Phase structure of finite temperature QCD in the heavy quark mass region

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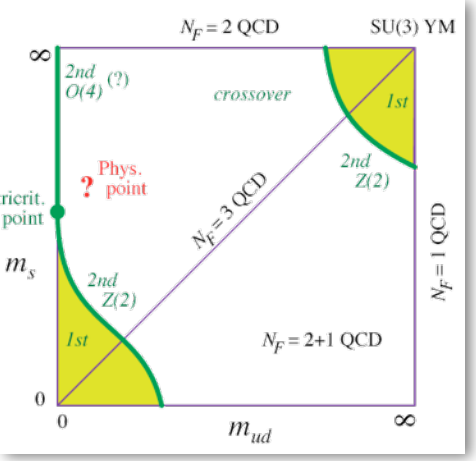
Introduction

- **QCD phase diagram**
  - Low temperature **Confinement**
    ↔ High temperature **QGP**
  - The order of the phase transition and critical point are important to understand the transition.
  - The order of the transition depends on quark mass.

K. Kanaya
arXiv:1012.4247



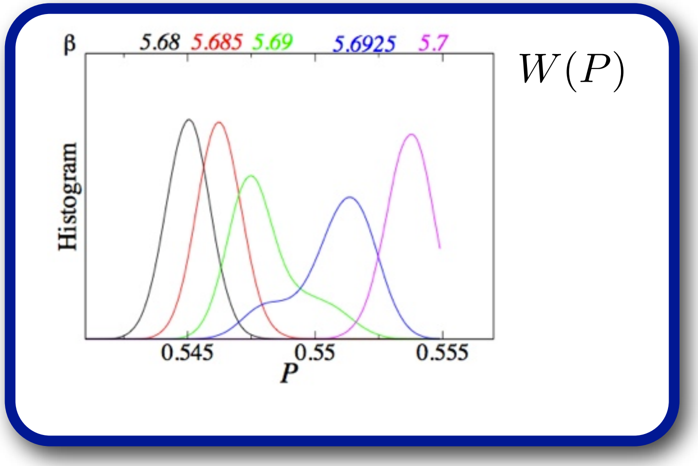
- **In this study**
  - The critical point for $N_f = 2+1$ case in heavy quark region
  - Test a method:
    { probability distribution function
      reweighting

Effective Potential

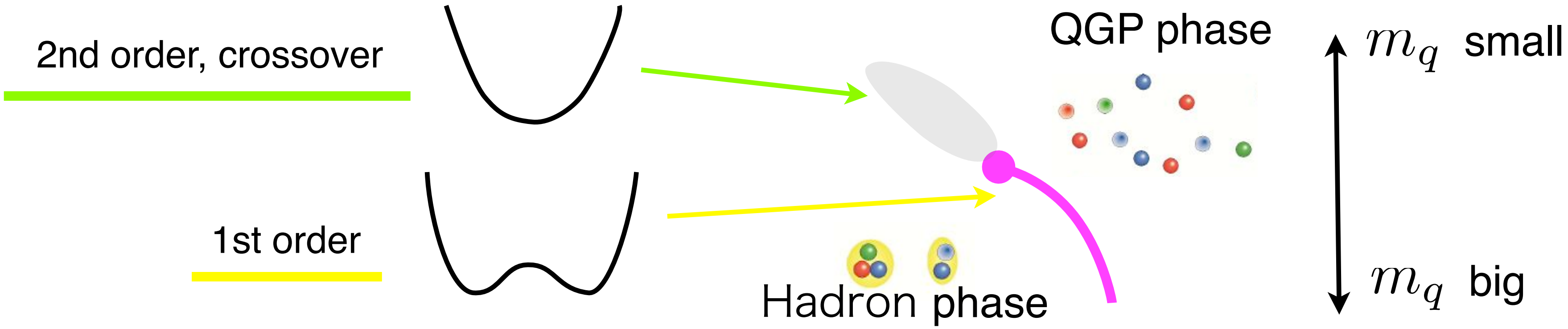
- Effective potential by histogram
  - Definition: $V_{eff}(P) = -\ln W(P)$

$$W(P') = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\,\delta(P(U)-P')e^{-S}$$

  - Ex. 1st order transition for pure gauge theory



- Quark mass dependence



2nd order, crossover

1st order

QGP phase

Hadron phase

m_q small

m_q big

Effective Potential (Quark mass dependence) 1

- Histogram

$$W(P') = R(P',\kappa)W_0(P',\beta)$$

$$\kappa = \frac{1}{2(m_q a + 4)}$$

$$W_0(P',\beta) \equiv \int \mathcal{D}U\,\delta(P(U) - P')e^{-S_g(\beta)}$$

$$R(P',\kappa) \equiv \frac{\int \mathcal{D}U\,\delta(P(U) - P')[\det M(U,\kappa)]^{N_f}}{\int \mathcal{D}U\,\delta(P(U) - P')}$$

- Effective potential

$$V_{\mathrm{eff}}(P,\kappa,\beta) = \boxed{-\ln R(P,\kappa)} + V_0(P,\beta)$$

Reweighting in κ

$$V_0(P,\beta) \equiv -\ln W_0(P,\beta)$$

- Action

Plaquette action:

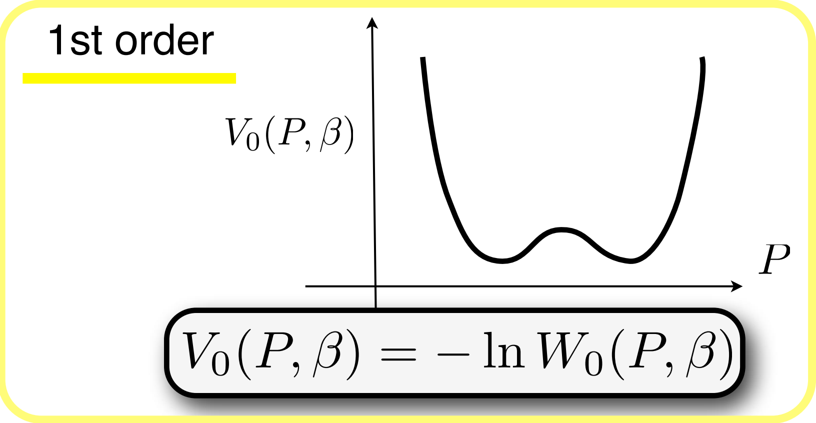
$$S_g = -\frac{\beta}{3}\sum_n \sum_{\mu<\nu} Re tr P_{\mu\nu} \equiv -6\beta N_{\mathrm{site}} P$$

Wilson Fermion:

$$S_q = \sum_n \bar{\psi}_n \psi_n - \kappa \sum_{n,\mu} \bar{\psi}_n[(r - \gamma_\mu)U_{n,\mu}\psi_{n+\hat{\mu}} + (r + \gamma_\mu)U^\dagger_{n,\mu}\psi_{n-\hat{\mu}}] \equiv \sum \bar{\psi} M(U,\kappa)\psi$$

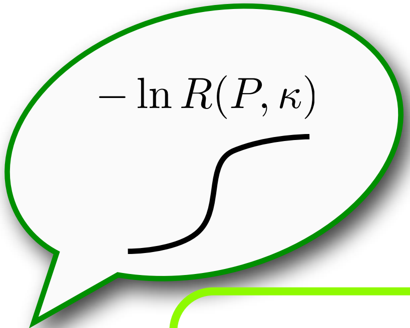
Effective Potential (Quark mass dependence) 2

- The shape of the V_{eff} discriminates the order of the transition.

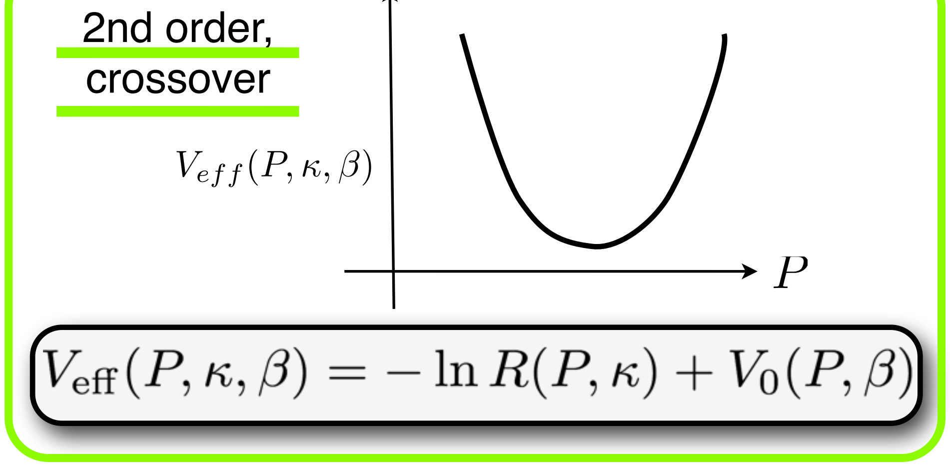


$$V_0(P,\beta) = -\ln W_0(P,\beta)$$

It is known that for pure gauge the order of the transition is 1st.



The transition becomes to be crossover when the effective potential comes to have only one minimum.



$$V_{\mathrm{eff}}(P,\kappa,\beta) = -\ln R(P,\kappa) + V_0(P,\beta)$$

Quark mass dependence

- hopping parameter expansion

$$N_{\mathrm{f}} \ln \left[\frac{\det M(\kappa)}{\det M(0)} \right] = N_{\mathrm{f}} \left[288 N_{\mathrm{site}} \kappa^4 P + 12 \cdot 2^{N_t} N_{\mathrm{s}}^3 \kappa^{N_t} \mathrm{Re}\Omega + \dots \right]$$

$$P = \frac{1}{6N_{\mathrm{site}}} \sum_{n,\mu<\nu} \frac{1}{3} \mathrm{Re\ tr} \left[U_{n,\mu} U_{n+\hat{\mu},\nu} U^{\dagger}_{n+\hat{\nu},\mu} U^{\dagger}_{n,\nu} \right]$$: plaquette

$$\Omega = \frac{1}{N_s^3} \sum_{\mathbf{n}} \frac{1}{3} \mathrm{tr} \left[U_{\mathbf{n},4} U_{\mathbf{n}+\hat{4},4} U_{\mathbf{n}+2\hat{4},4} \cdots U_{\mathbf{n}+(N_t-1)\hat{4},4} \right]$$

: Real part of polyakov loop

  - We can choose the κ values freely.

Simulation setup

- HeatBath

  $\det M(\kappa, P)$ is evaluated by using a hopping parameter expansion.

- β & Number of Configurations

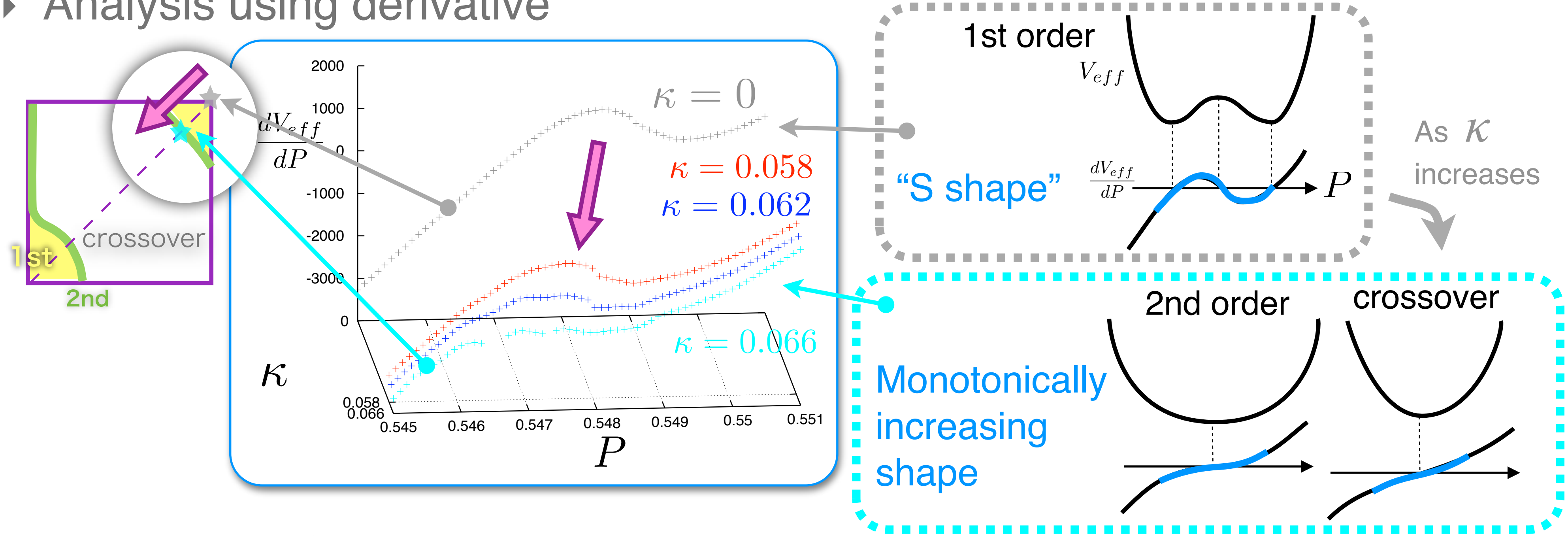
β	Conf. num.
5.68	100,000
5.685	430,000
5.69	500,000
5.6925	670,000
5.7	100,000

⇦ transition point

- Lattice size: 24^3 x 4

Derivative

- Analysis using derivative

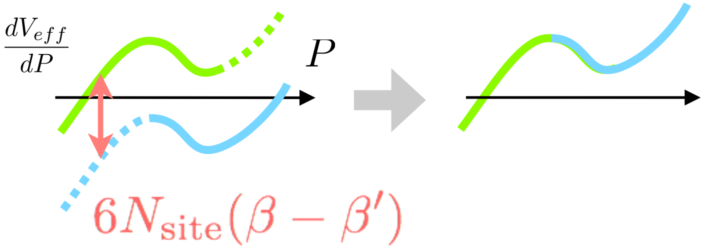


- Advantage of derivative

  By reweighting method

$$\frac{dV_0(P,\beta)}{dP} = \frac{dV_0(P,\beta')}{dP} - 6N_{\mathrm{site}}(\beta-\beta')$$

const.



Using this relation, we can obtain the derivative in wide range and S-shape indicating 1st order transition easier.

Result 1 - κ Dependence of the transition point -

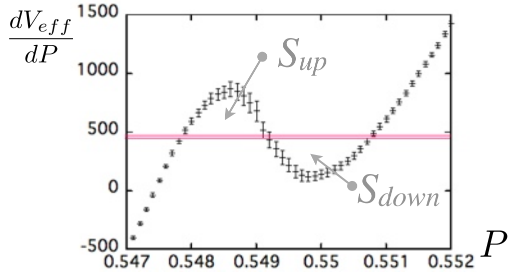
- Identifying the transition point

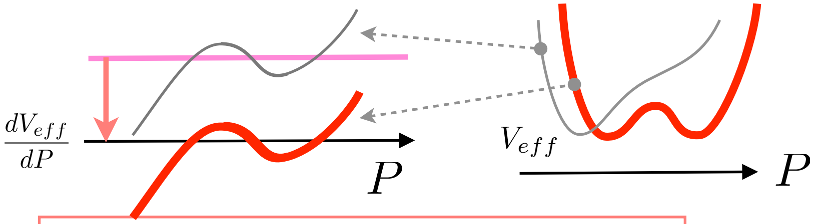
  1. draw a line which is parallel to horizontal line and which satisfy Maxwell construction between the S_{up} and S_{down}.

  2. Shift → Transition point

Maxwell construction

$S_{up}=S_{down}$

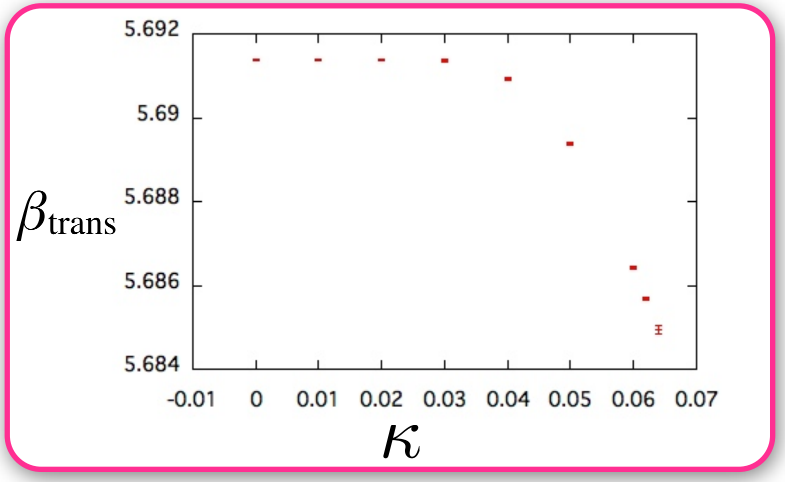




$$\frac{dV_0(P,\beta)}{dP} = \frac{dV_0(P,\beta')}{dP} - \boxed{6N_{\rm site}(\beta-\beta')}$$

- The transition point depends on the quark mass.

  As κ increases, $\beta_{\rm trans}$ value decreases.



Result 2 - Latent Heat -

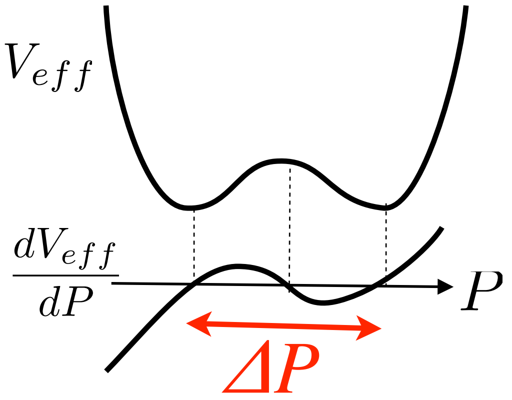
- Latent heat in pure gauge

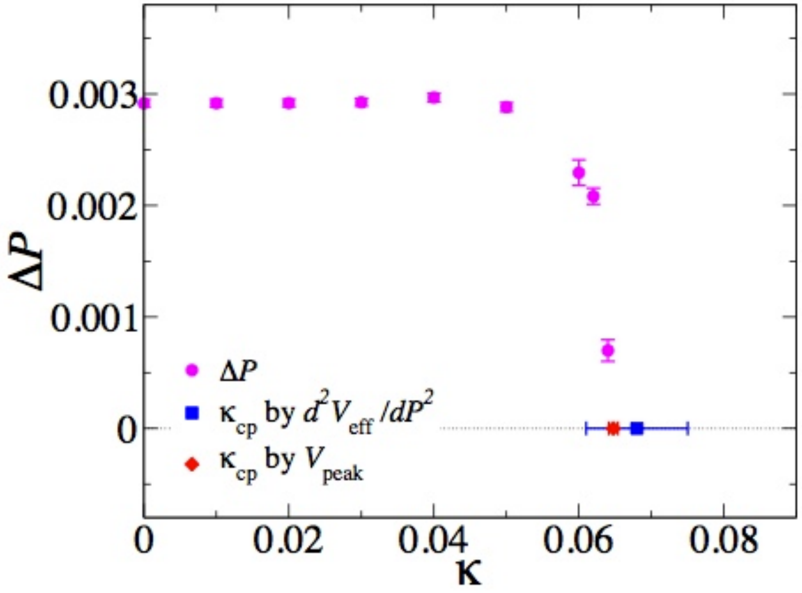
$$\frac{\varepsilon - 3p}{T^4} = -\frac{1}{VT^3} a \frac{\partial \ln \mathcal{Z}}{\partial a} - [T=0]$$

$$= -N_t^4 \left[N_{\rm f} a \frac{\partial \kappa}{\partial a} \left(\kappa^3 1152 \langle P \rangle + N_t^{-1} \kappa^{N_t - 1} 12 \times 2^{N_t} \langle {\rm Re}\Omega \rangle \right) + \underline{a \frac{\partial \beta}{\partial a} 6 \langle P \rangle} + O(\kappa^6) \right] - [T=0]$$

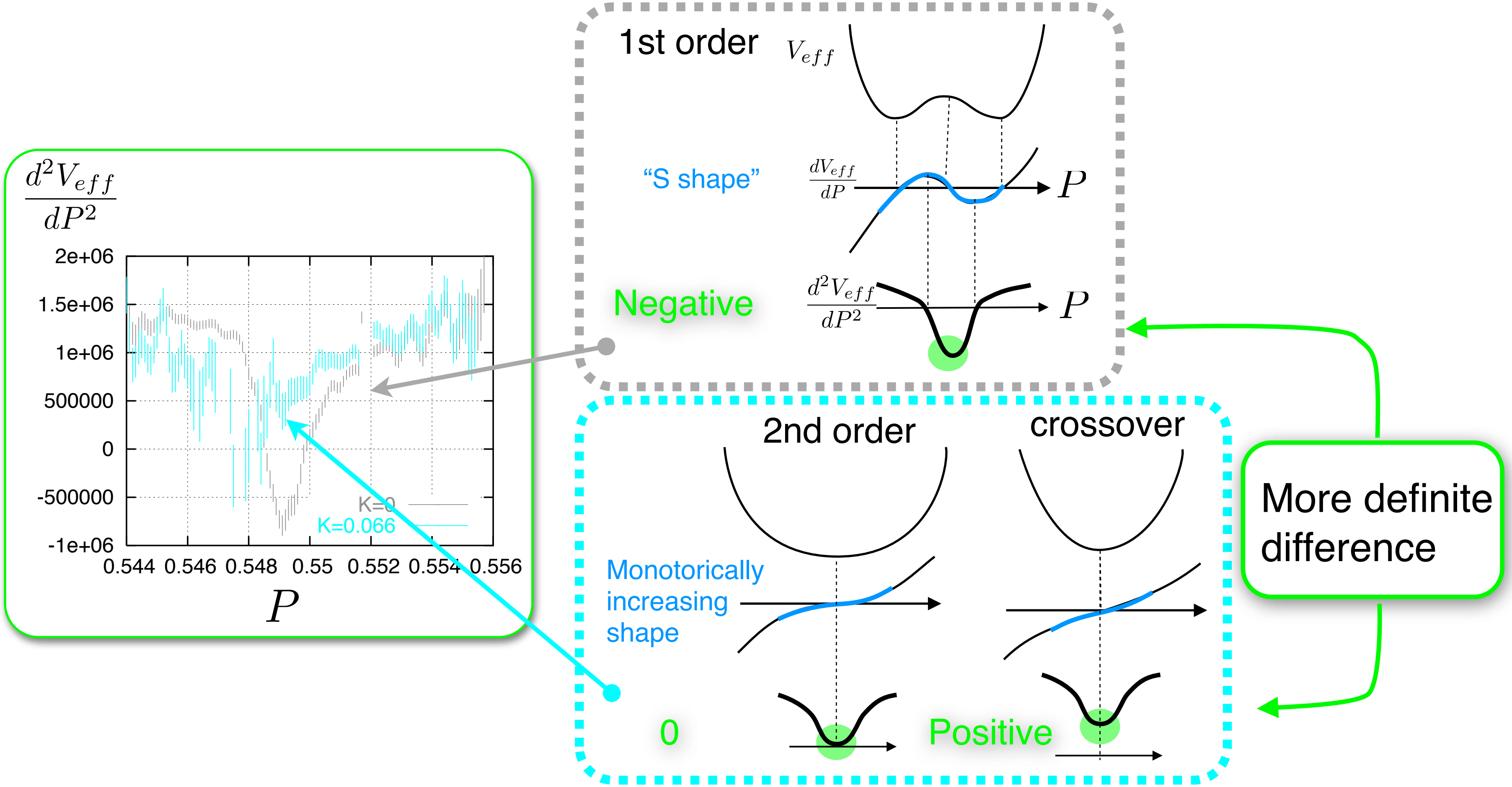
- In pure gauge $\kappa = 0$

$$\frac{\Delta\varepsilon}{T^4} = \frac{\Delta(\varepsilon - 3p)}{T^4} \approx -6 N_t^4 a \frac{\partial \beta}{\partial a} \langle \Delta P \rangle$$





Second Derivative



1st order

"S shape"

Negative

2nd order

crossover

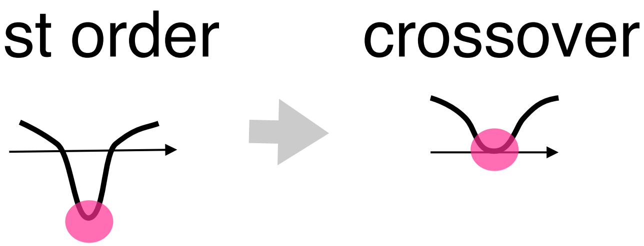
Monotorically increasing shape

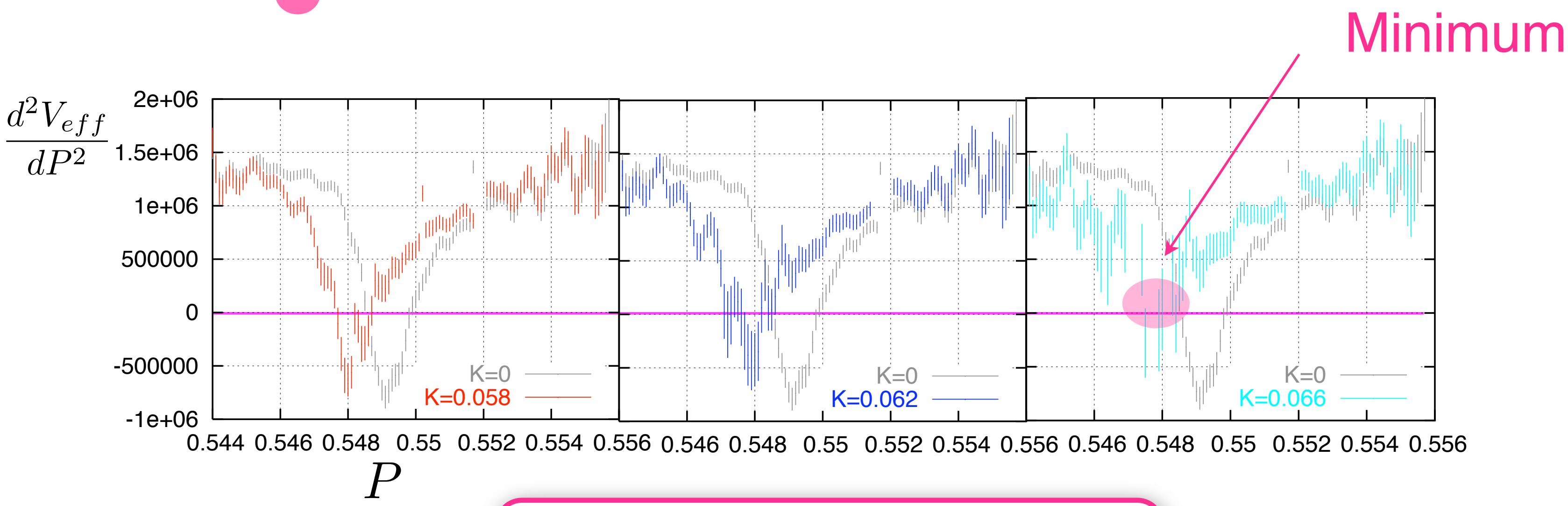
0

Positive

More definite difference

Second Derivative





Minimum

critical point

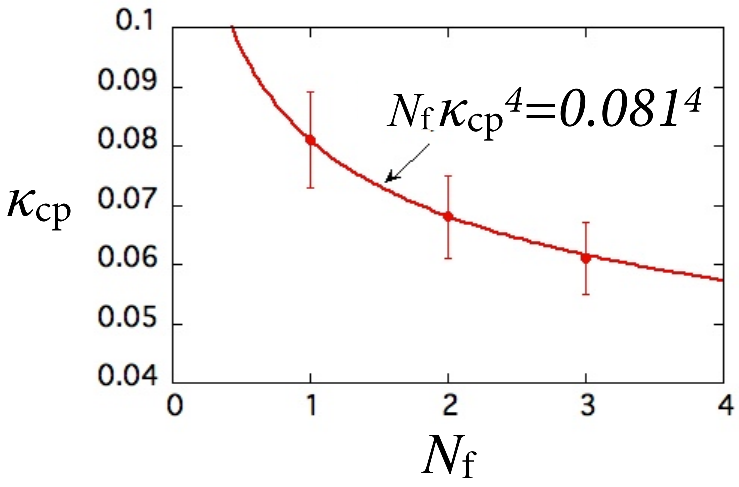
$\kappa_{\mathrm{cp}}=0.068(7)$ for $N_f=2$

Result 3 - Dependence on the number of flavors -

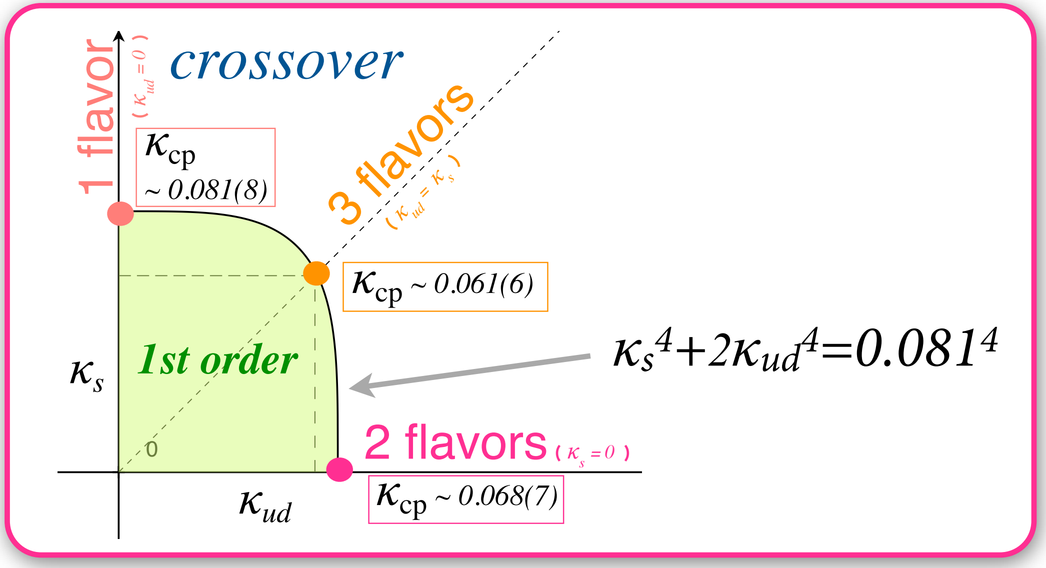
- Critical point in case of N_f=1, 2, 3
  - ✓ We can get the kappa critical for another -flavor case since in hopping parameter expansion the same value of this term cause the same phenomena.
  - ✓ As N_f increases, κ_{cp} decreases.

  Z(3) Effective model by Alexandrou(1990)

  $\kappa_{cp} \sim 0.08$ for $N_f = 1$



- In case of $N_f = 2+1$ case

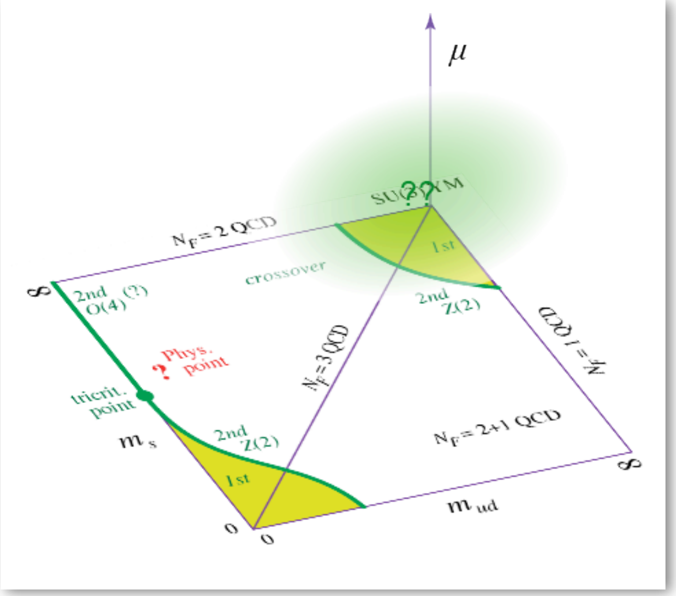


2nd Part ($\mu \neq 0$)

Introduction for Finite Density

Deconfinement phase transition at finite μ

- sign problem $[\det M(\mu)]^* \neq \det M(\mu)$
  - ✓ It is important to find out the feature of the complex phase.
- μ dependence of the critical point



In this study

- The critical point for $N_f = 2+1$ case in heavy quark mass region at finite μ
- To find out the complex phase at finite μ in heavy quark region

Finite chemical potential in heavy quark region

- finite chemical potential

$$\boxed{U_{n,4} \to e^{\mu_q a} U_{n,4} \quad U^\dagger_{n,4} \to e^{-\mu_q a} U^\dagger_{n,4}}$$

➡ $$\boxed{\Omega \to e^{\mu_q/T}\Omega \qquad \Omega^* \to e^{-\mu_q/T}\Omega^*}$$

- hopping parameter expansion

$$N_{\mathrm{f}} \ln \left[\frac{\det M(\kappa,\mu)}{\det M(0,0)} \right]$$

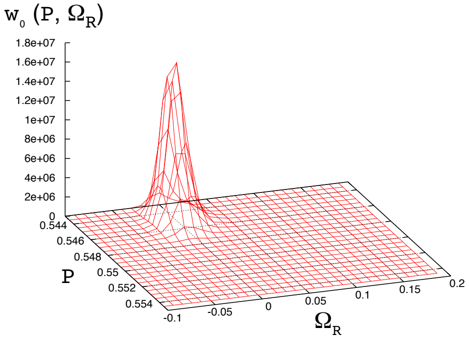
$$= N_{\mathrm{f}} \left[288 N_{\mathrm{site}} \kappa^4 P + 12 \cdot 2^{N_t} N_{\mathrm{s}}^3 \kappa^{N_t} \cosh\left(\frac{\mu}{T}\right) \left\{ \mathrm{Re}\Omega + \underbrace{i \tanh\left(\frac{\mu}{T}\right) \mathrm{Im}\Omega}_{\text{complex phase}} \right\} + \dots \right]$$

- probability distribution function : P, Ω fixed

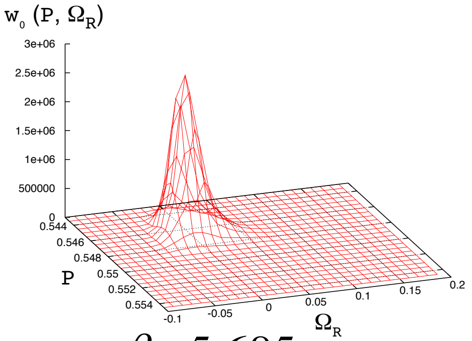
$$w(P, \Omega_R, \kappa, \beta, \mu)$$

$$= \int \mathcal{D}U \delta(P(U) - P')\delta(\Omega_R(U) - \Omega') \left[\det M(\kappa,\mu)\right]^{N_{\mathrm{f}}} e^{6N_{\mathrm{site}}\beta P}$$

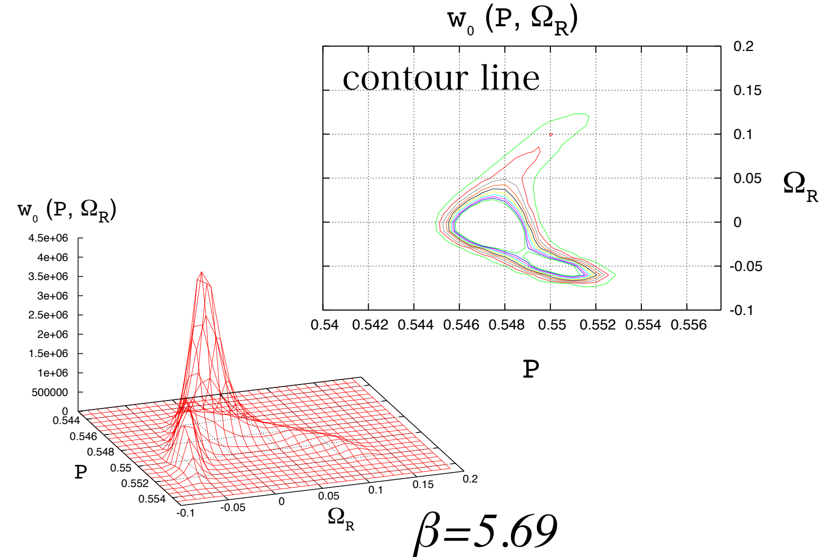
Probability distribution function (P, Ω fixed)



$\beta=5.68$

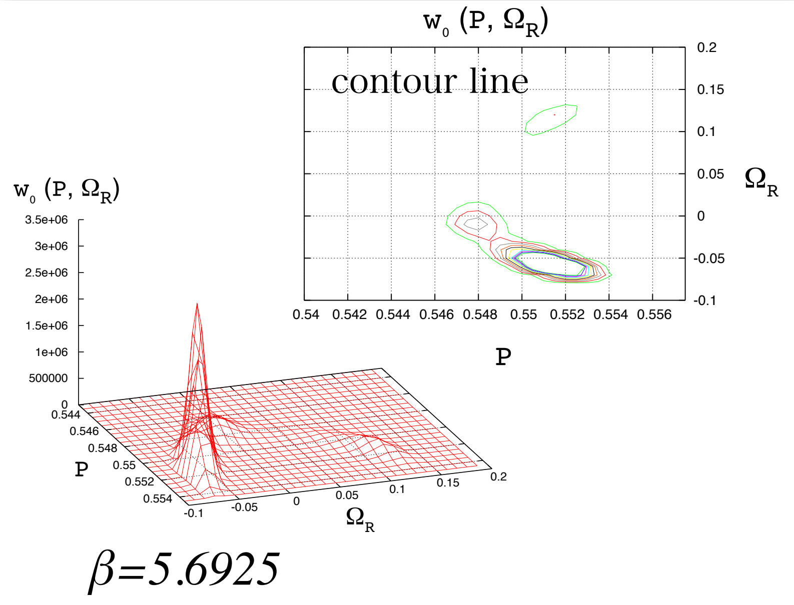


$\beta=5.685$

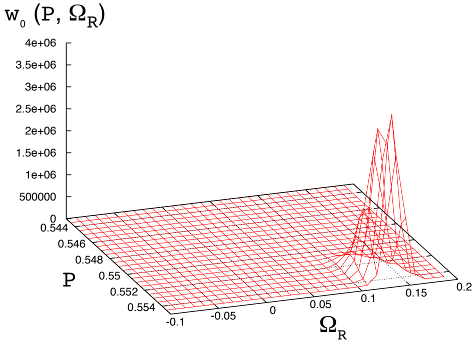


$\beta=5.69$

Two-peak structure



$\beta=5.6925$



$\beta=5.7$

Ex.) Isospin chemical potential in heavy quark region

- hopping parameter expansion

$$\ln\left[\frac{\det M(\kappa,\mu)\det M(\kappa,-\mu)}{\det M(0,0)^2}\right]$$
$$= 2\left[288N_{\rm site}\kappa^4 P + 12\cdot 2^{N_t} N_{\rm s}^3 \kappa^{N_t}\cosh(\frac{\mu}{T})\mathrm{Re}\Omega + \ldots\right]$$

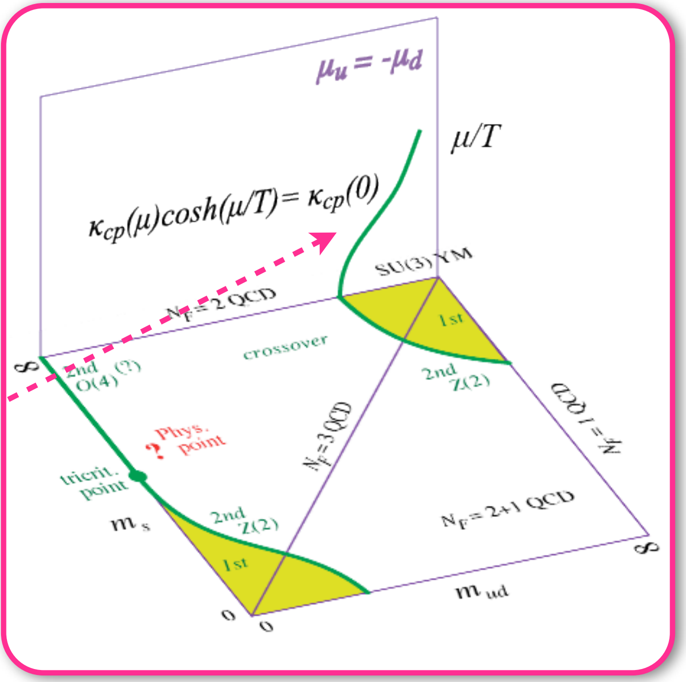
✓ isospin chemical potential

$$\mu_u = -\mu_d$$

✓ complex phase canceled!!

✓ $\kappa_{\rm cp}$ at finite μ (analytically evaluated)

$$\kappa_{\rm cp}(\mu)\cosh(\frac{\mu}{T}) = \kappa_{\rm cp}(0)$$



Cumulant expansion and the complex phase

- cumulant expansion

$\langle \cdots \rangle_{P,\ \Omega_R}$ expectation value fixed P and Ω_R

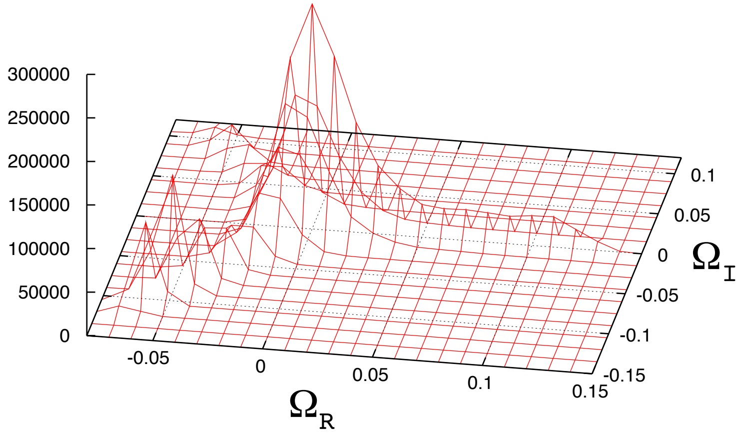
$$\left\langle \exp\left[i N_{\rm f} N_s^3 ab \Omega_I \right] \right\rangle_{P,\ \Omega_R}$$

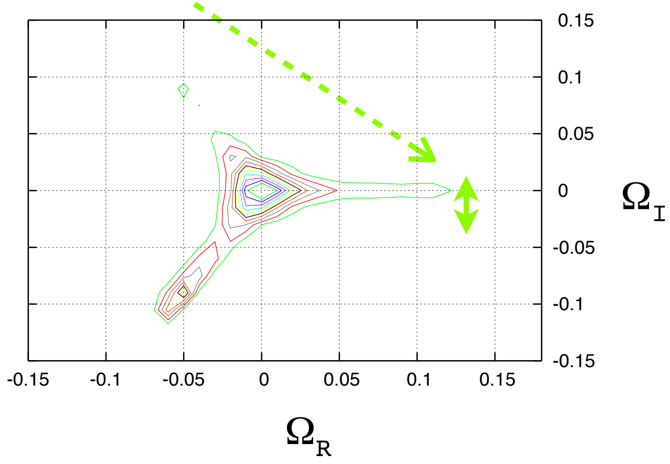
$$= \exp\left[-\frac{(N_{\rm f} N_s^3 ab)^2}{2} \left\langle \Omega_I^2 \right\rangle_{c\ P,\ \Omega_R} + \frac{(N_{\rm f} N_s^3 ab)^4}{4!} \left\langle \Omega_I^4 \right\rangle_{c\ P,\ \Omega_R} + \cdots \right]$$

$$\begin{cases} \left\langle \Omega_I^2 \right\rangle_{c\ P,\ \Omega_R} = \left\langle \Omega_I^2 \right\rangle_{P,\ \Omega_R} - \left\langle \Omega_I \right\rangle^2_{P,\ \Omega_R} \\ \qquad\qquad : \text{dispersion of } \Omega_{\rm I} \text{ (leading)} \\ \left\langle \Omega_I^4 \right\rangle_{c\ P,\ \Omega_R} = \left\langle \Omega_I^4 \right\rangle_{P,\ \Omega_R} - 3 \left\langle \Omega_I \right\rangle^4_{P,\ \Omega_R} \end{cases}$$

The odd term is vanished due to symmetry.

- distribution on (Ω_R, Ω_I) plane





$\kappa_{\rm cp}$ at finite μ can be evaluated !!

Summary

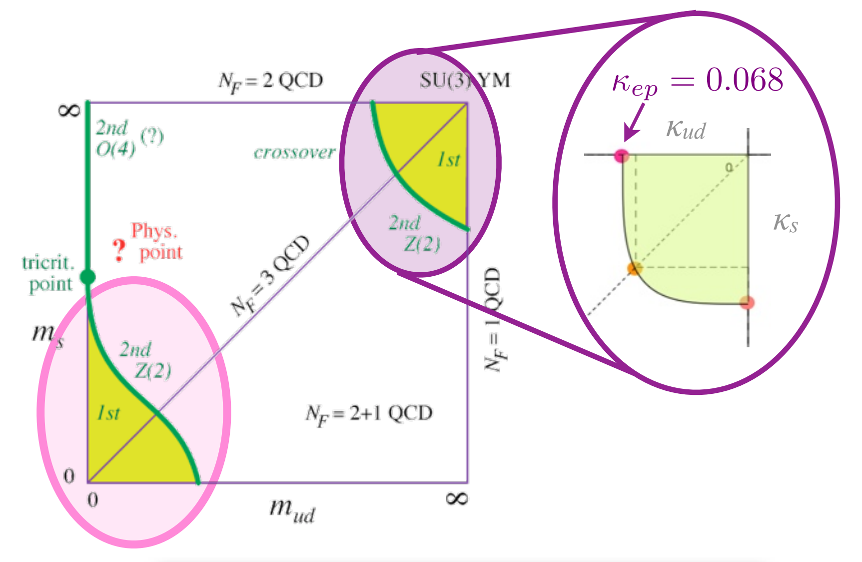
- In heavy quark region

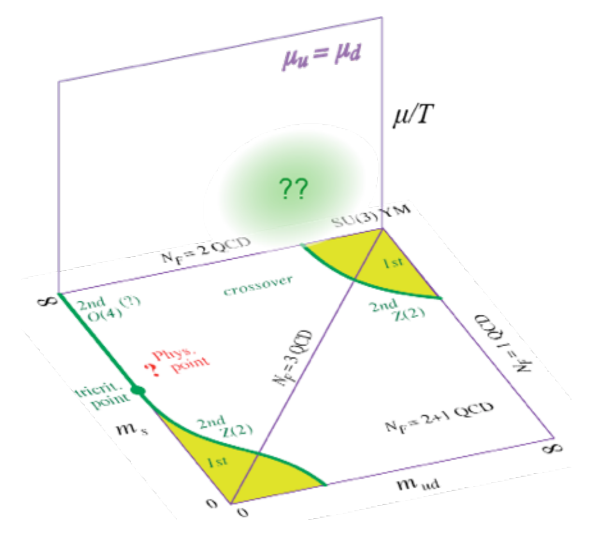
  $\kappa_{\rm cp}=0.068(7)$ for $N_{\rm f}=2$

- As $N_{\rm f}$ increases, $\kappa_{\rm cp}$ decreases.

  It is easy to indicate this nature, since a κ dependence is trivial in hopping parameter expansion. We could get the $\kappa_{\rm cp}$ for 1 flavor which is consist with Alexandrou et al.(1990).

- Latent heat in pure gauge
- Transition posits for each κ
- Reweighting in κ works well to find out $\kappa_{\rm cp}$.
- Critical point of Iso-spin chemical potential





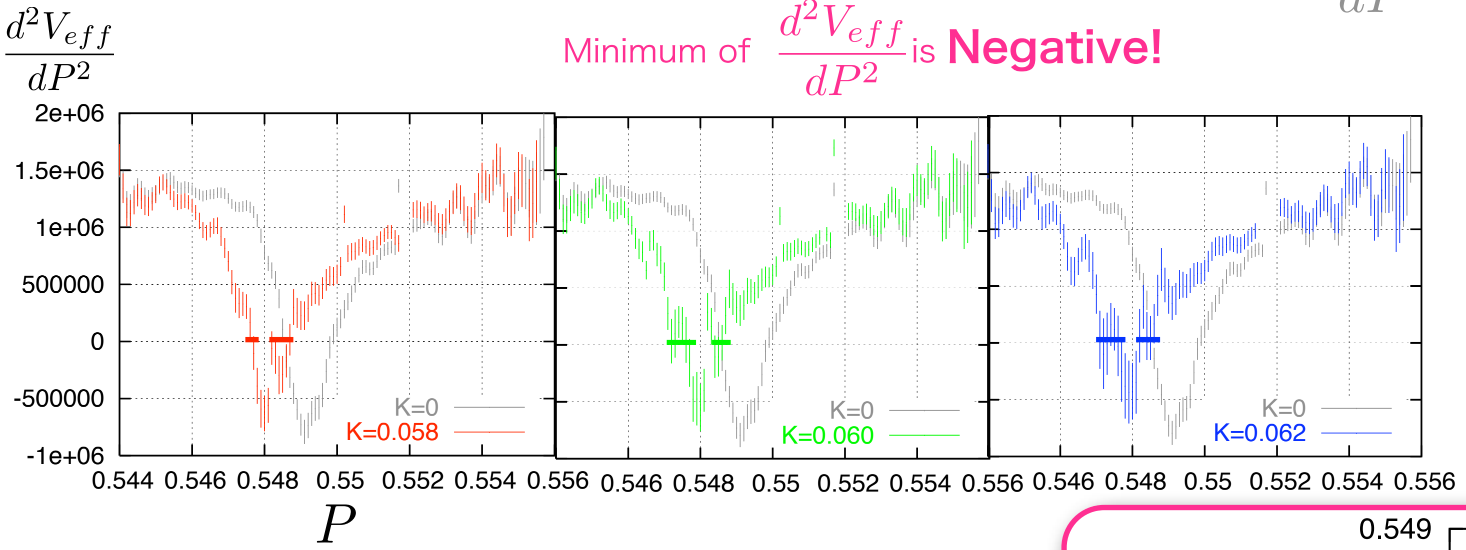
Future

- Critical point at finite μ for $N_{\rm f}=2$ in heavy quark region and light quark mass

Extra Slide

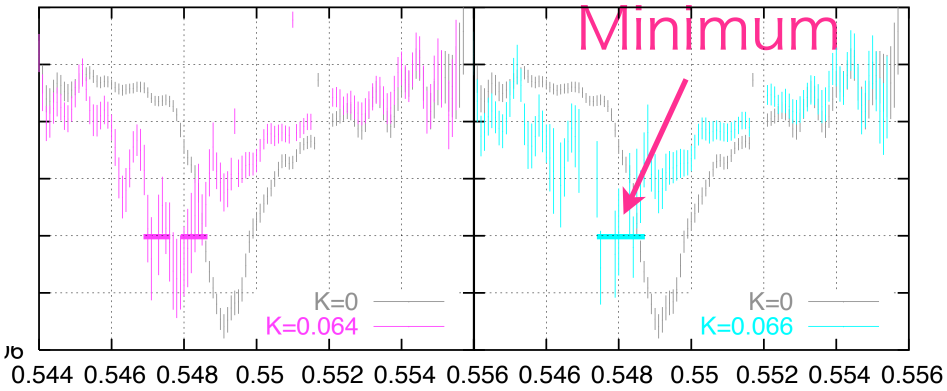
Result - Identification of κ_{ep} value (Detail 1)

- How to decide the candidates for minimum of $\frac{d^2V_{eff}}{dP^2}$.

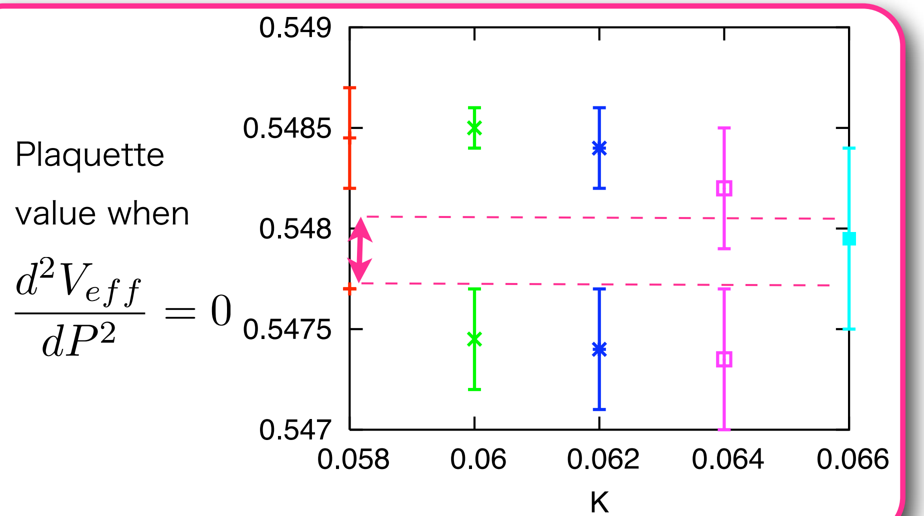


Minimum of $\frac{d^2V_{eff}}{dP^2}$ is **Negative!**

Using the plaquette value when $\frac{d^2V_{eff}}{dP^2} = 0$



Minimum of $\frac{d^2V_{eff}}{dP^2}$ is consistent with **0!**

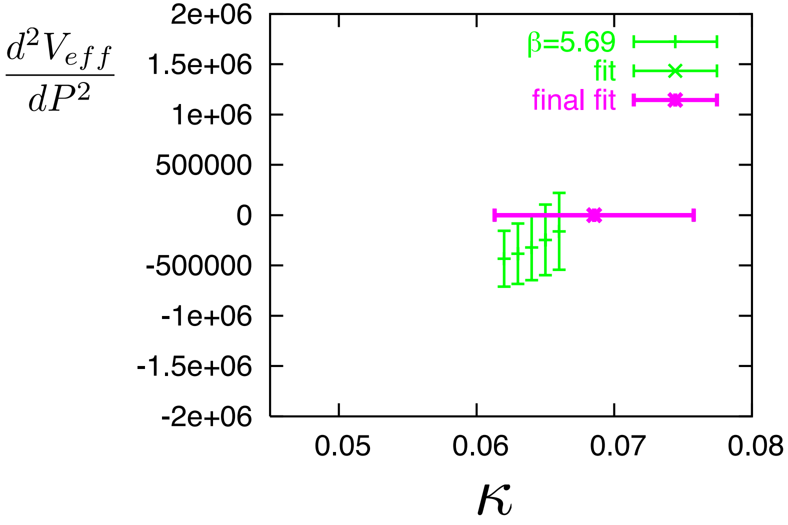


Result - Identification of κ_{ep} value (Detail 2)

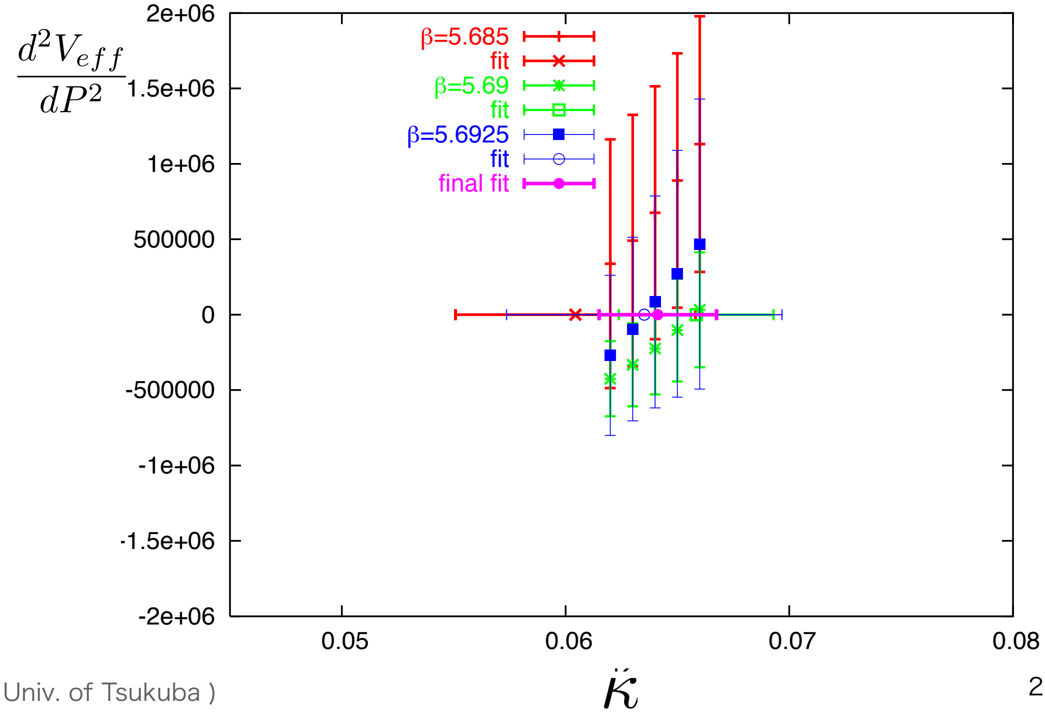
- How to decide the error of Kc

1. jacknife
2. χ^2 fit (for several β data)

single β data

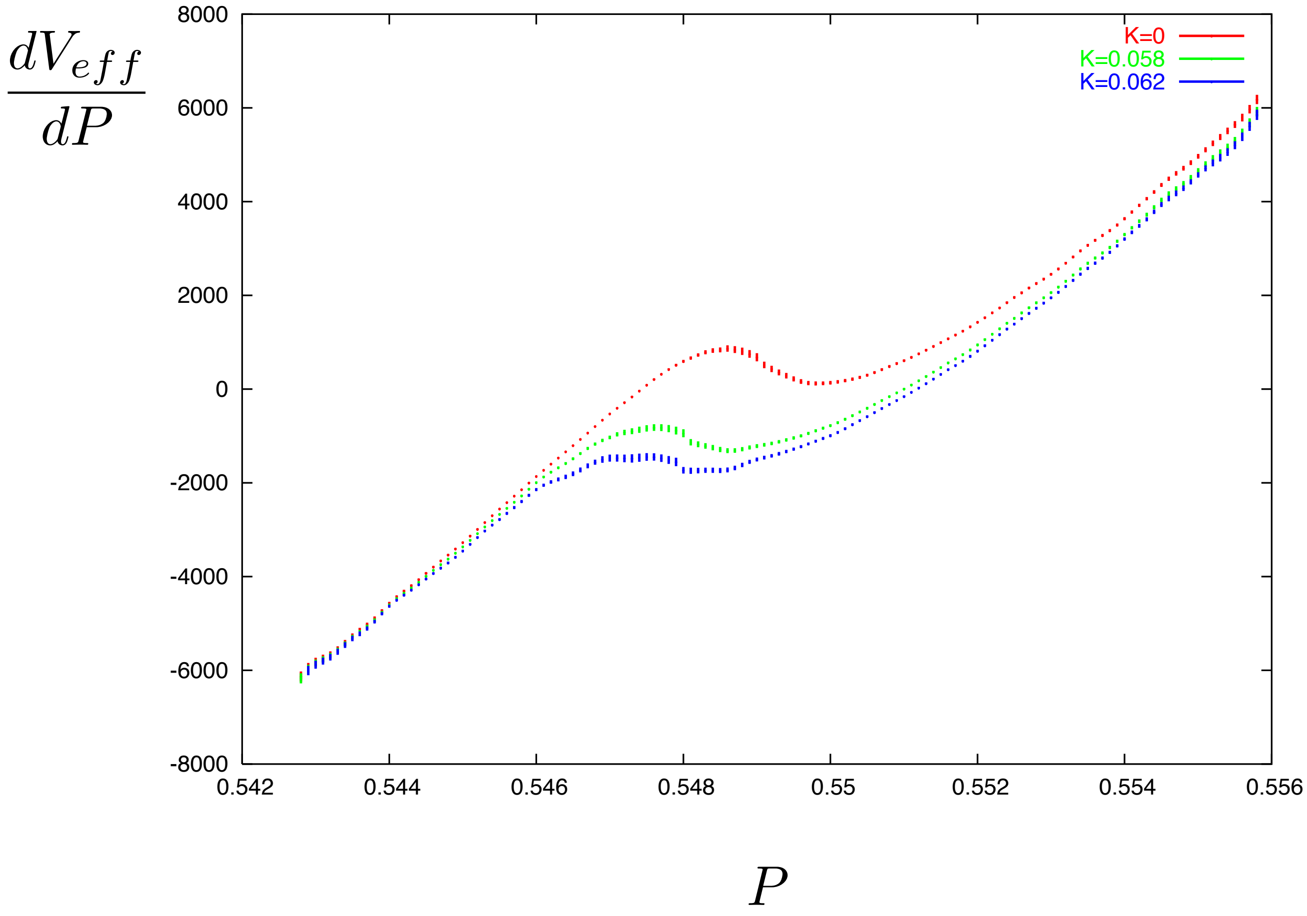


several β data



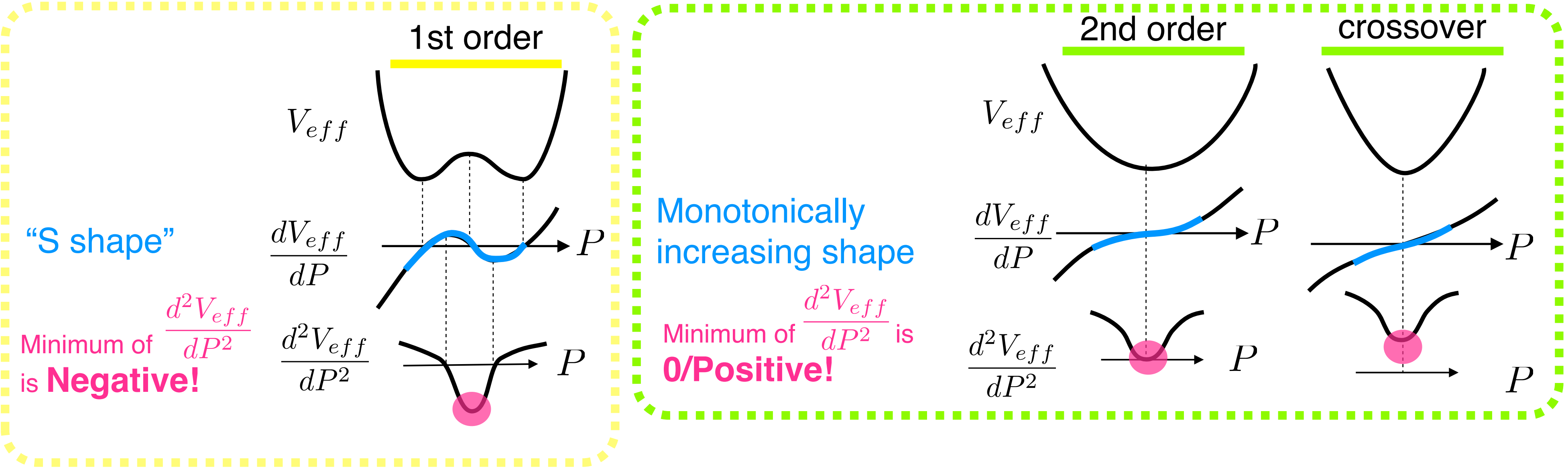
Result - 1st order transition disappears with error bar

- The derivative of V_{eff} with error



Derivative

- Analysis by using the derivative/secondary derivative



“S shape”

Minimum of $\frac{d^2V_{eff}}{dP^2}$ is **Negative!**

Monotonically increasing shape

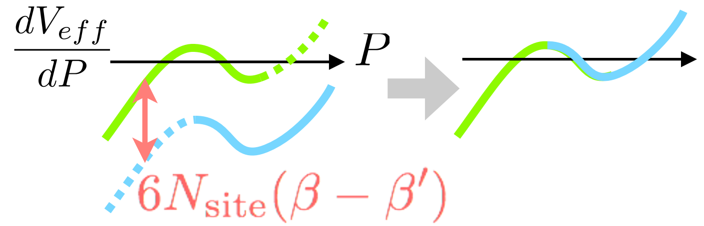
Minimum of $\frac{d^2V_{eff}}{dP^2}$ is **0/Positive!**

- To avoid a large number of confs to identify a 1st order transition

From a reweighting analysis,

$$\frac{dV_0(P,\beta)}{dP} = \frac{dV_0(P,\beta')}{dP} - \underbrace{6N_{\rm site}(\beta-\beta')}_{\text{const.}}$$

⇨ combine data at several β

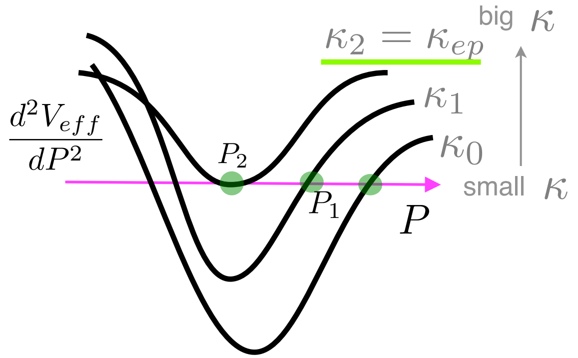


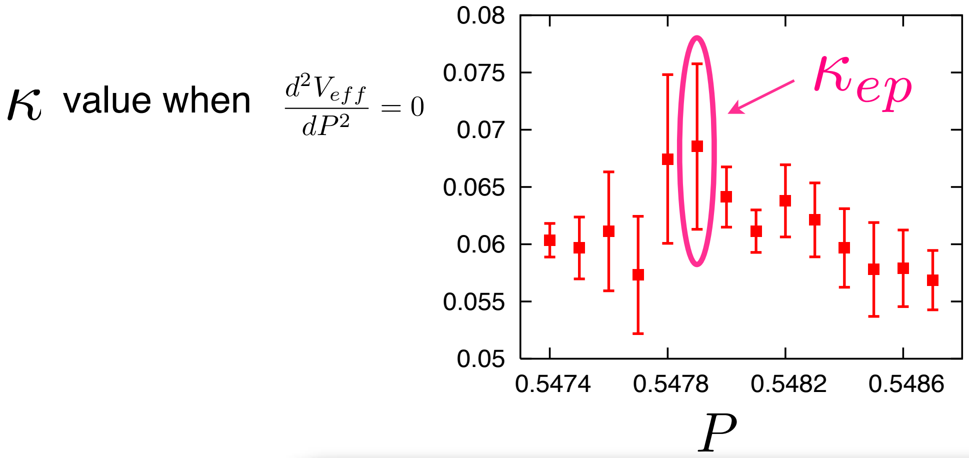
We obtain the derivative of $V_{\rm eff}$ in a wide range.

Result - Identification of κ_{ep} value 2

- Detailed analysis for the ● region

1. Assume that the secondary derivative of V_{eff} changes like this as κ increases.
2. Near the end point, search κ values where $\frac{d^2V_{eff}}{dP^2} = 0$ for each P ➡ ●
3. Maximum of these κ values is identified by κ_{ep}





$$N_f = 2 \qquad \kappa_{ep} = 0.068 \pm 0.007$$