# Exceptionals Points from a Hybrid Physical System:

Squeezing and Anti-Squeezing.

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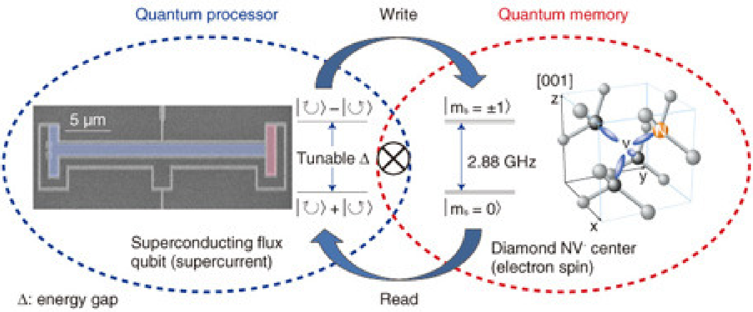
March $23^{\mathrm{th}}$ 2021.

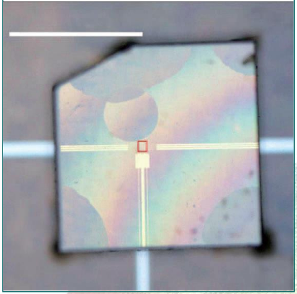
# The hybrid model.

| | Controllability, scalability | Memory time (coherence time) |
|---|---|---|
| **Superconducting qubits** | Good (various circuit designs) | Poor (coupling to noise environment) |
| **NV⁻ centers in diamond** | Poor (weak coupling between spins) | Good (isolated from external noise) |



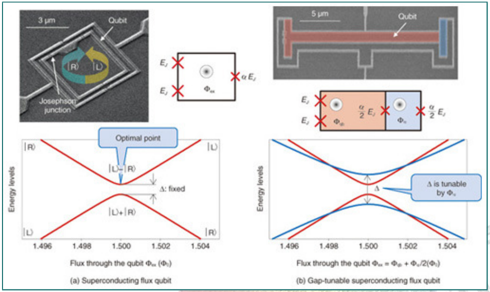

Zhu et. al, Nature**478**, 211 (2011).

# The hybrid model


Cristal de diamante colocado sobre un qubit

$$\mathbf{H} = \mathbf{H}_{\mathbf{qb}} + \mathbf{H}_{\mathbf{NV}} + \mathbf{H}_{\mathbf{int}}.$$

# The Superconducting Flux Qubit.




$$\mathbf{H}_{qb} = \epsilon\, s_x + \Delta\, s_x.$$

$$\begin{pmatrix} s_z \\ s_x \\ s_y \end{pmatrix} = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_z \\ \sigma_x \\ \sigma_y \end{pmatrix}.$$

$$\downarrow$$

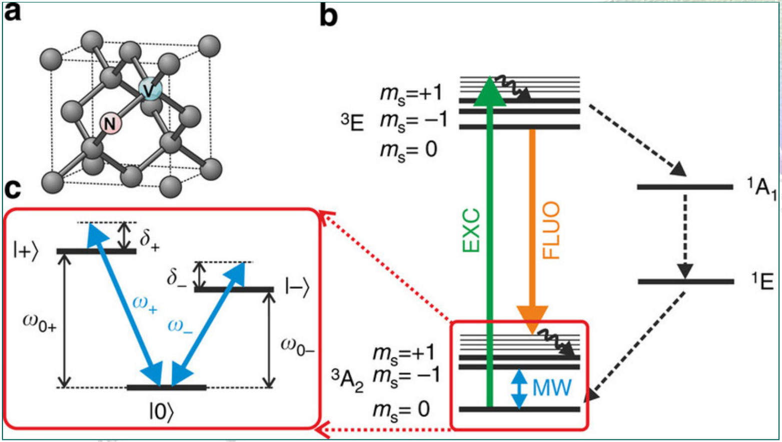
$$\mathbf{H}_{qb} = \frac{1}{2} E_{qb} \sigma_z.$$

$$\mathbf{E}_{qb} = \sqrt{\epsilon^2 + \Delta^2}, \quad \mathbf{c} = \epsilon / E_{qb}, \quad \mathbf{s} = -\Delta / E_{qb}.$$

$$[s_i, s_j] = \epsilon_{ijk} s_k, \quad [\sigma_i, \sigma_j] = \epsilon_{ijk} \sigma_k \quad (\mathrm{su}_2).$$

# The $NV^{-1}$ Centers in diamond.

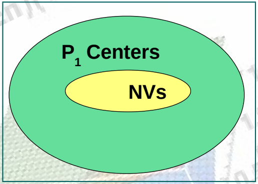
$$H_{NV} = D\, S_z^2 + E\,(S_x^2 - S_y^2).$$



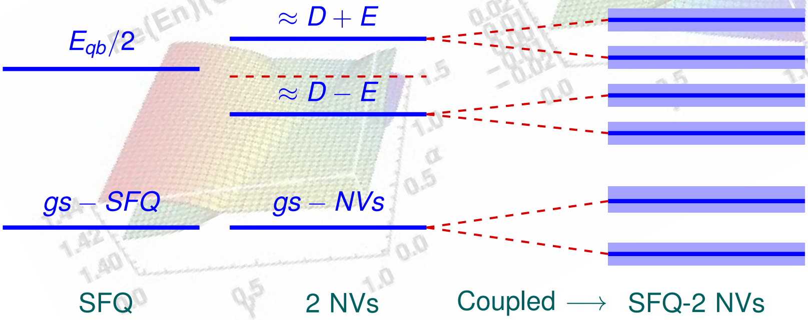
# NV$^{-1}$-FSQ Interaction.

$$\mathbf{H}_{int-qs} = 2g \ ( c \, \sigma_z + s \, \sigma_x ) (\alpha S_+ + S_-) \quad \Leftarrow$$

effective term




R. Ramírez, M. R., D. Tielas, Eur. Phys. J. D, 74, 193 (2020).



# Symmetries of **H**.

Parity Symmetry

$$\mathcal{P}\mathbf{S}\mathcal{P}^{-1} = \mathbf{S}$$

Time Reversal Symmetry

$$\mathcal{T}\mathbf{S}\mathcal{T}^{-1} = -\mathbf{S}, \quad \mathbf{i} \rightarrow -\mathbf{i}.$$

$$H = H_{NV} + H_{qb} + H_{int}.$$

non-T Symmetry

$$H_{NV} = D\, S_z^2 + E\,(S_x^2 - S_y^2) \quad \rightarrow \quad H_{NV}$$

$$H_{qb} = \frac{1}{2} E_{qb}\sigma_z \quad \rightarrow \quad -H_{qb}$$

$$H_{int} = 2g(c\sigma_z + s\sigma_x)(\alpha S_+ + S_-) \quad \rightarrow \quad -2g(c\sigma_z + s\sigma_x)(\alpha S_- + S_+).$$

# Symmetries of **H**.

## Similarity Transformation.

$H^\dagger = H^T \Rightarrow H$ and $H^\dagger$ are iso-spectral operators.

$$\left.\begin{array}{ll} H & = \widetilde{P} J \widetilde{P}^{-1} \\ H^T & = \overline{P} J \overline{P}^{-1}. \end{array}\right\} \Rightarrow H = \mathcal{S} H^T \mathcal{S}^{-1}, \quad \mathcal{S} = \widetilde{P}\overline{P}^{-1}.$$

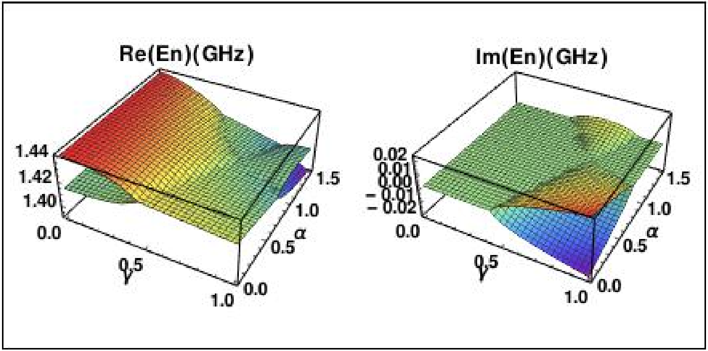
## The Basis.

$$\begin{array}{rcl} |k_{qb}, N_S, k_S\rangle & = & |k_{qb}\rangle \otimes |N_S, k_S\rangle. \\ \mathcal{P} \mid k_{qb}, N_S, k_S\rangle & = & (-1)^{k_S + k_{qb}} \mid k_{qb}, N_S, k_S\rangle. \end{array}$$

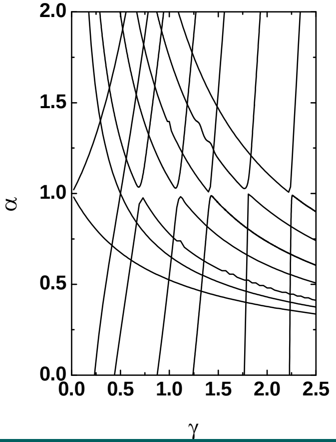
$$H = \left(\begin{array}{c|c} X_{++} & 0 \\ \hline 0 & Y_{--} \end{array}\right)$$

# Spectrum $SFQ - 2NVs$.

$D = 2.88[Ghz],\ E = 0.026[Ghz],\ \Delta = 2D,\ \gamma = g/E.$



## $D = 2.88[Ghz],\ E = 0.026[Ghz],\ \gamma = g/E.$




Defining $d = \left(1 - \frac{\Delta}{2D}\right)\frac{D}{E}$, from left to right, the curves correspond to values of $d = \ 1,\ 0.5,\ 0,\ -1.5,\ -3,\ -6,\ -9,$ respectively.

# Time Evolution and Green Function.

## Green Function

$$\mathbf{G}(\omega) = (\omega \mathbf{I} - H)^{-1},$$

## Fourier Transform G($\omega$)

$$\mathcal{F}(t) \quad = \quad \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi}\, \mathbf{G}(\omega) \mathrm{e}^{-\mathbf{i}\omega t}.$$

## Transition Matrix

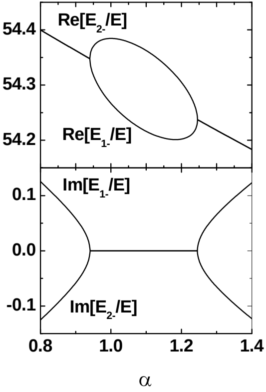
$$\mathrm{P}(t) = \mathcal{N}^2(t)\, \mathcal{F}^\dagger(t) \mathcal{F}(t)$$

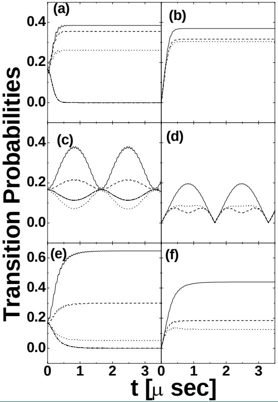
Its elements give the $p(t)$ among states.

$$\mathcal{N}(t) = \left(\mathrm{Tr}(\mathcal{F}^\dagger(t).\mathcal{F}(t))\right)^{-1/2}.$$

# Transition Probabilities SFQ-2NVs

$D = 2.88[Ghz],\ E = 0.026[Ghz],\ g = 0.02[Ghz],$
$\{\ |0{,}0\rangle,\ |1,1\rangle,\ |0,2\rangle, |1,0\rangle,\ |0,1\rangle,\ |1,2\rangle\}.$



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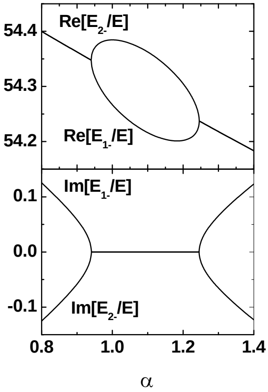
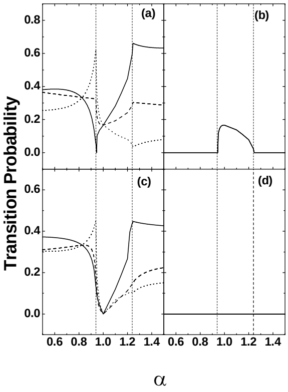


# Non Exponential Decay.

$$\mathbf{G}(\omega) = \tilde{P}^{-1}\mathbf{G_0}(\omega)\tilde{P}, \quad \mathbf{G_0}(\omega) = (\omega \mathbf{I} - J)^{-1}.$$

$$\mathcal{F}(\mathrm{t}) = \tilde{P}^{-1}\mathcal{F}_0(\mathrm{t})\tilde{P}, \quad |I(\mathrm{t})\rangle = \mathcal{N}(\mathrm{t})\ \mathcal{F}(\mathrm{t})|I(0)\rangle.$$

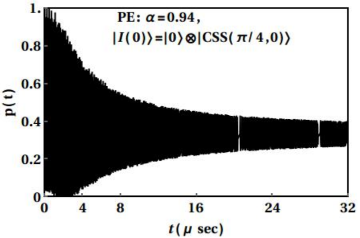
$$\mathcal{F}_0(\mathrm{t}) = \begin{pmatrix} e^{-\mathbf{i}E_{1-}t} & i e^{-iE_{1-}t}\, t & 0 & 0 & 0 & 0 \\ 0 & e^{-\mathbf{i}E_{1-}t} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-\mathbf{i}E_{3-}t} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\mathbf{i}E_{1+}t} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\mathbf{i}E_{2+}t} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\mathbf{i}E_{3+}t} \end{pmatrix}.$$

## As an Example: $|I(0)\rangle = |0\rangle \otimes |CSS(\pi/4,0)\rangle$.

$$\mathrm{p(t)} = |\langle I(t)|I(0)\rangle|^2.$$

$$\begin{aligned}\mathrm{p(t)} \quad = \quad & \Big(a_2\, t^2 - t\, \left(a_{11} \sin(\omega_1 t) + a_{12} \sin(\omega_2 t)\right) + \\ & a_{01}\, \cos(\omega_1 t) + a_{02}\, \cos(\omega_2 t) + \\ & a_{03}\, \cos(\omega_3 t) + a_0 \Big) \, / \\ & \quad \Big(b_2\, t^2 - t\, b_1 \sin(\omega_1 t) + b_{01} \cos(\omega_1 t) + b_0 \Big)\,,\end{aligned}$$

$$\omega_1 = E_{1-} - E_{3-}\,,\; \omega_2 = E_{1-} - E_{1+},\; \omega_3 = E_{3-} - E_{1+}$$



# Survival Probability.

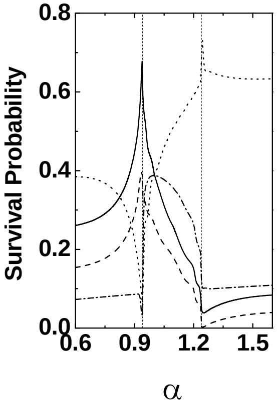
$$|I(0)\rangle = |I\rangle_{qb} \otimes |I\rangle_{NV}.$$

$$|I(t)\rangle = \mathcal{N}(t)\ \mathcal{F}|I(0)\rangle,$$

$$\mathrm{p(t)} = |\langle I(0)|I(t)\rangle|^2,$$

$$|I(0)\rangle = |k_{qb} = 0\rangle \otimes \mathcal{N}_S\ \mathrm{e}^{z_S S_+}\ |\ 0 >,$$

$$z_S = -\mathrm{e}^{\mathbf{i}\phi} \tan(\theta/2).$$



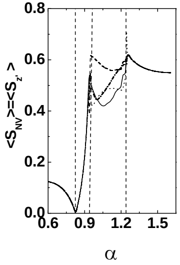
# Squeezing and Anti-squeezing.

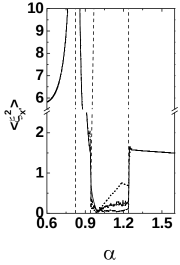
$$[S_{x'}, S_{y'}] = \mathbf{i} S'_z,$$

$$\Delta^2 S_{x'}\ \Delta^2 S_{y'} \ \geq\ \frac{1}{4} |\langle S_{z'} \rangle|^2.$$

$$\zeta^2_{x'} = \frac{2\Delta^2 S_{x'}}{|\langle S_{z'} \rangle|},\ \zeta^2_{y'} = \frac{2\Delta^2 S_{y'}}{|\langle S_{z'} \rangle|}.$$

Squeezing Parameter



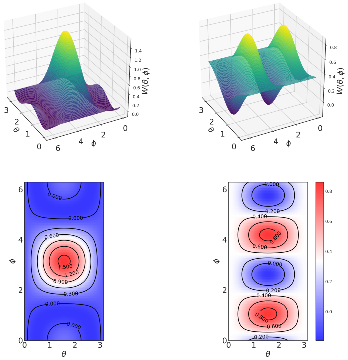



## Schödinger-cat states.

$$|I(0)\rangle = |CS(\pi/2,0)\rangle -> |\Psi(\phi \approx \pi/2)\rangle$$

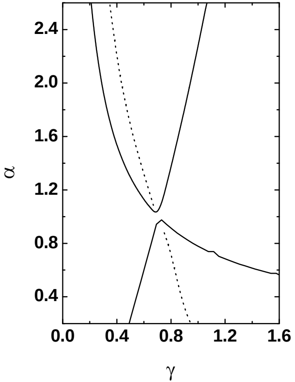
$$|\Psi\rangle = \frac{1}{\sqrt{2}}|I(\theta,\phi)\rangle + \frac{1}{\sqrt{2}}|I(\pi-\theta,\phi+\pi)\rangle,$$

$$\langle\Psi|\mathbf{S}|\Psi\rangle = 0$$

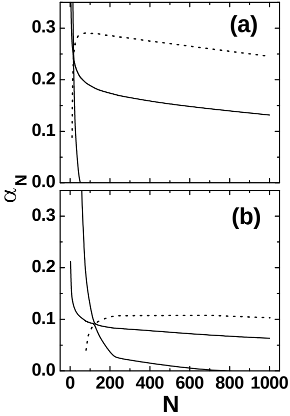


## Large number of NVs.

SFQ-2NVs




SFQ-N NVs.



# Conclusions and Perspective.

- Appearance of EPs in hybrid systems.
- Non-exponential decay in the vecinity of EPs.
- Generation of Steady Robust States, i.e. Schödinger Cat States.
- Next...Non-hermitian systems at finite temperature.

# Thanks!

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