Quantum first detection passage time

Eli Barkai

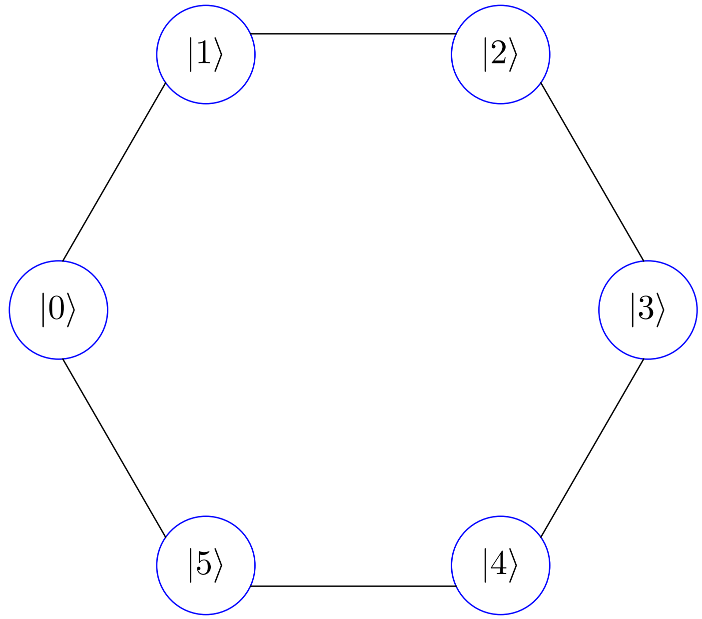
Friedman, Kessler, Mualem, Thiel

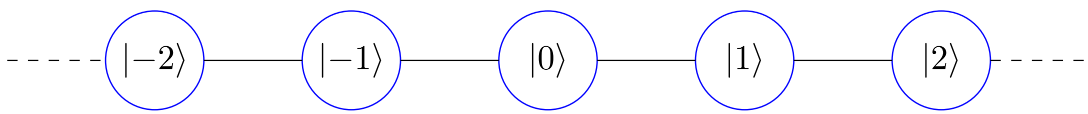
PRE 95, 032141 (2017)

PRL 120, 040502 (2018)

PRR 102, 02210 (2020)

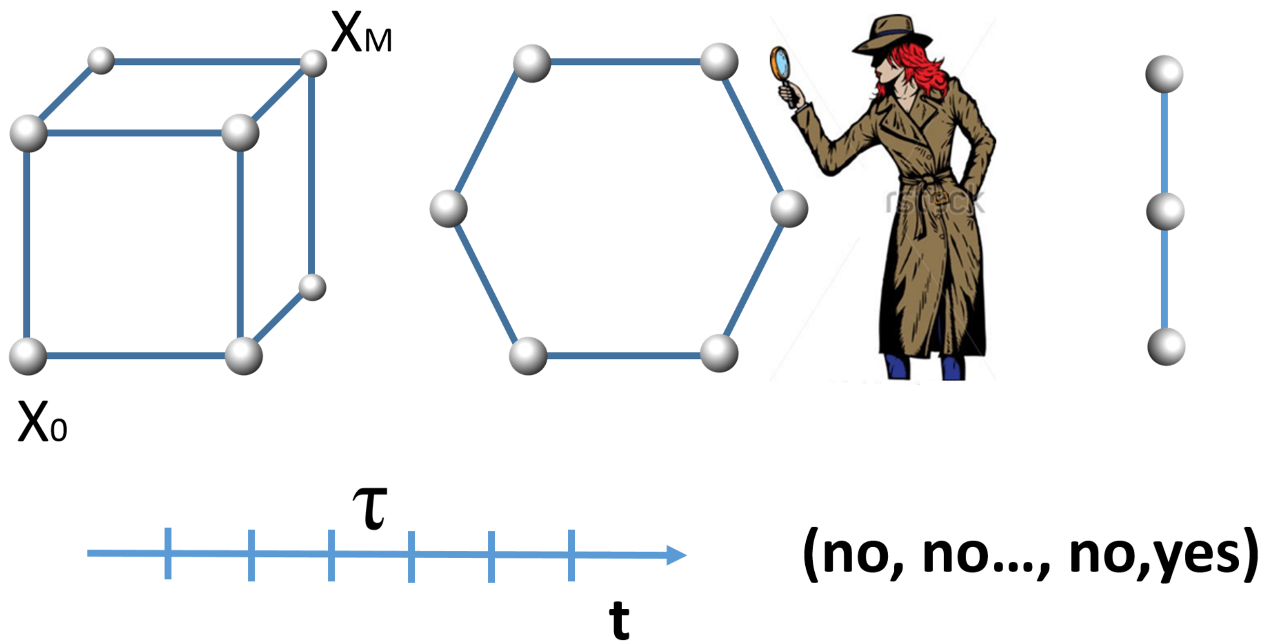
Quantum Walks, $U = \exp(-iH\tau)$





$$H = -\gamma \sum_i \left(|i\rangle\langle i+1| + |i+1\rangle\langle i| \right)$$

Measurement protocol



Quantum computing: is the quantum search superior to the classical random walk? How to choose τ? optimal sampling?

Ambainis et al (2001), Krovi and Brun (2008), Grunbaum et al (2013) Dhar et al (2015)

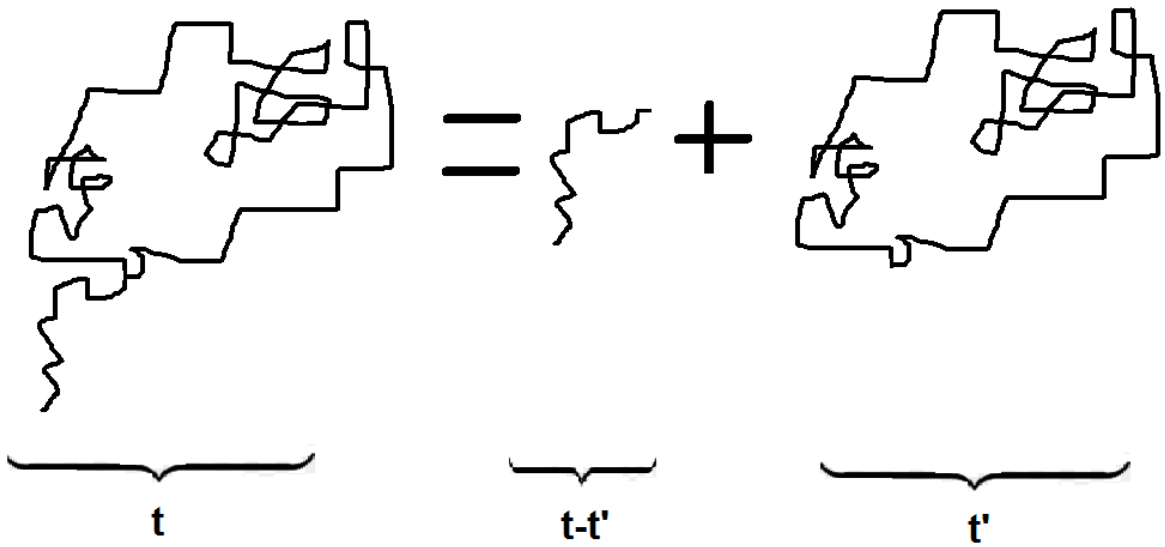
First Detection Time

- We get the string (no, no, ... yes) and in the n-th entry a yes.
- $n\tau$ is the random first detection event.

- What is non-trivial is that the measurement process "collapses" the wave-function, setting $\psi(x_M) = 0$.

- In operator language, we "project out" the x_M component of the state.

Classical First Passage



Particles arriving at x at time t, first arrived at x some earlier time $t-t'$ and returned there after t' additional steps.

Quantum Renewal Equation

- ϕ_n amplitude of first detection probability (Dhar).
- $F_n = |\phi_n|^2$ Prob. of first detection in the n-th attempt.

$$\langle x_M | U(n\tau) | \psi_{in} \rangle = \sum_{j=1}^{n} \langle x_M | U\left[(n-j)\tau\right] | x_M \rangle \phi_j$$

- For example: $\phi_1 = \langle x_M | U(\tau) | \psi_{in} \rangle$

$$\phi_2 = \langle x_M | U \left(1 - |x_M\rangle\langle x_M|\right) U | \psi_{in} \rangle.$$

Friedmann, DK, EB **PRE** (2017)

Detection probability

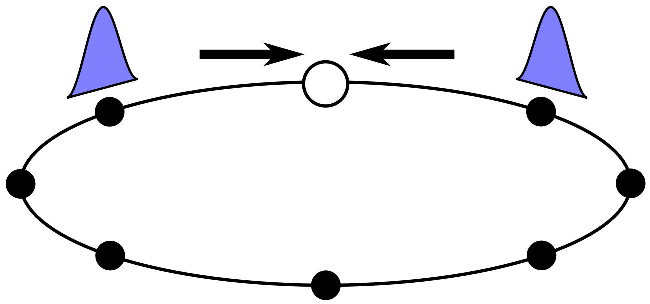
Even for finite space detection probability P_{det} can be less than one.

Quantum searches in these cases are suboptimal compared with classical search.

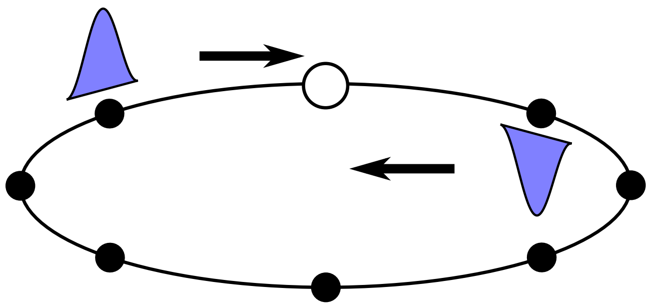
We will therefore consider

$$P_{det} = \sum_{n=1}^{\infty} F_n = \sum_{n=1}^{\infty} |\phi_n|^2$$

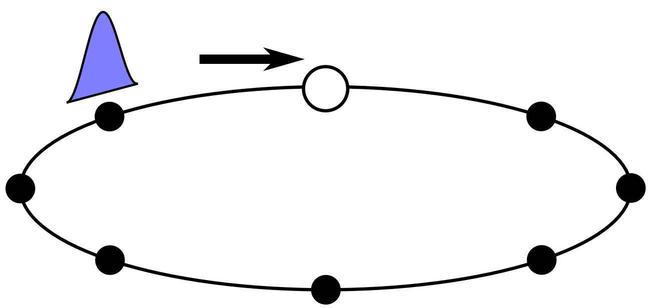
Friedmann, DK, EB **PRE** (2017)



Constructive interference

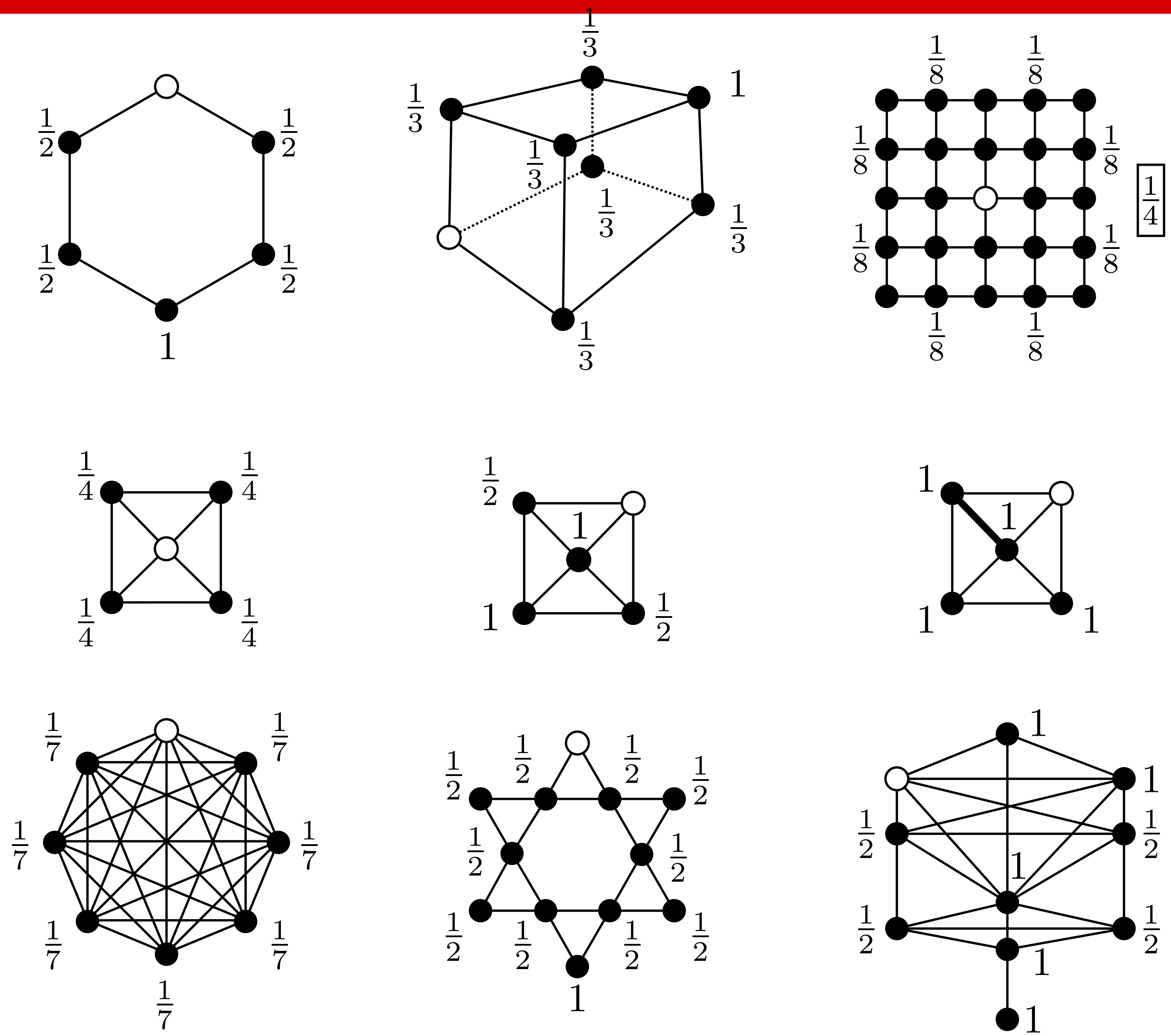


Destructive interference



$$P_{\text{det}} \leq \frac{1}{2}$$

Detection probability P_{det}



Detection probability

- Start in an initial state

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}}\left(|r_i\rangle + e^{i\delta}|r_j\rangle\right)$$

- From quantum renewal equation detection amplitudes sum up

$$\phi_n^{\psi_{in}\to x_M} = \frac{1}{\sqrt{2}}\left(\phi_n^{r_i\to x_M} + e^{i\delta}\phi_n^{r_j\to x_M}\right).$$

- If states $|r_j\rangle$ and $|r_j\rangle$ are equivalent with respect to detection. Then:

$$F_n^{\psi_{in}\to x_M} = (1+\cos\delta)F_n^{r_i\to x_M}$$

- Since $\sum_{n=1}^{\infty} F_n^{\psi_{in}\to x_M} \le 1$ we have

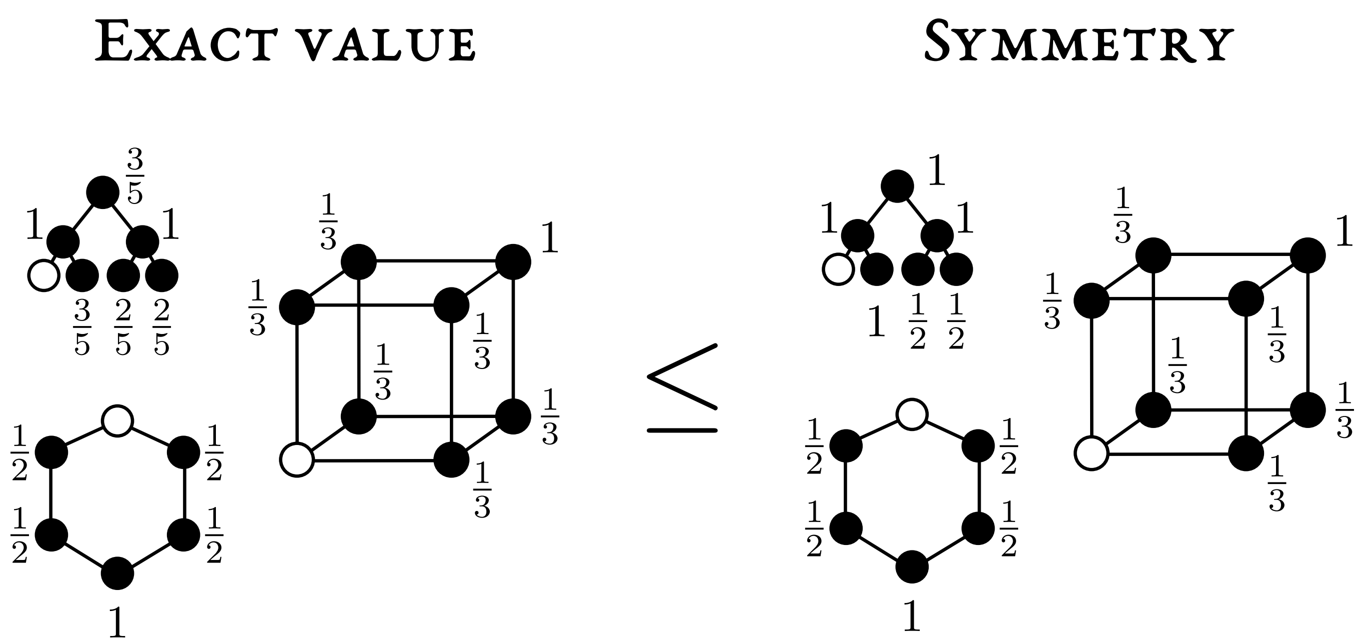
$$(1+\cos\delta)\sum_{n=1}^{\infty} F_n^{r_i\to x_M} \le 1.$$

Choose $\delta = 0$ so $P_{det}(r_i \to x_M) \le \frac{1}{2}$.

- More generally, consider a graph with ν initial sites all equivalent with respect to the detection. We have

$$\boxed{P_{det}^{r_i \to x_M} \leq 1/\nu}$$

- Quantum detection is less efficient if compared with classical random walks.



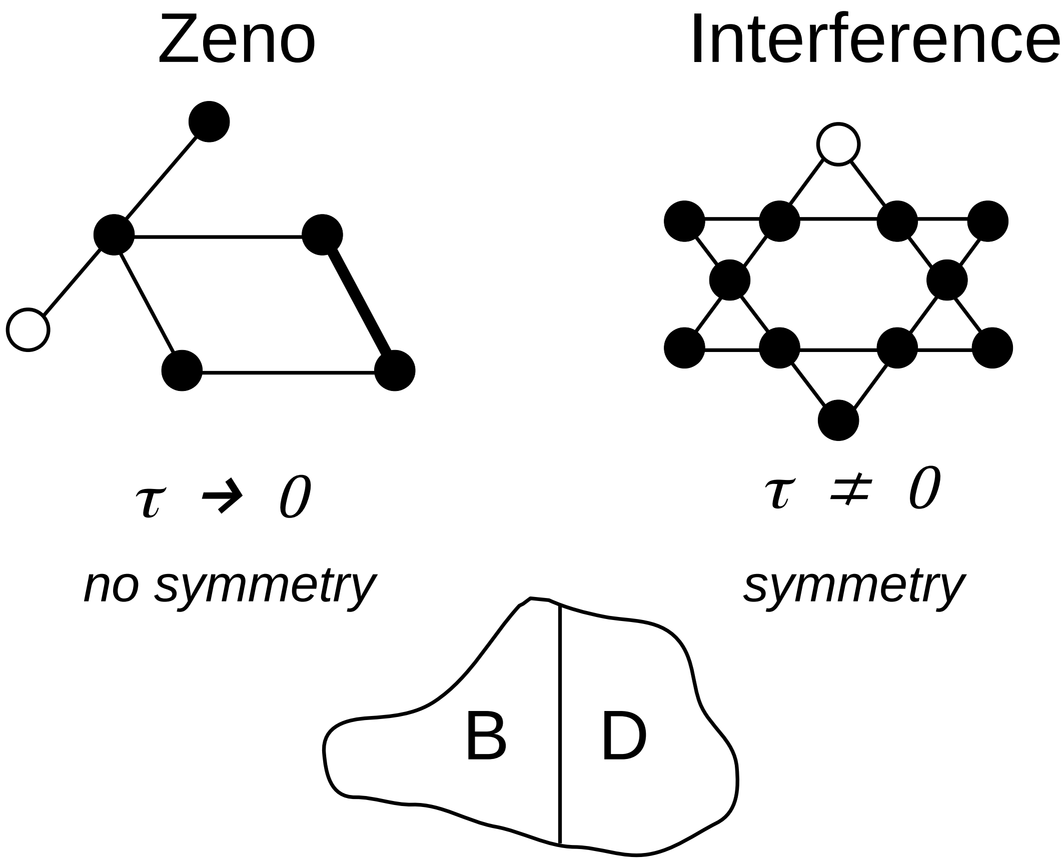
- More generally $P_{det} = \sum_l |\langle \text{bright}_l | \psi_{in} \rangle|^2$ and after the classification of the bright states we find

$$P_{det} = \sum_l{}' \frac{|\sum_{m=1}^{g_l} \langle x_M | E_{l,m} \rangle \langle E_{l,m} | \psi_{in} \rangle|^2}{\sum_{m=1}^{g_l} |\langle x_M | E_{l,m} \rangle|^2}.$$

- In the case of no degeneracy (removal of symmetry) detection is unity.
- Disorder is good for efficient search [Plenio - light harvesting systems].
- P_{det} is τ independent.
- Valid beyond the stroboscopic protocol.

Thiel, Mualem, DK, EB (2019).

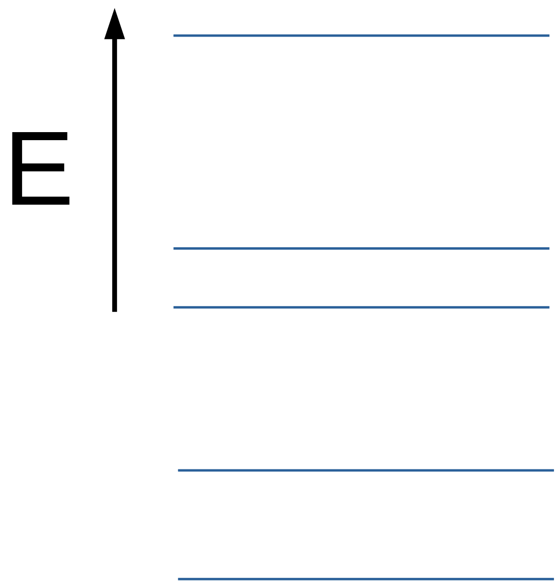
Hilbert space under repeated measurement



Dark and Bright

- A bright/dark state is detected with probability one/zero.
- The Hilbert space can be decomposed into bright and dark subspaces.
- For rapid measurement this is related to the Zeno effect.
- Dark states are related to degeneracy and hence to symmetry.
- For degenerate states: $|\psi\rangle = N\left(\langle x_M|E_2\rangle|E_1\rangle - \langle x_M|E_1\rangle|E_2\rangle\right)$ is dark.
- Also a non degenerate energy level can be dark if $\langle x_M|E_3\rangle = 0$.
- Alan Turing, Sudarshan, Misra, Plenio, Facchi, Paasazio, Caruso, Krovi ....

Sketch- Dark Bright States



g_l	# Bright	# Dark
1	1	0
5	1	4
8	1	7
7	1	6
1	1	0

Start $|\psi\rangle = N\left(\langle x_M|E_2\rangle|E_1\rangle - \langle x_M|E_1\rangle|E_2\rangle\right)$.
Find other orthogonal dark states total $g_l - 1$.
From here get the total detection probability P_{det}.

Uncertainty Relation

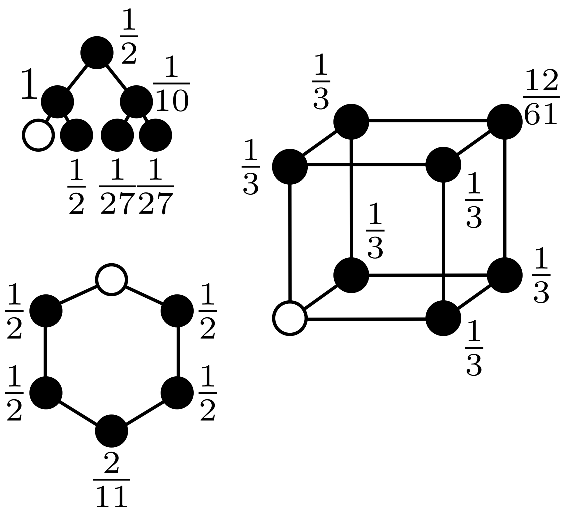
$$\Delta P \text{Var}(H)_M \geq |\langle x_M| [H, D] |\psi_{in}\rangle|^2$$

- ΔP is the deviation of the total detection probability from the initial probability of detection.

$$\Delta P = P_{det} - |\langle \psi_{in}|x_M\rangle|^2.$$

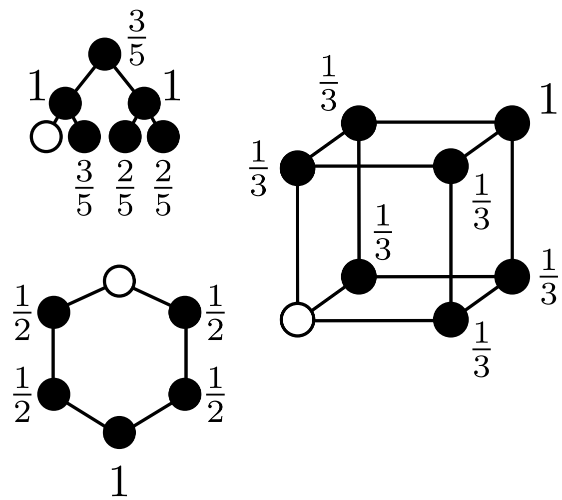
- $\text{Var}(H)_M$ is the variance of the energy in the detected state.
- $D = |x_M\rangle\langle x_M|$ is the measurement projector.
- On the RHS we connect the initial and final states.

Uncertainty



$$\leq$$

Exact value



Ref. Thanks

- Friedman, Kessler, EB *Quantum walks: the first detected passage time problem* **Phys. Rev. E. 95**, 032141 (2017) *Editor's suggestion*.
- Thiel, EB, and Kessler *First detected arrival of a quantum walker on an infinite line* **Phys. Rev. Lett. 120**, 040502 (2018).
- Ruoyu Yin, Ziegler, Thiel, EB *Large fluctuations of the quantum first return time* **Phys. Rev. Res. 1**, 033086 (2019) *Editor's suggestion*.
- Thiel, Mualem, Kessler EB *Uncertainty and symmetry bounds for the total detection probability of quantum walks* **Phys. Rev. Res.** 2, 023392, (2020).
- Quancheng Liu, Yin, Ziegler, EB *Quantum walks: the mean first detected transition time* **Phys. Rev. Res. 2**, 033113 (2020).