

6 dimensional superconformal field theories

Seok Kim

(Seoul National University)

9 January 2014

Asian Winter School 2014, Puri.

Plan

- 6d superconformal field theories, discovered via string/M-theory.
 - Hard to expect from the viewpoint of Lagrangian QFT. So very hard to study.
 - Understanding these CFTs are the key to understand M5-branes.
 - They are very novel as QFT. (tensor “gauge” theory, N^3 , higher dimensional QFT, ...)
 - Recently, we are finding many useful “applications” of this QFT via compactifications.

- I will discuss...
 - String theory approach, early attempts (mostly in late 90's) for microscopic studies [Lecture 1]
 - Recent studies on SUSY observables from 5d maximal SYM
 - Instanton partition function: technically, starts from Nekrasov (2002) [Lecture 2]
 - 5d SYM & 6d (2,0) partition functions on curved manifolds [Lecture 3]

Some references:

1995 – 1997 (some in early 2000):

- “discovery” of new 6d SCFT: [Witten](#); Strominger ; Witten; Seiberg, Witten ; [Seiberg](#);
- DLCQ, matrix models: Aharony, Kachru, Berkooz, Seiberg, Silverstein; [Aharony](#), [Berkooz](#), [Seiberg](#);
- deconstruction: Arkani-Hamed, Cohen, Kaplan, Karch, Motl; Lambert, Papageorgakis, Schmidt-Sommerfeld

(2007- : advanced techniques to study SUSY QFT [[Pestun](#)] ,)

(2008 - : formulation of M2-brane QFT's [[Bagger,Lambert](#)] [[Gustavsson](#)] [[ABJM](#)] ,)

Re-investigate 5d maximal super-Yang-Mills theory to study 6d (2,0):

- Douglas(2010); Lambert, Papageorgakis, Schmidt-Sommerfeld (2010);
- Bern, Carrasco, Dixon, Douglas, Hippel, Johansson (2012);

- instantons:

[Nekrasov](#) (2002); [Nekrasov](#), [Okounkov](#) (2003);

[H.-C. Kim](#), [S.K.](#), [E. Koh](#), [K. Lee](#), [S. Lee](#) (2011); [Haghighat](#), [Iqbal](#), [Kozcaz](#), [Lockhart](#), [Vafa](#) (2013);

- SUSY partition functions of 5d SYM (2012 -): [Kallen](#), ([Qiu](#),) [Zabzine](#); [Hosomichi](#), [Seong](#), [Terashima](#); [Imamura](#); ([Fukuda](#),) [Kawano](#), [Matsumiya](#); [Yagi](#); [Jafferis](#), [Cordova](#); [Lee](#), [Yamazaki](#); ...

[H.-C.Kim](#), [S.K.](#) (2012); [H.-C.Kim](#), [K. Lee](#) (2012); [Lockhart](#), [Vafa](#) (2012);

[H.-C.Kim](#), [J. Kim](#), [S.K.](#) (2012); [H.-C.Kim](#), [S.K.](#) [S.-S. Kim](#), [K. Lee](#) (2013);

6d SCFT: symmetries

- Superconformal field theories in higher dimensions (d=3,4,5,6) [Nahm]

d	symmetry	bosonic subgroup	\mathcal{N} (extended SUSY)
3	$OSp(\mathcal{N} 4)$	$\supset SO(3, 2) \times SO(\mathcal{N})$	1, 2, 3, 4, 5, 6, 8
4	$SU(2, 2 \mathcal{N})$	$\supset SO(4, 2) \times SU(\mathcal{N}) \times U(1)$	1, 2
4	$PSU(2, 2 4)$	$\supset SO(4, 2) \times SU(4)$	4
5	$F(4)$	$\supset SO(5, 2) \times SU(2)$	1
6	$OSp(8^* 2\mathcal{N})$	$\supset SO(6, 2) \times Sp(2\mathcal{N})$	1, 2

- Possible superconformal symmetries in 6d:

$$Q_\alpha^i, S_i^\alpha \text{ with } (i = 1, \dots, 2\mathcal{N}; \alpha = 1, \dots, 4, \text{ chiral})$$

$$\bar{Q}_j^\beta = Q_\alpha^i C^{\alpha\beta} \Omega_{ij}, \quad \bar{S}_\beta^j = S_i^\alpha C_{\alpha\beta} \Omega^{ij}$$

$$\{Q_\alpha^i, S_j^\beta\} = \delta_j^i \delta_\alpha^\beta (-iD) - \delta_j^i J_\alpha^\beta - 4\delta_\alpha^\beta R_i^j$$

- (2,0) theory: maximal SUSY, admits a good classification. my lectures will be on this CFT.
- (1,0) theory: we know some examples. Some systematic studies are recently being made. [Heckman, Morrison, Vafa] (F-theory) [Tomasiello et.al.] (massive IIA)

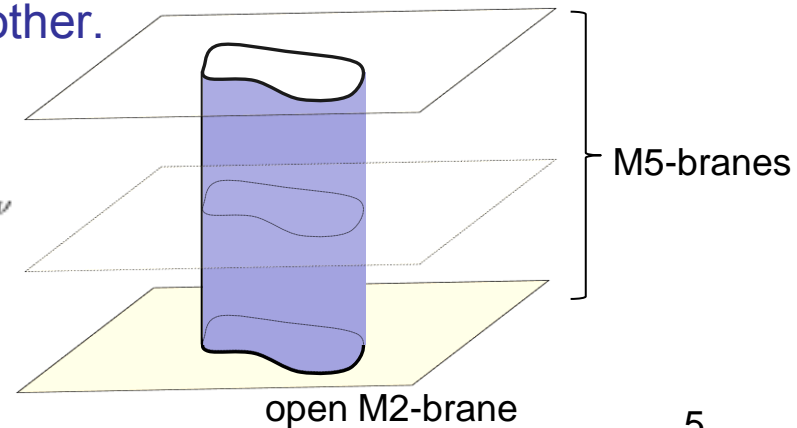
Discovery of (2,0) SCFT

- Witten (1995): take type IIB strings on $\mathbb{R}^{5,1} \times \mathbb{C}^2/\Gamma$ $\Gamma = A_n, D_n, E_6, E_7, E_8 \subset SU(2)$

A_n	$(z_1, z_2) \rightarrow \left(e^{\frac{2\pi i}{n+1}} z_1, e^{-\frac{2\pi i}{n+1}} z_2 \right)$
D_n	$(z_1, z_2) \rightarrow \left(e^{\frac{\pi i}{n-2}} z_1, e^{-\frac{\pi i}{n-2}} z_2 \right)$, $(z_1, z_2) \rightarrow (z_2, -z_1)$
E_6, E_7, E_8	more involved...

- 6d tensionless strings live on the tip of orbifold (D3's wrapping singular 2-cycles)
- decoupled from 10 dimensional degrees at low E
- Worldvolume theory on N M5-branes (N=n+1) [Strominger] [Witten]:
- 6d strings in “Coulomb phase”: end-locus of M2s, charged under N 2-forms
- Tensionless** when M5's are on top of each other.

$$\int_{\Sigma_2} B_{\mu\nu} dx^\mu dx^\nu$$



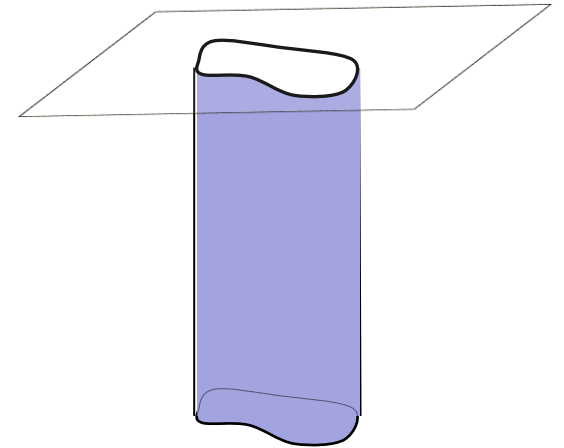
Abelian M5-brane theory

- Single M5-brane: massless tensor supermultiplet of (2,0) SUSY

$B_{\mu\nu}$: field strength $H = dB$ satisfies $H = \star_6 H$

Ψ_α^i : $\Gamma^7 \Psi^i = +\Psi^i$, $i = 1, 2, 3, 4$ for $Sp(4)_R = SO(5)_R$ spinor $(\bar{\Psi}_j = (\Psi^i)^T C \Omega_{ij}$: symplectic-Majorana)

ϕ^I : $I = 1, 2, 3, 4, 5$ for $SO(5)_R$ vector



- Couples to (infinite) open M2-brane sources.
- Equal electric/magnetic charges (should source H^+ only).

$$S_{\text{el}} = \int_{\Sigma_2} B = \int_{D_3} H, \quad S_{\text{mag}} = \int_{D_3} \star_6 H, \quad S_{\text{WZ}} = \int_{D_3} 2H^+$$

- Dirac quantization forces the couplings as above: **no coupling parameter**
- “Coulomb branch” of multiple M5’s: N tensors couple to finite-tension self-dual strings in the same way (tensions proportional to VEVs): **self-dual strings**

Basic properties: gauge theory...?

- Moduli space is $(\mathbb{R}^5)^{r_G}/W_G$ (r_G : rank, W_G : Weyl group of $G = U(N), SO(2N), E_{6,7,8}$)
- In the Coulomb branch, massive strings' charges are classified by ADE roots:

$$S_{\text{WZ}} = q_i \int_{D_3} 2H^{i+}$$

anomaly cancelation for $\delta B_{\mu\nu} = 2\partial_{[\mu}\epsilon_{\nu]}$ with self-dual strings: $q \cdot q = 2$ } all possibilities realized
 Dirac quantization: $q \cdot q' \in \mathbb{Z}$ } by type IIB models

[Henningson et.al.] (2004)

- Compactifying on S^1 , we get 5d SYM w/ ADE gauge groups. (or M5 to D4 for A_n)

reduce to “5d tensor” and then dualize to vector
 (precise sense is known only for Abelian case)

$$\frac{4\pi^2}{g_{YM}^2} = \frac{1}{r_1}$$

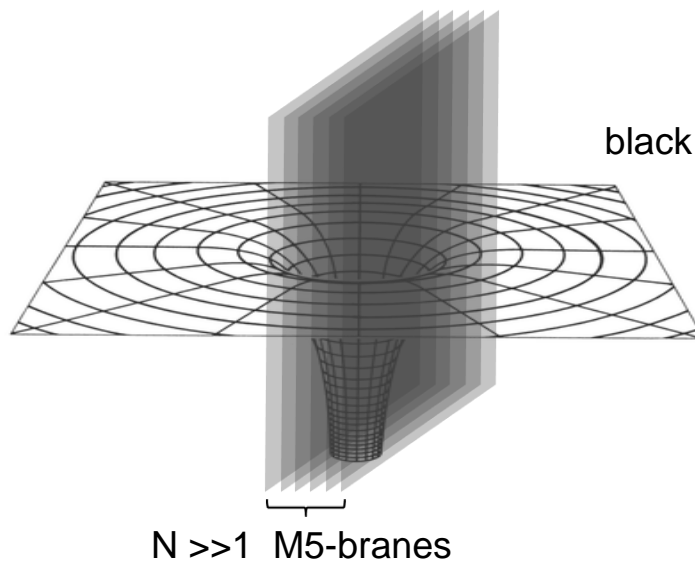
$$\frac{1}{2}\epsilon_{\mu\nu\rho\alpha\beta}F^{\alpha\beta} = H_{\mu\nu\rho}$$

- So, there are indirect notions in which ADE can be regarded as “gauge groups”
- However, a precise form of non-Abelian tensor gauge theory is unknown.

Strongly interacting theory

- No continuous parameters: M-theory only has 11d Planck scale. So at low E, M5-brane system has neither dimensionful nor dimensionless parameters.
- No perturbative approximation. We don't (yet) have a Lagrangian description.
- Some aspects of this theory (known from string/M-theory) are mysterious.

E.g. on N coincident M5-branes, there are indications that N^3 light degrees live on it.



$$\frac{S_{BH}}{(\text{volume})_5} \sim N^3 T^5$$

- Despite the absence of concrete Lagrangian description, this system is expected to obey basic properties (“axioms”) for **relativistic QFT's** [Seiberg] (1996)

Microscopic descriptions

- Microscopic descriptions: on subsectors, or breaking some symmetry, or EFT.
- Discrete light-cone quantization (DLCQ):

[Aharony, Berkooz, Kachru, Seiberg, Silverstein] [Aharony, Berkooz, Seiberg] (1997)

- Take $R^{1,1}$ part of $R^{5,1}$: $x^\pm \equiv x^0 \pm x^1$: compactify $x^- \sim x^- + 2\pi R^-$, regard x^+ as time
- This is compactifying on a spatial circle with $R \rightarrow 0$ [Sen] [Seiberg] (1997)
- Sector with k units of KK momenta at finite E ($v \sim$ rest mass): non-relativistic QM

- Matrix quantum mechanics of D0-D4, decoupled from the “bulk D0’s”.

$$L_{QM} = \frac{1}{g_{QM}^2} \text{tr}_{k,N} \left[\frac{1}{2} (D_t \varphi^I)^2 + \frac{1}{2} (D_t a_m)^2 + D_t q_{\dot{\alpha}} D_t \bar{q}^{\dot{\alpha}} + \frac{1}{4} [\varphi^I, \varphi^J]^2 + \frac{1}{2} [a_m, \varphi^I]^2 - (\varphi^I \bar{q}^{\dot{\alpha}})(q_{\dot{\alpha}} \varphi^I) - D^{\dot{\alpha}}_{\dot{\beta}} D^{\dot{\beta}}_{\dot{\alpha}} + \dots \right]$$

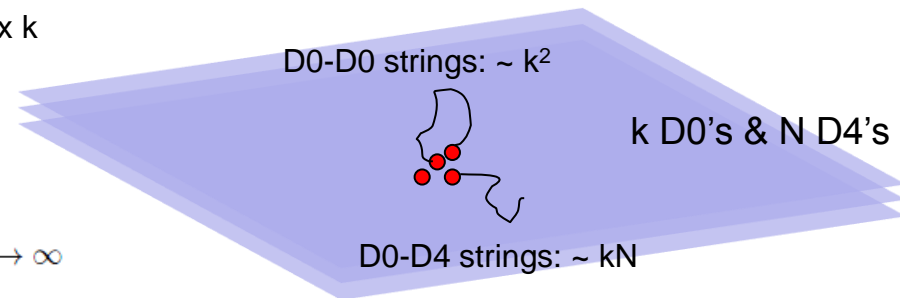
transverse k^2

longitudinal k^2

$N \times k$

$$D^{\dot{\alpha}}_{\dot{\beta}} = \bar{q}^{\dot{\alpha}} q_{\dot{\beta}} + \frac{1}{4} (\bar{\sigma}^{mn})^{\dot{\alpha}}_{\dot{\beta}} [a_m, a_n] - \frac{1}{2} \delta^{\dot{\alpha}}_{\dot{\beta}} (\text{trace})$$

The bulk modes decouple in the limit $\ell_p \rightarrow 0$, or $g_{QM}^2 = \frac{R^3}{\ell_p^6} \rightarrow \infty$



(More discussions on it later)

Microscopic descriptions

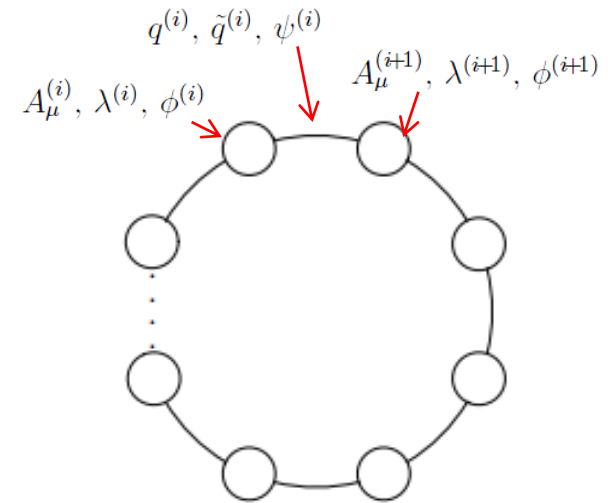
- Deconstruction: 4d N=2 SUSY $U(N)^K$ gauge theory, coupled to K bi-fundamental hypers.
- 4d superconformal theory (gauge coupling g_4)
- Go to Higgs branch by giving VEV $v \mathbf{1}_{N \times N}$ to K scalars.

• Claim [Arkani-Hamed, Cohen, Kaplan, Karch, Motl] (2001) :

Take $K, v, g_4 \rightarrow \infty$, with $g_5^2 \equiv \frac{g_4}{v} = \text{fixed}$, $2\pi R \equiv \frac{K}{g_4 v} = \text{fixed}$.

$a \equiv \frac{1}{g_4 v} = \frac{2\pi R}{K} \rightarrow 0$ in the limit

At $E \ll \frac{1}{a}$, the system is the 6d (2,0) theory on T^2 with radii $R, \frac{g_5^2}{4\pi^2}$



- “discretized” 5d maximal SYM on S^1 : low E theory on D4’s, S^1 compactified M5’s

Divide S^1 with radius R into K steps with length $a = \frac{2\pi R}{K}$.

Discretize all 5d fields: $\partial_5 \Phi(x^5 = an) \rightarrow \frac{\Phi^{(n+1)} - \Phi^{(n)}}{a}$

g_5 is the 5d gauge coupling \sim 6th circle’s radius

- Claims that “latticizing” (thus attempting to give sense to) the low E effective theory of (2,0) also provides a UV description of the latter.

5 dimensional maximal super-Yang-Mills

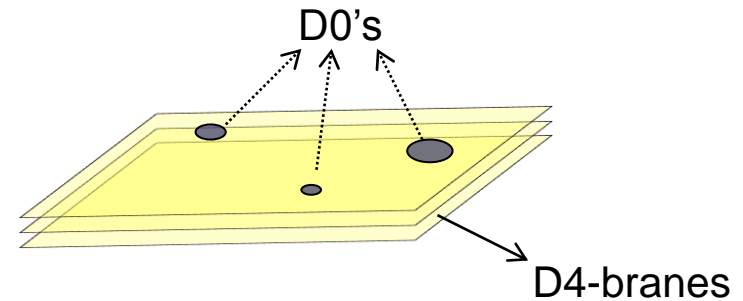
- All these have to do with 5d SYM, obtained by circle compactification. (M5 to D4)
- The Kaluza-Klein degrees: reduces to D0-branes, marginally bound to D4's.
- These are visible as 5d SYM solitons: “instanton” solitons

$$F_{\mu\nu} = \pm \star_4 F_{\mu\nu} \quad \text{on } \mathbb{R}^4$$

$$k = \frac{1}{8\pi^2} \int \text{tr}(F \wedge F) \in \mathbb{Z}$$

$$E = \frac{1}{4g_{YM}^2} \int d^4x \text{tr}(F_{\mu\nu}^2) = \frac{1}{8g_{YM}^2} \int d^4x \text{tr} (F_\mu \mp \star_4 F_{\mu\nu})^2 \pm \frac{1}{2g_{YM}^2} \int \text{tr}(F \wedge F) \geq \frac{4\pi^2 |k|}{g_{YM}^2}$$

The radius of 6th circle: $\frac{4\pi^2}{g_{YM}^2} = \frac{1}{r_1}$



- This makes 5d SYM interesting: UV degrees seen in the 5d effective theory.
- But the theory appears non-renormalizable. $[g_{YM}^2] = L^1$

5d SYM...?

- Other approaches I explained **try to give precise sense to the usage of 5d SYM.**
 - DLCQ: This is just the quantum mechanics for instanton solitons of 5d SYM (more on it later)
 - Deconstruction: 4d theory has monopoles & dyons, taking place of to 5d instantons
- In any of these approaches, one needs to do nonperturbative/strong coupling studies to understand 6d (2,0):
 - Technically difficult in all approaches.
 - Maybe even ill-defined from the “naïve 5d SYM” in general.
- Are there 6d observables directly computable even from the naïve 5d SYM?

BPS observables

Technical challenge

- Let me address the technical difficulty first: “computation at strong coupling”

- A cousin problem: strongly interacting M2-brane SCFT with $\text{OSp}(8|4)$

- QFT: ABJM (2008) $U(N) \times U(N)$ Chern-Simons gauge theory, A_μ, \hat{A}_μ : CS level ± 1
matters in bifundamental rep.: Φ^I, Ψ_α^I ($I = 1, 2, 3, 4$)

$$S = \frac{1}{4\pi} \int \text{tr} \left(AdA - \frac{2i}{3} A^3 \right) - \frac{1}{4\pi} \int \text{tr} \left(\hat{A}d\hat{A} - \frac{2i}{3} \hat{A}^3 \right) + (\text{interaction with matters})$$

- Learned a lot on M2 & M-theory from “BPS quantities” (2009 -):

2009: S.K. (superconformal index); Kapustin-Willet-Yaakov (S^3 partition function & Wilson loops);

2010: Drukker-Marino-Putrov ($N^{3/2}$ degrees); Jafferis (3d version of a-maximization);

- BPS observables in “old days”: many of them are computable from (almost-)free theory
- Recent: SUSY controls observables (so calculable), but **very sensitive to interactions**.
- E.g. many BPS observables are functions of the coupling constants.

BPS observables & 5d SYM

- BPS observables play 2 important roles in our studies from 5d SYM:
 - With incomplete low E effective theory, it is not clear a priori how to write down consistent expressions for observables within the description.
 - But **SUSY path integrals for BPS observables are well-defined** : due to a strong B/F cancelations, especially in the UV, they are free of UV divergence issues.

[Note. Whether the written-down 5d expression is the 6d observable we want to study is still highly unclear. In other words, we **empirically** find that “**very nontrivial SUSY observables are correctly calculable from 5d SYM**”, but we should better understand “**why it works.**” I will try to briefly comment on two possible resolutions to the last question, at the end of lecture 3.]

- Even with well-defined path integral, we need a strong-coupling calculus to see 6d physics.
- Question:
 - Can we learn about 6d (2,0) theory by exactly computing useful observables?**

5d SYM & stable particles

- Fields & action of 5d maximal SYM:

Fields: A_μ , λ_α^i ($i = 1, 2, 3, 4$ for $SO(5)_R$), ϕ^I ($I = 1, 2, 3, 4, 5$)

$$S = \frac{1}{g_{YM}^2} \int d^5x \operatorname{tr} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \phi^I D^\mu \phi^I - \frac{i}{2} \bar{\lambda}_i \gamma^\mu D_\mu \lambda^i + \frac{1}{4} [\phi^I, \phi^J]^2 - \frac{i}{2} \bar{\lambda}_i (\hat{\gamma}_I)^i_j [\phi^I, \lambda^j] \right]$$

- Massless W-bosons & superpartners; also, instanton solitons $F_{\mu\nu} = \pm \star_4 F_{\mu\nu}$
- **Coulomb phase**: massive BPS W-bosons & superpartners
- Marginal bounds of instantons & W-bosons: classical solitons [Lambert, Tong]

$$\begin{aligned} E &= \frac{1}{g_{YM}^2} \int d^4x \operatorname{tr} \left[\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} F_{\mu 0}^2 + \frac{1}{2} (D_\mu \phi)^2 + \frac{1}{2} (D_0 \phi)^2 \right] \\ &= \frac{1}{g_{YM}^2} \int d^4x \operatorname{tr} \left[\frac{1}{8} (F_{\mu\nu} - \star_4 F_{\mu\nu})^2 + \frac{1}{2} (F_{\mu 0} - D_\mu \phi)^2 + (D_0 \phi)^2 \right] + \frac{4\pi^2 k}{g_{YM}^2} + \operatorname{tr}(vq) \end{aligned}$$

$$\operatorname{tr}(vq) \equiv \frac{1}{g_{YM}^2} \int d^4x \partial^\mu \operatorname{tr}(\phi F_{\mu 0}) = \frac{1}{g_{YM}^2} \int_{S^3} dS^\mu \operatorname{tr}(v F_{\mu 0}) \sim g_{YM} \operatorname{tr}(v_{\text{can}} q_{\text{quant}})$$

- Each of them are $\frac{1}{2}$ -BPS. The bounds are $\frac{1}{4}$ -BPS.
- These uplift to self-dual strings of M5, wrapping S^1 with momenta.

[Their quantum bound are studied by Nekrasov's instanton partition function. (lecture 2)]

Properties of YM instantons

- The self-dual instantons have moduli: $4Nk$ real moduli for k instantons in $U(N)$ SYM

$$D_\mu \delta A_\nu - D_\nu \delta A_\mu = \epsilon_{\mu\nu\rho\sigma} D_\rho \delta A_\sigma, \quad D_\mu \delta A_\mu = 0 \rightarrow (\bar{\sigma}^\mu D_\mu)^{\dot{\alpha}\alpha} \delta A_{\alpha\dot{\beta}} = 0$$

$$\delta A_{\alpha\dot{\beta}} = (\sigma^\mu)_{\alpha\dot{\beta}} \delta A_\mu$$

pair of chiral Dirac equation for $\dot{\beta} = 1, 2$:
index theorem counts 0-modes

- Zero modes from fermions as well.
- Moduli space approximation yields a quantum mechanical system:

- Supersymmetric sigma-model:

coordinates of $4Nk$ dimensional
moduli space

$$S_{QM} = \int dt \left[g_{MN}(X) \dot{X}^M \dot{X}^N + \dots \right]$$

- reliable in strict sense only in special limits: low E scaling limit, DLCQ, ...
- However, this QM is useful more widely for BPS observables.
- E.g. Nekrasov's partition function on (Omega-deformed) $\mathbb{R}^4 \times S^1$:

$$Z[v, g_{YM}^2/r, \dots]_{\mathbb{R}^4 \times S^1} = \int [\mathcal{D}A_\mu(x^m, \tau) \dots] e^{-S_{SYM}} = \int [\mathcal{D}X^M(\tau) \dots] e^{-S_{QM}}$$

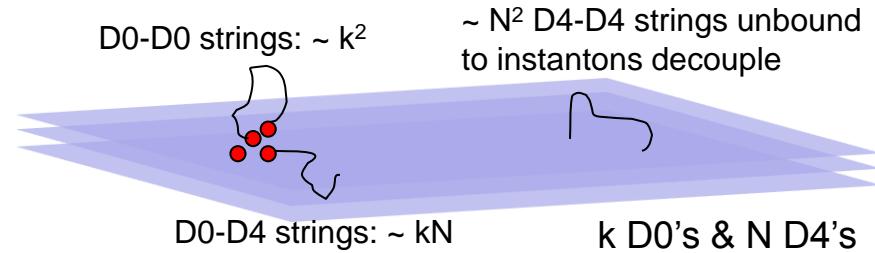
5d supersymmetric path integral

All apart from instanton zero modes on \mathbb{R}^4
cancel between B/F: reduces to 1d path integral

D-brane perspective

- D0-branes, marginally bound to D4-branes
- D0-D4-brane QM with 8 SUSY:

QM vector multiplet: $\varphi^I, A_t, \bar{\lambda}_{\dot{\alpha}}^i$
 $U(k)$ adjoint hyper: $a_m, \lambda_{\dot{\alpha}}^i$ } maximal SYM QM
 $U(N) \times U(k)$ bi-fundamental hyper: $q_{\dot{\alpha}}, \psi^i$



$$L_{QM} = \frac{1}{g_{QM}^2} \text{tr}_{k,N} \left[\frac{1}{2} (D_t \varphi^I)^2 + \frac{1}{2} (D_t a_m)^2 + \frac{1}{4} [\varphi^I, \varphi^J]^2 + \frac{1}{2} [a_m, \varphi^I]^2 \right. \\
 \left. + D_t q_{\dot{\alpha}} D_t \bar{q}^{\dot{\alpha}} - (\varphi^I \bar{q}^{\dot{\alpha}} - \bar{q}^{\dot{\alpha}} v^I)(q_{\dot{\alpha}} \varphi^I - v^I q_{\dot{\alpha}}) - D_{\dot{\beta}}^{\dot{\alpha}} D_{\dot{\alpha}}^{\dot{\beta}} + \dots \right]$$

instantons in Coulomb phase: UV theory same, changes vacuum

$$D_{\dot{\beta}}^{\dot{\alpha}} = \bar{q}^{\dot{\alpha}} q_{\dot{\beta}} - \frac{1}{2} \zeta^A (\tau^A)^{\dot{\alpha}}_{\dot{\beta}} + \frac{1}{4} (\bar{\sigma}^{mn})^{\dot{\alpha}}_{\dot{\beta}} [a_m, a_n] - \frac{1}{2} \delta_{\dot{\beta}}^{\dot{\alpha}} (\text{trace})$$

FI parameter: turning on NS-NS B-field on D4, changes UV theory, IR physics unaffected

$SO(4, 1) \rightarrow SO(4) = SU(2)_{1L} \times SU(2)_{1R}$ in massive particle background

$SO(5)_R \rightarrow SO(4)_R = SU(2)_{2L} \times SU(2)_{2R}$ with VEV $v^I = \delta_5^I v$

Coulomb branch decouples with either g_{QM} or ζ large.

- Higgs branch $4Nk + 4k^2 - 3k^2 - k^2 = 4Nk$: dimension of instanton moduli space

$$D_{\dot{\beta}}^{\dot{\alpha}} = \bar{q}^{\dot{\alpha}} q_{\dot{\beta}} - \frac{1}{2} \zeta^A (\tau^A)^{\dot{\alpha}}_{\dot{\beta}} + \frac{1}{4} (\bar{\sigma}^{mn})^{\dot{\alpha}}_{\dot{\beta}} [a_m, a_n] - \frac{1}{2} \delta_{\dot{\beta}}^{\dot{\alpha}} (\text{trace}) = 0 \quad \& \text{U}(k) \text{ gauge-fix}$$

- Coulomb branch decouples w/ g_{QM} or ζ large: Higgs branch yields SUSY σ -model

Moduli space dynamics

- Example: single U(N) instanton. The metric on 4N dimensional moduli space is

$$ds^2 = g_{MN}(X)dX^M dX^N = \boxed{ds^2(\mathbb{R}^4)} + d\lambda^2 + \lambda^2 \left[\boxed{ds^2(S^3/\mathbb{Z}_2)} + \boxed{ds^2(\mathcal{M}_{4N-8})} \right]$$

center-of-mass
instanton "size"
SU(2) orientation
 $\frac{SU(N)}{SU(2) \times U(N-2)}$

$$\lambda^2 \sim \bar{q}^{\dot{\alpha}} q_{\dot{\alpha}}$$

- The size moduli: non-compact moduli from internal degrees

- Large: continuous spectrum. Presumably reflects IR physics of 6d CFT, but not clarified yet.

We study observables which are insensitive to this IR issue:

- observables in **Coulomb phase** (instanton size frozen): e.g. Nekrasov's instanton partition function
- "gapped" CFT observables: local operator spectrum, compact space partition functions, ...

- Small: Metric is singular. Moduli space dynamics is incomplete. Needs more UV input on 5d SYM.

The singularity can be resolved, e.g. by putting it in B-field background: beyond 5d SYM or SUSY QM

$$U(2) : d\lambda^2 + \frac{\lambda^2}{4} [d\theta^2 + \sin^2 \theta d\phi^2 + (d\psi + \cos \theta d\phi)^2] \rightarrow \frac{1}{\sqrt{1 + \frac{4\zeta^2}{\lambda^4}}} \left[d\lambda^2 + \frac{\lambda^2}{4} (d\psi + \cos \theta d\phi)^2 \right] + \frac{\lambda^2}{4} \sqrt{1 + \frac{4\zeta^2}{\lambda^4}} [d\theta^2 + \sin^2 \theta d\phi^2]$$

$$\boxed{\mathbb{R}^4/\mathbb{Z}_2}$$

$$\boxed{\text{Eguchi-Hanson metric: } \mathbb{R}^2 \times S^2 \text{ near } \lambda = 0}$$

Plan for the next 2 lectures

- The supersymmetric partition function for the instantons: “Nekrasov”
- Partition function for the ADHM quantum mechanics in Coulomb phase

- Study them as the index of 6d KK states, bound to other BPS particles: can study the physics of self-dual strings wrapping S^1 factor: **both in Coulomb & symmetric phases**

- Study other observables of 6d (2,0) theory, from the 5d SYM partition functions: 6d superconformal index ($S^5 \times S^1$), brief comments on other observables