Non-Hermitian Physics Online ICTS

## Soliton Steering in Parity-Time Symmetric Nonlinear Couplers

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Department of Physics Indian Institute of Technology Guwahati Date: March 23 , 2021

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## My Research Group at IIT Guwahati

### Current Members



Dipti



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## Our Research Interests

Quantum Optics Nonlinear Optics PT-Physics Quantum Optomechanics Soliton Physics

## Our Recent Contributions to PT-Physics

I sincerely thank Prof. Demetrios Christodoulides for introducing this area to me!



### PHYSICAL REVIEW E 89, 052918 (2014)

### Continuous and discrete Schrödinger systems with parity-time-symmetric nonlinearities

Amarendra K. Sarma,<sup>1,\*</sup> Mohammad-Ali Miri,<sup>2</sup> Ziad H. Musslimani,<sup>3</sup> and Demetrios N. Christodoulides<sup>2</sup>
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**Research Article** 

Vol. 57, No. 5 / 10 February 2018 / Applied Optics 1119

## applied optics

### Highly amplified light transmission in a paritytime symmetric multilayered structure

#### JYOTI PRASAD DEKA AND AMARENDRA K. SARMA\*

Department of Physics, Indian Institute of Technology Guwahati, Guwahati, Assam 781039, India \*Corresponding author: aksarma@iitg.ernet.in

### PHYSICAL REVIEW A 100, 063846 (2019)

### Delayed sudden death of entanglement at exceptional points

Subhadeep Chakraborty<sup>\*</sup> and Amarendra K. Sarma<sup>†</sup> Department of Physics, Indian Institute of Technology Guwahati, Guwahati 781039, Assam, India

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Nonlinear Dyn (2019) 96:565–571 https://doi.org/10.1007/s11071-019-04806-z

ORIGINAL PAPER

## Chaotic dynamics and optical power saturation in parity-time (*PT*) symmetric double-ring resonator

Jyoti Prasad Deka • Amarendra K. Sarma🕞

Nonlinear Dyn https://doi.org/10.1007/s11071-020-05585-8

ORIGINAL PAPER

## Multifaceted nonlinear dynamics in $\mathcal{PT}\text{-symmetric}$ coupled Liénard oscillators

Jyoti Prasad Deka • Amarendra K. Sarma<sup>®</sup> • A. Govindarajan<sup>®</sup> • Manas Kulkarni

## **Optics Letters**

### Tailoring $\mathcal{PT}$ -symmetric soliton switch

A. GOVINDARAJAN,<sup>1,\*</sup> AMARENDRA K. SARMA,<sup>2</sup> O AND M. LAKSHMANAN<sup>1</sup>



## Present Talk: Collaborators

Prof. M. Lakshmanan

Dr. A. Govindarajan

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Dr. Ambaresh Sahoo

## Outline

- PT-symmetry in coupled waveguide system in Optics
- Nonlinear Coupler
- Soliton Steering in Nonlinear Coupler
- Troubles with Conventional Nonlinear Coupler
- Non-Hermitian Physics may be the saviour!
- Soliton Steering in Nonlinear Coupler
- Ultrashort pulse steering dynamics in PT-coupler with Kerr-nonlinearity
- Short pulse steering dynamics in PT-coupler with Saturating nonlinearity
- Conclusions

## Basics of QM

QM system

Described by Hamiltonian H

Otherwise, energy eigenvalues are Complex. H is Non-Hermitian:  $H \neq H^{\dagger}$ .

If Energy is experimentally measurable, energy eigenvalues of the system are Real. H is assumed to be Hermitian:  $H = H^{\dagger}$ .

Even a Non-Hermitian Hamiltonian can exhibit REAL eigenvalue, if H is PT-symmetric!

## PT-Symmetry in Quantum Mechanics

VOLUME 80, NUMBER 24

PHYSICAL REVIEW LETTERS

15 JUNE 1998

### Real Spectra in Non-Hermitian Hamiltonians Having $\mathcal{PT}$ Symmetry

Carl M. Bender<sup>1</sup> and Stefan Boettcher<sup>2,3</sup>

<sup>1</sup>Department of Physics, Washington University, St. Louis, Missouri 63130 <sup>2</sup>Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545 <sup>3</sup>CTSPS, Clark Atlanta University, Atlanta, Georgia 30314 (Received 1 December 1997; revised manuscript received 9 April 1998)

The condition of self-adjointness ensures that the eigenvalues of a Hamiltonian are real and bounded below. Replacing this condition by the weaker condition of  $\mathcal{PT}$  symmetry, one obtains new infinite classes of complex Hamiltonians whose spectra are also real and positive. These  $\mathcal{PT}$  symmetric theories may be viewed as analytic continuations of conventional theories from real to complex phase space. This paper describes the unusual classical and quantum properties of these theories. [S0031-9007(98)06371-6]

PACS numbers: 03.65.Ge, 02.60.Lj, 11.30.Er



### PT SYMMETRY IN QUANTUM PHYSICS: FROM A MATHEMATICAL CURIOSITY TO OPTICAL EXPERIMENTS

LETTERS PUBLISHED ONLINE: 24 JANUARY 2010 | DOI: 10.1038/NPHYS1515



### **Observation of parity-time symmetry in optics**

Christian E. Rüter<sup>1</sup>, Konstantinos G. Makris<sup>2</sup>, Ramy El-Ganainy<sup>2</sup>, Demetrios N. Christodoulides<sup>2</sup>, Mordechai Segev<sup>3</sup> and Detlef Kip<sup>1\*</sup>

One of the fundamental axioms of quantum mechanics is associated with the Hermiticity of physical observables<sup>1</sup>. In the case of the Hamiltonian operator, this requirement not only implies real eigenenergies but also guarantees probability  $(\varepsilon > \varepsilon_{\rm th})$ , the spectrum ceases to be real and starts to involve imaginary eigenvalues. This signifies the onset of a spontaneous *PT* symmetry-breaking, that is, a 'phase transition' from the exact to broken-*PT* phase<sup>7,20</sup>.

QMOptics
$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi$$
 $i\frac{\partial E(x,z)}{\partial z} = \left[-\frac{1}{2k_0n_0}\frac{\partial^2}{\partial x^2} + V(x)\right]E(x,z)$ • PT-symmetric if  $V(x) = V^*(-x)$ • Optical Potential:  $V(x) = k_0(n_R(x) + in_I(x))$ • PT-symmetric if  $n_R(x) = n_R(-x)$  and $n_I(x) = -n_I(-x)$ 

## COUPLED WAVEGUIDES



$$i\frac{da}{dt} - iga + \kappa b = 0, i\frac{db}{dt} + igb + \kappa a = 0$$
$$H = \begin{pmatrix} ig & -\kappa \\ -\kappa & -ig \end{pmatrix}$$
$$E = \sqrt{\kappa^2 - g^2}.$$

$$\begin{split} |1\rangle_{belowEP} &= \begin{pmatrix} 1\\ e^{i\Theta} \end{pmatrix}, |2\rangle_{belowEP} = \begin{pmatrix} 1\\ -e^{-i\Theta} \end{pmatrix} \qquad sin\Theta = g/\kappa \\ |1\rangle_{aboveEP} &= \begin{pmatrix} 1\\ ie^{\Theta} \end{pmatrix}, |2\rangle_{aboveEP} = \begin{pmatrix} 1\\ ie^{-\Theta} \end{pmatrix} \qquad cosh\Theta = g/\kappa \end{split}$$

## Launch condition is important in PT Systems!

c Conventional system



PT-symmetric system below threshold



PT-symmetric system above threshold





## THE EXPERIMENT



Figure 2 | Experimental set-up. An Ar<sup>+</sup> laser beam (wavelength 514.5 nm) is coupled into the arms of the structure fabricated on a photorefractive LiNbO<sub>3</sub> substrate. An amplitude mask blocks the pump beam from entering channel 2, thus enabling two-wave mixing gain only in channel 1. A CCD camera is used to monitor both the intensity and phases at the output.



(a) Front (top) and top (bottom) view of the PT-symmetric coupled system fabricated in LiNbO<sub>3</sub>. (b) Measured (normalized) intensities  $l_{12}$  at the output facet during optical pumping as a function of time *t* (normalized by the time constant  $\tau$  for build-up of gain). The upper/lower panel shows the situation when light is coupled into channel 1 and 2, respectively. Clearly, with increasing gain, the system behaves in a nonreciprocal manner. Blue dashed lines mark the symmetry-breaking threshold. Above that, light is predominantly guided in channel 1—thus experiencing gain—and the intensity in both channels depends solely on the magnitude of the gain. The power evolution is also depicted (last column) at various times.

## NONLINEAR COUPLER



Input pulses appear at different output ports depending on their peak powers

$$\frac{\partial A_1}{\partial z} + \frac{1}{v_g} \frac{\partial A_1}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_1}{\partial t^2} = i\kappa A_2 + i\gamma (|A_1|^2 + \sigma |A_2|^2) A_1,$$
  
$$\frac{\partial A_2}{\partial z} + \frac{1}{v_g} \frac{\partial A_2}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_2}{\partial t^2} = i\kappa A_1 + i\gamma (|A_2|^2 + \sigma |A_1|^2) A_2,$$

## Coupler: Low power optical beams (CW)



Power transfer to the second core occurs in a periodic fashion. The maximum power is transferred at distances such that  $\kappa_e z = m\pi/2$ , where *m* is an integer. The shortest distance at which maximum power is transferred to the second core for the first time is called the *coupling length* and is given by  $L_c = \pi/(2\kappa_e)$ 

## Coupler: Linear Pulse Switching

$$\frac{\partial A_1}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A_1}{\partial T^2} = i\kappa A_2$$
$$\frac{\partial A_2}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A_2}{\partial T^2} = i\kappa A_1$$

For  $L \ll L_D$  with  $L_D = T_0^2/|\beta_2|$ 

$$\begin{split} \kappa(\omega) &\approx \kappa_0 + (\omega - \omega_0)\kappa_1 + \frac{1}{2}(\omega - \omega_0)^2\kappa_2 \\ A_1(z,T) &= \frac{1}{2} \left[ A_0(T - \kappa_1 z)e^{i\kappa_0 z} + A_0(T + \kappa_1 z)e^{-i\kappa_0 z} \right] \\ A_2(z,T) &= \frac{1}{2} \left[ A_0(T - \kappa_1 z)e^{i\kappa_0 z} - A_0(T + \kappa_1 z)e^{-i\kappa_0 z} \right] \end{split}$$

When  $\kappa_1 = 0$ 

$$A_1(z,T) = A_0(T)\cos(\kappa_0 z), \qquad A_2(z,T) = A_0(T)\sin(\kappa_0 z)$$

## Nonlinear Coupler: Quasi-CW Switching



## Nonlinear Coupler: Nonlinear Switching

$$i\frac{\partial u}{\partial \xi} - \frac{s}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 u + Kv = 0$$
$$i\frac{\partial v}{\partial \xi} - \frac{s}{2}\frac{\partial^2 v}{\partial \tau^2} + |v|^2 v + Ku = 0$$

$$\xi = z/L_D, \quad \tau = T/T_0, \quad u = (\gamma L_D)^{1/2} A_1, \quad v = (\gamma L_D)^{1/2} A_2$$





## Troubles with Conventional Nonlinear Coupler



Extremely High power levels are needed for nonlinear switching in Fiber Couplers

## Non-Hermitian Physics may be the saviour!

#### Letter

## **Optics Letters**

## Tailoring $\mathcal{PT}$ -symmetric soliton switch

### A. GOVINDARAJAN,<sup>1,\*</sup> AMARENDRA K. SARMA,<sup>2</sup> O AND M. LAKSHMANAN<sup>1</sup>

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Different configurations of PT-symmetric coupler: (a) Type-1, (b) Type-2 coupler.

## Some notable works on Nonlinear PT-Coupler

PHYSICAL REVIEW A 85, 063837 (2012)

### Optical solitons in $\mathcal{PT}$ -symmetric nonlinear couplers with gain and loss

N. V. Alexeeva,<sup>\*</sup> I. V. Barashenkov,<sup>†</sup> Andrey A. Sukhorukov, and Yuri S. Kivshar Nonlinear Physics Centre, Australian National University, Canberra ACT 0200, Australia



$$iu_t + u_{xx} + 2|u|^2 u = -v + i\gamma u,$$
  

$$iv_t + v_{xx} + 2|v|^2 v = -u - i\gamma v.$$

PHYSICAL REVIEW A 86, 053809 (2012)

### Breathers in $\mathcal{PT}$ -symmetric optical couplers

I. V. Barashenkov,<sup>1,2,3</sup> Sergey V. Suchkov,<sup>1,4</sup> Andrey A. Sukhorukov,<sup>1</sup> Sergey V. Dmitriev,<sup>4</sup> and Yuri S. Kivshar<sup>1</sup> <sup>1</sup>Nonlinear Physics Centre, Research School of Physics and Engineering, Australian National University, Canberra, Australian Capital Territory 0200, Australia <sup>2</sup>Department of Mathematics, University of Cape Town, Rondebosch 7701, South Africa <sup>3</sup>Joint Institute for Nuclear Research, Dubna, Russia <sup>4</sup>Institute for Metal Superplasticity Problems, Russian Academy of Sciences, Ufa 450001, Russia



### Stability of solitons in parity-time-symmetric couplers

Rodislav Driben<sup>1,2,\*</sup> and Boris A. Malomed<sup>2</sup>

<sup>1</sup>Jerusalem College of Engineering—Ramat Beit HaKerem, P.O. Box 3566, Jerusalem 91035, Israel <sup>2</sup>Department of Physical Electronics, School of Electrical Engineering, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel

$$\begin{split} & iu_z + (1/2)u_{tt} + |u|^2 u - i\gamma u + \kappa v = 0, \\ & iv_z + (1/2)v_{tt} + |v|^2 v + i\Gamma v + \kappa u = 0, \end{split}$$



## PT-Nonlinear Coupler

Letter

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## **Optics Letters**

### Tailoring $\mathcal{PT}$ -symmetric soliton switch

### A. GOVINDARAJAN,<sup>1,\*</sup> AMARENDRA K. SARMA,<sup>2</sup> AND M. LAKSHMANAN<sup>1</sup>

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$$i\frac{\partial\Psi_1}{\partial\zeta} + \frac{1}{2}\frac{\partial^2\Psi_1}{\partial\tau^2} + |\Psi_1|^2\Psi_1 + \kappa\Psi_2 = i\Gamma\Psi_1, \qquad (1)$$

$$i\frac{\partial\Psi_2}{\partial\zeta} + \frac{1}{2}\frac{\partial^2\Psi_2}{\partial\tau^2} + |\Psi_2|^2\Psi_2 + \kappa\Psi_1 = -i\Gamma\Psi_2, \qquad (2)$$



(c) Steering dynamics of solitons in  $\mathcal{PT}$ -symmetric dimers



## Steering Dynamics of Solitons: Type 1



## Steering Dynamics of Solitons: Type 2





type 1  $\mathcal{PT}$  dimer with device length of  $2\pi$ 

### Soliton Propagation Dynamics: Conventional Coupler



32

### Soliton Propagation Dynamics: PT Nonlinear Coupler Type 1





type 1  $\mathcal{PT}$  symmetry configuration

Soliton Propagation Dynamics: PT Nonlinear Coupler Type 2





## Suggested Typical Parameters for Experimental Realizations

$$\beta_2 = -20 \ ps^2/km$$
$$\lambda = 1550 \ nm$$
$$T_0 = 50 \ fs$$
$$L_D = 12.5 \ cm$$
$$\gamma = 10 \ W^{-1}/km$$
$$P_{cr} = 8.56 \ W$$

## Dark Soliton Steering

### **Research Article**

Vol. 37, No. 11 / November 2020 / Journal of the Optical Society of America B 3443



**OPTICAL PHYSICS** 

## Dark soliton steering in $\mathcal{PT}$ -symmetric couplers with third-order and intermodal dispersions

### DIPTI KANIKA MAHATO,<sup>1,\*</sup> A. GOVINDARAJAN,<sup>2</sup> D AND AMARENDRA K. SARMA<sup>1</sup> D

<sup>1</sup>Department of Physics, Indian Institute of Technology Guwahati, Guwahati 781039, Assam, India <sup>2</sup>Centre for Nonlinear Dynamics, School of Physics, Bharathidasan University, Tiruchirapalli 620024, India



## Steering Dynamics of Dark Solitons



## TO SUM UP:

We demonstrate steering dynamics of dark solitons in a parity-time ( $\mathcal{PT}$ )-symmetric nonlinear directional coupler (NLDC) in the presence of third-order dispersion (TOD) and intermodal dispersion (IMD). A complete switch with an excellent efficiency at a very low critical power, even lower compared to the bright soliton switching, has been observed. The numerical results show that both TOD and IMD have no effect on soliton steering in  $\mathcal{PT}$ -symmetric couplers with coupling length  $\pi/2$ . But, as we increase the coupling length to  $2\pi$ , IMD shows marginal effects for dark soliton steering in  $\mathcal{PT}$ -symmetric couplers, while TOD shows no impact. Additionally, we have also studied the phase-controlled switching in  $\mathcal{PT}$ -symmetric couplers with two different coupling lengths and demonstrated its advantage over the power-controlled one. © 2020 Optical Society of America

https://doi.org/10.1364/JOSAB.402606

## Ultrashort Soliton Steering in PT-Nonlinear Coupler

The pulse propagation in a realistic PT-symmetric fiber-coupler is represented by generalised coupled NLSE:

$$\begin{split} &i\frac{\partial u}{\partial\xi} + \sum_{n=2}^{\infty} \delta_n \left( i\frac{\partial}{\partial\tau} \right)^n u - i\Gamma u + \left( 1 + is\frac{\partial}{\partial\tau} \right) \left( u(\xi,\tau) \int_{-\infty}^{\tau} R(\tau-\tau') |u(\xi,\tau')|^2 d\tau' \right) + \kappa v = 0, \\ &i\frac{\partial v}{\partial\xi} + \sum_{n=2}^{\infty} \delta_n \left( i\frac{\partial}{\partial\tau} \right)^n v + i\Gamma v + \left( 1 + is\frac{\partial}{\partial\tau} \right) \left( v(\xi,\tau) \int_{-\infty}^{\tau} R(\tau-\tau') |v(\xi,\tau')|^2 d\tau' \right) + \kappa u = 0. \end{split}$$

- u,v = field envelopes in the bar and cross channel
- $\delta_n$  = Normalised GVD and higher-order dispersion parameters
- s = Self-steepening parameter
- $R(\tau) =$  Nonlinear response function
- $\kappa$  = Normalized linear coupling coefficient
- $\Gamma = \text{Gain}/\text{Loss}$

$$\frac{\partial A}{\partial z} + \frac{1}{2} \left( \alpha(\omega_0) + i\alpha_1 \frac{\partial}{\partial t} \right) A - i \sum_{n=1}^{\infty} \frac{i^n \beta_n}{n!} \frac{\partial^n A}{\partial t^n} \\ = i \left( \gamma(\omega_0) + i\gamma_1 \frac{\partial}{\partial t} \right) \left( A(z, t) \int_0^\infty R(t') |A(z, t - t')|^2 dt' \right)$$

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} = i\gamma \left(|A|^2 A + \frac{i}{\omega_0}\frac{\partial}{\partial T}(|A|^2 A) - T_R A \frac{\partial|A|^2}{\partial T}\right)$$

## Effect of Intrapulse Raman Scattering (IRS)



## Effect of Intrapulse Raman Scattering (IRS)



## Effect of Higher-order dispersion





## Saturating nonlinearity (SNL)



- Critical power of switching  $(P_{cr})$  depends on the input peak power.
- For Silica,  $n_2$  value is relatively low, thus input pulse with high peak power is required for switching from one channel to the other.
- Use of materials such as Semiconductor doped Silica and organic polymers with high  $n_2$  value can decrease the requirement of high input power.

The pulse propagation in a PT-symmetric fiber-coupler with saturating nonlinearity is represented by coupled NLSE:

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + f(|u|^2)u + \kappa v = i\Gamma u \tag{1}$$

$$i\frac{\partial v}{\partial \xi} + \frac{1}{2}\frac{\partial^2 v}{\partial \tau^2} + f(|v|^2)v + \kappa u = -i\Gamma v$$
(2)

- $f(|u|^2) = \text{saturating nonlinearity function} = \frac{|u|^2}{1+s|u|^2}$  (F-Model)
- Eqs. (1) and (2) are solved Numerically using SSFT method.
- All the results shown are for the Type-1 PT-symmetric coupler in the Unbroken regime with  $\kappa(0.1) > \Gamma(0.05)$ .



PT with Saturating Nonlinearity wins!

Coupling length = $\pi/2$ , $\kappa=0.1$	Conventional coupler	Type 1 PT-coupler	Type 2 PT-coupler (not shown here)
Kerr Nonlinearity	0.83	0.38	Incomplete
Saturating Nonlinearity	1.11	0.30	3.65,16.77

# Soliton evolution inside Type-1 and Type-2 PT coupler



## Future Directions and Goals

- To understand the physics of Nonlinear coupler more deeply
- To come up with practical schemes for facilitating experiments
- To go beyond Fiber Coupler
- Soliton Steering and Switching in Quantum Regime?



## **THANK YOU!**