

PROBLEM SET 6

July 22, 2019

- (1) Show that $\Theta(e^{-\eta}h) = \Theta(h) + \partial\bar{\partial}\eta \otimes \mathbf{1}_E$.
- (2) Show that every bounded domain has a non-constant $\eta \in \text{SBG}$.
- (3) Show that if $\psi \in L^1_{\text{loc}}(\mathbb{D})$ satisfies

$$\Delta\psi \geq -\frac{1-\delta}{(1-|z|^2)^2}$$

then for all $f : \mathbb{D} \rightarrow \mathbb{C}$ such that

$$\int_{\mathbb{D}} |f|^2 e^{-\psi} \frac{dA(z)}{(1-|z|^2)^2} < +\infty,$$

there exist $u \in L^1_{\text{loc}}(\mathbb{D})$ such that $\frac{\partial u}{\partial z} = f$ and

$$\int_{\mathbb{D}} |u|^2 e^{-\psi} \leq C_{\delta} \int_{\mathbb{D}} |f|^2 e^{-\psi} \frac{dA(z)}{(1-|z|^2)^2}.$$

- (4) Let X be a complex manifold and $f \in \mathcal{O}(X)$ such that $\sup_X |f| \leq 1$. Show that $\eta := -\log \log \frac{1}{|f|^2} \in \text{SBG}(X-Z)$ where $Z = \{x \in X : f(x) = 0\}$. Is η in $\text{SBG}(X)$.