Gravitational Astronomy: The Big Picture

International Centre for Theoretical Sciences
Bangalore, India, June 25, 2013

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School of Physics and Astronomy, Cardiff University, UK
What are Gravitational Waves?

In Newton’s law of gravity the gravitational field satisfies the Poisson equation:

$$\nabla^2 \Phi(t, X) = 4\pi G \rho(t, X)$$

Gravitational field is described by a scalar field, the interaction is instantaneous and no gravitational waves.

In general relativity for weak gravitational fields, i.e.

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \quad |h_{\alpha\beta}| \ll 1$$

in Lorentz gauge, i.e. \(h^{\alpha\beta},_\beta = 0\), Einstein’s equations reduce to wave equations in the metric perturbation:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)h^{\alpha\beta} = -16\pi T^{\alpha\beta}.$$ 

Here \(h_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}\eta^{\mu\nu}h_{\mu\nu}\) is the trace-reverse tensor.
Luminosity = Asymmetry factor \times (v/c)^{10} = Asymmetry factor \times (M/R)^{5}

A strong function of velocity: During merger a binary black hole in gravitational waves outshines the entire Universe in light

Amplitude from a source of size R at a distance D is

\[ h = (\text{Asymmetry factor}) \times (M/D) \times (M/R) \]

Gravitational wave detectors are essentially detectors of neutron stars and black holes

Frequency of the waves is the dynamical frequency \( f \sim \sqrt{G\rho} \)

For binaries dominant gravitational wave frequency is twice the orbital frequency. A binary of 20 solar masses merges at a frequency of 200 Hz

Polarization is determined from a network of detectors

A single detector is sensitive only to a linear combination of the two polarizations
A.5-*18

Frequency-Mass Diagram For Compact Binaries

- BNS
- IMBBH
- SMBBH

Ground

Space

PTA

Change in frequency <1/10 years

No radiation expected

dynamical frequencies

- 44 QNM
- 33 QNM
- 22 QNM
- LSO

time to merge

- 1 min
- 1 day
- 3 years

Mass (solar mass)

Frequency (Hz)

10^0 10^2 10^4 10^6 10^8 10^10 10^12

10^4 10^2 10^0

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Overview of the Talk

Quantum fluctuations in the very early Universe
Binary supermassive black holes in galactic nuclei
Phase transitions in the early universe
Black holes, compact stars captured by supermassive holes in galactic nuclei
Binary stars in the galaxy and beyond
Merging binary neutron stars and stellar black holes in distant galaxies; fast pulsars with mountains

Sources
Age of the Universe
Years
Hours
Seconds
Msec

Wave Period
Frequency (Hz)

10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
10^{2}

Detectors

Inflation probe (NASA)
Polarization map of cosmic microwave background
Precision timing of millisecond pulsars (1982 - )
LISA (ESA/NASA, 2010)
Big Bang Obs (NASA)
GEO, LIGO, Virgo, TAMA (2002 - )

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Ultra Low Frequency

quantum fluctuations in the very early Universe

binary supermassive black holes in galactic nuclei

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Wave Period

AGE OF THE UNIVERSE

years

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frequency (Hz)

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DETECTORS

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laser tracking of drag-free proof mass in spacecraft orbiting the sun

laser interferometers on Earth (also bar detectors)
Planck Temperature Fluctuations
Primordial Background and New Physics

- Horizon scale stochastic radiation
- Gravitational waves can cause
  - Temperature anisotropies as well as specific polarization modes in CMB photons
- Detection can determine the energy scale of inflation
  - Larger the energy scale greater is the strength of the background
- New physics
  - Need to have extra dimensions required by string theory
Very Low Frequency

- Quantum fluctuations in the very early Universe
- Binary supermassive black holes in galactic nuclei
- Phase transitions in the early universe
- Black holes, compact stars captured by supermassive holes in galactic nuclei
- Binary stars in the galaxy and beyond
- Merging binary neutron stars and stellar black holes in distant galaxies; fast pulsars with mountains

Wave Period
- Frequency (Hz)
  - $10^{-16}$
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  - 1
  - $10^2$

Age of the Universe
- YEARS
- HOURS
- SECONDS
- MSEC

Detectors
- Inflation probe (NASA)
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Pulsar timing arrays:
Use millisecond pulsars (MSPs) to detect gravitational waves
Pulsar Timing Array: a galactic-scale gravitational wave detector.

Sensitive to very low frequency (~nHz) grav waves.
Pulsar Timing Arrays around the world:

Parkes Pulsar Timing Array (PPTA)

European Pulsar Timing Array (EPTA)

North American Nanohertz Observatory for Gravitational Waves (NANOGrav)

In combination, International Pulsar Timing Array (IPTA)!
Black Holes Undergo Frequent Merger
Upper Limits on GW Stochastic Background
LISA: Laser Interferometer Space Antenna
eLISA

- Consists of 3 spacecraft in heliocentric orbit
  - Distance between spacecraft ~ 1 million km
  - 10 to 30 degrees behind earth
- The three eLISA spacecraft follow Earth almost as a rigid triangle entirely due to celestial mechanics
  - The triangle rotates like a cartwheel as craft orbit the sun
The Gravitational Universe
A General Science Theme addressed by the eLISA Survey Mission observing the entire Universe

eLISA Survey Mission

The payload consists of four identical units, two on the mother spacecraft and one on each daughter spacecraft. Each unit contains a Gravitational Reference Sensor (GRS) with an embedded free-falling test mass that acts both as the end point of the optical length measurement and as a geodesic reference test particle. A telescope with 20 cm diameter transmits the laser light, about 2 W at 1064 nm, along the arm and also receives the weak light from the other end. Laser interferometry is performed on an optical bench in between the telescope and the GRS.
Growth of Supermassive Black holes

Mayer et al, Science 2007, 316, 1874
Visibility of SMBBH in eLISA

- Plot shows SNR contours as a function of intrinsic total mass and redshift.
- Cosmological redshift makes binaries appear more massive than they actually are.
- Even at $z=20$ SNRs can be pretty large.
Understanding Black Hole Populations

- Masses can be measured to an accuracy of 0.1% to 1%
- Absolute errors in dimensionless spin in the range 0.01 to 0.1
- For sources within $z=1$ distance could be measured to within 1 to 10%

![Figure](image)

Slide from E. Berti
Milky Way’s black hole – a 4 million solar mass monster
Measuring the Kerr Geometry

Large black hole:
- shown to scale
- 250 solar masses
- 80% maximal spin

Small black hole:
- shown enlarged
- 1.4 solar masses
- no spin

Trace duration:
- 10 seconds

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Between 2006–2010 larger detectors took 2 years worth of data at unprecedented sensitivity levels.

No detections so far but beginning to impact astrophysics.
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Advanced Detectors:
Ca 2015–2025
Baselines in light travel time (ms)

Detector Networks 2015-

Baselines in light travel time (ms)
Baselines in light travel time (ms)

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Baselines in light travel time (ms)
Detector Beam Pattern Function

- Gives the sensitivity of a detector to sources at different parts of the sky.
- For a single detector the beam is a quadrupole.
- For a network of 5 or more globally distributed detectors the pattern can essentially become isotropic.

Hanford

Hanford–Livingston
Challenge of Gravitational Wave Searches

- A network of gravitational wave detectors is always on and sensitive to most of the sky.
- Signals can be milliseconds long or last for years.
- Multiple signals could be in band but with different amplitudes.
- We can integrate and build SNR by coherently tracking signals in phase.

Hanford-Livingston-Virgo

Hanford-Livingston-Virgo-KAGRA-India

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How confident our detections are likely to be?

The Big Dog Event

Cumulative Rate (yr$^{-1}$)

Threshold $\rho_c$

- Background
- Foreground

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Advanced LIGO Sensitivity

LIGO Livingston S6 sensitivity
LIGO Hanford S6 sensitivity
Early AdvLIGO, 2015-16
Mid AdvLIGO, 2016-17
Near Final AdvLIGO, 2017-18
Final AdvLIGO

Aasi et al 2013

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Advanced Detectors: Schedule and Sensitivity

Advanced LIGO

Advanced Virgo

<table>
<thead>
<tr>
<th>Epoch</th>
<th>Run Duration</th>
<th>BNS range (Mpc)</th>
<th>Number of Detections</th>
<th>Median Area (deg²)</th>
<th>% localized within</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>LIGO</td>
<td>Virgo</td>
<td></td>
<td>5 deg²</td>
</tr>
<tr>
<td>2015</td>
<td>3 months</td>
<td>60 ± 20</td>
<td>—</td>
<td>0.0004 - 3</td>
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<td>2016–17</td>
<td>6 months</td>
<td>100 ± 20</td>
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<td>2022+ (India)</td>
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<td>130</td>
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Advanced Detectors: Schedule and Sensitivity

### Advanced LIGO

- **Early (2015, 60 ± 20 Mpc)**
- **Mid (2016–17, 100 ± 20 Mpc)**
- **Late (2017–18, 140 ± 30 Mpc)**
- **Final (2019, 200 Mpc)**
- **BNS-optimized (2020, 215 Mpc)**

### Advanced Virgo

- **Early (2016–17, 40 ± 20 Mpc)**
- **Mid (2017–18, 70 ± 15 Mpc)**
- **Late (2018–20, 100 ± 15 Mpc)**
- **Final (2021, 130 Mpc)**

### Table: Summary of Observing Schedule

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<th>BNS range (Mpc)</th>
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<th>% Localized within 5 deg²</th>
<th>% Localized within 20 deg²</th>
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<td>2015</td>
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Advanced Detectors: Schedule and Sensitivity

The operation of the LIGO/Virgo network over the next decade will improve both sky coverage and localisation capabilities beyond those envisioned here. KAGRA in this document, we note that the addition of KAGRA to the worldwide GW detector systems with advanced detector network will be designed to have a BNS range greater than \( 10^{11} \) Mpc at final sensitivity. While we do not consider configurations are shown in Fig. mj to a BNS range of \( 10^{11} \) Mpc circa nlnmj. The sensitivity curves for the various AdV stages is expected to bring AdV to a BNS range of \( 10^{12} \) Mpc circa nlnmj. The final design sensitivity corresponding is expected to bring AdV to a BNS range of \( 10^{12} \) Mpc circa nlnm. The sensitivity for each of these stages is shown in Fig. mj.

The commissioning timeline for AdV [p] is still being defined, but it is anticipated that in nlmq AdV may join the LIGO detectors in their first science run depending on the sensitivity attained. The site development would start in nlmq, assuming no unexpected problems. First runs are anticipated circa nlnl and design sensitivity at the same level as the Hm and Lm detectors with installation of the detector beginning in nlmj. Assuming no unexpected problems, first commissioning runs are anticipated circa nlnl and design sensitivity at the same level as the Hm and Lm detectors.

The overall progression represents our best current estimates. While both dates and sensitivity curves are subject to change, notions of the progression of sensitivity are given for early, middle, and late commissioning phases. The BNS range for a specific class of astrophysical signal, such as BNS, the average distance to which binary neutron star dBNSe signals could be seen is given in Mpc. Current

Detection rates are computed assuming a false alarm rate of...
Sources in advanced detectors

Signal strengths and sensitivities (Hz^{-1/2})

Frequency (Hz)

- BBH 250 Mpc
- BNS 450 Mpc
- Sco-X1
- Crab
- LMXBs
- NS (ε=10^{-6})
- aLIGO
- E=10^{12} M_⊙
- f-mode
- 1 kpc
- 10 kpc

Sources in advanced detectors
Beyond Advanced Detectors:
Einstein Telescope
Gravity Gradient Limits Detectors on Ground

- Early (2015, 40-80 Mpc)
- Mid (2016-17, 80-120 Mpc)
- Late (2017-2018, 120-170 Mpc)
- Design (2019, 200 Mpc)
- BNS-optimized (215 Mpc)
- Enhanced aLIGO (2023+)

Frequency (Hz) vs. Noise amplitude spectrum (per Hz)
Underground detectors should have Significant reduction in GG
ET Topology
ET’s Null Stream

- Given a network of two collocated and three or more non-collocated detectors it is possible to construct a linear combination of the responses that is completely devoid of any gravitational waves.
  - For detectors that are not collocated different linear combinations are required for different directions on the sky.
- For ET the linear combination is the same for all directions on the sky.
  - It is just the sum of the responses from the three triangular detectors.
  - This is called the null stream and contains no gravitational wave signals.
  - Extremely useful for understanding detector noise.
Sources in advanced detectors

![Graph showing signal strengths and sensitivities against frequency for various sources such as BBH, BNS, NS, LMXBs, and Crab in advanced detectors.](image-url)
Fundamental Physics, Astrophysics and Cosmology with Ground Based Detectors
Cosmology

- **Cosmography**
  - Strengthen existing distance calibrations at high $z$
  - Calibration–free measurements of distance and cosmological parameters

- **Black hole seeds**
  - Black hole seeds could be stellar mass or intermediate mass black holes
  - Explore hierarchical growth of central engines of black holes

- **Anisotropic cosmologies**
  - In an anisotropic Universe the distribution of $H$ on the sky should show residual quadrupole and higher–order anisotropies

- **Primordial gravitational waves**
  - Quantum fluctuations in the early Universe produce a stochastic $b/g$

- **Production of GW during early Universe phase transitions**
  - Phase transitions, pre–heating, re–heating, etc., could produce detectable stochastic GW
Probing black hole mergers at $z \sim 10^{-20}$

![Graph showing luminosity distance vs. total mass (in $M_\odot$) for observed and intrinsic mass, with lines for non-spinning and spinning systems, and redshift $z$.]

- Blue dashed line: Sky-ave. dist. vs Obs. M, $\nu=0.25$, $\chi=0$
- Red solid line: Sky-ave. dist. vs Phys. M, $\nu=0.25$, $\chi=0$
- Blue dotted line: Sky-ave. dist. vs Obs. M, $\nu=0.25$, $\chi=0.75$
- Red dotted line: Sky-ave. dist. vs Phys. M, $\nu=0.25$, $\chi=0.75$
is further augmented by a factor of 1.12. At this rate, we find that one year of observation should be enough to measure $H_0$ to an accuracy of $\sim 1\%$ if SHBs are dominated by beamed NS-BH binaries using the “full” network of LIGO, Virgo, AIGO, and LCGT—admittedly,
ET will observe 100’s of binary neutron stars and GRB associations each year.

GRBs could give the host location and redshift, GW observation provides $D_L$.

Figure 3. Scatter plot of the retrieved values for $(\Omega_\Lambda, w)$, with 1-$\sigma$, 2-$\sigma$ and 3-$\sigma$ contours, in the case where weak lensing is not corrected.
Measuring \( w \) and its variation with \( z \)

\[ w(z) \equiv \frac{p_{de}}{\rho_{de}} = w_0 + w_a z / (1 + z) \]
Fundamental Physics

- The two body problem in general relativity
- Properties of gravitational waves
  - Testing GR beyond the quadrupole formula
  - How many polarizations are there?
  - Do gravitational waves travel at the speed of light?
- EoS of dark energy
  - Black hole binaries are standard candles/sirens
- EoS of supra-nuclear matter
  - Signature of EoS in GW emitted when neutron stars merge
- Black hole no-hair theorem and cosmic censorship
  - Are BH candidates of nature BH of general relativity?
- An independent constraint/measurement of neutrino mass
  - Delay in the arrival times of neutrinos and gravitational waves
Binary black hole dynamics

- The signal from a binary black hole is characterized by:
  - slow adiabatic inspiral – the two bodies slowly spiral in towards each other; dynamics well described by post-Newtonian approximation
  - fast and luminous merger phase; requires numerical solutions to Einstein equations
  - rapid ringdown phase; newly black hole emits quasi-normal radiation
- The shape of the signal contains information about the binary
The shape of the signal is determined by masses, spins and eccentricity.

The amplitude and arrival times in different detectors are determined by the distance, direction, polarization and inclination.

See SB’s talk on July 5
Testing Black Hole No-Hair Theorem

- Deformed black holes are unstable; they emit energy in their deformation as gravitational waves

- Superposition of damped waves with many different frequencies and decay times

- In Einstein’s theory, frequencies and decay times all depend only on the mass $M$ and spin $j$ of the black hole

- Measuring two or modes would constrain Einstein’s theory or provide a smoking gun evidence of black holes

- If modes depend on other parameters (e.g., the structure of the central object), then test of the consistency between different mode frequencies and damping times would fail

- The amplitude of the modes carry additional information about what caused the deformity

Astrophysics

- Unveiling progenitors of short-hard GRBs
  - Understand the demographics and different classes of short-hard GRBs

- Understanding Supernovae
  - Astrophysics of gravitational collapse and accompanying supernova?

- Evolutionary paths of compact binaries
  - Evolution of compact binaries involves complex astrophysics

- Finding why pulsars glitch and magnetars flare
  - What causes sudden excursions in pulsar spin frequencies and what is behind ultra high-energy transients of EM radiation in magnetars

- Ellipticity of neutron stars as small as 1 part in a billion (10μm)
  - Mountains of what size can be supported on neutron stars?

- NS spin frequencies in LMXBs
  - Why are spin frequencies of neutron stars in low-mass X-ray binaries bounded?

- Onset/evolution of relativistic instabilities
  - CFS instability and r-modes
Binary Neutron Stars

- These are systems we know exist and we should see them
- Rates are highly uncertain
  - Advanced detectors could see events in the range 0.5 to 400 per year
- Observed event rates will constrain models of formation and evolution of compact binaries
- Can measure masses and spins and possibly equation of state of supra-nuclear matter

See SB’s talk on July 5
Progenitors of GRBs

- What causes these giant explosions?
- What are the different classes of GRBs?
- Synergy between EM and GW Astronomy
  - Distances measured with GW
  - Redshift measured with EM
  - Could potentially be very useful for cosmography

See SB’s talk on July 5

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Gravitational Astronomy

- We expect gravitational waves to be detected before the end of this decade.
  - Detections could come from either Pulsar Timing Arrays or interferometers.
- Scientific potential of future detectors, eLISA and ET, is huge.
- Fundamental Physics
  - Is the nature of gravitational radiation as predicted by Einstein?
  - Is Einstein theory the correct theory of gravity?
  - Are black holes in nature black holes of GR and are there naked singularities?
- Astrophysics
  - What is the nature of gravitational collapse?
  - What is the origin of gamma ray bursts?
  - What is the structure of neutron stars and other compact objects?
- Cosmology
  - How did massive black holes at galactic nuclei form and evolve?
  - What is dark energy?
  - What phase transitions took place in the early Universe?
  - What were the physical conditions at the big bang and what role did quantum gravity in the early evolution of the Universe?