Degenerated Liouvillians and controlling transport

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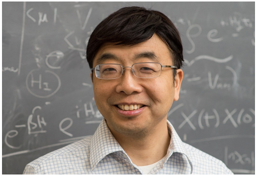
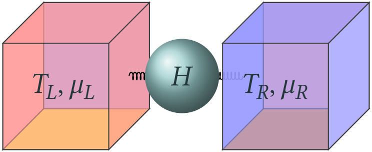


Outline

- Introduction to Symmetries in Open Systems
- Simple Examples
  - Minimal model
  - Benzene
- Obtaining Nonequilibrium Steady States
- Controlling Transport
  - Cubic networks: Magnetic field
  - SSH model: Lead position
- Summary

Open Quantum Systems



Lindblad Quantum Master Equation

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{\alpha=L,R} \sum_{k=p,d} \Gamma_\alpha^k \left(L_\alpha^k \rho(t) L_\alpha^{k\dagger} - \frac{1}{2}\{L_\alpha^{k\dagger} L_\alpha^k, \rho(t)\} \right)$$

Important Properties

- Complete Positivity: $\rho(t) = \sum_j M_j \rho(0) M_j^\dagger$.
- Trace Preserving: $\mathrm{Tr}[\rho] = 1$.
- Hermiticity Preserving: $\rho^\dagger = \rho$.

Symmetries: Open Systems

Weak Symmetry Condition

The open system dynamics $\dot{\rho}(t) = \mathcal{L}[\rho(t)]$

If $\exists$ a set of unitary super operators $\mathcal{P}_n$ s.t. $\hat{\mathcal{P}}_n^\dagger \hat{\mathcal{P}}_n = \mathbb{1}$ and

$$\boxed{[\hat{\mathcal{L}}, \hat{\mathcal{P}}_n] = 0} \quad \forall\, n$$

Strong Symmetry Condition

If $\exists$ a set of unitary operators Π_n s.t. $\Pi_n^\dagger \Pi_n = \mathbb{1}$ and

$$\boxed{[H, \Pi_n] = [L_\alpha^k, \Pi_n] = 0} \quad \forall\, n, k$$

$\implies$ $\hat{\mathcal{L}}$ (matrix map of $\mathcal{L}$) can be block diagonalized

The system evolution is restricted to invariant subspaces

$\implies$ Multiple Steady-states

D. Manzano and P. I. Hurtado, Adv. Phys. **67**, 1 (2018).

Liouvillian Spectrum

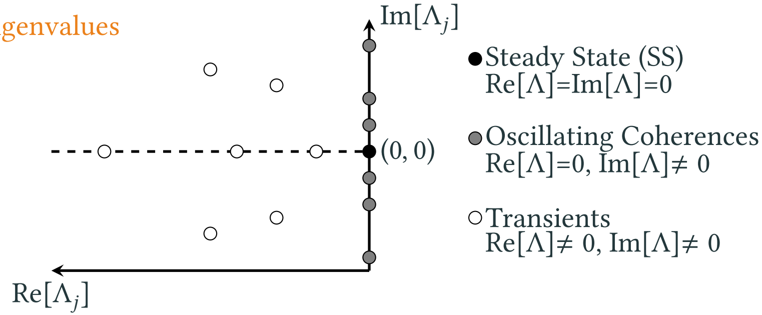
Spectral Analysis

$$\dot{\rho} = \mathcal{L}[\rho]$$

$$|\rho(t)\rangle = \sum_j e^{\Lambda_j t} \langle \phi_j^l | \rho(0) \rangle | \phi_j^r \rangle$$

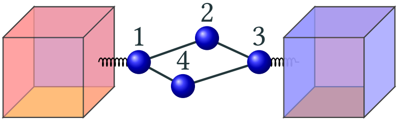
$$\hat{\mathcal{L}} |\phi_j^r\rangle = \Lambda_j |\phi_j^r\rangle; \qquad \langle \phi_j^l | \hat{\mathcal{L}} = \Lambda_j \langle \phi_j^l |$$

Eigenvalues



Simple Examples

Minimal Model



Tight-Binding Hamiltonian

$$H = \varepsilon \sum_{i=1}^{4} |i\rangle\langle i| + t \sum_{<i,j>} |i\rangle\langle j| + \text{h.c.}$$

Lindblad Jump Operators

$$L_L^p = (L_L^d)^\dagger = |1\rangle\langle 0|; \quad L_R^p = (L_R^d)^\dagger = |3\rangle\langle 0|$$

Strong Symmetry Operator

$$\Pi = \sum_{i=0,1,3} |i\rangle\langle i| + |2\rangle\langle 4| + |4\rangle\langle 2|$$

Minimal Model

Basis Transformation

$$|\tilde{0}\rangle = |0\rangle \quad \text{Groundstate } (g)$$

$$|\tilde{1}\rangle = |1\rangle; \quad |\tilde{2}\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle + |4\rangle\right); \quad |\tilde{3}\rangle = |3\rangle \quad \text{Symmetric subspace } (s)$$

$$|\tilde{4}\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle - |4\rangle\right) \quad \text{Antisymmetric subspace } (a)$$

Transformed Lindblad Equation (cross-correlations decay)

$$\dot{\rho}_{gg} = -\frac{1}{2}\{L_p^\dagger L_p, \rho_{gg}\} + L_d^\dagger \rho_{ss} L_d$$

$$\dot{\rho}_{ss} = -i[H_{ss}, \rho_{ss}] - \frac{1}{2}\{L_d L_d^\dagger, \rho_{ss}\} + L_p \rho_{gg} L_p^\dagger$$

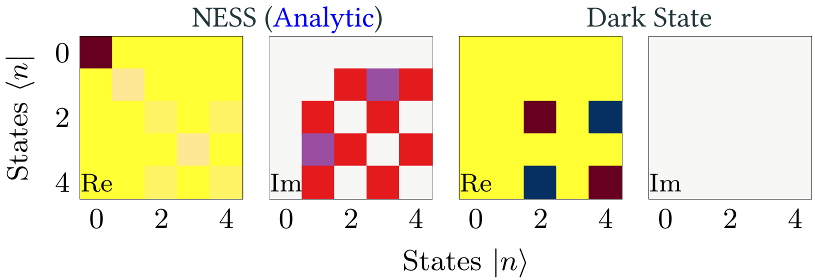
$$\dot{\rho}_{aa} = 0 \quad \text{Decoherence free subspace}$$

$L_x = \sqrt{\Gamma_L^x}|\tilde{1}\rangle + \sqrt{\Gamma_R^x}|\tilde{3}\rangle$ and $H_{ss} = \varepsilon \sum_{i=1}^{3} |\tilde{i}\rangle\langle\tilde{i}| + \sqrt{2}t \sum_{i=1}^{2} |\tilde{i}\rangle\langle\tilde{i}+1| + \text{h.c.}$

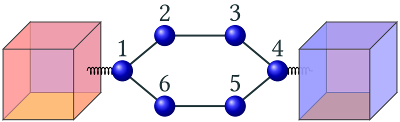
Two NESS

$$\rho^1 = |\psi\rangle\langle\psi|; \quad |\psi\rangle = |\tilde{4}\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle - |4\rangle\right)$$

Pure (dark-)state emerging from decoherence free subspace.



Benzene Ring



Tight-Binding Hamiltonian

$$H = \varepsilon \sum_{i=1}^{6} |i\rangle\langle i| + t \sum_{<i,j>} |i\rangle\langle j| + \text{h.c.}$$

Lindblad Jump Operators

$$L_L^p = (L_L^d)^\dagger = |1\rangle\langle 0|; \quad L_R^p = (L_R^d)^\dagger = |4\rangle\langle 0|$$

Strong Symmetry Operator

$$\Pi = \sum_{i=0,1,4} |i\rangle\langle i| + |2\rangle\langle 6| + |3\rangle\langle 5| + \text{h.c.}$$

Benzene Ring

Basis Transformed Lindblad Equation (cross-correlations decay)

$$
\begin{aligned}
\dot{\rho}_{gg} &= -\frac{1}{2}\{L_p^\dagger L_p, \rho_{gg}\} + L_d^\dagger \rho_{ss} L_d \\
\dot{\rho}_{ss} &= -i[H_{ss}, \rho_{ss}] - \frac{1}{2}\{L_d L_d^\dagger, \rho_{ss}\} + L_p \rho_{gg} L_p^\dagger \\
\dot{\rho}_{aa} &= -i[H_{aa}, \rho_{aa}] \quad \text{Decoherence free subspace}
\end{aligned}
$$

$$L_x = \sqrt{\Gamma_L^x}|\tilde{1}\rangle + \sqrt{\Gamma_R^x}|\tilde{4}\rangle$$

$$H_{ss} = \varepsilon \sum_{i=1}^{4} |\tilde{i}\rangle\langle\tilde{i}| + \sqrt{2}t \sum_{i=1}^{3} |\tilde{i}\rangle\langle\tilde{i}+1| + \text{h.c.}$$

$$H_{aa} = \varepsilon \sum_{i=5}^{6} |\tilde{i}\rangle\langle\tilde{i}| + t|\tilde{5}\rangle\langle\tilde{6}| + \text{h.c.}$$

Benzene Ring: Nonequilibrium Steady States (NESS)

Three NESS

$$\rho^1 = |\psi_1\rangle\langle\psi_1|; \quad |\psi_1\rangle = \frac{1}{2}\left(|2\rangle + |3\rangle - |5\rangle - |6\rangle\right)$$

$$\rho^2 = |\psi_2\rangle\langle\psi_2|; \quad |\psi_2\rangle = \frac{1}{2}\left(|3\rangle + |6\rangle - |5\rangle - |2\rangle\right)$$

Two pure (dark-)state emerging from decoherence free subspace

Not possible to obtain the general NESS analytically except special cases

Oscillating Coherence (cross terms in antisymmetric subspace)

$$\rho_{oc} = e^{2it\tau}|\psi_1\rangle\langle\psi_2| + e^{-2it\tau}|\psi_2\rangle\langle\psi_1|$$

mechanism underlying Dissipative Time Crystals

C. Booker, B. Buča, and D. Jaksch, N. J. Phys. **22**, 085007 (2020).

Obtaining Nonequilibrium Steady States

What's the problem?

If ρ^i are the steady-state reduced density matrices

$$\begin{aligned} \mathcal{L}[\rho^i] &= 0 \quad \forall i = 1, \cdots, M, \\ \mathrm{Tr}[\rho^i \rho^j] &= 0 \quad \forall i \neq j. \end{aligned}$$

Any linear combination $\tilde{\rho}^i = \sum_j c_{ij}\rho^j$ is also a steady-state solution.

$\tilde{\rho}^i$ need not be density matrices

Exact diagonalization to obtain density matrix using eigenvectors of 0 eigenvalue fails

- Either this yields initial condition dependent steady states
- It yields $\tilde{\rho}^i$ which are linear combinations

How to create density matrices given $\tilde{\rho}^i$?

Constructing ρ^i using $\tilde{\rho}^i$

- Step 1: Construct Hermitian operators

$$\rho_H^i = \tilde{\rho}^i + \left(\tilde{\rho}^i\right)^\dagger .$$

- Step 2: Orthogonalize the operators, i.e., $\mathrm{Tr}[\rho_O^i \rho_O^j] = 0 \; \forall \; i \neq j$

$$\begin{aligned}
\rho_O^1 &= \rho_H^1, \\
\rho_O^2 &= \rho_H^2 - \frac{\mathrm{Tr}[\rho_O^1 \rho_H^2]}{\mathrm{Tr}[\rho_O^1 \rho_O^1]} \rho_O^1, \\
\rho_O^3 &= \rho_H^3 - \frac{\mathrm{Tr}[\rho_O^1 \rho_H^3]}{\mathrm{Tr}[\rho_O^1 \rho_O^1]} \rho_O^1 - \frac{\mathrm{Tr}[\rho_O^2 \rho_H^3]}{\mathrm{Tr}[\rho_O^2 \rho_O^2]} \rho_O^2, \\
&\vdots \\
\rho_O^M &= \rho_H^M - \sum_{j=1}^{M-1} \frac{\mathrm{Tr}[\rho_O^j \rho_H^N]}{\mathrm{Tr}[\rho_O^j \rho_O^j]} \rho_O^j.
\end{aligned}$$

Constructing ρ^i using $\tilde{\rho}^i$

- Step 3: Rotate the operators ρ_O^i to make them semi-positive.

Rotate the vector $|\rho_O\rangle\rangle = \{\rho_O^i\}$ using Euler angles
$\vec{\chi} = \left\{\chi_1, , \cdots , \chi_{\frac{M^2-M}{2}}\right\}$

$$|\rho(\vec{\chi})\rangle\rangle = U(\vec{\chi})|\rho^O\rangle\rangle \qquad U(\vec{\chi}) \in \mathrm{SO}(M)$$

The Euler angles can be fixed ($\vec{\chi}^*$) by maximizing the functional

$$F\left[\{\rho_i(\vec{\chi})\}\right] = \sum_{i=1}^{M}\sum_{j=1}^{N} \overbrace{v_j^{\rho_i(\vec{\chi})}}^{\text{ev of } \rho_i(\vec{\chi})} - \left|v_j^{\rho_i(\vec{\chi})}\right|$$

s.t. $F\left[\{\rho_i(\vec{\chi}^*)\}\right] \approx 0.$

Constructing ρ^i using $\tilde{\rho}^i$

- Step 4: Obtain the semi-positive operators $\left\{\rho_P^i\right\} = |\rho_P\rangle\rangle$

$$|\rho_P\rangle\rangle = U(\vec{\chi}^*)|\rho^O\rangle\rangle$$

- Step 5: Normalize the semi-positive operators to obtain density matrices

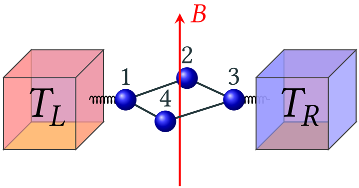
$$\rho^i = \frac{\rho_P^i}{\mathrm{Tr}[\rho_P^i]}$$

$\therefore\ \rho^i$ are density matrices

J. Thingna and D. Manzano, arXiv:2101.10236 (2021).

Controlling Transport

Symmetry Breaking: Magnetic Field



Tight-Binding Hamiltonian

$$H = \varepsilon \sum_{i=1}^{4} |i\rangle\langle i| + \sum_{<i,j>} t_{ij} |i\rangle\langle j| + \text{h.c.}$$

$$t_{12} = t_{34} = t;\ t_{14} = t_{23} = te^{-i\pi B}$$

The presence of magnetic field breaks the geometric symmetries of the system.

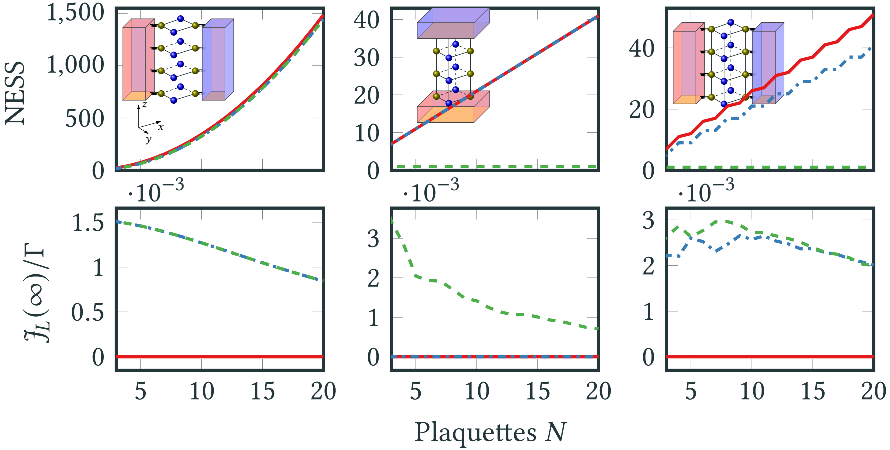
Particle Currents

$$\mathcal{J}_L(t) = \overbrace{\Gamma_L^p \mathrm{Tr}[(L_L^p)^\dagger L_L^p \rho(t)]}^{\text{Current Injected}} - \overbrace{\Gamma_L^d \mathrm{Tr}[(L_L^d)^\dagger L_L^d \rho(t)]}^{\text{Current Extracted}}$$

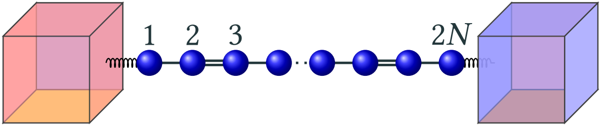
J. Thingna, D. Manzano and J. Cao, New J. Phys. **22**, 083026 (2020).

Cubic Networks: Magnetic Field

$\vec{B} = (0,0,0)$ $\vec{B} = (0,0,B)$ $\vec{B} = (B,B,B)$



SSH Model



Su-Schrieffer-Heeger Model

$$H = w \sum_{i=even} |i\rangle\langle i+1| + v \sum_{i=odd} |i\rangle\langle i+1|.$$

Lindblad Jump Operators

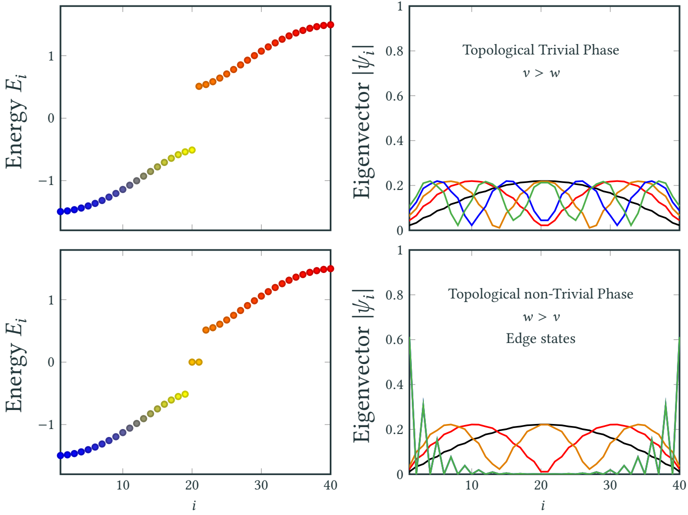
$$L_L^p = (L_L^d)^\dagger = |1\rangle\langle 0|; \quad L_R^p = (L_R^d)^\dagger = |2N\rangle\langle 0|$$

NO simple strong symmetry operators.

Chiral (sublattice symmetry) is not a strong symmetry operator.

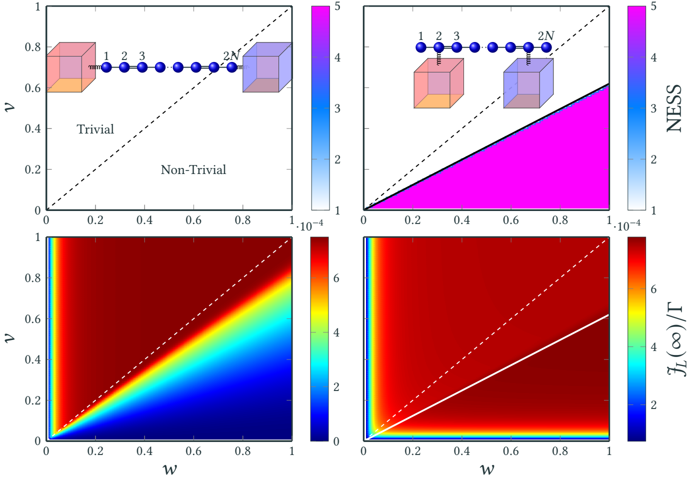
SSH Model: Closed System Phases

Toplogical Phase Boundary at $v = w$



SSH Model: Lead Position

Open Symmetry Breaking Boundary at $v \approx 0.6w$



Summary

Summary

- Symmetries in open quantum systems lead to multiple steady states.
- Using symmetry operators one can simplify the open system problem allowing to analytically calculate the NESS and oscillating coherence.
- A computationally friendly approach to obtain the steady state reduced density matrices was introduced.
- Direction of the magnetic field controls the number of steady states in complex cubic networks and hence controls transport.
- The connection of leads to the system can control currents in the topological SSH model and give rise to an open-system symmetry breaking boundary which is distinct from the topological phase boundary.

Steady States

Dark steady state $\rho_1 = \frac{1}{2}\Big(|2\rangle\langle 2| + |4\rangle\langle 4| - |2\rangle\langle 4| - |4\rangle\langle 2|\Big)$

Nonequilibrium steady state ρ_2

Initial Condition

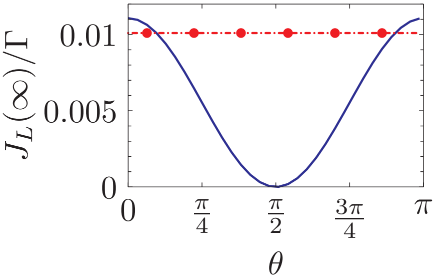
$\rho(0) = \sin^2(\theta)\rho_1 + \cos^2(\theta)\rho_2$

Pure system:

$\varepsilon_2 = \varepsilon_4 = \varepsilon_0$

Disordered system:

$\varepsilon_i = \varepsilon_0 + \xi_i$

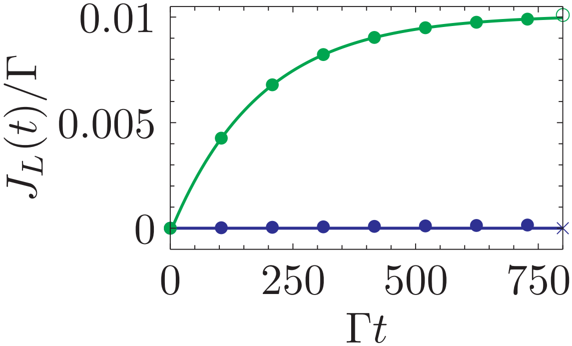


J. Thingna, D. Manzano, and J. Cao, Sci. Rep. **6**, 28027 (2016).

Initial Conditions

$$\rho(0) = \frac{1}{2}\Big(|2\rangle\langle 2| + |4\rangle\langle 4| - |2\rangle\langle 4| - |4\rangle\langle 2|\Big)$$

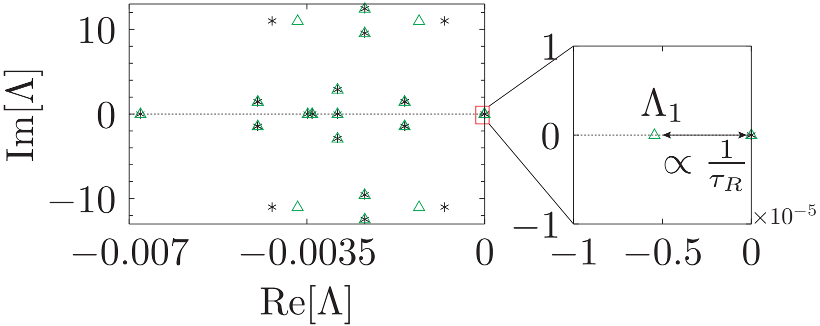
$$\rho(0) = \frac{1}{2}\Big(|2\rangle\langle 2| + |4\rangle\langle 4| + |2\rangle\langle 4| + |4\rangle\langle 2|\Big)$$



Minimal Model: Metastable States

* : Pure System

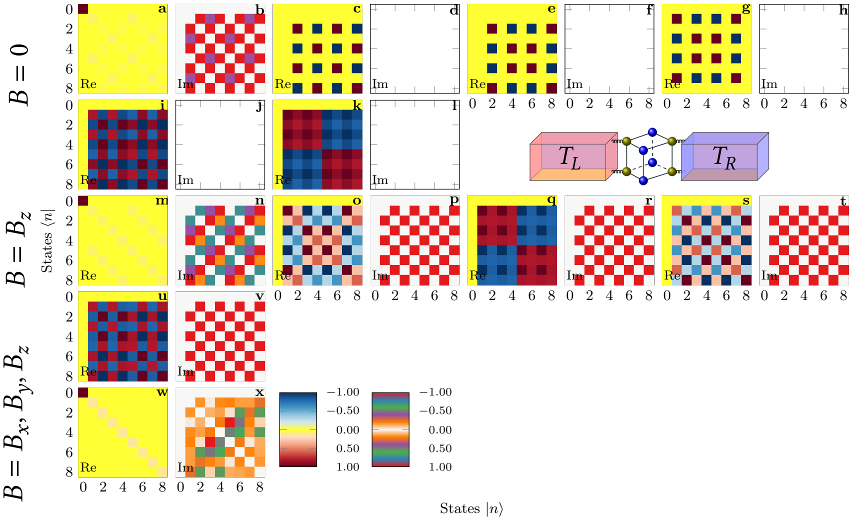
△: Disordered system



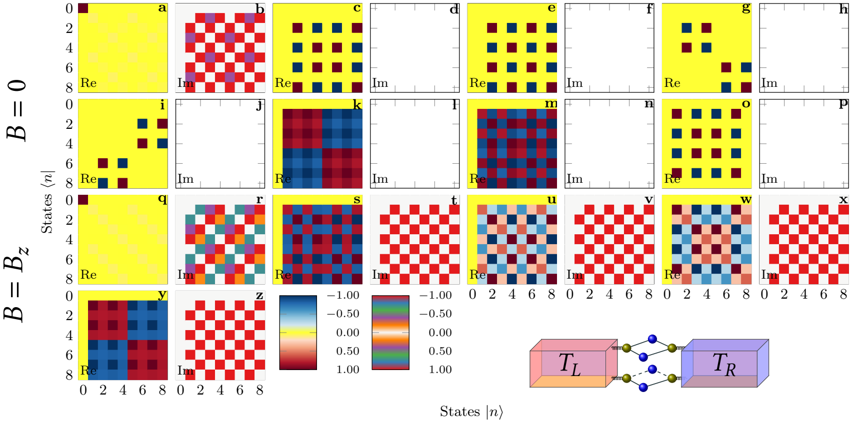
Metastable states have extremely long lifetimes

K. Macieszczak, M. Guţă, I. Lesanovsky, and J. P. Garrahan, Phys Rev. Lett. **116**, 240404 (2016).

Cube: Steady States



Cube: Steady States



J. Thingna, D. Manzano and J. Cao, NJP (2020)