Open with Adobe Acrobat Reader for animation.

The Geometry of Quantum States

C.-Y. Ju

Department of Physics

National Chung Hsing University

March 24, 2021

國立中興大學

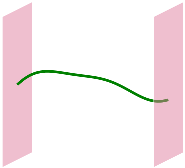
National Chung Hsing University

$$i\partial_t\psi = H\psi$$

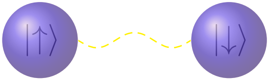
$$i\partial_t\psi = H\psi$$

Quantum Physics

Hydrogen atom



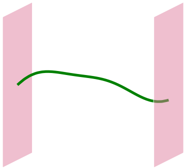
String theories



Quantum information

Quantum Physics

Hydrogen atom



String theories

Quantum information

No-go

No-Go Theorems

- No-cloning
- No-deleting
- No-signaling (no-communication)
- Entanglement invariance under local unitary transformation
- No entanglement increasing under local operations
- $\vdots$

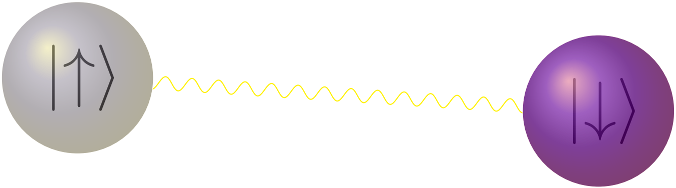
No-Go Theorems

- No-cloning
- No-deleting
- No-signaling (no-communication)
- Entanglement invariance under local unitary transformation
- No entanglement increasing under local operations
- ⋮

No-Signaling Theorem Explained

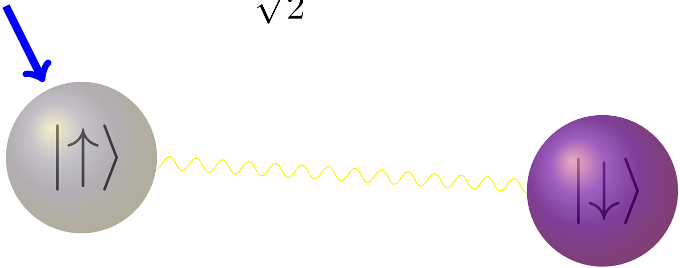
No-Signaling Theorem Explained

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



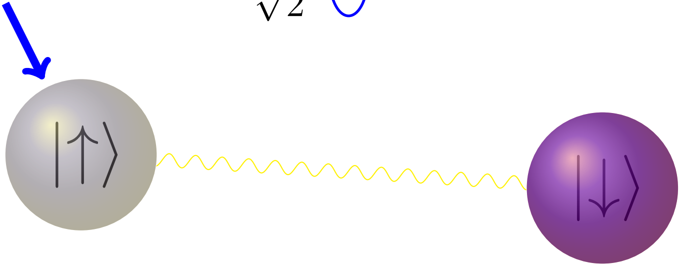
No-Signaling Theorem Explained

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

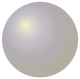


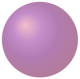
No-Signaling Theorem Explained

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



A Quick Observation





$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle\right)$$

A Quick Observation

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

A Quick Observation

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

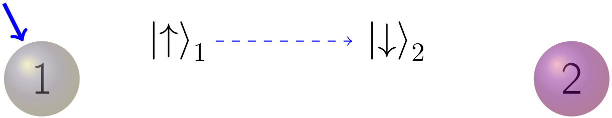
A Quick Observation

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

A Quick Observation

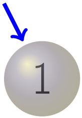
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

A Quick Observation



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

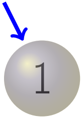
A Quick Observation



$|\uparrow\rangle_1 \dashrightarrow |\downarrow\rangle_2$: 50%

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

A Quick Observation

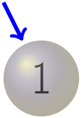


$|\uparrow\rangle_1 \dashrightarrow |\downarrow\rangle_2$: 50%

$|\downarrow\rangle_1 \dashrightarrow |\uparrow\rangle_2$: 50%

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

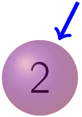
A Quick Observation



$|\uparrow\rangle_1 \dashrightarrow |\downarrow\rangle_2$: 50%

$|\downarrow\rangle_1 \dashrightarrow |\uparrow\rangle_2$: 50%

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



A Quick Observation

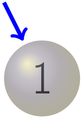
$|\uparrow\rangle_1 \dashrightarrow |\downarrow\rangle_2$: 50%

$|\downarrow\rangle_1 \dashrightarrow |\uparrow\rangle_2$: 50%

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$|\uparrow\rangle_1 \dashleftarrow |\downarrow\rangle_2$: 50%

A Quick Observation



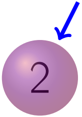
$|\uparrow\rangle_1 \dashrightarrow |\downarrow\rangle_2$: 50%

$|\downarrow\rangle_1 \dashrightarrow |\uparrow\rangle_2$: 50%

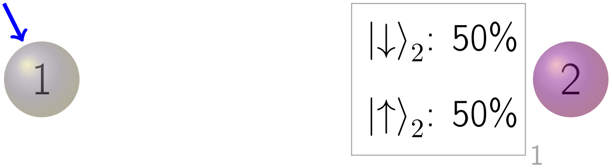
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$|\uparrow\rangle_1 \dashleftarrow |\downarrow\rangle_2$: 50%

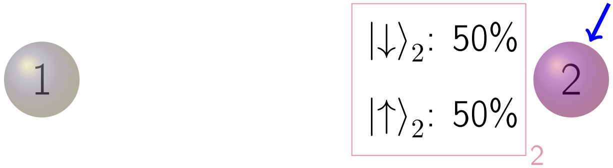
$|\downarrow\rangle_1 \dashleftarrow |\uparrow\rangle_2$: 50%



A Quick Observation



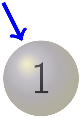
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



A Quick Observation

$$|\psi\rangle = 0.8\,|\uparrow\downarrow\rangle + 0.6\,|\downarrow\uparrow\rangle$$

A Quick Observation



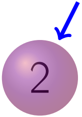
$|\uparrow\rangle_1 \dashrightarrow |\downarrow\rangle_2$: 64%

$|\downarrow\rangle_1 \dashrightarrow |\uparrow\rangle_2$: 36%

$$|\psi\rangle = 0.8\,|\uparrow\downarrow\rangle + 0.6\,|\downarrow\uparrow\rangle$$

$|\uparrow\rangle_1 \dashleftarrow |\downarrow\rangle_2$: 64%

$|\downarrow\rangle_1 \dashleftarrow |\uparrow\rangle_2$: 36%



A Quick Observation

$|\uparrow\rangle_1 \dashrightarrow |\downarrow\rangle_2$: 64%

$|\downarrow\rangle_1 \dashrightarrow |\uparrow\rangle_2$: 36%

No differences!

$|\uparrow\rangle_1 \dashleftarrow |\downarrow\rangle_2$: 64%

$|\downarrow\rangle_1 \dashleftarrow |\uparrow\rangle_2$: 36%

Non-Hermitian Systems?

A Schrödinger equation,

$$i\partial_t |\psi\rangle = H |\psi\rangle ,$$

with

$$H = H^\dagger$$

can be physical.

Non-Hermitian Systems?

A Schrödinger equation,

$$i\partial_t |\psi\rangle = H |\psi\rangle ,$$

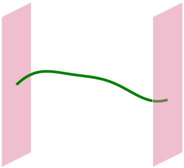
~~with~~

$$\cancel{H = H^\dagger}$$

can be physical.

Quantum Physics

Hydrogen atom



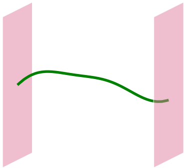
String theories

Quantum information

No-go

Quantum Physics

Hydrogen atom



String theories

Quantum information

Violation of No-Signaling?

PRL **112,** 130404 (2014) PHYSICAL REVIEW LETTERS week ending 4 APRIL 2014

Local $\mathcal{PT}$ Symmetry Violates the No-Signaling Principle

Yi-Chan Lee,[1,2,*] Min-Hsiu Hsieh,[2] Steven T. Flammia,[3] and Ray-Kuang Lee[1,4]
[1]*Physics Department, National Tsing-Hua University, Hsinchu City 300, Taiwan*
[2]*Centre for Quantum Computation & Intelligent Systems, Faculty of Engineering and Information Technology, University of Technology, Sydney, New South Wales 2007, Australia*
[3]*School of Physics, University of Sydney, Sydney, New South Wales 2006, Australia*
[4]*Institute of Photonics Technologies, National Tsing-Hua University, Hsinchu City 300, Taiwan*
(Received 12 December 2013; published 3 April 2014)

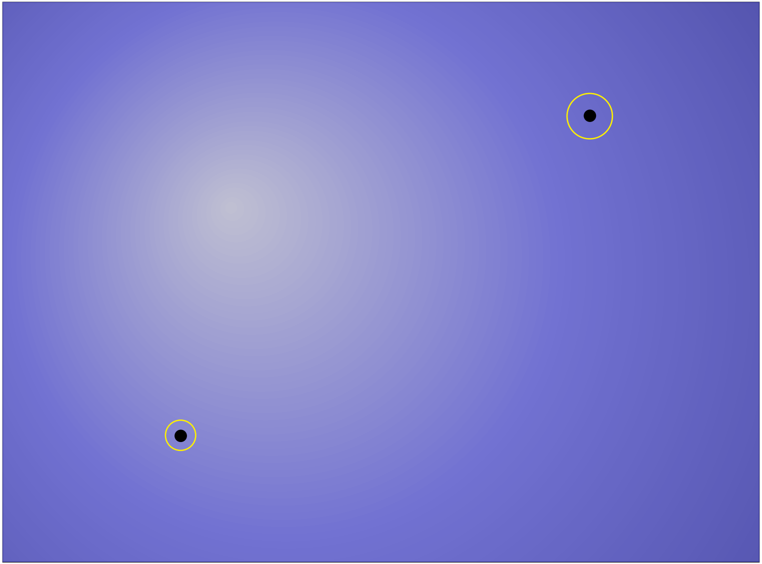
Bender *et al.* [Phys. Rev. Lett. **80**, 5243 (1998)] have developed $\mathcal{PT}$-symmetric quantum theory as an extension of quantum theory to non-Hermitian Hamiltonians. We show that when this model has a local $\mathcal{PT}$ symmetry acting on composite systems, it violates the nonsignaling principle of relativity. Since the case of global $\mathcal{PT}$ symmetry is known to reduce to standard quantum mechanics A. Mostafazadeh [J. Math. Phys. **43**, 205 (2001)], this shows that the $\mathcal{PT}$-symmetric theory is either a trivial extension or likely false as a fundamental theory.

DOI: 10.1103/PhysRevLett.112.130404 PACS numbers: 03.65.Xp, 03.65.Ca, 03.67.Ac, 03.67.Hk

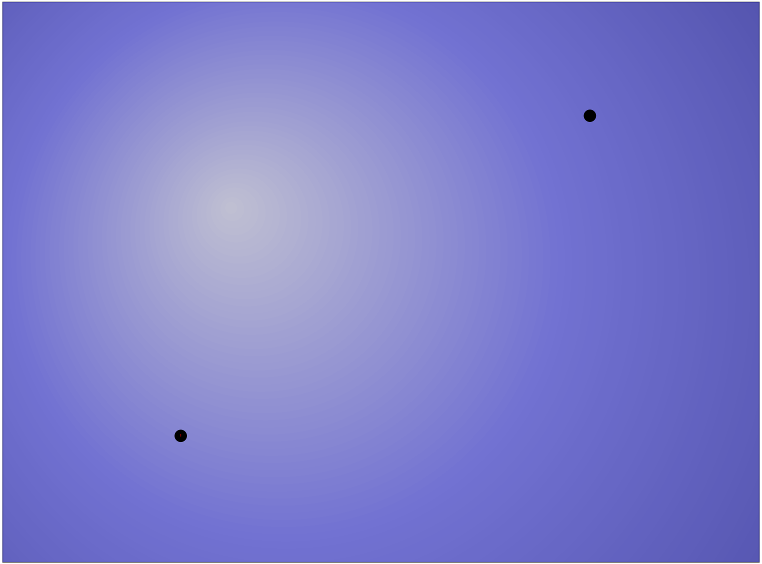
No-Go Theorems

- No-cloning
- No-deleting
- No-signaling (no-communication)
- Entanglement invariance under local unitary transformation
- No entanglement increasing under local operations
- $\vdots$

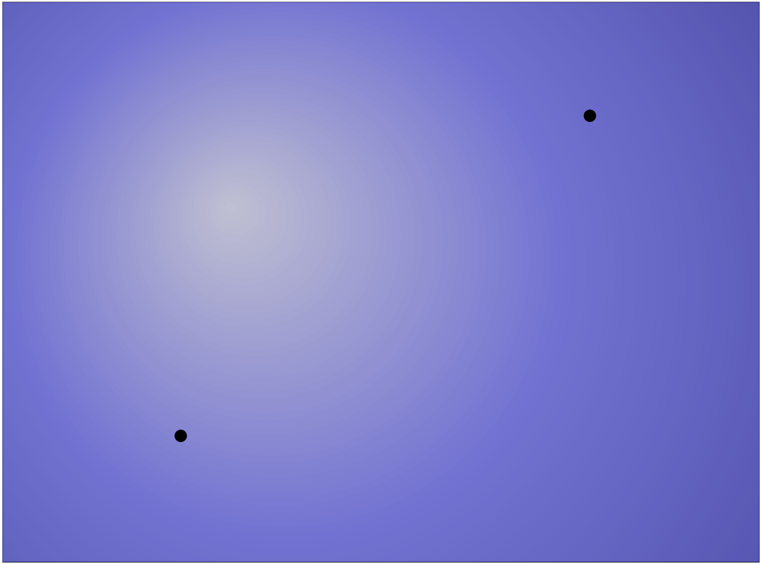
The Geometry Matters



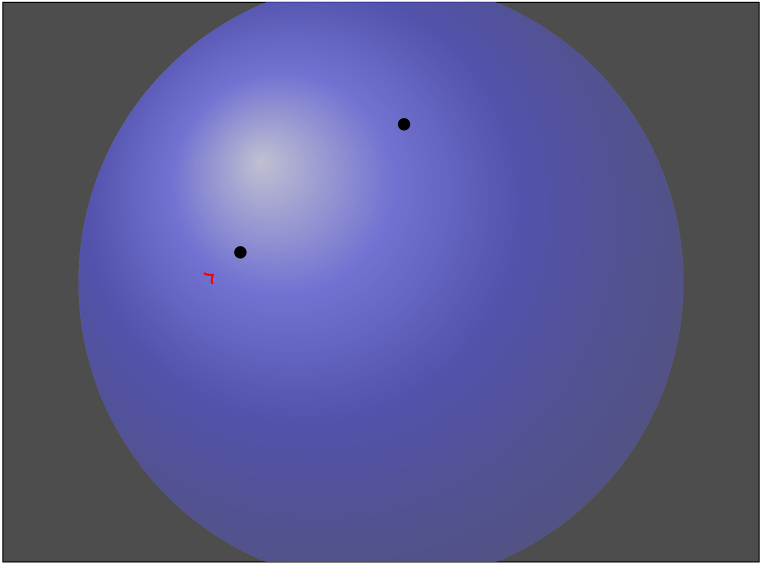
The Geometry Matters



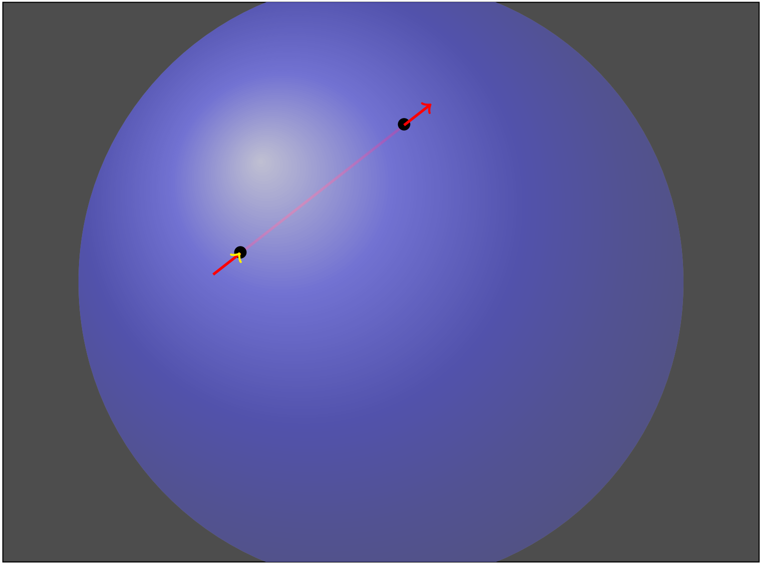
The Geometry Matters



The Geometry Matters



The Geometry Matters



“Non-Euclidean” Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$$

“Non-Euclidean” Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^{\mathsf{T}} V,$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^{\mathsf{T}} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^\mathsf{T} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

$\rightarrow$ "(Non-)Euclidean" space:

$$\langle \vec{u}, \vec{v} \rangle = g_{11} u_1 v_1 + g_{12} u_2 v_1 + g_{21} u_1 v_2 + g_{22} u_2 v_2$$

“Non-Euclidean” Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^{\mathsf{T}} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

$\rightarrow$ “(Non-)Euclidean” space:

$$\langle \vec{u}, \vec{v} \rangle = g_{11} u_1 v_1 + g_{12} \underline{u_2 v_1} + g_{21} \underline{u_1 v_2} + g_{22} u_2 v_2$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^{\mathsf{T}} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

$\rightarrow$ "(Non-)Euclidean" space:

(Fiber)-metric

$$\langle \vec{u}, \vec{v} \rangle = \underline{g_{11}} u_1 v_1 + \underline{g_{12}} u_2 v_1 + \underline{g_{21}} u_1 v_2 + \underline{g_{22}} u_2 v_2$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^\mathsf{T} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

$\rightarrow$ "(Non-)Euclidean" space:

$$\begin{aligned} \langle \vec{u}, \vec{v} \rangle &= g_{11} u_1 v_1 + g_{12} u_2 v_1 + g_{21} u_1 v_2 + g_{22} u_2 v_2 \\ &= \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \end{aligned}$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^\mathsf{T} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

$\rightarrow$ "(Non-)Euclidean" space:

$$\begin{aligned} \langle \vec{u}, \vec{v} \rangle &= g_{11} u_1 v_1 + g_{12} u_2 v_1 + g_{21} u_1 v_2 + g_{22} u_2 v_2 \\ &= \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = U^\mathsf{T} G V. \end{aligned}$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^{\mathsf{T}} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

$\rightarrow$ "(Non-)Euclidean" space:

$$\begin{aligned} \langle \vec{u}, \vec{v} \rangle &= g_{11} u_1 v_1 + g_{12} u_2 v_1 + g_{21} u_1 v_2 + g_{22} u_2 v_2 \\ &= \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = U^{\mathsf{T}} V. \end{aligned}$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^{\mathsf{T}} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

$\rightarrow$ "(Non-)Euclidean" space:

$$\begin{aligned} \langle \vec{u}, \vec{v} \rangle &= g_{11} u_1 v_1 + g_{12} u_2 v_1 + g_{21} u_1 v_2 + g_{22} u_2 v_2 \\ &= \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = U^{\mathsf{T}} G V. \end{aligned}$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^\mathsf{T} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

→ "(Non-)Euclidean" space:

$$\begin{aligned} \langle \vec{u}, \vec{v} \rangle &= g_{11} u_1 v_1 + g_{12} u_2 v_1 + g_{21} u_1 v_2 + g_{22} u_2 v_2 \\ &= \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = U^\mathsf{T} G V. \end{aligned}$$

"Non-Euclidean" Space

Euclidean plane:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = U^\mathsf{T} V,$$

$$\text{where} \quad U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad V \equiv \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

→ "(Non-)Euclidean" space:

$$\begin{aligned} \langle \vec{u}, \vec{v} \rangle &= g_{11} u_1 v_1 + g_{12} u_2 v_1 + g_{21} u_1 v_2 + g_{22} u_2 v_2 \\ &= \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = U^\mathsf{T} G V. \end{aligned}$$

The Hilbert Space Metric

"Euclidean" $\rightarrow$ "(Non-)Euclidean" Hilbert space:

$$\langle\psi_1|\psi_2\rangle \rightarrow \langle\langle\psi_1\,|\,\psi_2\rangle\rangle \equiv \langle\psi_1|\,G\,|\psi_2\rangle\,,$$

where $\langle\psi_1| \equiv (|\psi_1\rangle)^\dagger$.

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Hilbert Space Metric

"Euclidean" $\rightarrow$ "(Non-)Euclidean" Hilbert space:

$$\langle\psi_1|\psi_2\rangle \rightarrow \langle\!\langle\psi_1\,|\,\psi_2\rangle\!\rangle \equiv \langle\psi_1|\,G\,|\psi_2\rangle\,,$$

where $\langle\psi_1| \equiv (|\psi_1\rangle)^\dagger\,.$

Hilbert space properties:

I) $\langle\!\langle\psi_1\,|\,\psi_2\rangle\!\rangle = \overline{\langle\!\langle\psi_2\,|\,\psi_1\rangle\!\rangle}$

II) $\langle\!\langle\psi\,|\,\psi\rangle\!\rangle \geq 0$

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Hilbert Space Metric

"Euclidean" $\rightarrow$ "(Non-)Euclidean" Hilbert space:

$$\langle\psi_1|\psi_2\rangle \rightarrow \langle\!\langle\psi_1\,|\,\psi_2\rangle\!\rangle \equiv \langle\psi_1|\,G\,|\psi_2\rangle\,,$$

where $\langle\psi_1| \equiv (|\psi_1\rangle)^\dagger$.

Hilbert space properties:

I) $\langle\!\langle\psi_1\,|\,\psi_2\rangle\!\rangle = \overline{\langle\!\langle\psi_2\,|\,\psi_1\rangle\!\rangle}$

II) $\langle\!\langle\psi\,|\,\psi\rangle\!\rangle \geq 0$

$\Leftrightarrow$

Hilbert space conditions:

i) $G = G^\dagger$.

ii) G is positive-definite.

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Hilbert Space Metric

"Euclidean" $\rightarrow$ "(Non-)Euclidean" Hilbert space:

$$\langle\psi_1|\psi_2\rangle \rightarrow \langle\!\langle\psi_1\,|\,\psi_2\rangle\!\rangle \equiv \langle\psi_1|\,G\,|\psi_2\rangle\,,$$

where $\langle\psi_1| \equiv (|\psi_1\rangle)^\dagger$.

Hilbert space properties:

I) $\langle\!\langle\psi_1\,|\,\psi_2\rangle\!\rangle = \overline{\langle\!\langle\psi_2\,|\,\psi_1\rangle\!\rangle}$

II) $\langle\!\langle\psi\,|\,\psi\rangle\!\rangle \geq 0$

$\Leftrightarrow$

Hilbert space conditions:

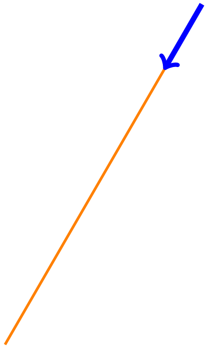
i) $G = G^\dagger$.

ii) G is positive-definite.

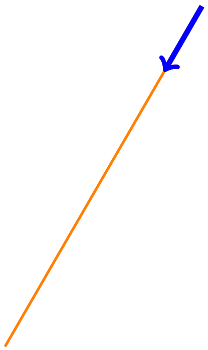
C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

Parallel Transport

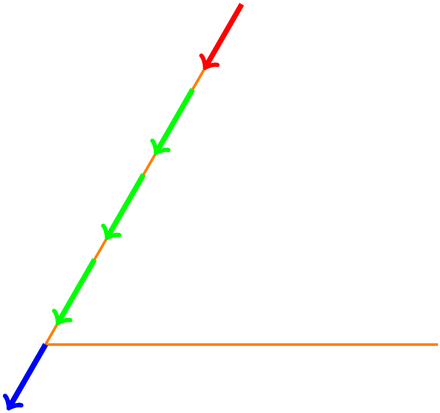
Parallel Transport



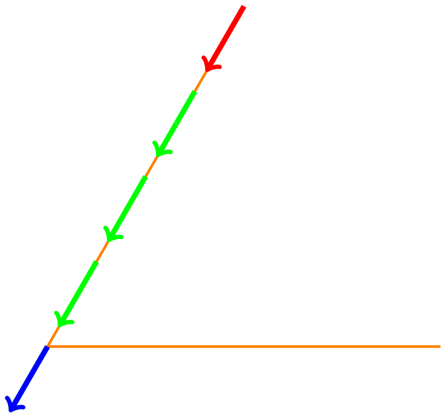
Parallel Transport



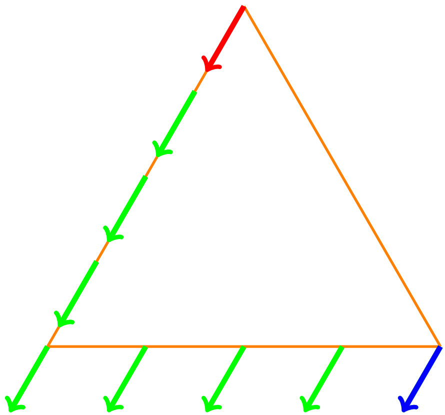
Parallel Transport



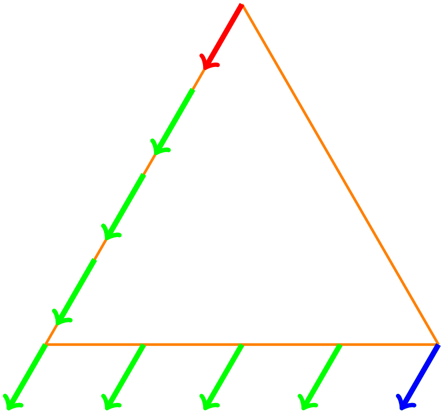
Parallel Transport



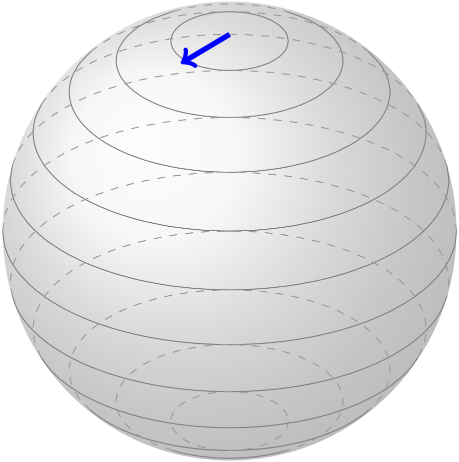
Parallel Transport



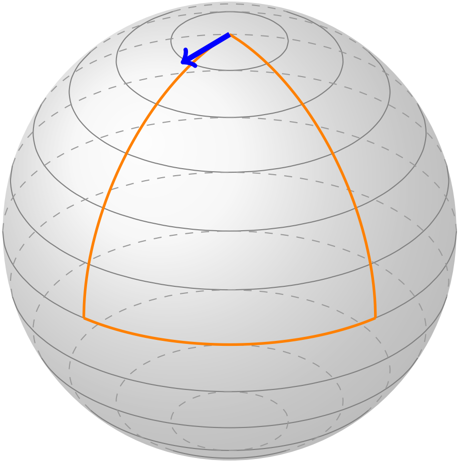
Parallel Transport



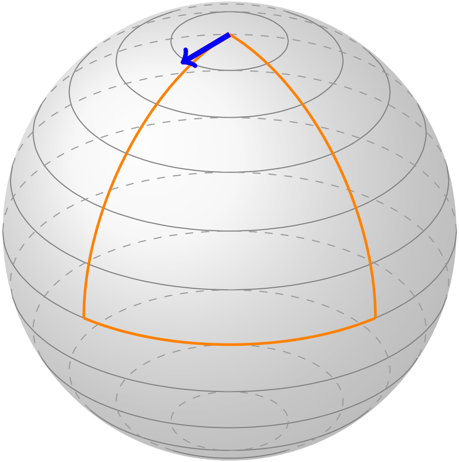
Parallel Transport



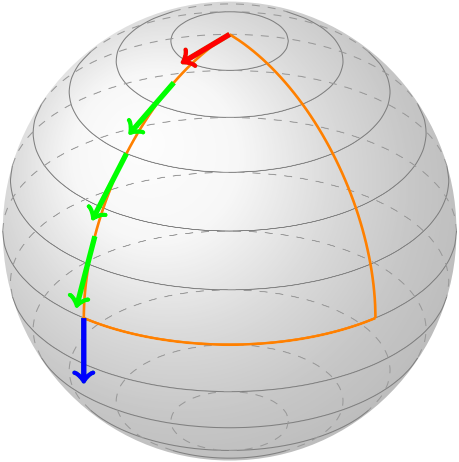
Parallel Transport



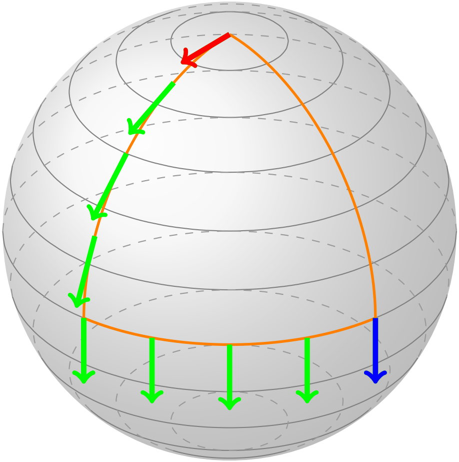
Parallel Transport



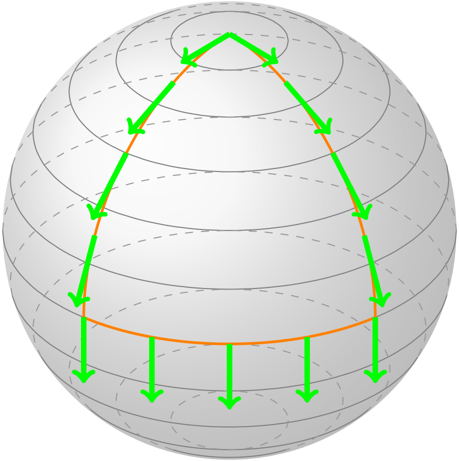
Parallel Transport



Parallel Transport

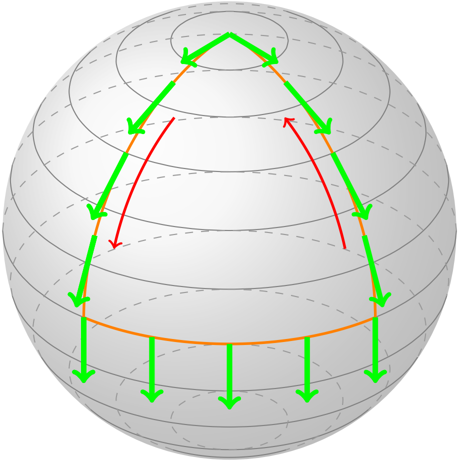


Parallel Transport



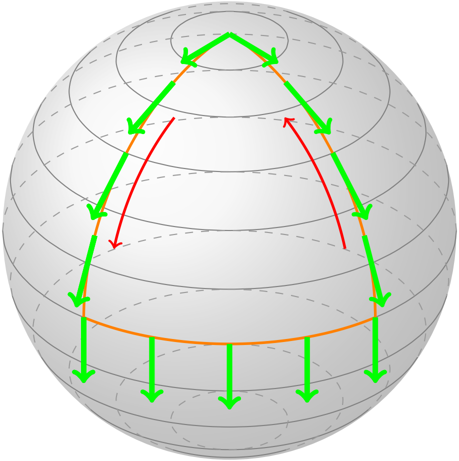
$$V = \begin{pmatrix} v_\theta \\ v_\phi \end{pmatrix}$$

Parallel Transport



$$V = \begin{pmatrix} v_\theta \\ v_\phi \end{pmatrix}$$

Parallel Transport



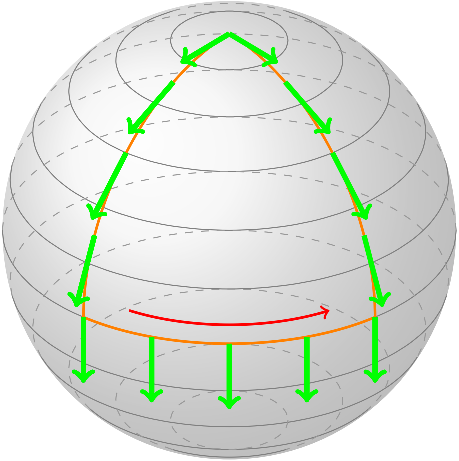
$$V = \begin{pmatrix} v_\theta \\ v_\phi \end{pmatrix}$$

Parallel transport:

$\nabla_\theta V = 0$,

where $\nabla_\theta V \equiv \partial_\theta V + \Gamma_\theta V$.

Parallel Transport



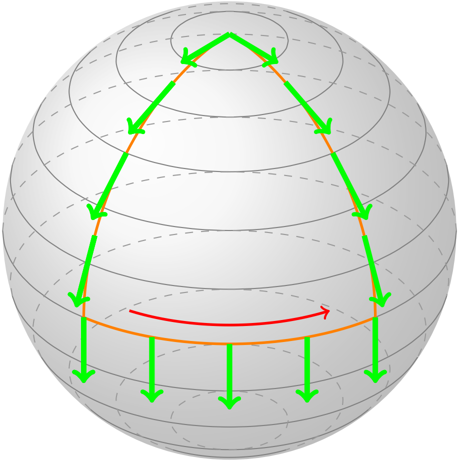
$$V = \begin{pmatrix} v_\theta \\ v_\phi \end{pmatrix}$$

Parallel transport:

$\nabla_\phi V = 0$,

where $\nabla_\phi V \equiv \partial_\phi V + \Gamma_\phi V$.

Parallel Transport



$$V = \begin{pmatrix} v_\theta \\ v_\phi \end{pmatrix}$$

Parallel transport:

$\nabla_\phi V = 0$,

where $\underbrace{\nabla_\phi V \equiv \partial_\phi V + \Gamma_\phi V}$.

Covariant derivative
(Connection)

The Metric Operator Field Equation

$$\nabla_t \left|\psi\right\rangle \equiv (\partial_t + iH) \left|\psi\right\rangle$$

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Metric Operator Field Equation

$$\nabla_t \left|\psi\right\rangle \equiv (\partial_t + iH) \left|\psi\right\rangle = 0.$$

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Metric Operator Field Equation

$$\nabla_t |\psi\rangle \equiv \boxed{(\partial_t + iH)|\psi\rangle = 0.}$$

← parallel transport

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Metric Operator Field Equation

$$\boxed{\nabla_t} |\psi\rangle \equiv \boxed{(\partial_t + iH)\,|\psi\rangle = 0.}$$

connection/covariant derivative

parallel transport

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Metric Operator Field Equation

$$\boxed{\nabla_t} |\psi\rangle \equiv \boxed{(\partial_t + iH)\,|\psi\rangle = 0.}$$

parallel transport

connection/covariant derivative

"Connection-compatible-metric" $\Leftrightarrow 0 = \nabla_t \langle\!\langle \psi_1 \,|\, \psi_2 \rangle\!\rangle$

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Metric Operator Field Equation

$$\boxed{\nabla_t}|\psi\rangle \equiv \boxed{(\partial_t + iH)\,|\psi\rangle = 0.}$$

parallel transport

connection/covariant derivative

"Connection-compatible-metric" $\Leftrightarrow$ $$0 = \nabla_t \langle\!\langle \psi_1 \,|\, \psi_2 \rangle\!\rangle$$
$$= \nabla_t \langle \psi_1 |\, G \,| \psi_2 \rangle$$

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

The Metric Operator Field Equation

$$\boxed{\nabla_t} |\psi\rangle \equiv \boxed{(\partial_t + iH) |\psi\rangle = 0.}$$

parallel transport

connection/covariant derivative

"Connection-compatible-metric" $\Leftrightarrow 0 = \nabla_t \langle\langle \psi_1 | \psi_2 \rangle\rangle$

$$= \nabla_t \langle \psi_1 | G | \psi_2 \rangle$$

$$\Rightarrow \partial_t G = i \left(GH - H^\dagger G \right).$$

C.-Y. Ju, A. Miranowicz, G.Y. Chen, F. Nori, Phys. Rev. A **100**, 062118 (2019).

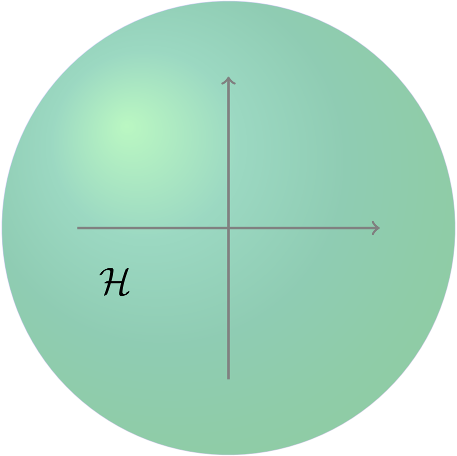
No-Go Theorems

- ✓ No-cloning
- ✓ No-deleting
- ✓ No-signaling (no-communication)
- ✓ Entanglement invariance under local unitary transformation
- ✓ No entanglement increasing under local operations
- $\vdots$

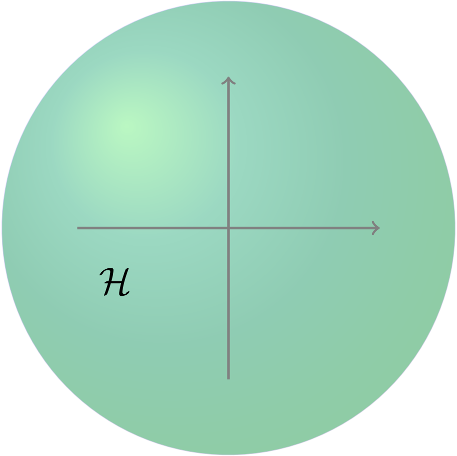
No-Go Theorems

- ✗ No-cloning
- ✗ No-deleting
- ✗ No-signaling (no-communication)
- ✗ Entanglement invariance under local unitary transformation
- ✗ No entanglement increasing under local operations
- $\vdots$

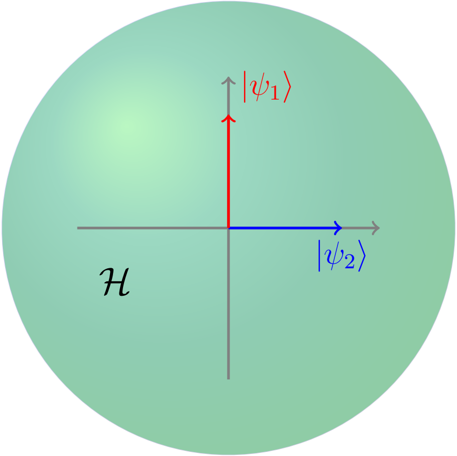
Proving No-Go Theorems in NHQM



Proving No-Go Theorems in NHQM

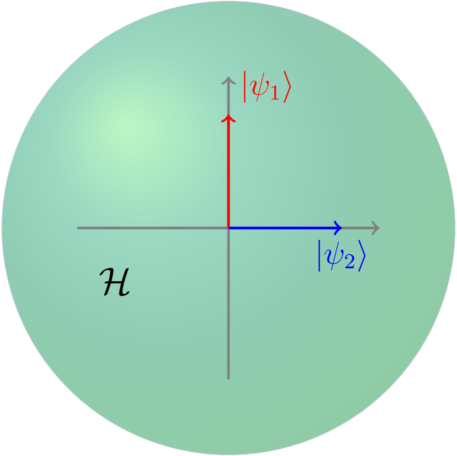


Proving No-Go Theorems in NHQM



Proving No-Go Theorems in NHQM

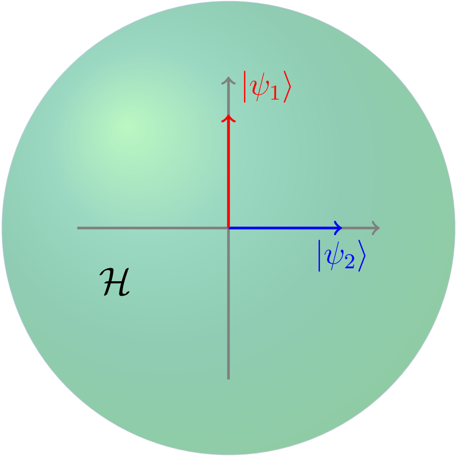
$$\langle\psi_1(t_0)|\psi_2(t_0)\rangle = 0$$



Proving No-Go Theorems in NHQM

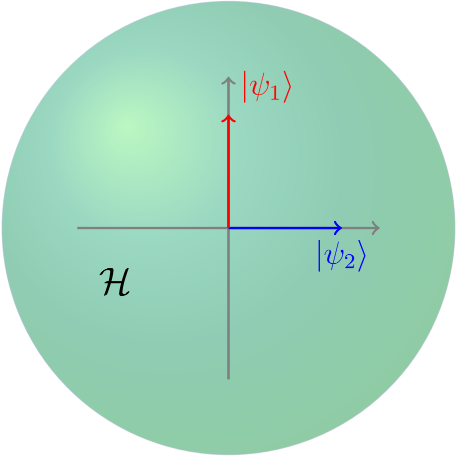
$$\langle \psi_1(t_0) | \psi_2(t_0) \rangle = 0$$

$$\Rightarrow \langle \psi_1(t) | \psi_2(t) \rangle = 0$$



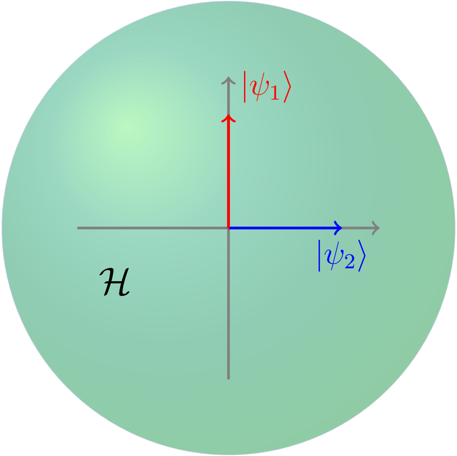
Proving No-Go Theorems in NHQM

$$\langle \psi_1(t_0) | \psi_2(t_0) \rangle = 0$$



Proving No-Go Theorems in NHQM

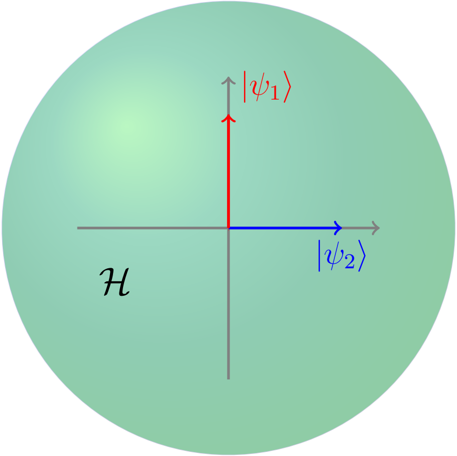
$$\langle\psi_1(t_0)|\psi_2(t_0)\rangle = 0$$



Proving No-Go Theorems in NHQM

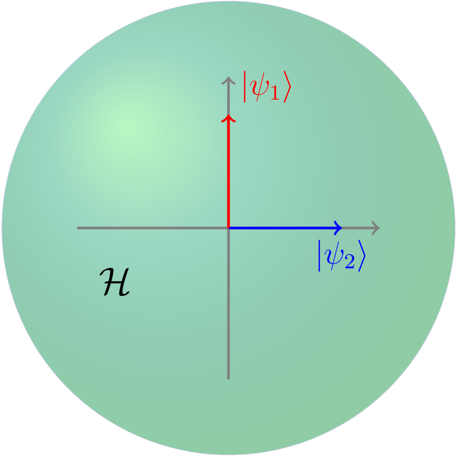
$$\langle \psi_1(t_0)|\psi_2(t_0)\rangle = 0$$

$$\not\Rightarrow \langle \psi_1(t)|\psi_2(t)\rangle = 0$$



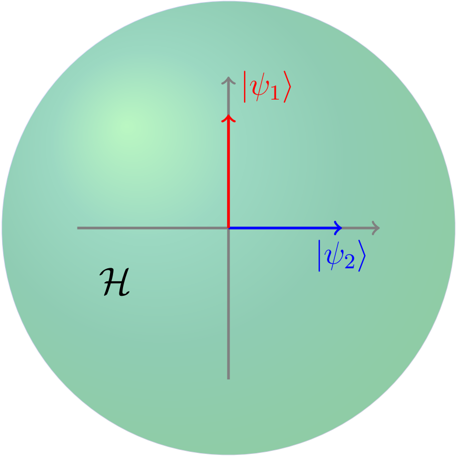
Proving No-Go Theorems in NHQM

$$\langle\!\langle \psi_1(t_0) \,|\, \psi_2(t_0) \rangle\!\rangle = 0$$



Proving No-Go Theorems in NHQM

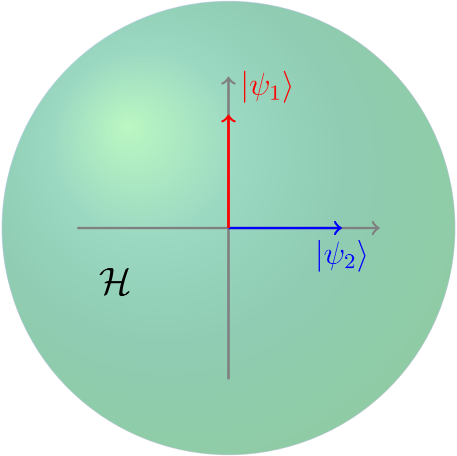
$$\langle\!\langle \psi_1(t_0) \,|\, \psi_2(t_0) \rangle\!\rangle = 0$$



Proving No-Go Theorems in NHQM

$$\langle\!\langle \psi_1(t_0) \,|\, \psi_2(t_0) \rangle\!\rangle = 0$$

$$\Rightarrow \langle\!\langle \psi_1(t) \,|\, \psi_2(t) \rangle\!\rangle = 0$$



No-Go Theorems

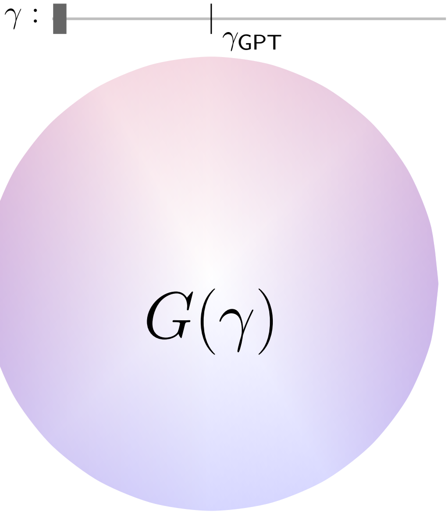
- ✗ No-cloning
- ✗ No-deleting
- ✗ No-signaling (no-communication)
- ✗ Entanglement invariance under local unitary transformation
- ✗ No entanglement increasing under local operations
- ⋮

No-Go Theorems

- ✓ No-cloning
- ✓ No-deleting
- ✓ No-signaling (no-communication)
- ✓ Entanglement invariance under local unitary transformation
- ✓ No entanglement increasing under local operations
- $\vdots$

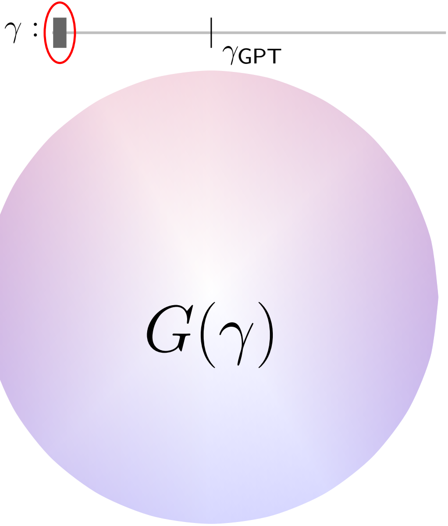
Deforming a Hilbert Space

$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$



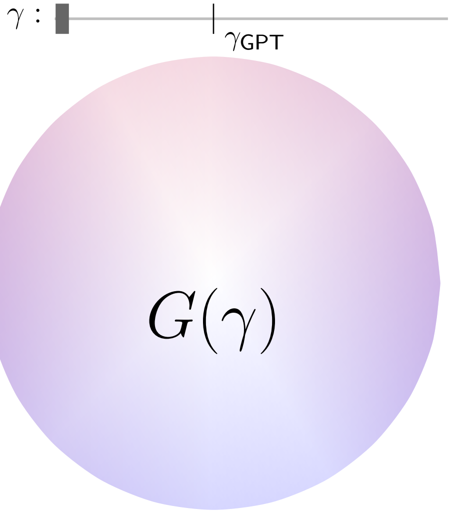
Deforming a Hilbert Space

$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$



Deforming a Hilbert Space

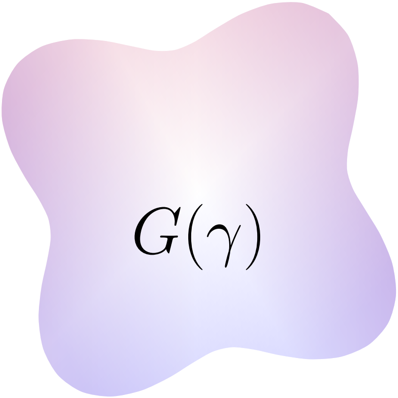
$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$



Deforming a Hilbert Space

$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$

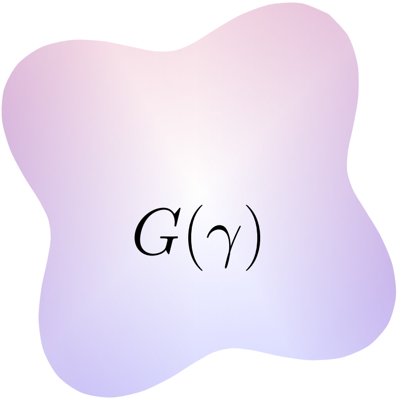
γ :



Deforming a Hilbert Space

$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$

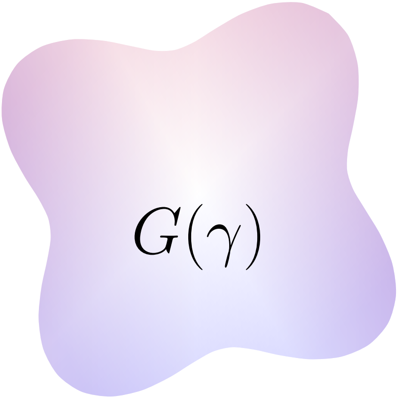
γ :



Deforming a Hilbert Space

$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$

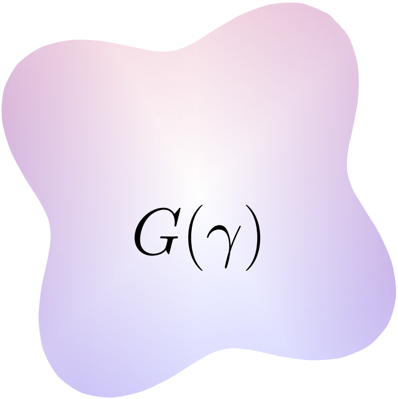
γ :



Deforming a Hilbert Space

$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$

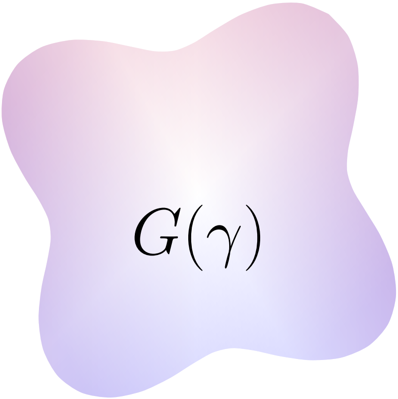
γ :



Deforming a Hilbert Space

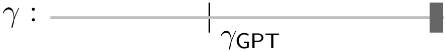
$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$

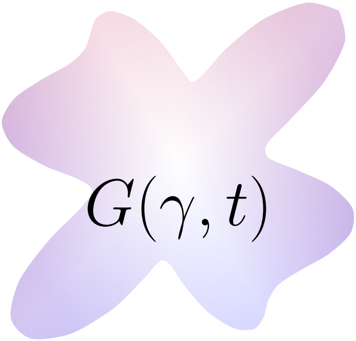
γ :



Deforming a Hilbert Space

$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$

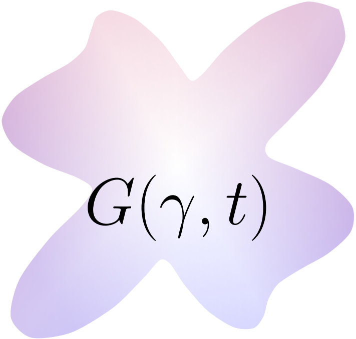




Deforming a Hilbert Space

$$\partial_t G = i\left(GH(\gamma) - H^\dagger(\gamma)G\right)$$

γ :



Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)\big|\Psi(\gamma)\rangle \langle\Psi(\gamma)\big|\Psi(\gamma+\epsilon)\rangle$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle$

Ground state of $H(\gamma)$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\underline{\Psi(\gamma)}\rangle\langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle$

Ground state of $H(\gamma + \epsilon)$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle \langle\Psi(\gamma)|\underline{\Psi(\gamma+\epsilon)}\rangle$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle\langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle$

$$= 1 - \epsilon^2\chi(\gamma) + \mathcal{O}\left(\epsilon^3\right)$$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle$

Groundstate fidelity:
$$\begin{aligned}\mathcal{F} &= \langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle \langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle \\ &= 1 - \epsilon^2 \underline{\chi(\gamma)} + \mathcal{O}\left(\epsilon^3\right)\end{aligned}$$

Fidelity susceptibility

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle \langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle$

$$= 1 - \epsilon^2 \underline{\chi(\gamma)} + \mathcal{O}\left(\epsilon^3\right)$$

Fidelity susceptibility

Phase transition: $\chi(\gamma) \to \infty$ when $\gamma \to \gamma_{\mathsf{PT}}$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle$

Groundstate fidelity:
$$\begin{aligned} \mathcal{F} &= \langle\Psi(\gamma+\epsilon)\big|\Psi(\gamma)\rangle \langle\Psi(\gamma)\big|\Psi(\gamma+\epsilon)\rangle \\ &= 1 - \epsilon^2\chi(\gamma) + \mathcal{O}\left(\epsilon^3\right) \end{aligned}$$

Phase transition: $\chi(\gamma) \to \infty$ when $\gamma \to \gamma_{\mathsf{PT}}$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle \rightarrow \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle\langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle$

$$= 1 - \epsilon^2\chi(\gamma) + \mathcal{O}\left(\epsilon^3\right)$$

Phase transition: $\chi(\gamma) \rightarrow \infty$ when $\gamma \rightarrow \gamma_{\mathsf{PT}}$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\,\langle\phi|\psi\rangle \;\; \rightarrow \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\,\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

$$= \langle\psi|\,G\,|\phi\rangle\,\langle\phi|\,G\,|\psi\rangle$$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle\,\langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle$

$$= 1 - \epsilon^2\chi(\gamma) + \mathcal{O}\left(\epsilon^3\right)$$

Phase transition: $\chi(\gamma) \rightarrow \infty$ when $\gamma \rightarrow \gamma_{\mathsf{PT}}$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle \;\; \to \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\,\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

$$= \langle\psi|\,G\,|\phi\rangle\,\langle\phi|\,G\,|\psi\rangle$$

?

Groundstate fidelity: $\mathcal{F} = \underline{\langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle\,\langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle}$

$$= 1 - \epsilon^2\chi(\gamma) + \mathcal{O}\left(\epsilon^3\right)$$

Phase transition: $\chi(\gamma) \to \infty$ when $\gamma \to \gamma_{\mathsf{PT}}$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle \;\to \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$
$= \langle\psi|\,G\,|\phi\rangle\langle\phi|\,G\,|\psi\rangle$

Ground state of $H(\gamma)$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\underline{\Psi(\gamma)}\rangle\langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle$

$$= 1 - \epsilon^2\chi(\gamma) + \mathcal{O}\left(\epsilon^3\right)$$

Phase transition: $\chi(\gamma) \to \infty$ when $\gamma \to \gamma_{\mathsf{PT}}$

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle \;\rightarrow \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\,\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

$$= \langle\psi|\,G\,|\phi\rangle\,\langle\phi|\,G\,|\psi\rangle$$

Ground state of $H(\gamma+\epsilon)$

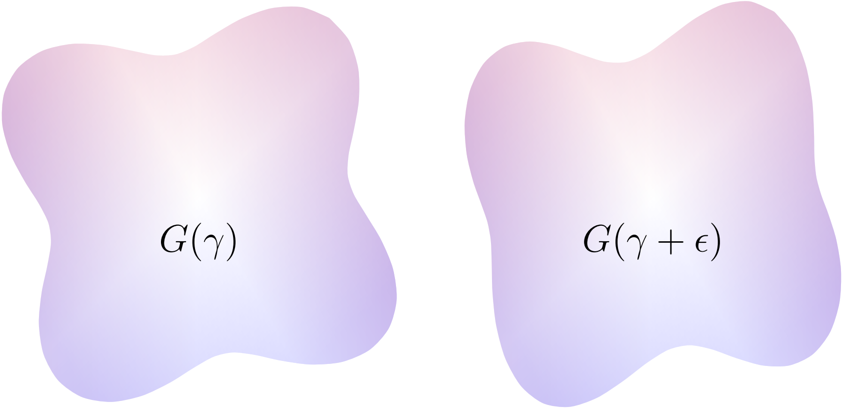
Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle\,\langle\Psi(\gamma)|\underline{\Psi(\gamma+\epsilon)\rangle}$

$$= 1 - \epsilon^2\chi(\gamma) + \mathcal{O}\left(\epsilon^3\right)$$

Phase transition: $\chi(\gamma) \rightarrow \infty$ when $\gamma \rightarrow \gamma_{\mathsf{PT}}$

Generalizing the Fidelity

$$\partial_t G = i\left(GH - H^\dagger G\right)$$



Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle \; \rightarrow \mathcal{F} = \langle\!\langle \psi \,|\, \phi \rangle\!\rangle \langle\!\langle \phi \,|\, \psi \rangle\!\rangle$
$= \langle\psi| \, G \, |\phi\rangle \langle\phi| \, G \, |\psi\rangle$

Groundstate fidelity: $\mathcal{F} = \langle\Psi(\gamma+\epsilon)|\Psi(\gamma)\rangle \langle\Psi(\gamma)|\Psi(\gamma+\epsilon)\rangle$

Y-C. Tzeng, C.-Y. Ju, G.Y. Chen, W.M. Huang, PRResearch 3, 013015 (2021)

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle \ \rightarrow \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle \langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

$$= \langle\psi|\, G \,|\phi\rangle \langle\phi|\, G \,|\psi\rangle$$

Groundstate fidelity: $\mathcal{F} = \langle\!\langle\Psi(\gamma+\epsilon)\,|\,\Psi(\gamma)\rangle\!\rangle \langle\!\langle\Psi(\gamma)\,|\,\Psi(\gamma+\epsilon)\rangle\!\rangle$

Y-C. Tzeng, C.-Y. Ju, G.Y. Chen, W.M. Huang, PRResearch 3, 013015 (2021)

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle \langle\phi|\psi\rangle \rightarrow \mathcal{F} = \langle\langle\psi\,|\,\phi\rangle\rangle \langle\langle\phi\,|\,\psi\rangle\rangle$

$$= \langle\psi|\, G \,|\phi\rangle \langle\phi|\, G \,|\psi\rangle$$

Groundstate fidelity: $\mathcal{F} = \langle\langle\Psi(\gamma+\epsilon)\,|\,\Psi(\gamma)\rangle\rangle \langle\langle\Psi(\gamma)\,|\,\Psi(\gamma+\epsilon)\rangle\rangle$

$$= \langle\Psi(\gamma+\epsilon)|\, G(\gamma+\epsilon) \,|\Psi(\gamma)\rangle \times \langle\Psi(\gamma)|\, G(\gamma) \,|\Psi(\gamma+\epsilon)\rangle$$

Y-C. Tzeng, C.-Y. Ju, G.Y. Chen, W.M. Huang, PRResearch 3, 013015 (2021)

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle \;\rightarrow \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

$$= \langle\psi|\,G\,|\phi\rangle\langle\phi|\,G\,|\psi\rangle$$

Groundstate fidelity:

$$\begin{aligned}\mathcal{F} &= \langle\!\langle\Psi(\gamma+\epsilon)\,|\,\Psi(\gamma)\rangle\!\rangle\langle\!\langle\Psi(\gamma)\,|\,\Psi(\gamma+\epsilon)\rangle\!\rangle \\ &= \langle\Psi(\gamma+\epsilon)|\,G(\gamma+\epsilon)\,|\Psi(\gamma)\rangle \\ &\quad\times\langle\Psi(\gamma)|\,G(\gamma)\,|\Psi(\gamma+\epsilon)\rangle \\ &= 1-\epsilon^2\chi(\gamma)+\mathcal{O}\left(\epsilon^3\right)\end{aligned}$$

Y-C. Tzeng, C.-Y. Ju, G.Y. Chen, W.M. Huang, PRResearch 3, 013015 (2021)

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle \;\to \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\,\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

$$= \langle\psi|\,G\,|\phi\rangle\,\langle\phi|\,G\,|\psi\rangle$$

Groundstate fidelity: $\mathcal{F} = \langle\!\langle\Psi(\gamma+\epsilon)\,|\,\Psi(\gamma)\rangle\!\rangle\,\langle\!\langle\Psi(\gamma)\,|\,\Psi(\gamma+\epsilon)\rangle\!\rangle$

$$= \langle\Psi(\gamma+\epsilon)|\,G(\gamma+\epsilon)\,|\Psi(\gamma)\rangle$$
$$\times\,\langle\Psi(\gamma)|\,G(\gamma)\,|\Psi(\gamma+\epsilon)\rangle$$
$$= 1 - \epsilon^2\underline{\chi(\gamma)} + \mathcal{O}\left(\epsilon^3\right)$$

Generalized fidelity susceptibility

Y-C. Tzeng, C.-Y. Ju, G.Y. Chen, W.M. Huang, PRResearch 3, 013015 (2021)

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle \;\rightarrow \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\,\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

$$= \langle\psi|\,G\,|\phi\rangle\,\langle\phi|\,G\,|\psi\rangle$$

Groundstate fidelity: $\mathcal{F} = \langle\!\langle\Psi(\gamma+\epsilon)\,|\,\Psi(\gamma)\rangle\!\rangle\,\langle\!\langle\Psi(\gamma)\,|\,\Psi(\gamma+\epsilon)\rangle\!\rangle$

$$= \langle\Psi(\gamma+\epsilon)|\,G(\gamma+\epsilon)\,|\Psi(\gamma)\rangle \times \langle\Psi(\gamma)|\,G(\gamma)\,|\Psi(\gamma+\epsilon)\rangle$$

$$= 1 - \epsilon^2\underline{\chi(\gamma)} + \mathcal{O}\left(\epsilon^3\right)$$

Generalized fidelity susceptibility

Phase transition: $\chi(\gamma) \rightarrow \infty$ when $\gamma \rightarrow \gamma_{\mathsf{PT}}$

Y-C. Tzeng, C.-Y. Ju, G.Y. Chen, W.M. Huang, PRResearch 3, 013015 (2021)

Generalizing the Fidelity

Fidelity: $\mathcal{F} = \langle\psi|\phi\rangle\langle\phi|\psi\rangle \;\rightarrow \mathcal{F} = \langle\!\langle\psi\,|\,\phi\rangle\!\rangle\langle\!\langle\phi\,|\,\psi\rangle\!\rangle$

$$= \langle\psi|\,G\,|\phi\rangle\langle\phi|\,G\,|\psi\rangle$$

Groundstate fidelity: $\mathcal{F} = \langle\!\langle\Psi(\gamma+\epsilon)\,|\,\Psi(\gamma)\rangle\!\rangle\langle\!\langle\Psi(\gamma)\,|\,\Psi(\gamma+\epsilon)\rangle\!\rangle$

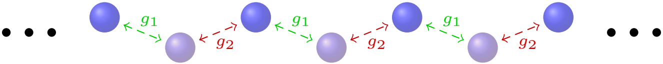
$$\begin{aligned} &= \langle\Psi(\gamma+\epsilon)|\,G(\gamma+\epsilon)\,|\Psi(\gamma)\rangle \\ &\quad\times\langle\Psi(\gamma)|\,G(\gamma)\,|\Psi(\gamma+\epsilon)\rangle \\ &= 1-\epsilon^2\underline{\chi(\gamma)}+\mathcal{O}\left(\epsilon^3\right)\end{aligned}$$

Generalized fidelity susceptibility

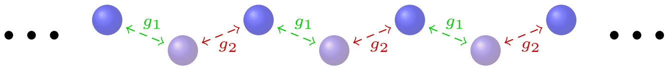
"Phase transition": $\begin{cases} \chi(\gamma)\rightarrow\infty \text{ when } \gamma\rightarrow\gamma_{\mathsf{PT}} \\ \chi(\gamma)\rightarrow-\infty \text{ when } \gamma\rightarrow\gamma_{\mathsf{GPT}} \end{cases}$

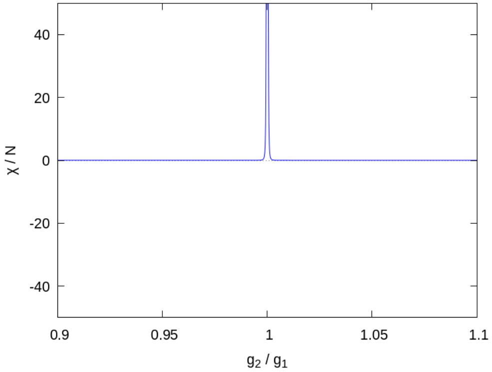
Y-C. Tzeng, C.-Y. Ju, G.Y. Chen, W.M. Huang, PRResearch 3, 013015 (2021)

Generalized Fidelity Susceptibility

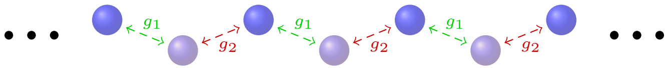


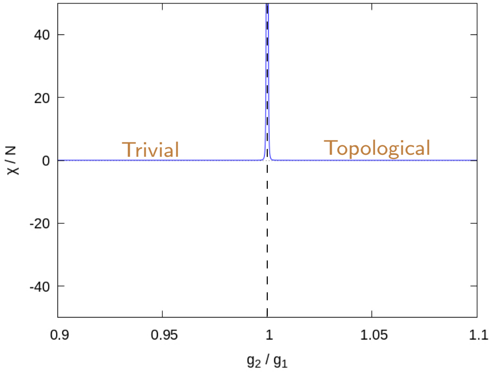
Generalized Fidelity Susceptibility



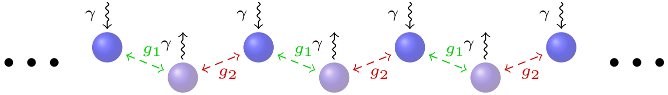


Generalized Fidelity Susceptibility

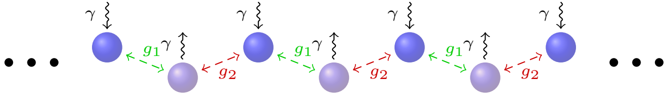


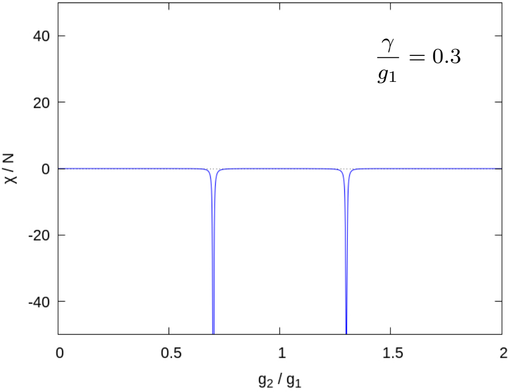


Generalized Fidelity Susceptibility

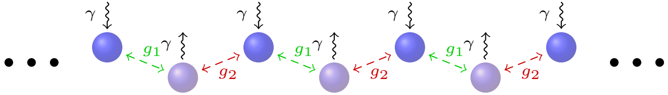


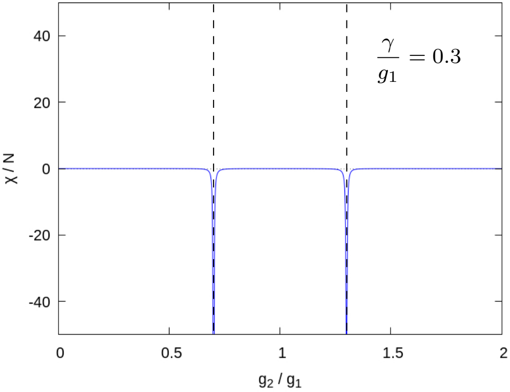
Generalized Fidelity Susceptibility



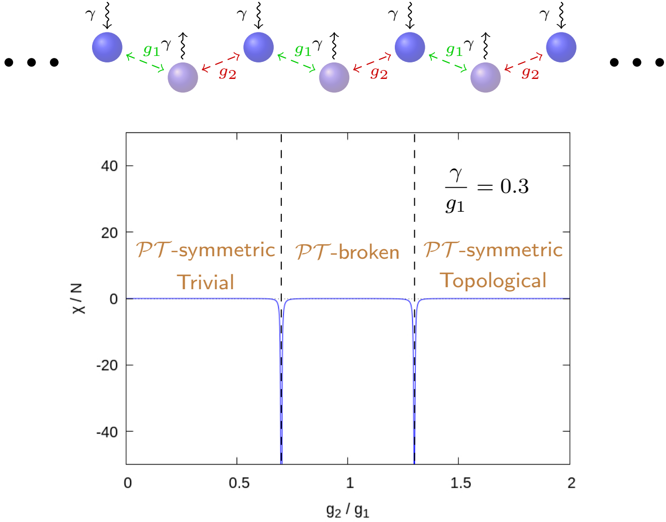


Generalized Fidelity Susceptibility





Generalized Fidelity Susceptibility



Summary

- ☆ A geometric realization of Schrödinger's equation
- ☆ The geometries of the Hilbert spaces
- ☆ A generalization of fidelity
- ☆ A geometry caused phase transition-like phenomenon

Summary

- ☆ A geometric realization of Schrödinger's equation
- ☆ The geometries of the Hilbert spaces
- ☆ A generalization of fidelity
- ☆ A geometry caused phase transition-like phenomenon

Summary

- ☆ A geometric realization of Schrödinger's equation
- ☆ The geometries of the Hilbert spaces
- ☆ A generalization of fidelity
- ☆ A geometry caused phase transition-like phenomenon

Summary

- ☆ A geometric realization of Schrödinger's equation
- ☆ The geometries of the Hilbert spaces
- ☆ A generalization of fidelity
- ☆ A geometry caused phase transition-like phenomenon

Summary

- ☆ A geometric realization of Schrödinger's equation
- ☆ The geometries of the Hilbert spaces
- ☆ A generalization of fidelity
- ☆ A geometry caused phase transition-like phenomenon

Summary

- ☆ A geometric realization of Schrödinger's equation
- ☆ The geometries of the Hilbert spaces
- ☆ A generalization of fidelity
- ☆ A geometry caused phase transition-like phenomenon