Transport and fractality in boundary driven chains

Vipin Kerala Varma

(CUNY)

Outline

- Transport in (disordered) chains

Agarwal et al., *arXiv 1408.3413*
Vosk-Altman-Huse, *arXiv 1412.3117*
VKV et al., *arXiv 1511.09144*

- Lindblad boundary driving

Prosen-Znidaric, *JSTAT* (2009)

- Anomalous hydrodynamics in a ...
  - Disordered + interacting chain

    Znidaric-Scardicchio-VKV, *PRL* (2016)

  - Quasidisordered + free chain

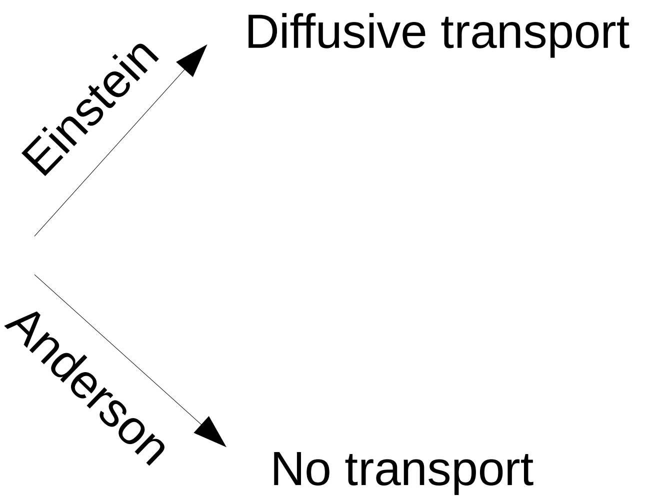
    VKV-Pilati-Kravtsov, *PRB* (2016)
    VKV-Mulatier-Znidaric, *arXiv* (2017)

  - Clean chain (≈ integrable)

    Sanchez-VKV-Oganesyan, *soon* (2017)

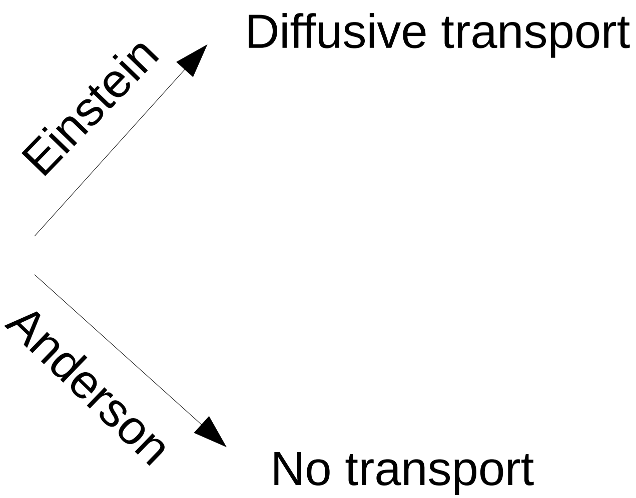
Transport (in disordered systems)

- Usually:
  - Disordered system



Transport (in disordered systems)

- Usually:
  - Disordered system



$$\frac{\partial \phi(x,t)}{\partial t} = -\nabla . \vec{j}(x,t)$$

→ Continuity equation (exact)

$$\vec{j}(x,t) = -\mathcal{D}_s \nabla \phi(x,t)$$

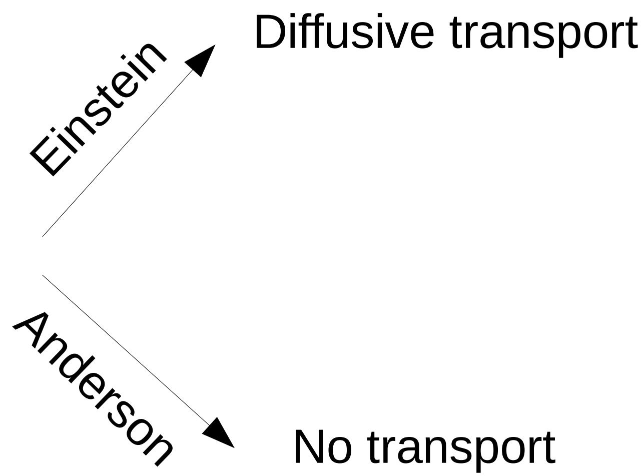
→ Fick's first law (assumption)

$$\Longrightarrow \frac{\partial \phi(x,t)}{\partial t} = \mathcal{D}_s \Delta \phi(x,t)$$

→ Transport equation (phenomenological)

Transport (in disordered systems)

- Usually:
  - Disordered system



- BUT it can be anomalous:

$$\frac{\partial \phi(x,t)}{\partial t} = \tilde{D}\nabla^{2+b}\phi(x,t),$$

$$b > 0 : \text{subdiffusion}$$

Levy flights: Derrida et al. (1982 - ...)

Criticality: Ohtsuki et al. (1997)

Griffiths fractality: Huse et al. (2016)

…

Boundary driven chain

- Lindblad's master equation: driving by 2 “baths”

$$\mathrm{d}\rho/\mathrm{d}t = \mathrm{i}[\rho, H] + \frac{1}{4}\sum_{k=1}^{4}\Big([L_k\rho, L_k^\dagger] + [L_k, \rho L_k^\dagger]\Big)$$

Unitary part

Dissipative part

See also: S. Weinberg, “The Trouble with QM”, *NY-Book Review* (2017)

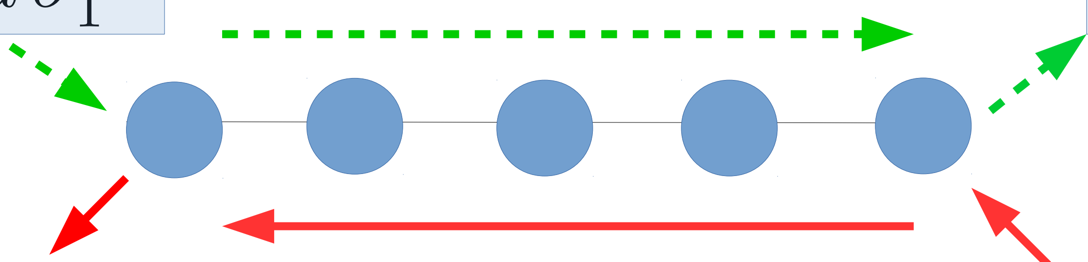
Boundary driven chain

- Lindblad's master equation: driving by 2 "baths"

$$\mathrm{d}\rho/\mathrm{d}t = \mathrm{i}[\rho, H] + \frac{1}{4}\sum_{k=1}^{4}\left([L_k\rho, L_k^\dagger] + [L_k, \rho L_k^\dagger]\right)$$

$$L_1 = \sqrt{1-\mu}\,\sigma_1^+$$

$$L_4 = \sqrt{1-\mu}\,\sigma_L^-$$



$$L_2 = \sqrt{1+\mu}\,\sigma_1^-$$

$$L_3 = \sqrt{1+\mu}\,\sigma_L^+$$

Left Lindblad terms

(Linear response)

$$0 < \mu \ll 1$$

Right Lindblad terms

Steady state current

- Protocol
  - Initialise density matrix $\rho(0)$

(Infinite temperature)

  - Evolve with t-DMRG: $\rho_\infty = \frac{1}{\mathcal{N}}(\mathbb{1} + \mu \ldots)$
  - Measure final current: $\mathrm{tr}(j_k \rho_\infty) = (j \rho_\infty)$

$$j_k := i[s_k^z, H_{k,k+1}] = s_k^x s_{k+1}^y - s_k^y s_{k+1}^x$$

Steady state current

- Scaling of steady state current

$$j \sim \frac{1}{L^\gamma} \sim D(L)\frac{\langle s_L^z \rangle - \langle s_1^z \rangle}{L}$$

- Spreading of spin excitations

$$x^2 \sim t^{2\beta} \sim D(t)t$$

$$\beta = \frac{1}{1+\gamma}$$

Superdiffusion:	$1/2 < \beta < 1 \rightarrow 0 < \gamma < 1$
Diffusion:	$\beta = 1/2 \rightarrow \gamma = 1$
Subdiffusion:	$0 < \beta < 1/2 \rightarrow 1 < \gamma < \infty$

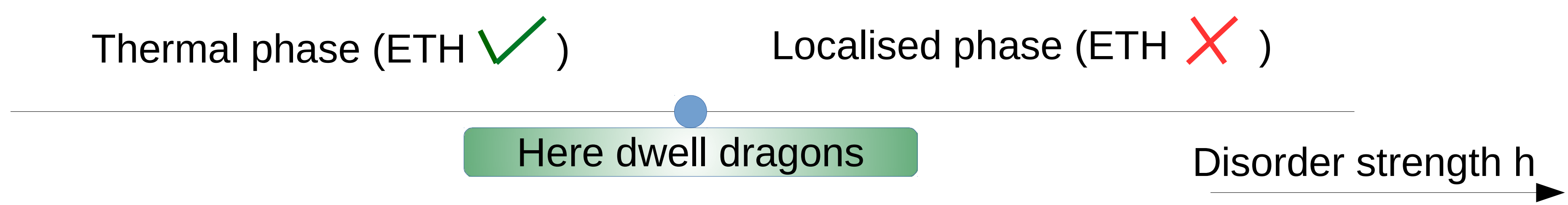
Disordered Heisenberg model

- Standard model of 1D physics

$$H = \sum_{k=1}^{L-1} H_{k,k+1}$$

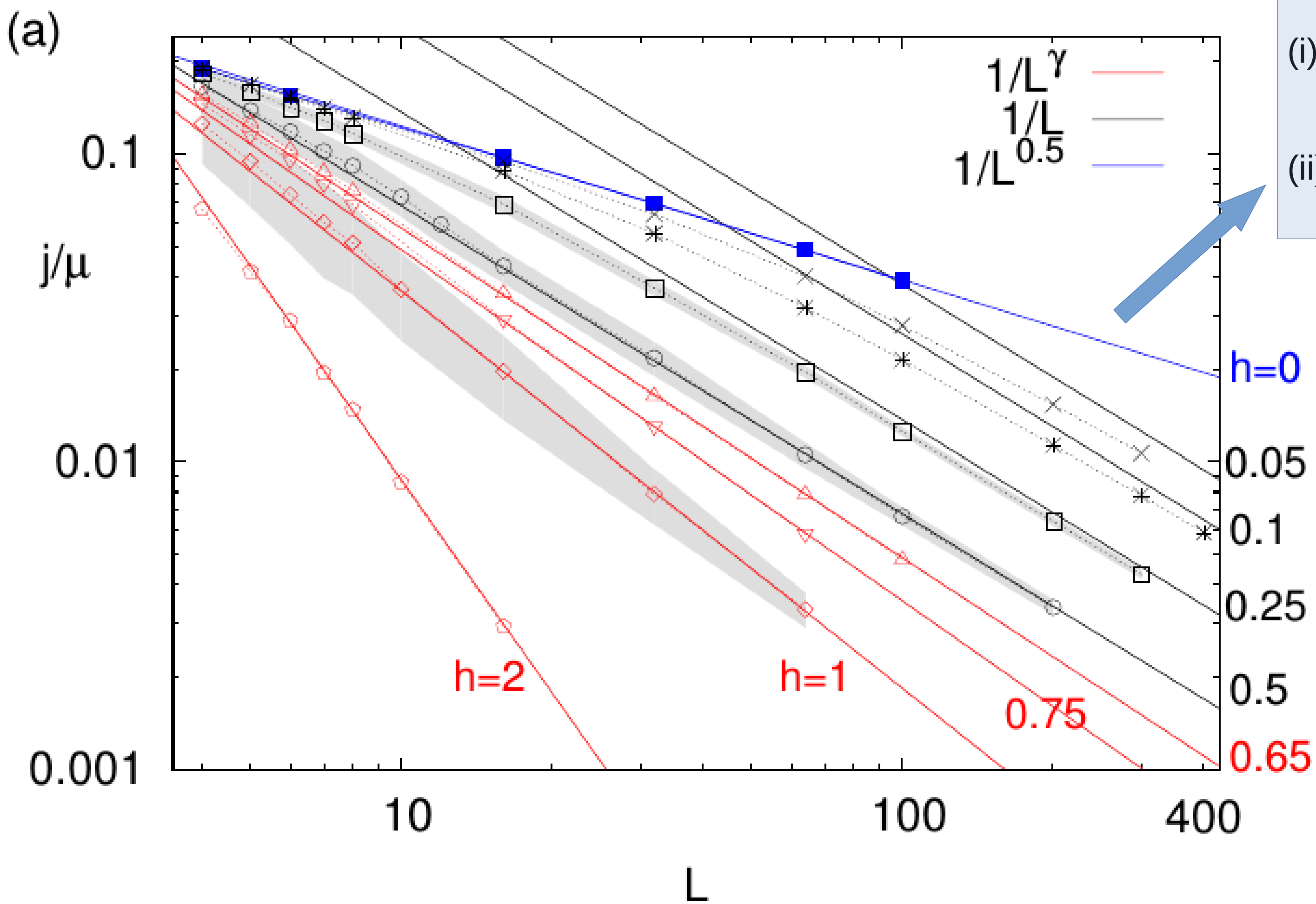
$$H_{r,p} = s_r^x s_p^x + s_r^y s_p^y + \Delta s_r^z s_p^z + \frac{h_r}{2} s_r^z + \frac{h_p}{2} s_p^z, \qquad h_i \in [-h, h]$$

- (In)finite temperature quantum phase transition



Oganesyan-Huse, *PRB* (2008)

Isotropic point: steady state current



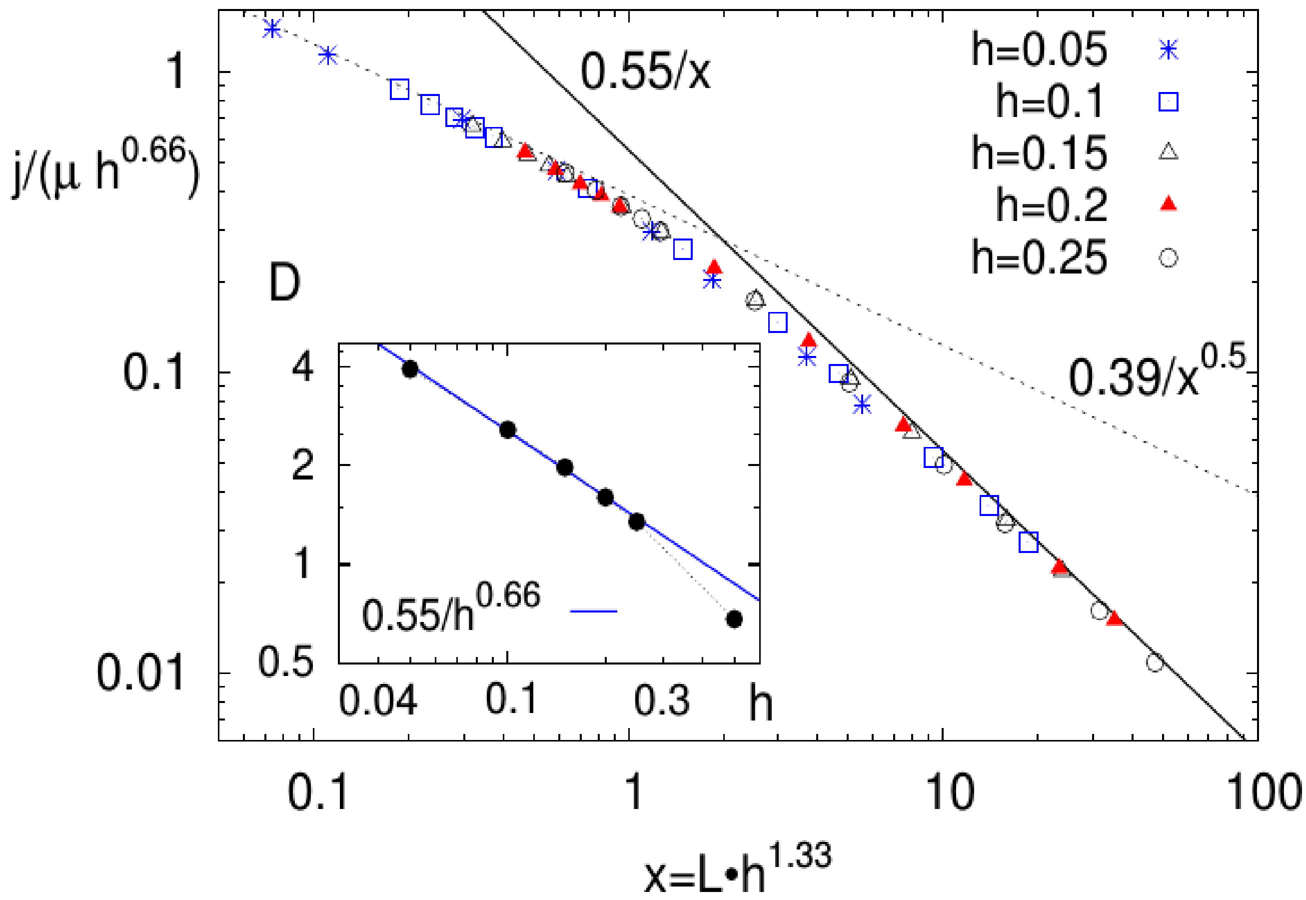
h=0 quantitatively agrees with:

(i) long-time (t ~ 150) unitary t-DMRG Ljubotina et al., *Nat. Comm.* (2017)

(ii) memory function, and Kubo response Sanchez-VKV-Oganesyan, *soon* (2017)

UPSHOT: Critical length scale L* below which disorder is irrelevant

Isotropic point: scaling collapse



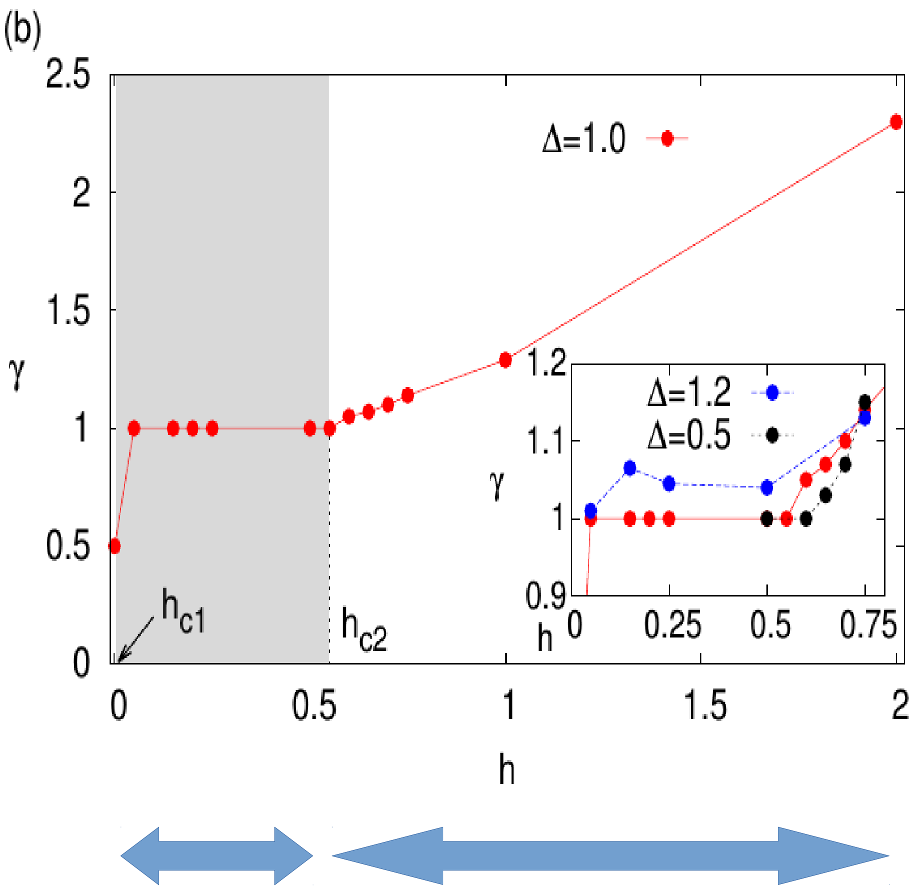
$$L_* \sim \frac{1}{h^{2\beta}}$$

Estimable from FGR

“Pretends” to be clean

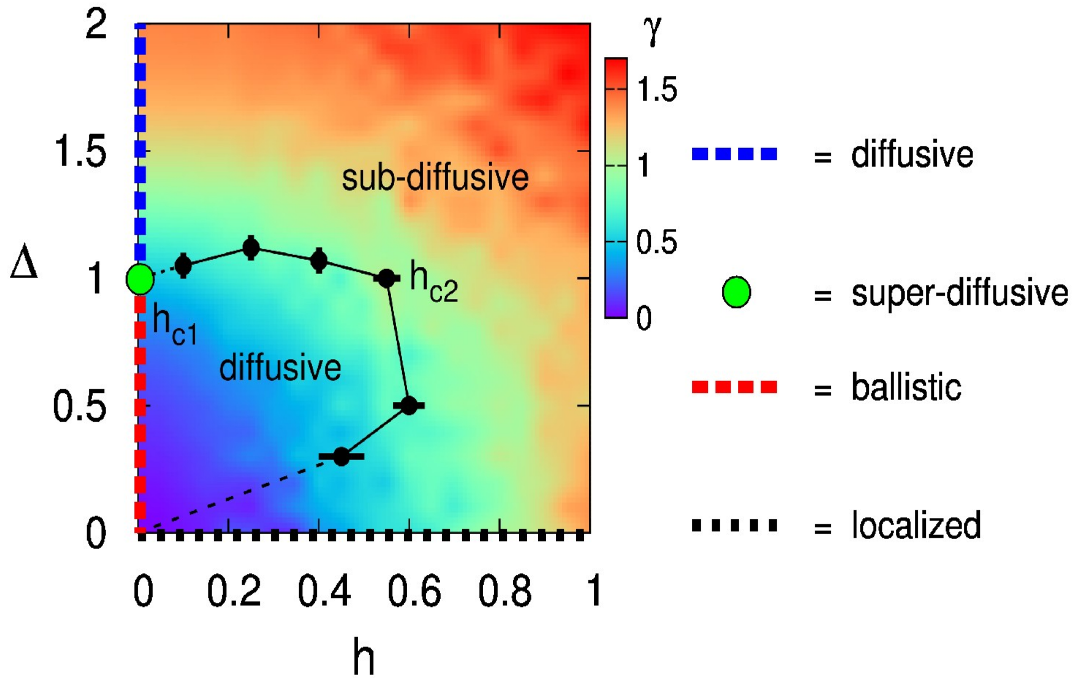
Disorder is felt

Dynamical exponent



Diffusive

Subdiffusive



Dynamical phase diagram

Similar (and more!) effects for energy:

VKV-Lerose-Pietracaprina-Goold-Scardicchio, *JSTAT* (2017)

Arenas-Znidaric-VKV-Goold-Clark-Scardicchio, *soon(?)* (2017)

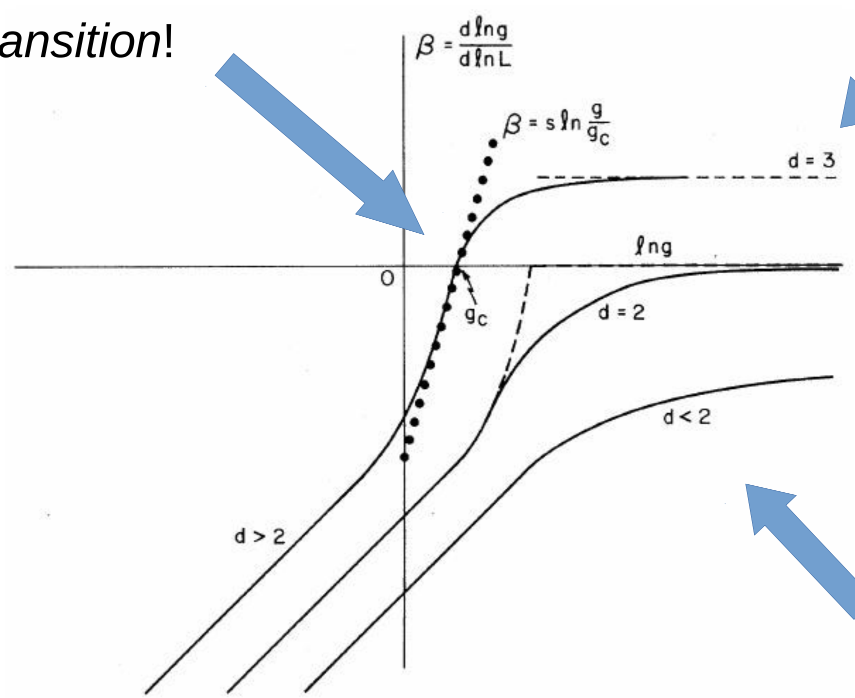
Question: since anomalous transport has long been seen in quasiperiodic systems (80's and 90's)*, perhaps we should revisit it in this new light?

* Hiramoto-Abe, *JPSJ* (1988)
Piechon, *PRL* (1996)
Ketzmerick et al., *PRL* (1997)

Disordered systems

- Generically no localization transition in 1D

Transition!



Metal: $g \sim L^{2-d}$

Insulator: $g \sim \mathrm{e}^{-L/\xi}$

Gang of four (1979)

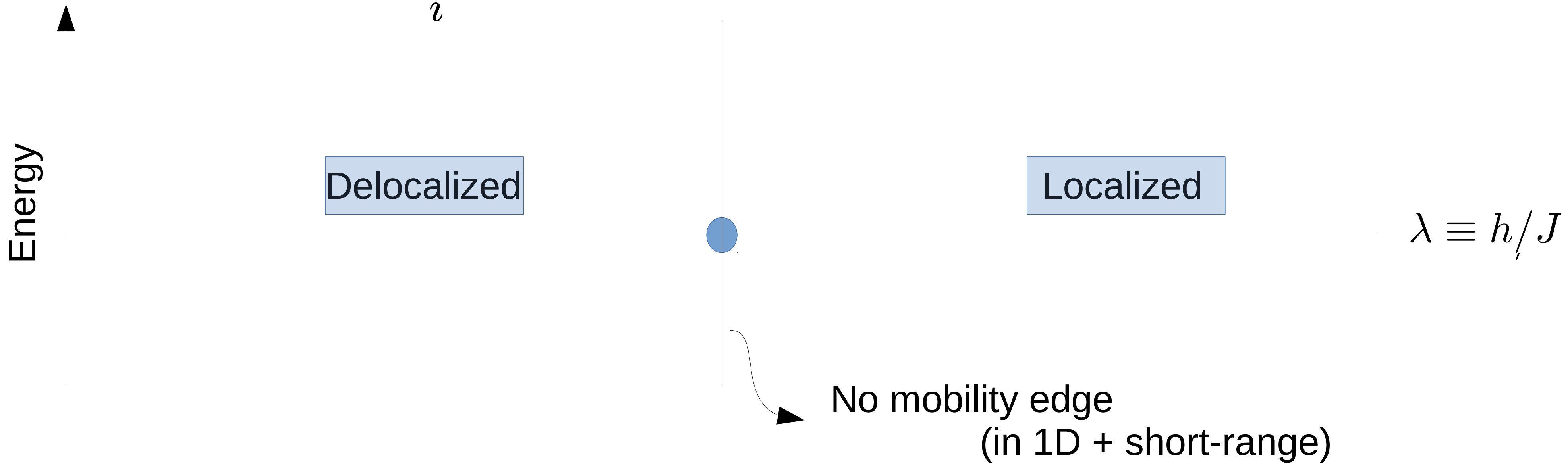
Quasiperiodic systems

- Delocalizable even in 1D

$$\mathcal{H}_i = J s_i^x s_{i+1}^x + J s_i^y s_{i+1}^y + h \cos\left(2\pi g i + \phi\right) s_i^z$$

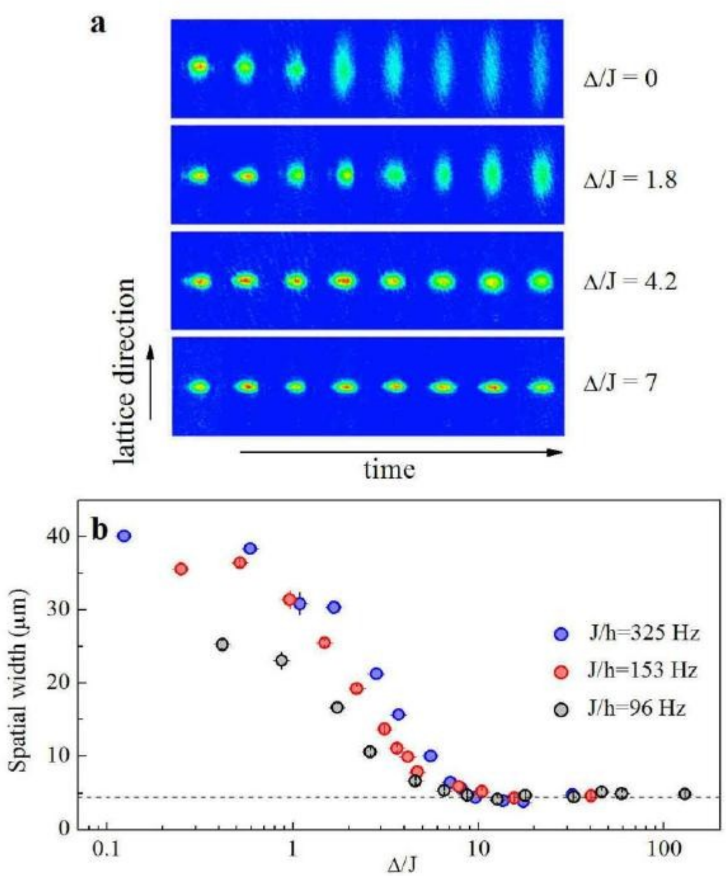
$$\mathcal{H} = \sum_i \mathcal{H}_i$$

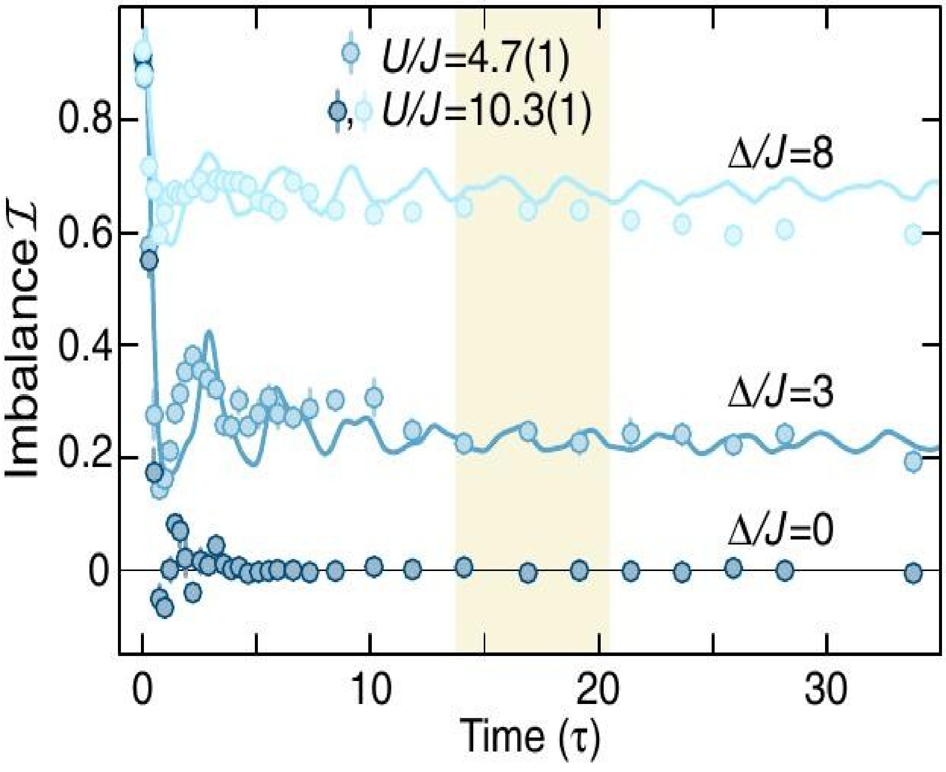
Aubry and André (1980)



Quasiperiodic systems

- Boson localization
- Fermion localization



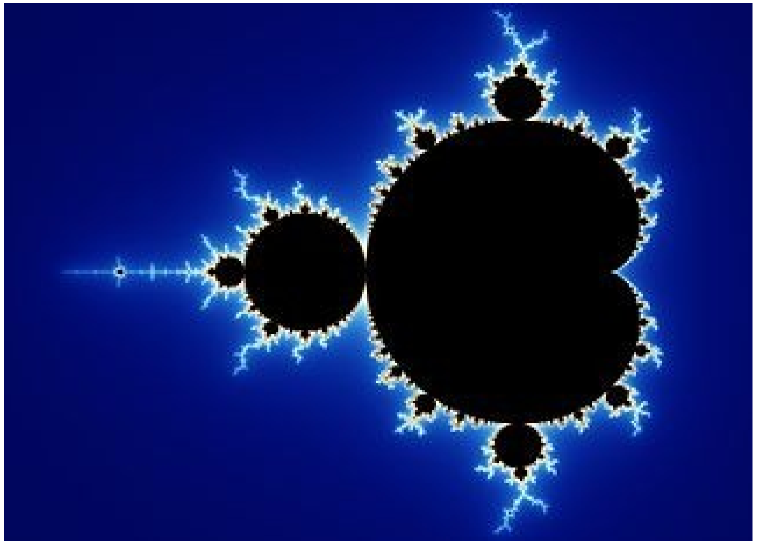


For atoms: Roati et al. (Nature 2008)
For light: Lahini et al. (PRL 2009)

Schreiber et al. (Science 2015)

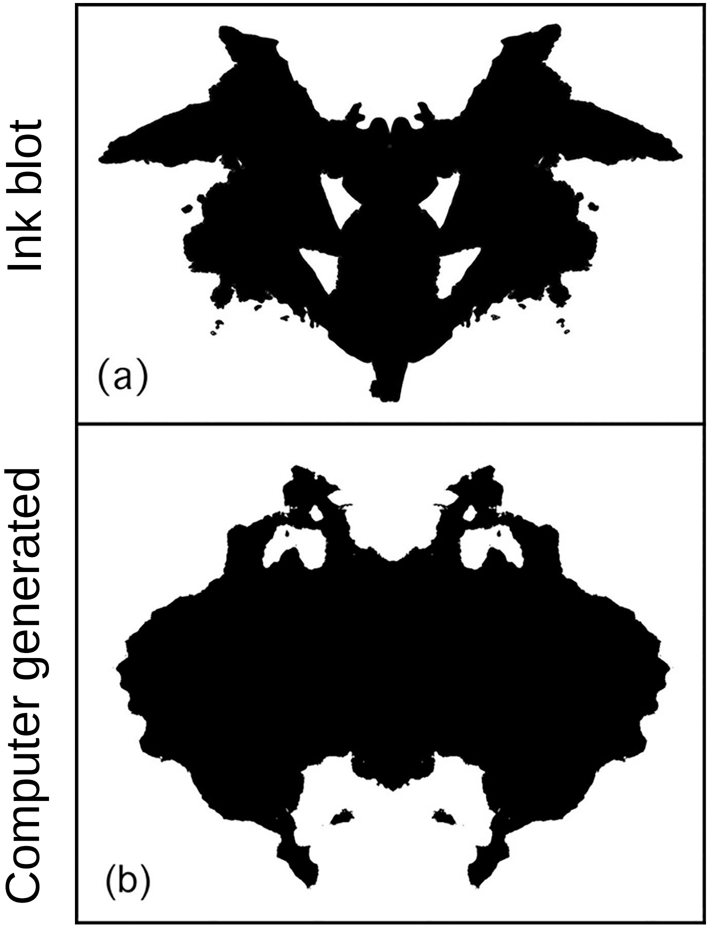
Fractality

- Generic example



Benoit B. Mandelbrot set

Taylor et al., PLOS ONE (2017)



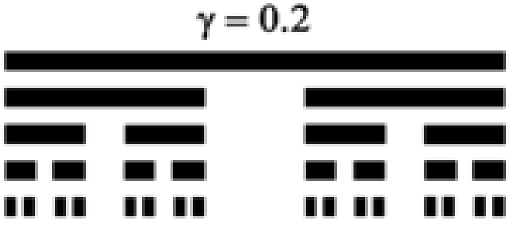
Fractality

- Mathematics example

Generalized Cantor set

-- Fractal dimension = function of γ

-- Spectra of quasiperiodic systems



Wikipedia

Avila, Fields Medal (2014)

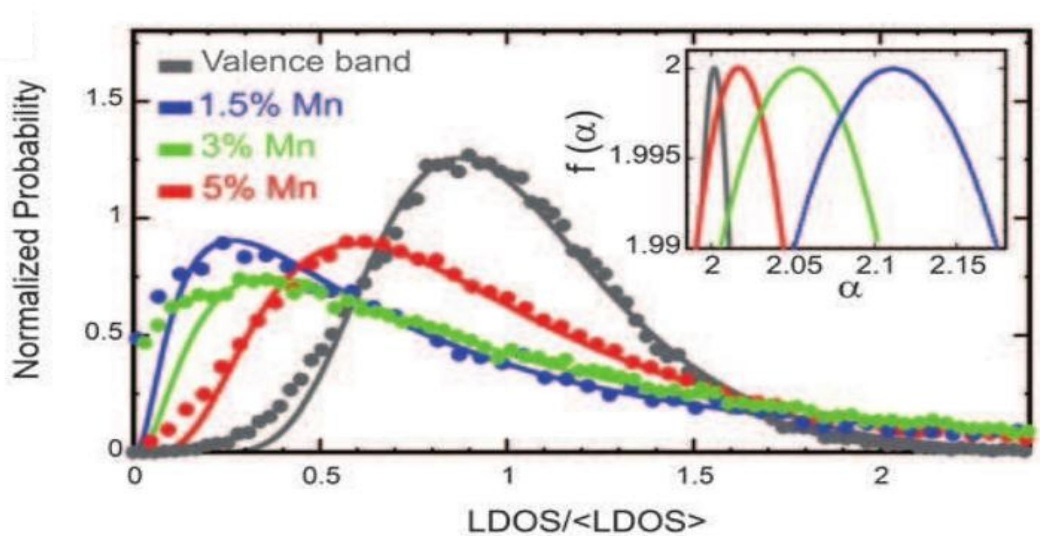
Fractality

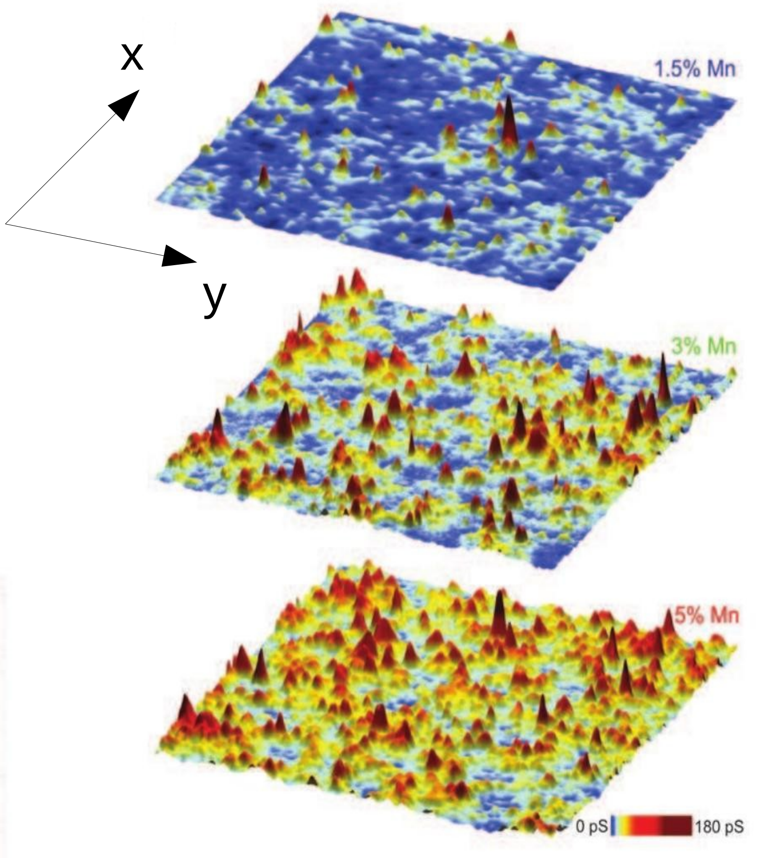
- Physics example

Metal-insulator transitions

-- Doped Gallium Arsenide

-- LDOS at E_F using STEM

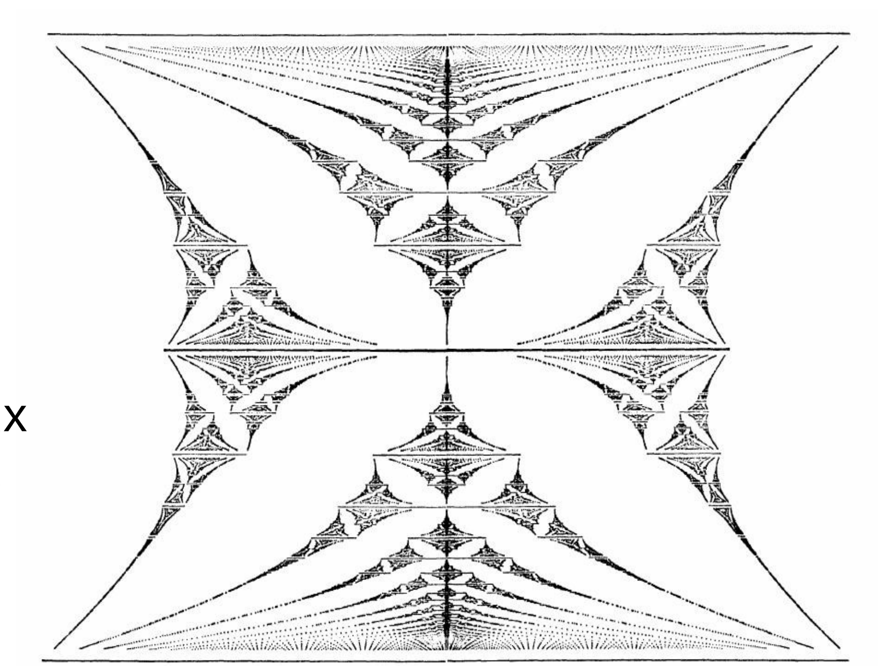




Richardella et al., Science (2010)

Fractality

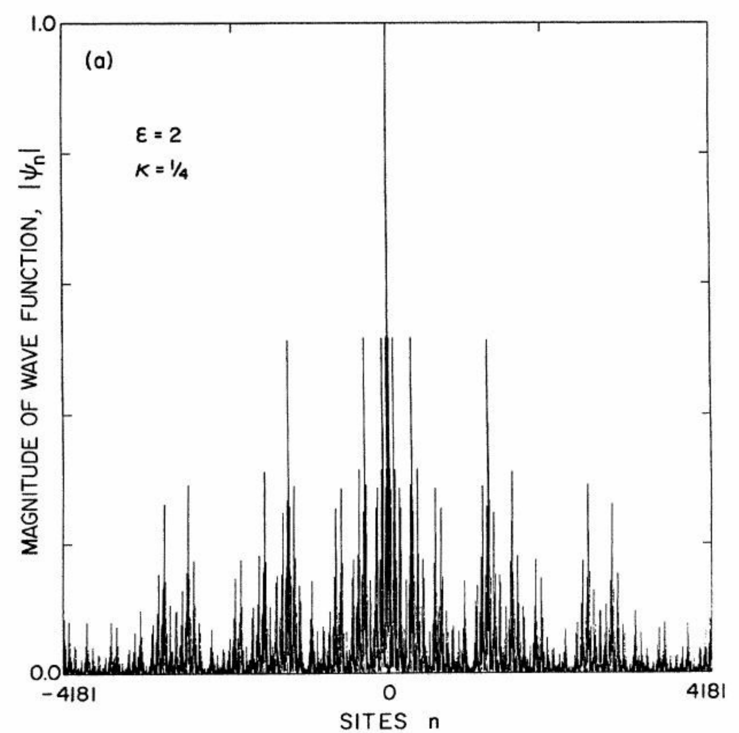
Fractal spectrum vs. Fractal eigenfunctions



flux

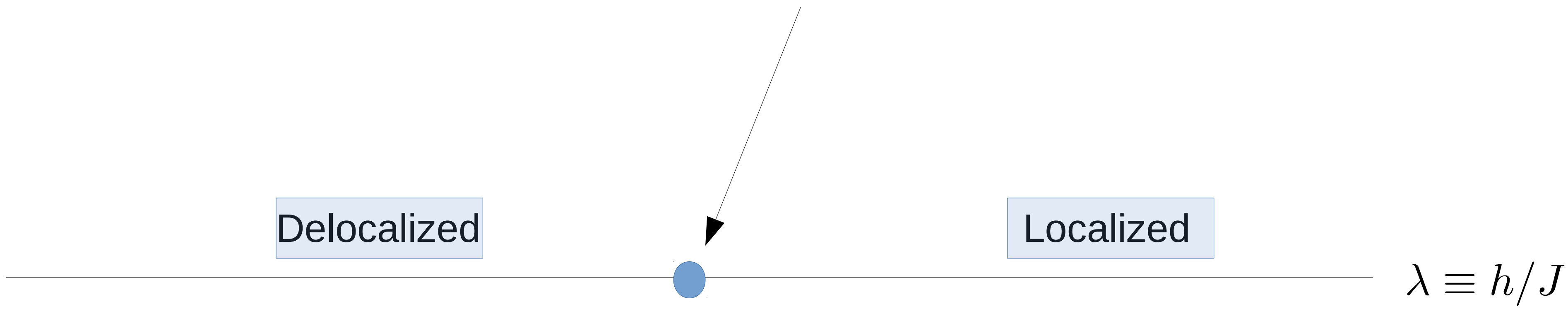
energy

Hofstadter's butterfly (1976)



Ostlund and Pandit, PRB (1984)

Question: how stable are these fractal properties to driving at the transition?



Exact NESS solution (or Why open ≠ closed?)

- NESS dependent on e-functions and e-values!

$$j = -16\,\mathrm{Im}\left[\sum_{j,k} \frac{1}{\lambda_j + \lambda_k^*} (\psi_1^{(Rj)})^2 (\psi_1^{(Rk)} \psi_2^{(Rk)})^*\right],$$

$$\langle \sigma_p^z \rangle = -1 + 4\left[\sum_{j,k} \frac{1}{\lambda_j + \lambda_k^*} (\psi_1^{(Rj)} \psi_p^{(Rj)})(\psi_1^{(Rk)} \psi_p^{(Rk)})^*\right]$$

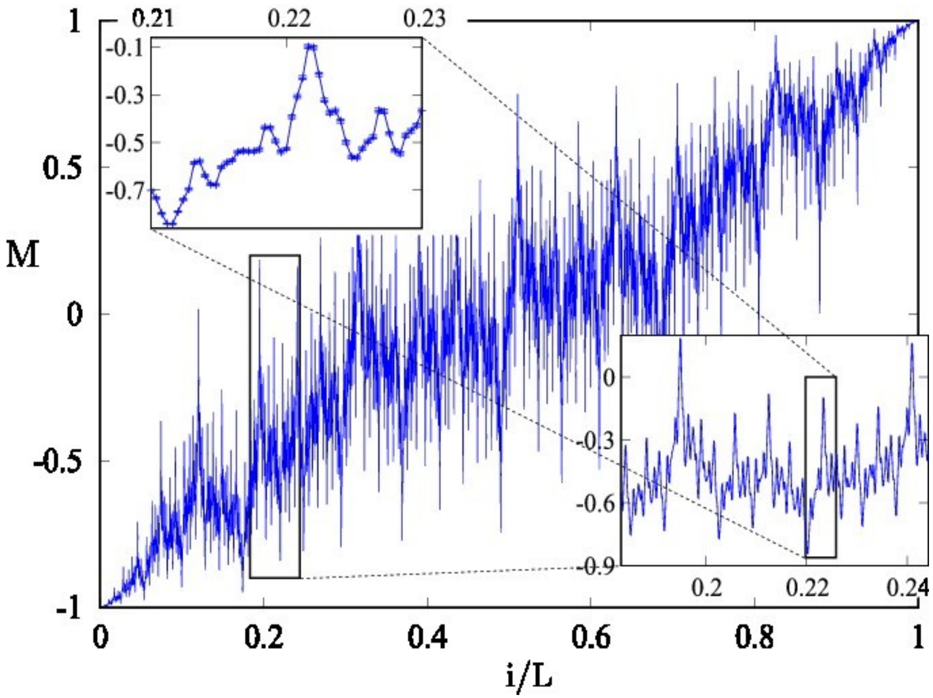
Eigenvalues

Eigenfunctions

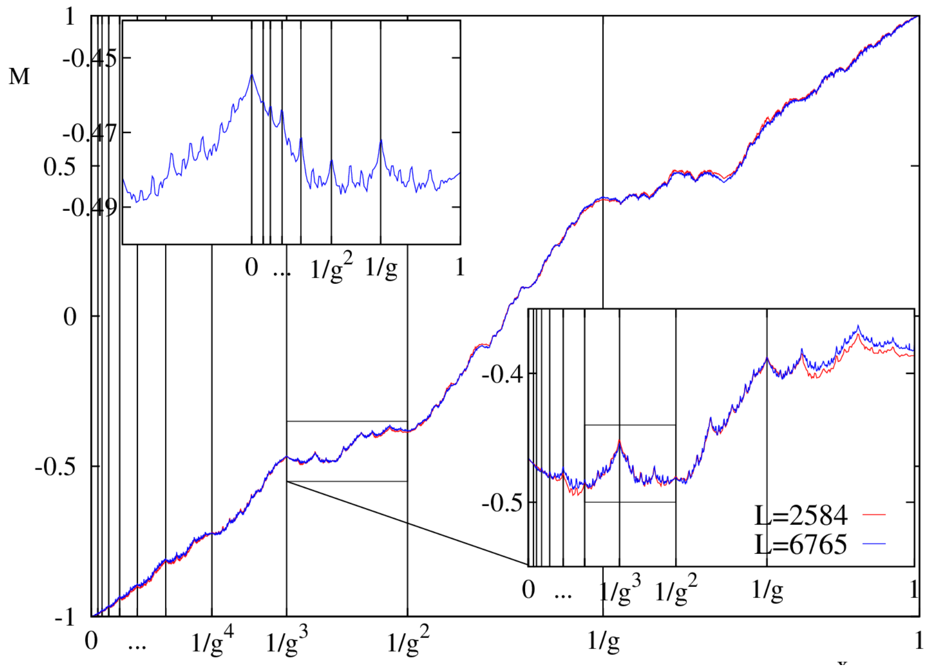
RECALL: e-function fractality ≠ e-value fractality

Fractality in nonequilibrium steady states of quasiperiodic systems (I)

Steady state magnetization

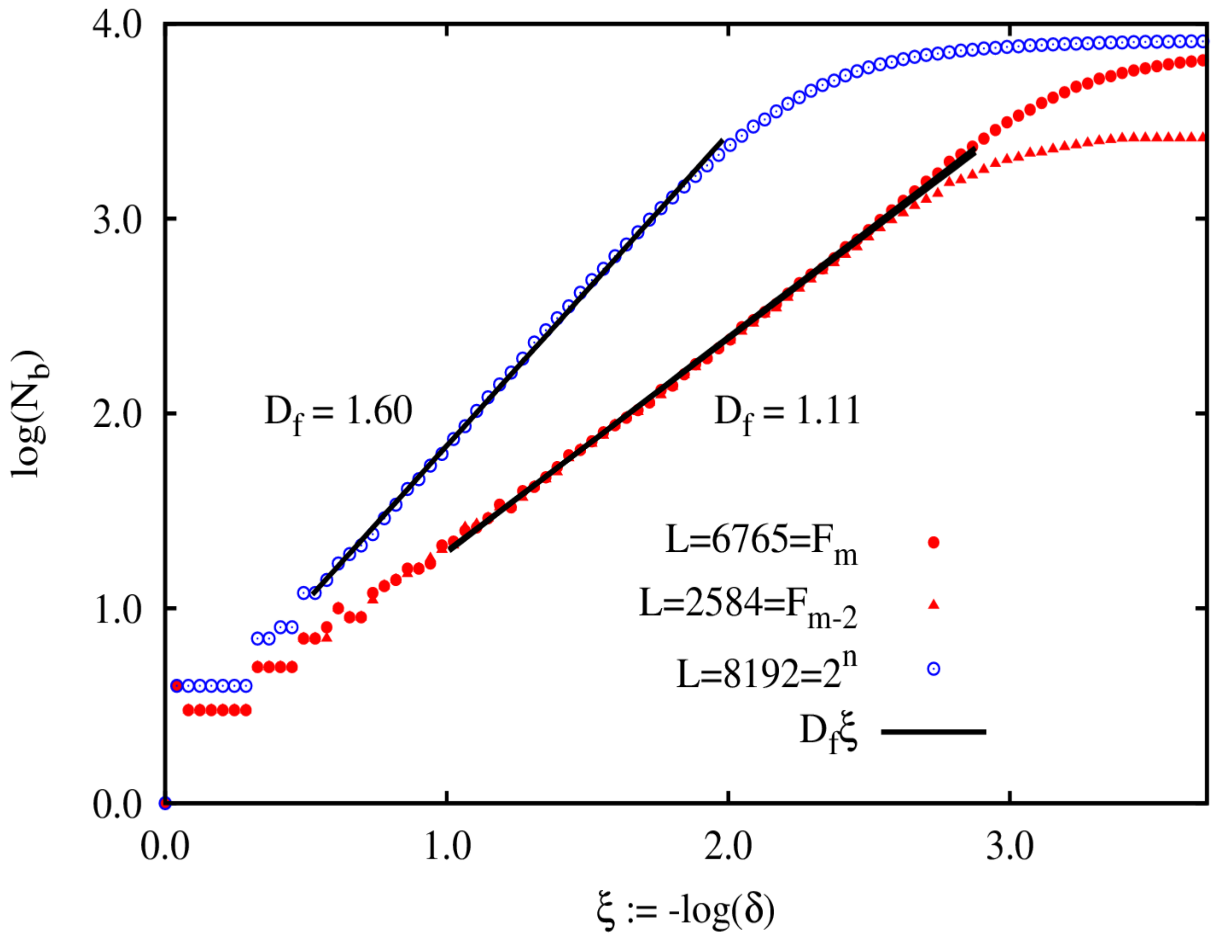


$L \neq F_n$



$L = F_n$

Fractality in nonequilibrium steady states of quasiperiodic systems (II)



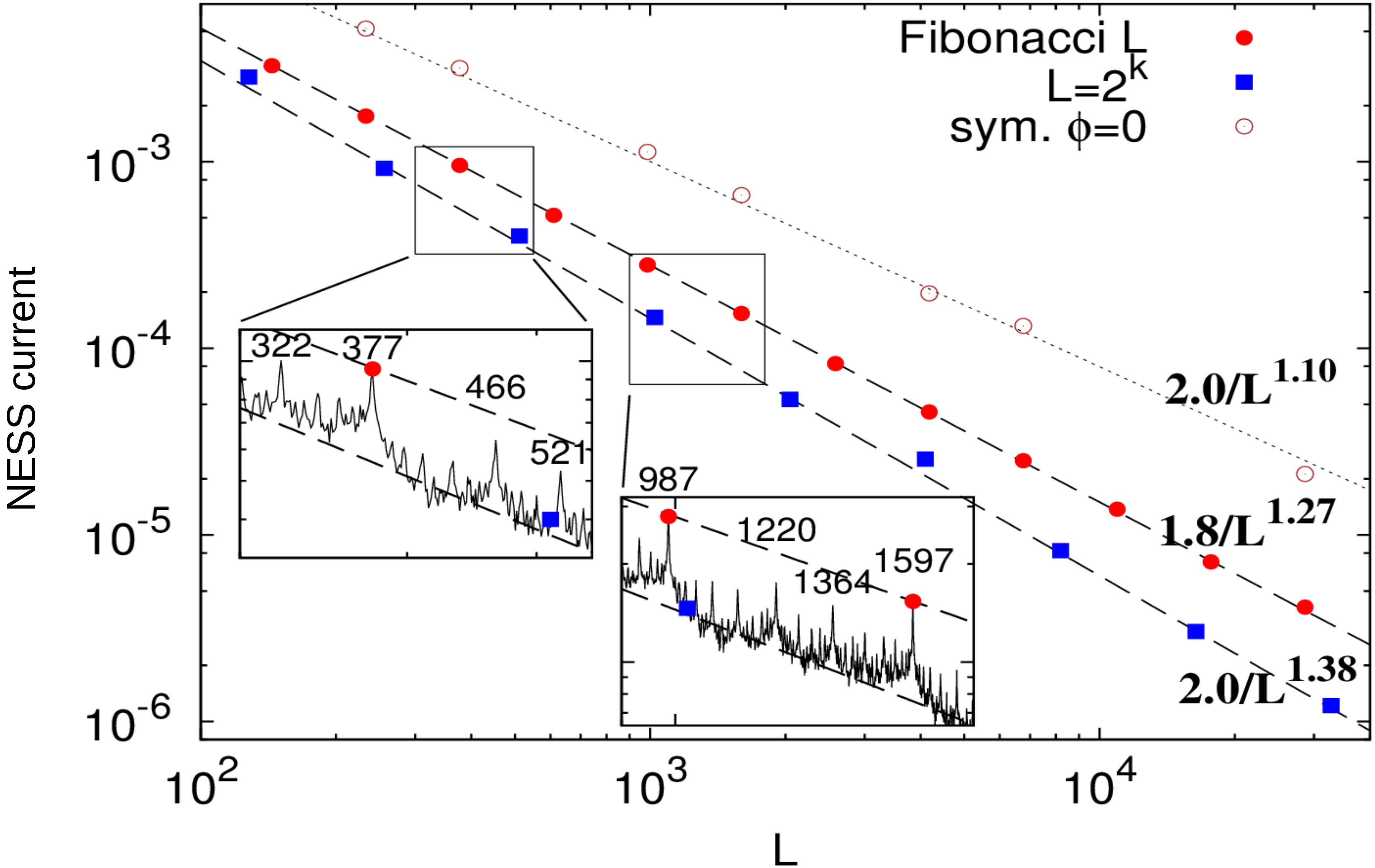
\# boxes

$$D_\mathrm{f} = -\frac{\log N_\mathrm{b}}{\log \delta}$$

box size

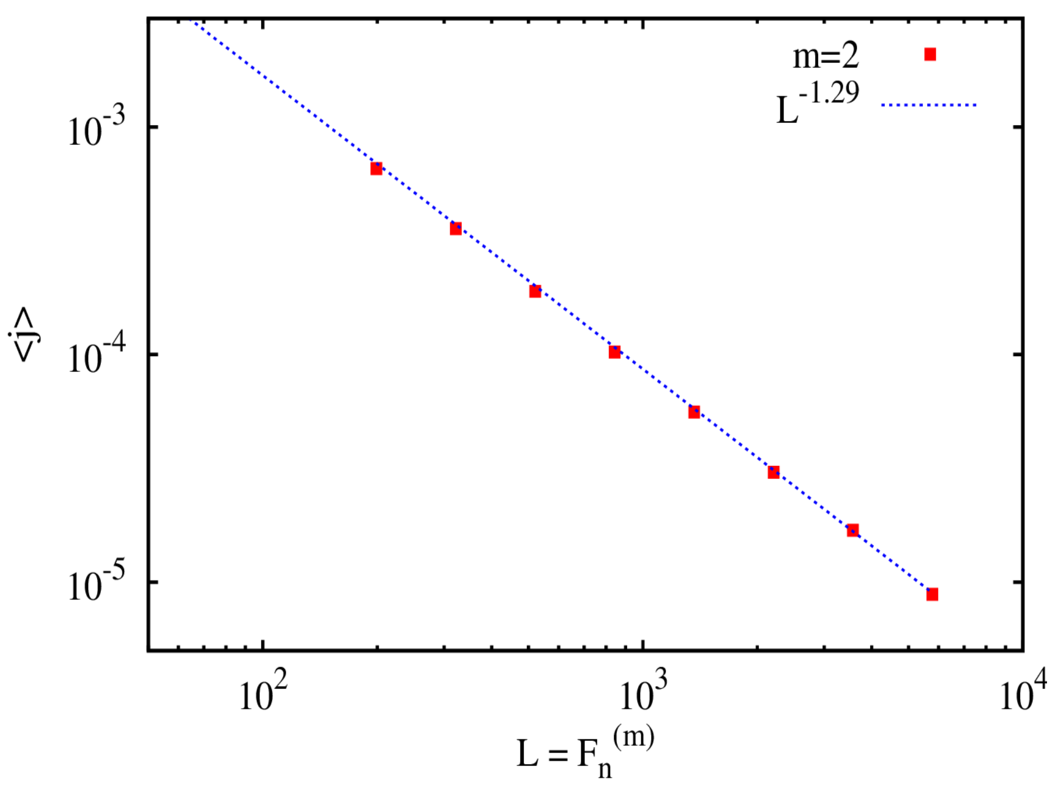
Box counting: fractal dimension

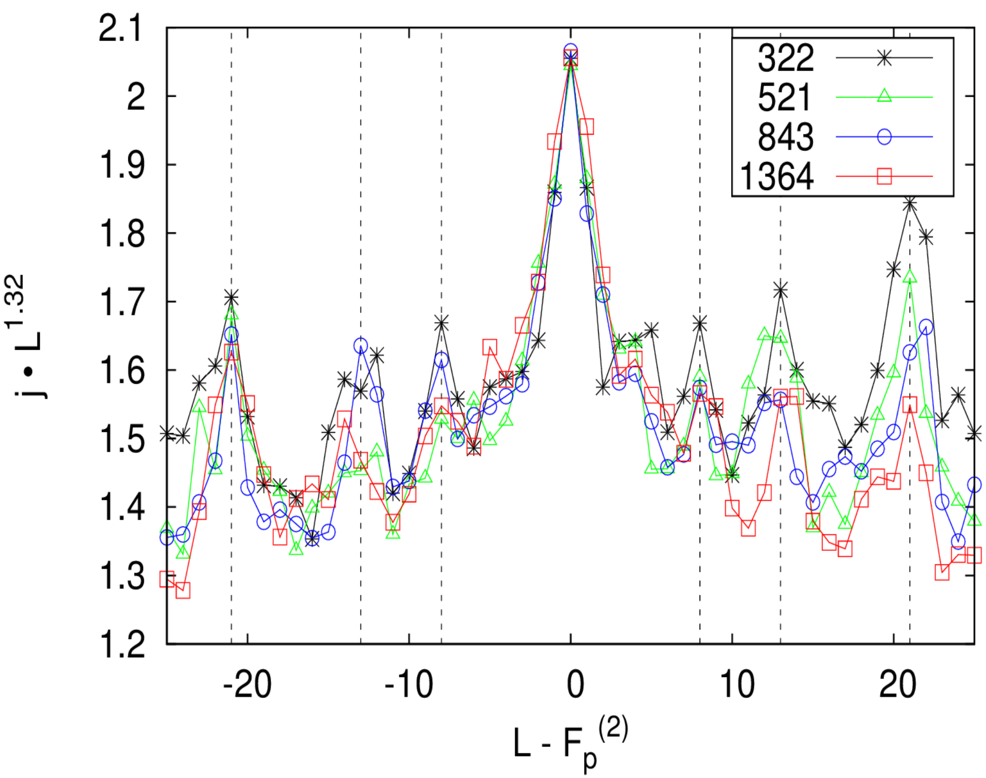
Open system dynamics (I)



Open system dynamics (II)

- Secondary resonances $F_n^{(m)} = F_{n-1} + F_{n-m-1}$

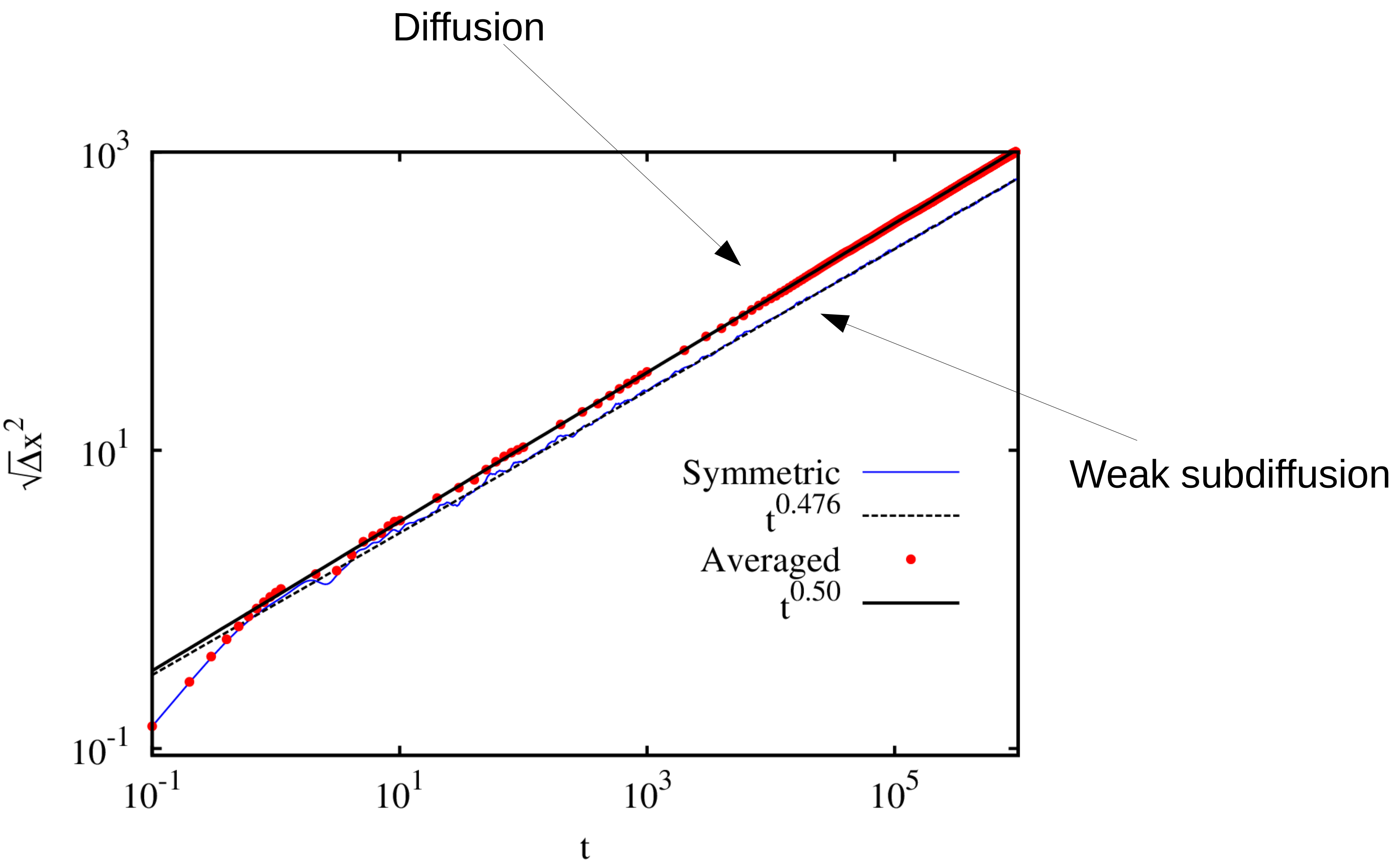




Closed system dynamics (I)

- Initialize wavepacket at chain centre
- Unitarily evolve: $|\psi(t)\rangle = \exp(-\mathrm{i}tH)|\psi(0)\rangle$
- Measure its spread: $\Delta x^2(t) = \left[x - \frac{L}{2}\right]^2 |\langle x|\psi(t)\rangle|^2$

Closed system dynamics (II)



Conclusions

- Anomalous transport in Lindblad-driven
  - Interacting (+disordered) systems
  - Noninteracting (+quasidisordered) systems
- Nonequilibrium fractality