Quantum Fields, Geometry and Representation Theory ICTS-TIFR Bengaluru, July 16-27, 2018

Book of Abstracts

July 11, 2018

List of Speakers

- **Mini-Courses :**

  Sergey Cherkis (Univ of Arizona, Tucson)
  Tudor Dimofte (Univ of California, Davis)
  Jacques Distler (Univ of Texas, Austin)
  Ron Donagi (Univ of Pennsylvania, Philadelphia)
  Sergei Gukov (Caltech, Pasadena)
  Lotte Hollands (Heriot-Watt Univ, Edinburgh)
  Michiaki Inaba (Kyoto University)
  Takuro Mochizuki (RIMS, Kyoto)
  Tony Pantev (Univ of Pennsylvania, Philadelphia)
  Masa-Hiko Saito (Kobe University, Kobe)
  Piotr Sułkowski (Univ of Warsaw, Warsaw)

- **Research Talks :**

  Sujay Ashok (IMSc, Chennai)
  Lakshya Bhardwaj (Perimeter Institute, Waterloo)
  Satoshi Nawata (Fudan University, Shanghai)
  Du Pei (CQGMS, Aarhus)
  P. Ramadevi (IIT Bombay, Mumbai)
  Meng-Chwan Tan (NUS, Singapore)

Mini-Courses

Lectures notes, review/survey articles are shown in bold.

1. **Instantons and Monopoles**

   by **Sergey Cherkis.**

   Abstract: Yang-Mills instantons and Yang-Mills-Higgs monopoles play prominent role in quantum gauge theory. These lectures will describe a systematic way of constructing instantons and monopoles and explore their moduli spaces. We shall further explore a generalization producing Yang-Mills instantons on curved manifolds (called gravitational instantons).

   Background References:

   (a) M. Atiyah, "**Geometry of Yang-Mills Fields**" Pisa Lectures, 1979 (or in Michael Atiyah: Collected works, vol. 5* 75-173).

   (b) W. Nahm, "**Self-dual monopoles and calorons**," Lecture Notes in Physics 201 (1984).

   (c) M.K. Prasad, "Instantons and Monopoles in Yang-Mills Gauge Theories," Physica 1D (1980) 167–191.

   (d) C.D.A. Blair and S.A. Cherkis "Singular Monopoles from Cheshire Bows," Nucl.Phys. B845 (2011) 140-164; e-Print: arXiv:1010.0740 [hep-th]

   (e) S.A. Cherkis, "Instantons on Gravitons," Commun.Math.Phys. 306 (2011) 449-483; e-Print: arXiv:1007.0044 [hep-th].

2. **Higher algebra in SUSY QFT**

   by **Tudor Dimofte**

   Outline:

   Lecture 1: Secondary products in SUSY QFT
   Lecture 2: G actions in SUSY QM; or, the Fukaya category of point/G
   Lecture 3: Line operators and geometry in 3d N=4 gauge theory

   Abstract:

   A mathematical treatment of TQFT, based on category theory, was initiated in the early 90's. In more modern times the mathematics of TQFT has come to use advanced techniques from derived geometry and higher algebra (à la Lurie). This series of talks is loosely aimed at explaining how some of these advanced techniques apply in familiar examples of supersymmetric field theory and its topological twists. For physics, this will lead to some surprising new structure, as well as some useful organizing principles.

   The first and second lectures are based on work with C. Beem, D. Ben-Zvi, M. Bullimore, and A. Neitzke. The first lecture will re-examine operator algebras in topological twists of supersymmetric field theories. In d dimension, the algebras naturally come equipped with a Lie bracket of degree 1-d, which can be realized very concretely via topological descent. We'll look at examples of this bracket, in 2d, 3d, and 4d. We'll also use topological descent

to give a new interpretation of the statement that turning on on Omega background is a form of quantization.

The second lecture focuses on another sort of homological/descent operation, this time in the context of SUSY quantum mechanics with G symmetry. We'll find that Hilbert spaces in (de Rham) SQM come equipped with a natural homological action of G, i.e. an action of the exterior algebra $H_*(G)$. This action controls the process of gauging the G symmetry. A nice physical way to understand the $H_*(G)$ action and gauging/ungauging comes from considering SUSY boundary conditions for 2d G gauge theory; this will lead us to a definition of the "Fukaya category of point/G." The mathematical structures involved come from work of Goresky, Kottwitz, and MacPherson. Given time, we'll discuss some higher-dimensional examples, and the physics of Koszul duality.

The third lecture applies some of the ideas from the first two in the specific context of 3d N=4 gauge theories with linear matter. It is joint work with N. Garner, M. Geracie, and J. Hilburn. Our goal will be to define the category of line operators in the A and B twists of these theories, and to understand – in a concrete and computational way – the algebras of local operators bound to a line. These algebras get quantized in an Omega background; and they act on modules defined by boundary conditions. In the special case of an A twist and the trivial (identity) line operator, we will recover the Braverman-Finkelberg-Nakajima construction of the Coulomb-branch chiral ring.

Background References:

For Lecture 1

(a) Getzler, "Batalin-Vilkovisky algebras and two-dimensional topological field theories" hep-th/9212043

(b) Freed, "**The Cobordism Hypothesis**" arXiv:1210.5100 (for some perspective on the categorical approach to TQFT)

For Lecture 2

(a) Witten, "Supersymmetry and Morse theory," J.Diff.Geom. 17 (1982) no.4, 661-692

(b) Goresky, Kottwitz, and MacPherson, "Equivariant cohomology, Koszul duality, and the localization theorem" Invent. Math. 131 (1997) 24-83

(c) Guillemin and Sternberg "**Supersymmetry and equivariant de Rham theory**"

(d) Gaiotto, Moore, and Witten, "An introduction to the web-based formalism" arXiv:1506.04086 (more advanced and beyond what we need here... but actually a really nice illustration of derived concepts!)

For Lecture 3

(a) Braverman and Finkelberg, **Lecture notes on Coulomb branches** (available at `http://www.salafrancesco.altervista.org/uploads/8/3/1/9/83195312/braverman_finkelberg_-_coulomb_branches_of_3-dimensional_gauge__theories_and_related_structures.pdf`)

(b) Section 9-10 of Kapustin and Witten, "Electric-magnetic duality and the Geometric Langlands program" hep-th/0604151

(c) Assel and Gomis, "Mirror symmetry and loop operators" arXiv:1506.01718

(d) Bullimore, Dimofte, Gaiotto, Hilburn, and Kim, "Vortices and Vermas" arXiv:1609.04406 (a particular illustration of the framework I'll discuss)

3. **An Introduction to Class-$\mathcal{S}$ and Tinkertoys**

   by **Jacques Distler**

   Abstract: $\mathcal{N} = 2$ supersymmetric field theories in $D = 4$ provide a fruitful ground for the intersection between physics and mathematics. A particularly interesting subset, theories of "Class-$\mathcal{S}$," arise via compactification from $D = 6$. Many properties of these theories are encoded in the geometry of the compactification. In these lectures, I will try to explain the preceding sentences and give an overview of the classification of such theories.

   Background References:

   (a) Chacaltana, Distler, Trimm, Zhu, "Tinkertoys for the E8 Theory", arXiv:1802.09626

   (b) Chalcaltana, Distler, Trimm, Zhu, "Tinkertoys for the E7 Theory", arXiv:1704.07890

   (c) Chacaltana, Distler, Tachikawa, "Nilpotent orbits and codimension-two defects of 6d N=(2,0) theories", arXiv:1203.2930

   (d) Chacaltana, Distler, "Tinkertoys for Gaiotto Duality", arXiv:1008.5203

4. **The Geometric Langlands conjecture and non-abelian Hodge theory**

   by **Ron Donagi**

   Course Outline:

   1. Curves and their Jacobians
   2. Picard varieties
   3. Local systems
   4. Class field theory
   5. Hecke operators, correspondences
   6. Geometric Langlands
   7. Other Langlands
   8. G-bundles
   9. Hitchin's system
   10. Non abelian Hodge theory Ramified GLC
   11. GLC and HMS
   12. Abelianization via Hitchin's system
   13. Duality for Hitchin systems
   14. The gerbe of Higgs bundles
   15. GLC as deformation of abelianized GLC
   16. GLC as hyper Kahler rotation of abelianized GLC

17. Open non abelian Hodge theory
18. Wobbly bundles
19. Outline of a program
20. Prospects

Background References:

Ari04 D.Arinkin, Moduli of connections with a small parameter on a curve, arXiv:math/0409373.

Ari10 D.Arinkin, Autoduality of compactified Jacobians for curves with plane singularities, arXiv:1001.3868.

AD12 D.Arinkin, D.Gaitsgory, Singular support of coherent sheaves and the geometric Langlands conjecture, arXiv:1201.6343.

BD04 A. Beilinson and V. Drinfeld, Chiral Algebras, AMS Colloquium Publications, vol. 51, 2004, ISBN-10: 0-8218-3528-9.

Don93 R. Donagi, Decomposition of spectral covers. Asterisque, 218:145–175, 1993.

Don95 R. Donagi, Spectral covers, In Current topics in complex algebraic geometry (Berkeley, CA, 1992/93), MSRI Publ. 28, pp. 65–86, Cambridge Univ. Press, Cambridge, 1995.

DP06 R.Donagi and T.Pantev: Langlands duality for Hitchin systems, Inv Math, arXiv:math/0604617

DDP07 D.E. Diaconescu, R. Donagi, and T. Pantev, Intermediate Jacobians and ADE Hitchin systems. Math. Res. Lett. 14 (2007), no. 5, 745–756.

DG02 R. Donagi and D. Gaitsgory, The gerbe of Higgs bundles. Transform. Groups, 7(2):109–153, 2002.

Dr83 V. Drinfeld, Two-dimensional l-adic representations of the fundamental group of a curve over a finite field and automorphic forms on GL(2). Amer. J. Math. 105 (1983), no. 1, 85–114.

FGV02 E. Frenkel, D. Gaitsgory and K. Vilonen, On the geometric Langlands conjecture, J. Amer. Math. Soc. 15 (2002), no. 2, 367-417

G02 D. Gaitsgory, Geometric Langlands correspondence for GLn, Proceedings of the International Congress of Mathematicians, Vol. II (Beijing, 2002), 571-582.

GW07 S.Gukov and E.Witten, Gauge Theory, Ramification, And The Geometric Langlands Program, arXiv:hep-th/0612073. [Hit87] N. Hitchin, Stable bundles and integrable systems, Duke Math. J., 54(1):91–114, 1987.

HT03 T. Hausel and M. Thaddeus, Mirror symmetry, Langlands duality, and the Hitchin system, Invent. Math., 153(1):197–229, 2003. [KW07] A. Kapustin and E. Witten, Electric-magnetic duality and the geometric Langlands program, Commun. Number Theory Phys. 1(2007), no. 1, 1–236.

Lau87 G. Laumon, Correspondance de Langlands geometrique pour les corps de fonctions, Duke Math. J. 54, (1987), 309-359.

Sim94 C. Simpson, Moduli of representations of the fundamental group of a smooth projective variety - I, Publications Mathematiques de l'I.H.E.S., 79: 47–129, 1994; II, Publications Mathematiques de l'I.H.E.S., 80: 5–79, 1995.

5. **Lecture 1: Geometric Langlands and S-duality in $\mathcal{N}=4$ SYM**

   followed by

   Mini-Course : "VOA[$\mathbf{M_4}$]"

   by **Sergei Gukov**

   Abstract:

   Lecture 1 : This will be an introduction to the physics perspective on Geometric Langlands.

   Mini-course : The main goal of these lectures is to see explicitly, "in action" how holomorphic twists of 3d $N=2$ theories and 2d $N=(0,2)$ theories enjoy non-trivial equivalence relations (dualities) that a low-dimensional topologist can recognize as Kirby moves, i.e. equivalence relations in constructions of 3-manifolds and 4-manifolds, respectively.

   This provides a bridge between physics, algebra, and topology, which we explore from various perspectives and in many different examples. In particular, we will study the half-index of the combined 2d-3d system, first introduced in a joint work with A.Gadde and P.Putrov in 2013. Following that line of work, we also review the first non-trivial duality in a non-abelian 2d $N=(0,2)$ gauge theory, the so-called triality of 2d $N=(0,2)$ SQCD, and discuss how it leads to a triality symmetry of the corresponding VOAs.

   We then discuss various ways of "gluing" vertex operator algebras that correspond to different gluing operations in the world of smooth 4-manifolds and illustrate how topological invariants of 4-manifolds arise as chiral correlation functions in the resulting algebras VOA[M4]

   Background References:

   (a) Introductory Chapters of Kapustin-Witten ("*Electric-Magnetic Duality And The Geometric Langlands Program*") and Gukov-Witten I ("*Gauge Theory, Ramification, And The Geometric Langlands Program*") for Lecture 1.

   (b) A. Gadde, S. Gukov, P. Putrov,"Fivebranes and 4-manifolds"

   (c) S. Gukov, P. Putrov, C. Vafa, "Fivebranes and 3-manifold homology"

   (d) M. Dedushenko, S. Gukov, P. Putrov, " Vertex algebras and 4-manifold invariants"

6. **Introduction into spectral networks**

   by **Lotte Hollands**

   Abstract: First I try to explain the relation between four-dimensional N=2 quantum field theories and (quantum) Hitchin integrable systems, following references [a], [b] and [c]. Then I introduce spectral networks, and motivate why they are relevant in physics (for instance, in understanding the BPS spectrum of N=2 theories) as well as in mathematics (for instance, in generating Darboux coordinate systems on Hitchin moduli spaces), following references [b] and [d]

   The second talk will be about "*Spectral problems for the E6 Minahan-Nemeschansky theory*". According to Nekrasov and Shatashvili, the Coulomb vacua of four-dimensional N=2 theories of "class S", subjected to the Omega background in two of the four dimensions, correspond to the eigenstates of a quantisation of a Hitchin integrable system. The vacua may be found as the intersection between two Lagrangian branes in the Hitchin moduli space, one of which is the space of opers (or quantum Hamiltonians) and one is defined in terms of a system of Darboux coordinates on the corresponding moduli space of flat connections. I will introduce such a system of Darboux coordinates on the moduli space of SL(3) flat connections on the three-punctured sphere through a procedure called abelianization and describe the spectral problem characterising the corresponding quantum Hitchin system. This talk is based on work to appear with Andrew Neitzke.

Background References:

(a) Neitzke, **Hitchin systems in $\mathcal{N} = 2$ theories**, arXiv:1412.7145.

(b) Gaiotto, Moore and Neitzke, Wall-crossing, Hitchin Systems, and the WKB Approximation, arXiv:0907.3987v2.

(c) Nekrasov and Shatashvili, Quantization of Integrable Systems and Four Dimensional Gauge Theories, arXiv:0908.4052v1.

(d) Hollands and Neitzke, Spectral networks and Fenchel-Nielsen coordinates, 1312.2979v2.

7. **Lectures on Parabolic bundles, Parabolic Connections and Geometric Langlands**

by **M. Inaba, M-H Saito,**

Lecture **1** by M-H. Saito on *"Moduli spaces of parabolic connections and parabolic bundles and Geometric Langlands"*

Abstract: Moduli spaces of stable parabolic connections on curves are very interesting objects which are related to different area of mathematics like algebraic geometry, integrable systems, mathematical physics and Geometric Langlands conjecture. In this lecture, we will explain about an explicit geometry of the moduli spaces of stable parabolic connections on curves introduced and constructed by Inaba, Iwasaki and Saito and Inaba. Then we will review on a work of Arinkin and Lysenko on a rank 2 connections on the projective line with 4 singular points, which is related to Geometric Langlands conjecture in this case. We then explain about the joint work on the moduli space of rank 2 parabolic bundles on the projective line with Simpson and Loray. If time permits, related works of Geometric Langlands conjecture in these cases may be discussed.

Background References:

(a) D. Arinkin, *Orthogonality of natural sheaves on moduli stacks of* SL(2)*-bundles with connections on* $\mathbb{P}^1$ *minus 4 points*, Selecta Math. (N.S.) **7**, no. 2, (2001), 213–239.

(b) D. Arinkin, S. Lysenko, *On the moduli of* SL_2*-bundles with connections on* $\mathbb{P}^1 \backslash \{x_1, \ldots, x_4\}$. Internat. Math. Res. Notices 1997, no. 19, 983–999.

(c) M. Inaba, K. Iwasaki, and M. -H. Saito, *Moduli of stable parabolic connections, Riemann-Hilbert correspondence and geometry of Painlevé equation of type VI. I*, Publ. Res. Inst. Math. Sci. **42** (2006), no. 4, 987–1089.

(d) M. Inaba, K. Iwasaki, and M. -H. Saito, *Moduli of stable parabolic connections, Riemann-Hilbert correspondence and geometry of Painlevé equation of type VI. II*, Moduli spaces and arithmetic geometry (Tokyo), Adv. Stud. Pure Math., vol. 45, Math. Soc. Japan, Tokyo, 2006, pp. 387–432.

(e) A. Komyo, M. -H. Saito, *Explicit description of jumping phenomena on moduli spaces of parabolic connections and Hilbert schemes of points on surfaces* to appar in Kyoto Journal of Mathematics, arXiv:1611.00971

(f) F. Loray, M.-H. Saito, C. T. Simpson, *Foliations on the moduli space of rank two connections on the projective line minus four points.* to appear in "Seminaires et Congre" of the Societe Mathematique de France (SMF), arXiv:1012.3612v2.

(g) F. Loray, M.-H. Saito, *Lagrangian Fibrations in Duality on Moduli Spaces of Rank 2 Logarithmic Connections Over the Projective Line.* Int. Math. Res. Not. (IMRN), No. 4 (2015), 995–1043.

Lecture **2** by M. Inaba on *"Moduli space of regular singular parabolic connections and isomonodromic deformation."*

Abstract: In the joint work with Iwasaki and Saito, we introduced the moduli space of regular singular parabolic connections on smooth projective curves and proved the properness of the Riemann-Hilbert morphism which is a holomorphic map from the moduli space of regular singular parabolic connections to the moduli space of the representations of a fundamental group. As a corollary we get the geometric Painlevé property of the isomonodromic deformation on the moduli space of regular singular parabolic connections. In this talk I will give a quick introduction to the moduli space of regular singular parabolic connections on a smooth projective curve and its properties. After that I will explain the isomonodromic deformation on the moduli space of regular singular parabolic connections and its geometric Painlevé property. In the case of rank two connections on the projective line with regular singularities at 4 points, the isomonodromic deformation coincides with the Painlevé equation of type VI. If time permits, I will give an algebraic construction of the isomonodromy equation on the moduli space of regular singular parabolic connections.

Background References:

(a) M. Inaba, K. Iwasaki and M.-H. Saito, Moduli of stable parabolic connections, Riemann-Hilbert correspondence and geometry of Painlevé equation of type VI, I, Publ. Res. Inst. Math. Sci., 42 (2006) no. 4, 987–1089.

(b) M. Inaba, Moduli of parabolic connections on curves and the Riemann-Hilbert correspondence, J. Algebraic. Geom. 22 (2013), no. 3, 407–480.

Lecture **3** by M. Inaba on *" Unfolding of the moduli space of unramified irregular singular connections."*

Abstract: In this talk I will give a relative moduli space of connections on smooth projective curves whose generic fiber is a moduli space of regular singular connections and whose special fiber is a moduli space of unramified irregular singular connections. In the construction, we avoid using the parabolic structure but we use another parametrization of the local exponents. If we fix a diagonal matrix N with the distinct eigenvalues, the data of local exponents is given by a polynomial in N. Using this idea, we define a space of local exponents and we can construct over this base space a family of moduli spaces of connections.

On the moduli space of generic unframified irregular singular connections, we construct, joint with M.-H. Saito, a generalized isomonodromic deformation via patching local forms of Jimbo-Miwa-Ueno equations. On an unfolded family of moduli spaces of connections, I expect to construct an unfolded generalized isomonodromic deformation based on the theory by Hurtubise, Lambert and Rousseau, but my proof is not completed yet. I will explain fundamental solutions of an unfolded linear differential equation with an asymptotic nature given in the theory by Hurtubise, Lambert and Rousseau.

Background References:

(a) J. Hurtubise, C. Lambert and C. Rousseau, Complete system of analytic invariants for unfolded differential linear systems with an irregular singularity of Poincaré rank k, Mosc. Math. J. 14 (2) (2014) 309–338.

(b) J. Hurtubise and C. Rousseau, Moduli space for generic unfolded differential linear systems, Adv. Math. 307 (2017), 1268–1323.

8. **Kobayashi-Hitchin correspondence for wild harmonic bundles**

by **Takuro Mochizuki**

Abstract: This will be a mini-course on the Kobayashi-Hitchin correspondence between wild harmonic bundles and good filtered Higgs bundles on compact Riemann surfaces.

Background References:

(a) T. Mochizuki, Harmonic bundles and Toda lattices with opposite sign, arXiv:1301.1718

(b) C. T. Simpson, Constructing variations of Hodge structure using Yang-Mills theory and applications to uniformization, J. Amer. Math. Soc. 1 (1988), 867–918.

(c) C. T. Simpson, Harmonic bundles on noncompact curves, J. Amer. Math. Soc. 3 (1990), 713–770.

9. **Modular spectral covers and Hecke eigensheaves on interesections of quadrics** by **Tony Pantev**

Abstract: In these talks I will review the Geometric Langlands Conjecture in the unramified and tamely ramified cases and will connect it to the homological mirror correspondence for the moduli of Higgs bundles on a curve. I will outline a program which uses non-abelian Hodge theory and Fourier-Mukai duality on the Hitchin system to construct automorphic D-modules on the moduli of bundles and objects in the Fukaya category on the moduli of Higgs bundles.

I will discuss specific examples of the construction building automorphic sheaves on moduli spaces of bundles that are realized as intersections of quadrics. I will explain the resulting algebraic geometric question and will show how it can be solved explicitly by a higher dimensional version of the spectral cover construction and some interesting calculations with parabolic Chern classes. The focus will be on the projective geometry of the moduli spaces involved, and on the singularities and geometric subtleties needed for the correct formulation of the correspondence. This is a joint work with Ron Donagi and Carlos Simpson.

Background References:

(a) R. Donagi, T. Pantev "**Geometric Langlands and non-abelian Hodge theory**", Surveys in differential geometry. Vol. XIII. , 85–116, Int. Press, 2009. https://www.icmat.es/seminarios/langlands/school/handouts/pantev.pdf

(b) R. Donagi, T. Pantev "Langlands duality for Hitchin systems", Invent. Math. 189 (2012), no. 3, 653–735. https://arxiv.org/abs/math/0604617.

(c) R. Donagi, T. Pantev, C. Simpson "Direct Images in Non Abelian Hodge Theory", https://arxiv.org/abs/1612.06388.

(d) C. Pauly, A. Peón-Nieto "Very stable bundles and properness of the Hitchin map", https://arxiv.org/abs/1710.10152.

(e) S. Pal, C. Pauly "The wobbly divisors of the moduli space of rank-2 vector bundles", https://arxiv.org/abs/1803.11315.

10. **Topological strings, knots, and quivers**

by **Piotr Sułkowski**

Abstract: In the past three decades intimate links between knot theory and theoretical physics have been discovered. They include interpretation of polynomial knot invariants as partition functions of statistical models or expectation values in Chern-Simons quantum field theory, generalization of these relations to brane systems in topological string theory, identification of knot homologies with spaces of BPS states, relations to matrix models and topological recursion, etc. In the first lecture I will summarize some of these relations, and report some recent results in those contexts. In the second lecture I will show how (some of) these relations, as well as various knot invariants, are unified by relating them to quiver representation theory, in a way that we refer to as the knots-quivers correspondence. This correspondence is motived by various string theory constructions involving BPS states, and its consequences include the proof of the famous Labastida-Marino-Ooguri-Vafa conjecture (for symmetric representations), explicit (and unknown before) formulas for colored HOMFLY polynomials for various knots, new viewpoint on knot homologies and categorification, new dualities between quivers, new links with topological strings and statistical models, etc. While the knots-quivers correspondence has already led to surprising new results, at the same time it poses new deep and interesting questions, which I will also summarize.

Background Literature:

(a) Edward Witten, "Quantum field theory and the Jones polynomial", Commun. Math. Phys. 121 (1989) 351.

(b) Edward Witten, "Chern-Simons gauge theory as a string theory", Prog. Math. 133 (1995) 637, arXiv: hep-th/9207094.

(c) Hirosi Ooguri, Cumrun Vafa, "Knot invariants and topological strings", Nucl. Phys. B577 (2000) 419, arXiv: hep-th/9912123.

(d) J. Labastida, Marcos Marino, Hirosi Ooguri, Cumrun Vafa, "Knots, links and branes at large N", JHEP 0011 (2000) 007, arXiv: hep-th/0010102.

(e) Marcos Marino, "**Chern-Simons theory and topological strings**", Rev. Mod. Phys. 77 (2005) 675-720, arXiv: hep-th/0406005.

(f) Sergei Gukov, Albert Schwarz, Cumrun Vafa, “Khovanov-Rozansky homology and topological strings”, Lett.Math.Phys. 74 (2005) 53-74, arXiv: hep-th/0412243.

(g) Sergei Gukov, Ingmar Saberi, “**Lectures on knot homology and quantum curves**”, arXiv: 1211.6075 [hep-th].

(h) Sergei Gukov, Piotr Sułkowski, ”A-polynomial, B-model, and Quantization”, JHEP 1202 (2012) 070, arXiv: 1108.0002 [hep-th].

(i) Hiroyuki Fuji, Sergei Gukov, Piotr Sułkowski, ”Super-A-polynomial for knots and BPS states”, Nucl. Phys. B867 (2013) 506, arXiv: 1205.1515 [hep-th].

(j) Stavros Garoufalidis, Piotr Kucharski, Piotr Sułkowski, ”Knots, BPS states, and algebraic curves”, Commun. Math. Phys. 346 (2016) 75-113, arXiv: 1504.06327 [hep-th].

(k) Piotr Kucharski, Markus Reineke, Marko Stosic, Piotr Sułkowski, ”BPS states, knots and quivers”, Phys. Rev. D 95 (2017) 121902(R), arXiv: 1707.02991 [hep-th].

(l) Piotr Kucharski, Markus Reineke, Marko Stosic, Piotr Sułkowski, ”Knots-quivers correspondence”, arXiv: 1707.04017 [hep-th].

(m) Miłosz Panfil, Marko Stosic, Piotr Sułkowski, ”Donaldson-Thomas invariants, torus knots, and lattice paths”, arXiv: 1802.04573 [hep-th].

Research Talks

1. **Surface operators, dual quivers and contours**

   by **Sujay Ashok**

   Abstract:

   We study half-BPS surface operators in four dimensional $\mathcal{N} = 2$ SU(N) gauge theories. We calculate the ramified instanton partition function using equivariant localization and extract the low-energy effective action on the four dimensional Coulomb branch. We also study surface operators as coupled 2d/4d quiver gauge theories with an SU(N) flavour symmetry. In this description, the same surface operator can be described by different quivers that are related to each other by two dimensional Seiberg duality. We argue that these dual quivers correspond, on the localization side, to distinct integration contours that can be determined by the relative magnitudes and signs of the Fayet-Iliopoulos parameters of the two dimensional gauge nodes. We verify the proposal by mapping the solutions of the twisted chiral ring equations of the 2d/4d quivers onto individual residues of the localization integrand.

   Background References:

   (a) S. Gukov, Surface Operators, [arXiv:1412.7127].

   (b) D. Gaiotto, S. Gukov, and N. Seiberg, Surface Defects and Resolvents, [arXiv:1307.2578].

2. **TBA**

   by **Lakshya Bhardwaj**

   Abstract:

   Background References:

   (a) TBA

3. **Representations of DAHA from Hitchin moduli space**

   by **Satoshi Nawata**

   Abstract: I will talk about physics approach to understand representation theory of double affine Hecke algebra (DAHA). DAHA can be realized as an algebra of line operators in 4d N=2* theory and therefore it appears as quantization of coordinate ring of Hitchin moduli space over once-punctured torus. Using 2d A-model on the Hitchin moduli space, I will explain relationship between representation category of DAHA and Fukaya category of the Hitchin moduli space.

   Background References:

   (a) Gukov Witten, Branes and Quantization (0809.0305)

   (b) Cherednik, **Double Affine Hecke Algebra, London Mathematical Society**

4. **On mirror symmetry of (B, A, A)-branes**

   by **Du Pei**

   Abstract: Picking a real form G_r of a complex Lie group G defines a "(B, A, A)-brane" inside the moduli space of G-Higgs bundles. Under mirror symmetry, this (B, A, A)-brane will be mapped to a hyperholomorphic sheaf – a (B, B, B)-brane – over the moduli space of $G^\vee$-Higgs bundles, where $G^\vee$ is the Langlands dual group of G. In this talk, I will discuss how to construct these hyperholomorphic sheaves, and show how these proposals can be tested by computing equivariant indices. In particular, I will give computational evidence to Nigel Hitchin's proposal for the case of $G = GL_2$ and $G_r = U(1, 1)$. This talk is based on joint work with Tamas Hausel and Anton Mellit.

   Background References:

   (a) D.Pei, T. Hausel and A. Mellit, "Mirror symmetry with branes by equivariant Verlinde formulae", arXiv:1712.04408

5. **Knot polynomials from Chern-Simons field theory and their string theoretic interpretation**

   by **P. Ramadevi**

   Abstract: We will discuss the construction of knot polynomials from Chern-Simons field theory. We will indicate our computational status and limitations towards tackling classification of knots. We will briefly discuss the developments within topological strings and intersecting brane model.

   Background References:

   (a) R.K. Kaul, T.R. Govindarajan, P. Ramadevi, "Chern-Simons Theory as a theory of knots and links," Nucl. Phys. B402 (1993)548-566

   (b) H. Ooguri, C. Vafa, "Knot Invariants and Topological Strings,"Nucl.Phys.B577:419-438,2000

   (c) E.Witten, "Five branes and knots," arXiv:1101.3216 (hep-th)

6. **Quasi-topological gauged sigma models, the geometric Langlands program, and knots**

   by **Meng-Chwan Tan**

   Abstract: I will explain how a certain quasi-topological N = (0,2) gauged sigma model physically realizes the mathematical theory of "Twisted Chiral Differential Operators". In turn, I will give a non-gauge theoretic interpretation of the geometric Langlands correspondence for any simply-connected, simple, complex Lie group. I will also explain how worldsheet twisted-instantons can trivialize the chiral algebra of the sigma model completely, whence we would be able to connect the vanishing of the Witten genus on string manifolds with positive Ricci curvature to the conditions for the existence of Hecke eigensheaves in the geometric Langlands correspondence at genus zero. If time permits, I will also explain the connections to knot homologies and quantum groups as suggested by the physics of the sigma model.

   Background References:

(a) https://arxiv.org/abs/1111.0691

(b) https://arxiv.org/abs/0810.4964

(c) https://arxiv.org/abs/hep-th/0512172

Additional background material

This is a further collection of pedagogical texts that can help serve as a bridge to some of the courses/talks. This was compiled by organizers with input from the speakers and other participants.

Classical theory of Riemann surfaces :

1. R. Miranda, "Algebraic Curves and Riemann Surfaces", AMS Graduate Studies in Mathematics, Vol 5 (1995).
2. H. M. Farkas, Kra.I, "Riemann Surfaces", Springer Graduate Texts in Mathematics
3. Gunning, R.C, "Lectures on Riemann Surfaces", Princeton Univ Press.

Theory of Vector bundles/Principal bundles :

1. P. E. Newstead, *Introduction to Moduli Problems and Orbit spaces*, TIFR Lectures on Mathematics Vol. 51
2. Atiyah, M. F., and Raoul Bott. "*The Yang-Mills equations over riemann surfaces.*" Phil. Trans. R. Soc. Lond. A 308.1505 (1983): 523-615

The Hitchin integrable system/Higgs Bundles :

1. Bradlow, Steven B., Oscar García-Prada, and Peter B. Gothen. "*What is... a Higgs bundle.*" Notices of the AMS 54, no. 8 (2007)
2. Hitchin, Nigel. "Stable bundles and integrable systems." Duke mathematical journal 54.1 (1987): 91-114.
3. Hitchin, Nigel J., Graeme B. Segal, and Richard Samuel Ward. *Integrable systems: Twistors, loop groups, and Riemann surfaces. Vol. 4.* OUP Oxford, 2013
4. Wentworth, Richard. "*Higgs bundles and local systems on Riemann surfaces.*" Geometry and quantization of moduli spaces. Birkhäuser, Cham, 2016. 165-219

Differential equations, Irregular singularities, Stokes Phenomena :

1. Hille, Einar. Ordinary differential equations in the complex domain. Courier Corporation, 1997.

Knot Invariants :

1. Kauffman, Louis H. *Knots and physics. Vol. 1.* World scientific, 2001.

Quantum Field Theory (QFT) from mathematical point of view :

1. *Quantum Fields and Strings: A Course For Mathematicians (P. Deligne, P. Etingof, D.S. Freed, L. Jeffrey, D. Kazhdan, J. Morgan, D.R. Morrison and E. Witten, eds.)*, 2 vols., American Mathematical Society, Providence, 1999
2. Daniel S. Freed, *Five Lectures on Supersymmetry*, American Mathematical Society, Providence, 1999

QFTs with Supersymmetry (SUSY) :

1. Argyres, Philip, "*Introduction to Supersymmetry*" (1996), Lectures notes available at author's webpage : `http://homepages.uc.edu/~argyrepc/cu661-gr-SUSY/index.html` .

2. Argyres, Philip, "*Non-perturbative dynamics of four dimensional supersymmetric field theories*" (Istanbul Lectures also available at above webpage)

3. Tachikawa, Yuki, "*N=2 supersymmetric dynamics for pedestrians*" (arXiv:1312.2684)

SUSY sigma models and Mirror Symmetry :

1. Hori, Kentaro, et al, "*'Mirror symmetry*", Clay Mathematical Monographs Vol 1, American Mathematical Soc., 2003
   (Available online at `http://www.claymath.org/library/monographs/cmim01c.pdf`)

2. Aspinwall, Paul et al, "*Dirichlet Branes and Mirror Symmetry*" , Clay Mathematical Monographs Vol 4, American Mathematical Soc., 2009
   (Available online at `http://www.claymath.org/library/monographs/cmim04.pdf`)

Topologically Twisted SUSY QFTs

1. Witten, Edward. "Topological quantum field theory." Communications in Mathematical Physics 117.3 (1988): 353-386.

2. Witten, Edward. "Dynamics of Quantum Field Theory" (esp Lectures 14, 19) in Vol 2 of *Quantum Fields and Strings: A Course For Mathematicians (P. Deligne, P. Etingof, D.S. Freed, L. Jeffrey, D. Kazhdan, J. Morgan, D.R. Morrison and E. Witten, eds.).*

Schedule of courses (Prelim)

Week 1 (July 16-20)

1. *Instantons and Monopoles*, Sergey Cherkis
2. *An Introduction to Class-S and Tinkertoys*, Jacques Distler
3. *The Geometric Langlands conjecture and non-abelian Hodge theory*, Ron Dongai
4. *Introduction into spectral networks*, Lotte Hollands
5. *Lectures on Parabolic bundles, Parabolic Connections and Geometric Langlands*, Michiaki Inaba and Masa-Hiko Saito
6. *Kobayashi-Hitchin correspondence for wild harmonic bundles*, Takuro Mochizuki

Week 2 (July 23-27)

1. *Higher algebra in SUSY QFT*, Tudor Dimofte
2. *Geometric Langlands and S-duality in N=4 SYM* followed by lectures on "*VOA[M4]*" , Sergei Gukov
3. *Modular spectral covers and Hecke eigensheaves on interesections of quadrics*, Tony Pantev
4. *Topological strings, knots, and quivers*, Piotr Sułkowski

Full Schedule

See webpage.