6d SCFT's III

- (2,0)/SYM partition functions on curved spaces -

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Plan

- Want to study (2,0) theory observables on curved manifolds…

  - Superconformal index: partition function on $S^5 \times S^1$ → I will focus on this.

  - $S^4 \times M_2$: [Alday, Gaiotto, Tachikawa] ……
  - $S^1 \times S^3 \times M_2$: [Rastelli et.al] [Gaiotto, Rastelli, Razamat] ……
  - $S^3 \times M_3$: [Dimofte, Gaiotto, Gukov] ……
  - $S^1 \times S^2 \times M_3$: [Dimofte, Gaiotto, Gukov] ……
  - etc.

  } Some of these observables depend much less on geometry

- With a circle factor, reduce to 5d & study them using suitable 5d SYM.

Issues

- Putting (2,0) or (1,0) theories on curved manifolds.

- Lorentzian: no conceptual issue, just needs to check if the space admit SUSY
- Euclidean: It is a priori unclear what is the Euclidean version of the "self-dual" tensor theory one can write down, on general 6-manifolds.

$B_{\mu\nu}$: field strength $H = dB$ satisfies $H = \star_6 H$: Lorentzian notion. What to do depends on how one physically motivates the Euclidean theory.

- The theory on $M^5 \times S^1$ would appear in the thermal partition function (or index). Existence of Euclidean theory is implied if there is a Lorentzian theory on $M^5 \times R$.

[Presumably, working with a non-covariant formalism with chosen S^1 direction will do.]

The superconformal index

- A natural Witten index partition function for radially quantized SCFT's.
- Put the theory on S^5 x R: energy E ; SO(6) j_1, j_2, j_3 ; $SO(5)_R$ R_1, R_2
- Choose a pair of Q, S (= Q^+)

$$Q^{(R_1,R_2)}_{(j_1,j_2,j_3)} \rightarrow Q^{(\frac{1}{2},\frac{1}{2})}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}} \; : \; \text{BPS bound } E = 2R_1 + 2R_2 + j_1 + j_2 + j_3$$

[Kinney,Maldacena,Minwalla,Raju] [Bhattacharya,Bhattacharyya,Minwalla,Raju] [Romelsberger]

- Index partition function on S^5 x S^1: or counts local BPS operators on R^6

$$I(\beta, m, \epsilon_1, \epsilon_2) = \text{Tr}\left[(-1)^F e^{-\beta'\{Q,S\}} e^{-\beta(E-\frac{R_1+R_2}{2})} e^{\beta m(R_1-R_2)} e^{-\gamma_1(j_1-j_3)} e^{-\gamma_2(j_2-j_3)}\right]$$

- Four charges in OSp(8*|4) designed to commute with Q, S.
- β plays a similar role as the "inverse-temperature" variable in the index.
- It compactifies the Euclidean time direction.

5d SYM perspective 1

- start from high temperature regime. small circle.
- S^5 QFT interpretation: reduction on S^1 with twistings (clear in Abelian theory)

1. β ~ S^1 radius ~ 5d or "type IIA" coupling: $\frac{4\pi^2}{g_{YM}^2} = \frac{1}{r_1} = \frac{2\pi}{r\beta}$
2. m: adjoint hyper mass $\frac{\partial}{\partial\tau} \to \frac{\partial}{\partial\tau} - ia_i\frac{\partial}{\partial\phi_i} + \frac{R_1+R_2}{2} - m(R_1-R_2)$
3. a_i = (a, b, c), satisfying a+b+c=0, squash S^5: $e^{-\gamma_1(j_1-j_3)}e^{-\gamma_2(j_2-j_3)} = e^{-\beta(aj_1+bj_2+cj_3)}$

- Spatial chemical potentials squash S5. $n_1^2+n_2^2+n_3^2 = 1$

$$ds_6^2 = r^2\left[dn_1^2 + n_1^2\left(d\phi_1+\frac{ia_1}{r}d\tau\right)^2 + dn_2^2 + n_2^2\left(d\phi_2+\frac{ia_2}{r}d\tau\right)^2 + dn_3^2 + n_3^2\left(d\phi_3+\frac{ia_3}{r}d\tau\right)^2\right] + d\tau^2$$

$$ds_6^2 = ds_5^2 + \alpha^{-2}\left(d\tau + C\right)^2$$

→ 5d background: $ds_5^2 = dn_i^2 + n_i^2 d\phi_i^2 + \alpha^2(a_i n_i^2 d\phi_i)^2$

$\alpha^{-2} = 1 - a_i^2 n_i^2 \quad C = i\sum_{i=1}^{3} a_i n_i^2 d\phi_i$

- "Instantons" appear on S^5 as saddle points of path integral (wrap contractible circles)

$$Z[S^5] \sim \sum_{k=0}^{\infty} Z_k(\beta)e^{-\frac{4\pi^2 k}{\beta}}$$

$$Z_k = \sum_n a_n^{(k)}\beta^n$$

at weak couplng: β << 1 → should re-expand it in $e^{-\beta}$ at β >> 1

5d SYM perspective 2

- We take the Hopf fiber of S^5 and try to reduce to 5d SYM on CP^2 x R.
- Start from weak coupling: impose an extra Z_K orbifold, fractional shift on Hopf fiber
- SUSY KK reduction on S^1/Z_K fiber:

2π/K rotation with $$k \equiv j_1 + j_2 + j_3 + \frac{3}{2}(R_1 + R_2) + n(R_1 - R_2)$$

- Half-an-odd integer n: twisted reductions, infinitely many 5d QFT
- Our interest: strong-coupling QFT at K=1: instantons provide KK towers
- 6d chemical index = 5d index (time direction kept)
- Here, instantons are solitonic particles on CP2.
- Benefit of this approach: "index nature " would be manifest (manifestly an expansion in fugacities.)

SUSY QFT actions

- SYM action on S^5: "off-shell" version (bosonic terms) $D = 2(a_1^2 + a_2^2 + a_3^2)\alpha^2 \quad V_{ab} = (dC)_{ab}$

$$\left.\begin{aligned} g_{YM}^2 e^{-1}\mathcal{L} \;=\; & \frac{1}{2}\left(\frac{3}{16\alpha^2}V^2 + \frac{1}{4}R + D\right)\alpha\phi^2 - \frac{1}{4\alpha}\phi^2 V^2 - \frac{1}{2}\phi V^{ab}F_{ab} \\ & -2\phi\left(-\frac{1}{4}V^{ab}F_{ab} - \frac{1}{2}\partial^a\alpha D_a\phi + \frac{i}{4}\alpha^2(\sigma^3)_{ij}D^{ij}\right) \\ & -\alpha\left(-\frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}D^a\phi D_a\phi - \frac{1}{4}D_{ij}D^{ij}\right) + e^{-1}\frac{i}{8}\epsilon^{\mu\nu\lambda\rho\sigma}C_\mu F_{\nu\lambda}F_{\rho\sigma} \end{aligned}\right\} \text{vector multiplet}$$

$$\left. +|D_\mu q^i|^2 + \left(4 - \frac{\alpha^2}{4}\right)|q^i|^2 - \bar{F}_{i'}F^{i'} + ([\bar{q}_i,\phi] - im\alpha\bar{q}_i)([\phi,q^i] - im\alpha q^i) - \bar{q}_i(\sigma^I)^i{}_j([D^I,q^i] + m\alpha^2\delta_3^I q^j)\right\} \text{adjoint hypermultiplet}$$

- On CP^2 x R: (can also make it off-shell)

$$\begin{aligned} S \;=\; & \frac{1}{\tilde{g}_{YM}^2}\int d^5x\sqrt{g}\,\mathrm{tr}\Big[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi^I D^\mu\phi^I - \frac{i}{2}\lambda^\dagger\gamma^\mu D_\mu\lambda - \frac{1}{4}[\phi^I,\phi^I]^2 - \frac{i}{2}\lambda^\dagger\hat{\gamma}^I[\lambda,\phi^I] \\ & + \frac{2}{r^2}(\phi_I)^2 - \frac{1}{2r^2}(M_n\phi^I)^2 + \frac{1}{8r}\lambda^\dagger J_{\mu\nu}\gamma^{\mu\nu}\lambda - \frac{i}{2r}\lambda^\dagger M_n\lambda - \frac{i}{r}(3-2n)\underbrace{[\phi^1,\phi^2]}_{R_1}\phi^3 - \frac{i}{r}(3+2n)\underbrace{[\phi^4,\phi^5]}_{R_2}\phi^3 \\ & - \frac{i}{2r\sqrt{g}}\epsilon^{\mu\nu\lambda\rho\sigma}\left(A_\mu\partial_\nu A_\lambda - \frac{2i}{3}A_\mu A_\nu A_\lambda\right)J_{\rho\sigma}\Big] \end{aligned}$$

$$\tilde{g}_{YM}^2 = 4\pi^2 r/K \qquad M_n \equiv \tfrac{3}{2}(R_1+R_2) + n(R_1 - R_2)$$

- In the latter, the effect of chemical potentials is simply twisted B.C. on S^1.

Localization & saddle points

- Localization of SUSY path integral:

$$Z(\beta) = \int e^{-S-tQV} \ : \ t \text{ independent}$$

V chosen to satisfy $[Q^2, V] = 0$

- Saddle points on round S^5:

$$\xi^\mu = \epsilon^\dagger \gamma^\mu \epsilon = \sum_{i=1}^{3} \partial^\mu_{\phi_i}$$

self-dual instantons on CP^2 base

$$\boxed{F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta\gamma}F^{\alpha\beta}\xi^\gamma} \ , \quad F_{\mu\nu}\xi^\nu = 0 \ , \quad D_\mu \phi = 0 \ , \quad D = i\phi\sigma^3$$

all other fields = 0

- CP^2 x S^1: without angular momentum chemical potentials on CP^2

$$D^1 = D^2 = 0 \ , \quad \boxed{F^- = \frac{2s}{r^2}J} \ , \quad \frac{\phi}{r} + D = \frac{4s}{r^2} \ , \quad D + \frac{\xi}{r}\phi = 0$$

anti-self-dual instantons allowed on CP^2, proportional to Kahler 2-form

$$\oint_{S^1} A = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_N) \ , \quad \lambda_i \sim \lambda_i + 2\pi$$

Holonomy of gauge field on S^1

- With squashing on S^5 or chemical potentials on CP^2, the self-dual instantons' moduli get lifted to fixed points of rotations (later)…

Results

- S^5: $\lambda = r\phi$ (W: Weyl group, r: rank)

Each factor takes the form of Nekrasov partition function on R^4 x S^1

$$Z(\beta, m, a_i) = \frac{1}{|W|}\int_{-\infty}^{\infty}\left[\prod_{i=1}^{r} d\lambda_i\right]\exp\left[-\frac{2\pi^2 \mathrm{tr}\lambda^2}{\beta(1+a)(1+b)(1+c)}\right] Z^{(1)}_{\rm pert}Z^{(1)}_{\rm inst}\cdot Z^{(2)}_{\rm pert}Z^{(2)}_{\rm inst}\cdot Z^{(3)}_{\rm pert}Z^{(3)}_{\rm inst}$$

$$\begin{aligned}
1 &: (\epsilon_1,\epsilon_2) = \left(\frac{2\pi i(b-a)}{1+a}, \frac{2\pi i(c-a)}{1+a}\right), \quad m_0 = \frac{2\pi i(m+\frac{3}{2}(1+a))}{1+a}, \quad \mu = \frac{2\pi\phi}{1+a}, \quad q = e^{-\frac{4\pi^2}{\beta(1+a)}} \\
2 &: (\epsilon_1,\epsilon_2) = \left(\frac{2\pi i(c-b)}{1+b}, \frac{2\pi i(a-b)}{1+b}\right), \quad m_0 = \frac{2\pi i(m+\frac{3}{2}(1+b))}{1+b}, \quad \mu = \frac{2\pi\phi}{1+b}, \quad q = e^{-\frac{4\pi^2}{\beta(1+b)}} \\
3 &: (\epsilon_1,\epsilon_2) = \left(\frac{2\pi i(a-c)}{1+c}, \frac{2\pi i(b-c)}{1+c}\right), \quad m_0 = \frac{2\pi i(m+\frac{3}{2}(1+c))}{1+c}, \quad \mu = \frac{2\pi\phi}{1+c}, \quad q = e^{-\frac{4\pi^2}{\beta(1+c)}}
\end{aligned}$$

$\beta << 1$

- CP^2 x S^1: we did U(N), but other groups should be similar

$$\frac{1}{N!}\sum_{s_1,s_2,\cdots s_N=-\infty}^{\infty}\oint\left[\frac{d\lambda_i}{2\pi}\right] e^{\frac{\beta}{2}\sum_{i=1}^{N}s_i^2 - i\sum_i s_i\lambda_i} Z^{(1)}_{\rm pert}Z^{(1)}_{\rm inst}\cdot Z^{(2)}_{\rm pert}Z^{(2)}_{\rm inst}\cdot Z^{(3)}_{\rm pert}Z^{(3)}_{\rm inst}$$

$$\begin{aligned}
1 &: (\epsilon_1,\epsilon_2) = (\beta(b-a),\beta(c-a)), \quad m_0 = \beta(m+n(1+a)), \quad \mu = \beta\sigma - i\lambda + \beta s a, \quad q = e^{-\beta(1+a)} \\
2 &: (\epsilon_1,\epsilon_2) = (\beta(c-b),\beta(a-b)), \quad m_0 = \beta(m+n(1+b)), \quad \mu = \beta\sigma - i\lambda + \beta s b, \quad q = e^{-\beta(1+b)} \\
3 &: (\epsilon_1,\epsilon_2) = (\beta(a-c),\beta(b-c)), \quad m_0 = \beta(m+n(1+c)), \quad \mu = \beta\sigma - i\lambda + \beta s c, \quad q = e^{-\beta(1+c)}
\end{aligned}$$

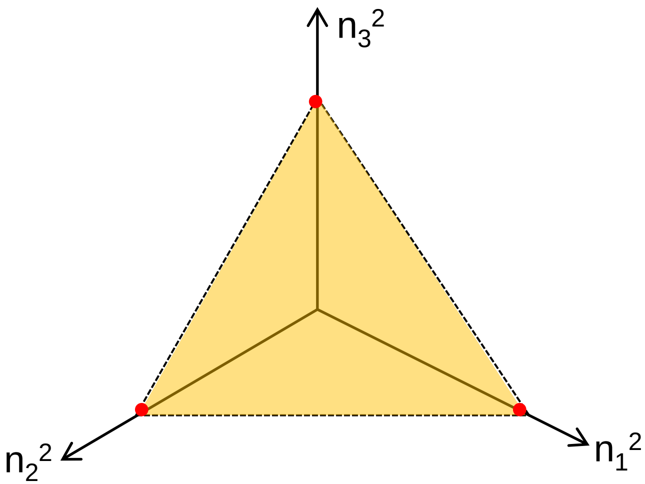
$\beta >> 1$

[There is a subtle choice of integral contours, which I don't explain here.]

Nekrasov's partition functions

- Both expressions contain Nekrasov's partition function on R4 x S1.
- Factorization: [Atiyah] [Pestun]

- Self-dual instantons at the saddle point singularly localizes to the 3 fixed points on CP2, after deforming the path integral by chemical potential or squasing.
- Gaussian determinant also factorizes into 3.



$$\mathbb{C}^3 : \; Z_i = n_i e^{i\phi_i} \quad , \quad n_1^2 + n_2^2 + n_3^2 = 1 \quad , \quad (i = 1, 2, 3)$$

$$(n_1, n_2, n_3) \; = \; (1,0,0), \; (0,1,0) \text{ or } (0,0,1)$$

Turning on rotation parameters, the self-dual instanton profiles on CP2 are also (singularly) supported at fixed points.

- Near fixed points, the local QFT's on R4 x S1 take the same form as Nekrasov's one with Omega deformations, upon suitable parameter identifications

Detailed study: $Z[S^5]$

- A key issue is whether one can perform a strong-coupling re-expansion.
- "bulk version" of this issue studied w/ topological strings for type IIA compactified on CY3: re-interpret as M-theory index at strong coupling [Gopakumar, Vafa] (1998)

- One should know how to do re-expansion with $Z[R^4 \times S^1]$.
- Now one can study the full index this way. some work in progress [Jungmin Kim, S.K.]

[This basically should be showing that $Z[S^5] = Z[CP^2 \times S^1]$ …]

- Here, I will explain something simpler, to give you some feeling on how it works.
- At special points in fugacity space, more SUSY commute with the measure in the index. So there are extra B/F cancelations, making calculations easier.

Unrefined indices

- 16 (maximal) SUSY at m = ½ or – ½ & a = b = c = 0;

- from 6d index: $\mathrm{tr}[(-1)^F e^{-\beta(E-R_1)}]$ $\quad Q^{(+\frac{1}{2},R_2)}_{(j_1,j_2,j_3)}$

- from 5d SYM: maximal SYM at this point with SU(4|2) (a,b,c=1,2,3; i=4,5)

$$S = \frac{1}{g_{YM}^2}\int d^5x\sqrt{g}\,\mathrm{tr}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}D_\mu\phi^I D^\mu\phi^I+\frac{i}{2}\lambda^\dagger\gamma^\mu D_\mu\lambda-\frac{1}{4}[\phi^I,\phi^J]^2-\frac{i}{2}\lambda^\dagger\hat{\gamma}^I[\lambda,\phi^I]\right.$$
$$\left.+\frac{4}{2r^2}(\phi^a)^2+\frac{3}{2r^2}(\phi^i)^2-\frac{i}{4r}\lambda^\dagger\hat{\gamma}^{45}\lambda-\frac{1}{3r}\epsilon_{abc}\phi^a[\phi^b,\phi^c]\right]$$

- For simplicity, we take this limit by: take c to be zero first, and then a, b to zero.

- Simplifications of three Z[R^4 x S^1]'s at this point: extra SUSY, cancelation.

$$Z^{(1)}_{\mathbb{R}^4\times S^1}\ ,\ \ Z^{(2)}_{\mathbb{R}^4\times S^1}\to\prod_{\alpha:\text{ positive roots}}2\sinh\pi\alpha(\lambda)\qquad U(N),SO(2N):\ Z^{(3)}_{\mathbb{R}^4\times S^1}\to\frac{1}{\eta(\tau=2\pi i/\beta)^N}$$

$$Z(\beta)=\frac{1}{|W|}\int d\lambda\, e^{-\frac{2\pi^2\mathrm{tr}(\lambda^2)}{\beta}}\prod_\alpha 2\sinh^2(\pi\alpha(\lambda))\cdot\eta(2\pi i/\beta)^{-N}$$

Result

- U(N):

$$Z^{U(N)} = e^{\beta\left(\frac{N(N^2-1)}{6}+\frac{N}{24}\right)} \prod_{n=0}^{\infty}\prod_{s=1}^{N} \frac{1}{1-e^{-\beta(n+s)}}$$

- SO(2N):

$$Z^{SO(2N)} = e^{\beta\left(\frac{c_2|G|}{6}+\frac{N}{24}\right)} \prod_{n=0}^{\infty}\left[\frac{1}{1-e^{-\beta(n+N)}}\prod_{s=1}^{N-1}\frac{1}{1-e^{-\beta(n+2s)}}\right]$$

- They are indeed indices.

- General form: (at least for ADE)

$$Z^{ADE} = e^{\beta\left(\frac{c_2|G|}{6}+\frac{r}{24}\right)} \prod_{n=0}^{\infty}\prod_{\text{Casimir op.}} \frac{1}{1-e^{-\beta(n+d)}}$$

d: degree of the Casimir operator

[We can only calculate Z_{pert} for E_6, E_7, E_8, but we conjecture that all ADE index takes the above form. The calculated Z_{pert} for E_n is consistent with this, provided that $Z_{\rm inst}^{E_n} = \eta(e^{-\frac{4\pi^2}{\beta}})^{-n}$.]

Vacuum “energy”

- The prefactor takes the form of “vacuum energy.” $e^{-\beta\epsilon_0} \equiv e^{\beta\left(\frac{c_2|G|}{6}+\frac{r}{24}\right)}$
- However, vacuum energy has to be understood with great care.

- Let us consider a simple example of free QFT on S^n x R.

$$\epsilon_0 \equiv \mathrm{tr}\left[(-1)^F \frac{E}{2}\right] = \sum_{\text{bosonic modes}} \frac{E}{2} - \sum_{\text{fermionic modes}} \frac{E}{2}$$

- regularize/renormalize the infinite sum: symmetries of the problem constrain it.

$$\epsilon_0 = \lim_{\beta'\to 0} \mathrm{tr}\left[(-1)^F \frac{E}{2} e^{-\beta' E}\right]$$

- In the index, these are constrained by different symmetries: Maximal SUSY

$$(\epsilon_0)_{\text{index}} \equiv \mathrm{tr}\left[(-1)^F \frac{E-R_1}{2}\right] = \sum_{\text{bosonic modes}} \frac{E-R_1}{2} - \sum_{\text{fermionic modes}} \frac{E-R_1}{2}$$

$$(\epsilon_0)_{\text{index}} = \lim_{\beta'\to 0} \mathrm{tr}\left[(-1)^F \frac{E-R_1}{2} e^{-\beta'(E-R_1)}\right]$$

Vacuum energy

- Let us compare the free tensor multiplet on S5 x R:

- "conventional" Casimir energy:

canceled by a counterterms ~ $\Lambda^2 \int_{S^5\times S^1} d^6x\sqrt{g}\; R^2$

$$\mathrm{tr}\left[(-1)^F \frac{E}{2} e^{-\beta' E}\right] = \frac{5r}{16(\beta')^2} - \frac{25}{384r} + r^{-3}\mathcal{O}(\beta')^2$$

Casimir "energies"

- What should appear in our index:

$$\mathrm{tr}\left[(-1)^F \frac{E-R_1}{2} e^{-\beta'(E-R_1)}\right] = \frac{r}{2(\beta')^2} - \frac{1}{24r} + r^{-3}\mathcal{O}(\beta')^2$$

- So, these are just two different observables. (although conceptually similar)

- $(\epsilon_0)_{\mathrm{index}} = -\frac{N^3-N}{6} - \frac{N}{24}$ from 5d SYM:

- A small check at N=1: agrees with the 6d calculation above.
- Another observation: large N vacuum energies

$$(\epsilon_0)_{\mathrm{index}} = -\frac{N^3}{6} \neq (\epsilon_0)_{\mathrm{gravity}} = -\frac{5N^3}{24}$$

Calculated from AdS_7 dual [Awad, Johnson] (2000)

Their holographic renormalization calculus should have steps which do not respect SUSY

The index from $CP^2 \times S^1$

- This quantity is manifestly taking an index form.
- To use semi-classical instanton expansion of Nekrasov, set 4 fugacities to obey:

$$q = e^{-\beta} \ll 1 \,, \quad \overbrace{y = e^{\beta\hat{m}} = e^{\beta(m+n)} \,, \quad y_i = e^{-\beta a_i}}^{\text{keep } O(1)}$$

- q can be regarded as a fugacity conjugate to the instanton number:

$$\begin{aligned} k &\equiv \frac{1}{8\pi^2}\int_{\mathbb{CP}^2} \mathrm{tr} F\wedge F = \frac{1}{8\pi^2}\int_{\mathbb{CP}^2} \mathrm{tr} F^+\wedge F^+ + \frac{1}{8\pi^2}\int_{\mathbb{CP}^2} \mathrm{tr} F^-\wedge F^- \\ &= k_{SD} + \frac{1}{2\pi^2}\sum_{i=1}^{N} s_i^2 \int_{\mathbb{CP}^2} J\wedge J = k_{SD} - \frac{1}{\pi^2}\sum_{i=1}^{N} s_i^2 \,\mathrm{vol}(\mathbb{CP}^2) = k_{SD} - \frac{1}{2}\sum_{i=1}^{N} s_i^2 \end{aligned}$$

- We studied it in some low energy expansion (~ instanton number expansion)

Z[CP^2 x S^1] at “large” N

- At k < N, we find that the index is independent of N.
- E ~ N is indeed a threshold energy, beyond which finite effects are expected to show up from the AdS gravity dual: “giant gravitons”

- The indices in this regime (N larger than k):

- k=0 (vacuum): $I_{k=0} = e^{\beta(1-\hat{m})\frac{N(N^2-1)}{6}}$ → the “vacuum energy” $(\hat{m} \equiv m - \frac{1}{2})$

- k = 1:

$$I_{k=0}\left(Ne^{-\beta}e^{\beta\hat{m}} - (N-1)e^{-\beta}e^{\beta\hat{m}}\right) = I_{k=0}\cdot e^{-\beta}e^{\beta\hat{m}}$$

- k = 2:

$$q^2\left[\frac{N(N+1)}{2}y^2 + Ny(y_1+y_2+y_3) - N\left(y_1^{-1}+y_2^{-1}+y_3^{-1}\right) + Ny^{-1}\right]$$
$$-(N-1)(N-2)q^2y^2 - (N-1)q^2\left[y^2 + y(y_1+y_2+y_3) - (y_1^{-1}+y_2^{-1}+y_3^{-1}) + y^{-1}\right]$$
$$+\frac{(N-2)(N-3)}{2}q^2y^2 \qquad = q^2\left[2y^2 + y(y_1+y_2+y_3) - (y_1^{-1}+y_2^{-1}+y_3^{-1}) + y^{-1}\right]$$

All results completely agree with SUGRA index on AdS_7 x S^4

Z[CP^2 x S^1] at finite N

- At k > N, indices shows finite N deviation from large N gravity index.
- So far, we only studied up to k=3 with U(2), U(3), so not too much to say about it.

U(2) at k = 3 :

$$q^3\left[\boxed{2y^3}+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)-\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\frac{y_2}{y_3}+\frac{y_3}{y_2}+\frac{y_3}{y_1}+\frac{y_1}{y_3}\right)+y^{-1}(y_1+y_2+y_3)\right]$$

SUGRA (large N) at k = 3 :

$$q^3\left[\boxed{3y^3}+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)-\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\frac{y_2}{y_3}+\frac{y_3}{y_2}+\frac{y_3}{y_1}+\frac{y_1}{y_3}\right)+y^{-1}(y_1+y_2+y_3)\right]$$

- Gauge theories like 4d N=4 SYM:

- One possible finite N corrections appear as trace relations: reduced # of states
- There may be more states (especially if one expects black holes in AdS dual).

- At E >> N, even the reduction pattern might provide interesting data for 6d (2,0), as we don’t know what kind of “gauge theory” it is.

Concluding remarks

- Perhaps a more intuitive understanding on 5d quantum instantons is needed.
- More observables on other 6-manifolds, also with application to d≤4 QFT's:

  Z[S^3 x S^1 x M_2]: [Kawano, Matsumiya] [Fukuda, Kawano, Matsumiya] (2012)

  Z[S^1 x S^2 x M_3]: [Yagi] [Lee, Yamazaki] (2013)

  Z[S^3 x M_3]: [Cordova, Jafferis] (2013)
- Study on 6d (1,0) CFT's. Some (2,0) techniques applicable.

- Question: "Why do these work…?" Why not $\mathcal{L}_{SYM} + \sum_n a_n (g_{YM})^{2n} \hat{\mathcal{O}}_{\Delta=n+5}$?

- 1: If all operators are Q-exact, then it will not affect the result. Then it works because the observable (e.g. superconformal index) is so specially chosen.
- 2: If irrelevant operator allowed by symmetry isn't Q-exact, then it may be working because 5d maximal SYM is "special". [Douglas] [Lambert, Papageorgakis, Schmidt-Sommerfeld]