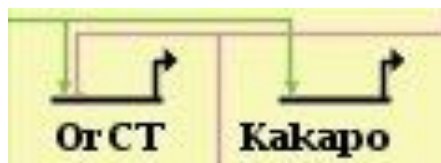


# Evolving networks *in silico*

## Goals:

use mutation-selection to numerically evolve networks of differential equations that perform a specified phenotype

We change both topologies and kinetic parameters



See PF, V. Hakim (2004) and multiple publications since then

# Equations

Mass-action laws

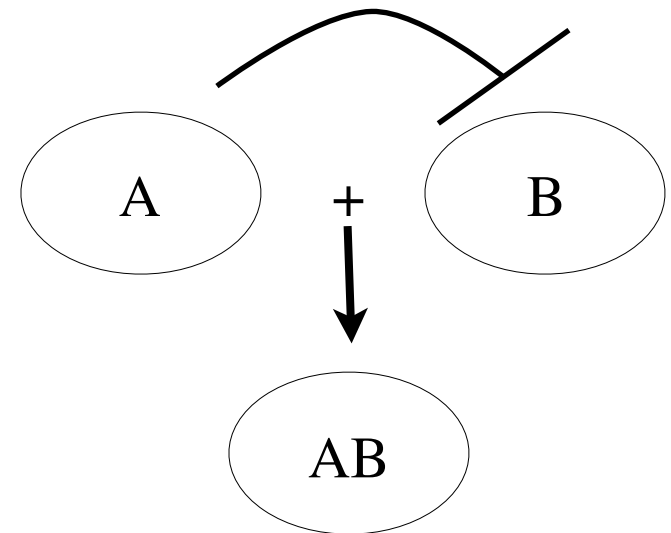
+

Hill functions for transcription

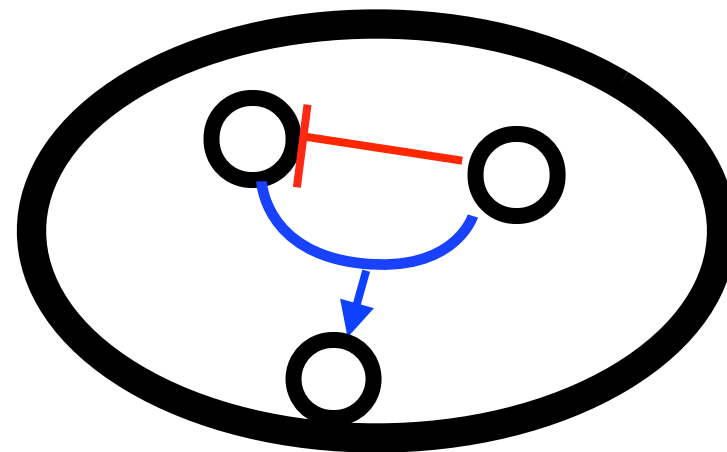
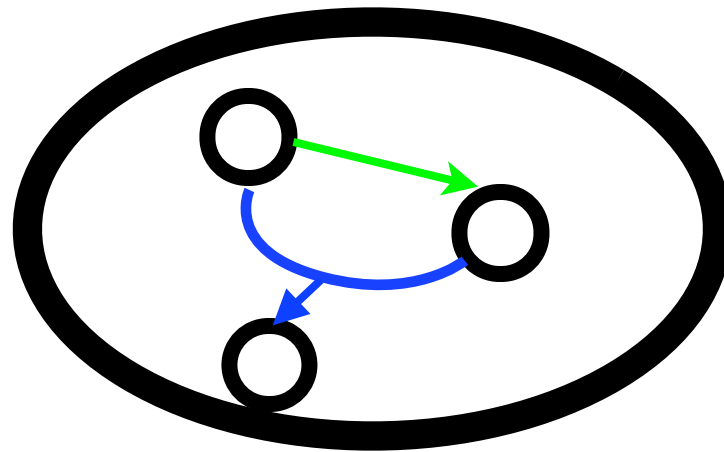
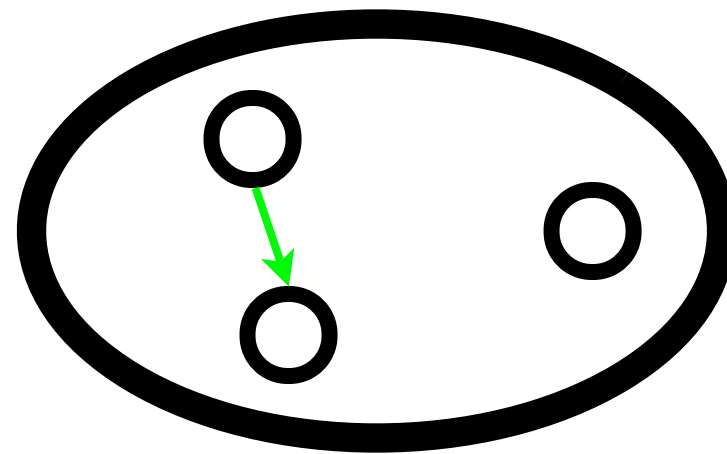
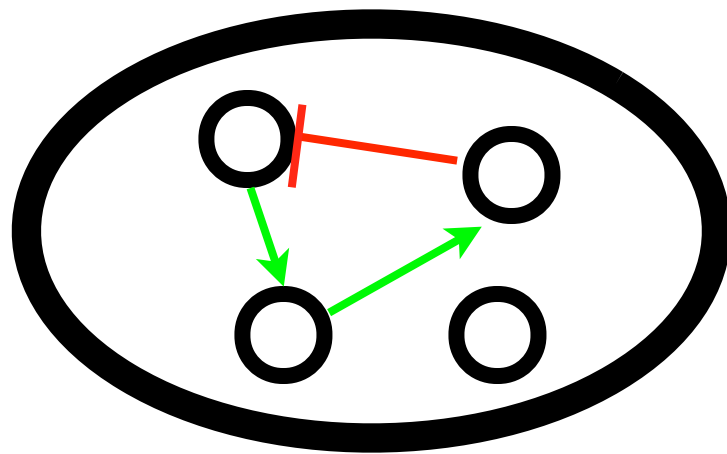
$$\frac{d[A]}{dt} = \dots$$

$$\frac{d[B]}{dt} = \frac{\rho}{1 + ([A]/A_0)^n} - \gamma[A][B] + \theta[AB] - \delta_B[B]$$

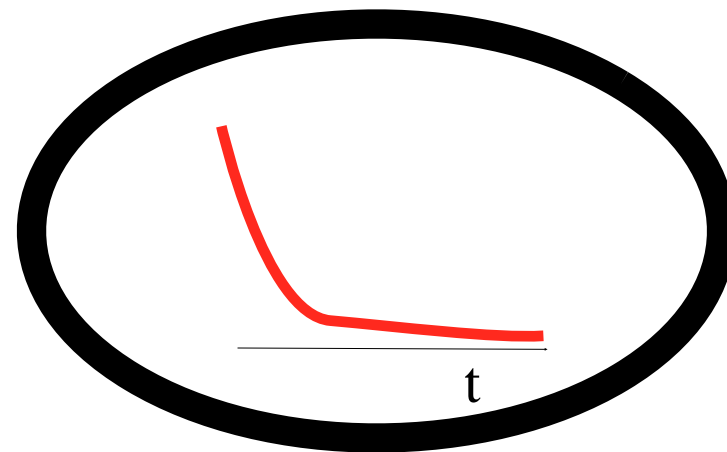
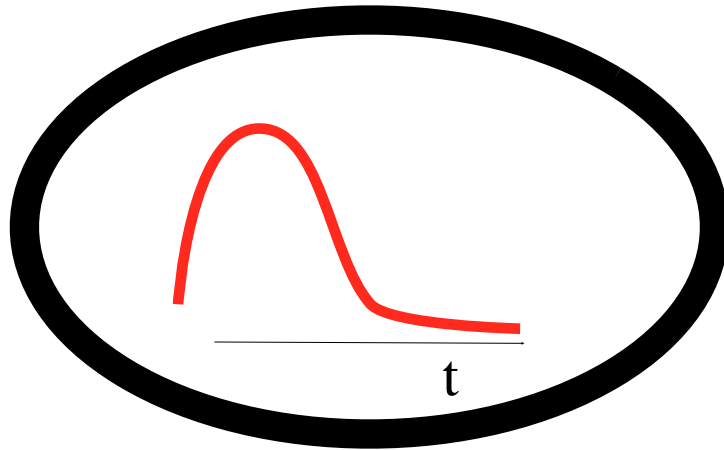
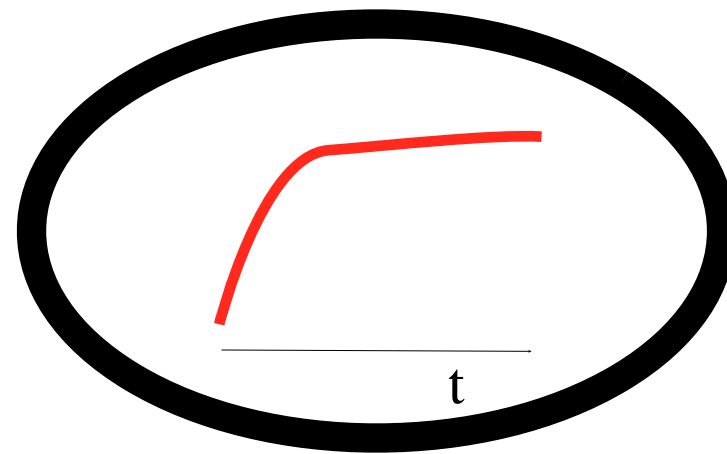
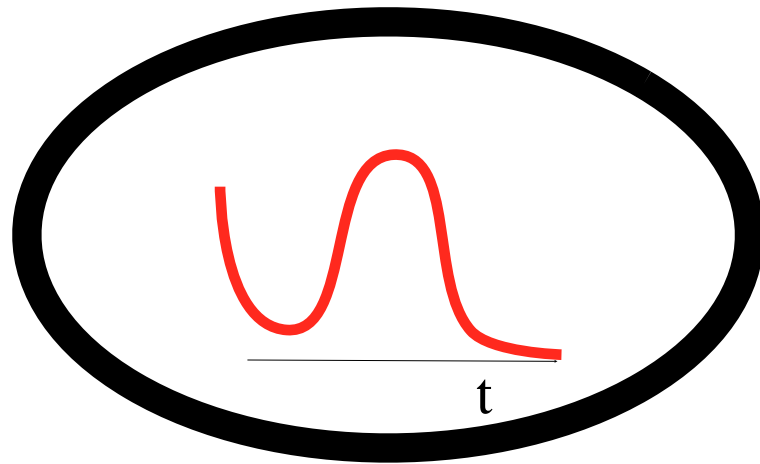
$$\frac{d[AB]}{dt} = \dots$$



# Population of different networks

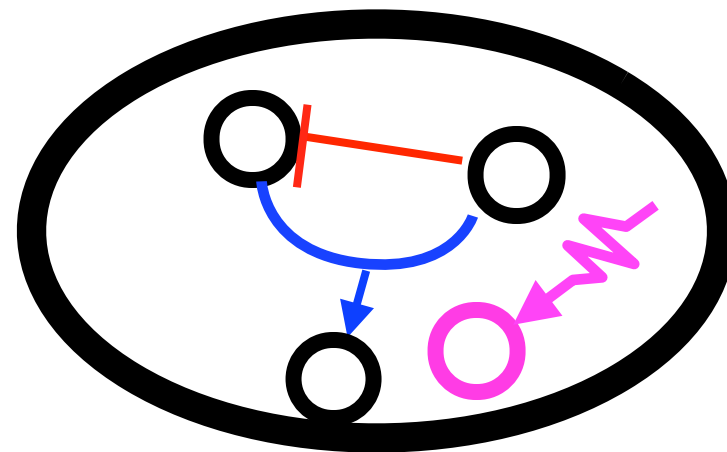
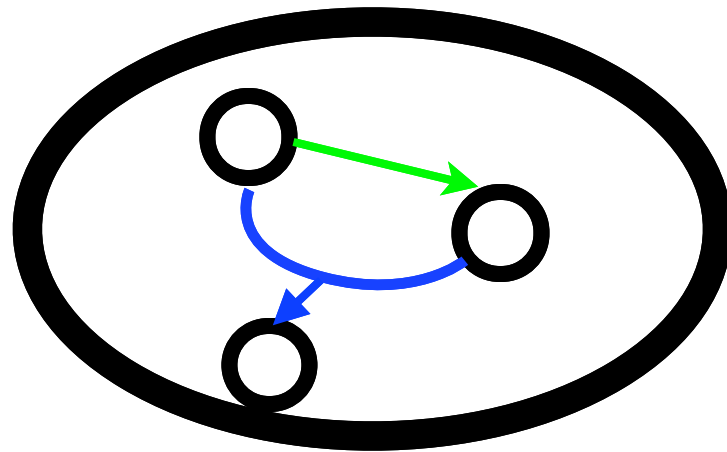
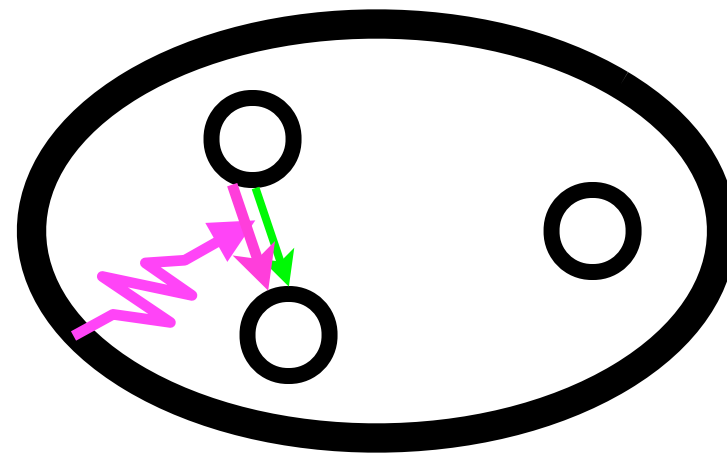
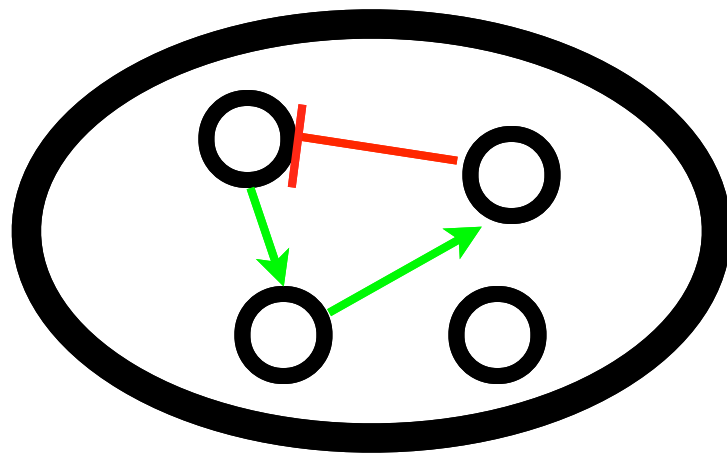


# Time integration : selection for oscillators ...

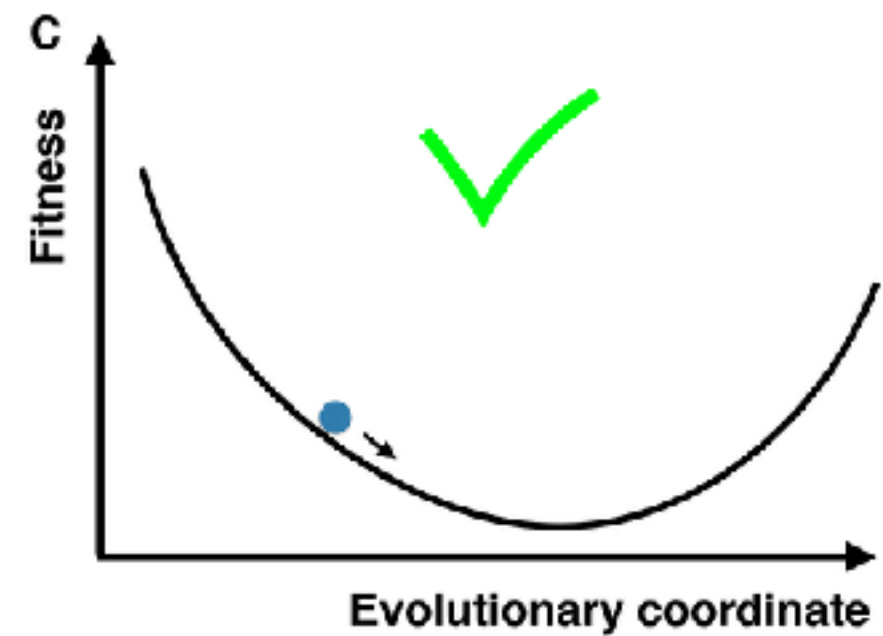
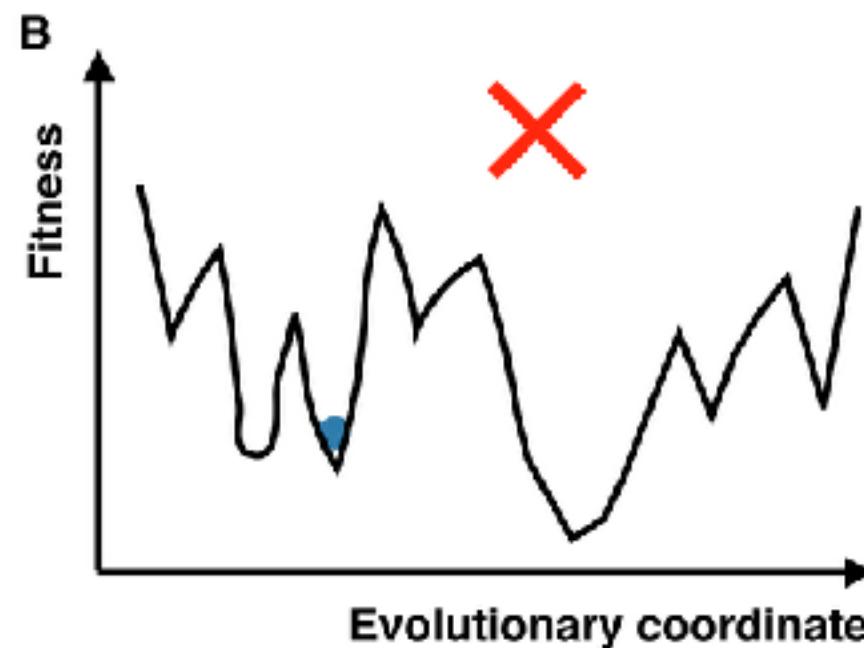
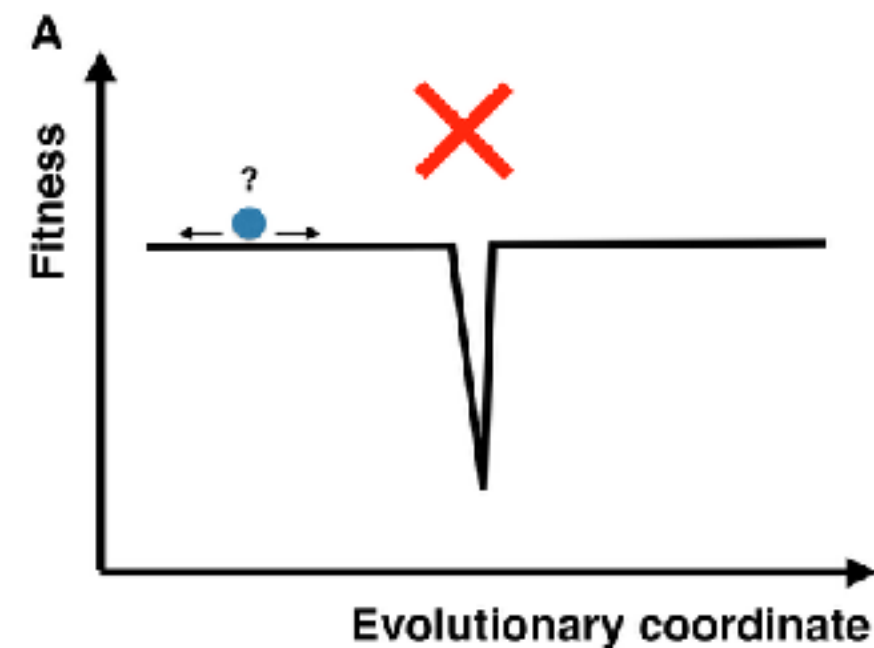


One reporter, here we compare its dynamics to an “ideal” behaviour

# Population of different networks



# How to define a good “fitness” ?



Strategy: coarse-grained description of phenotypes.

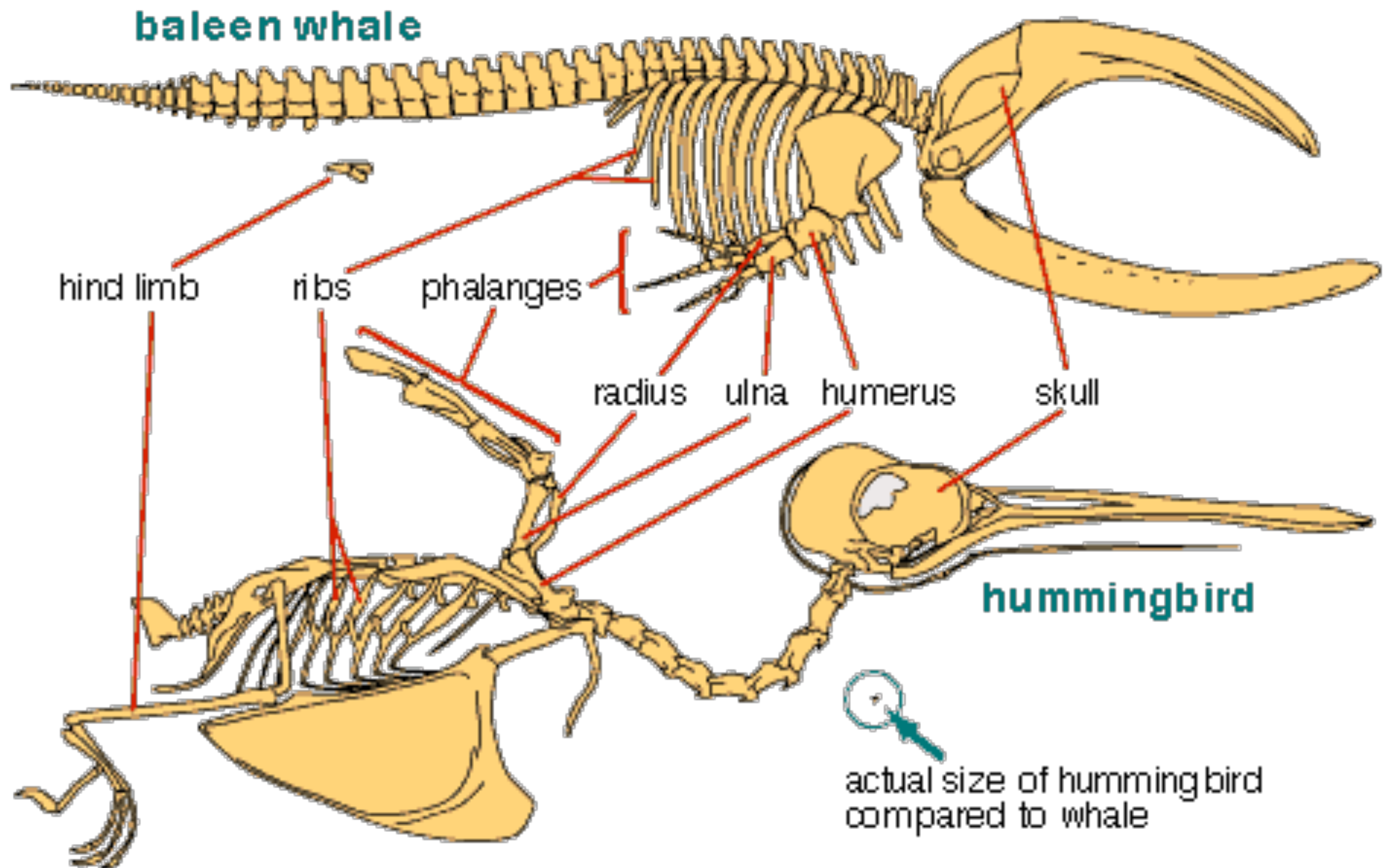
Multidimensional problem: mutual information based fitness works pretty well

Also: Pareto evolution (Warmflash, PF, Siggia, 2012)



<http://phievo.readthedocs.io/en/latest/>

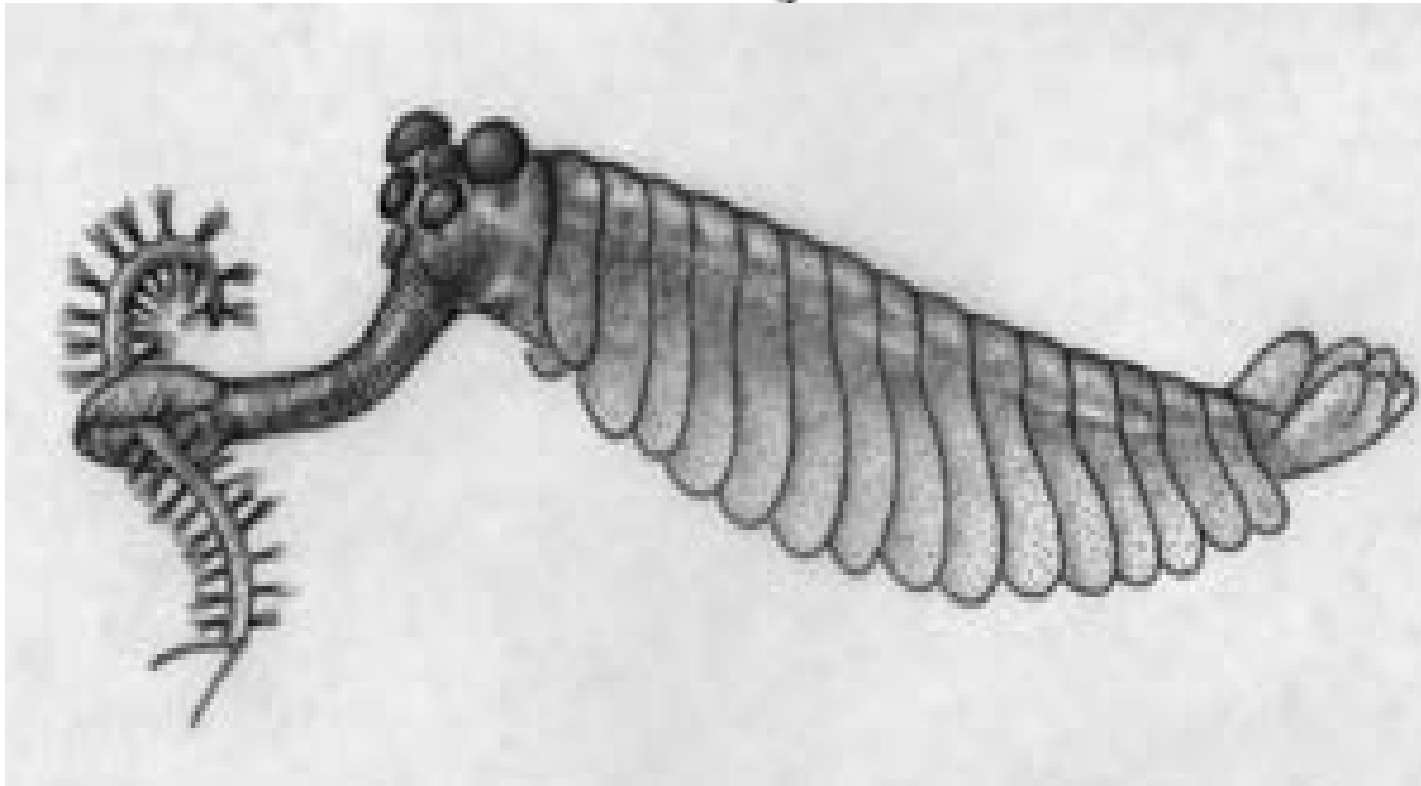
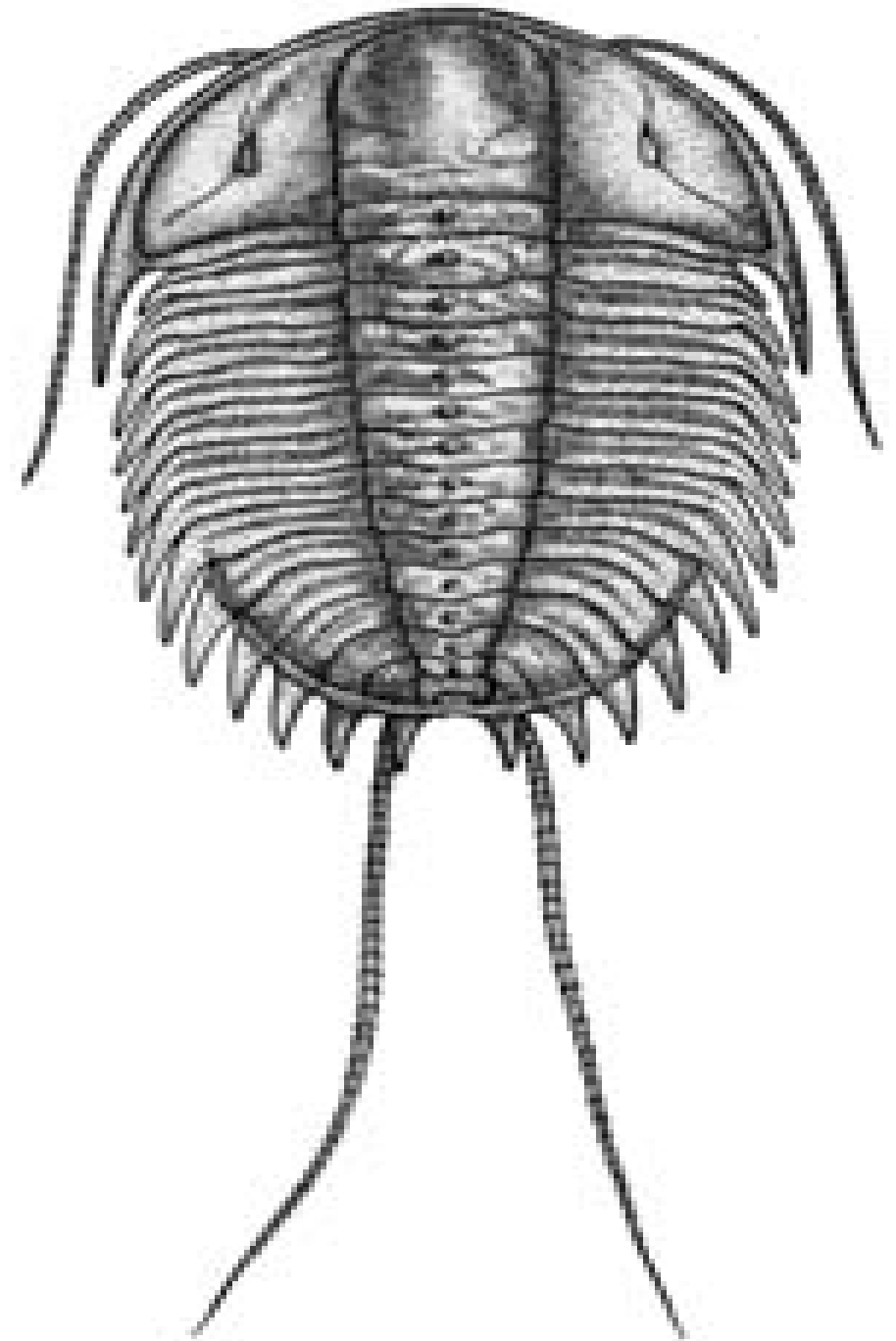
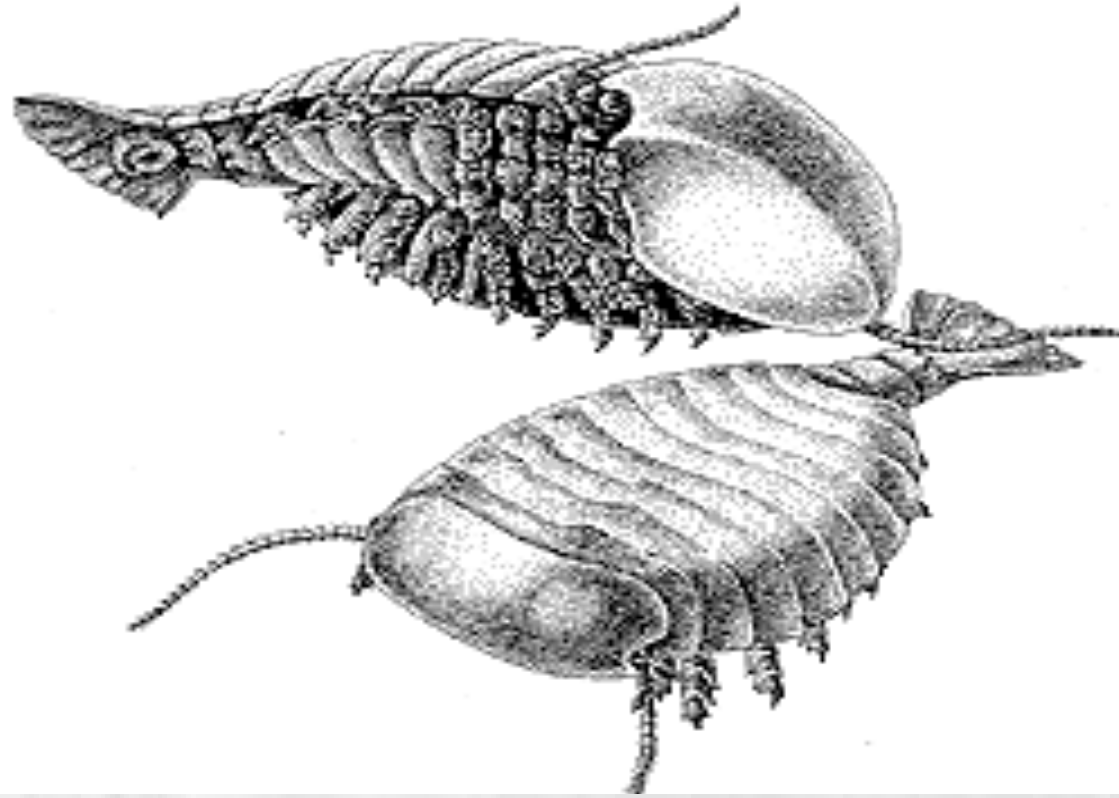
[https://www.youtube.com/watch?v=FRPc\\_hWm\\_JI](https://www.youtube.com/watch?v=FRPc_hWm_JI)



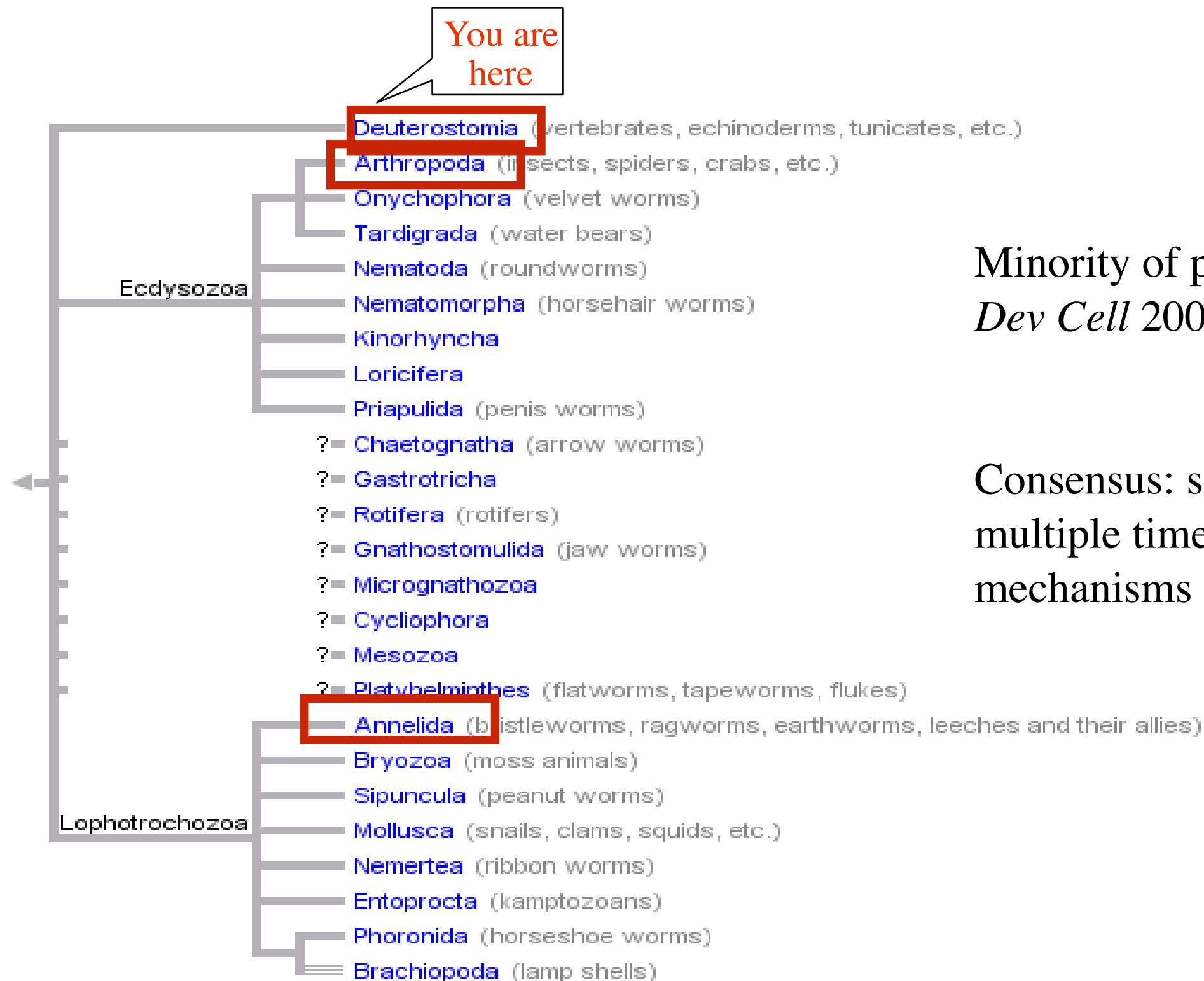
source: [evolution.berkeley.edu](http://evolution.berkeley.edu)



# Segmentation almost as old as animals (Cambrian)



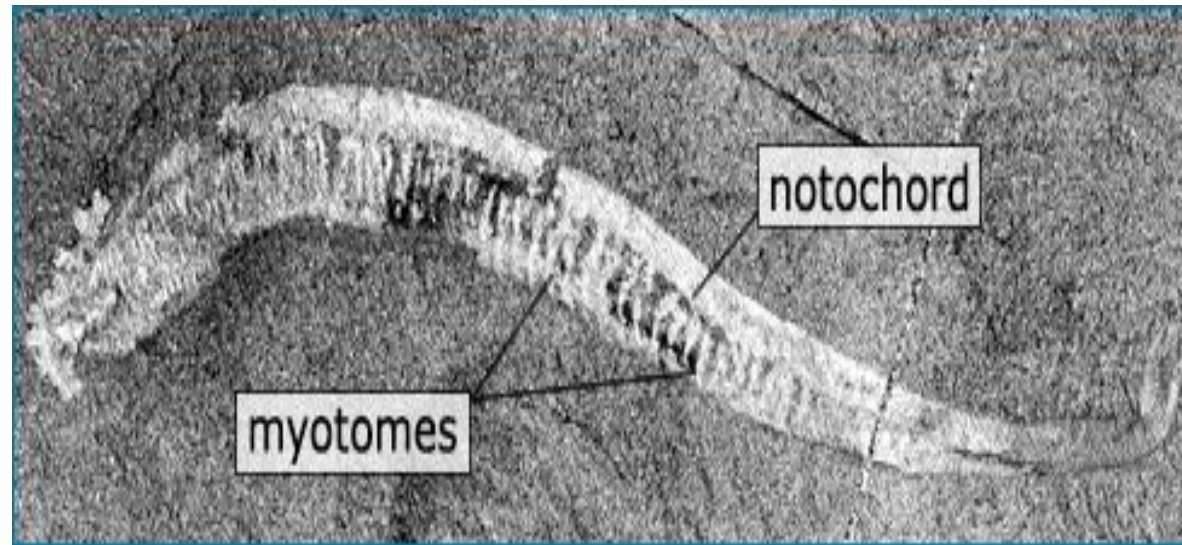
# Bilaterians (tolweb.org)



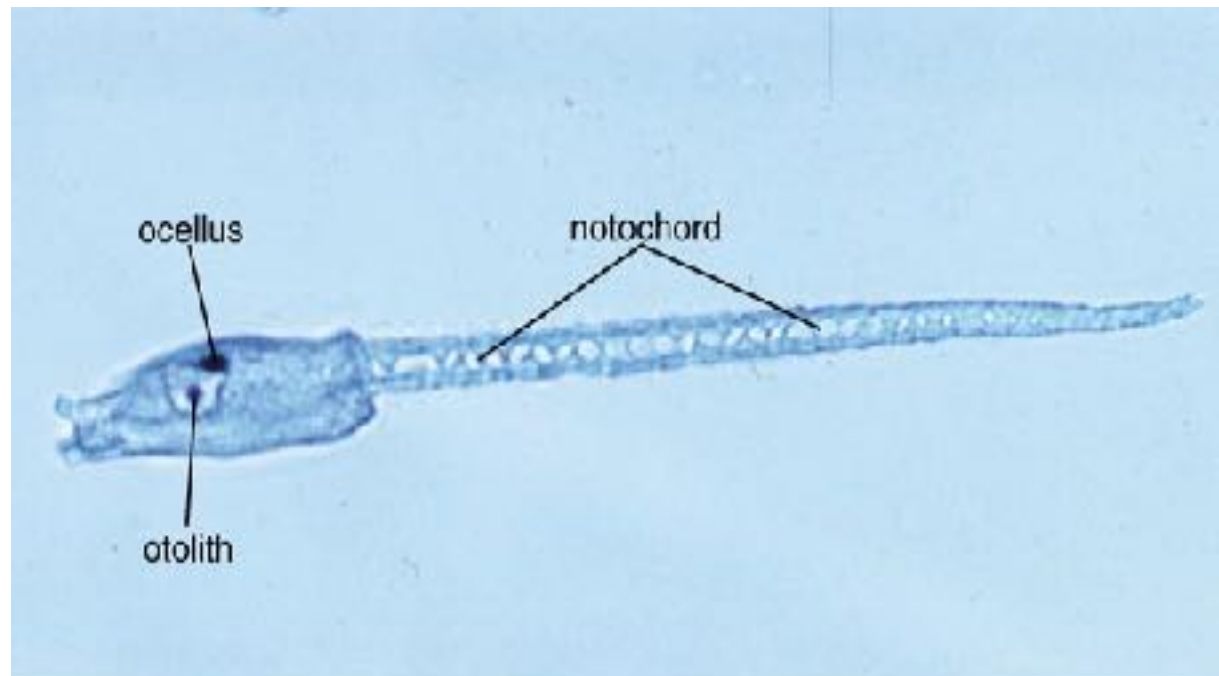
Minority of phyla segmented (Tautz  
*Dev Cell* 2004 )

Consensus: segmentation evolved  
multiple times, molecular  
mechanisms similar within phyla

# Chordate segmentation



*Pikaia* possible cambrian ancestor of ascidians ([//evolution.berkeley.edu](http://evolution.berkeley.edu))



Ascidians, non-vertebrate chordates: tadpole larvae

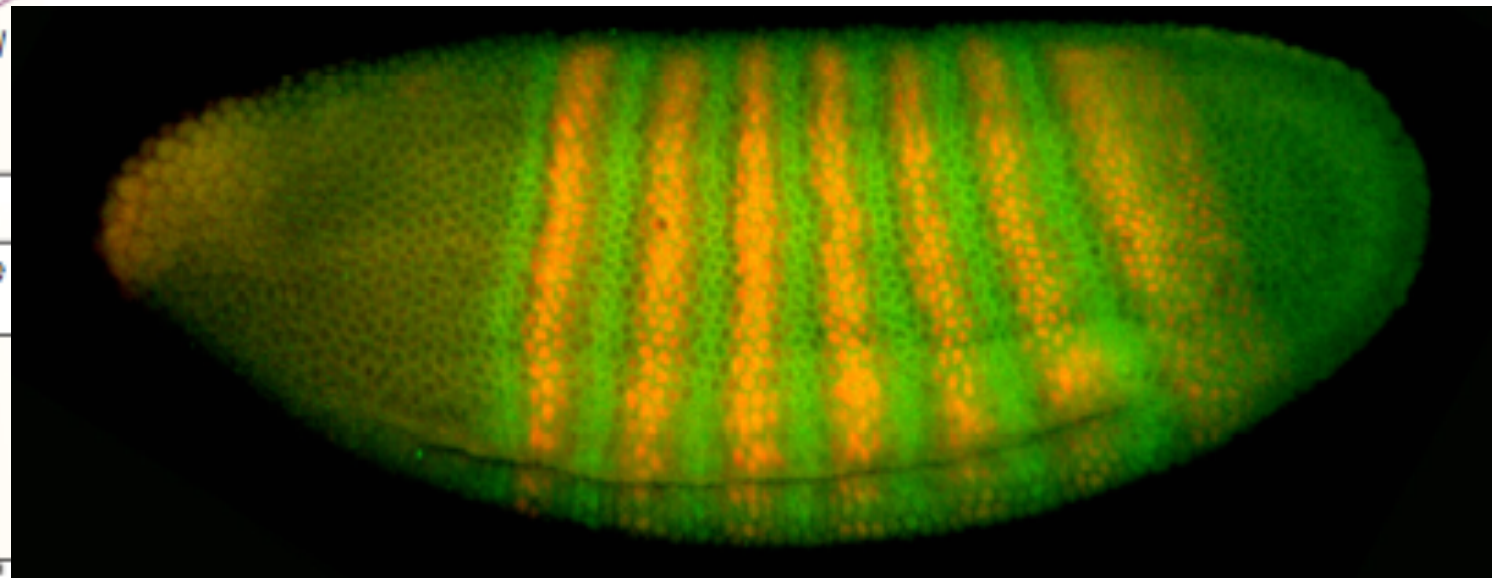
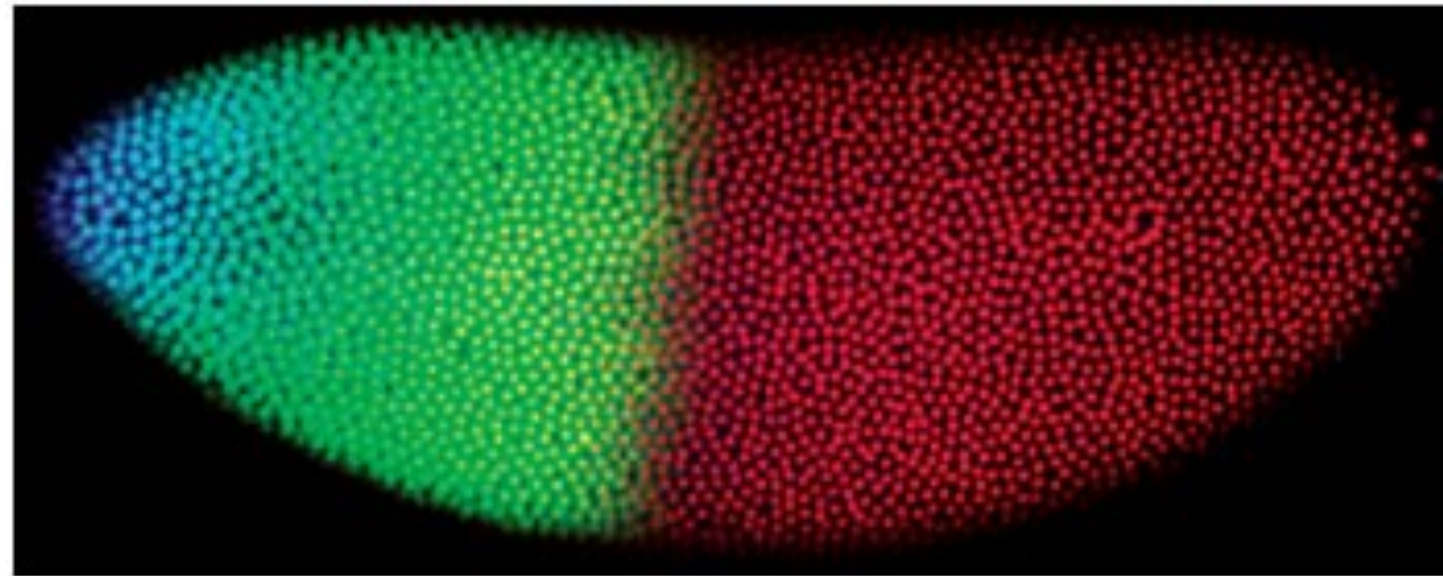
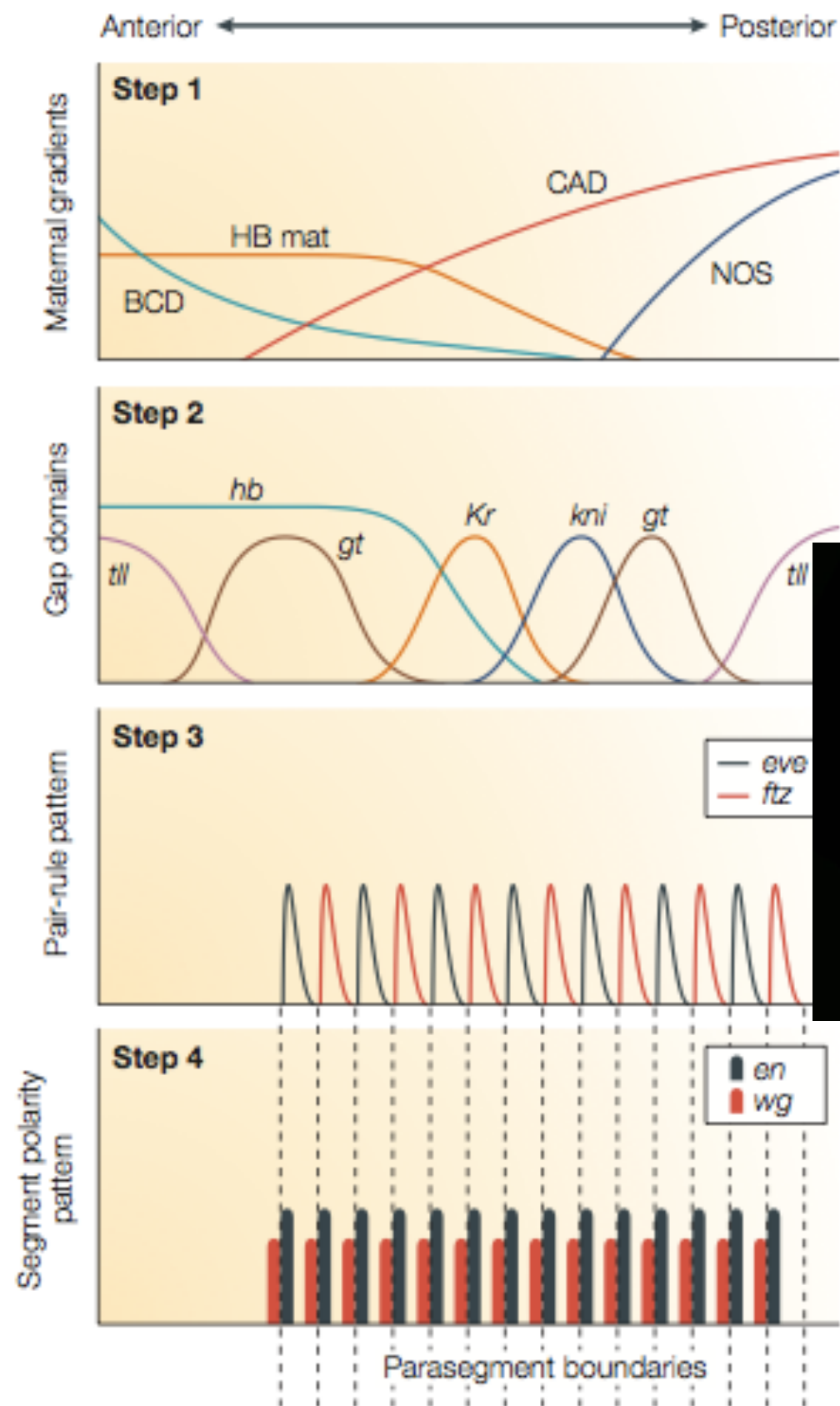


Lemaire, *Science* 2006

Frog tadpole



# A classical paradigm: fly

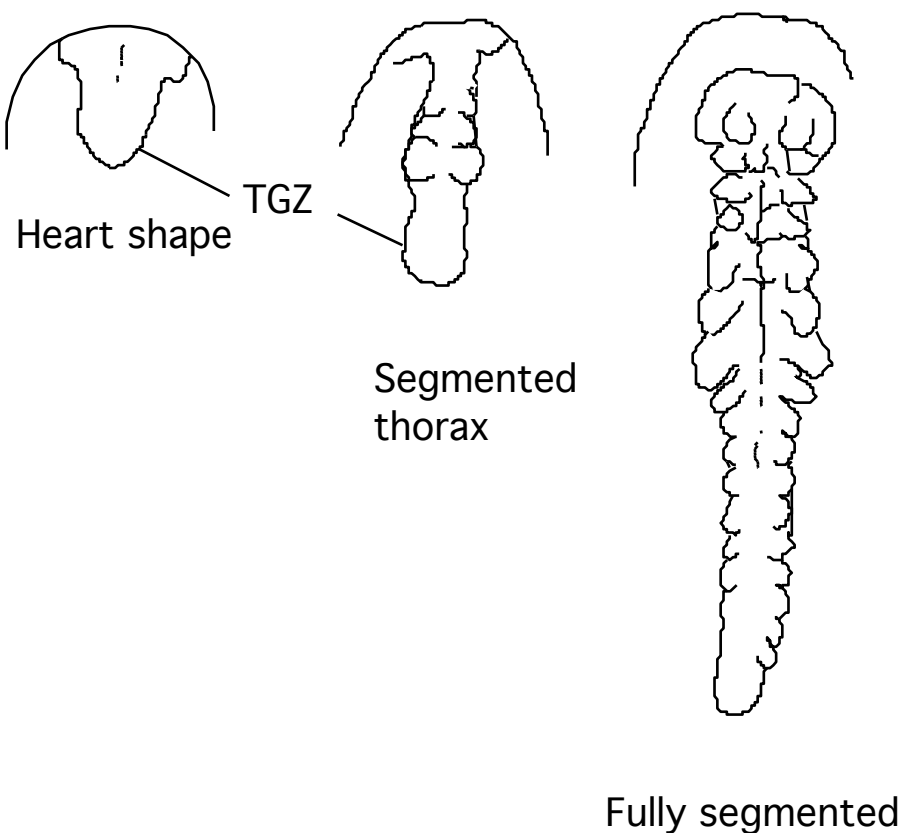


# THE FLY

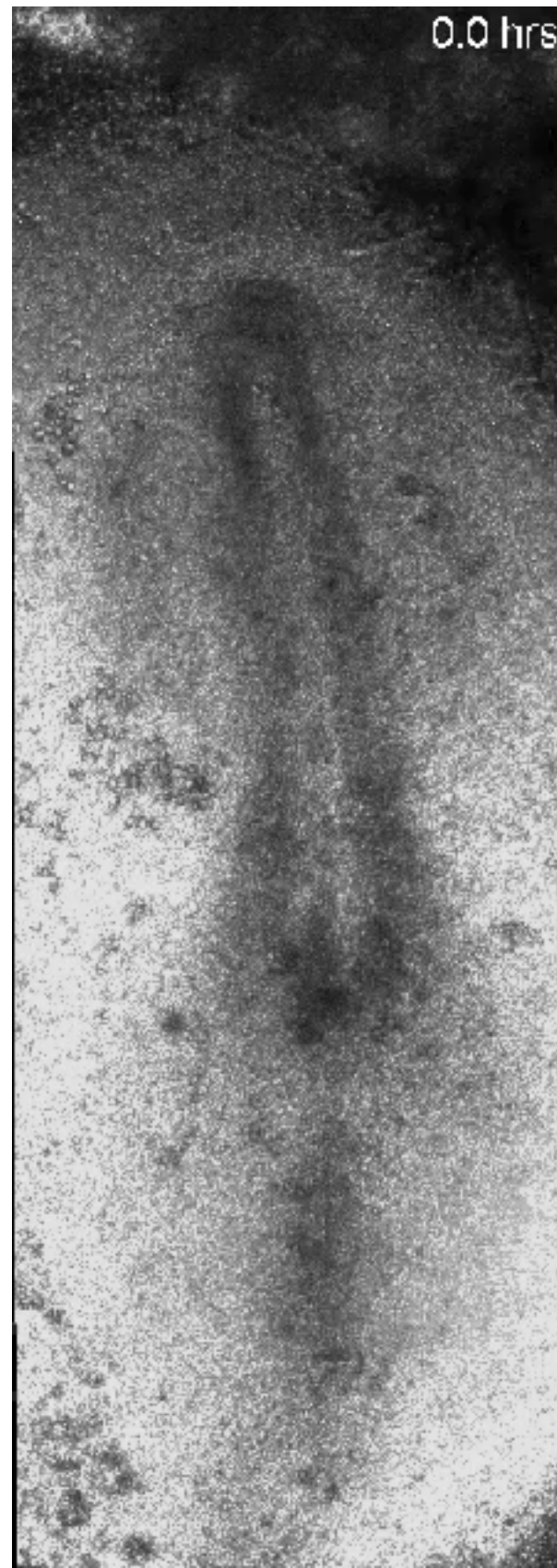




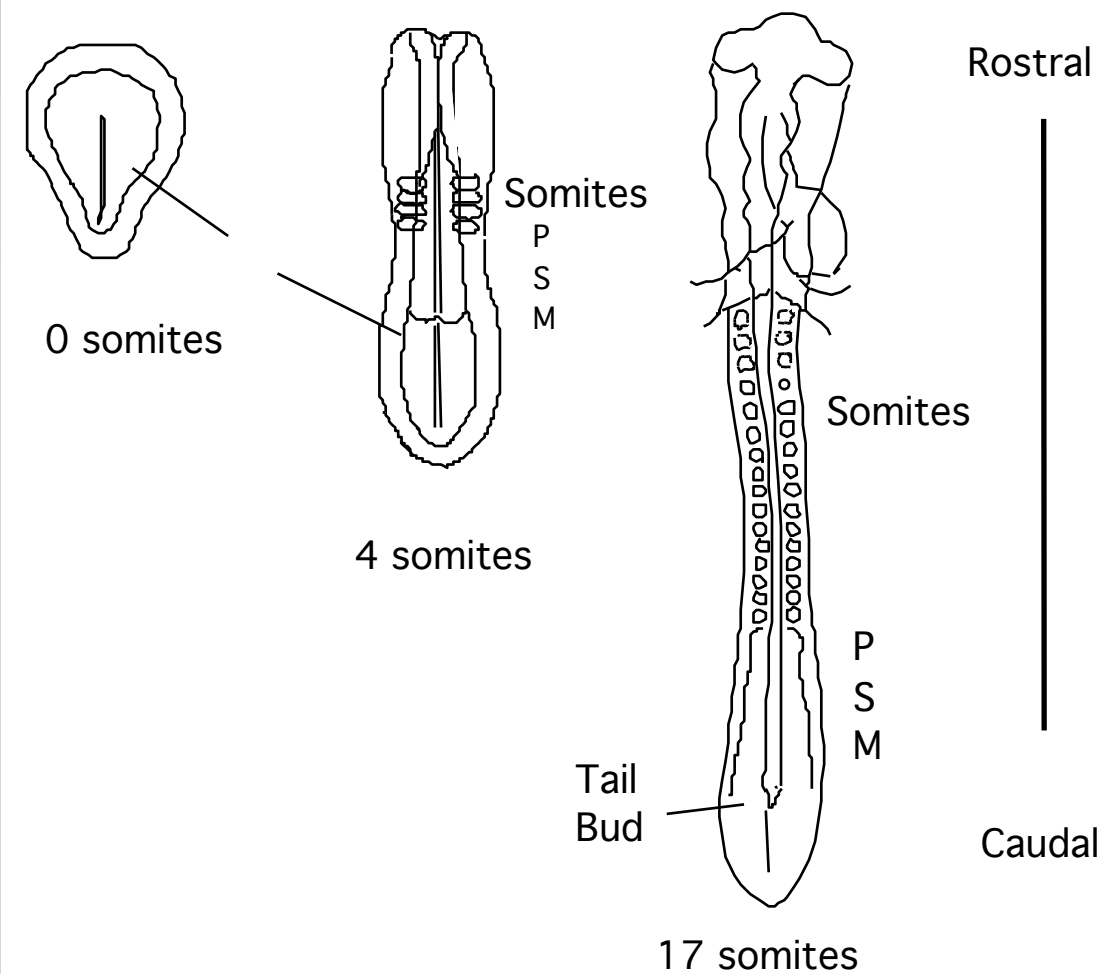
In most animal species, the body axis progressively forms by posterior elongation



Most arthropods

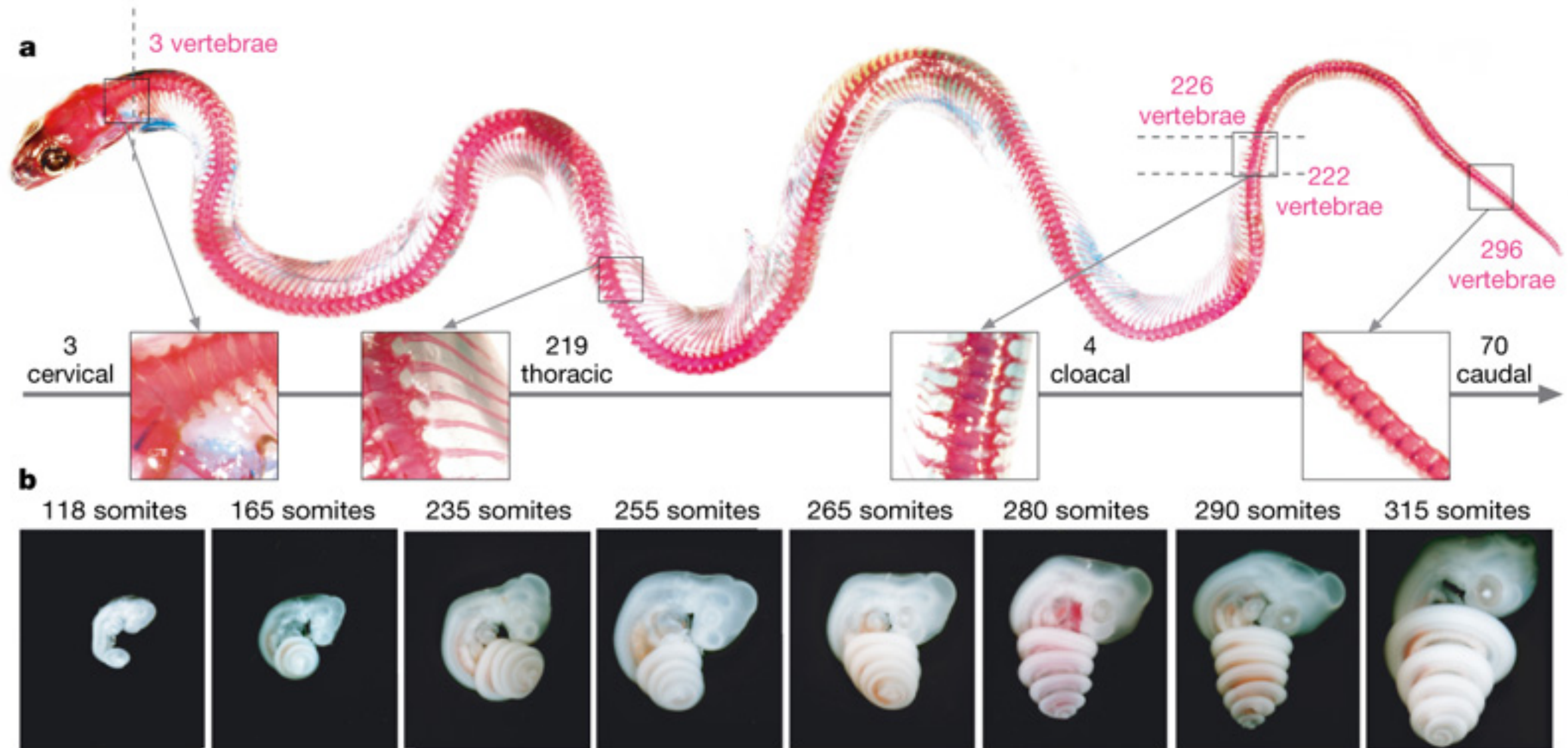


Chick



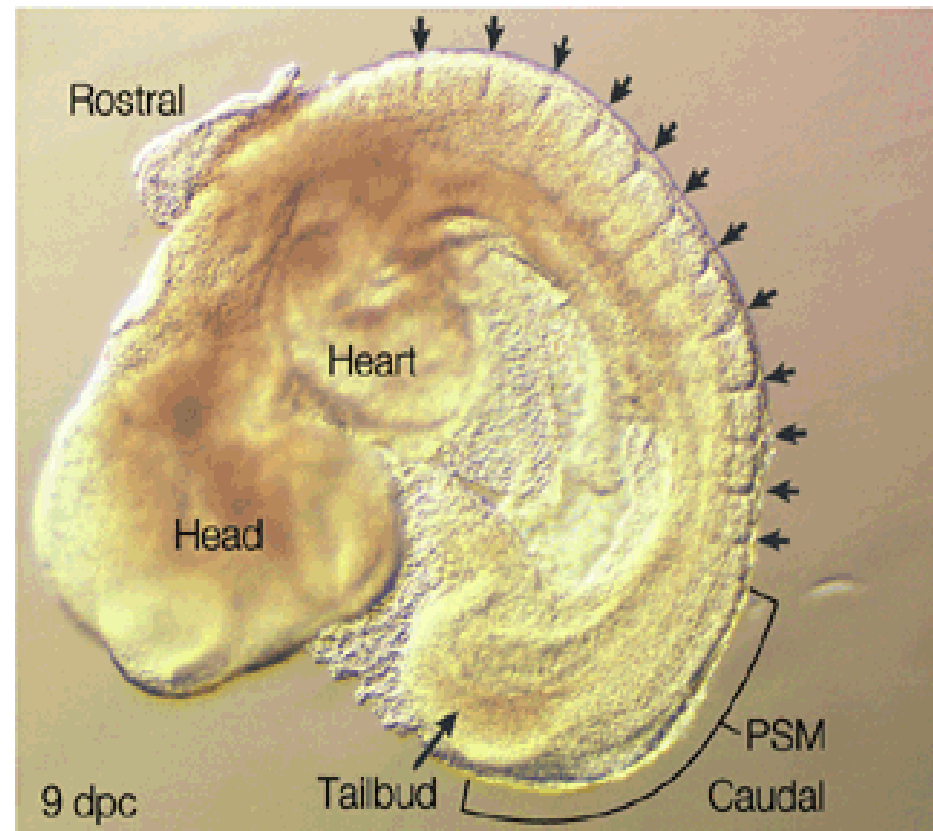
(Movie : Pourquoié's lab)

# Gomez et al. Nature, 2008

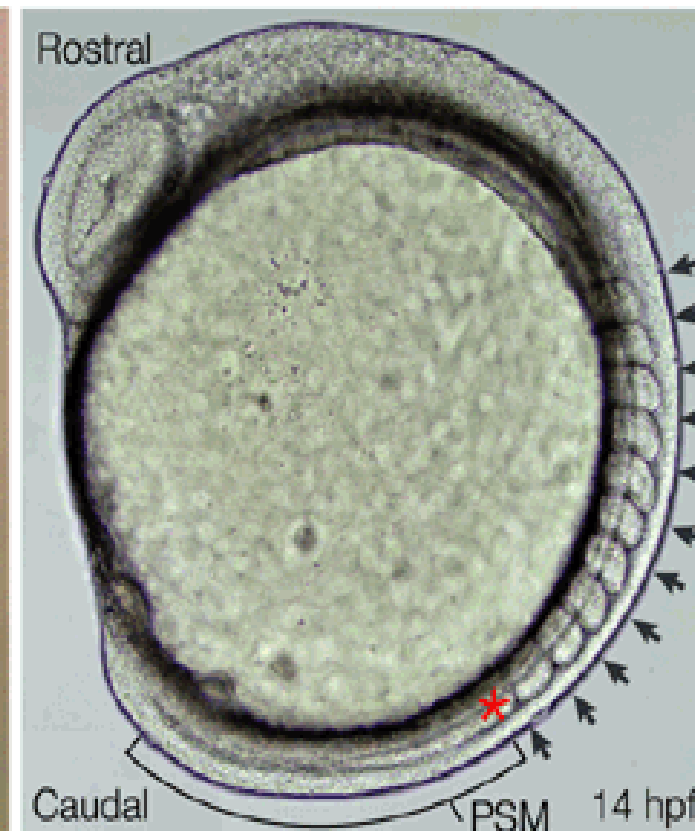


# Somitogenesis : sequential segmentation from PSM

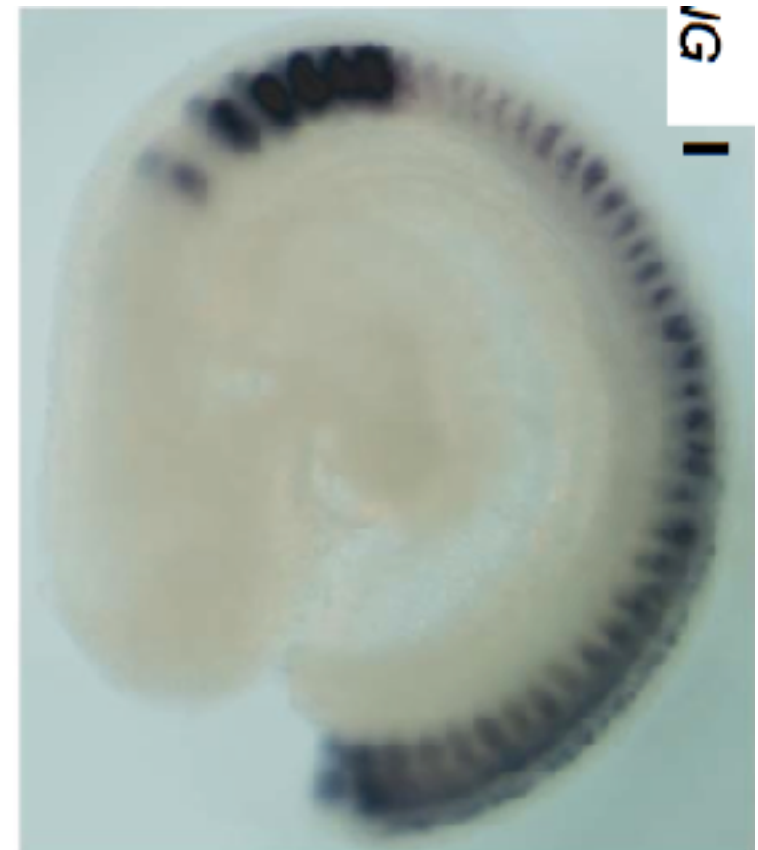
Mouse embryo



Zebrafish embryo



Snake embryo



Nature Reviews | **Genetics**

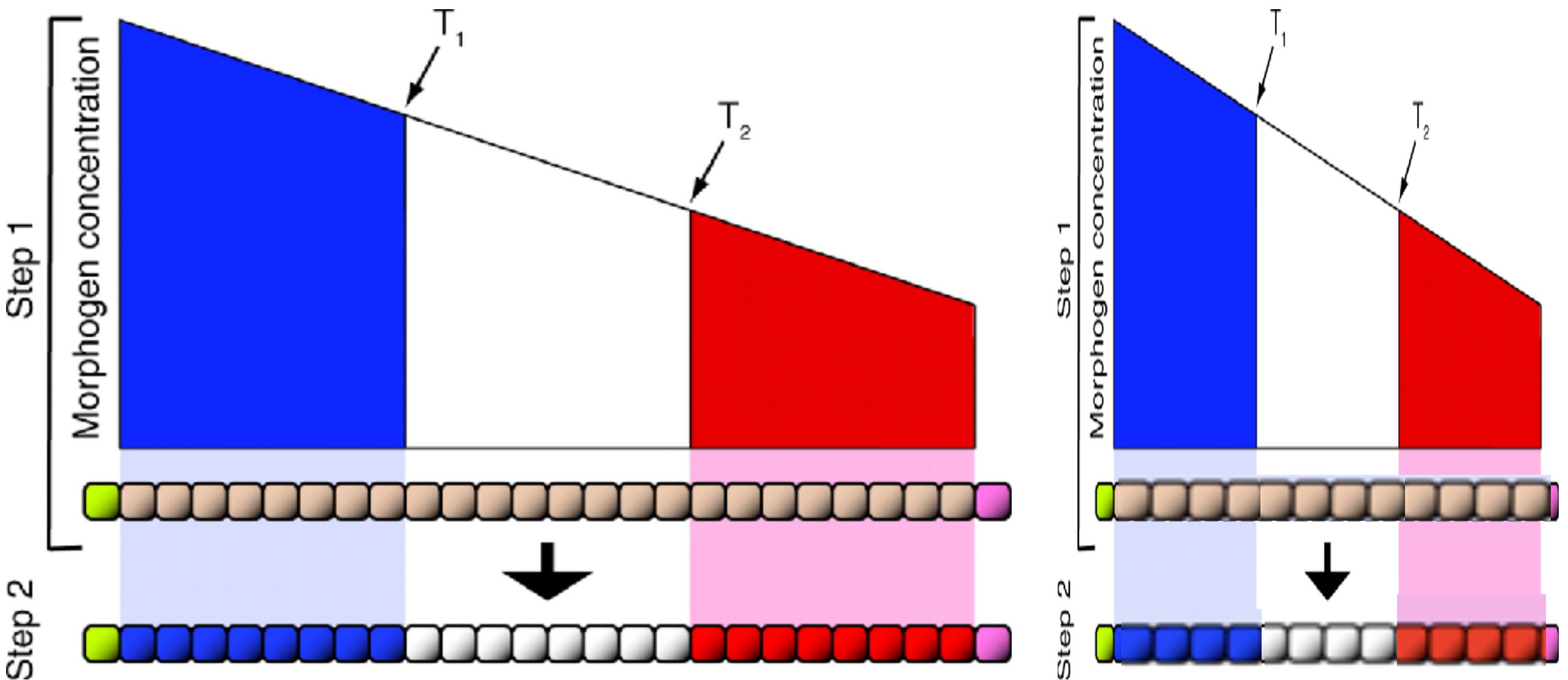
Saga & Takeda 2001

Gomez et al 2008

PSM= Pre Somitic Mesoderm



# A solution ? French Flag Model (~ fly)



Cooke and Zeeman consider it implausible: too many thresholds to define (think about the snake !),  
number of somites not so strongly controlled within body regions, etc..

# The Clock and Wavefront model

*J. theor. Biol.* (1976) **58**, 455–476

## **A Clock and Wavefront Model for Control of the Number of Repeated Structures during Animal Morphogenesis**

J. COOKE†

*National Institute for Medical Research,  
The Ridgeway, Mill Hill, London NW7 1AA, England*

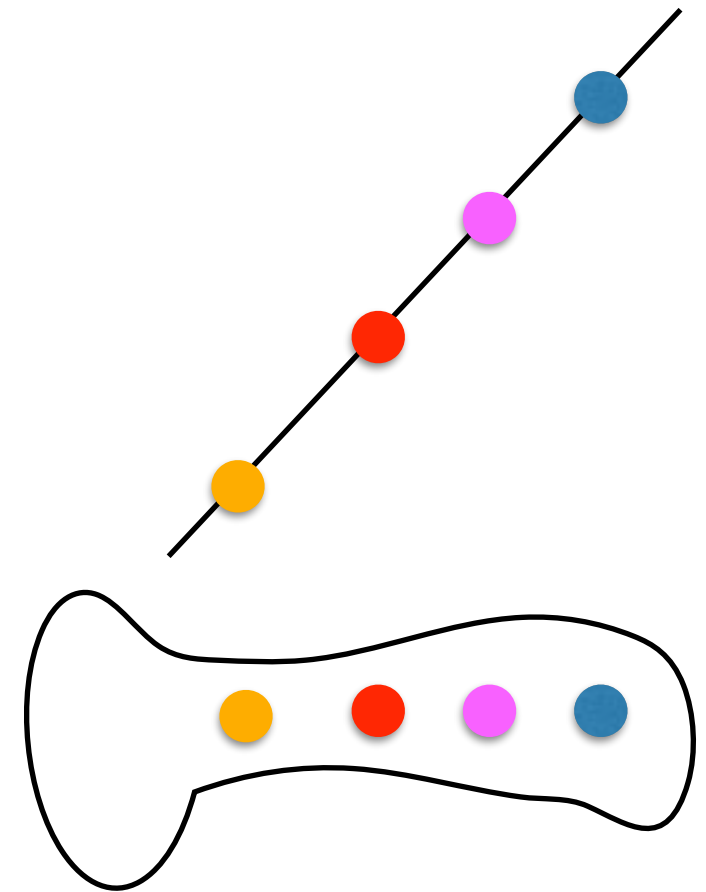
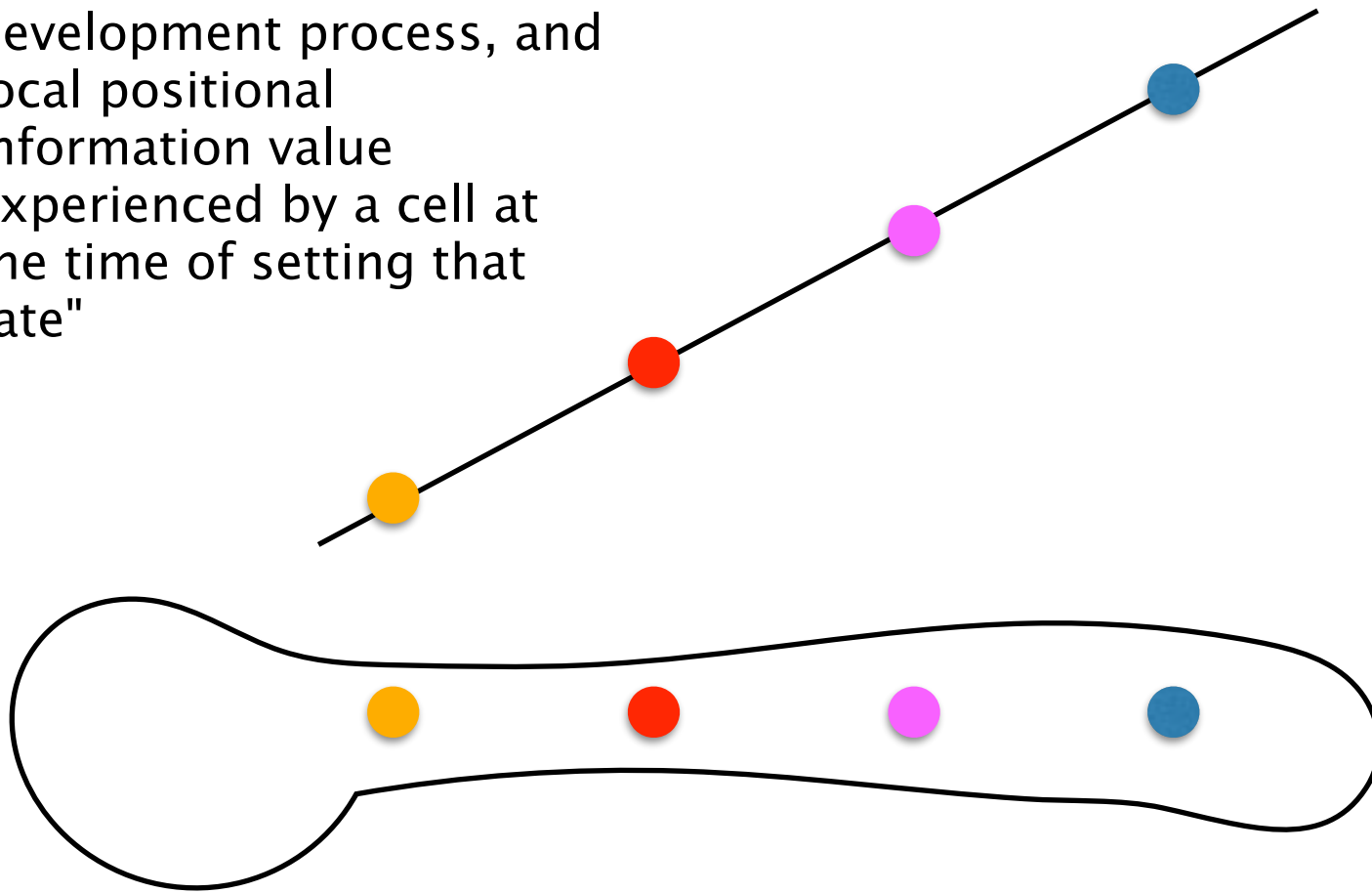
AND

E. C. ZEEMAN

*Institute of Mathematics, University of Warwick,  
Coventry, Warwick, England*

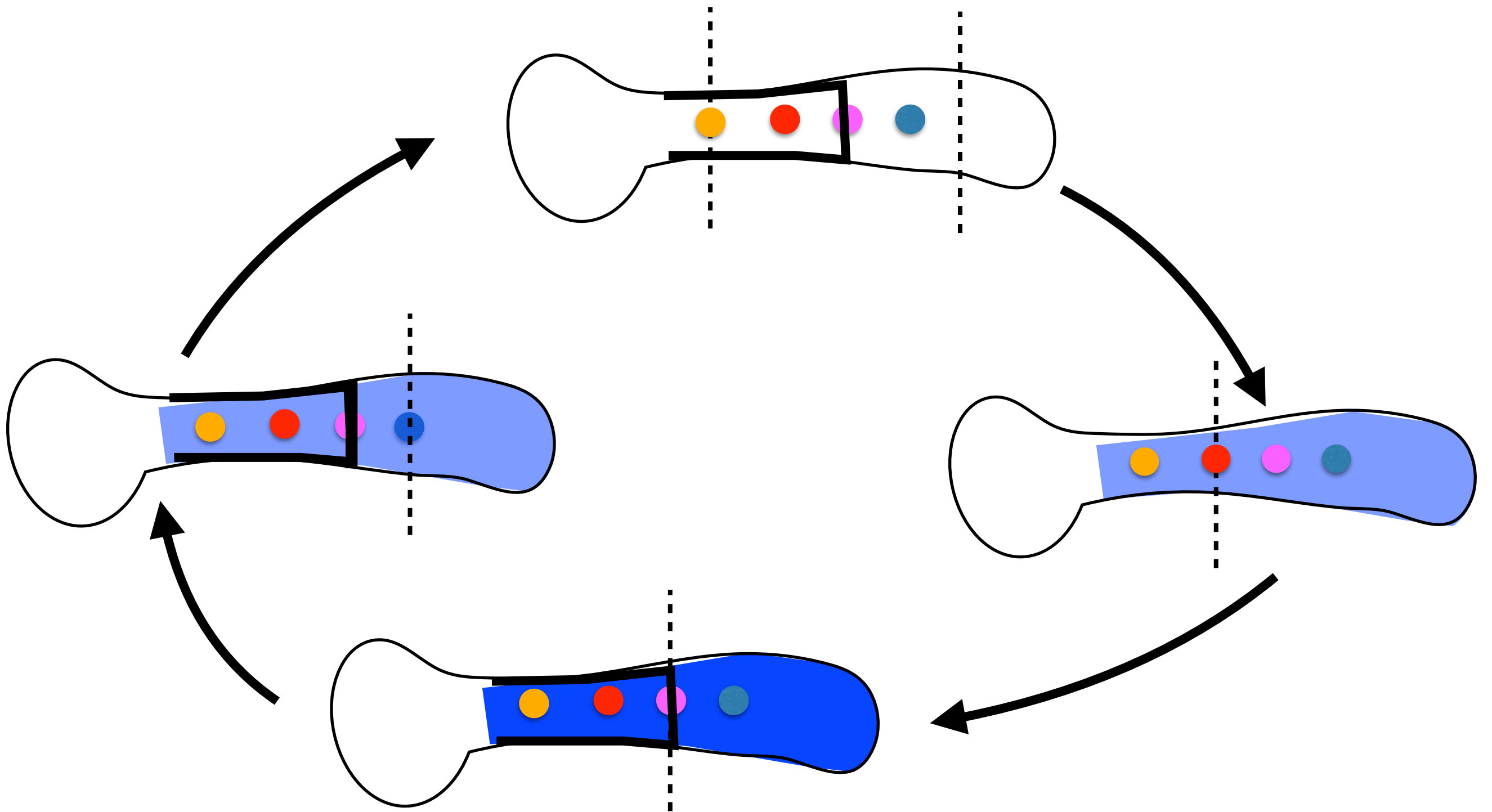
# Idea 1: French Flag Model on a continuous dynamical variable

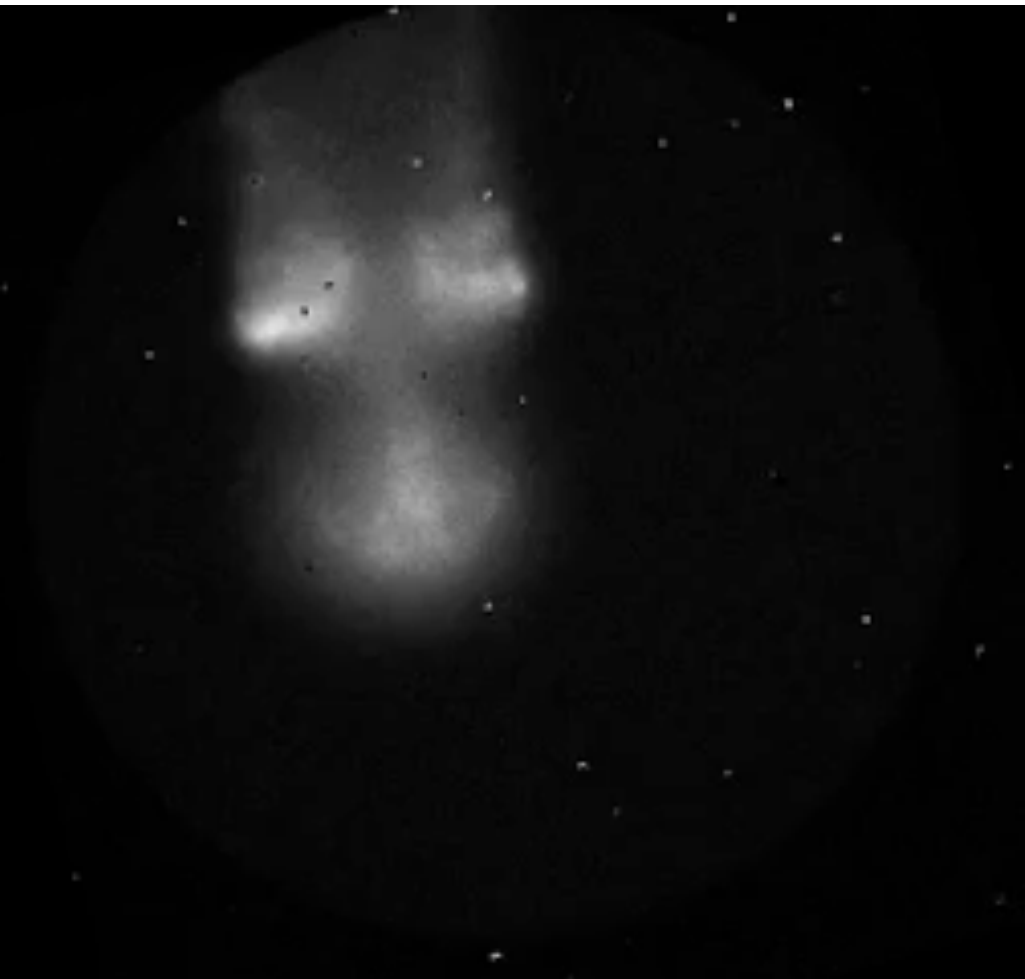
"We shall assume a fixed monotonic relation between **RATE** of an intracellular evolution of development process, and local positional information value experienced by a cell at the time of setting that rate"



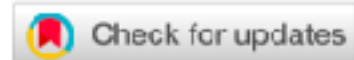
A cell exposed to concentration  $c$  is “committed” at time  $t(c)$   
This defines a temporal coordinate along the embryo called “wavefront”.

# The original picture





🏠 > Current issue > vol. 103 no. 5 > Yoshito Masamizu, 1313–1318, doi: 10.1073/pnas.C508658103



## Real-time imaging of the somite segmentation clock: Revelation of unstable oscillators in the individual presomitic mesoderm cells

Yoshito Masamizu<sup>\*</sup>, Toshiyuki Ohtsuka<sup>\*</sup>, Yoshiki Takashima<sup>\*</sup>, Hiroki Nagahara<sup>†</sup>, Yoshiko Takenaka<sup>†</sup>, Kenichi Yoshikawa<sup>†</sup>, Hitoshi Okamura<sup>‡</sup>, and Ryoichiro Kageyama<sup>\*§</sup>

**nature**  
International journal of science

Access provided by NIH Library

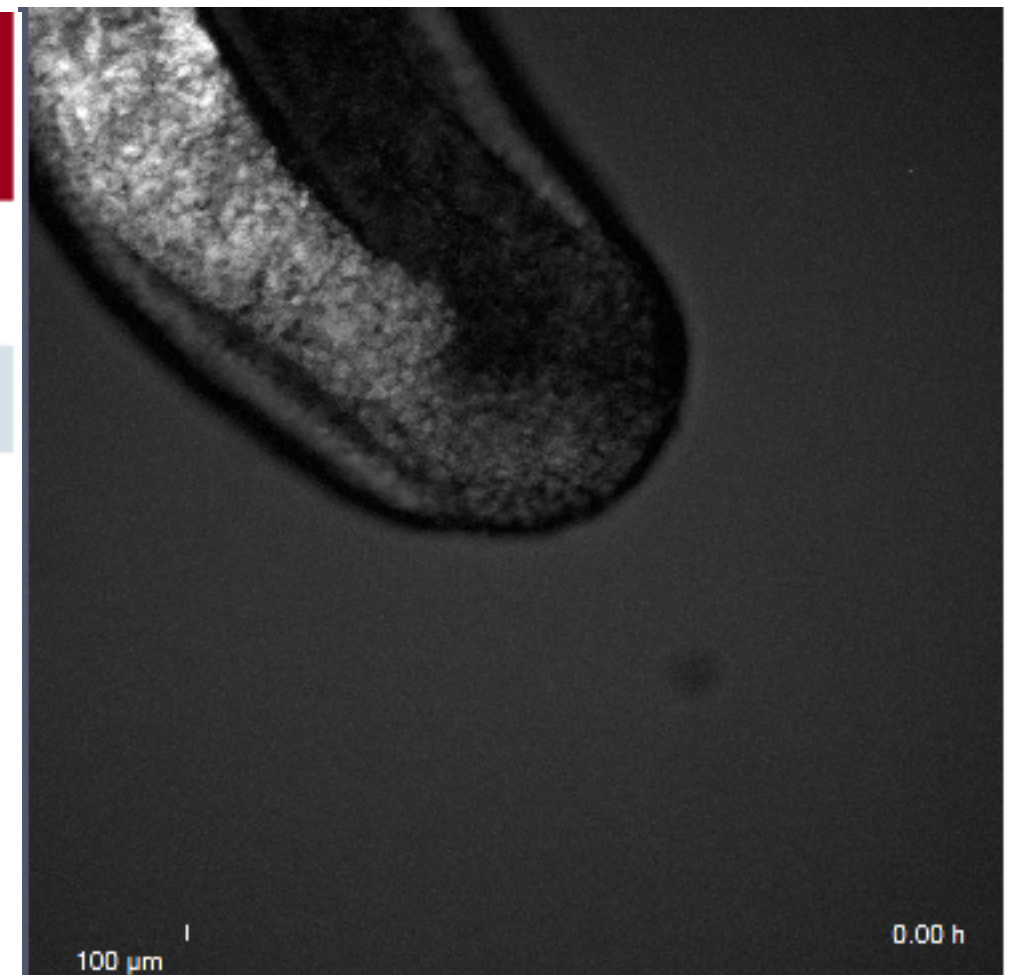
Altmetric: 19 Citations: 57

[More detail >>](#)

Letter

## Scaling of embryonic patterning based on phase-gradient encoding

Volker M. Lauschke, Charisios D. Tsiarris, Paul François & Alexander Aulehla

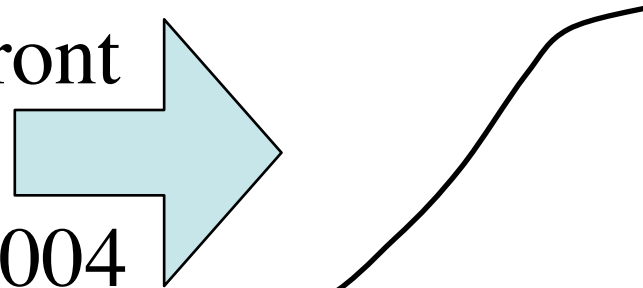




# Segmentation in zebrafish

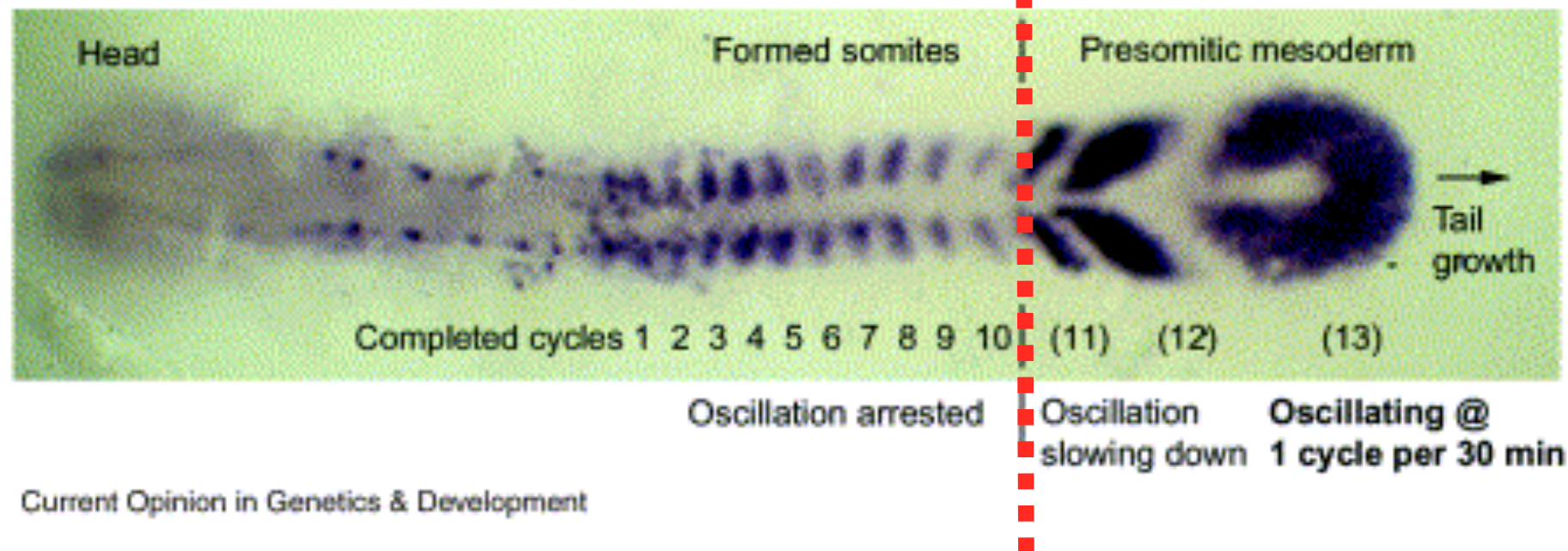
FGF8 morphogen front  
moving with tail

Dubrulle, Science 2004



“Frozen” expression  
Spatial oscillation

“Dynamical” expression  
Temporal oscillation

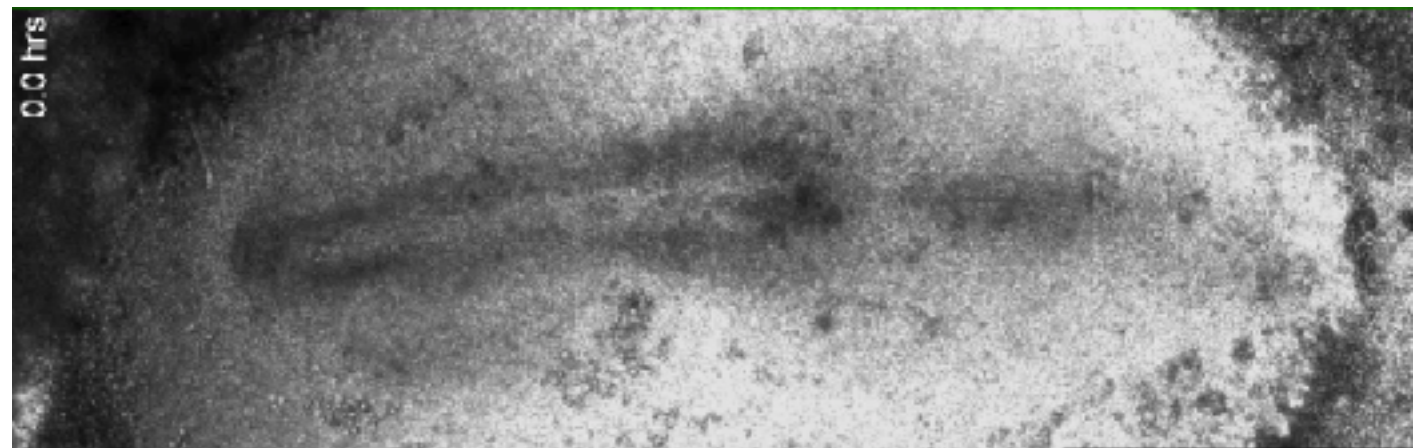
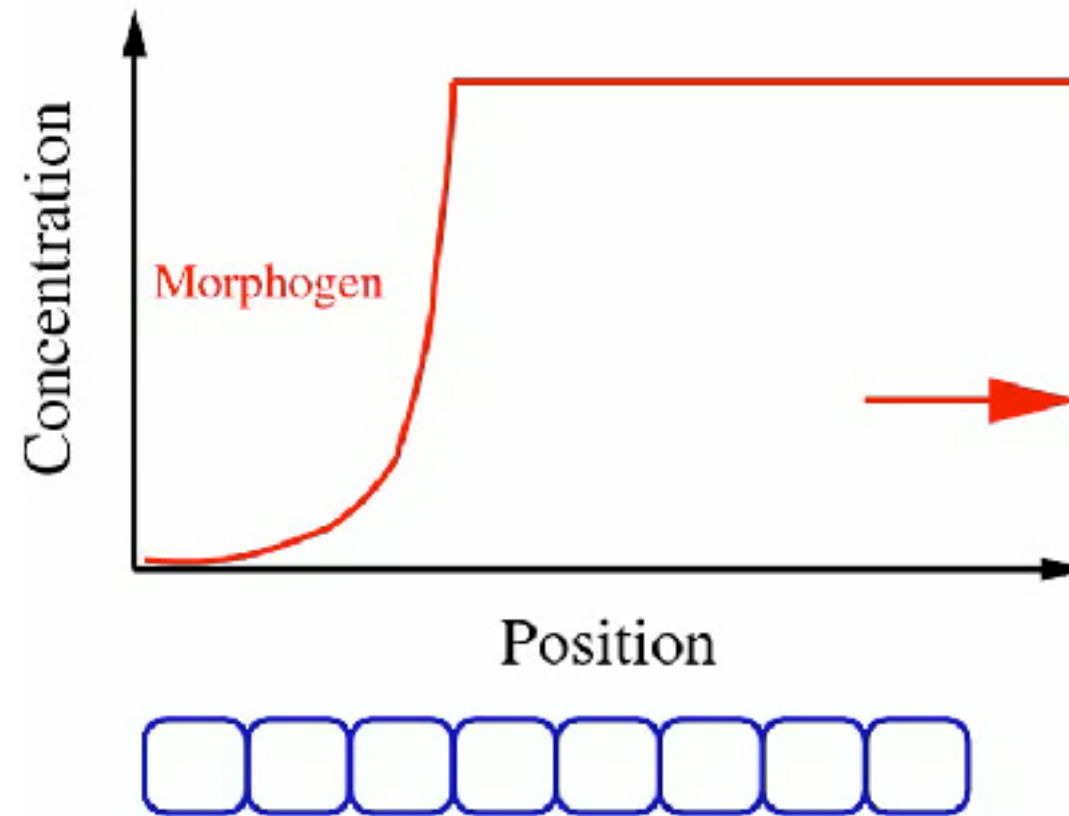


J Lewis,  
*Curr Bio* 2003

Speed of the front x Period of the clock =  
Typical size of the pattern (Cooke & Zeeman 1976)

# Evolving segmentation

- Embryo: a line of identical cells with common network
- Imposed morphogen translating to model growth zone
- Interactions: regulation of transcription + degradation, cell autonomous

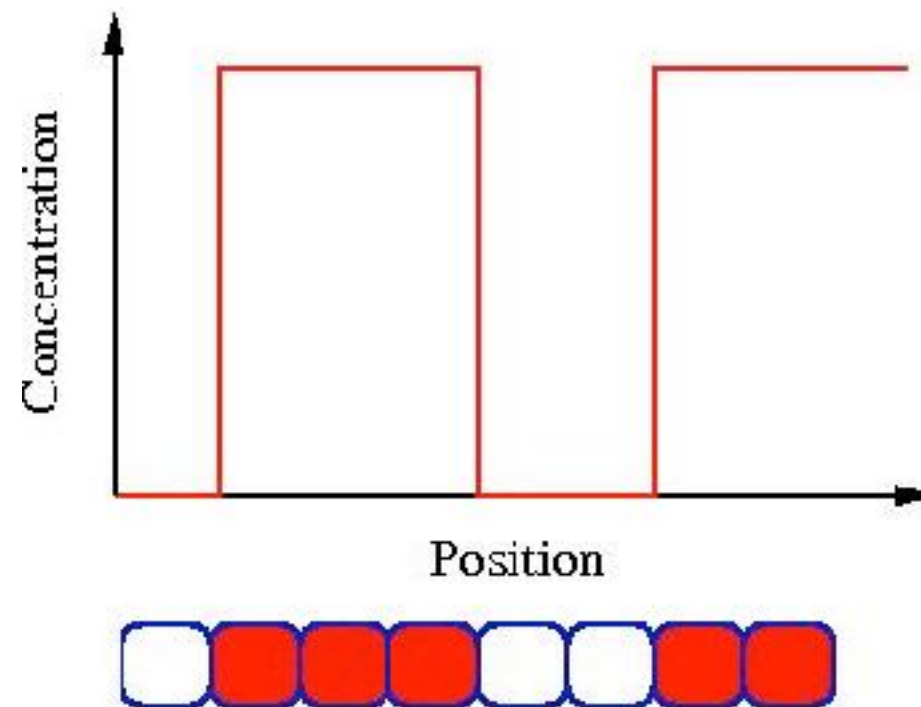


Fitness function : “More is better”

Level of protein E defines segments

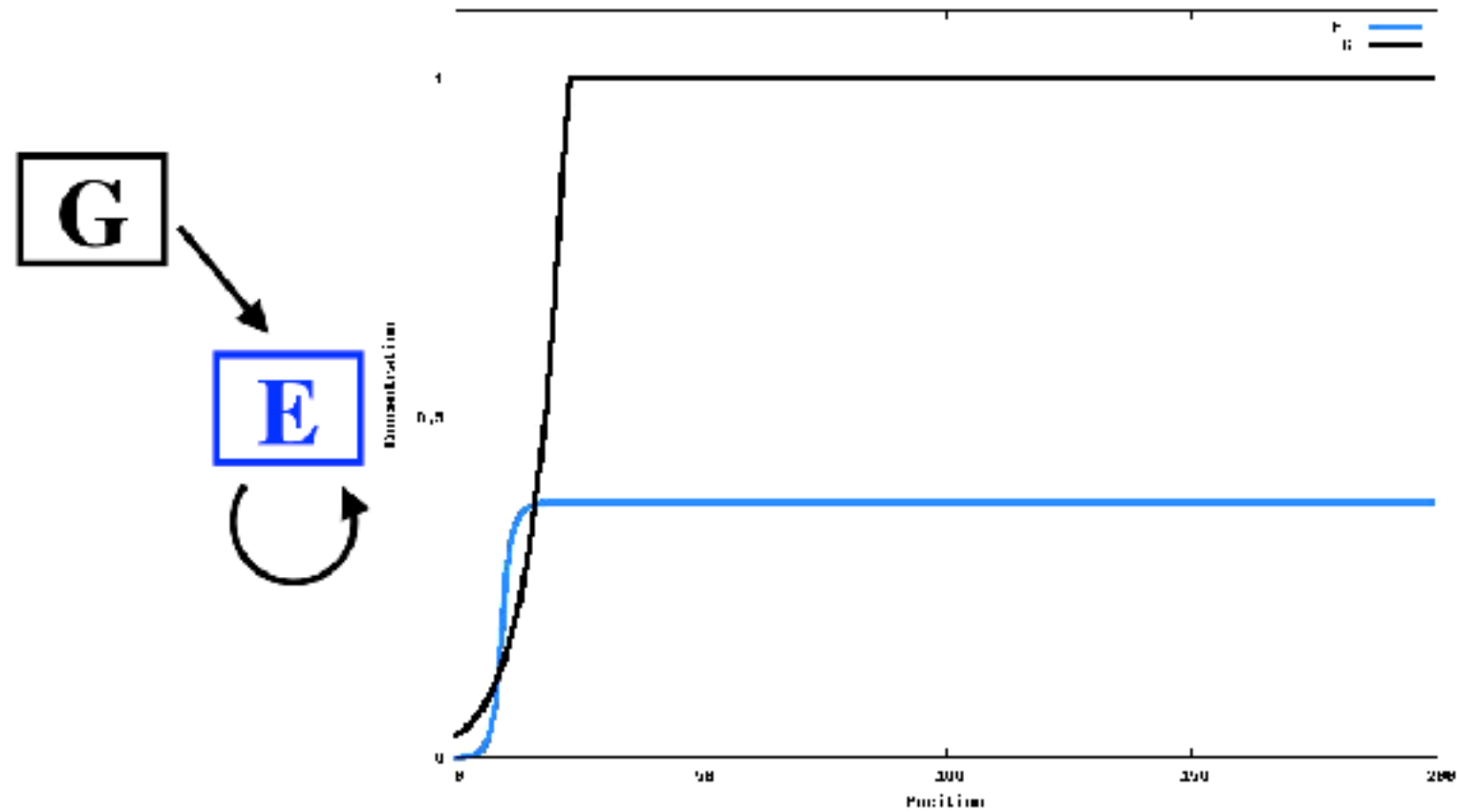
Fitness = number of up or down transitions in E at end of development.

e.g. here,  $F=3$



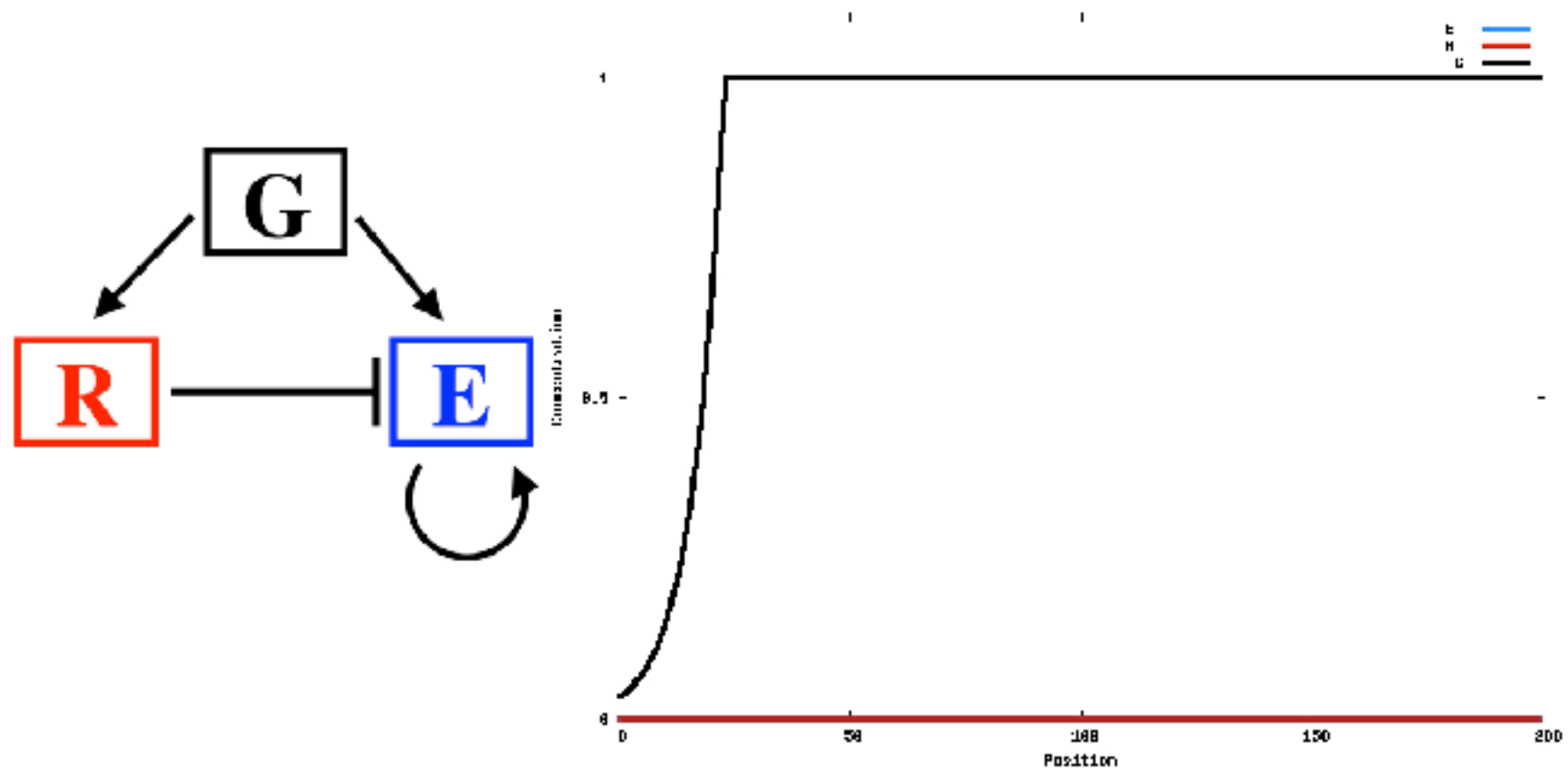


# Dynamic gradient : main pathway 1



$$F=1$$

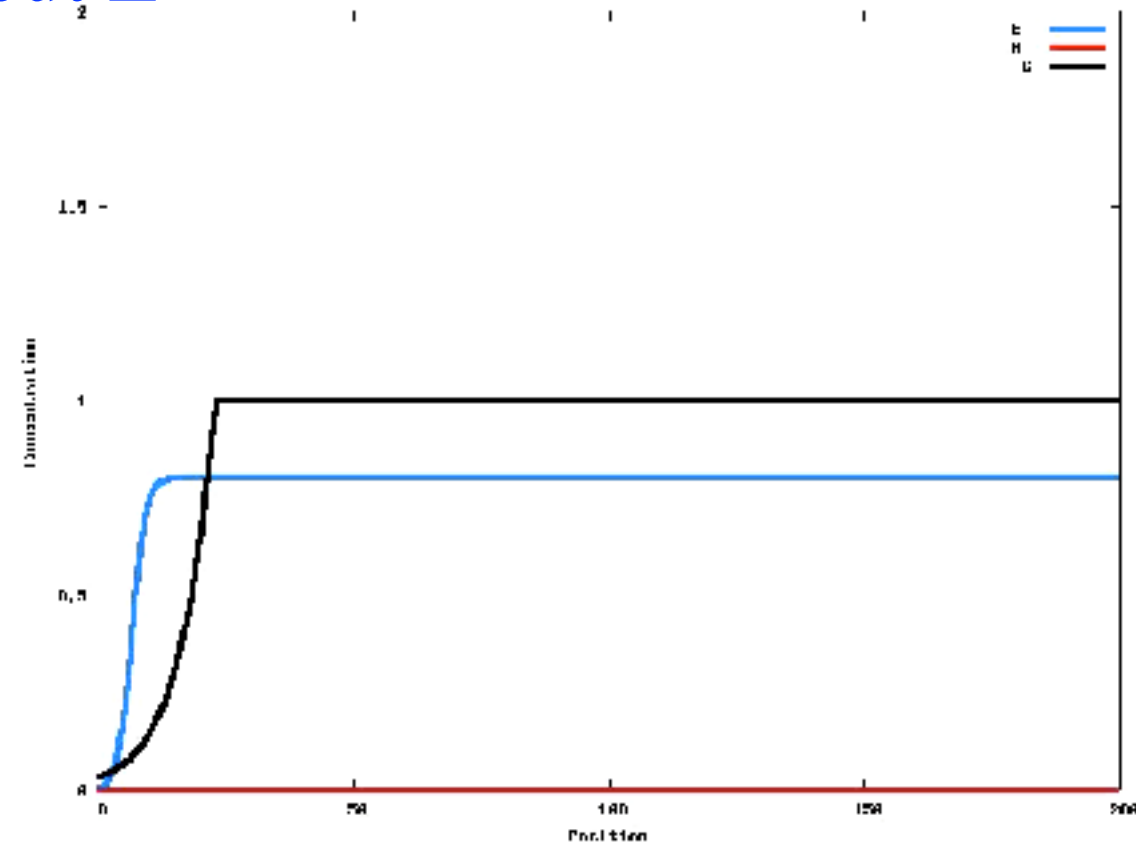
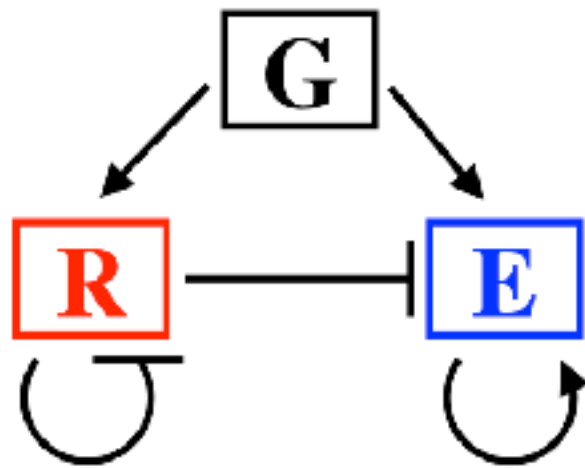
## Dynamic gradient : main pathway 2



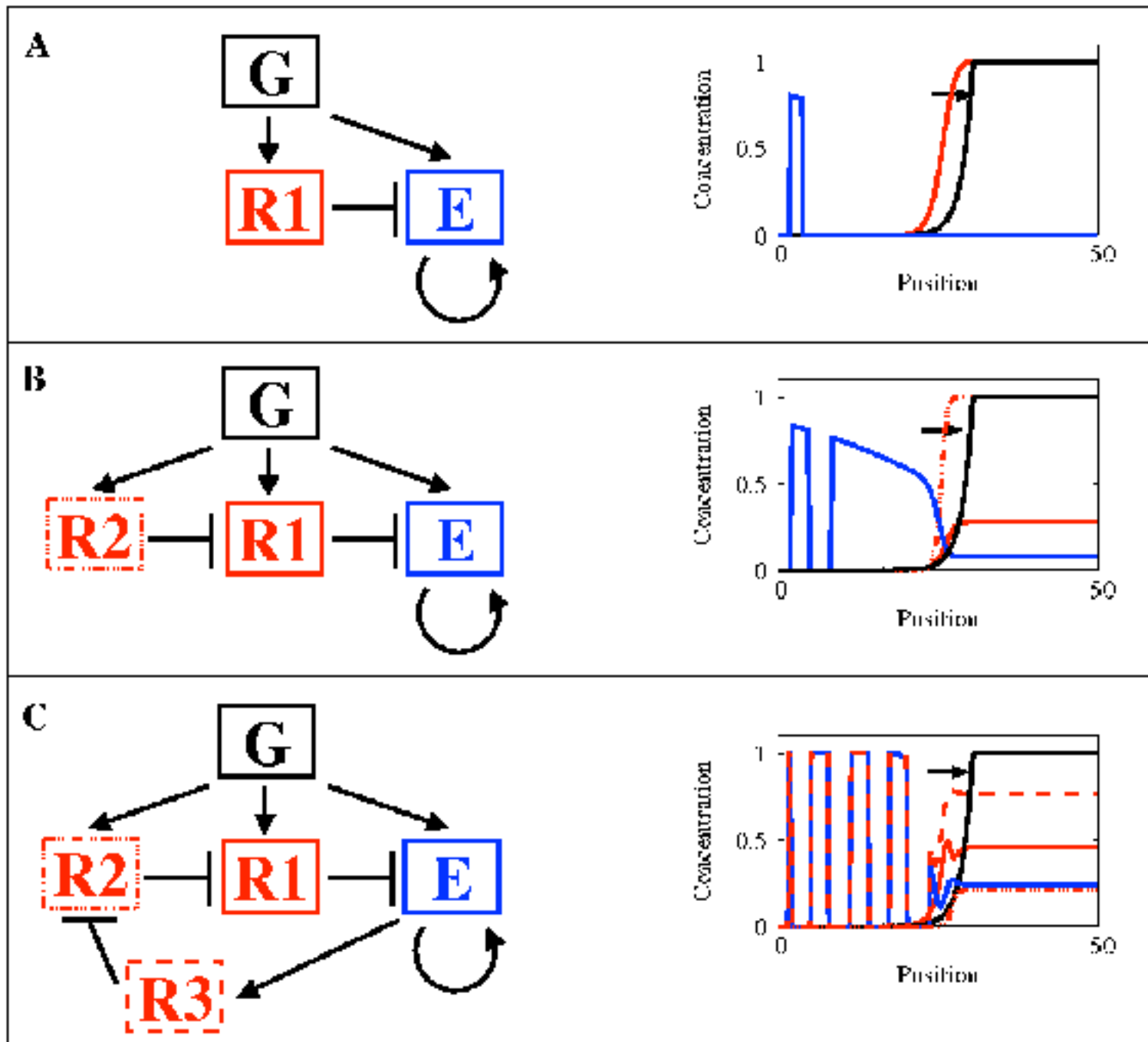
$F=2$

## Dynamic gradient : main pathway 3

Gradient G, Repressor R, Output E



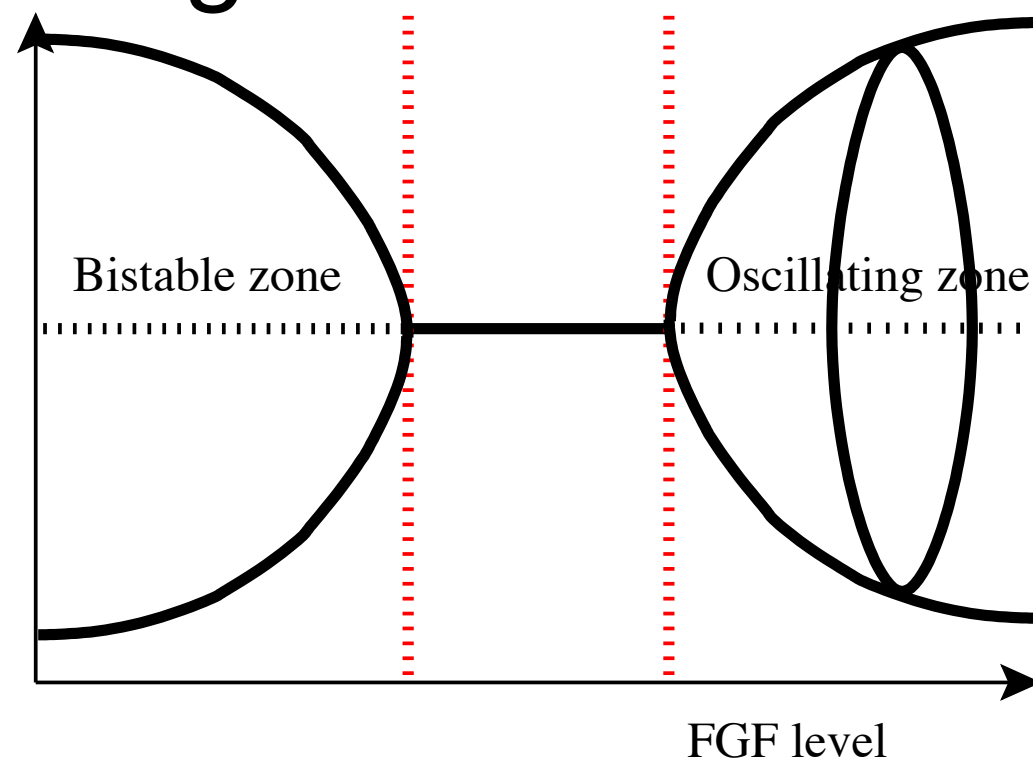
## Similar dynamics, but different topology



Instead of creating oscillator by neg. feedback of R1 on itself, make chain of repressors.. and finally oscillator via  $E \rightarrow R3$

## Stochasticity vs determinism

- Stochastic : network topology
- Reproducible (deterministic ?) : bifurcation diagrams



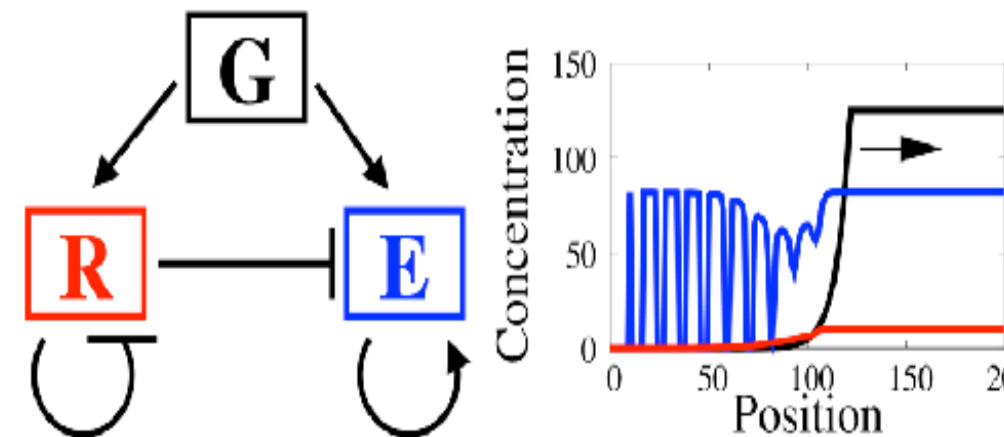
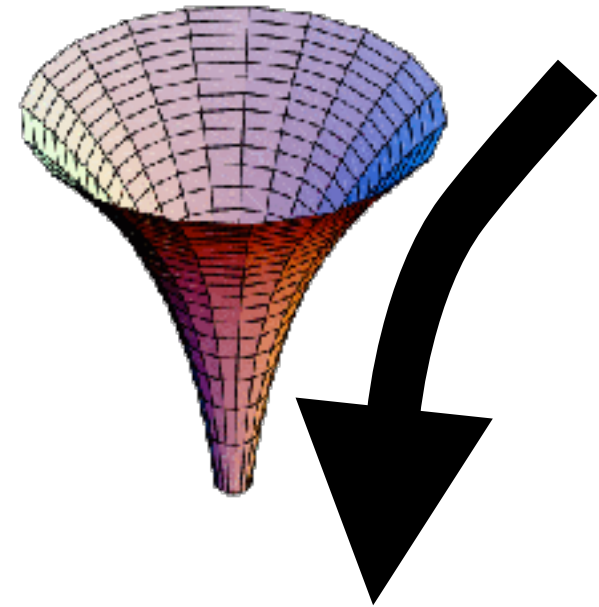
This means that the bifurcation level might be the “right” one to make predictions on evolution in that case.

(how to build a theory of this ?)

# Property of these evolutionary simulations

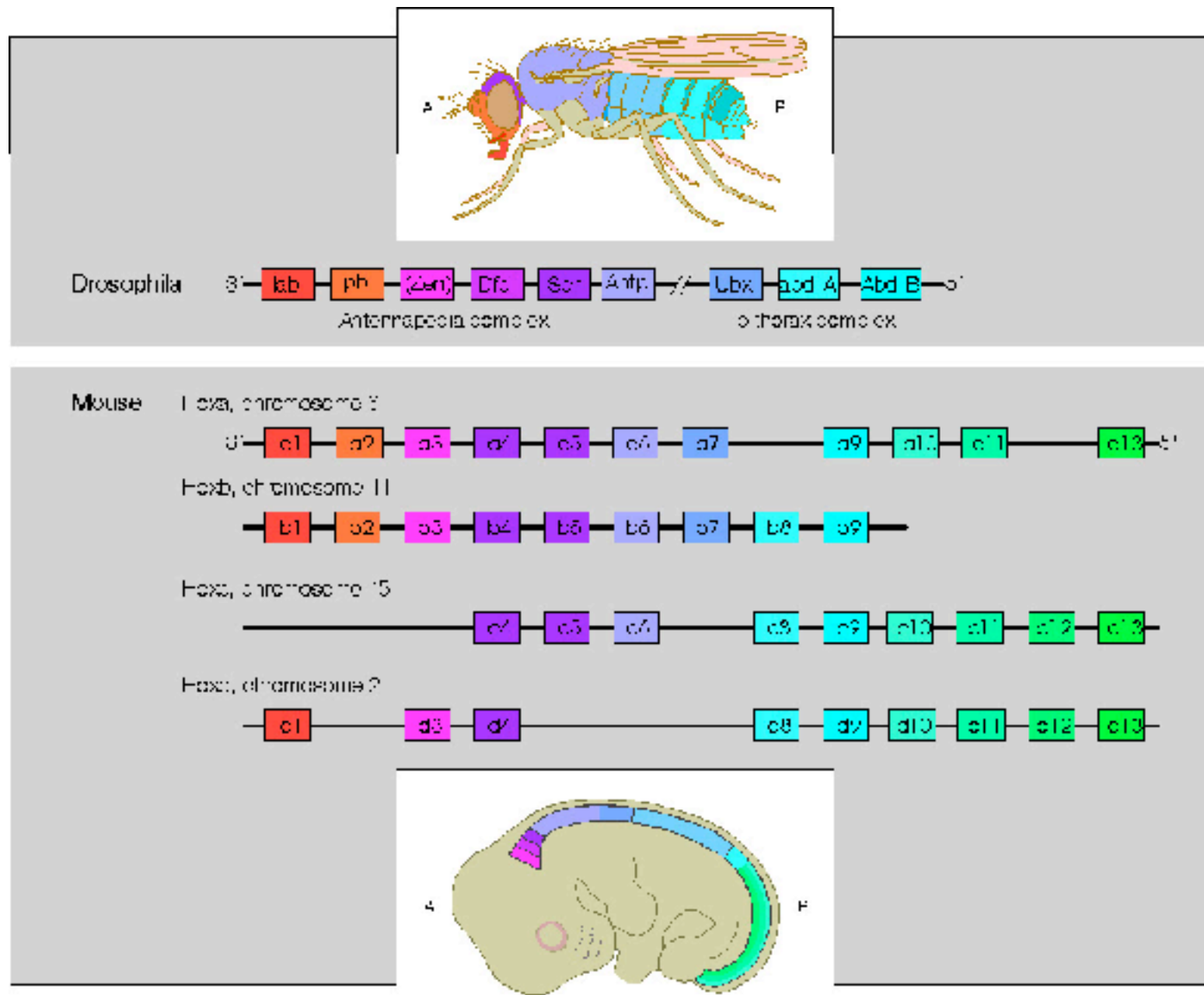
Evolution is reproducible from one simul. to the other

- “Parallel” evolution, a unique path :
  - Step 1 : bistable E to remember initial value.
  - Step 2 : Repressor added after E
  - Step 3 : Oscillator built from repressor
- Several solutions but similar phenotype.
- Clock and wavefront model comes “free”
- First model of the complete system.



PF, Vincent Hakim, Eric D. Siggia,  
Molecular Systems Biology (2007)

# Can we “guess” anything on hox patterning ?



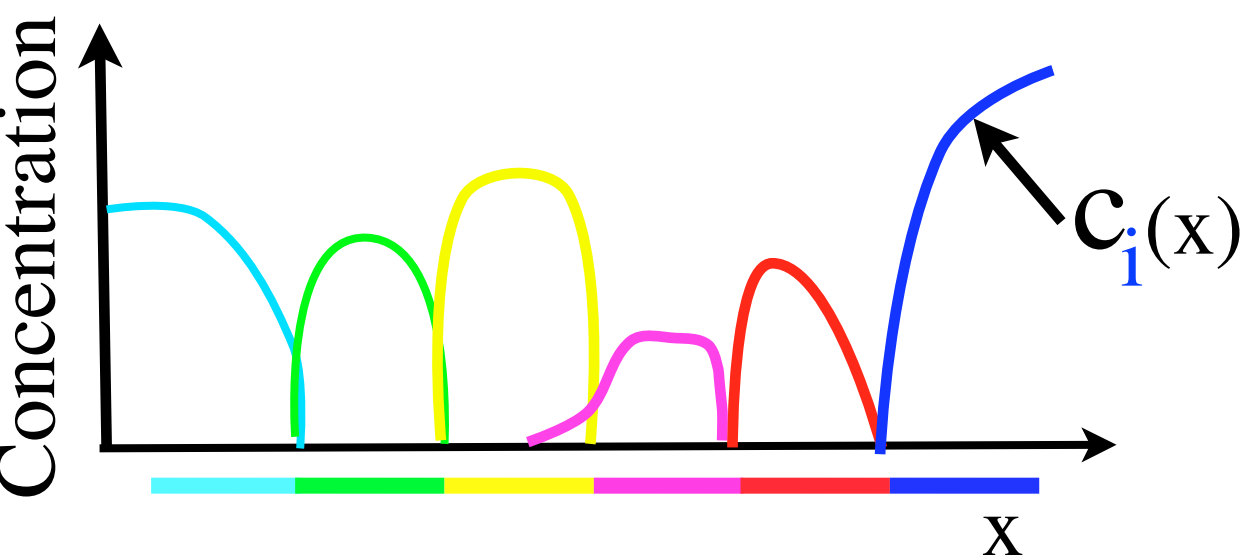
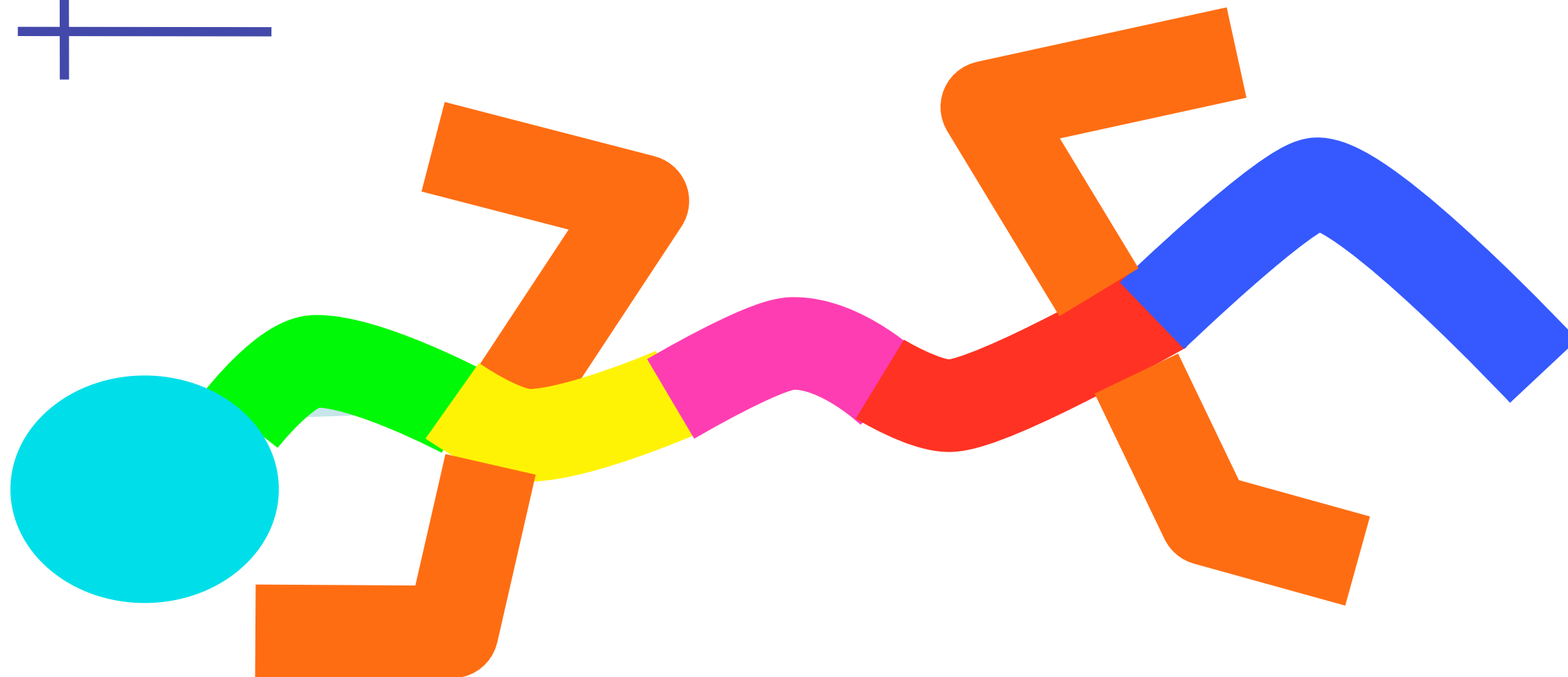
Hox genes : different parts of the body are defined by different hox genes.

“Cellular Zip Code”

Very complex system

Does any network structure emerge ?

# Patterning a field of cells



Selector genes hypothesis

Sox2 for neurons, MyoD for muscles...





# Patterning and entropy

Connecting patterning to entropy

- Rich patterning -> high **global** disorder, high entropy at the global level
- Precision of patterning -> high **local** order, low *conditional* entropy

Fitness I

$$p_{ij} = \frac{c_i(j)}{\sum_k c_k(j)}$$

i : selector gene

j : position

sum over i=1 -> we normalize  
by 1/N, N being the number of  
cells

## Fitness II Mutual information

Maximizing variability :  $\sum p_i \log(p_i) = -H(State)$

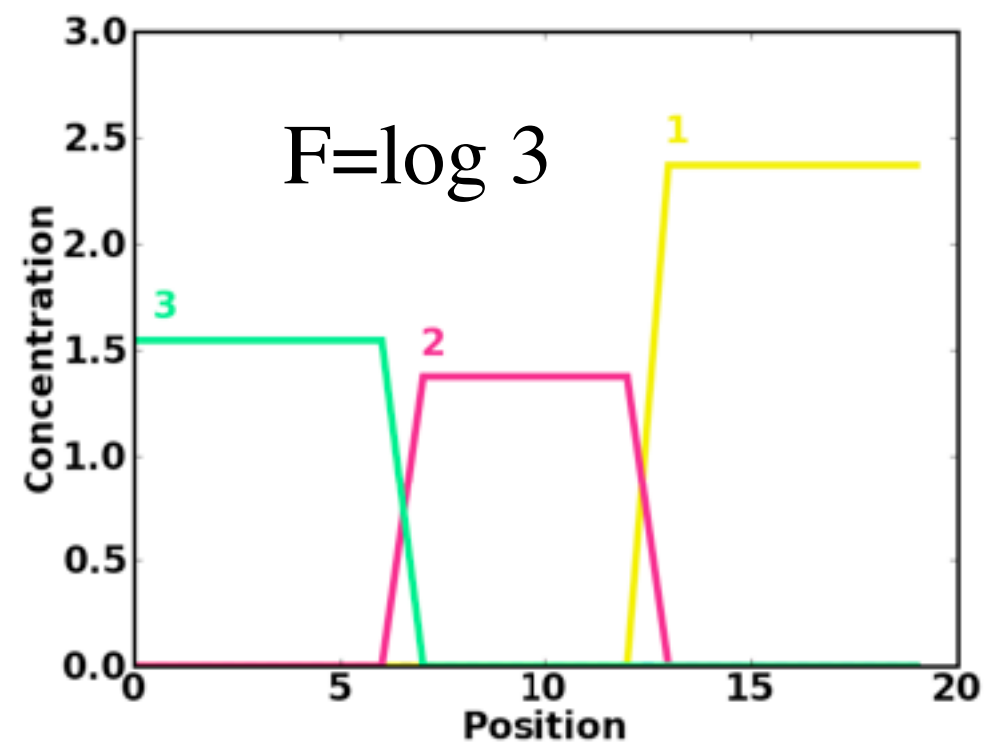
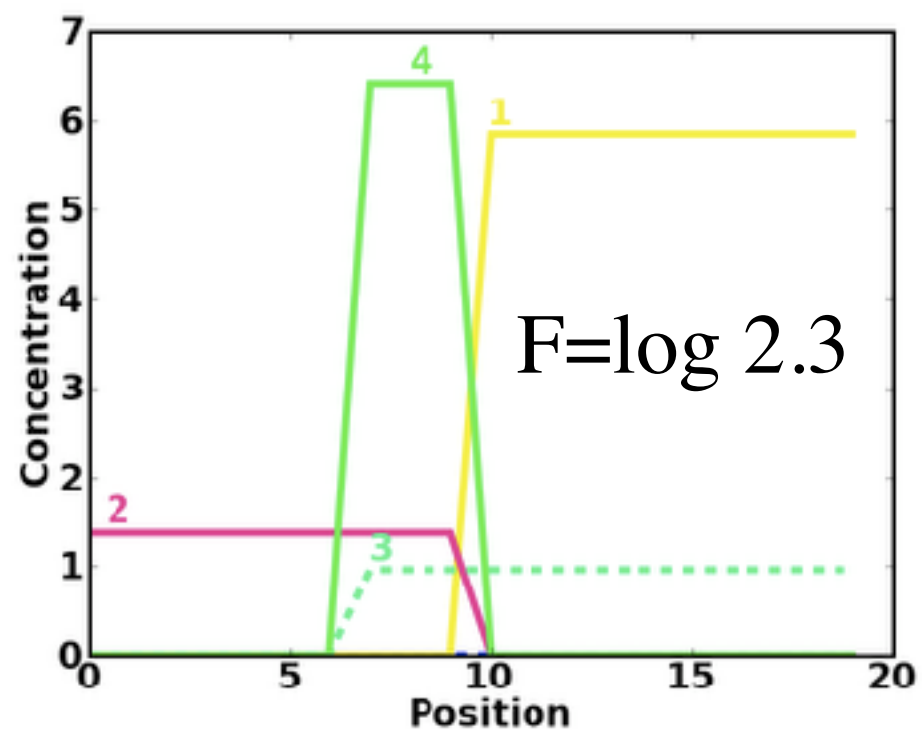
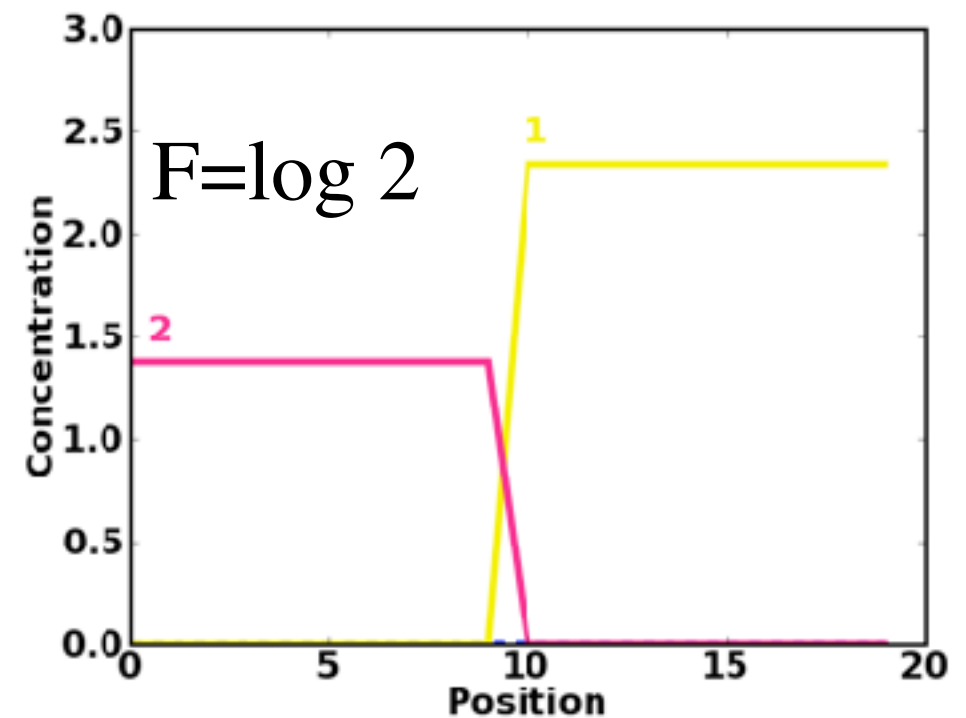
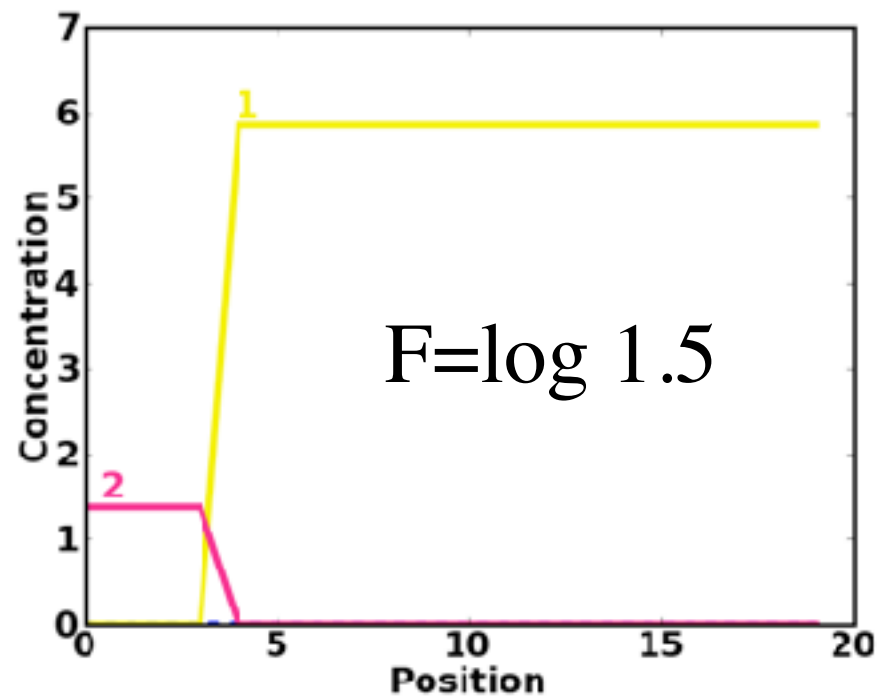
Minimizing entropy of State knowing Position :

$$\sum p_{ij} \log(p_{ij}) - \sum p_j \log(p_j) = -H(State|Position)$$

One Possible  
Fitness :  $\sum p_i \log(p_i) + \sum p_j \log(p_j) - \sum p_{ij} \log(p_{ij})$

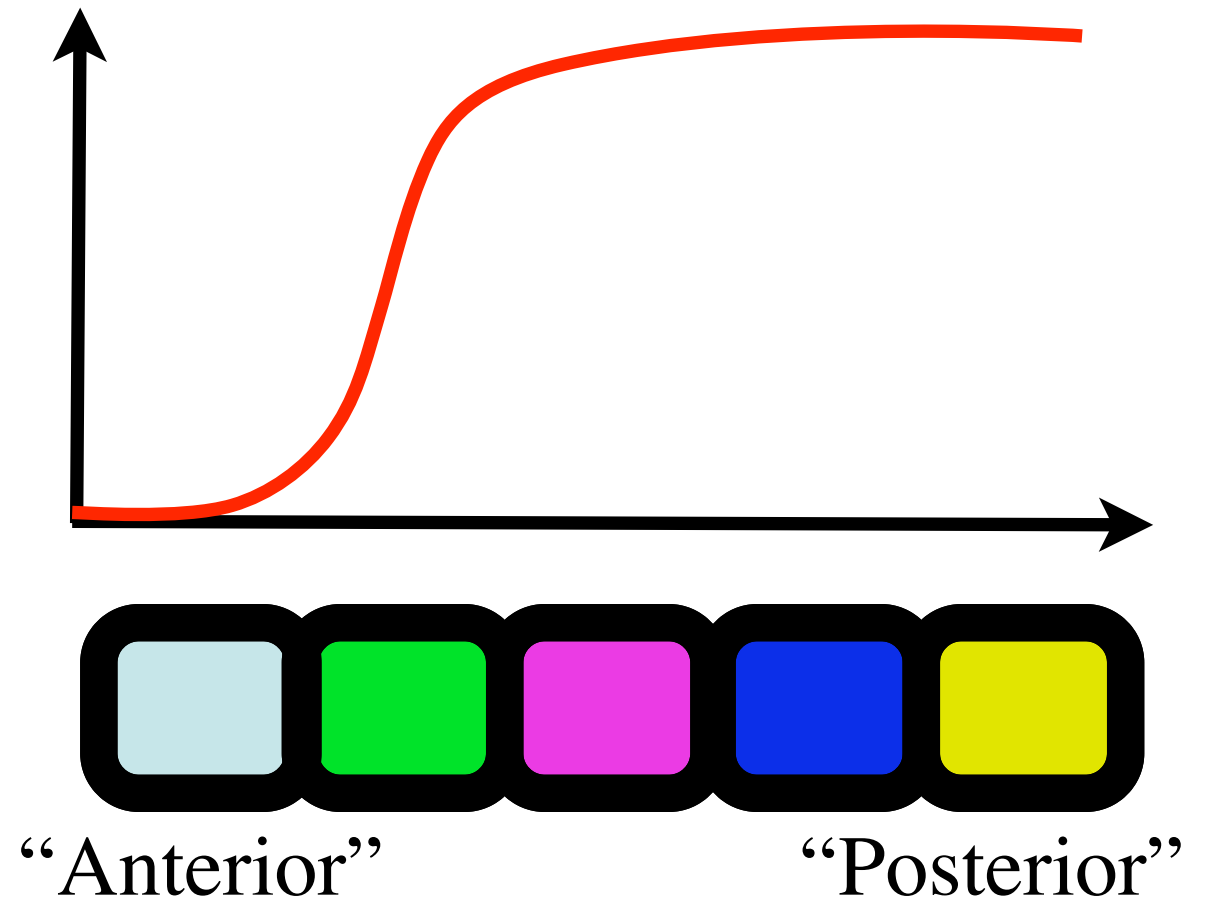
“From its fate, the cell can determine position, and vice-versa”

# Quantifying “patterning” : mutual info between expression and position

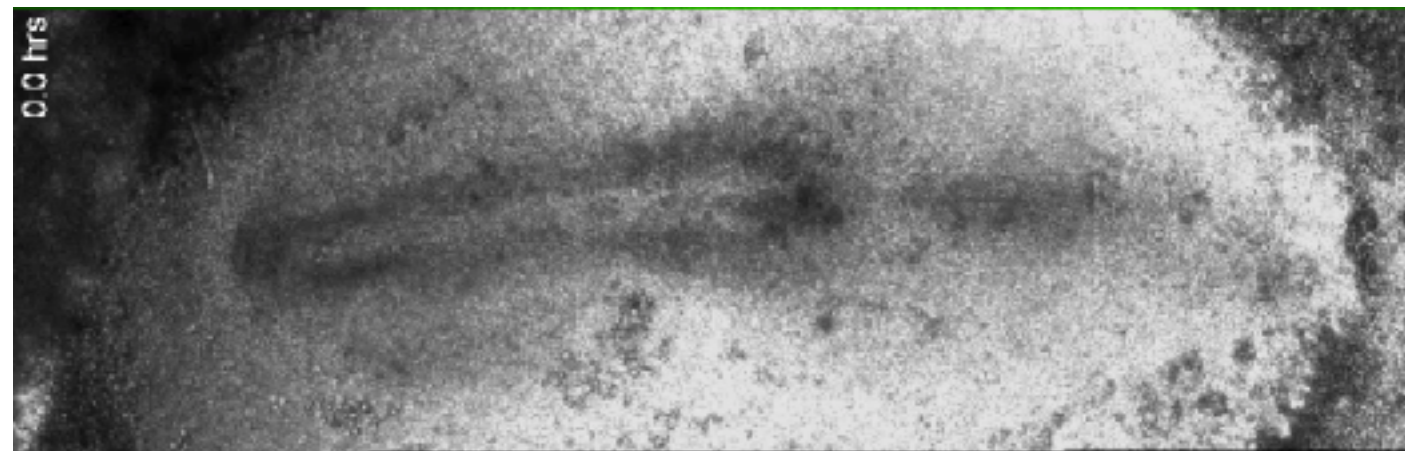


## Patterning a field of cells : AP growth

- Impose translating morphogen defining growth zone (or frog organizer)
- Gene duplication possible (“creates” a selector gene)



Difference between cells =  
time spent before involuting

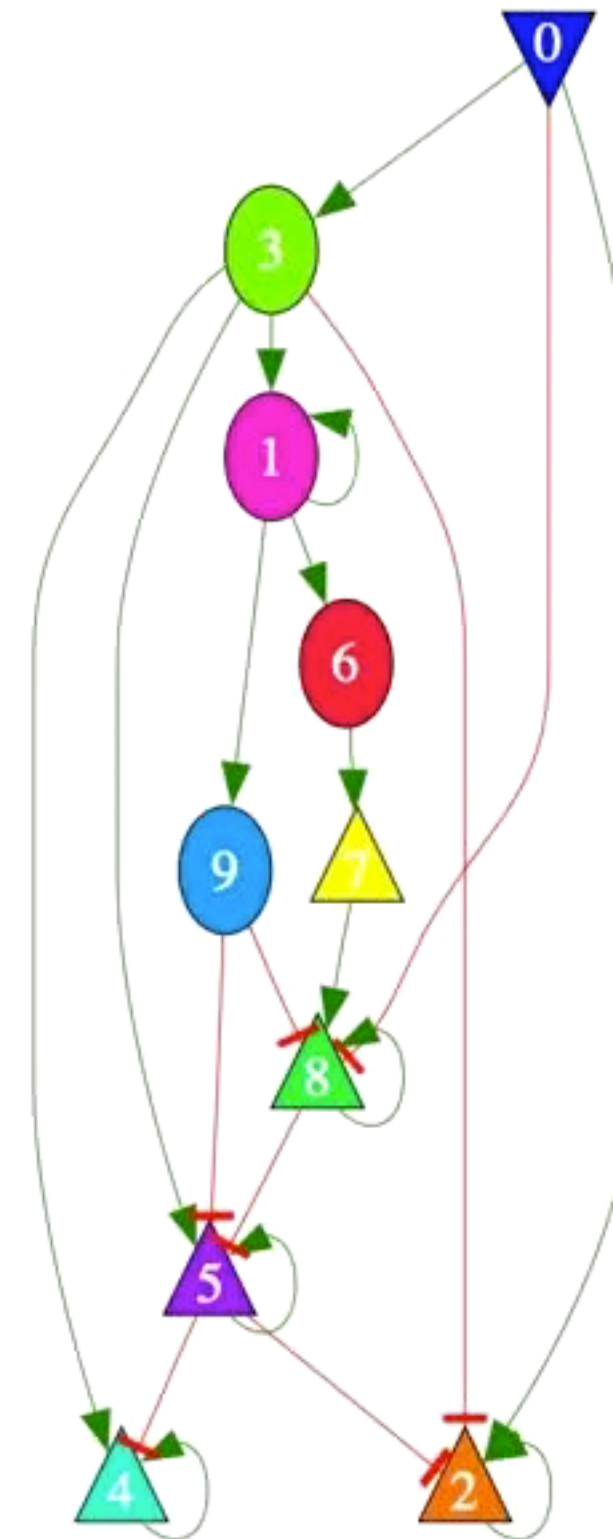


# Patterning a field of cells

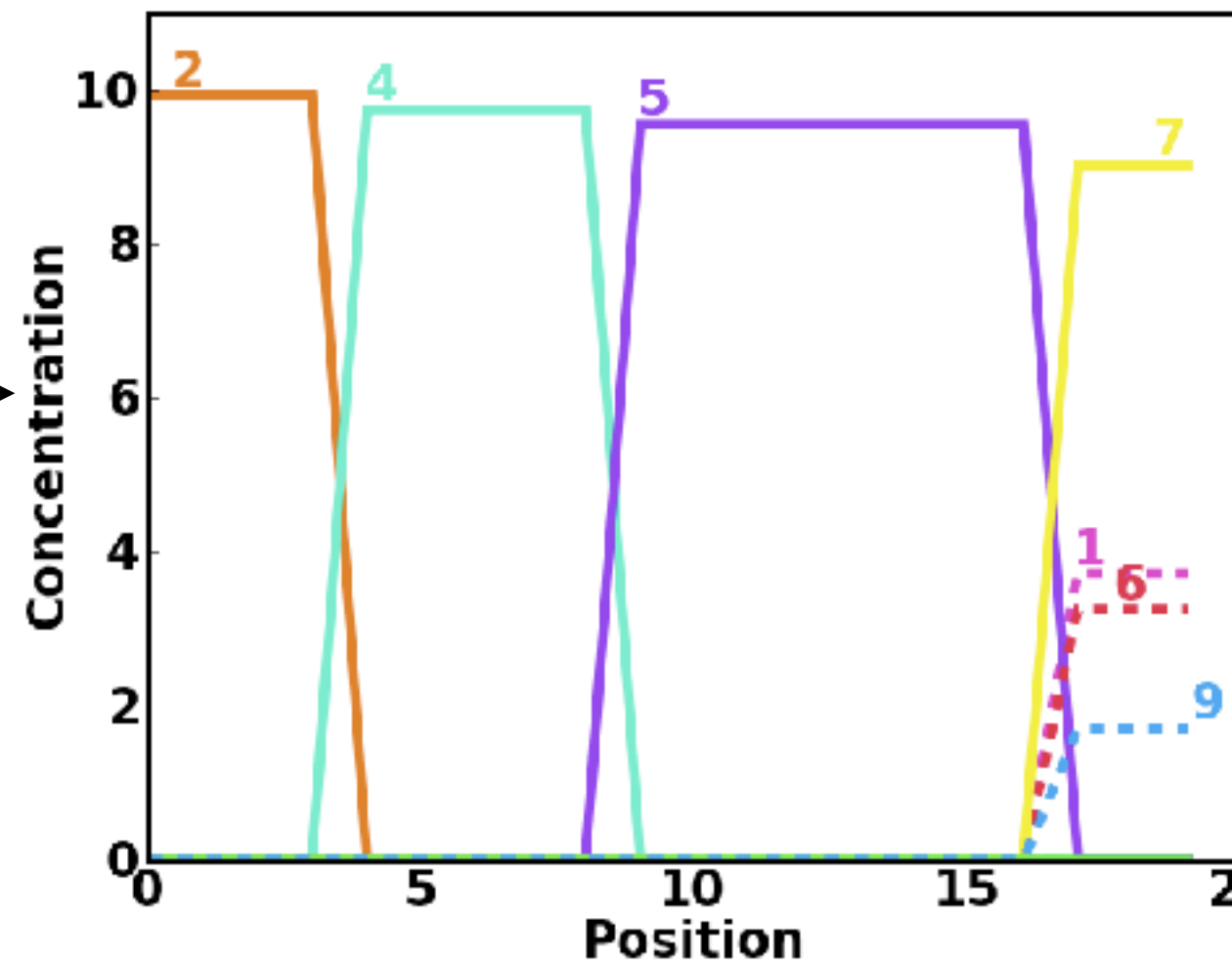
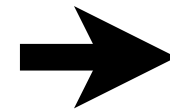
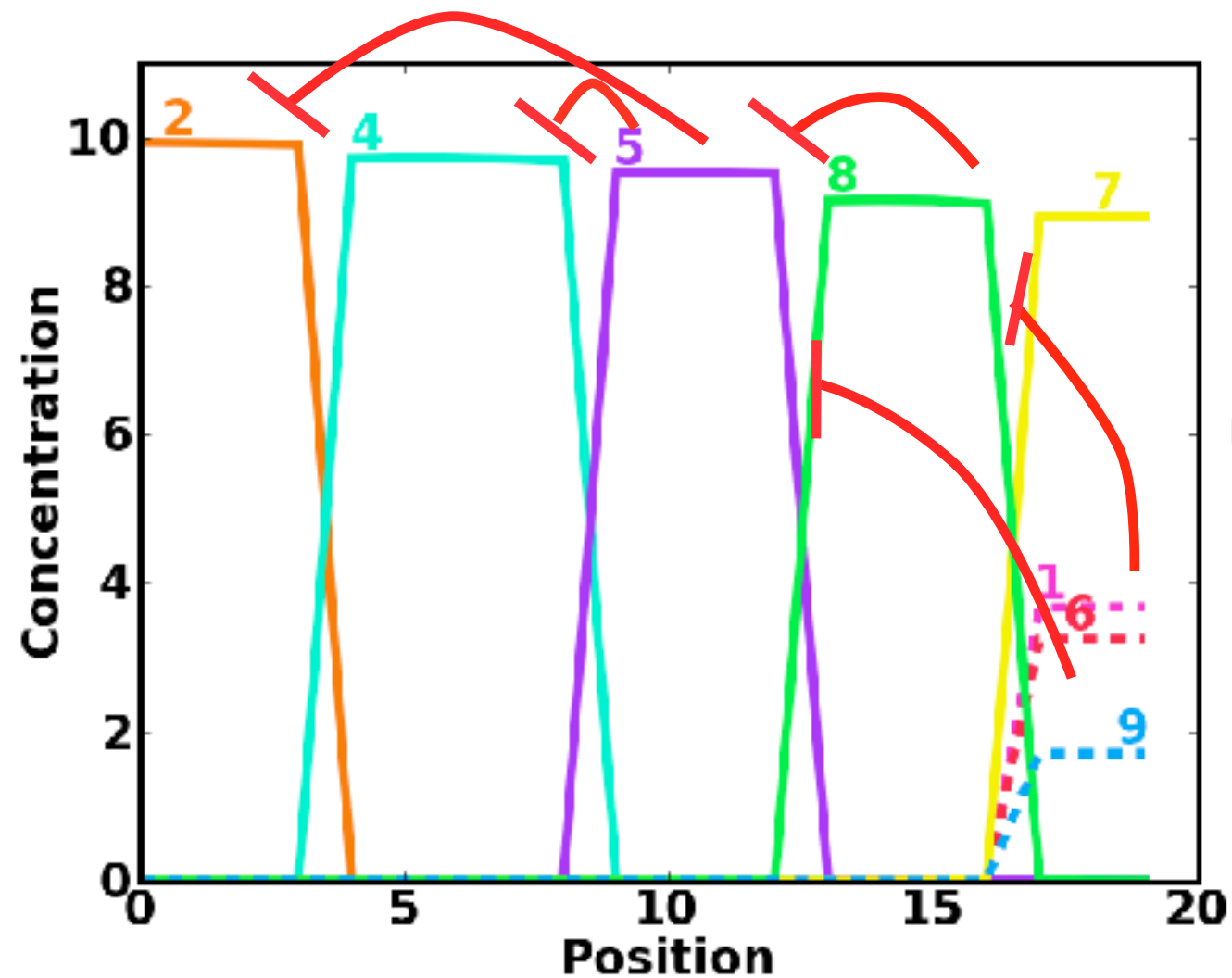
## Dynamics of the network topology displayed on Fig.5

Sliding input is in dashed blue,  
simulating the presence of cells  
in a growth zone

P. François, E. Siggia  
pfrancois@rockefeller.edu

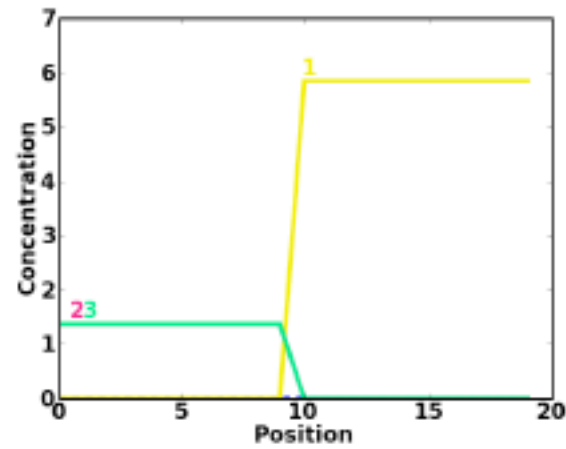
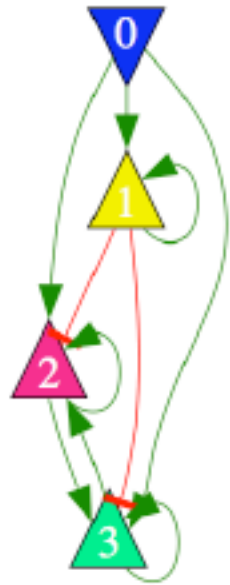


‘Anterior homeotic transformation’ : removal of gene  
8 -> similar to posterior prevalence

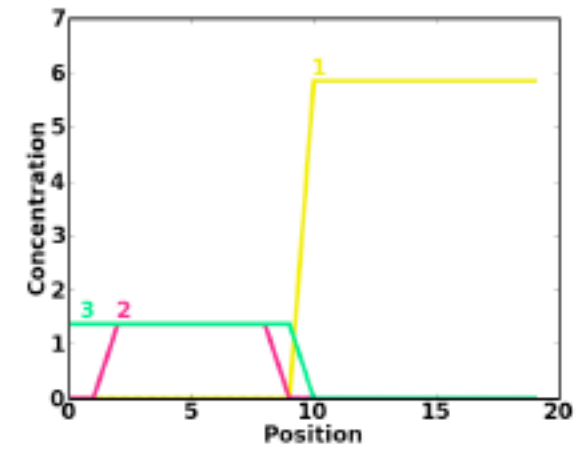
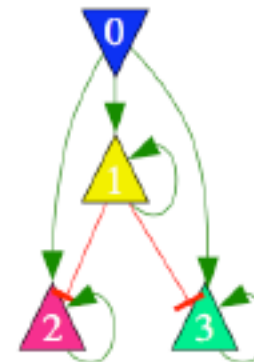


# Typical evolutionary pathways

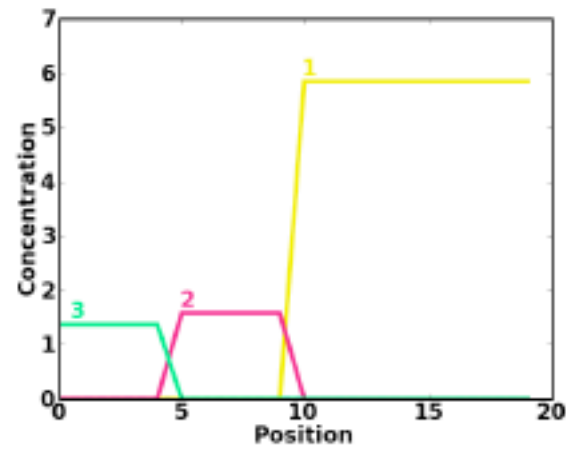
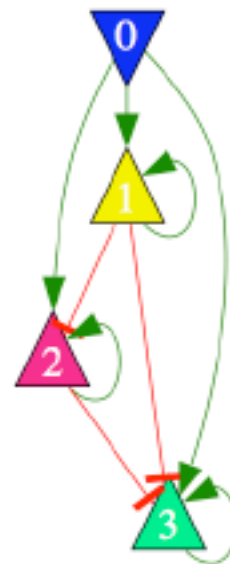
A



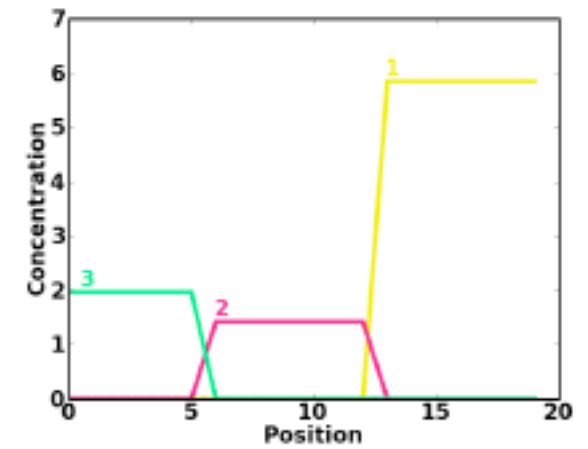
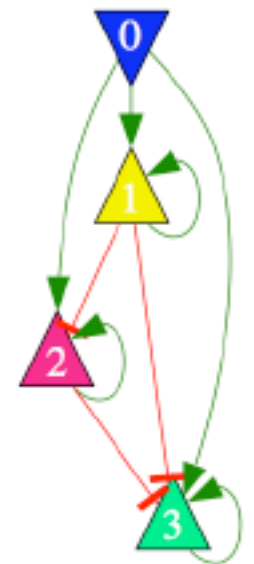
B



C



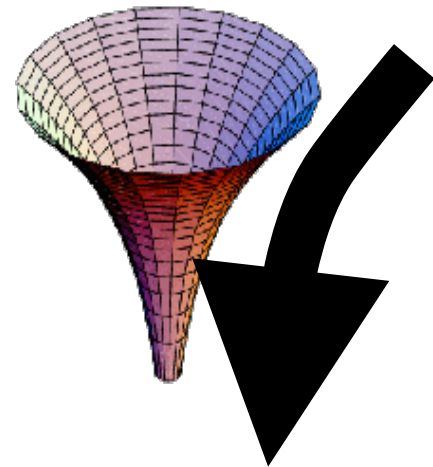
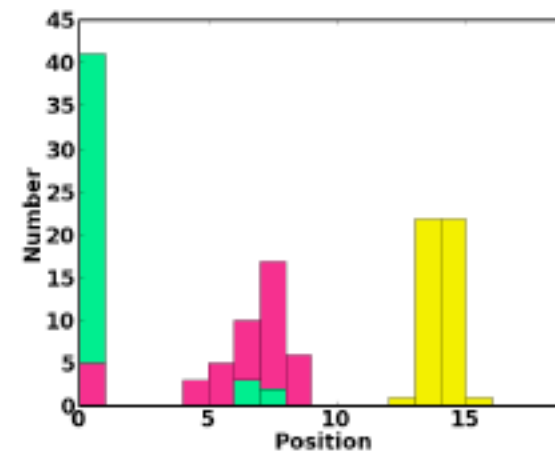
D



E



F





## Conclusions hox-like patterning

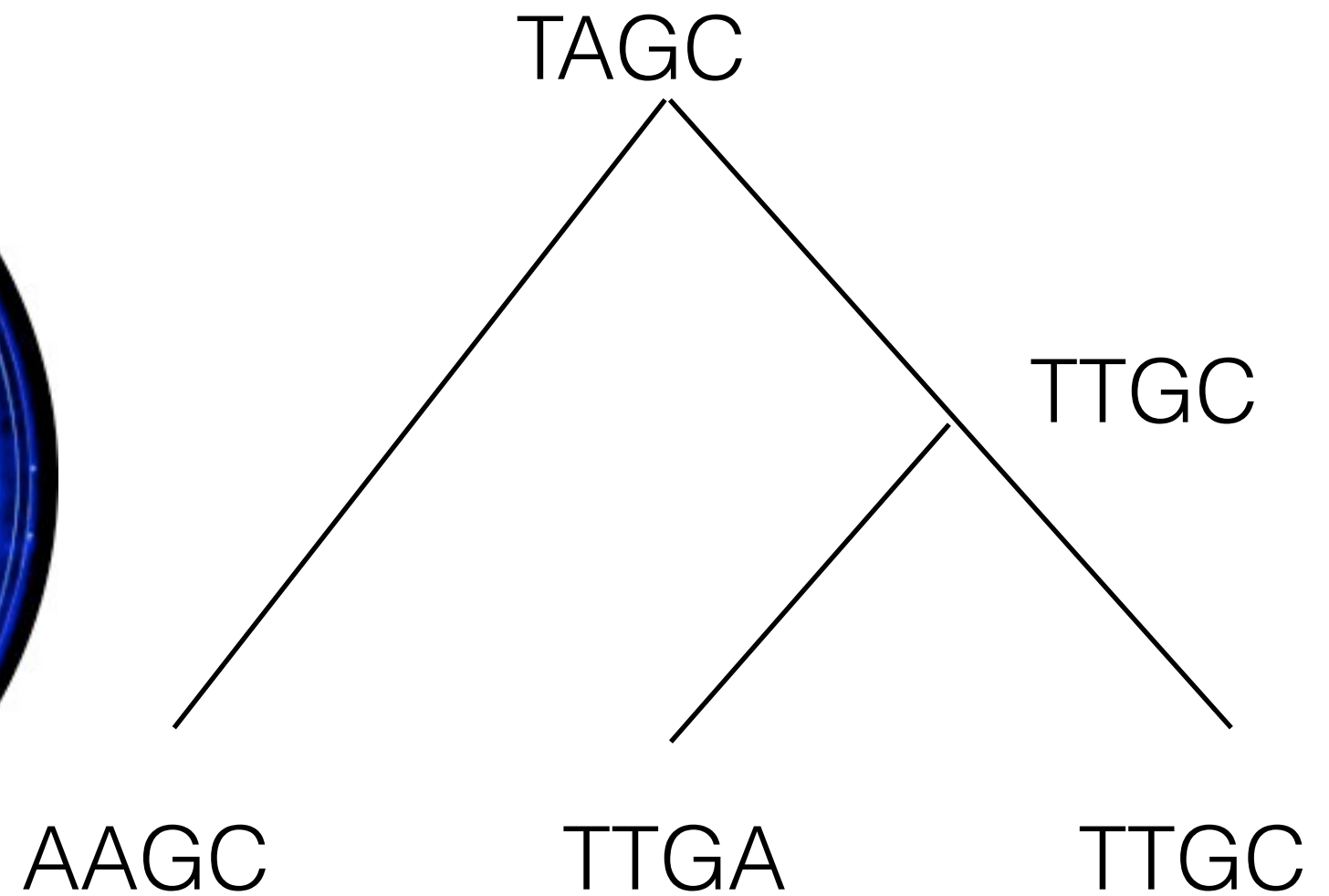
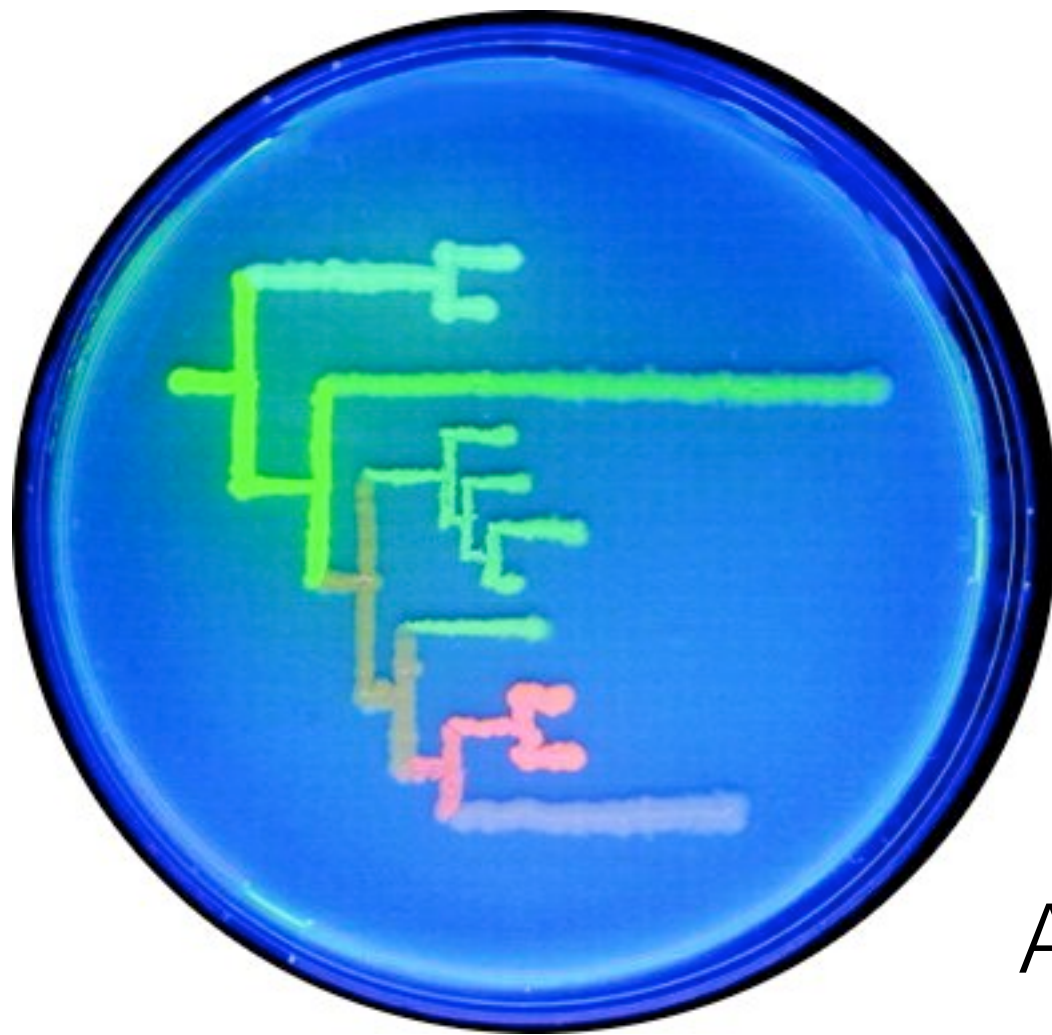
- A mutual information fitness can be defined to quantify “diversity” and “precision” in patterning
- Evolutionary steps can be understood mathematically
- Computational evolution suggests transitory forms

### Reproducible aspects of evolution :

- Networks evolve by successive gene duplications
- Networks are complex in general, but some clear rules evolve :
  - anterior boundaries set up by activation threshold by timer, we predict “timer gene” acting like morphogen
  - posterior boundaries set up by repression by posterior/late genes

-> *“posterior dominance” + temporal colinearity as consequences* PF, Eric D. Siggia, Development (2010)

# Reconstructing evolutionary history



# Predicting ancestral species ?

Ur-Dipteran ?



Drosophila



Clogmia



Anopheles

Not really clear what the cost function should be

Predicting ancestral species

(Evolutionary)  
Machine Learning

Ur-Dipteran ?

**Predictions**



Drosophila



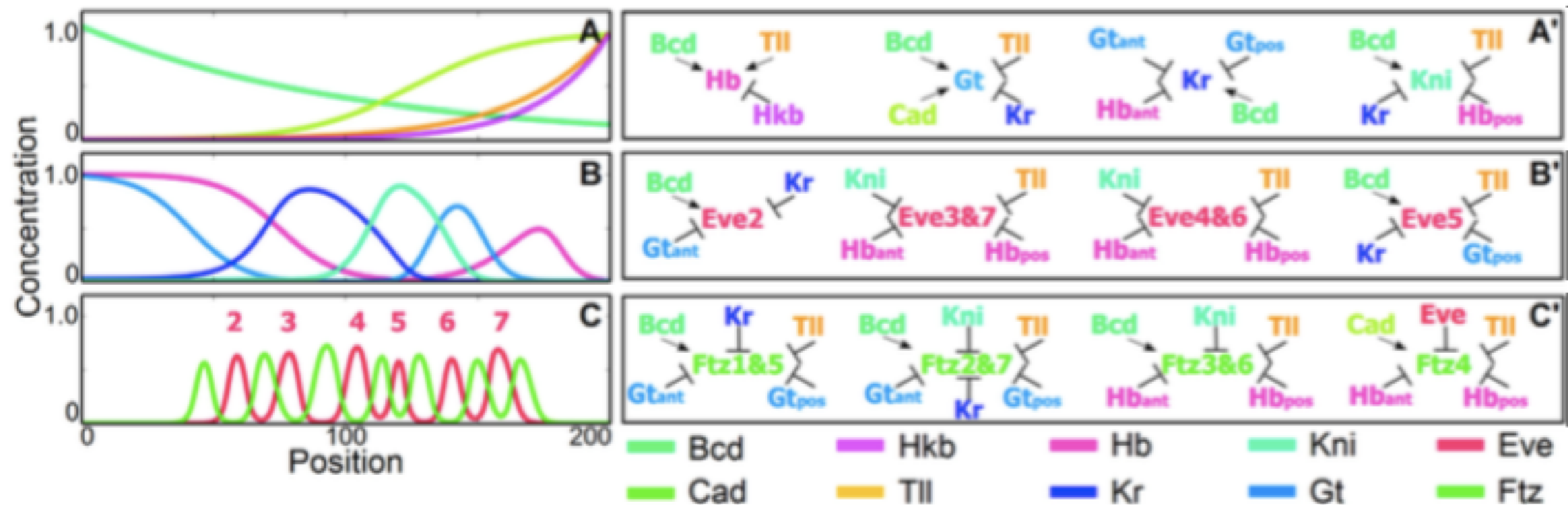
Clogmia



Anopheles

Score: conservation of “some” striped pattern

# A minimum semi-phenomenological model of the fly cascade



- gap genes repress each other
- eve stripes are sandwiched between local gap genes
- standard biochemical kinetics

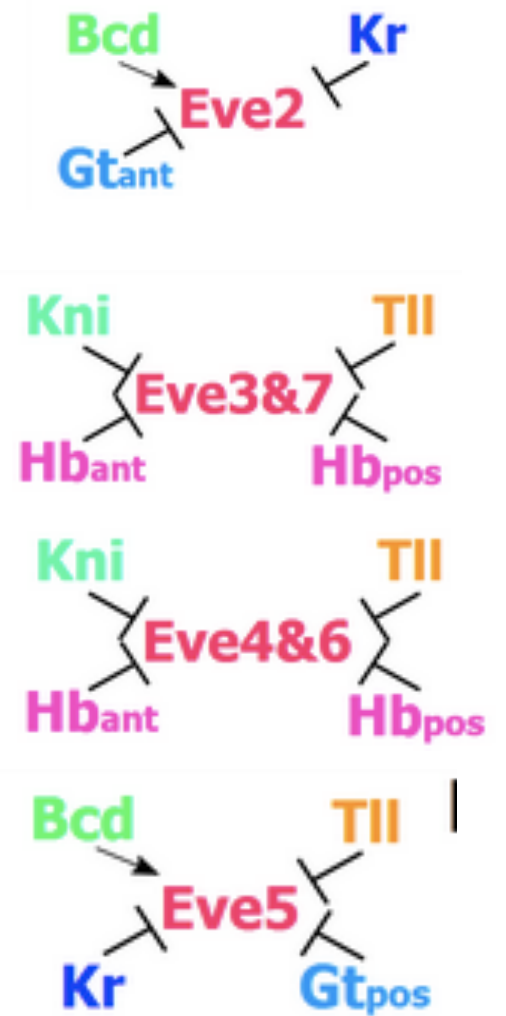
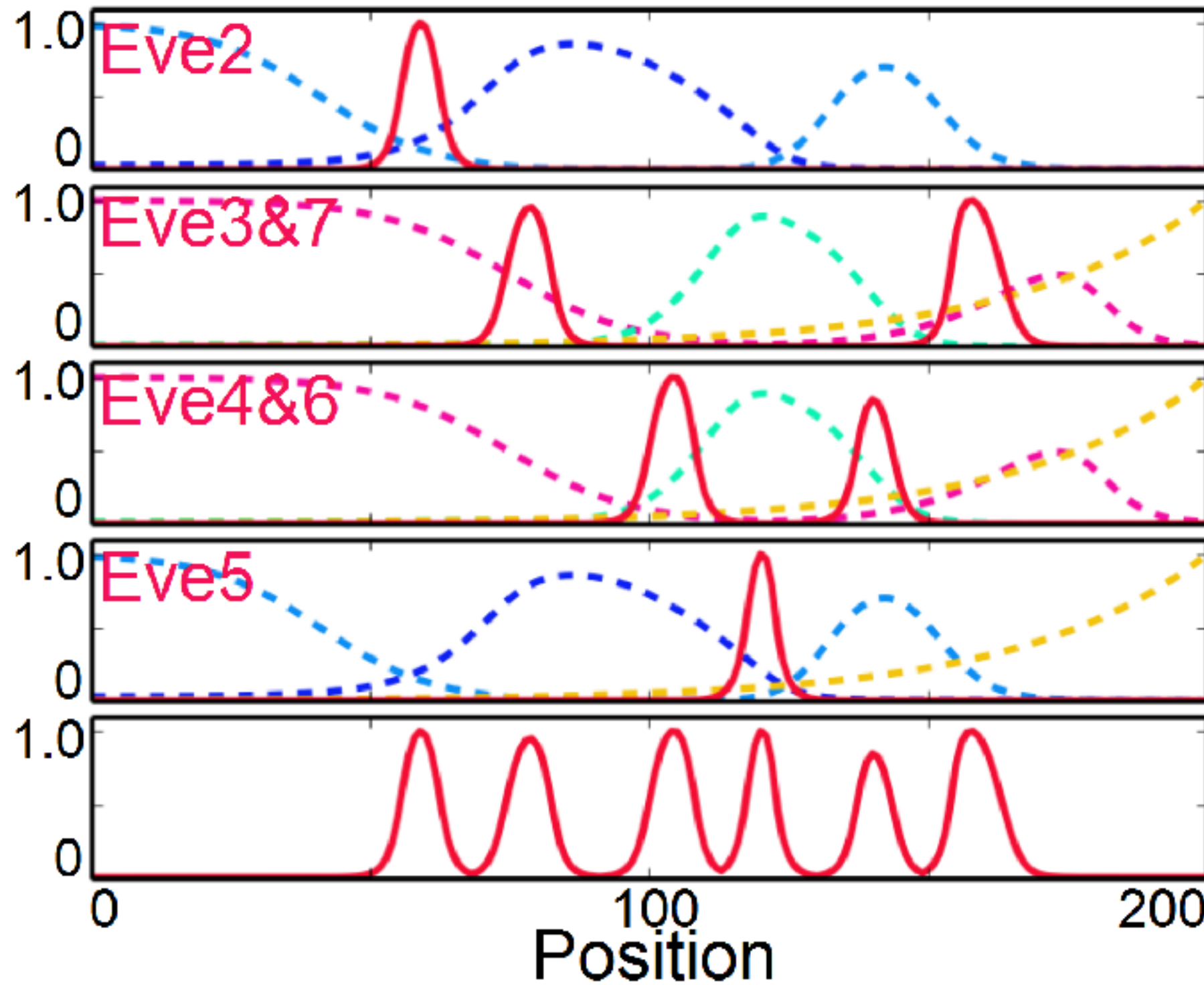
$$\frac{dGt}{dt} = \frac{Bcd^n}{Bcd^n + b^n} \frac{k^m}{Kr^m + k^m} - \delta Gt$$



# Eve regulations (from both genetics and bioinformatics)

1A

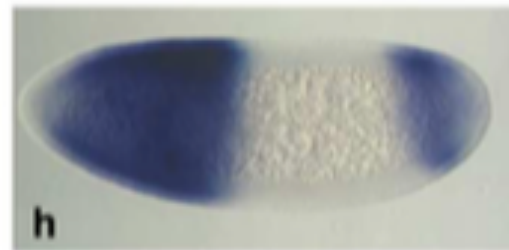
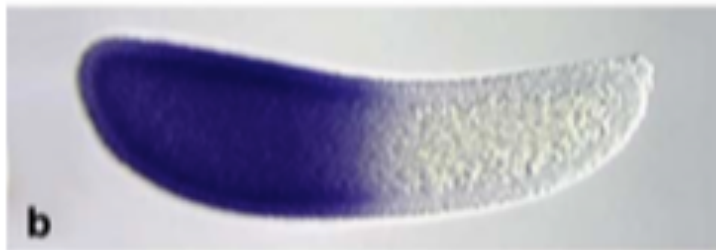
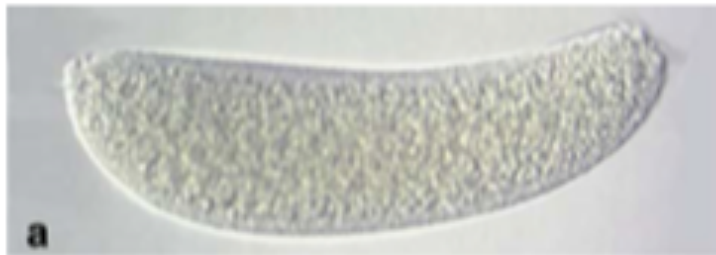
Concentration



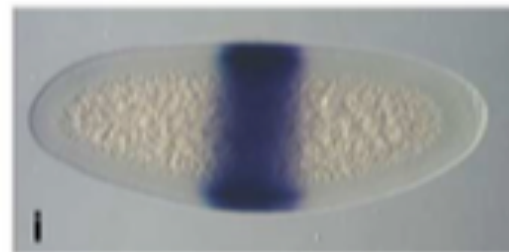
# Mosquito side: Goltsev et al., 2004

Mosquito

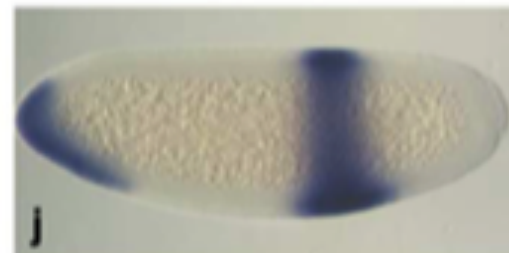
Fly



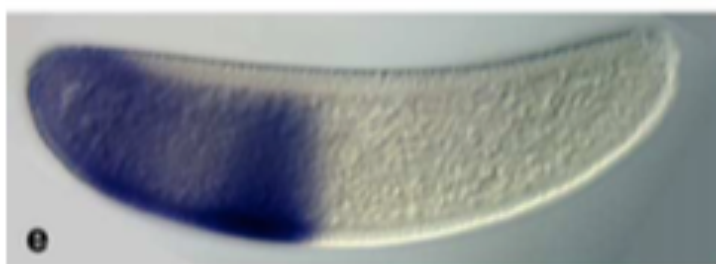
hb



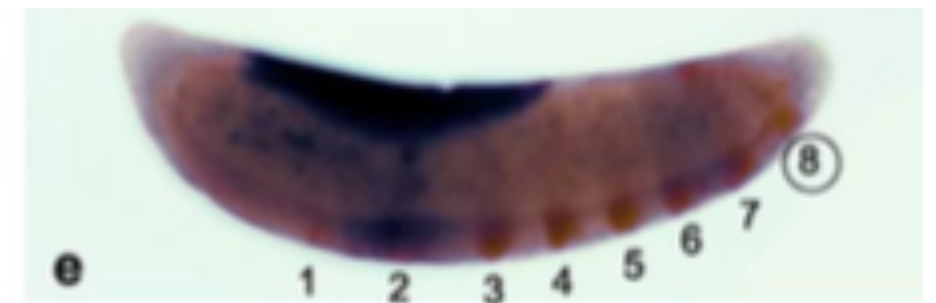
Kr



kni



gt



Up to 8 eve stripes !

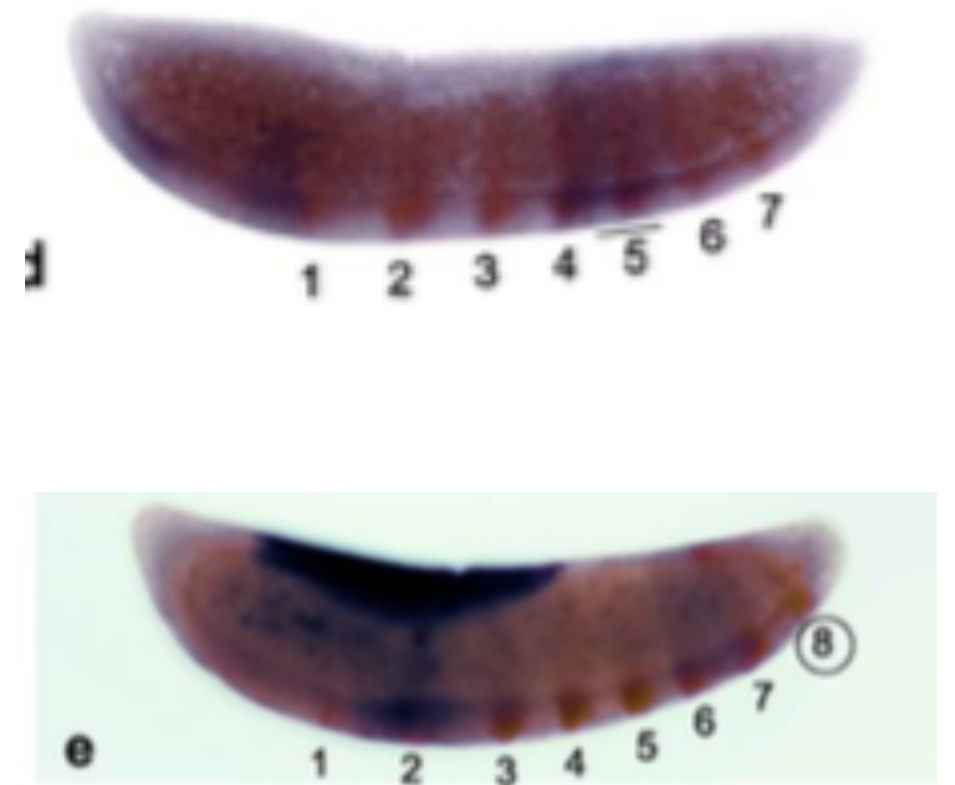
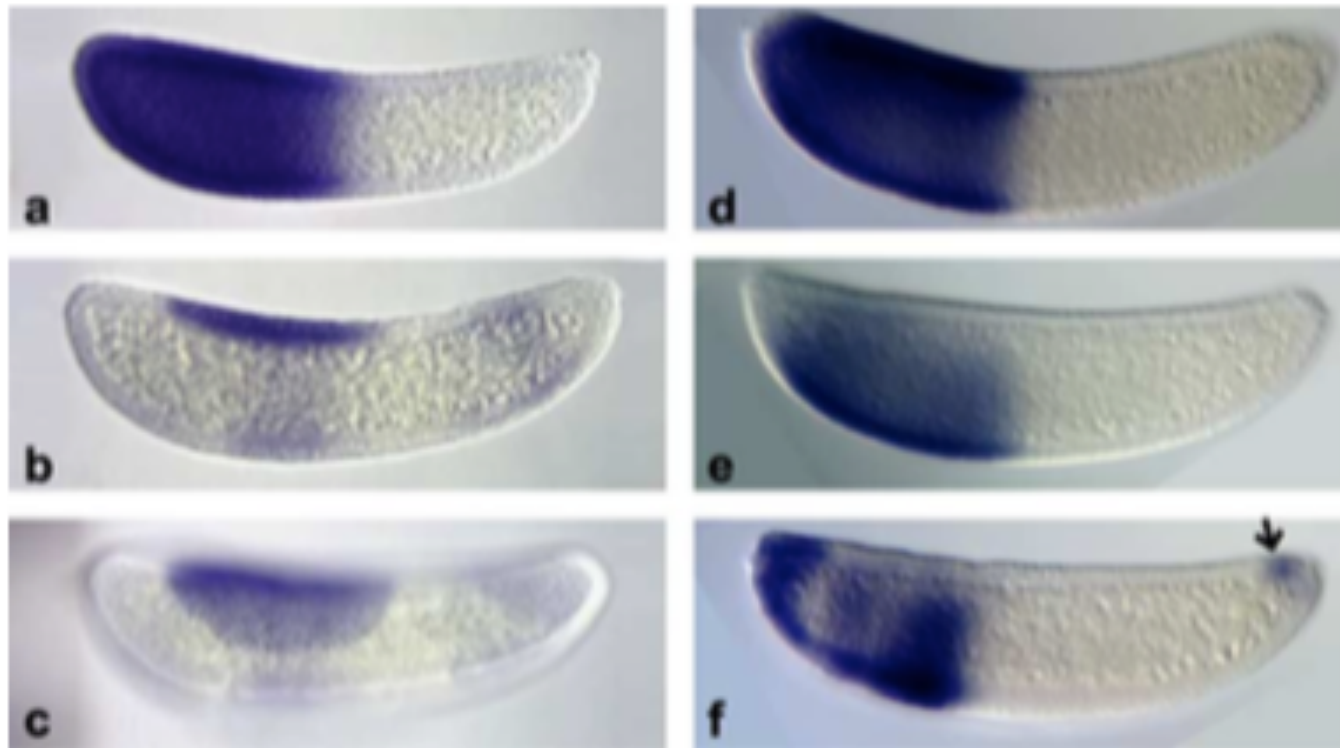
Posterior gap genes  
different

# Later patterning in the posterior

hb

gt

eve

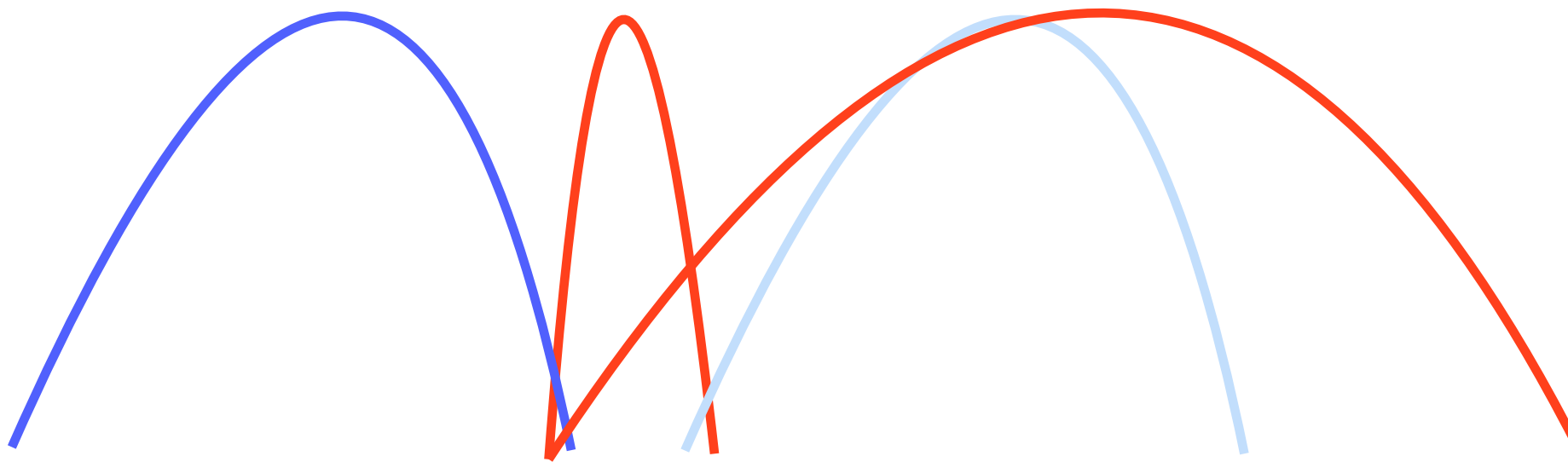


hb+eve



## The “eve 5 conundrum”

Kr Eve 5 Gt



Eve regulations can not be conserved.  
However some of the anterior stripes look the same ->  
some enhancers must be functionally conserved

# Score computation

There is surprising plasticity regarding stripes !



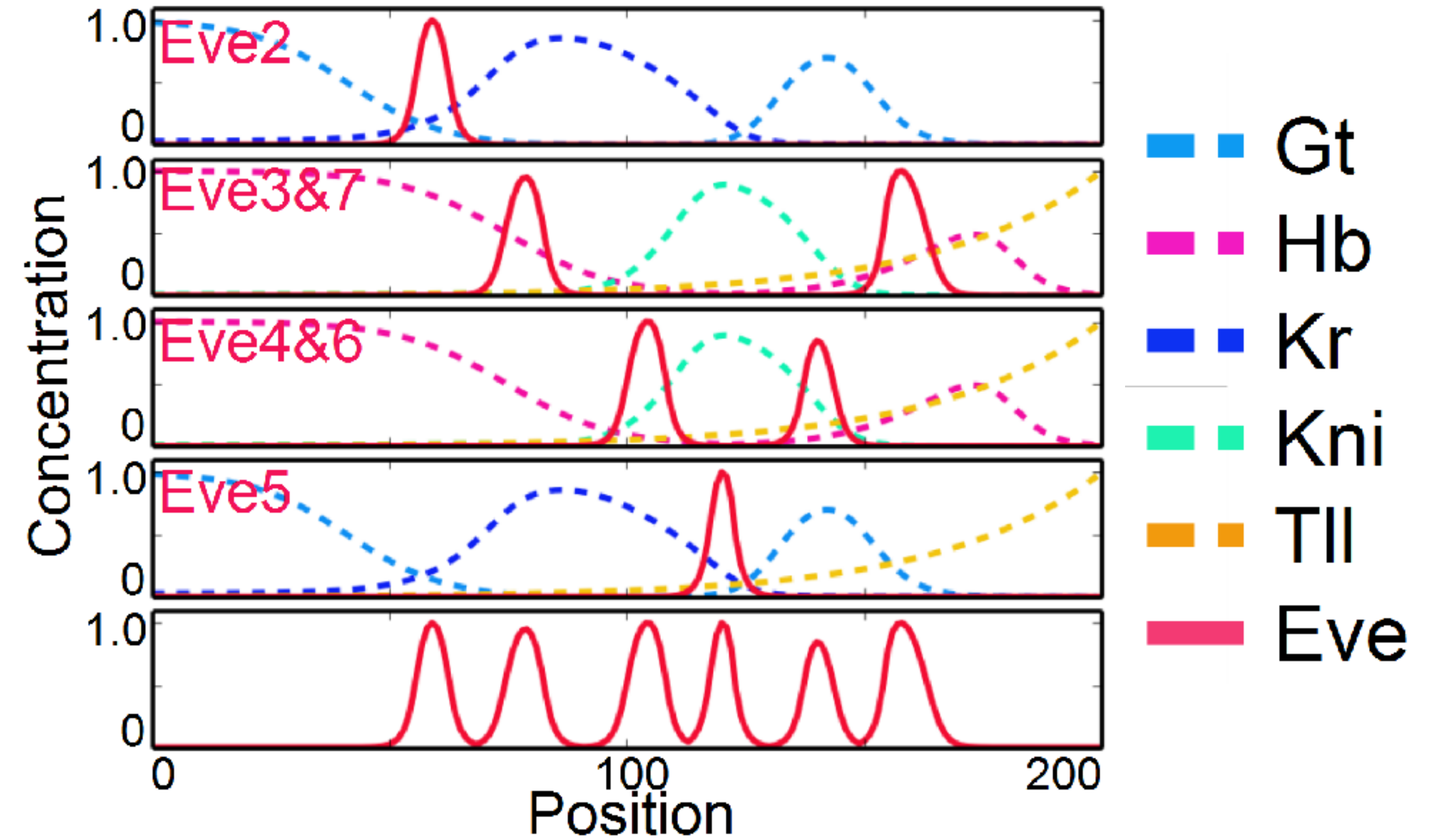
Fujioka et al, Development, 2002

- Score 2: simply counting Eve stripes and making sure we have **at least** 7. “Coarse-grained” fitness. We assume some compensatory mechanism can adjust problems with exact size positioning.

No constraint on size, etc...

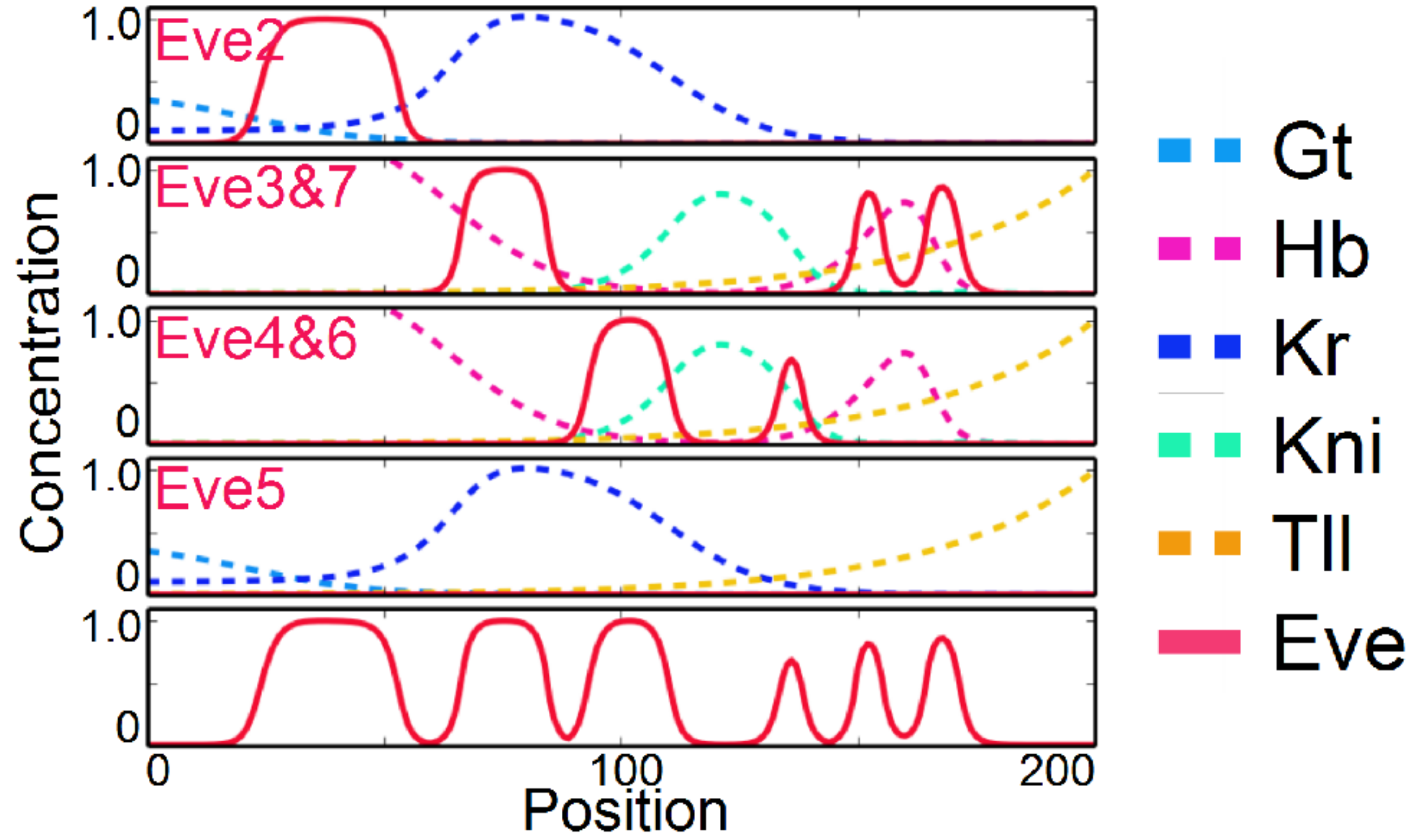
# Evolutionary scenario

F1A



Evolutionary scenario 1

F1C



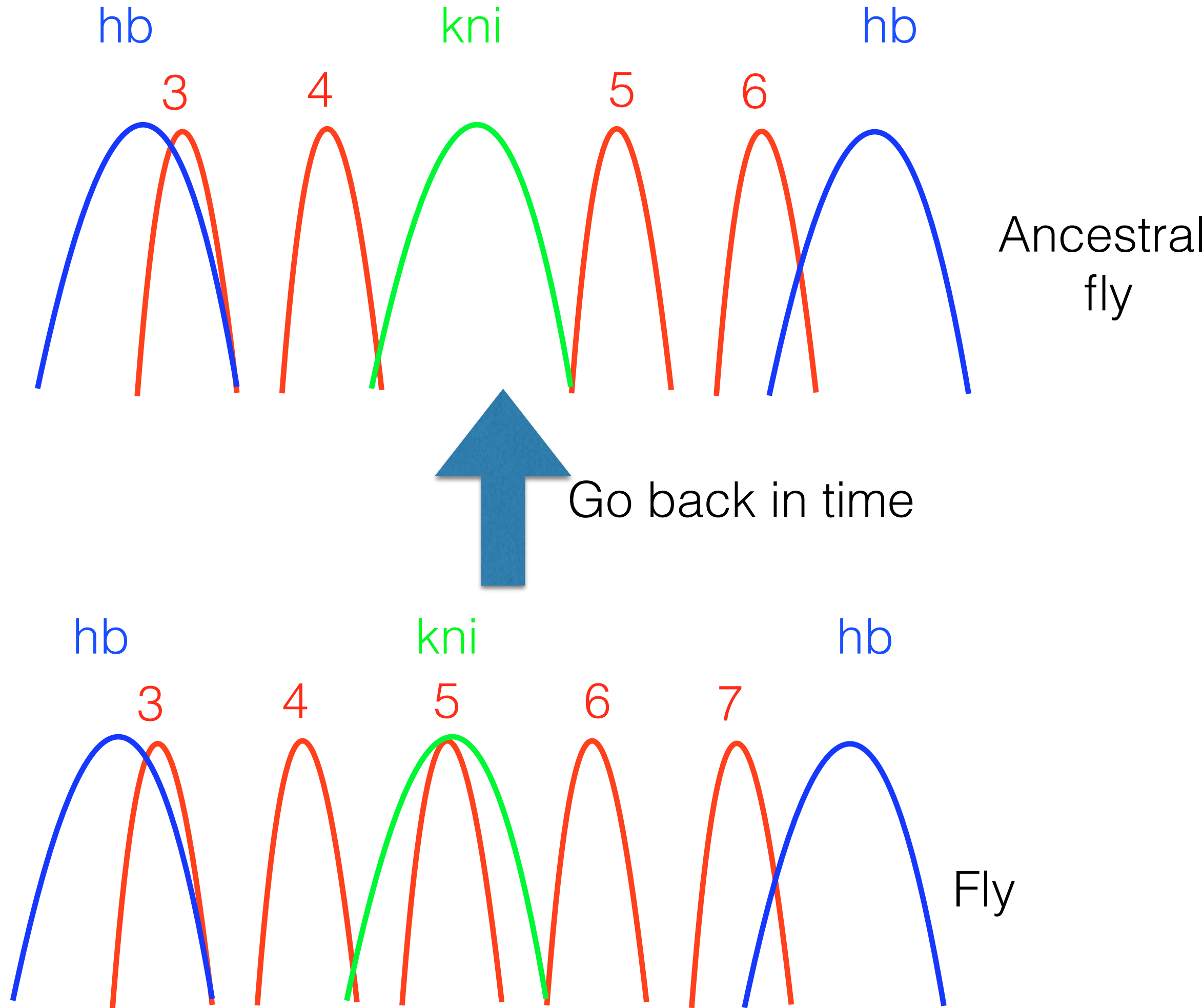
# Evolutionary scenarios

(from fly to mosquito)

- Creates an extra stripe in the posterior because hb getting more anterior
- Suppress eve 5

## **PREDICTION**

**eve 4-5 stripes in mosquito homologous to eve 4-6 in fly**





Predicting ancestral species

**(Evolutionary)  
Machine Learning**

Ur-Dipteran ?

**Predictions**



Drosophila



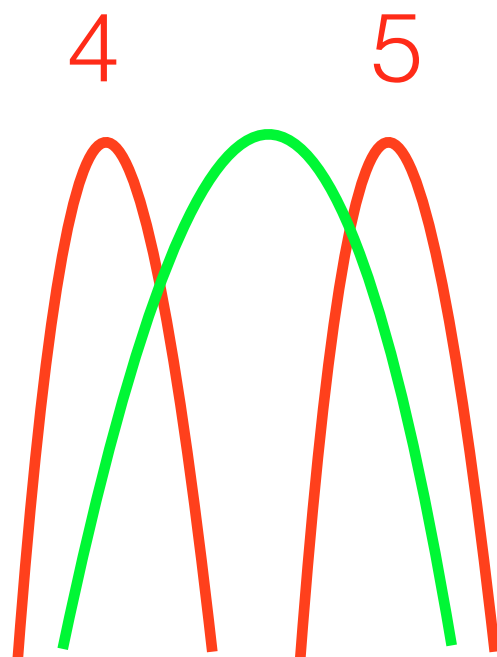
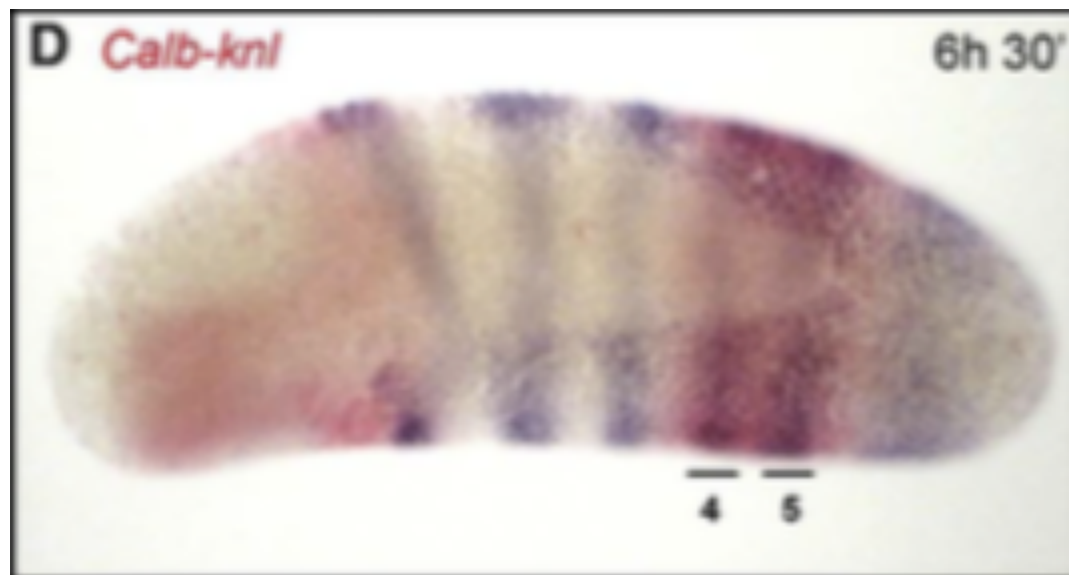
Clogmia



Anopheles

# Clogmia

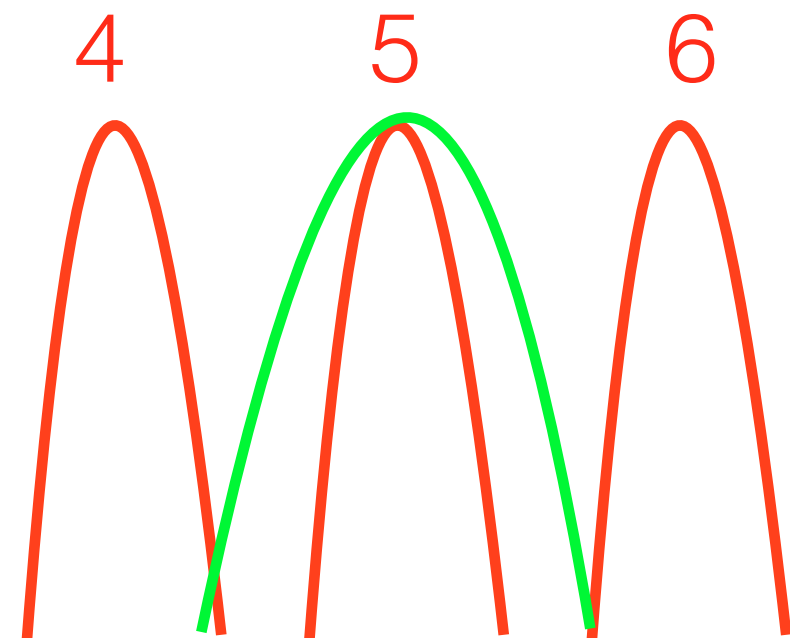
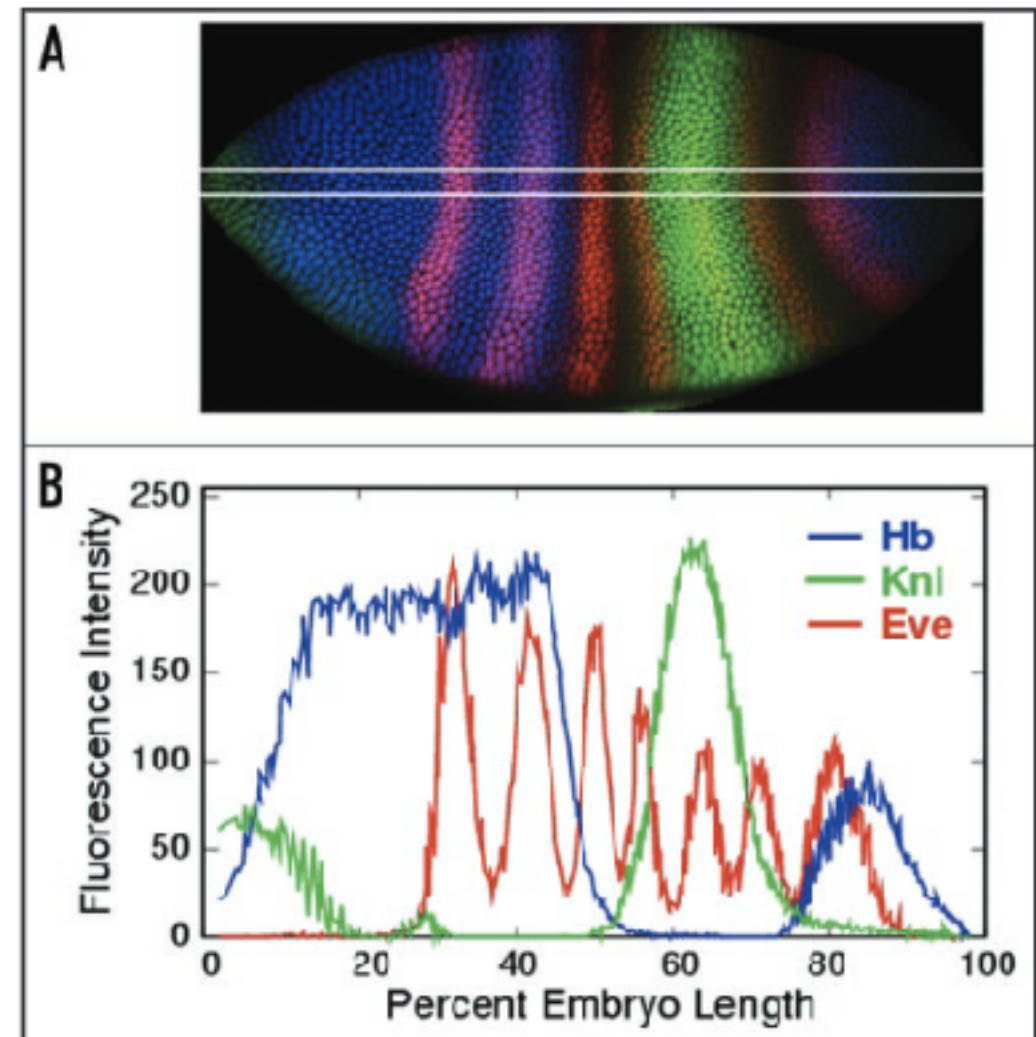
Garcia-Solache et al,  
Dev Biol 2014



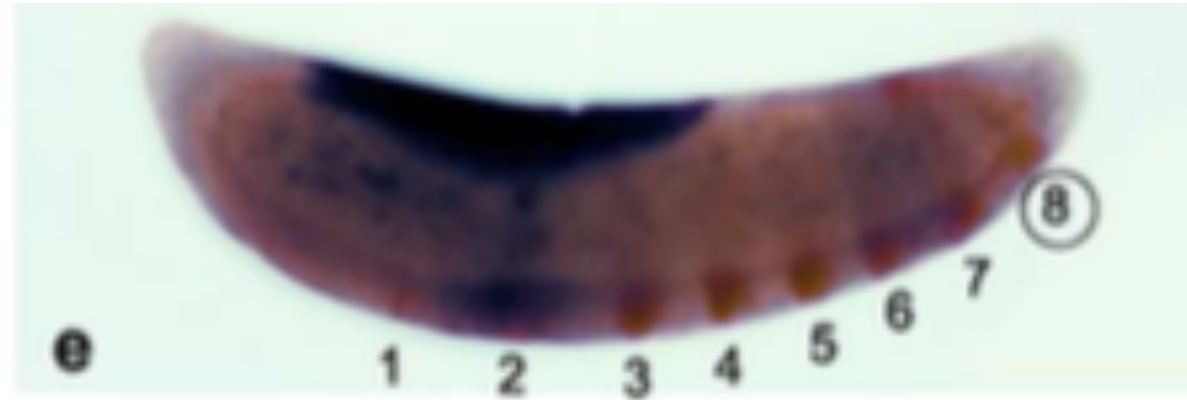
kni

# Drosophila

Surkova et al, Fly, 2008



# Another prediction: a proposal for mosquito Eve 8 stripe



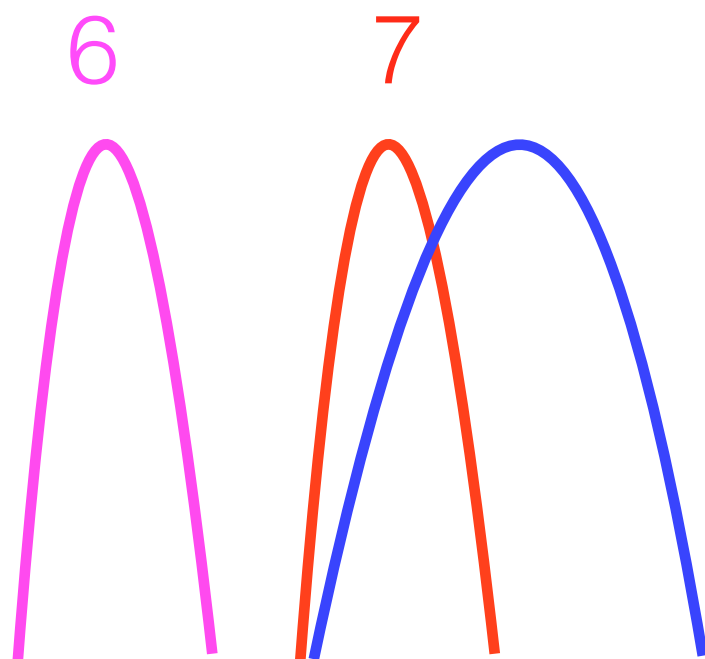
hb+eve

Either eve 6 or 7 are reused during our simulations.

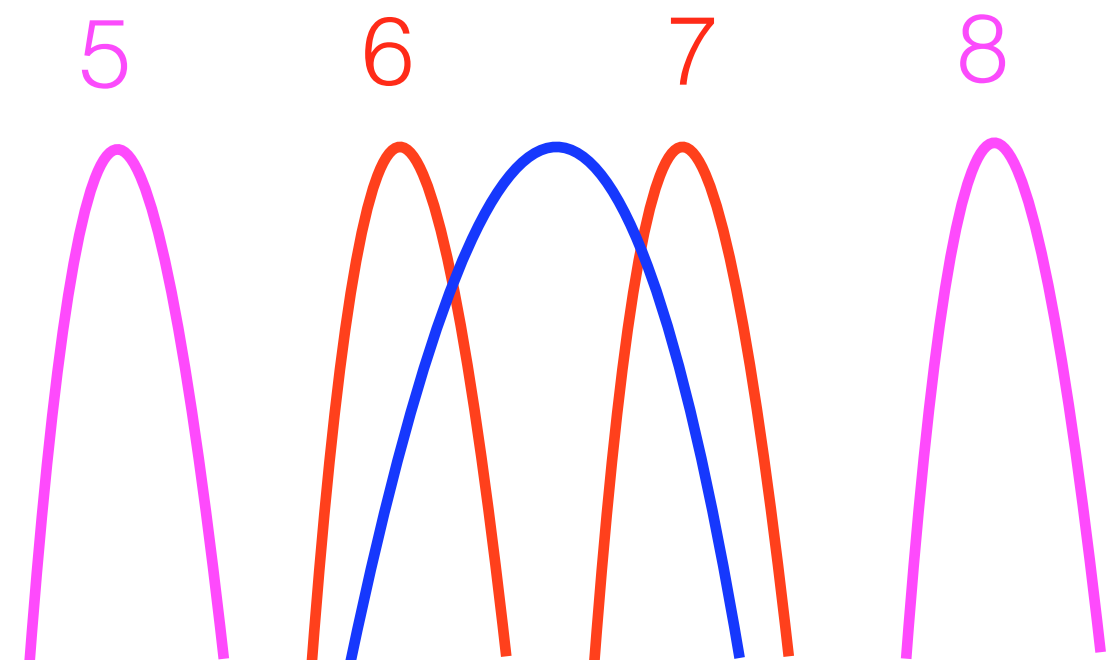
What if both are reused in the posterior?

Prediction: mosquito eve 5/6 are mirror images of eve 7/8 w.r. to hb ***(you can see it on in situs !)***

Fly



Mosquito



- You find “more than you put” (e.g. you correctly predict positions of eve stripes relative to gap genes even though that was unconstrained by design !!) - “spandrel”
- Computational evolution is surprisingly efficient and reproducible
- Evolutionary machine learning on sparse data works very well to generate predictions/