ICTS, Non-Hermitian physics 23/03/21

Stochastic description of noisy open quantum systems

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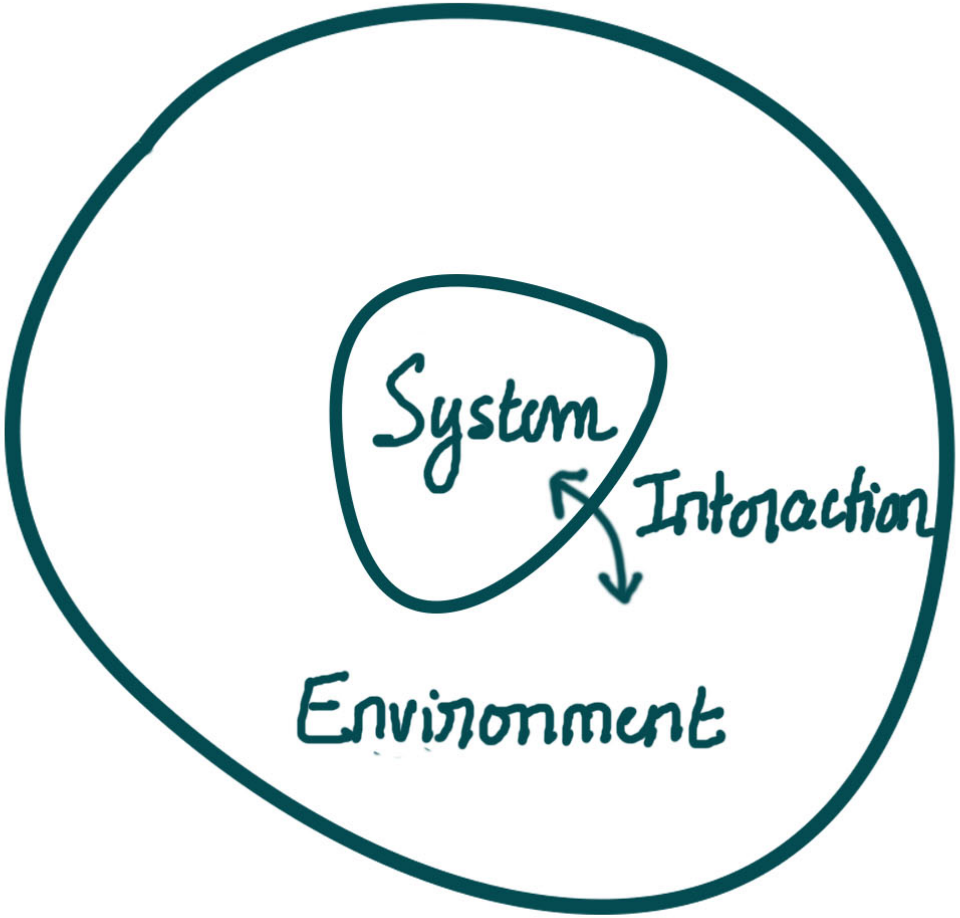
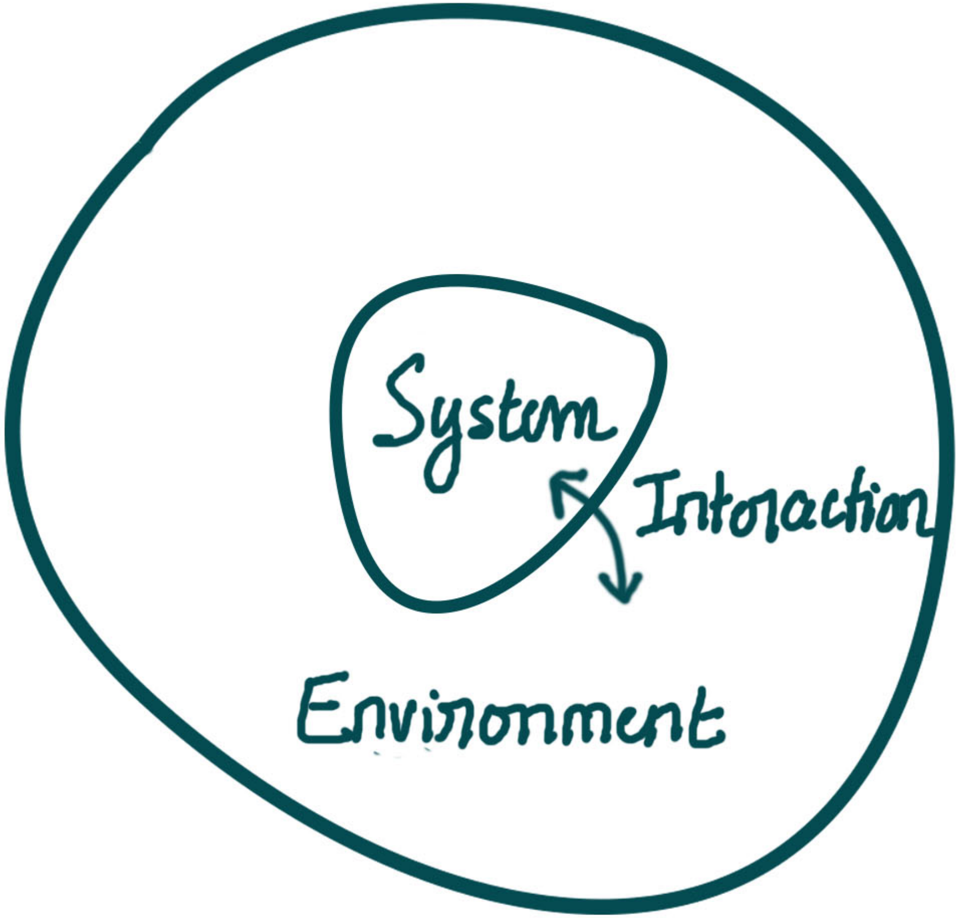
Open systems



Open systems



- In general very difficult
  ex: Kondo problem

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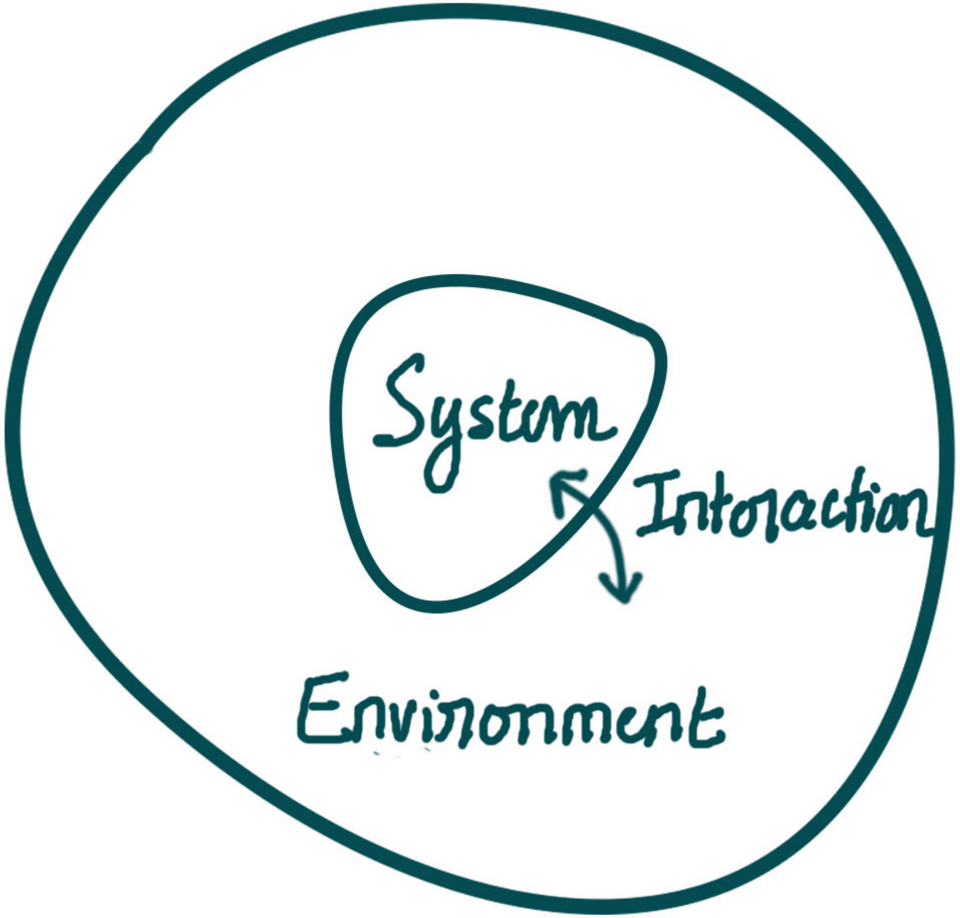

- In general very difficult
  ex: Kondo problem
- Simplifying hypothesis
  ↳ Markov property

GKSL equation (Gorini, Kossakowski, Sudarshan, Lindblad)

Map s.t: * completely positive * trace-preserving

* Markov property

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$\hookrightarrow$:= Quantum channel Φ

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↳ := Quantum channel Φ

$\mathcal{L}(\rho)$ <u>generator</u> of the map : $e^{\mathcal{L}t}[\rho] := \Phi[\rho]$

$$\boxed{\frac{d}{dt}\rho = -i[H,\rho] + \sum_i \gamma_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\} \right)}$$

↳ "Non-Hermitian" complex spectrum

Lifting the Lindblad equation

Stinespring's theorem:

Always possible to enlarge the space enough to get Φ as a reduced unitary evolution

$$\Phi(\rho) = \mathrm{Tr}_B\left(U \rho \otimes \rho_E U^\dagger\right)$$

"Noise"

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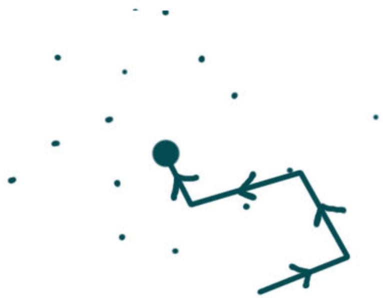
"Noise"

Stochastic unitary $\xrightarrow{\text{average}}$ Quantum channel

Stochastic Hamiltonian $\rightarrow$ Lindblad

Classical

Brownian motion



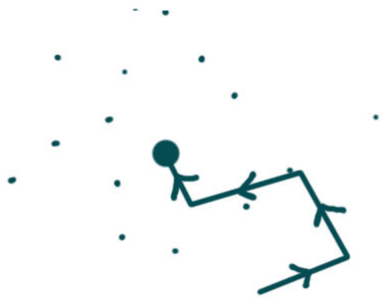
Langevin eq:

$$\boxed{m\,dv = -\alpha v\,dt + dB_t}$$

$\frac{dB_t}{dt}$ White noise

Classical

Brownian motion



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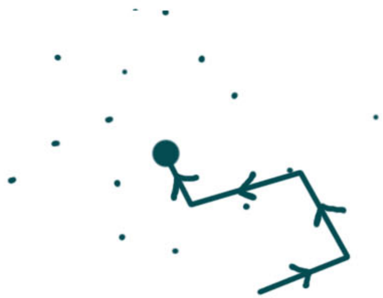

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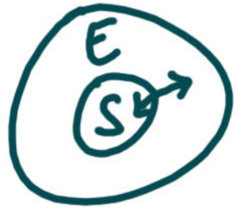
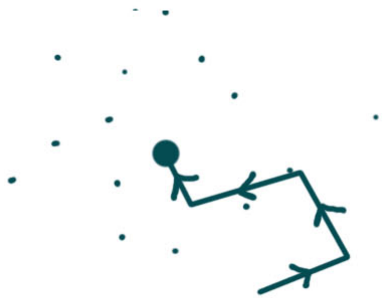


Stochastic Hamiltonian:

$$\boxed{|\psi_{t+dt}\rangle = e^{-i\,dH_t}|\psi_t\rangle}$$

Classical

Brownian motion



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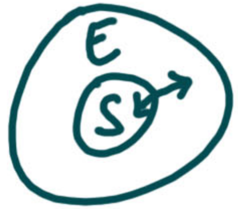

Stochastic Hamiltonian:

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$\downarrow$ average

Lindblad eq:

$$\boxed{\frac{d}{dt}\rho = \mathcal{L}(\rho)}$$

Lift

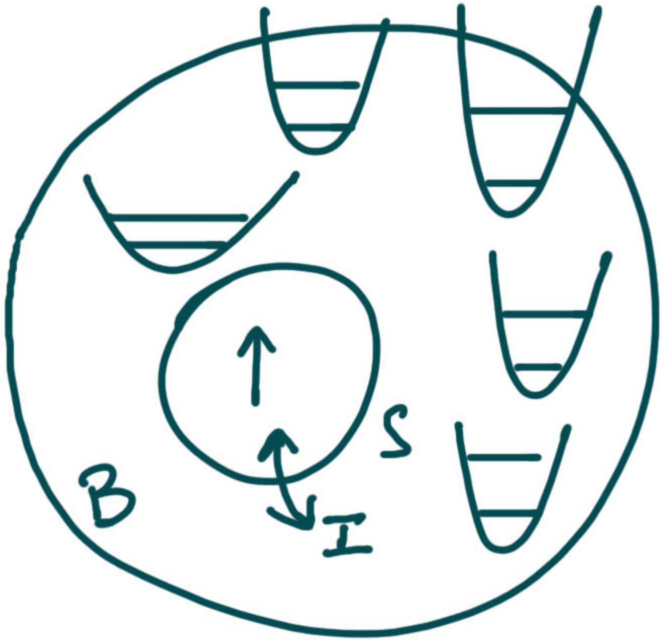
Disclaimer: Similar tools but

$\neq$ weak measurements

Why should you care?

- Useful to have unitary evolution
- Rich physics beyond the mean
- Stochastic process in the Hilbert space

Spin-boson (Caldeira-Leggett)



$$H = H_S + H_B + H_I$$

$$H_S = 0$$

$$H_B = \int \frac{dk}{2\pi} \, \varepsilon_k \, a_k^\dagger a_k$$

$$H_I = \int \frac{dk}{2\pi} \, z_k \left(a_k^\dagger \sigma_S^- + a_k \sigma_S^+ \right)$$

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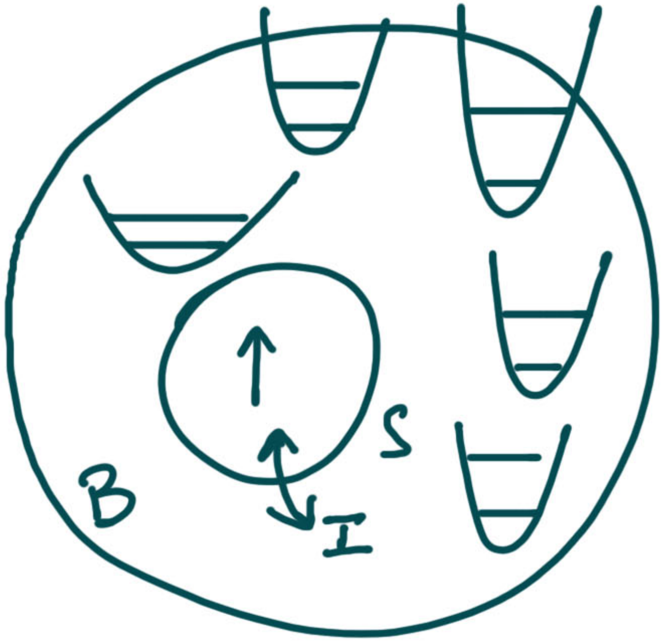

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$$|\psi_{t+dt}\rangle = e^{-idH_t} |\psi_t\rangle$$

$\downarrow$ Markov hypothesis

$$\boxed{dH_t = \sigma_S^+ d\overline{W}_t + \sigma_S^- dW_t}$$

$$dW_t = dB_t^1 + i\, dB_t^2$$

$$d\overline{W}_t = dB_t^1 - i\, dB_t^2$$

$$dH_t = \sigma_s^+ d\overline{W_t} + \sigma_s^- dW_t$$

$$|\psi_{t+dt}\rangle = e^{-idH_t} |\psi_t\rangle$$

$$\Big\downarrow \text{average}$$

$$\frac{d\rho}{dt} = \sigma^+ \rho_t \sigma^- - \frac{1}{2}\{\sigma^-\sigma^+, \rho_t\}$$

$$+ \sigma^- \rho_t \sigma^+ - \frac{1}{2}\{\sigma^+\sigma^-, \rho_t\}$$

$$dH_t = \sigma_s^+ d\overline{W}_t + \sigma_s^- dW_t$$

$$|\psi_{t+dt}\rangle = e^{-i\,dH_t} |\psi_t\rangle$$

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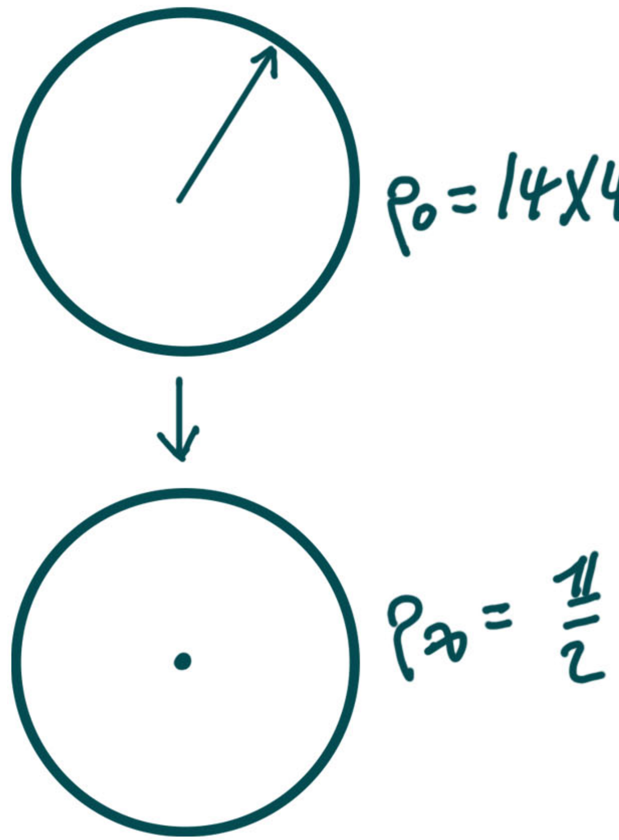
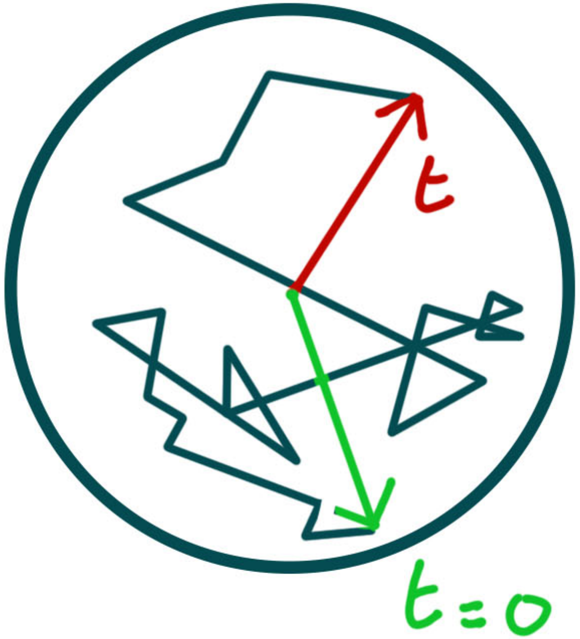
$$\frac{d\rho}{dt} = \sigma^+ \rho_t \sigma^- - \frac{1}{2}\{\sigma^-\sigma^+, \rho_t\} + \sigma^- \rho_t \sigma^+ - \frac{1}{2}\{\sigma^+\sigma^-, \rho_t\}$$



Decoherence towards the centre of the Bloch sphere.

Stochastic picture

Ergodic random walk on the Bloch sphere



No decoherence at the stochastic level

$$\text{Ex:} \quad \langle \mathrm{tr}(\rho S^j)^2 \rangle = 1/3 \qquad j = x, y, z$$

Stationary distribution

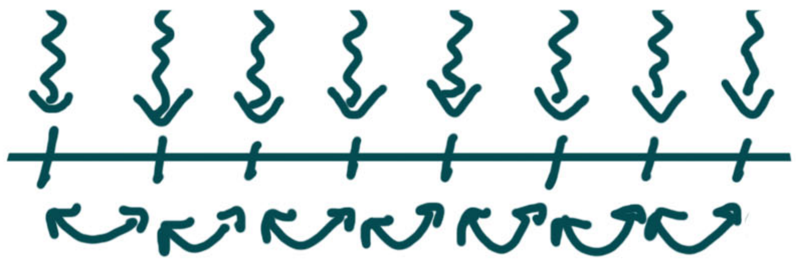
$$Z(A) := \langle \exp(\operatorname{tr} A \rho) \rangle$$

$$= \int_{U \in SU(2)} d\eta(U)\, e^{\operatorname{tr}(A U^{\dagger} \rho_0 U)}$$

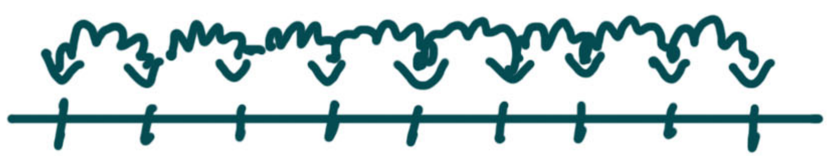
$\hookrightarrow$ The momenta probe the structure of the invariants of $SU(2)$

Many-body systems

Stochastic dephasing model



Quantum exclusion processes.



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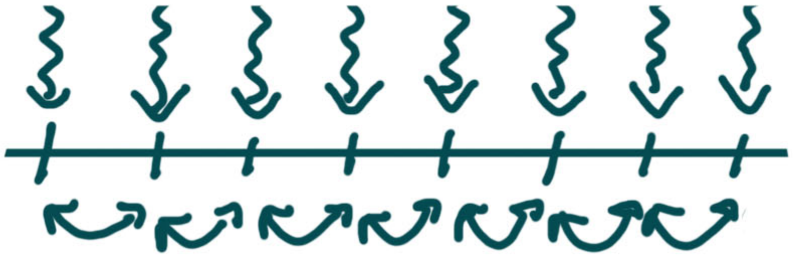

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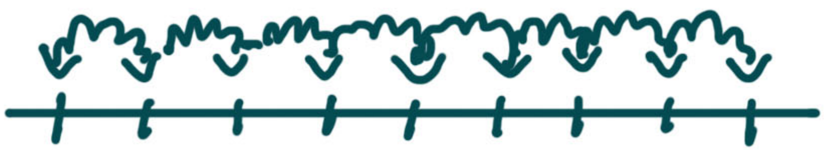

- Diffusive transport
- Rich structure beyond the mean
- First step towards quantum MFT?

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Thank you!