Quantum Correlations in PT-symmetric systems

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Non-Hermitian Physics Online, March 2021

[Quantum Sci. Technol. **6** 025005 (2021)]

Teaser

- Increasing interest in **non-Hermitian** Quantum Mechanics

C. M. Bender and S. Boettcher, Phys. Rev. Lett. 80, 5243 (1998).
C. M. Bender, Rep. Prog. Phys. 70, 947 (2007).
C. M. Bender, D. C. Brody, and H. F. Jones, American Journal of Physics 71, 1095 (2003).

- **Experimental** realisations of non-Hermitian **P**arity-**T**ime-symmetric systems

R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, Nat. Phys. 14, 11 (2018).
L. Feng, R. El-Ganainy, and L. Ge, Nat. Photonics 11, 752 (2017).
S. Longhi, Euro Phys. Lett. 120, 64001 (2017).
C. E. Rueter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Nat. Phys. 6, 192 (2010).
A. Regensburger, C. Bersch, M.-A. Miri, G. Onishchukov, D. N. Christodoulides, and U. Peschel, Na- ture 488, 167 (2012).
B. Peng, S. K. Ozdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, C. M. Bender, and L. Yang, Nat. Phys. 10, 394 (2014).

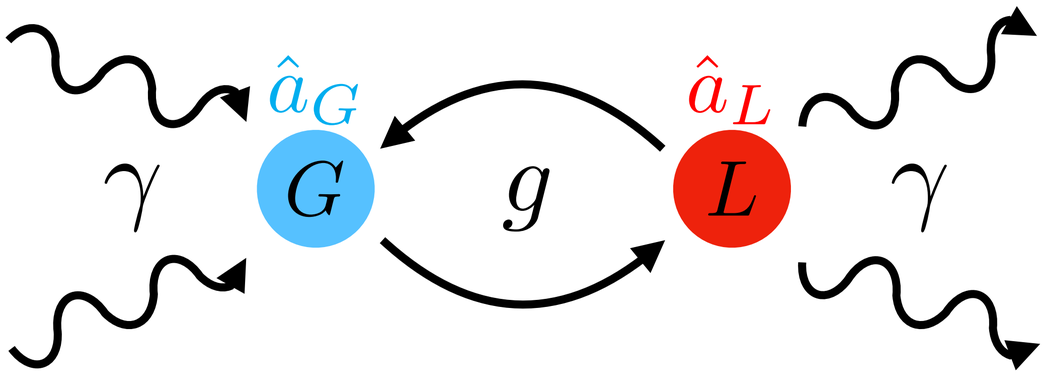
… mostly **classical**

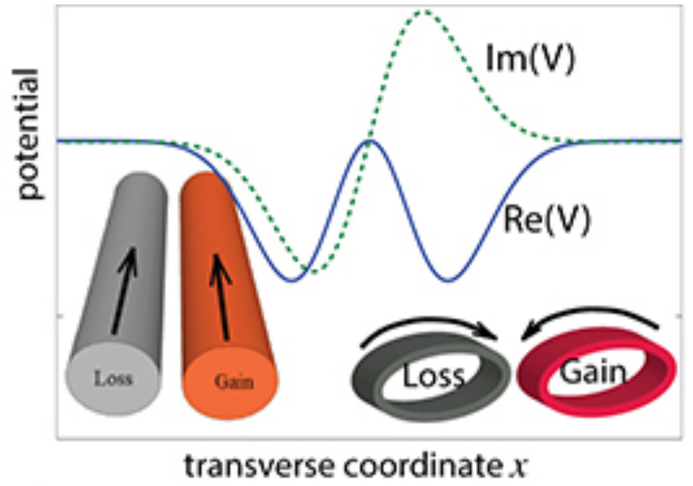
- **Quantum** character of PT symmetric systems is still an **open problem**

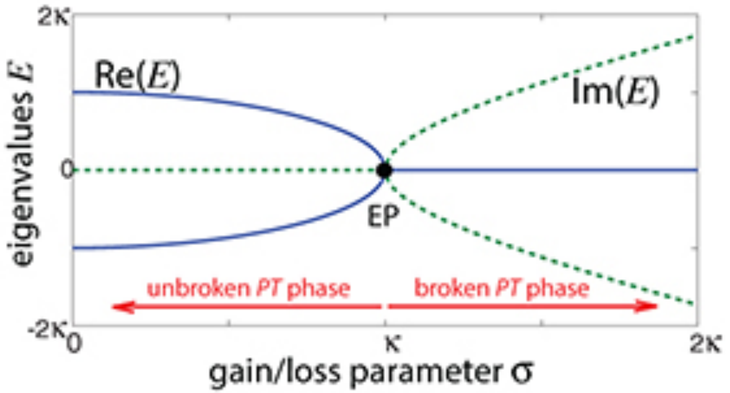
W. Cao, X. Lu, X. Meng, J. Sun, H. Shen, and Y. Xiao, Phys. Rev. Lett. 124, 030401.
Fring, Andreas, and Thomas Frith. "Eternal life of entropy in non-Hermitian quantum systems." *arXiv preprint arXiv:1905.07348* (2019).
Chakraborty, Subhadeep, and Amarendra K. Sarma. "Delayed sudden death of entanglement at exceptional points." *arXiv preprint arXiv:1906.00222* (2019).
H. Schomerus, Phys. Rev. Lett. 104, 233601 (2010). G. Yoo, H.-S. Sim, and H. Schomerus, Phys. Rev. A 84, 063833 (2011).
G. S. Agarwal and K. Qu, Phys. Rev. A 85, 031802 (2012). S. Longhi, Opt. Lett. 43, 5371 (2018).
S. Vashahri-Ghamsari, B. He, and M. Xiao, Phys. Rev. A 96, 033806 (2017) and Phys. Rev. A 99, 023819 (2019).

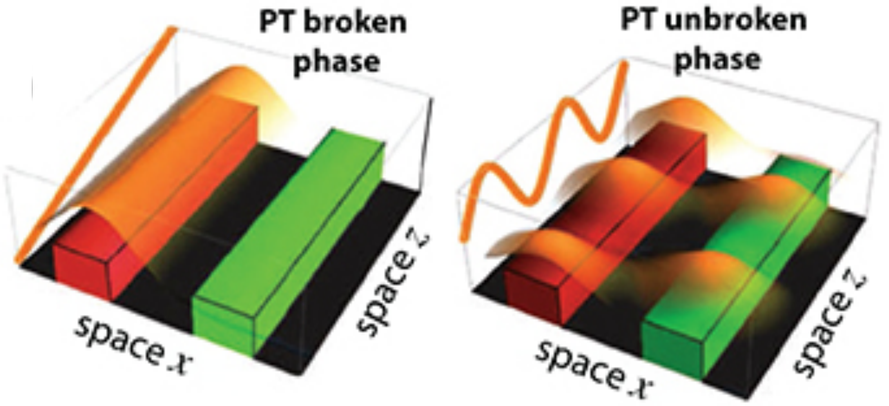
Gain-Loss System

- Coupled mode theory for
  - Coupled optical waveguides subject to gain/loss
  - Coupled active/lossy ring resonators



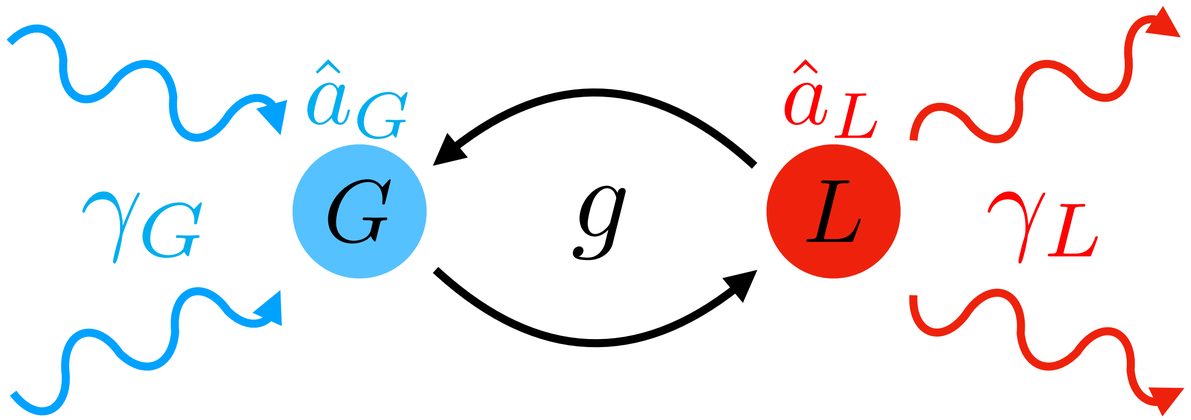






Stefano Longhi 2017 *EPL* **120** 64001

Gain-Loss System



(PT symmetric if $\gamma_G = \gamma_L \equiv \gamma$)

Mean-field dynamics

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\langle\hat{a}_L\rangle\\ \langle\hat{a}_G\rangle\end{pmatrix}=\begin{pmatrix}-i\gamma_L & g\\ g & i\gamma_G\end{pmatrix}\begin{pmatrix}\langle\hat{a}_L\rangle\\ \langle\hat{a}_G\rangle\end{pmatrix}$$

When **PT symmetric**:

UNBROKEN PHASE

$$\gamma < g$$

- Real eigenvalues
- Non-orthogonal eigenvectors

EXCEPTIONAL POINT

$$\gamma = g$$

- Coalescing eigenvalues
- Parallel eigenvectors

R. El-Ganainy et al., Nat. Phys. 14, 11 (2018).
C. E. Rueter et al., Nat. Phys. 6, 192 (2010).

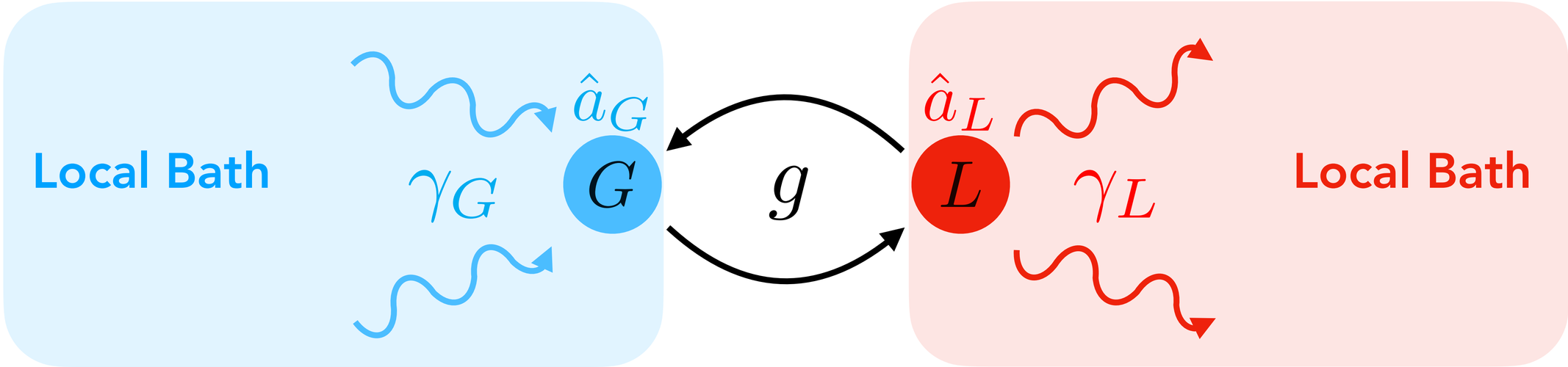
BROKEN PHASE

$$\gamma > g$$

- Imaginary eigenvalues
- Non-orthogonal eigenvectors

Fully quantum dynamics

$$H = g\left(\hat{a}_L^\dagger \hat{a}_G + \hat{a}_L \hat{a}_G^\dagger\right).$$



$$\dot{\rho} = \boxed{-i[H,\rho]} + \boxed{2\,\gamma_L\left(\hat{a}_L\,\rho\,\hat{a}_L^\dagger - \tfrac{1}{2}\{\hat{a}_L^\dagger\hat{a}_L,\rho\}\right)} + \boxed{2\,\gamma_G\left(\hat{a}_G^\dagger\,\rho\,\hat{a}_G - \tfrac{1}{2}\{\hat{a}_G\hat{a}_G^\dagger,\rho\}\right)}$$

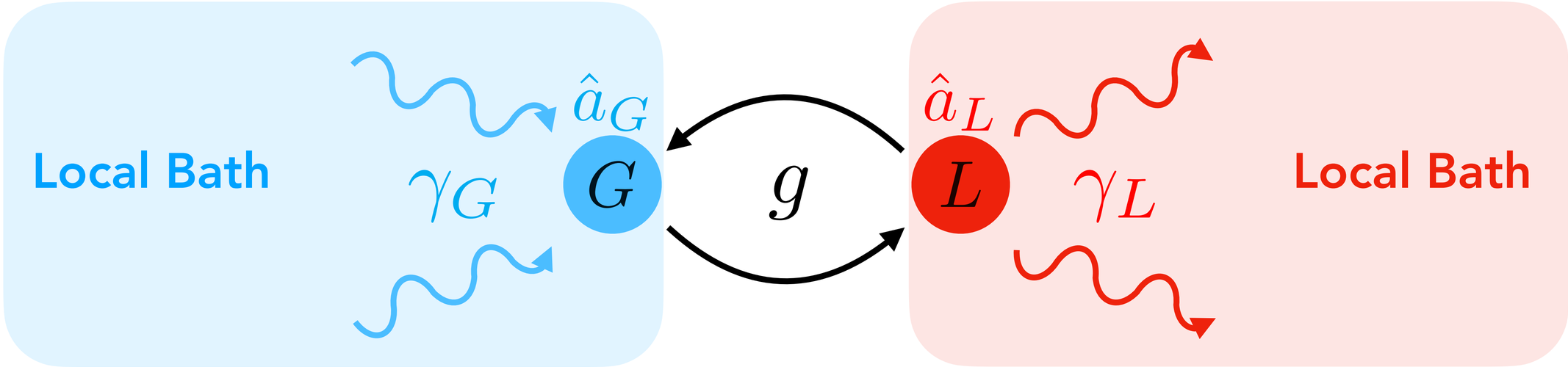
Von Neumann

Dissipator

"Exciter"

Fully quantum dynamics

$$H = g\left(\hat{a}_L^\dagger \hat{a}_G + \hat{a}_L \hat{a}_G^\dagger\right).$$

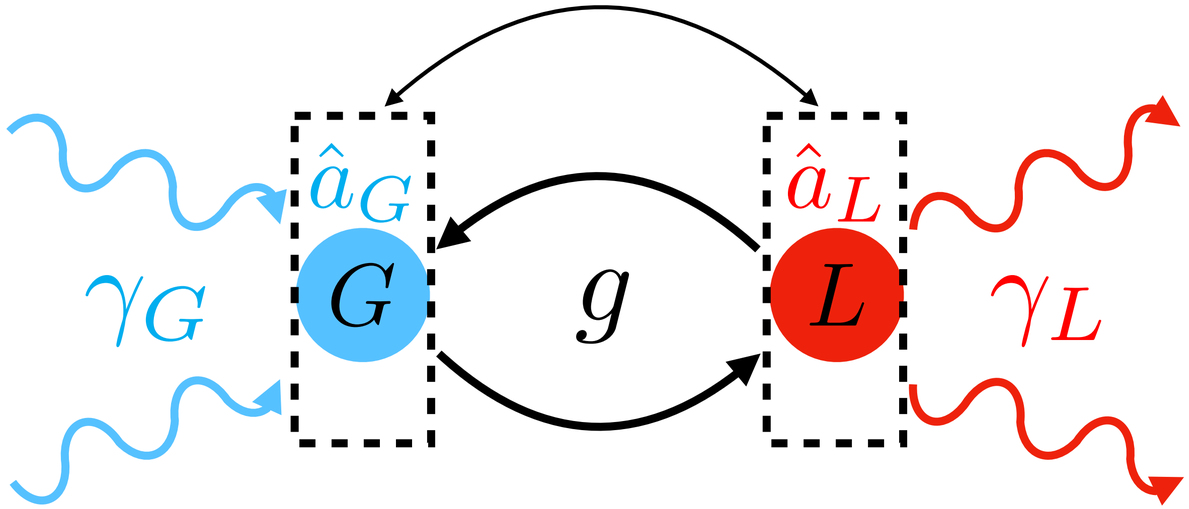


$$\dot{\rho} = -i[H,\rho] + 2\,\gamma_L\left(\hat{a}_L\,\rho\,\hat{a}_L^\dagger - \tfrac{1}{2}\{\hat{a}_L^\dagger\hat{a}_L,\rho\}\right) + 2\,\gamma_G\left(\hat{a}_G^\dagger\,\rho\,\hat{a}_G - \tfrac{1}{2}\{\hat{a}_G\hat{a}_G^\dagger,\rho\}\right)$$

Focus on Gaussian states! (e.g. a coherent state is Gaussian)

Looking for quantumness

In the correlations!

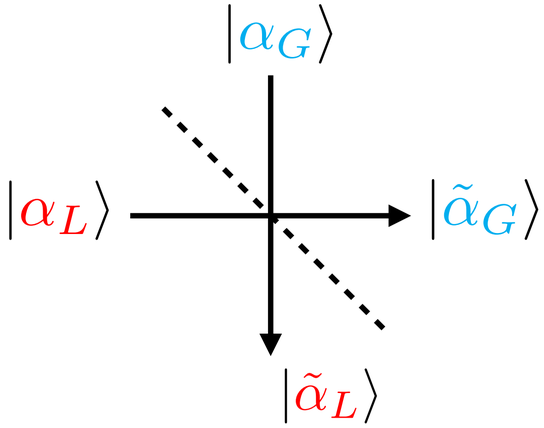


- Most famous Quantum Correlation (QC):

Entanglement

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

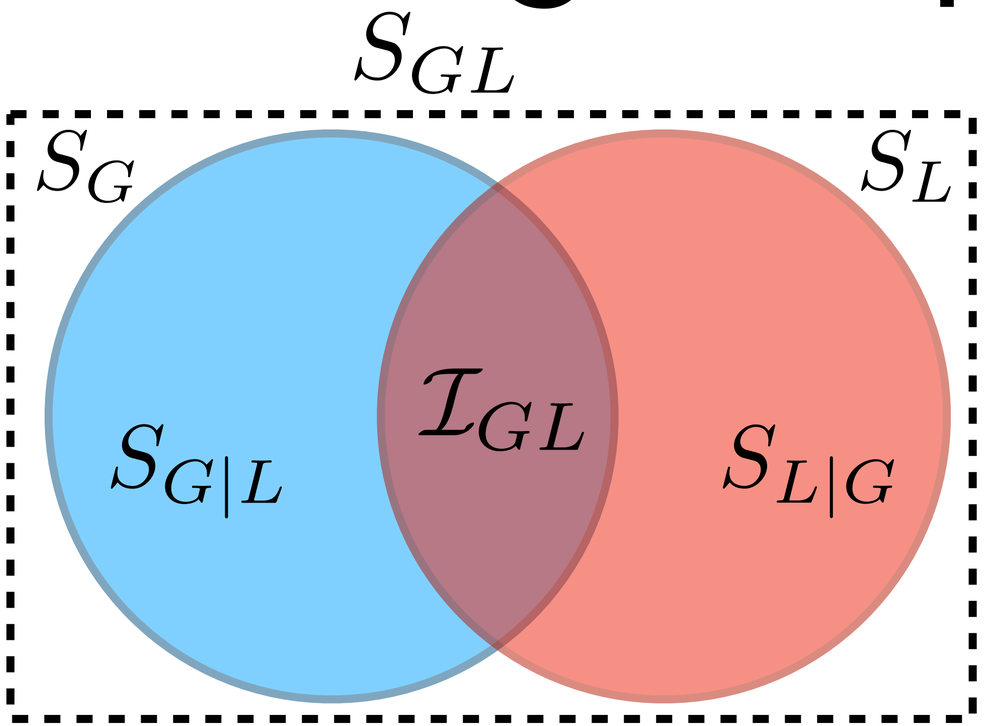
However…



$\Longrightarrow$ **NO Entanglement!**

M. S. Kim, W. Son, V. Buzek, and P. L. Knight, Phys. Rev. A 65, 032323 (2002).

Looking for quantumness



S_i = amount of uncertainty about i before we "look at it"

$$\begin{aligned}\mathcal{I}_{GL} &= S_G + S_L - S_{GL} \\ &= S_G - (S_{GL} - S_L) \\ &= S_G - S_{G|L} \\ &= \mathcal{C}_{GL} \equiv \mathcal{C}_{LG}\end{aligned}$$

Total amount of correlations | **Classical Corr.**

Quantum: $\mathcal{I} = S(\rho_G) + S(\rho_L) - S(\rho) \neq \mathcal{C}_{GL} \neq \mathcal{C}_{LG}$

Asymmetric!

mismatch (purely quantum!) Quantum Discord: $\mathcal{D}_{GL} = \mathcal{I} - \mathcal{C}_{GL}$

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001).

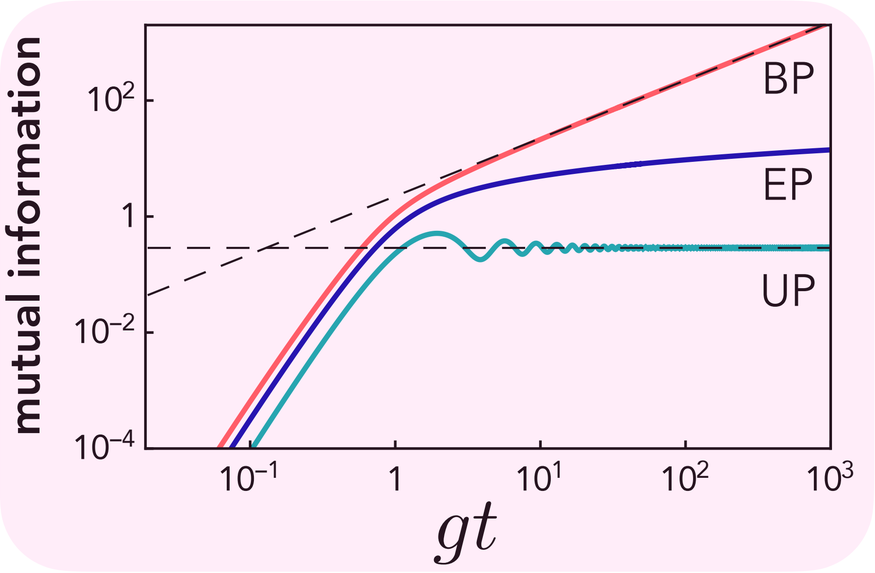
Correlations Dynamics

Initial state:

$\rho_0 = |\alpha_G\rangle\langle\alpha_G| \otimes |\alpha_L\rangle\langle\alpha_L|$ (totally uncorrelated)

PT symmetric case:

$\gamma_G = \gamma_L \equiv \gamma$



UNBROKEN PHASE (UP)	EXCEPTIONAL POINT (EP)	BROKEN PHASE (BP)
$\mathcal{I} \approx \log(\frac{g^2}{g^2-\gamma^2})$	$\mathcal{I} \approx \log(\frac{4g^2}{3}t^2)$	$\mathcal{I} \approx 2t\sqrt{\gamma^2-g^2}$

Correlations Dynamics

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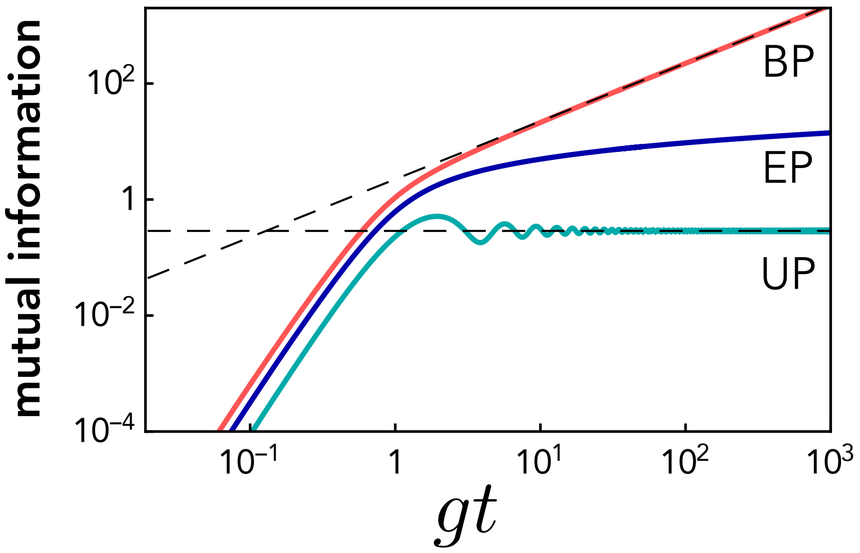

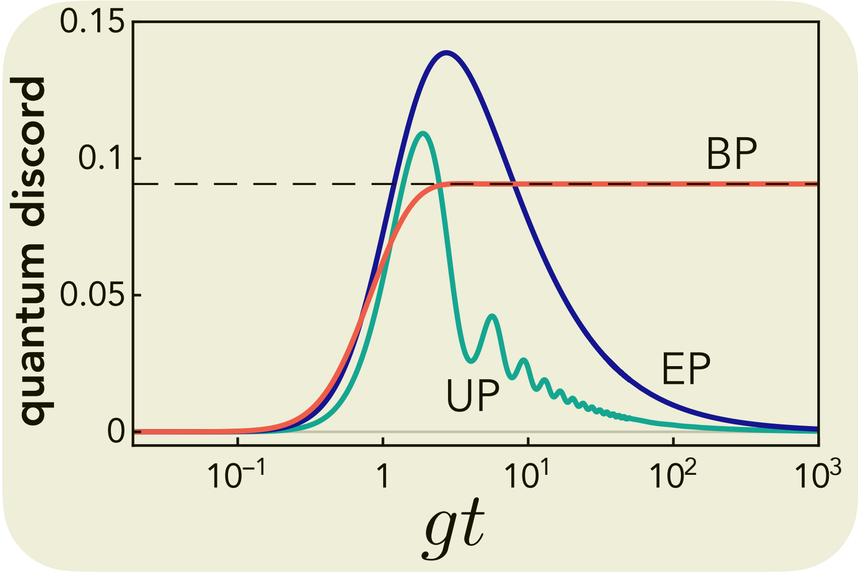

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$\mathcal{D}_{LG}, \mathcal{D}_{GL} \approx \frac{\gamma}{2g^2t}$	$\mathcal{D}_{LG}, \mathcal{D}_{GL} \approx \frac{1}{gt}$	$\mathcal{D}_{LG} \approx \log\left(\frac{\gamma(\gamma+\Omega)+g^2}{2\gamma^2}\right)$

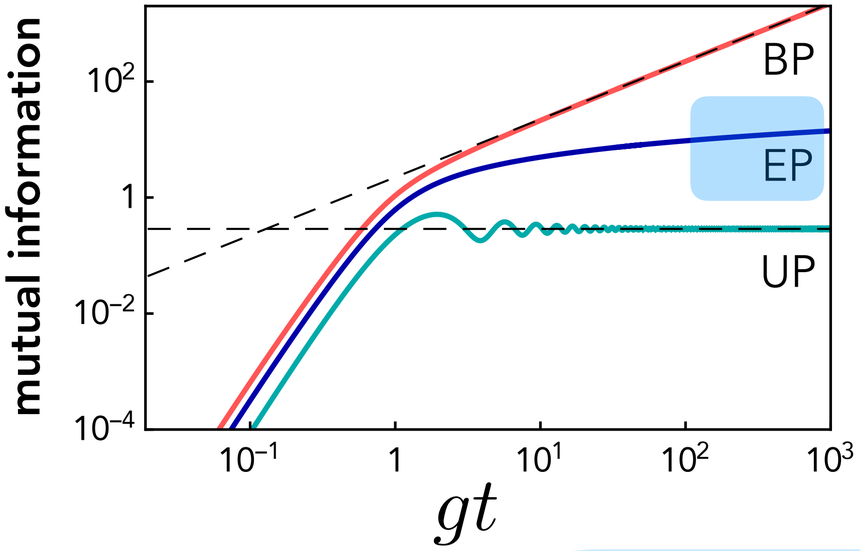
Correlations Dynamics

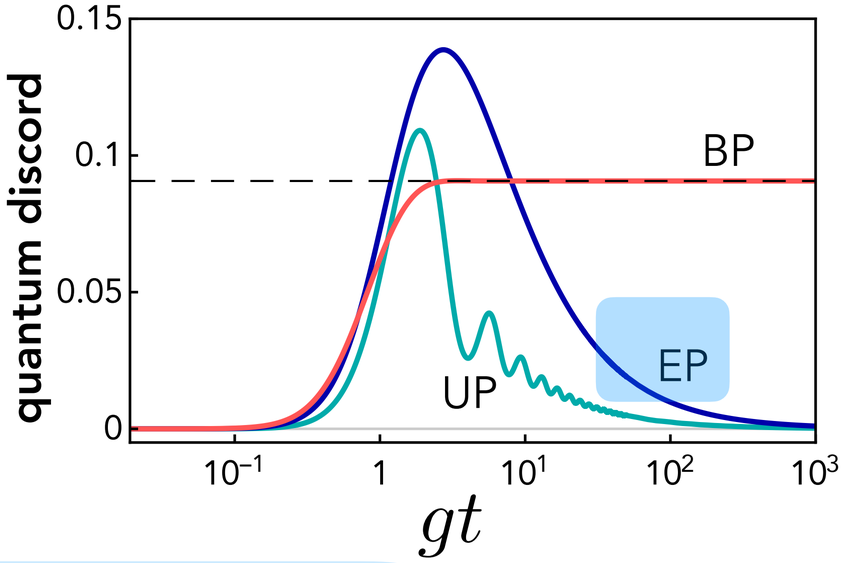
Initial state:

$\rho_0 = |\alpha_G\rangle\langle\alpha_G| \otimes |\alpha_L\rangle\langle\alpha_L|$ (totally uncorrelated)

PT symmetric case:

$\gamma_G = \gamma_L \equiv \gamma$





UNBROKEN PHASE (UP)

$$\mathcal{I} \approx \log\left(\frac{g^2}{g^2-\gamma^2}\right)$$

$$\mathcal{D}_{LG}, \mathcal{D}_{GL} \approx \frac{\gamma}{2g^2 t}$$

EXCEPTIONAL POINT (EP)

$$\mathcal{I} \approx \log\left(\frac{4g^2}{3}t^2\right)$$

$$\mathcal{D}_{LG}, \mathcal{D}_{GL} \approx \frac{1}{gt}$$

BROKEN PHASE (BP)

$$\mathcal{I} \approx 2t\sqrt{\gamma^2 - g^2}$$

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Correlations Dynamics

Initial state:

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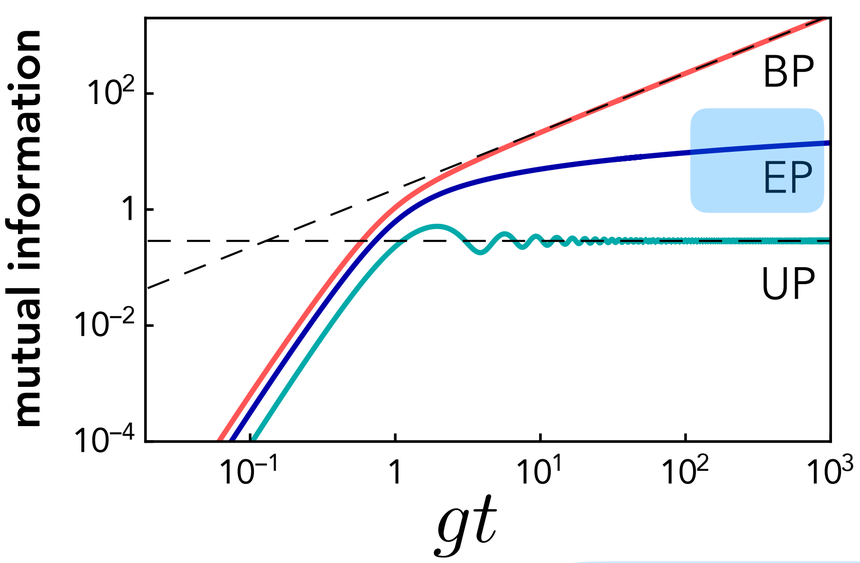

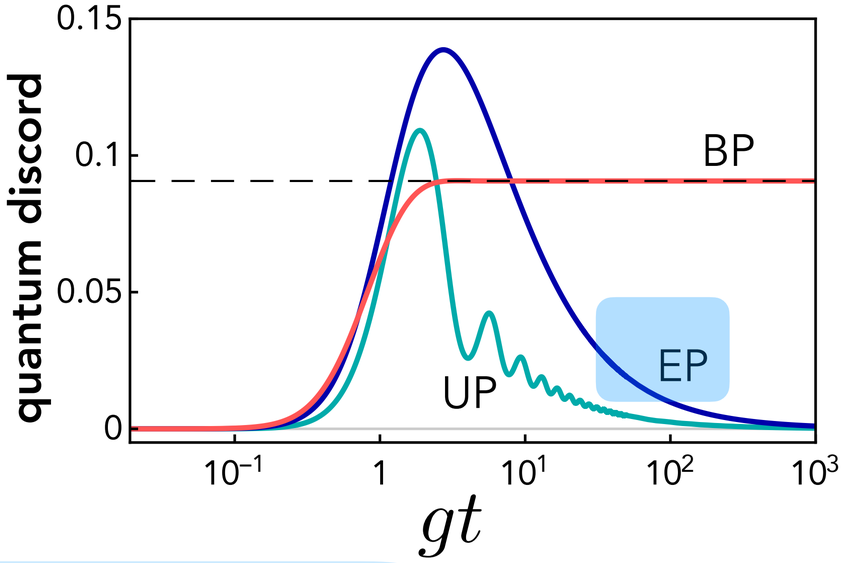

most classical configuration

EXCEPTIONAL POINT (EP)

$$\mathcal{I} \approx \log(\frac{4g^2}{3}t^2)$$

$$\mathcal{D}_{LG}, \mathcal{D}_{GL} \approx \frac{1}{gt}$$

Correlations Dynamics

Initial state:

$\rho_0 = |\alpha_G\rangle\langle\alpha_G| \otimes |\alpha_L\rangle\langle\alpha_L|$ (totally uncorrelated)

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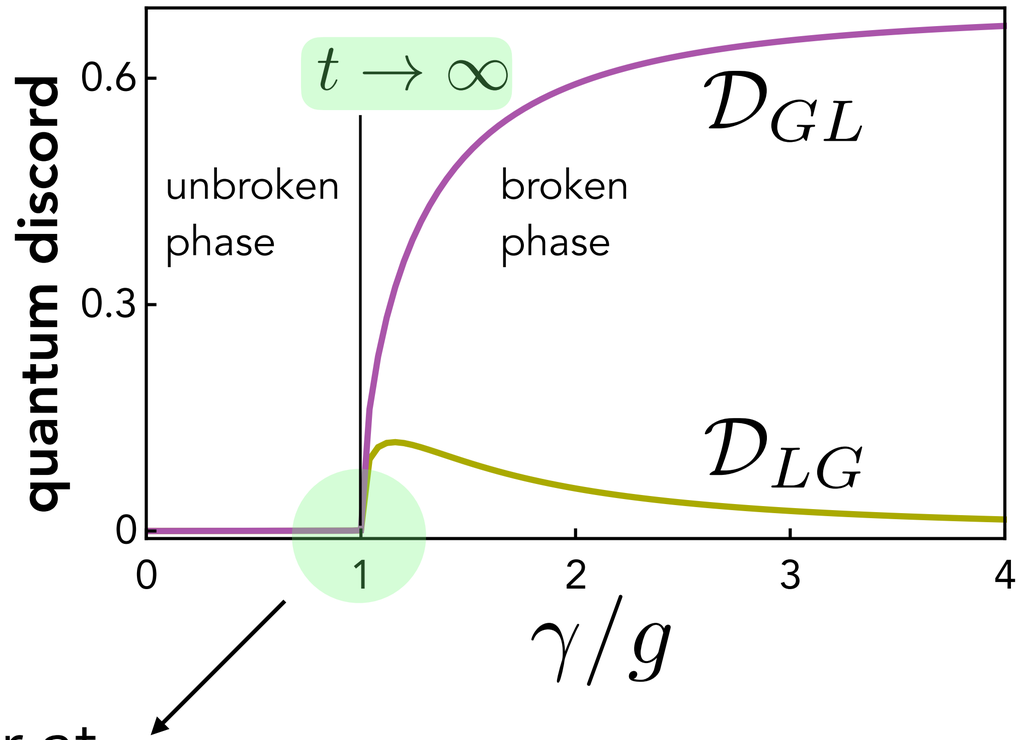


critical behavior at Exceptional Point

Physical mechanism

Generation of Quantum Discord

LOSS $|\alpha_L\rangle \otimes |\alpha_G\rangle \rightarrow |\eta\alpha_L\rangle \otimes |\alpha_G\rangle \qquad \eta < 1$

Purity unaffected

COUPLING $|\alpha_L\rangle \otimes |\alpha_G\rangle \rightarrow |\tilde{\alpha}_L\rangle \otimes |\tilde{\alpha}_G\rangle$

GAIN $|\alpha_G\rangle\langle\alpha_G| \rightarrow \int \mathrm{d}^2\alpha'_G \, P(\alpha'_G) |\alpha'_G\rangle\langle\alpha'_G|$

Purity diminished.
Superposition of
non orthogonal states

Combined action: generally discordant state

Physical mechanism

Long-time correlations dynamics: an entropic argument

$$\mathcal{D}_{LG} = \log\left(1 + \frac{e^{\mathcal{I}} - 1}{e^{S_G} + 1}\right)$$

$$\mathcal{D}_{LG} \approx \log\left(1 + e^{-(S - S_L)} - e^{-S_G}\right)$$

I decide!

I disappear..

$\mathcal{PT}$ line	UP	EP	BP
S	$\log\left(\frac{4\gamma^2 g^2}{g^2-\gamma^2}\, t^2\right)$	$\log\left(\frac{4g^4}{3} t^4\right)$	$2\Omega\, t + \log\left(\frac{\gamma^3(\gamma+\Omega)}{\Omega^4}\right)$
S_L	$\log\left(\frac{2\gamma g^2}{g^2-\gamma^2}\, t\right)$	$\log\left(\frac{4g^3}{3} t^3\right)$	$2\Omega\, t + \log\left(\frac{\gamma g^2}{2\Omega^3}\right)$
S_G			$2\Omega\, t + \log\left(\frac{\gamma(\gamma+\Omega)^2}{2\Omega^3}\right)$

- Competition of local and global entropies
- Unbroken Phase:

$$S > S_L$$

vanishing discord

Physical mechanism

Long-time correlations dynamics: an entropic argument

$$\mathcal{D}_{LG} = \log\left(1 + \frac{e^{\mathcal{I}} - 1}{e^{S_G} + 1}\right)$$

$$\mathcal{D}_{LG} \approx \log\left(1 + e^{-(S - S_L)} - e^{-S_G}\right)$$

I decide! (S − S_L)

I disappear.. e^{−S_G}

$\mathcal{PT}$ line	UP	EP	BP
S	$\log\left(\frac{4\gamma^2 g^2}{g^2-\gamma^2}\, t^2\right)$	$\log\left(\frac{4g^4}{3} t^4\right)$	$2\Omega\, t + \log\left(\frac{\gamma^3(\gamma+\Omega)}{\Omega^4}\right)$
S_L	$\log\left(\frac{2\gamma g^2}{g^2-\gamma^2}\, t\right)$	$\log\left(\frac{4g^3}{3} t^3\right)$	$2\Omega\, t + \log\left(\frac{\gamma g^2}{2\Omega^3}\right)$
S_G			$2\Omega\, t + \log\left(\frac{\gamma(\gamma+\Omega)^2}{2\Omega^3}\right)$

- Competition of local and global entropies
- Broken Phase:

$$S - S_L \to \text{const.}$$

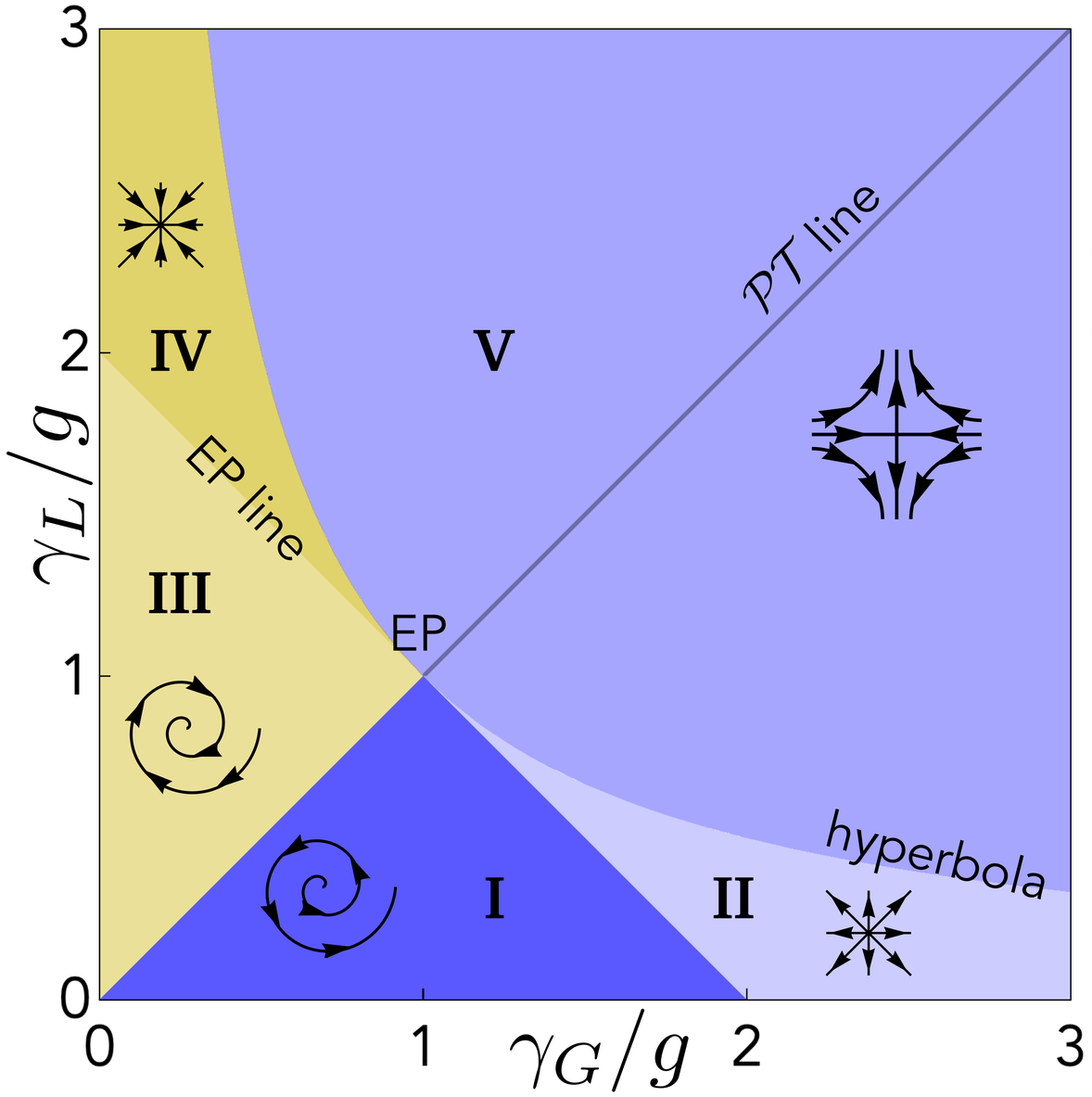
asymptotic finite discord

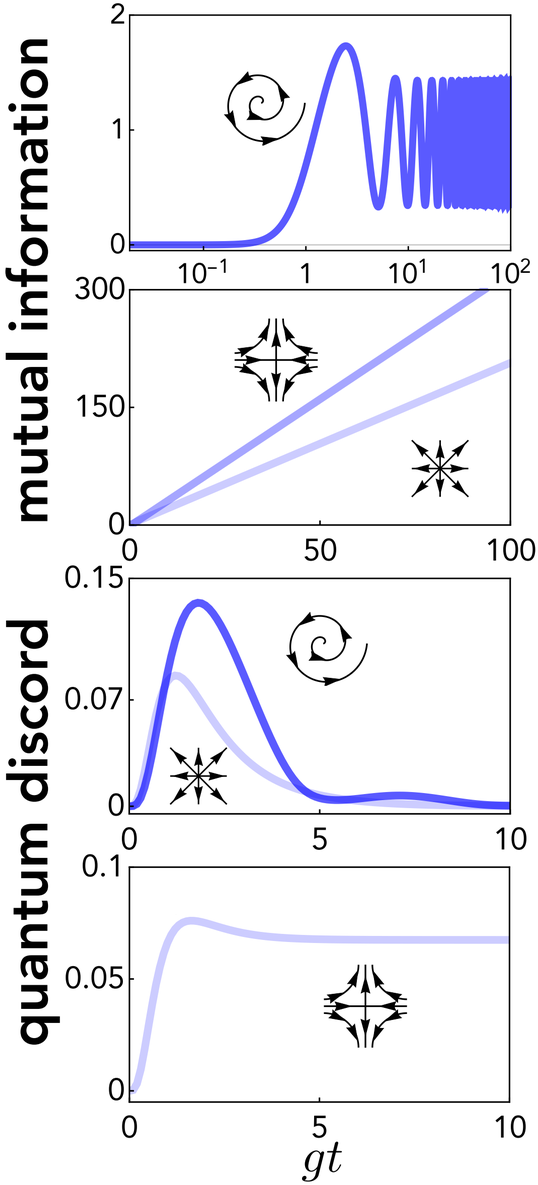
Correlations Dynamics

Initial state:

$$\rho_0 = |\alpha_G\rangle\langle\alpha_G| \otimes |\alpha_L\rangle\langle\alpha_L|$$

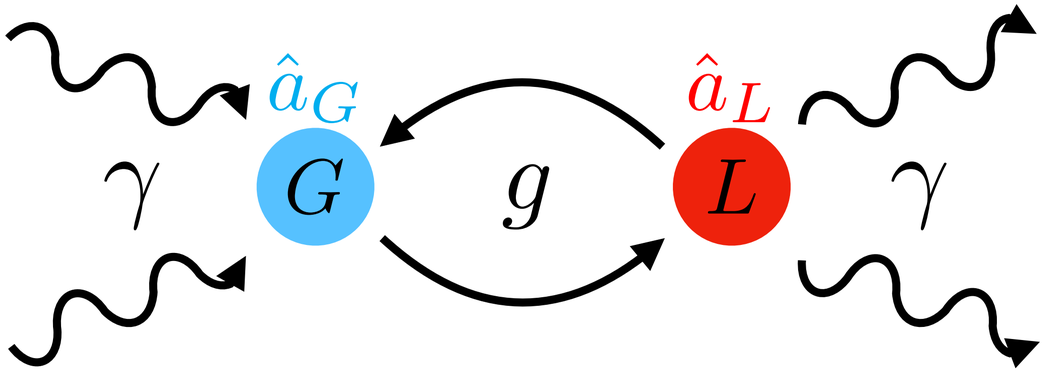
- Beyond balanced gain/loss
- I+II: vanishing Discord
- I+III+IV: finite mutual information
- III+IV: stable dynamics

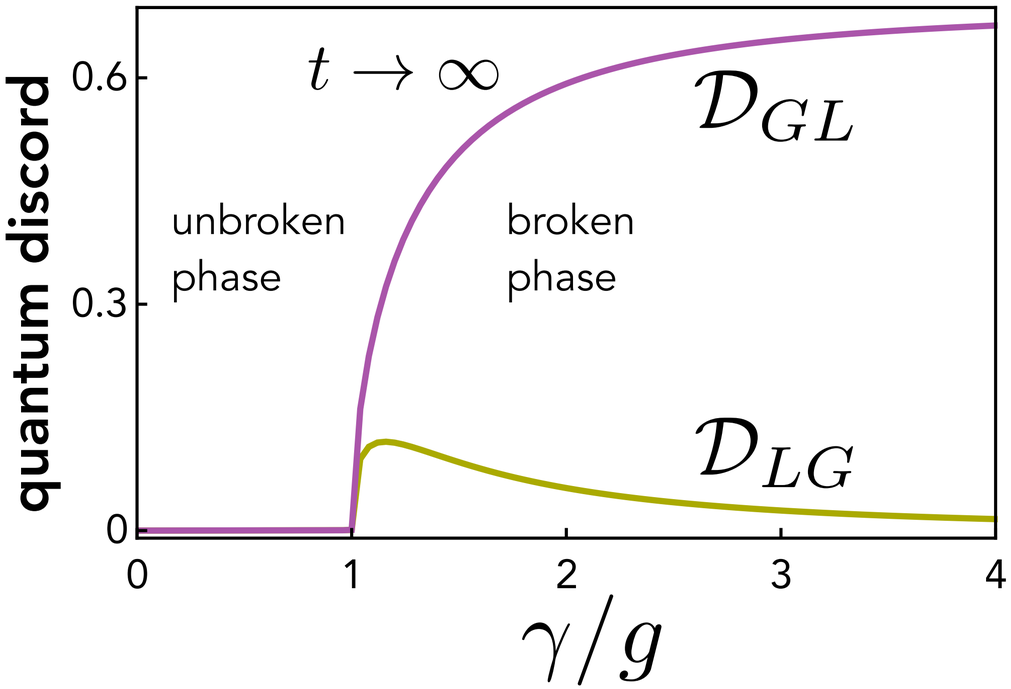




Take-home messages

- Quantum critical behavior of Exceptional Point detected by Quantum Discord
- Finite Quantum Correlations in PT Broken Phase
- Creation of Quantum Correlations: Gain
- Long-time behavior: competition of local and global entropies





Thank you!