6d SCFT's II

- the instanton partition function & the (2,0) theory -

Seok Kim

(Seoul National University)

10 January 2014

Asian Winter School 2014, Puri.

Plan

- D0-D4 quantum mechanics: brief review & expansion of yesterday's talk.

- Supersymmetric index of 5d maximal SYM and instanton QM on $R^4 \times S^1$:

  [Nekrasov] (2002); [Nekrasov, Okounkov] (2003); [Shadchin] (2005), ……

  [H.-C. Kim, S.K, E. Koh, K. Lee, S. Lee] (2011)

- (2,0) theory on $R^4 \times T^2$, in Coulomb phase: self-dual strings, KK modes, S-duality, …

  [H.-C. Kim, S.K, E. Koh, K. Lee, S. Lee] (2011);

  [Haghighat, Iqbal, Kozcaz, Lockhart Vafa] (2013)

- DLCQ M5-branes in the symmetric phase:

  [Aharony, Berkooz, Seiberg] (1997); [H.-C. Kim, S.K, E. Koh, K. Lee, S. Lee] (2011);

D0-D4 quantum mechanics

- k D0's quantum mechanics, with N D4's.

$$L_{QM} = \frac{1}{g_{QM}^2}\mathrm{tr}_{k,N}\Big[\frac{1}{2}(D_t\varphi^I)^2 + \frac{1}{2}(D_t a_m)^2 + \frac{1}{4}[\varphi^I,\varphi^J]^2 + \frac{1}{2}[a_m,\varphi^I]^2$$
$$+D_t q_{\dot\alpha} D_t \bar q^{\dot\alpha} - (\varphi^I \bar q^{\dot\alpha} - \bar q^{\dot\alpha} v^I)(q_{\dot\alpha}\varphi^I - v^I q_{\dot\alpha}) - D^{\dot\alpha}{}_{\dot\beta} D^{\dot\beta}{}_{\dot\alpha} + \cdots\Big]$$

$$D^{\dot\alpha}{}_{\dot\beta} = \bar q^{\dot\alpha} q_{\dot\beta} - \frac{1}{2}\zeta^A(\tau^A)^{\dot\alpha}{}_{\dot\beta} + \frac{1}{4}(\bar\sigma^{mn})^{\dot\alpha}{}_{\dot\beta}[a_m,a_n] - \frac{1}{2}\delta^{\dot\alpha}_{\dot\beta}(\mathrm{trace})$$

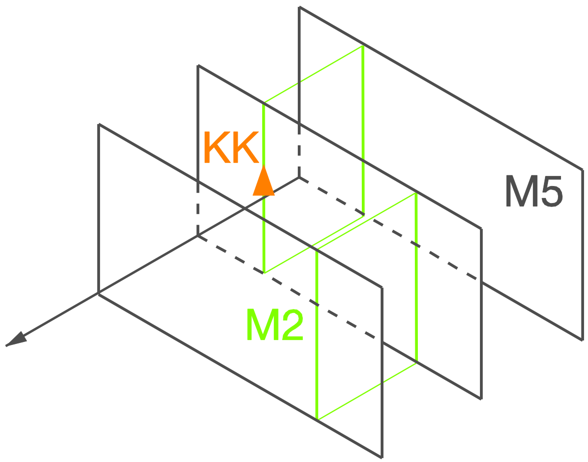
- D4's could possibly be separated (Coulomb phase), but not necessarily.

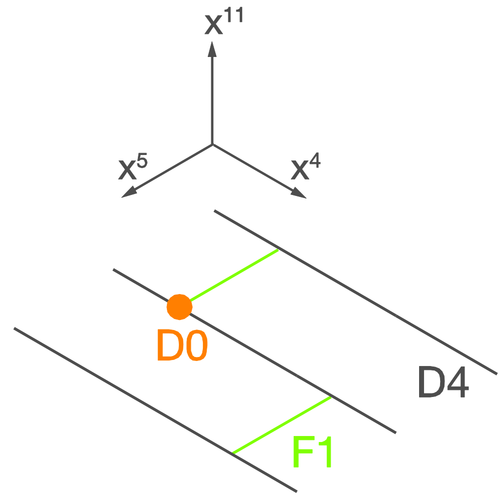
- Generally, bulk degrees are decoupled from "Higgs branch" only in special limits.
- BPS observables (e.g. Witten index): independent of continuous parameters

- FI term: Coulomb branch degrees carry mass^2 ~ ζ: Index probes Higgs branch.
- "UV calculation" of SUSY sigma model index for instantons: resolves small instantons

Nekrasov' partition function

- Nekrasov considered the index of this QM in 5d SYM's Coulomb phase.
- Counts the following BPS bound states:





- ¼-BPS: $Q^i_\alpha = \{ Q^a_\alpha \, , \, Q^{\dot a}_\alpha \} \; ; \; \overline{Q}^i_{\dot\alpha} = \{ \overline{Q}^a_{\dot\alpha} \, , \, \overline{Q}^{\dot a}_{\dot\alpha} \}$

preserved by ½-BPS W-bosons

preserved by ½-BPS self-dual instanton

- Will use 2 SUSY to define/study a Witten index. $Q \equiv \overline{Q}^{\dot 1}_{\dot 1} \, , \quad Q^\dagger = \overline{Q}^{\dot 2}_{\dot 2}$

Nekrasov used $Q^{\dot\alpha}_{\dot\alpha} = Q + Q^\dagger$ to define his TQFT.

The index

- The index: w/ 3 fugacities for spacetime symmetries, and N-1 for electric charges

$$Z_k(\epsilon_1,\epsilon_2,m,\mu^i) = \mathrm{Tr}\left[(-1)^F e^{-\beta'\{Q,Q^\dagger\}} e^{-\epsilon_1(j_1+J_R)} e^{-\epsilon_2(j_2+J_R)} e^{2mJ_L} \prod_{i=1}^{N} e^{-\mu^i s_i}\right]$$

$$Z[q] = \sum_{k=0}^{\infty} q^k Z_k \quad (Z_0 \equiv 1)$$

$$j_1, j_2 \in SO(4) \subset SO(4,1)$$

$$J_L \in SU(2)_L,\ J_R \in SU(2)_R,\ SU(2)_L \times SU(2)_R = SO(4)_R \subset SO(5)_R$$

$\{s_i\}$: $U(1)^N \subset U(N)$ electric charges ($\sum_i s_i = 0$)

(Nekrasov: extract Seiberg-Witten prepotential of 5d mass-deformed maximal SYM on R^4 x S^1)

$$Z[q;\epsilon_1,\epsilon_2,\mu^i,m] \sim \exp\left[\frac{\mathcal{F}(q,\mu^i,m)}{\epsilon_1\epsilon_2}\right]$$

Take $\varepsilon_{1,2}$ small, identify μ, m with Coulomb VEV & hypermultiplet mass of the Seiberg-Witten theory.

- This index is also useful in symmetric phase. I'll explain only one interpretation (free of large instanton issues). I think there are more to be learned from it.
- This can be understood in 2 contexts:

  1. index of a QM describing a decoupled sector of 5d SYM/6d (2,0),

  2. reduction of 5d SUSY path integral to QM path integrals

Calculation

- SUSY path integral: periodic B.C. on S1, twisted by chemical potentials

$$Z_k = \int [\mathcal{D}\phi^I \mathcal{D}a_m \mathcal{D}q_{\dot{\alpha}} \cdots] \exp[-S_{QM}]$$

- SUSY path integral: measure/B.C. preserve 2 SUSY $Q \equiv \overline{Q}^{\dot{1}}_{1}\ ,\quad Q^\dagger = \overline{Q}^{\dot{2}}_{2}$
- Index independent of continuous parameters of the theory, and regulator β': take suitable limit of parameters, so the path integral is computed by Gaussian "approximation"

$$\{|B\rangle, |F\rangle\} : Q|B\rangle = |F\rangle\ ,\quad Q^\dagger|F\rangle \sim |B\rangle$$

Pair of B/F states leaves or joins the BPS sector at the same time

- Nekrasov: integrals for all but $\phi \equiv \phi^5 - iA_\tau$ are localized to a Gaussian one

$$\sim \frac{1}{k!}\oint \prod_{I=1}^{k}\left(d\phi_I \prod_{i=1}^{N} \frac{\sinh(\phi_I - a_i + m)\sinh(\phi_I - a_i - m)}{\sinh(\phi_I - a_i - \frac{\epsilon}{2})\sinh(\phi_I - a_i + \frac{\epsilon}{2})}\right) \prod_{I\neq J}\sinh\phi_{IJ}$$

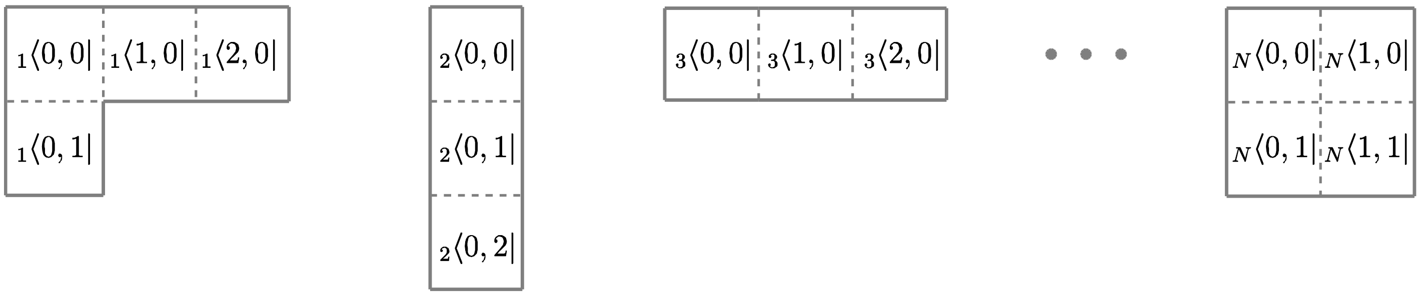
$$\times \prod_{I,J} \frac{\sinh(\phi_{IJ} - \epsilon)}{\sinh(\phi_{IJ} - \epsilon_1)\sinh(\phi_{IJ} - \epsilon_2)} \cdot \frac{\sinh(\phi_{IJ} + m + \frac{\epsilon_1 - \epsilon_2}{2})\sinh(\phi_{IJ} + m - \frac{\epsilon_1 - \epsilon_2}{2})}{\sinh(\phi_{IJ} + m - \frac{\epsilon}{2})\sinh(\phi_{IJ} + m + \frac{\epsilon}{2})}$$

- Contour choice for k eigenvalues of $\phi \equiv \phi^5 - iA_\tau$ is tricky, but found by Nekrasov.

Result

- Need to identify the poles inside the contour, and sum over the residues.

- poles: labeled by the N-colored Young diagrams with k boxes.



- sum of residues: [Bruzzo, Fucito, Morales, Tanzini] (2002)

$$Z_k = \sum_{Y;\ \sum_i |Y_i| = k} \prod_{i,j=1}^{N} \prod_{s \in Y_i} \frac{\sinh \frac{\beta(E_{ij}+m_0-\epsilon_+)}{2} \sinh \frac{\beta(E_{ij}-m_0-\epsilon_+)}{2}}{\sinh \frac{\beta E_{ij}}{2} \sinh \frac{\beta(E_{ij}-2\epsilon_+)}{2}}$$

$$E_{ij} = \mu_i - \mu_j - \epsilon_1 h_i(s) + \epsilon_2 (v_j(s) + 1)$$

- h_i : distance from box “s” to the right end of i’th Young diagram

- v_j : distance from box “s” to the lower end of the j’th Young diagram

6d & 11d

- Should count self-dual strings wrapping 6th circle, with k momenta: 6d physics is emergent from the instanton viewpoint.
- M5 version of the emergence of 11d physics M-theory from type IIA & D0-branes.

- Fundamental issue in the early days of M-theory, although many people forgot.
- Bulk problem: "Coulomb branch index" in our QM problem.

[Yi] [Sethi, Stern] (1997) [Moore, Nekrasov, Shatashvili] (1998)

$$L_{\rm bulk} = \frac{1}{g_{QM}^2}{\rm tr}_k\left[\frac{1}{2}(D_t X^M)^2 + \frac{1}{4}[X^I, X^J]^2 + \cdots\right] \qquad X^M = (a_m, \varphi^I)$$

$$\begin{aligned} L_{QM} &= \frac{1}{g_{QM}^2}{\rm tr}_{k,N}\Big[\frac{1}{2}(D_t\varphi^I)^2 + \frac{1}{2}(D_t a_m)^2 + \frac{1}{4}[\varphi^I, \varphi^J]^2 + \frac{1}{2}[a_m, \varphi^I]^2 \\ &\quad + D_t q_{\dot\alpha} D_t \bar q^{\dot\alpha} - (\varphi^I \bar q^{\dot\alpha} - \bar q^{\dot\alpha} v^I)(q_{\dot\alpha}\varphi^I - v^I q_{\dot\alpha}) - D^{\dot\alpha}{}_{\dot\beta} D^{\dot\beta}{}_{\dot\alpha} + \cdots\Big] \end{aligned}$$

- Counting marginal bound states is very subtle: translational 0-modes, continuum.

[Also true here, but… we decoupled Coulomb branch "in the index" with FI terms, and then could give mass to ALL zero modes with chemical potentials.]

D0 bound states & U(1) instantons

- Pure momentum bounds: should form 6d U(1) tensor multiplet or 11d SUGRA fields.
- 6d/11d prediction: KK modes for single multiplet, "unique" bound state for all D0 number.
- Nekrasov's U(1) result can be rewritten [Iqbal, Kozcaz, Shabbir] [Awata, Kanno] (2008)

$$Z[q] = PE\left[I_-(\epsilon_{1,2}, m)\frac{q}{1-q}\right] = 1+qI_-+q^2\left(\frac{I_-^2+I_-(\cdot^2)}{2}+I_-\right)+q^3\left(\frac{I_-^3+3I_-I_-(\cdot^2)+I_-(\cdot^3)}{6}+I_-^2+I_-\right)+\cdots$$

$$PE\left[f(x)\right] = \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}f(x^n)\right] \qquad I_-(e^{-\epsilon_{1,2}}, e^m) = \frac{\sinh\frac{m+\epsilon_-}{2}\sinh\frac{m-\epsilon_-}{2}}{\sinh\frac{\epsilon_1}{2}\sinh\frac{\epsilon_2}{2}}$$

- The single particle index: index for one 5d massive tensor multiplet for every k.

8 SUSY broken by instanton generate massive tensor

	$SU(2)_{1L}$	$SU(2)_{1R}$	$SU(2)_{2L}$	$SU(2)_{2R}$
B_2	**3**	**1**	**1**	**1**
ϕ_I	**1**	**1**	**2**	**2**
	1	**1**	**1**	**1**
λ	**2**	**1**	**2**	**1**
	2	**1**	**1**	**2**

$$Q^i_\alpha = \{\ Q^a_\alpha\ ,\ Q^{\dot a}_\alpha\ \}$$

$$I_{\text{tensor}} = \frac{\sinh\frac{m+\epsilon_-}{2}\sinh\frac{m-\epsilon_-}{2}\sinh\frac{\epsilon_++\epsilon_-}{2}\sinh\frac{\epsilon_+-\epsilon_-}{2}}{\sinh^2\frac{\epsilon_1}{2}\sinh^2\frac{\epsilon_2}{2}}$$

U(2) & higher gauge groups

- Charged bounds for U(2): consider bounds of 1 W-boson with many instantons
- Consider the single particle index: up to very high order in q expansion, we find

$$\left(\frac{\sin\frac{\gamma_R+\gamma_2}{2}\sin\frac{\gamma_R-\gamma_2}{2}}{\sin\frac{\gamma_1+\gamma_R}{2}\sin\frac{\gamma_1-\gamma_R}{2}}\right)\prod_{n=1}^{\infty}\frac{(1-q^n e^{i(\gamma_2+\gamma_R)})(1-q^n e^{i(\gamma_2-\gamma_R)})(1-q^n e^{i(-\gamma_2+\gamma_R)})(1-q^n e^{i(-\gamma_2-\gamma_R)})}{(1-q^n e^{i(\gamma_1+\gamma_R)})(1-q^n e^{i(\gamma_1-\gamma_R)})(1-q^n e^{i(-\gamma_1+\gamma_R)})(1-q^n e^{i(-\gamma_1-\gamma_R)})}$$

$$-\epsilon_1 = i\frac{\gamma_1-\gamma_R}{2}\ , \quad \epsilon_2 = i\frac{\gamma_1+\gamma_R}{2} \qquad m = i\frac{\gamma_2}{2}$$

- This is an index for a free 2d CFT having R^4 as the target space.
- W-boson particle uplifted to 2d free CFT on one self-dual string suspended between 2 M5-branes.

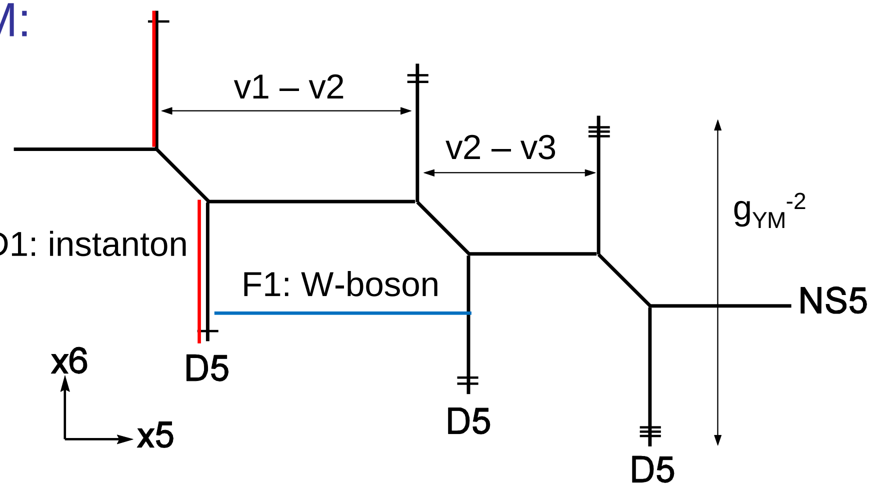
- Going to higher gauge group or electric charges is difficult.
- More exact formulae inferred from high order expansions in q, showing an emergent 2d index nature: but this is extremely cumbersome.

The "M-string" trick

- The instanton summation in $Z_{Nekrasov}$ can be done explicitly by the following constructions. [Haghighat, Iqbal, Kozcaz, Lockhart, Vafa] (2013)
- IIB branes for mass-deformed U(N) SYM:

S^1 compactification with a twist ~ m

	0	1	2	3	4	5	6	7	8	9
NS5	o	o	o	o	o	o				
N D5s	o	o	o	o	o		o			



- S-dual: two 5d indices should be the same.

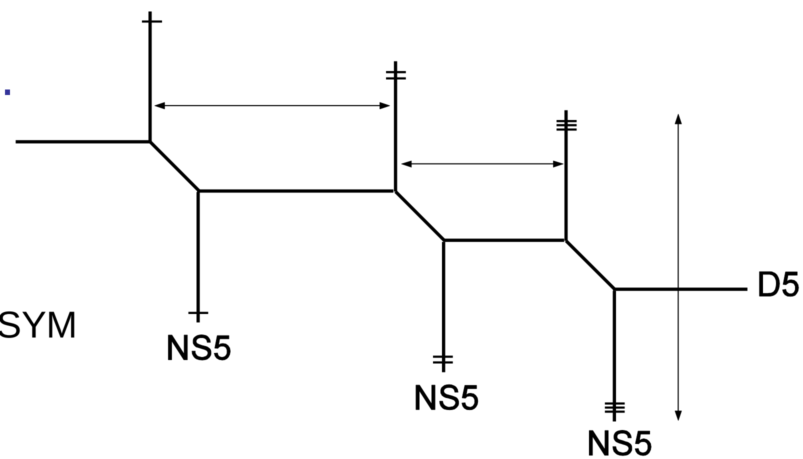
U(1)$^{N-1}$ SYM w/ 2 fundamental & N-2 bi-fundamental hypers

F1's for N hypers & N-1 vectors can wind around the circle.

D1 to winding F1's bound to N NS5's

F1 to U(1)$^{N-1}$ instantons

T-dualize on x^6: 6d SYM



Details

- We expect:

$$Z^{U(N)}_{\text{pert}}(e^{-\mu}) Z^{U(N)}_{\text{inst}}(q, e^{-\mu}) = Z^{U(1)^{N-1}}_{\text{pert}} \boxed{(q)} Z^{U(1)^{N-1}}_{\text{inst}} (e^{-(\mu_i - \mu_{i+1})} \boxed{, q)}$$

On RHS, q is geometrized into T-dualized 6th circle's radius.

- Easy parts:

$$Z^{U(N)}_{\text{pert}} = PE\left[\frac{\sinh \frac{m+\epsilon_+}{2} \sinh \frac{m-\epsilon_+}{2}}{\sinh \frac{\epsilon_1}{2} \sinh \frac{\epsilon_1}{2}} \sum_{i<j} e^{-(\mu_i - \mu_j)} \right]$$

$$Z^{U(1)^{N-1}}_{\text{pert}} = \left[\prod_{n=1}^{\infty} \frac{1}{1-q^n} \right]^N$$

- One can easily compute Z_{inst} on RHS by using the 6d uplift of Nekrasov partition function on $R^4 \times T^2$ (not the T2 where (2,0) is living): [Hollowood, Iqbal, Vafa] [Shadchin] .
- Z_{inst} in 5d is sum of ratios of sin or sinh (from S^1 KK modes): further include 6d KK

$$5\text{d}: \; 2\sin(\pi i z) \to \; 6\text{d}: \; -i(e^{\pi i z} - e^{-\pi i z}) \prod_{n=1}^{\infty} (1 - q^n e^{\pi i z})(1 - q^n e^{-\pi i z})$$

$$\sim \theta_1(z, q) = -i q^{\frac{1}{4}} (e^{\pi i z} - e^{-\pi i z}) \prod_{n=1}^{\infty} ()1 - q^n (1 - q^n e^{\pi i z})(1 - q^n e^{-\pi i z})$$

Results

- Z_{inst} on RHS for U(2) is

from 2 fundamental hypers

$$Z_{\rm inst}^{U(1)} = \sum_{Y} e^{-\mu_{12}|Y|} \prod_{(i,j)} \frac{\theta_1\left(\frac{\epsilon_1(Y_i-j)-\epsilon_2(i-1)+\epsilon_+ +m}{2\pi i};q\right)\theta_1\left(\frac{\epsilon_1(Y_i-j)-\epsilon_2(i-1)+\epsilon_+ +m}{2\pi i};q\right)}{\theta_1\left(\frac{\epsilon_1(Y_i-j)-\epsilon_2(Y_j^T-i)-\epsilon_2}{2\pi i};q\right)\theta_1\left(\frac{\epsilon_1(Y_i-j)-\epsilon_2(Y_j^T-i)+\epsilon_1}{2\pi i};q\right)}$$

from 6d U(1) vector multiplet

[If one doubts, one can expand it in q to check it agrees with the original index.]

- Similar (but a bit lengthier) results for U(N) can be obtained.

- Benefits:

- The KK modes' sum on T2 part is manifestly visible: It is really Z[R4 x T2].
- The U(2) string I explained all comes from here, but the index for N > 3 and/or higher electric charges has more intricate structure to be explored.
- It also makes the SL(2,Z) transformation property clear: another requirement for Z[R4 x T2] which depends on T2's complex structure only.

Symmetric phase

- In the index calculus, nonzero μ (electric charge chemical potential) provides masses to some important degrees: bi-fundamental hyper $[q_{\dot\alpha}]_{N\times k}$
- The calculation I explained all goes through at v = 0 (symmetric phase)
- We weight the states with non-Abelian U(N) charges.

- One can view it as an IR regulator, giving mass to (among others) instanton sizes.
- Very roughly, the lengths of the k row vectors of $[q_{\dot\alpha}]_{N\times k}$ are the instanton sizes.

$$L_{QM} = \frac{1}{g_{QM}^2}\mathrm{tr}_{k,N}\left[\frac{1}{2}(D_t\varphi^I)^2+\frac{1}{2}(D_t a_m)^2+\frac{1}{4}[\varphi^I,\varphi^J]^2+\frac{1}{2}[a_m,\varphi^I]^2 \right.$$
$$\left. + D_t q_{\dot\alpha} D_t\bar q^{\dot\alpha} - (\varphi^I\bar q^{\dot\alpha}-\bar q^{\dot\alpha}v^I)(q_{\dot\alpha}\varphi^I - v^I q_{\dot\alpha}) - D^{\dot\alpha}{}_{\dot\beta}D^{\dot\beta}{}_{\dot\alpha}+\cdots\right]$$

twisted B.C. can be untwisted to yield background gauge field

$$D_\tau q_{\dot\alpha} \to D_\tau q_{\dot\alpha} + \frac{\mu}{\beta}q_{\dot\alpha}+\cdots$$

Both provide potential for the size moduli of instantons.

- It is the interpretation (& proper usage) of this quantity that remains.

The DLCQ M5-branes

- Lightlike compactification: compactify on vanishingly small circle [Sen] [Seiberg]
- momentum k sector: non-relativistic matrix QM. This is a superconformal QM.
- subset of OSp(8*|4) commuting with $P_- = k/R_-$

- SO(6,2): $M_{AB},\ A, B = 0, 1, 2, \cdots, 6, 7 \longleftrightarrow \begin{cases} M_{\mu\nu} \text{ with } \mu, \nu = 0, 1, \cdots, 5 \\ P_\mu = M_{6\mu} + M_{7\mu}\ ,\quad K_\mu = -M_{6\mu} + M_{7\mu}\ ,\quad \Delta = M_{67} \end{cases}$

$$x^\pm = x^0 \pm x^5$$

$H \sim P_+\ ,\quad P_i\ ,\quad M_{ij}\ ,\quad G_i \sim M_{-i}\ ,\quad K \sim K_-\ ,\quad D = \Delta - M_{05}$ commute with $P_- = M_{6-} + M_{7-}$

$\boxed{SL(2,\mathbb{R}) \text{ subgroup}}$ $\quad [D, H] = -2iH\ ,\quad [D, K] = 2iK\ ,\quad [K, H] = -iD$

$$L_0 = aH + a^{-1}K\ ,\quad L_{\pm 1} = \frac{1}{2}(aH - a^{-1}K \mp iD) \quad [L_0, L_{\pm 1}] = \pm 2L_{\pm 1}\ ,\quad [L_{+1}, L_{-1}] = -L_0$$

- SUSY: 24 out of 32 commute with it. (It contains all supercharges that we shall discuss.)

- A “UV” gauged quantum mechanics description: the D0-D4 quantum mechanics.

The DLCQ index

- The spectrum of gauge invariant L_0 eigenstates (or eigen-operators of D)

$$M^{-1}(iD)M = H + K \qquad M = e^{H/2}e^{-K}$$

- K provides harmonic potential ~ λ^2 on instanton moduli space (actually for general conformal quantum mechanics). So the spectrum of L_0 is discrete.

- Nonrelativistic superconformal index [Nakayama] [Lee, Lee, Lee]

$$2i\{\bar{Q}^{\dot{a}}_{\dot{\alpha}}, \bar{S}^{\dot{\beta}}_{\dot{b}}\} = iD - 4\delta^{\dot{\beta}}_{\dot{\alpha}}(J_{2R})^{\dot{a}}{}_{\dot{b}} - 2\delta^{\dot{a}}_{\dot{b}}(J_{1R})_{\dot{\alpha}}{}^{\dot{\beta}} \longrightarrow 2i\{Q, S\} = iD \mp (4J_{2R} + 2J_{1R})$$

use either $Q \equiv \overline{Q}^{\dot{1}}_{\dot{1}}$, $Q^\dagger = \overline{Q}^{\dot{2}}_{\dot{2}}$

$$M^{-1}QM = Q - iS \equiv \hat{Q} \ , \quad M^{-1}SM = -i/2(Q + iS) = -\frac{i}{2}\hat{S} \qquad \{\hat{Q}, \hat{S}\} = L_0 \mp (4J_{2R} + 2J_{1R})$$

$$I_k(\epsilon_{1,2}, m) = \mathrm{Tr}\left[(-1)^F e^{-\beta\{\hat{Q},\hat{S}\}} e^{-2\epsilon_+(J_{1R}+J_{2R})} e^{-2\epsilon_- J_{1L}} e^{2mJ_{2L}}\right]$$

- Localize path integral for this: one finds same Z_k as before, but now subject to gauge invariance condition.

$$I_k = \frac{1}{N!}\oint \left[\prod_{i=1}^{N}\frac{d\alpha_i}{2\pi}\right]\prod_{i<j}\left(2\sin\frac{\alpha_i - \alpha_j}{2}\right)^2 Z_k\left(\epsilon_{1,2}, m, \mu_i = i\alpha_i\right)$$

The DLCQ index

- U(N) at k=1: $-\epsilon_1 = i\frac{\gamma_1 - \gamma_R}{2}\,, \quad \epsilon_2 = i\frac{\gamma_1 + \gamma_R}{2} \qquad m = i\frac{\gamma_2}{2} \qquad t \equiv e^{\mp\epsilon_+}$

$$I_{k=1} = \frac{e^{i\gamma_2} + e^{-i\gamma_2} - e^{i\gamma_1} - e^{-i\gamma_1}}{(1 - te^{i\gamma_1})(1 - te^{-i\gamma_1})} \left[\sum_{n=0}^{N-1} \chi_{\frac{n}{2}}(\gamma_2) t^{n+1} - \sum_{n=1}^{N-1} \chi_{\frac{n-1}{2}}(\gamma_2) t^{n+2} \right]$$

$$\chi_j(\gamma_2) = e^{2ji\gamma_2} + e^{2(j-2)i\gamma_2} + \cdots + e^{-2ji\gamma_2} = \frac{e^{(2j+1)i\gamma_2} - e^{-(2j+1)i\gamma_2}}{e^{i\gamma_2} - e^{-i\gamma_2}}$$

- Large N limit:

$$I_{N\to\infty,k=1} = \frac{e^{i\gamma_2} + e^{-i\gamma_2} - e^{i\gamma_1} - e^{-i\gamma_1}}{(1 - te^{i\gamma_1})(1 - te^{-i\gamma_1})} \frac{t - t^3}{(1 - te^{i\gamma_2})(1 - te^{-i\gamma_2})}$$

- Test: SUGRA index (although DLCQ is subtle in SUGRA, index is presumably OK…)

	D	J_{1L}	J_{2L}	$2(J_{1R}+J_{2R})$	boson/fermion
$p \geq 1$	$2p$	0	$\frac{p}{2}$	p	b
$p \geq 1$	$2p+1$	0	$\frac{p-1}{2}$	$p+1$	f
$p \geq 1$	$2p$	$\frac{1}{2}$	$\frac{p-1}{2}$	p	f
$p \geq 2$	$2p+1$	$\frac{1}{2}$	$\frac{p-2}{2}$	$p+1$	b
$p \geq 2$	$2p$	0	$\frac{p-2}{2}$	p	b
$p \geq 3$	$2p+1$	0	$\frac{p-3}{2}$	$p+1$	f
·	3	0	0	2	b (fermionic constraint)

agrees

Table 1: BPS fields of supergravity

Comments & the next lecture

- Although our QM was derived for a decoupled sector of instantons in 5d SYM (or D4-brane) system, wider applicability with BPS observables in mind.

- Nekrasov's partition function is such a particular observable.
- SUSY QFT path integral: reduces to 1d QM path integral, after cancelations between bose/fermi modes on R^4.

- Z_{Nekrasov} often appears as a building block of curved space partition functions.
- Example: $Z[S^4]$ of 4d gauge theories [Pestun]

$$Z_{S^4}(g_{YM}, m) = \int [d\phi] e^{-\frac{4\pi^2 \mathrm{tr}(\phi^2)}{g_{YM}^2}} \left| Z^{\mathbb{R}^4}_{\text{Nekrasov}}\left(q, \epsilon_+ = \frac{i}{r}, \epsilon_- = 0, m, \phi\right)\right|^2$$

- Similar role played by Z_{Nekrasov} on R^4 x S^1 in 5d SYM partition functions (tomorrow)