Tutorial 1: Thermal emission from expanding gas

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Physical constants and astrophysical units.

$$
\begin{aligned}
&G && 6.67384 \times 10^{-8}\ \mathrm{cm}^3\ \mathrm{g}^{-1}\mathrm{s}^{-2} \\
&c && 2.99792458 \times 10^{10}\ \mathrm{cm\ s}^{-1} \\
&h && 6.626070040 \times 10^{-27}\ \mathrm{erg\ s} \\
&\hbar && 1.054571628 \times 10^{-27}\ \mathrm{erg\ s} \\
&m_p && 1.6726217 \times 10^{-24}\ \mathrm{g} \\
&m_u && 1.6605389 \times 10^{-24}\ \mathrm{g} \\
&m_e && 9.10938291 \times 10^{-28}\ \mathrm{g} \\
&e && 4.80320425 \times 10^{-10}\ \mathrm{erg}^{1/2}\mathrm{cm}^{1/2} \\
&\alpha = \frac{e^2}{\hbar c} && \frac{1}{137.035999139} \\
&\sigma_{\mathrm{T}} = \frac{8\pi e^4}{m_e^2 c^4} && 6.6524574 \times 10^{-25}\ \mathrm{cm}^2 \\
&a_{\mathrm{B}} = \frac{\hbar}{m_e c\alpha} && 5.2917721067 \times 10^{-9}\ \mathrm{cm} \\
&k_{\mathrm{B}} && 1.3806488 \times 10^{-16}\ \mathrm{erg\ K}^{-1} \\
&\sigma_{\mathrm{SB}} && 5.6704 \times 10^{-5}\ \mathrm{erg\ cm}^{-2}\mathrm{s}^{-1}\mathrm{K}^{-4} \\
&a && 7.5657 \times 10^{-15}\ \mathrm{erg\ cm}^{-3}\mathrm{K}^{-4} \\
&G_F/(\hbar c)^3 && 1.1663787 \times 10^{-5}\ \mathrm{GeV}^{-2} \\
&M_\odot && 1.9884 \times 10^{33}\ \mathrm{g} \\
&GM_\odot && 1.32712440018 \times 10^{26}\ \mathrm{cm}^3\mathrm{s}^{-2} \\
&R_\odot && 6.955 \times 10^{10}\ \mathrm{cm} \\
&L_\odot && 3.828 \times 10^{33}\ \mathrm{erg/s} \\
&\mathrm{Jy} && 10^{-23}\ \mathrm{erg\ s}^{-1}\mathrm{cm}^{-2}\mathrm{Hz}^{-1} \\
&\mathrm{AU} && 1.495978707 \times 10^{13}\ \mathrm{cm} \\
&\mathrm{pc} && 3.08568 \times 10^{18}\ \mathrm{cm}
\end{aligned} \tag{1}
$$

1 Thermodynamics of expanding gas

Consider an expanding gas with radiative cooling due to the photon diffusion and with a time-dependent heating rate $\dot{Q}(t)$.

(1) Find a formal solution of the evolution of the internal energy of the gas, $E(t)$, by solving

$$\frac{dE}{dt} = -\frac{E}{t} - \frac{E}{t_{\rm rad}} + \dot{Q}(t), \tag{2}$$

where

$$t_{\rm rad} = \frac{3\kappa M}{4\pi tcv} \equiv \frac{t_{\rm diff}^2}{t}. \tag{3}$$

Here κ is the opacity, M is the ejecta mass, and v is the ejecta expansion velocity.

Supernovae

(2) Derive the specific radioactive heating rate of the decay chain of ^{56}Ni $\rightarrow^{56}$ Co $\rightarrow^{56}$ Fe. Here you may use $t_{1/2}(^{56}\mathrm{Ni}) = 6.075$ day and the energy release of 2.13 MeV, and $t_{1/2}(^{56}\mathrm{Co}) = 77.236$ day and the energy release of 4.57 MeV.

(3) Calculate a light curve arising from an ejecta with mass of $3M_{\odot}$, and the initial mass of ^{56}Ni of $0.1M_{\odot}$, the expansion velocity of $10^{-2}c$, and $\kappa = 0.1\,\mathrm{cm}^2/\mathrm{g}$ without the initial internal energy contribution.

(4) Calculate the contribution of the initial energy to the light curve assuming that $E_0 = 10^{51}$ erg at a radius of $R_0 = R_{\odot}$.

(5) Calculate the effective temperature evolution.

(6) Find the peak luminosity and peak time relation by setting $M = M_{\rm Ni}$ and show the region where Nickel powered transients can exist.

Neutron star mergers

(7) Calculate a light curve with an ejecta mass of $0.03M_{\odot}$, and a power-law radioactive heating rate of $0.5 \cdot 10^{10}(t/\mathrm{day})^{-1.3}$ erg/s/g, the expansion velocity of $0.1c$, and $\kappa = 10\,\mathrm{cm}^2/\mathrm{g}$.

(8) Show the contribution of the initial internal energy when $E_0 = 10^{51}$ erg at a radius of $R_0 = 10^7$ cm.

(9) Find the parameters (M, v, κ, E_0, R_0), with which the light curve is consistent with the earliest observed data point of GW170817, $L \approx 10^{42}$ erg/s and $t \approx 0.5$ day.

(10) Calculate the effective temperature evolution.