# PROBLEM SET 5

July 19, 2019

(1) Show that if $X$ is strongly pseudoconvex (which means that there is a smooth strictly plurisubharmonic exhasution function on $X$) then for any distinct points $x, y \in X$ there exists $f \in \mathcal{O}(X)$ such that $f(x) \neq f(y)$. Is this true if $X$ is weakly pseudoconvex?

(2) Show that if $X$ is strongly pseudoconvex then for any $p \in X$ there exist $f_1, \ldots, f_n \in \mathcal{O}(X)$, $n = \dim_{\mathbb{C}} X$, such that $df_1(p) \wedge \ldots \wedge df_n(p) \neq 0$.

(3) Let $X$ be compact complex manifold and $L \to X$ a holomorphic line bundle with metric $h$ having strictly positive Chern curvature. Show that any holomorphic line bundle $H \to X$ has a non-identically zero meromorphic section.

(4) Using the Bochner-Kodira-Moory-Kohn-Hörmander formula, directly show that if $\Omega \subset \mathbb{C}^n$ is a bounded strictly pseudoconvex domain (i.e., Levi-form is strictly positive) then $\mathcal{H}^{p,q}(\Omega) = 0$ for $q \geq 1$.

(5) Compute the Levi forms of the domains

$$\Omega_1 = \{|z_1| < 1\} \subset \mathbb{C}^2,$$
$$\Omega_2 = \{x_1^2 + x_2^2 < 1\} \subset \mathbb{C}^2.$$