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Non-Hermiticity: a new paradigm for model building in particle physics

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A few quick thank yous

To **the organisers** and (in advance) **to you** for listening in.

To my collaborators:

Jean Alexandre (King’s College London)

Carl M. Bender (University of Washington, St Louis)

John Ellis (King’s College London)

Dries Seynaeve (former PhD student at King’s College London)

New physics?

Despite the success of the Standard Model of particle physics, big questions remain:

- **Origin of neutrino masses**
- **Matter-antimatter asymmetry**
- **Hierarchy problem**
- **Strong CP problem**
- **Dark matter**
- **Dark energy**

Model building strategies

To go beyond the Standard Model of particle physics, we can:

- **Add new degrees of freedom**: extra gauge singlets, extra Higgs doublets, heavy neutrinos, SUSY partners, hidden sectors, …
- **Relax assumptions**: number of spatial dimensions, Lorentz invariance, locality, CPT invariance, ...

Outline

- Why not **non-Hermiticity** as a **model-building strategy**?

- What are the **subtleties**?

- What are some **potential implications**?

A scalar playground

A simple **scalar model** with **real *c*-number Lagrangian parameters**:

[Alexandre, PM & Seynaeve '17]

$$\mathcal{L} = \partial_\nu \phi_i^* \partial^\nu \phi_i - m_i^2 \phi_i^* \phi_i - \mu^2(\phi_1^* \phi_2 - \phi_2^* \phi_1) \qquad i = 1{,}2$$

"Naïve" $\mathcal{PT}$ symmetry if we have a complex scalar and a complex pseudoscalar, i.e.,

$$\mathcal{P}\colon\ \phi_1 \longrightarrow +\phi_1, \qquad \phi_2 \longrightarrow -\phi_2$$
$$\mathcal{T}\colon\ \phi_1 \longrightarrow +\phi_1^*, \qquad \phi_2 \longrightarrow +\phi_2^*$$

A scalar playground: matrix model

Non-Hermitian squared mass matrix: $M^2 = \begin{pmatrix} m_1^2 & \mu^2 \\ -\mu^2 & m_2^2 \end{pmatrix}$

With the $\mathcal{PT}$ symmetry of $\mathcal{L}$ translating into the **pseudo-Hermiticity** of M^2:

$$[P \cdot M^2 \cdot P]_{ij} = [M^2]_{ji} \qquad P = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$$

Mass spectrum: $M_\pm^2 = \frac{m_1^2+m_2^2}{2} \pm \left[\left(\frac{m_1^2-m_2^2}{2}\right)^2 - \mu^4\right]^{1/2} \in \mathbb{R} \text{ if } \eta \equiv \frac{2|\mu^2|}{|m_1^2-m_2^2|} \leq 1$

M^2 is defective at the **exceptional point** at $\eta = 1$.

A scalar playground: matrix model

Eigenvectors $(m_1^2 - m_2^2, \mu^2 > 0)$:

$$\boldsymbol{e}_+ = N\begin{pmatrix} \eta \\ -1+\sqrt{1-\eta^2} \end{pmatrix} \qquad \boldsymbol{e}_- = N\begin{pmatrix} -1+\sqrt{1-\eta^2} \\ \eta \end{pmatrix} \qquad \eta = \frac{2\mu^2}{m_1^2 - m_2^2}$$

- **Not** orthogonal with respect to the Dirac inner product: $\boldsymbol{e}_\pm^* \cdot \boldsymbol{e}_\mp \neq 0$.
- **Not** orthonormal with respect to the $\mathcal{PT}$ inner product: $\boldsymbol{e}_\pm^* \cdot P \cdot \boldsymbol{e}_\pm \ngtr 0$.
- Orthonormal with respect to the $\mathcal{C}'\mathcal{PT}$ inner product: $\boldsymbol{e}_\pm^* \cdot C' \cdot P \cdot \boldsymbol{e}_\pm = 1$.

  [Bender, Brody & Jones ’02; see also Alexandre, Ellis & PM ’20b; cf. Mannheim ‘18]

A scalar playground: matrix model

$$C' = R \cdot P \cdot R^{-1} = \frac{1}{\sqrt{1-\eta^2}}\begin{pmatrix} 1 & -\eta \\ \eta & -1 \end{pmatrix} \qquad R \cdot M^2 \cdot R^{-1} = \widehat{M}^2 = \begin{pmatrix} M_+^2 & 0 \\ 0 & M_-^2 \end{pmatrix}$$

The structure of C' follows from its relation to the **similarity transformation** to the corresponding Hermitian model $\widehat{M}^2 = e^{-Q/2} \cdot M^2 \cdot e^{Q/2}$:

$$C' = e^{-Q} \cdot P \qquad e^{-Q} = C' \cdot P = R^2 = \frac{1}{\sqrt{1-\eta^2}}\begin{pmatrix} 1 & \eta \\ \eta & 1 \end{pmatrix}$$

$$Q = \ln R^2 = -\operatorname{arctanh}(\eta)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

[Alexandre, Ellis & PM ’20b]

But we want to work directly with the **non-Hermitian Hamiltonian** …

Variational procedure

The action is not Hermitian, so

$$\frac{\partial \mathcal{L}}{\partial \phi_i^*} - \partial_\nu \frac{\partial \mathcal{L}}{\partial \partial_\nu \phi_i^*} = 0 \;\not\Leftrightarrow\; \frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\nu \frac{\partial \mathcal{L}}{\partial \partial_\nu \phi_i} = 0$$

Prescription: choose one of these pairs of **Euler-Lagrange equations** to fix the dynamics.

The choices are **physically equivalent**; they are equivalent up to a field redefinition.

[Alexandre, PM & Seynaeve ‘17]

Noether’s theorem

If the **Euler-Lagrange equations** are not mutually consistent, conserved currents **do not** correspond to symmetries of the Lagrangian.

$$\delta\mathcal{L} = \underbrace{\left(\frac{\partial\mathcal{L}}{\partial\phi_i} - \partial_\nu \frac{\partial\mathcal{L}}{\partial\partial_\nu\phi_i}\right)}_{\neq\,0}\delta\phi_i + \delta\phi_i^* \underbrace{\left(\frac{\partial\mathcal{L}}{\partial\phi_i^*} - \partial_\nu \frac{\partial\mathcal{L}}{\partial\partial_\nu\phi_i^*}\right)}_{=\,0} + \partial_\nu j_\delta^\nu$$

The current is conserved if

[Alexandre, PM & Seynaeve ‘17]

$$\delta\mathcal{L} = \left(\frac{\partial\mathcal{L}}{\partial\phi_i} - \partial_\nu \frac{\partial\mathcal{L}}{\partial\partial_\nu\phi_i}\right)\delta\phi_i$$

Noether's theorem

For our scalar model, the **conserved current** is

[Alexandre, PM & Seynaeve '17]

$$j^{\nu} = i[\phi_1^* \partial^{\nu} \phi_1 - (\partial^{\nu} \phi_1^*)\phi_1] - i\,[\phi_2^* \partial^{\nu} \phi_2 - (\partial^{\nu} \phi_2^*)\phi_2]$$

corresponding to $(U(1) \times U(1))$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \longrightarrow e^{i\alpha P} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} e^{+i\alpha} \phi_1 \\ e^{-i\alpha} \phi_2 \end{pmatrix}$$

with

$$\mathcal{L} \longrightarrow \partial_{\nu} \phi_i^* \partial^{\nu} \phi_i - m_i^2 \phi_i^* \phi_i - \mu^2 \left(e^{-2i\alpha} \phi_1^* \phi_2 - e^{+2i\alpha} \phi_2^* \phi_1 \right)$$

$$\delta \mathcal{L} = \left(\frac{\partial \mathcal{L}}{\partial \phi_j} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial \partial_{\nu} \phi_j} \right) \delta \phi_j = 2i\alpha\mu^2 (\phi_1^* \phi_2 - \phi_2^* \phi_1)$$

The Goldstone theorem

The existence of the conserved current is sufficient to ensure **the Goldstone theorem** continues to hold in the case of a **spontaneously broken global symmetry**:

[Alexandre, Ellis, PM & Seynaeve '18]

$$\mathcal{L} = \partial_\nu \phi_i^* \partial^\nu \phi_i + m_1^2 |\phi_1|^2 - m_2^2 |\phi_2|^2 - \mu^2 (\phi_1^* \phi_2 - \phi_2^* \phi_1) - \frac{g}{4} |\phi_1|^4, \qquad m_1^2, m_2^2 > 0$$

$$\left.\begin{aligned} \left.\frac{\partial U}{\partial \phi_1^*}\right|_{\phi_a = v_a} &= \frac{g}{2} |v_1|^2 v_1 - m_1^2 v_1 + \mu^2 v_2 = 0 \\ \left.\frac{\partial U}{\partial \phi_2^*}\right|_{\phi_a = v_a} &= m_2^2 v_2 - \mu^2 v_1 = 0 \end{aligned}\right\} \Rightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \sqrt{2 \frac{m_1^2 m_2^2 - \mu^4}{g m_2^2}} \begin{pmatrix} 1 \\ \frac{\mu^2}{m_2^2} \end{pmatrix} e^{i\alpha}$$

Away from the exceptional point, we have a **single massless, Goldstone mode**.

The Englert-Brout-Higgs mechanism

Gauging the global $U(1)$ symmetry (which is itself subtle):

$$\mathcal{L} = -\frac{1}{4}F_{\nu\rho}F^{\nu\rho} + D_\nu^*\phi_i^* D^\nu\phi_i + m_1^2|\phi_1|^2 - m_2^2|\phi_2|^2 - \mu^2(\phi_1^*\phi_2 - \phi_2^*\phi_1) - \frac{g}{4}|\phi_1|^4$$
$$D_\nu = \partial_\nu - iqA_\nu$$

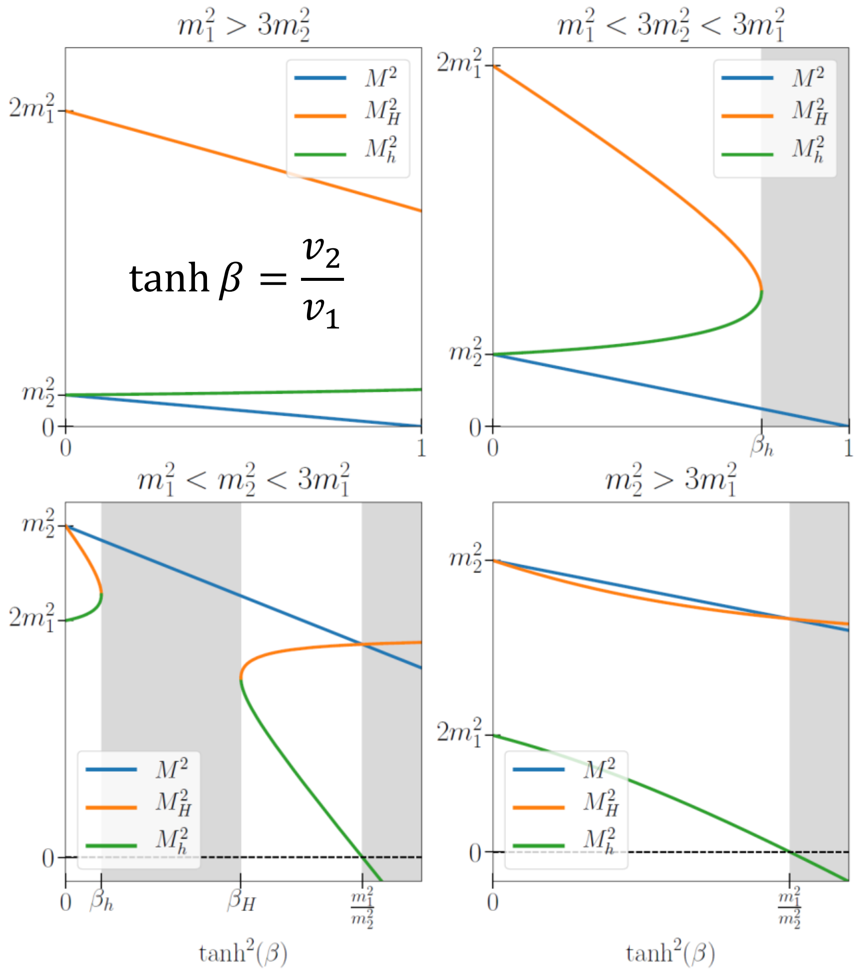
The **Englert-Brout-Higgs mechanism** is still borne out …

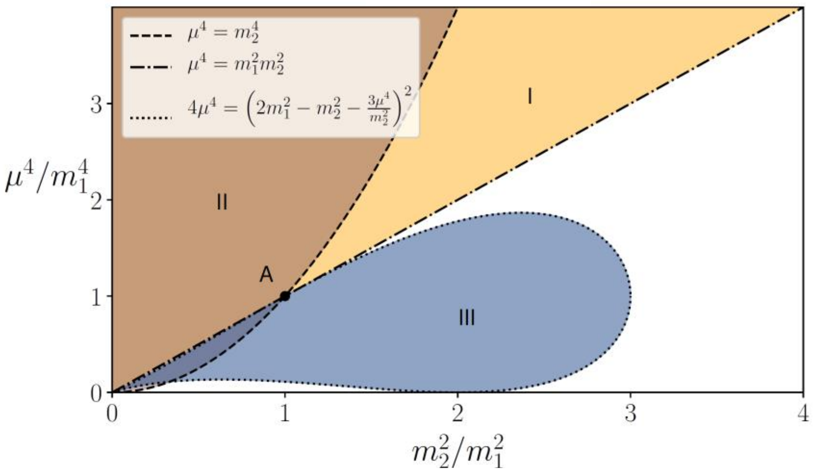
… all the way to the exceptional point:
[Alexandre, Ellis, PM & Seynaeve '19 & '20]

$$M_A^2 = 2q^2\big(|v_1|^2 + |v_2|^2\big) \longrightarrow 2q^2\big(|v_1|^2 + |v_1|^2\big) = \frac{4}{g}\big(m_1^2 - m_2^2\big)\ \text{at}\ \mu^2 = \pm m_2^2$$

The $\mathcal{C}'\mathcal{PT}$ norm of **the Goldstone mode** is ill defined, but its Hermitian norm is defined!
[cf. Mannheim '19, Fring & Taira '20a, '20b & '20c, based on a similarity transformation of this model, for which $|v_2|^2 \longrightarrow -|v_2|^2$.]

(Non-Abelian) Englert-Brout-Higgs mechanism

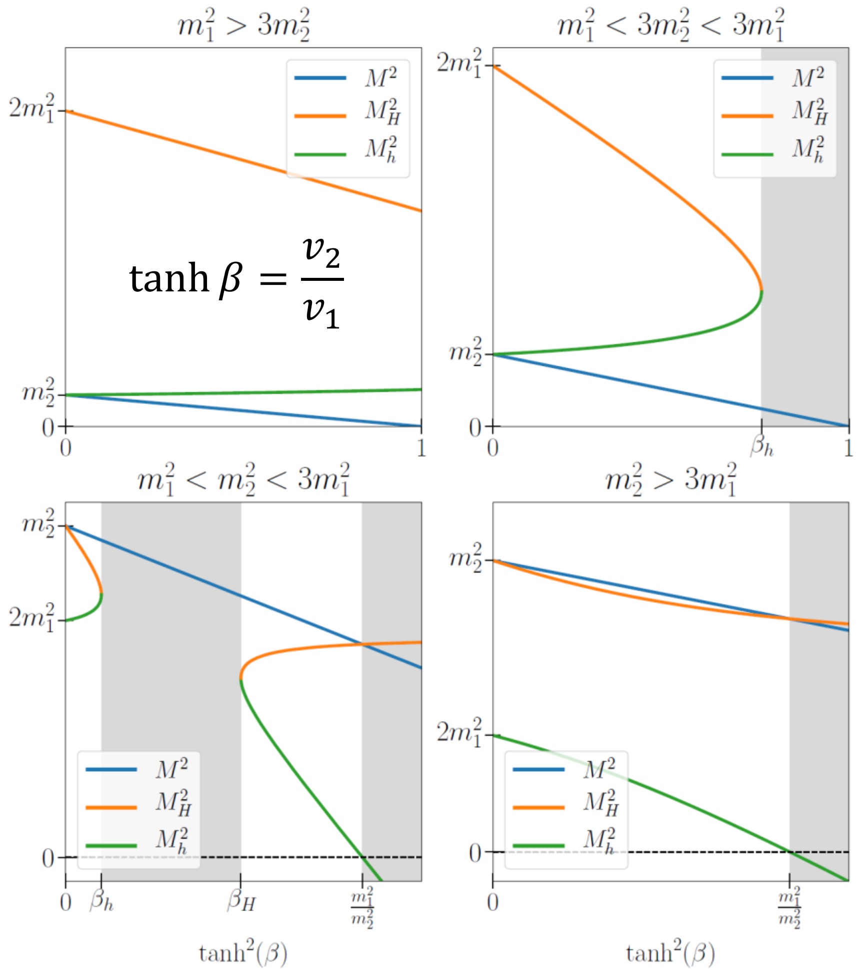




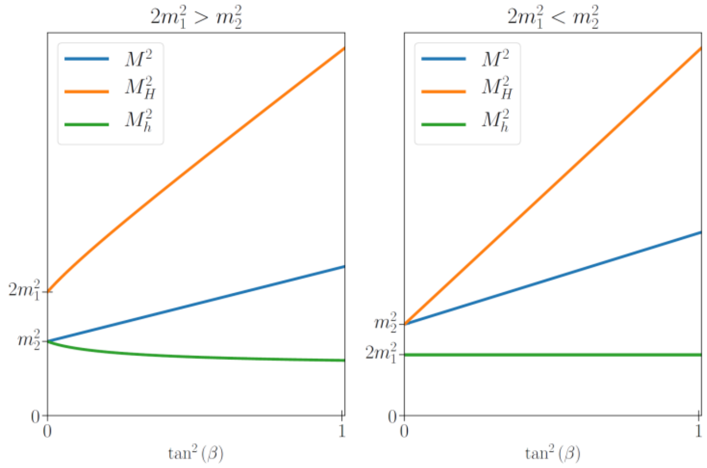
I: symmetric $SU(2) \times U(1)$ phase
II: $\mathcal{PT}$ broken phase ($M^2 < 0$ for $H^\pm, D$)
III: $\mathcal{PT}$ broken phase ($M_h^2, M_H^2 \notin \mathbb{R}$ for h, H)
Unshaded: physical SSB phase

[Alexandre, Ellis, PM & Seynaeve '20]

(Non-Abelian) Englert-Brout-Higgs mechanism



$$\tanh\beta = \frac{v_2}{v_1}$$



cf. Hermitian case

[Alexandre, Ellis, PM & Seynaeve ‘20]

Canonical variables

$\Phi \equiv (\phi_1, \phi_2)$ evolves with H $\Leftrightarrow$ $\dot{\Phi}^\dagger \equiv (\dot{\phi}_1, \dot{\phi}_2)^\dagger$ evolves with $H^\dagger \neq H$

But canonical variables must both evolve with **the same** H (or $H^\dagger$)!

Canonical variables

The **self-consistent non-Hermitian deformation** is

[Alexandre, Ellis & PM ’20b]

$$\mathcal{L} = \partial_\nu \tilde{\phi}_i^* \partial^\nu \phi_i - m_i^2 \tilde{\phi}_i^* \phi_i - \mu^2 \left(\tilde{\phi}_1^* \phi_2 - \tilde{\phi}_2^* \phi_1 \right)$$

The Euler-Lagrange equations are now mutually consistent:

$$\frac{\partial \mathcal{L}}{\partial \tilde{\phi}_i^*} - \partial_\nu \frac{\partial \mathcal{L}}{\partial \partial_\nu \tilde{\phi}_i^*} = 0 \iff \frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\nu \frac{\partial \mathcal{L}}{\partial \partial_\nu \phi_i} = 0$$

The **tilde-conjugated fields** are defined via

$$\mathcal{P}\colon\ \phi_i(t, \boldsymbol{x}) \longrightarrow \phi_i'(t, -\boldsymbol{x}) = P_{ij} \tilde{\phi}_j(t, \boldsymbol{x})$$

But there is still a choice!

Second quantisation

In the flavour basis, the matrix-valued energy is non-Hermitian, and we need:

[see Alexandre, Ellis & PM ’20b, also for full details of how the discrete symmetry properties $\mathcal{C},\mathcal{C}',\mathcal{P},\mathcal{T}$ are borne out.]

$$\hat{\phi}_i(x) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}[2E(\boldsymbol{p})]_{ij}^{-1/2}\Big[\left(e^{-ip\cdot x}\right)_{jk}\hat{a}_{k,\boldsymbol{p}}(0) + \left(e^{ip\cdot x}\right)_{jk}\check{c}^{\dagger}_{k,\boldsymbol{p}}(0)\Big]$$

$$\check{\phi}_i^{\dagger}(x) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}[2E(\boldsymbol{p})]_{ji}^{-1/2}\Big[\left(e^{-ip\cdot x}\right)_{kj}\hat{c}_{k,\boldsymbol{p}}(0) + \left(e^{ip\cdot x}\right)_{kj}\check{a}^{\dagger}_{k,\boldsymbol{p}}(0)\Big]$$

The **hatted** (ˆ) and **checked** (ˇ) **fields** are related via **parity**:

$$P_{ij}\check{\phi}_j(\mathcal{P}x) = \hat{\mathcal{P}}\hat{\phi}_i(x)\hat{\mathcal{P}}^{-1}$$

The **second-quantised Lagrangian** is

$$\hat{\mathcal{L}} = \partial_\nu\check{\phi}_i^{\dagger}\partial^\nu\hat{\phi}_i - m_i^2\check{\phi}_i^{\dagger}\hat{\phi}_i - \mu^2\left(\check{\phi}_1^{\dagger}\hat{\phi}_2 - \check{\phi}_2^{\dagger}\hat{\phi}_1\right)$$

Second quantisation

Why this doubling?

- To ensure a **consistent variational procedure**, and to construct **canonical conjugate variables**.

- In terms of the **mass "eigenfields"** or the **similarity transformation** to the Hermitian "frame":

$$\hat{\xi}_i = R_{ij}\hat{\phi}_j \not\Leftrightarrow \hat{\xi}_i^\dagger = \hat{\phi}_j^\dagger R_{ji}^{-1}$$

instead:

$$\hat{\xi}_i = R_{ij}\hat{\phi}_j \Leftrightarrow \hat{\xi}_i^\dagger = \check{\phi}_i^\dagger R_{ji}^{-1}$$

Flavour oscillations

Mass eigenstates: $|\boldsymbol{p},+(-),t\rangle, \quad (|\boldsymbol{p},+(-),t\rangle)^{\S}, \quad \S \equiv \mathcal{C}'\mathcal{P}\mathcal{T} \circ \mathrm{T}$

Flavour eigenstates: [Alexandre, Ellis & PM '20b]

$$|\breve{\boldsymbol{p}},1(2),t\rangle = N\left\{\eta|\boldsymbol{p},+(-),t\rangle - \left[1-\sqrt{1-\eta^2}\right]|\boldsymbol{p},-(+),t\rangle\right\}$$

$$\langle\widehat{\boldsymbol{p}},1(2),t| = N\left\{\eta(|\boldsymbol{p},+(-),t\rangle)^{\S} + \left[1-\sqrt{1-\eta^2}\right](|\boldsymbol{p},-(+),t\rangle)^{\S}\right\}$$

Orthonormality:

$$\langle\widehat{\boldsymbol{p}},i,t|\breve{\boldsymbol{p}}',j,t\rangle = (2\pi)^3\delta_{ij}\delta^3(\boldsymbol{p}-\boldsymbol{p}')$$

$$(|\boldsymbol{p},\pm,t\rangle)^{\S}|\boldsymbol{p}',\pm,t\rangle = (2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{p}')$$

$$(|\boldsymbol{p},\pm,t\rangle)^{\S}|\boldsymbol{p}',\mp,t\rangle = 0$$

Flavour oscillations

Transition "probability":

$$\Pi_{i\to j}(t) = \frac{1}{V}\int \frac{\mathrm{d}^3\boldsymbol{p}'}{(2\pi)^3}\langle \hat{\boldsymbol{p}}, j, t|\breve{\boldsymbol{p}}', i, 0\rangle\langle \hat{\boldsymbol{p}}', i, 0|\breve{\boldsymbol{p}}, j, t\rangle, \qquad V \equiv (2\pi)^3\delta^3(\boldsymbol{0})$$

Oscillation and **"survivial probabilities":** [Alexandre, Ellis & PM '20b]

$$\Pi_{1(2)\to 2(1)}(t) = -\frac{\eta^2}{1-\eta^2}\sin^2\left[\frac{1}{2}\big(E_+(\boldsymbol{p}) - E_-(\boldsymbol{p})\big)t\right] \notin [0,1]$$

$$\Pi_{1(2)\to 1(2)}(t) = 1 + \frac{\eta^2}{1-\eta^2}\sin^2\left[\frac{1}{2}\big(E_+(\boldsymbol{p}) - E_-(\boldsymbol{p})\big)t\right] \notin [0,1]$$

Unitarity: $\Pi_{1(2)\to 1(2)}(t) + \Pi_{1(2)\to 2(1)}(t) = 1$

[cf. the similar issue found in Ohlsson & Zhou '20 & '21]

Flavour oscillations

Resolution: experimental **observables** are **scattering matrix elements**.

[Alexandre, Ellis & PM '20b]

We must source states consistent with the $\mathcal{PT}$ symmetry:

$$\mathcal{L}_{\mathrm{int}} = J_A \check{\phi}_1^\dagger + J_A^\dagger \hat{\phi}_1 - J_B \check{\phi}_2^\dagger + J_B^\dagger \hat{\phi}_2$$

"Squared" matrix element for $A \longrightarrow B$:

$$\mathcal{M}_{A\longrightarrow B}^{\mathcal{C}'\mathcal{PT}} \mathcal{M}_{A\longrightarrow B} = VT(2\pi)^4 \delta^4(p_A - p_B) \frac{\mu^4}{\left(p_A^2 - M_+^2\right)^2 \left(p_A^2 - M_-^2\right)^2} > 0$$

remaining real and perturbative all the way up to the exceptional point ($M_+^2 = M_-^2$).

What about fermions?

An example: non-Hermitian extension of the **Dirac theory**

[Bender, Jones & Rivers ’05; Alexandre, Bender & PM ‘15; Alexandre, PM & Seynaeve ‘17]

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\nu}\partial_{\nu} - m - \mu\gamma^{5}\right)\psi, \qquad \gamma^{5} = \left(\gamma^{5}\right)^{\dagger}$$

Eigenmasses: $M^2 = m^2 - \mu^2$

The **conserved current** is [Alexandre & Bender ‘15]

$$j^{\nu} = \bar{\psi}\gamma^{\nu}\left(1 + \frac{\mu}{m}\gamma^{5}\right)\psi = \left(1 - \frac{\mu}{m}\right)\psi_L^{\dagger}\bar{\sigma}^{\nu}\psi_L + \left(1 + \frac{\mu}{m}\right)\psi_R^{\dagger}\bar{\sigma}^{\nu}\psi_R$$

corresponding to

$$\psi \longrightarrow \psi' = \exp\left[i\alpha\left(1 + \frac{\mu}{m}\gamma^{5}\right)\right]\psi$$

with

$$\delta\mathcal{L} = -2\mu\bar{\psi}\gamma^{5}\delta\psi \neq 0$$

What about fermions?

- **Exceptional points** $\mu = +(-)m \implies$ a **massless theory** of right(left) chiral Weyl fermions.
  [Alexandre, Bender & PM ’15, cf. Chernodub ‘17]
- Gauging this model, the full **vector plus axial vector gauge symmetry** is recovered at the **exceptional points**.
  [Alexandre, Bender & PM ‘15; Alexandre, PM & Seynaeve ‘17]
- Massless fermions can undergo **flavour oscillations**.
  [Jones-Smith & Mathur ‘14]
- The same model can be obtained from a **non-Hermitian Higgs-Yukawa theory**.
  [Alexandre, Bender & PM ’15 & ’17; see also Alexandre & Mavromatos ‘20]

$$\mathcal{L}_{\text{Yuk}} = -y_- \bar{L}_L \tilde{H} \nu_R - y_+ \bar{\nu}_R \tilde{H}^\dagger L_L$$

- A non-Hermitian explanation for the smallness of the **light neutrino masses**?

SUSY embedding?

Two $\mathcal{N} = 1$ scalar chiral superfields: Φ_1, Φ_2

Superpotential: [Alexandre, Ellis & PM ’20a]

$$W_\pm = \frac{1}{2} m_{11} \Phi_1^2 \mp m_{12} \Phi_1 \Phi_2 + \frac{1}{2} m_{22} \Phi_2^2, \qquad \mathcal{L} = \mathcal{L}_K + \int d^2\theta \; W_+ + \int d^2\theta^\dagger \, W_-^\dagger \neq \mathcal{L}^\dagger$$

On-shell (two complex scalars, two Majorana fermions, $a = 1,2$):

$$\mathcal{L}_{\text{scal}} = \partial_\nu \phi_a^* \partial^\nu \phi_a - \left(m_{aa}^2 - m_{12}^2\right) \phi_a^* \phi_a - m_{12}(m_{22} - m_{11})(\phi_1^* \phi_2 - \phi_2^* \phi_1)$$

$$\mathcal{L}_{\text{ferm}} = \frac{1}{2} \bar{\psi}_a i \gamma^\nu \partial_\nu \psi_a - \frac{1}{2} m_{aa} \bar{\psi}_a \psi_a - \frac{1}{2} m_{12} \left(\bar{\psi}_1 \gamma^5 \psi_2 + \bar{\psi}_2 \gamma^5 \psi_1 \right), \qquad \gamma^5 = \left(\gamma^5\right)^\dagger$$

But $M^2_{\text{scal},\pm} \neq M^2_{\text{ferm},\pm} \;\Rightarrow$ supersymmetry breaking!

Closing remarks

- **Noether's theorem**, **Goldstone theorem** and **Englert-Brout-Higgs mechanism** borne out.
- **Parametric dependence** very different to similar Hermitian models.
- **Second quantisation** in the non-Hermitian "frame" is subtle.
- Formulate **non-Hermitian flavour oscillations** consistent with perturbative unitarity.
- A new possibility for **SUSY breaking**.
- Potential implications for the **neutrino sector**.
- **CP violation?** [Dale, Mason & PM in prep]; **fermionic second quantisation?** [Alexandre, Ellis & PM in prep].

Thank you and stay well

Questions or **comments**?

Message me on **twitter @pwmillington** or **email** me at **p.millington@nottingham.ac.uk**.

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Please see references therein.

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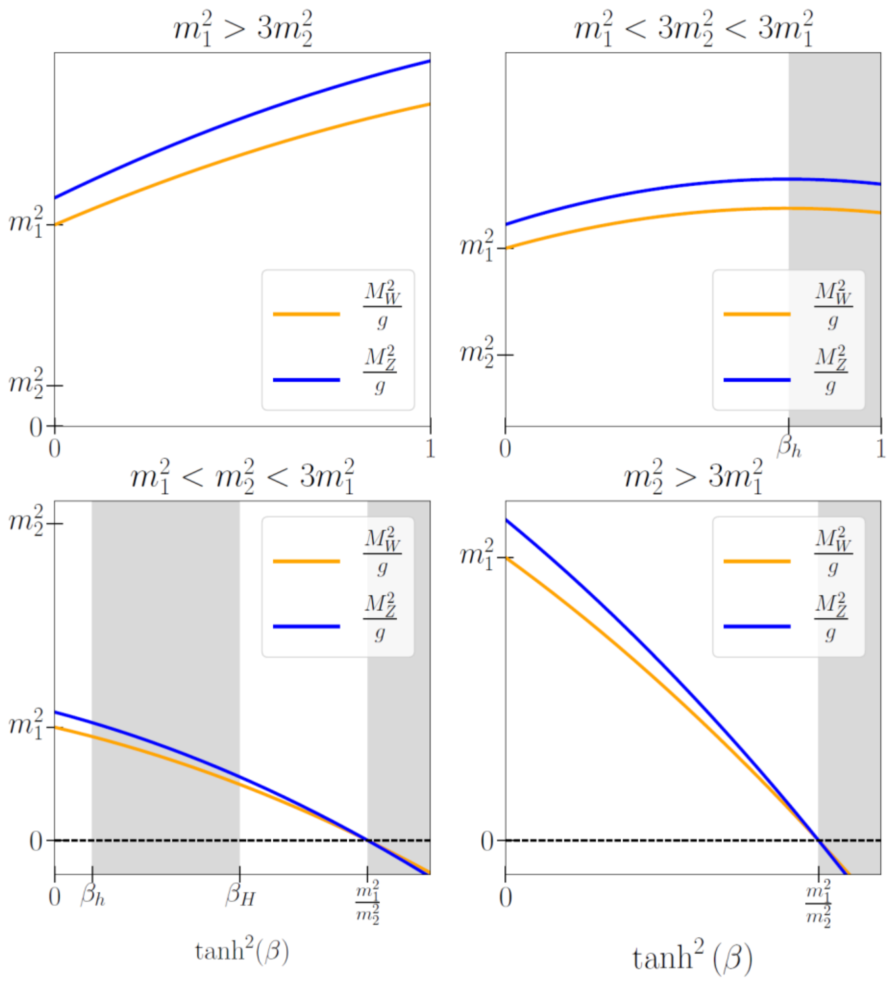
Back up slides

(Non-Abelian) Englert-Brout-Higgs mechanism



[Alexandre, Ellis, PM & Seynaeve '20]

Maxwell equations

Since we must couple to a **non-conserved current**

$$\partial_\nu j^\nu = -2iq\mu^2(\phi_1^*\phi_2 - \phi_2^*\phi_1)$$

The **Maxwell equations** are inconsistent, since $\partial_\nu \partial_\rho F^{\nu\rho} = 0$ identically.

Resolution: Adding a covariant **gauge fixing term** is sufficient: [Alexandre, Ellis, PM & Seynaeve '19]

$$\mathcal{L} \supset -\frac{1}{2\zeta}(\partial_\nu A^\nu)^2$$

This leads to the constraint (cf. Stueckelberg case)

$$\frac{1}{\zeta}\Box\, \partial_\nu A^\nu = -2iq\mu^2(\phi_1^*\phi_2 - \phi_2^*\phi_1)$$

precluding the **Lorenz gauge condition**.