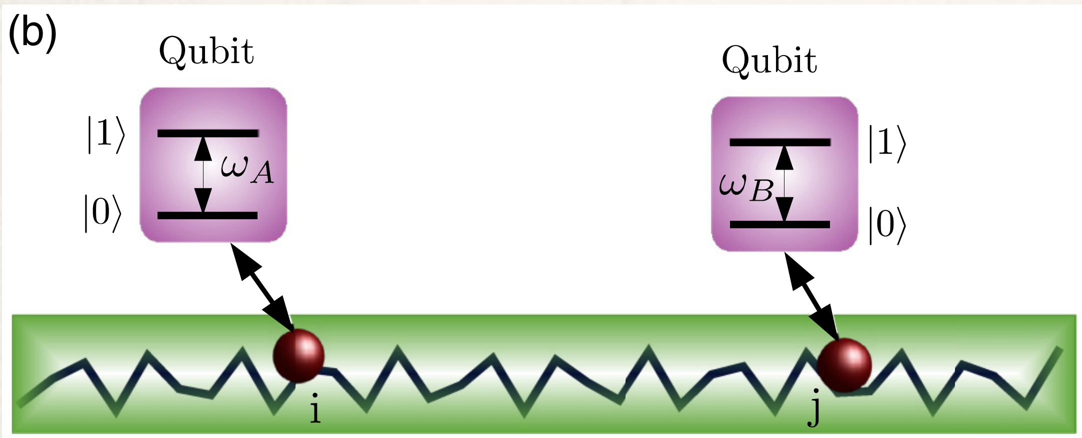
localized initial state width evolution

$$w(t) \sim t^{\eta} \begin{cases} \eta = 1 & \text{Ballistic, Extended} \\ \eta = 0.5 & \text{Diffusive, Critical} \\ \eta = 0 & \text{Localized} \end{cases}$$



$$\hat{H}^{SB}_{2q} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \frac{\omega_B}{2}\hat{\sigma}_z^j + \sum_{k=1}^{N} \omega_k \hat{\eta}_k^\dagger \hat{\eta}_k + \sum_{k=1}^{N} \hat{\sigma}_z^i \left(g_k^i \hat{\eta}_k^\dagger + g_k^{i*} \hat{\eta}_k\right) + \sum_{k=1}^{N} \hat{\sigma}_z^j \left(g_k^j \hat{\eta}_k^\dagger + g_k^{j*} \hat{\eta}_k\right)$$

Two Qubits Dynamics Solution



$$\hat{H}_{2q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \frac{\omega_B}{2}\hat{\sigma}_z^j + \sum_{k=1}^{N} \omega_k \hat{\eta}_k^\dagger \hat{\eta}_k + \sum_{k=1}^{N} \hat{\sigma}_z^i \left(g_k^i \hat{\eta}_k^\dagger + g_k^{i*} \hat{\eta}_k\right) + \sum_{k=1}^{N} \hat{\sigma}_z^j \left(g_k^j \hat{\eta}_k^\dagger + g_k^{j*} \hat{\eta}_k\right)$$

$$\mathrm{Cov}(\hat{\sigma}_-^i \hat{\sigma}_+^j) = e^{-i(\omega_A - \omega_B)t}\Big(\langle \hat{\sigma}_-^i(0)\hat{\sigma}_+^j(0)\rangle e^{-\Gamma_{ij}(t)} - \langle \hat{\sigma}_-^i(0)\rangle \langle \hat{\sigma}_+^j(0)\rangle \Big\langle e^{-i\Delta\Omega_-(t)\hat{\sigma}_z^j + i\Delta\Omega_+(t)\hat{\sigma}_z^i}\Big\rangle e^{-[\Gamma_i(t)+\Gamma_j(t)]}\Big)$$

$$\Delta\Omega_\pm(t) = \sum_{k=1}^{N} \frac{4}{\omega_k^2}\Big([\sin(\omega_k t) - \omega_k t]\,\mathrm{Re}\left[g_k^i g_k^{j*}\right] \pm [1 - \cos(\omega_k t)]\,\mathrm{Im}\left[g_k^i g_k^{j*}\right]\Big).$$

Gen. Lamb Shift

$$\Gamma_{ij}(t) = 4\sum_{k=1}^{N} \left|g_k^i - g_k^j\right|^2 \coth\left(\frac{\beta\omega_k}{2}\right)\frac{1-\cos(\omega_k t)}{\omega_k^2}$$

Correlated Dephasing

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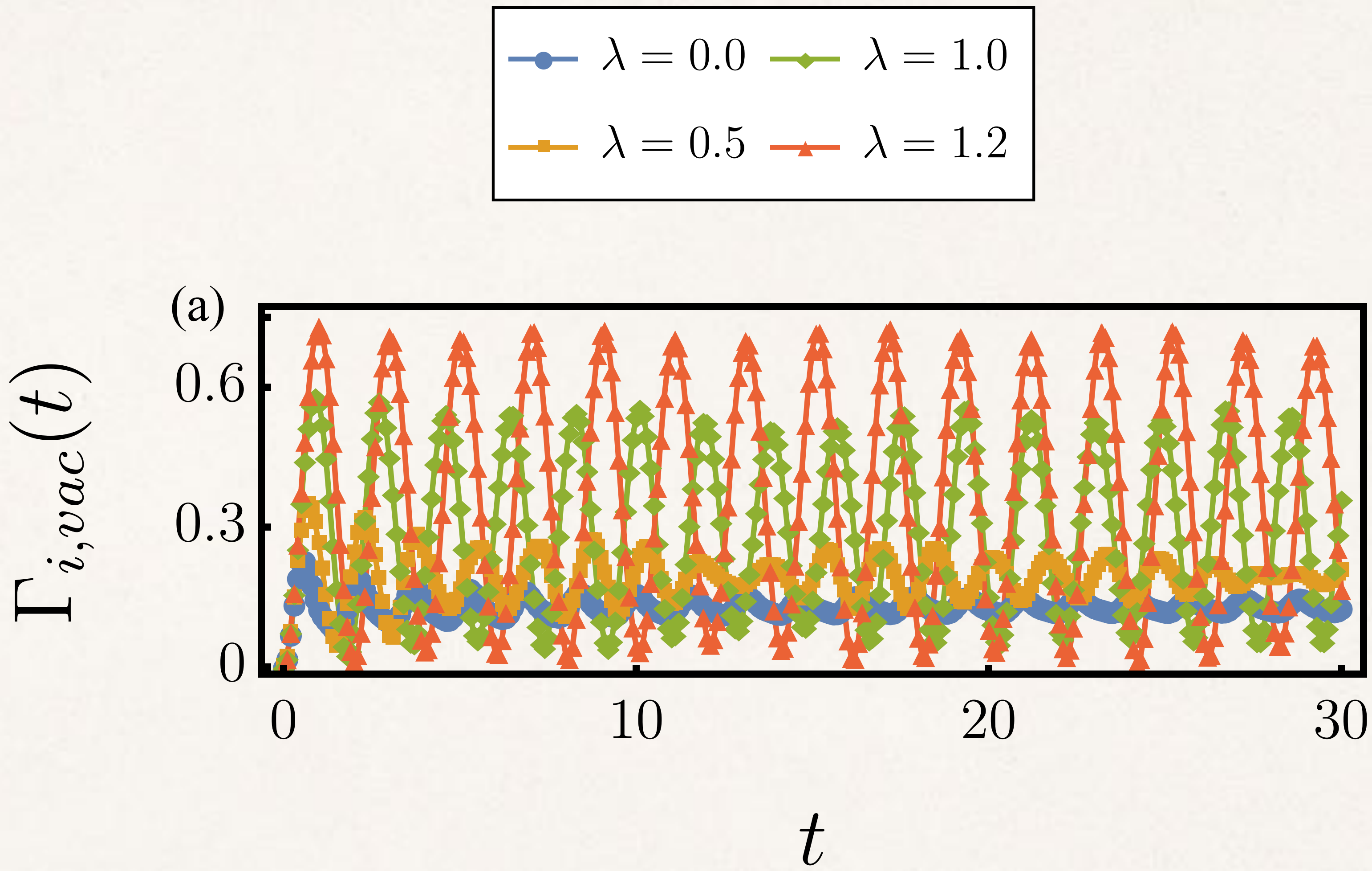
Single Qubit Dephasing Dynamics

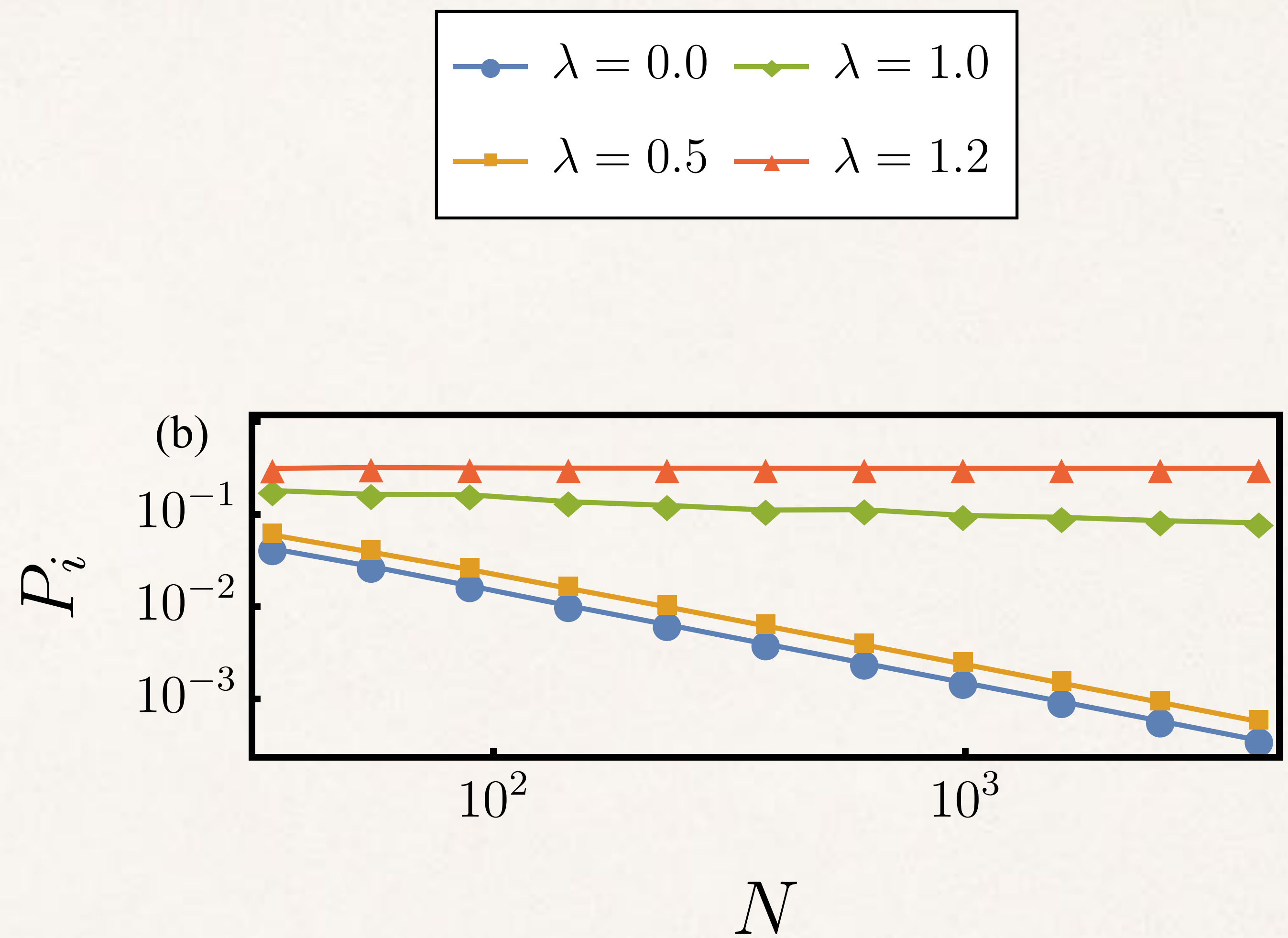
AAH Model
N = 610



$$\Gamma_i(t) = 4\sum_{k=1}^{N} \left|g_k^i\right|^2 \coth\left(\frac{\beta\omega_k}{2}\right)\frac{1-\cos(\omega_k t)}{\omega_k^2}$$

$$\hat{c}_i = \sum_{k=1}^{N} S_{i,k}\hat{\eta}_k \qquad g_k^i = gS_{i,k}$$

Single Qubit Dephasing Dynamics



AAH Model

N = 610

$$P_i \equiv \sum_{k=1}^{N} |g_k^i|^4$$

$$\Gamma_i(t) = 4 \sum_{k=1}^{N} \left|g_k^i\right|^2 \coth\left(\frac{\beta\omega_k}{2}\right) \frac{1-\cos(\omega_k t)}{\omega_k^2}$$

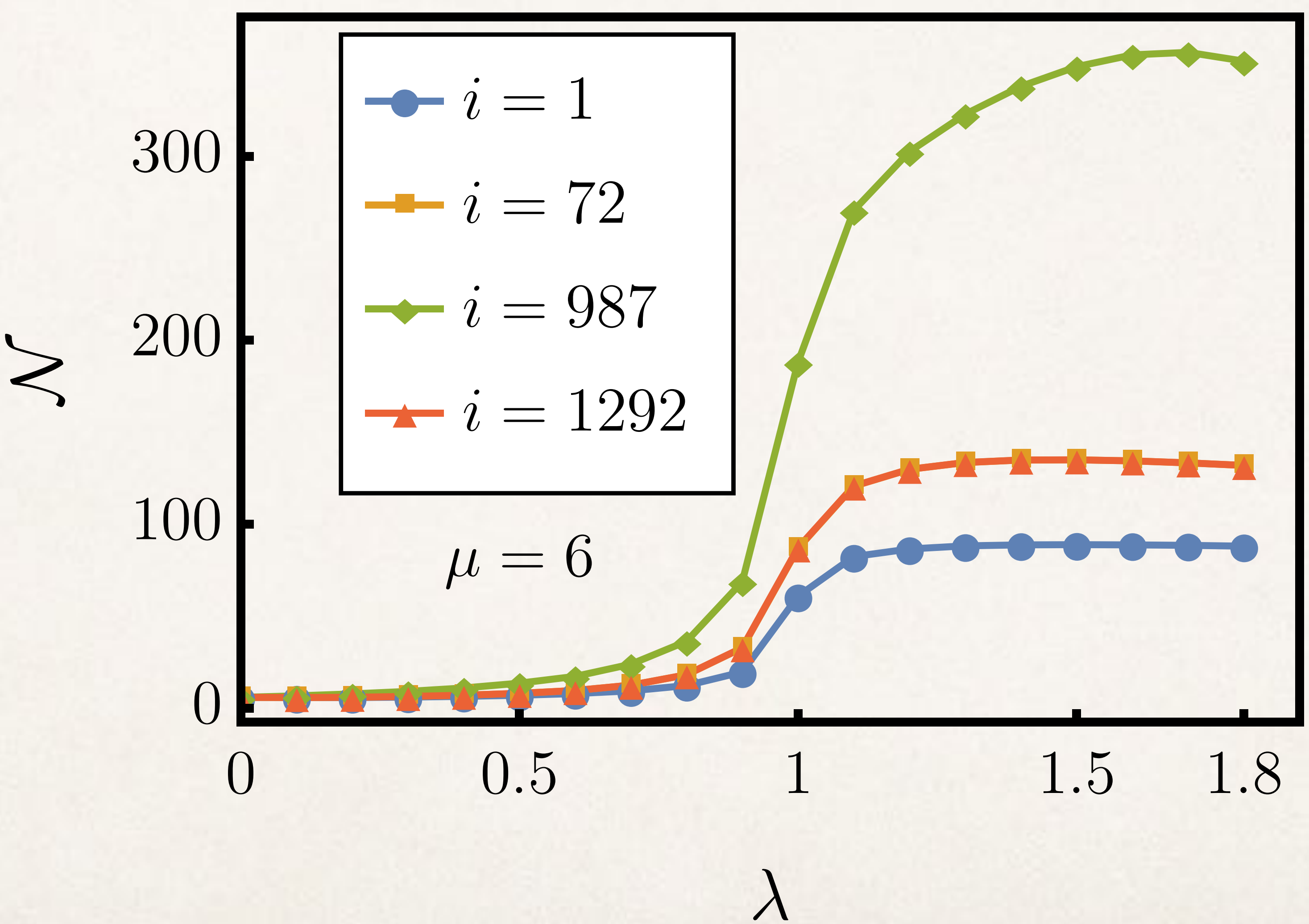
$$\hat{c}_i = \sum_{k=1}^{N} S_{i,k} \hat{\eta}_k \qquad g_k^i = g S_{i,k}$$

Non-Markovianity as SPE indicator

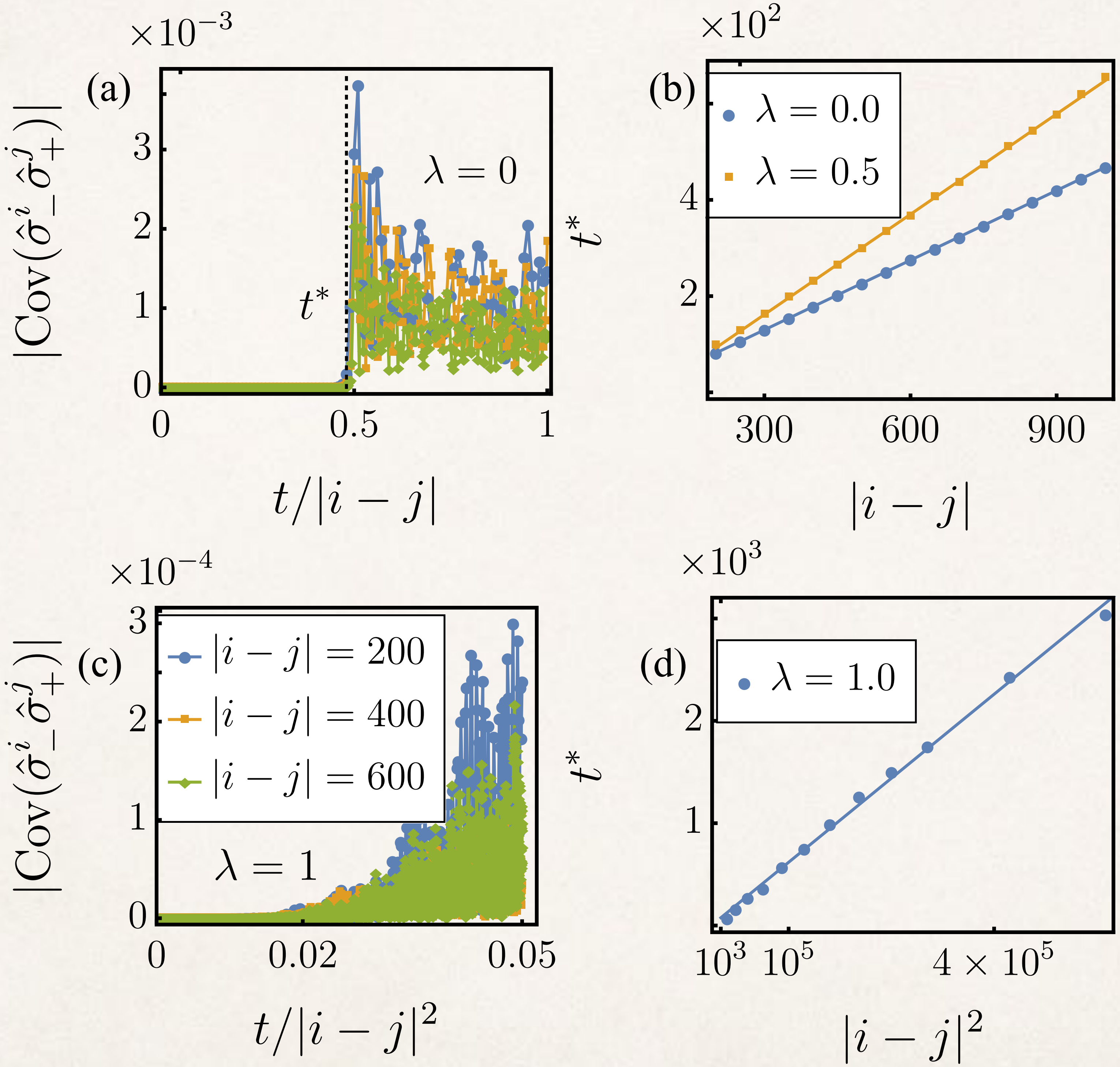
$$\mathcal{N} = \sum_{p=1}^{N_{\max}} \left(e^{-\Gamma(t_p^f)} - e^{-\Gamma(t_p^i)}\right)$$

sum is over all intervals $[t_p^i, t_p^f]$ $\gamma(t) < 0$ $\gamma(t) = \dot{\Gamma}(t)/2$

AAH Model



Transport Readout: Two qubit correlations



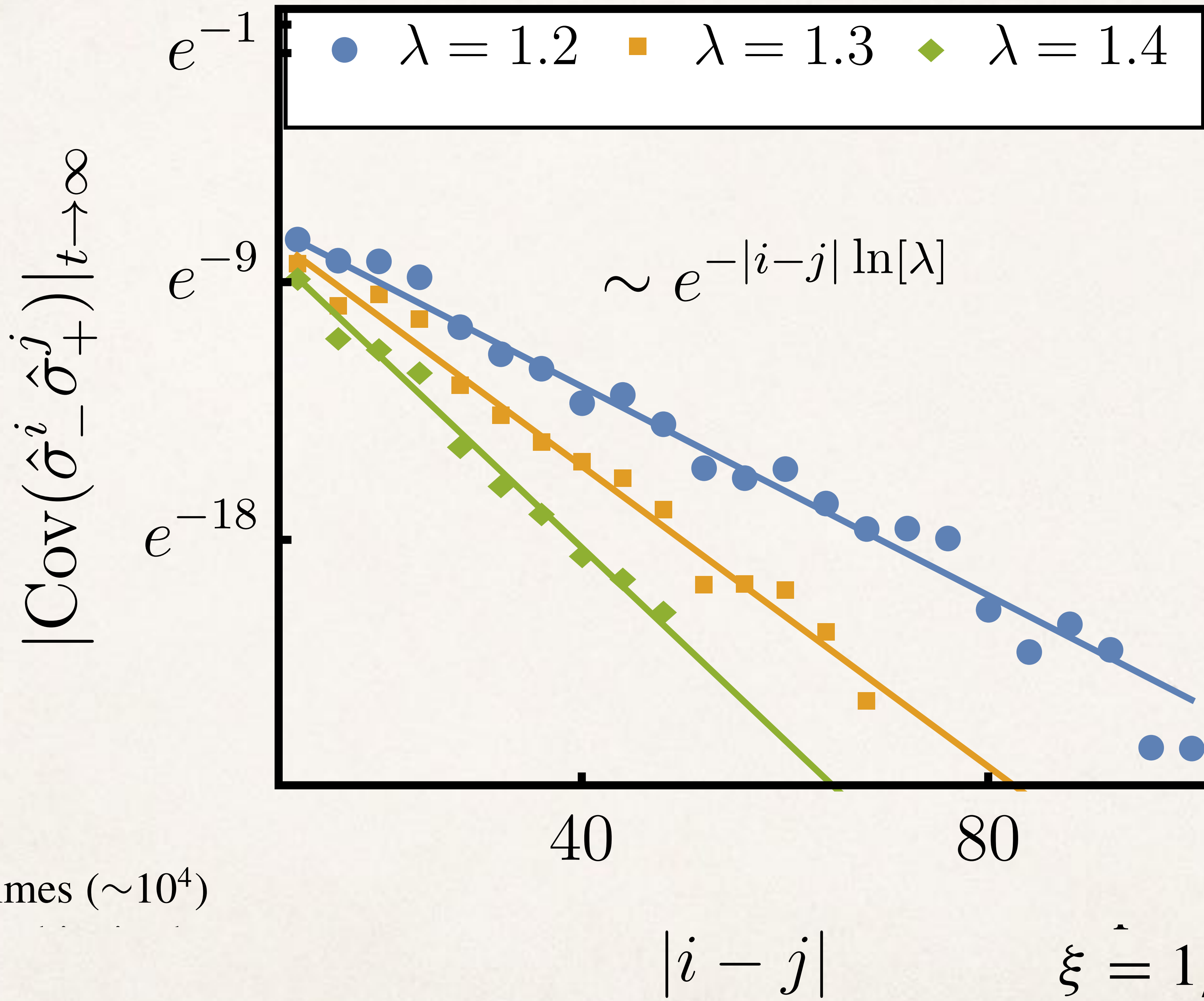
Ballistic

Diffusive

$N = 400$, 800, and 1200

$i = N/4 \quad j = 3N/4$

Correlations: Localized Regime



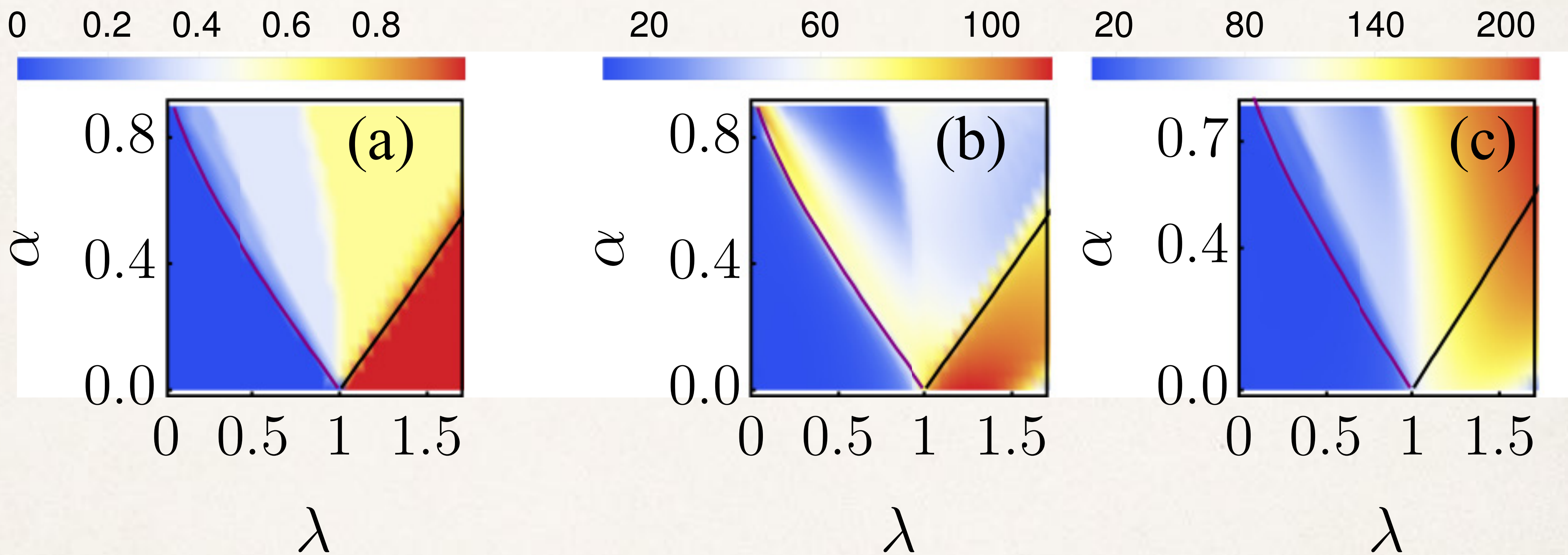
long times ($\sim 10^4$)

$\xi = 1/\ln[\lambda]$

Localization Length Readout

Generalized AAH Model : Non-Markovianity

Fraction of localized states $\qquad \mathcal{N} \quad i = 1 \qquad \mathcal{N} \quad i = 19$



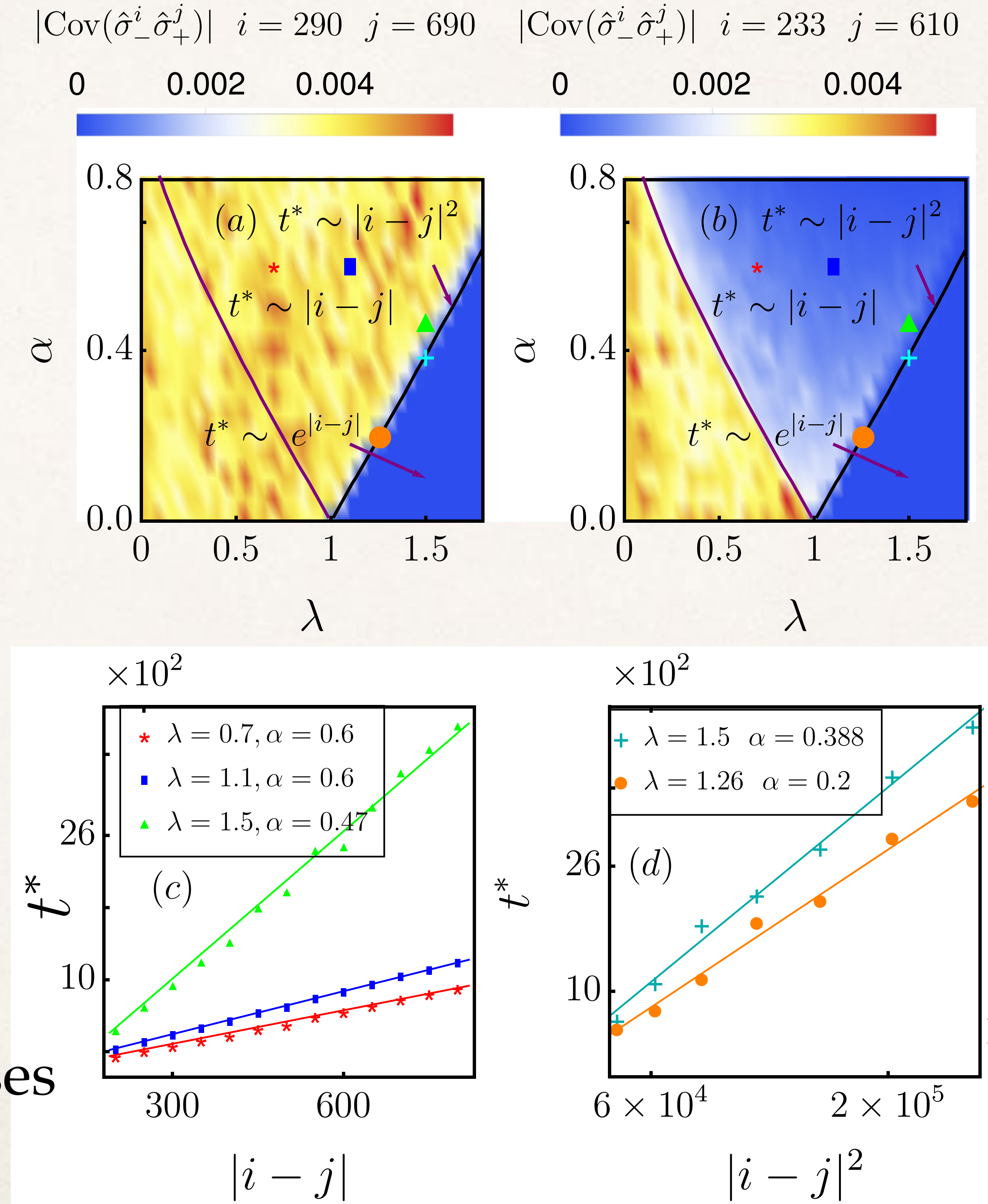
with mobility edge: site dependence of $\mathcal{N}$

$E = \mu + 2(J - \lambda)/\alpha$

Generalized AAH Model: Two qubit Correlations

long-time correlations

site-dependence with mobility edge



slope increases with λ

mixture of critical+localized

Summary/Conclusion

- Readout of GAAH chain by coupling to qubits
- Single Qubit: Non-markovianity of dephasing, nature of SPEs
- Two Qubits: Transport properties from correlations
- Experimental Implementation: Single qubit good prospect with ultracold atoms, multiple qubits better to also look at polaritonic lattices + solid state qubits

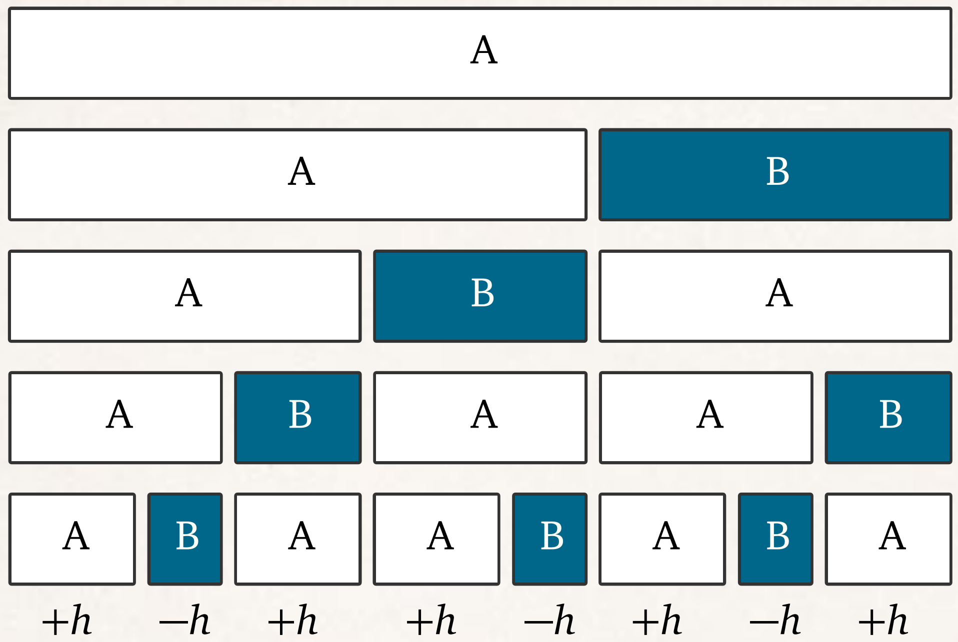
Outlook

- Non-dephasing coupling: AAH and GAAH Bath Thermodynamics, readout of current (direct signature of transport)
- Back-action of qubit on chain?

Thank you!

Please also visit Madhumita's poster on this topic

Extra



$$\sigma : \begin{cases} A \to AB \\ B \to A \end{cases} \qquad (h_i = \pm h)$$

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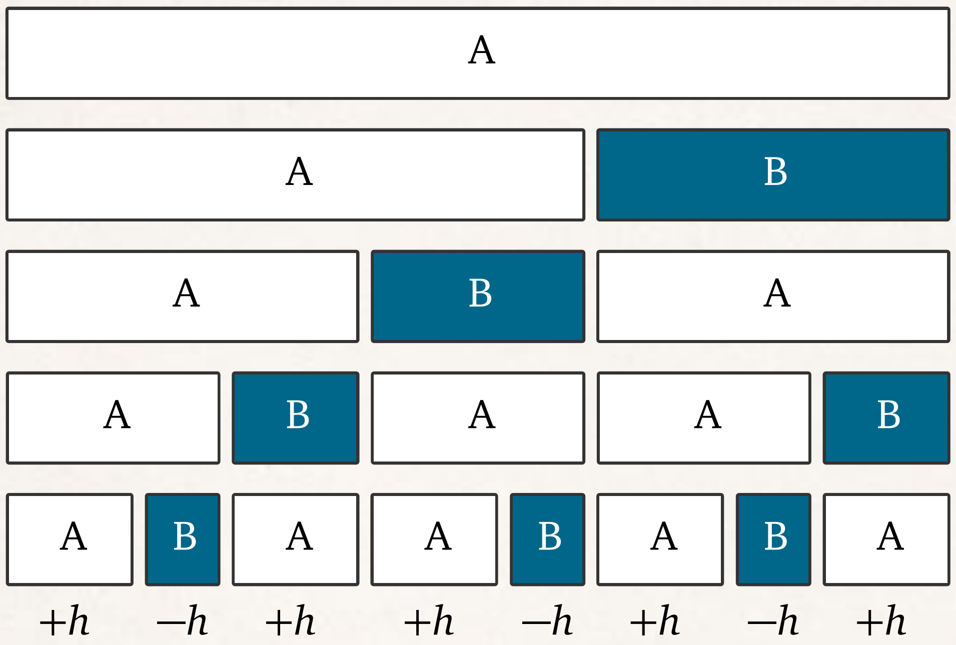

$$\sigma : \begin{cases} A \to AB \\ B \to A \end{cases} \qquad (h_i = \pm h)$$

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