



Open quantum systems, August 3, 2017

Non-Markovian dynamics of a qubit due to single-photon scattering in a waveguide

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pics: Palermo Cathedral (top left) vs Duke Chapel (bottom right)

joint work with



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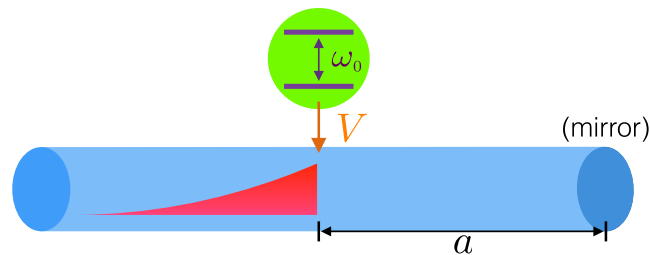


Harold U. Baranger

Duke
UNIVERSITY

in this talk

open dynamics of a qubit undergoing
single-photon scattering in a waveguide



how does such open dynamics look like?

how **non-Markovian** (NM) is it?

where does NM behavior stem from?

outline

1. why are these questions important

3 slides

2. review of theoretical tools to answer these (non-Markovianity measures)

12 slides

H.-P. Breuer, E.-M. Laine, J. Piilo, and B. Vacchini, Rev. Mod. Phys. 88, 021002 (2016)
I. de Vega and D. Alonso, Rev. Mod. Phys. 89, 015001 (2017)

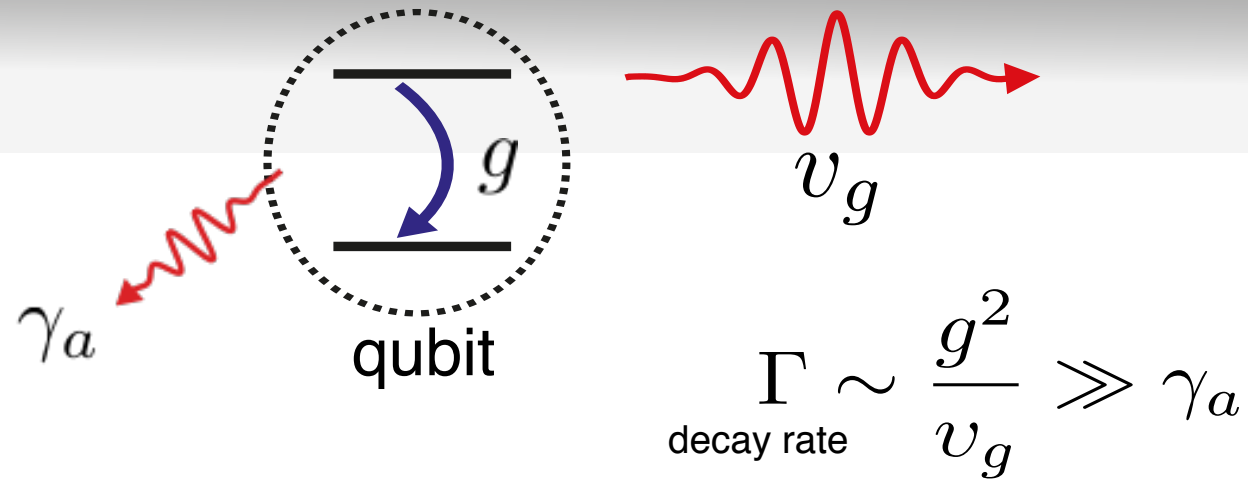
3. what we did, outcomes & interpretations

Y.-L. L. Fang, F. Ciccarello H. U. Baranger, arXiv:1707.05946

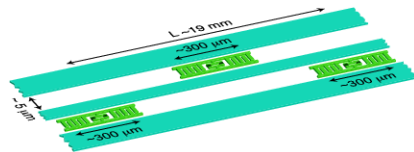
7 slides

waveguide QED

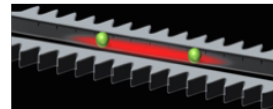
1D waveguide



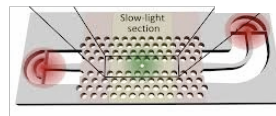
circuit QED



photonic crystal waveguides

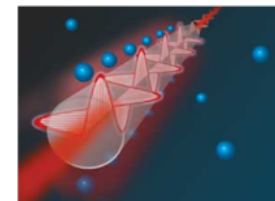


(J. Kimble)



(P. Lodahl)

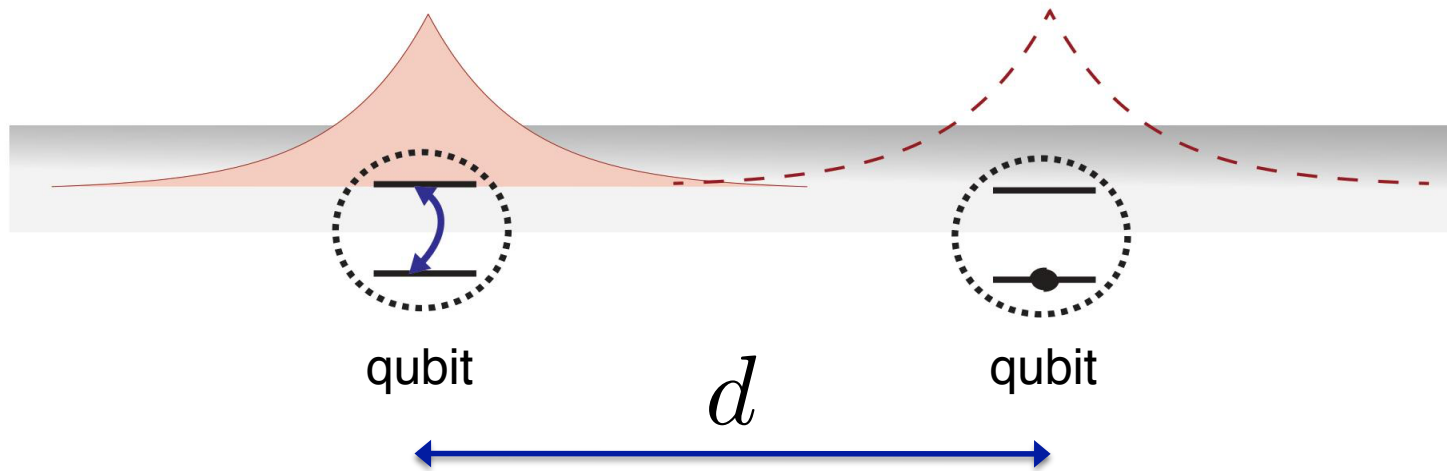
nanofibers



(A. Rauschenbeutel)

- P. Lodahl, S. Mahmoodian, and S. Stobbe, Rev. Mod. Phys. 87, 347 (2015).
Z. Liao, X. Zeng, H. Nha, and M S.Zubairy, Physica Scripta 91, 063004 (2016).
D. Roy, C. M. Wilson, and O. Firstenberg, Rev. Mod. Phys. 89, 021001 (2017).
X. Gu, A. Frisk Kockum, A. Miranowicz, Y.-X. Liu, and F. Nori, arXiv: 1707.02046v1.

waveguide QED



delay time

$$\tau \sim \frac{d}{v}$$

negligible time delays
($\Gamma\tau \ll 1$)



standard
regime

non-negligible time delays



largely
unexplored!

N. Roch et al., PRL 112, 170501 (2014)
M. V. Gustafsson et al., Science 346, 207 (2014)

effects of time delays

entanglement generation via
spontaneous emission spoiled

C Gonzalez-Ballester, F J Garcia-Vidal, and E. Moreno, New J. Phys. 15, 073015 (2013)
T. Ramos, B. Vermersch, P. Hauke, H. Pichler, and P. Zoller, Phys. Rev. A 93, 062104 (2016)

photon-photon correlations in
scattering processes deeply affected

H. Zheng and H. U. Baranger, Phys. Rev. Lett. 110, 113601 (2013)
M. Laakso and M. Pletyukhov, Phys. Rev. Lett. 113, 183601 (2014)
Y.-L. L. Fang and H. U. Baranger, Phys. Rev. A 91, 053845 (2015)
Tao Shi, D. E Chang, and J I. Cirac, Phys. Rev. A 92, 053834 (2015).

Lindblad master equation no longer holds

G. Calajò, F. Ciccarello, D. Chang, and P. Rabl, Phys. Rev. A 93, 033833 (2016)

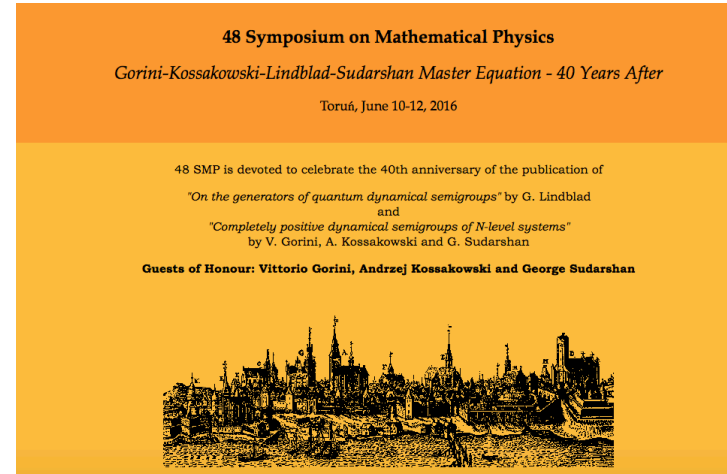
our case study



infinite-waveguide case: expected to be Markovian

semi-infinite-waveguide case: mirror $\Rightarrow$ delay time $\Rightarrow$ expected to be NM

what does “non-Markovian” mean?



Lindblad master equation (ME)

$$\dot{\rho} = -i[\hat{H}, \rho] + \sum_{\nu} \gamma_{\nu} \left(\hat{L}_{\nu} \rho \hat{L}_{\nu}^{\dagger} - \frac{1}{2} [\hat{L}_{\nu}^{\dagger} \hat{L}_{\nu}, \rho]_{+} \right) \quad \text{time-local!}$$
$$\gamma_{\nu} > 0$$

instance: 2-level-atom spontaneous emission

$$\dot{\rho} = -i [\omega_0 \hat{\sigma}_+ \hat{\sigma}_-, \rho] + \gamma \left(\hat{\sigma}_- \rho \hat{\sigma}_+ - \frac{1}{2} [\hat{\sigma}_+ \hat{\sigma}_-, \rho]_{+} \right)$$

Gorini, V., Kossakowski, A., Sudarshan, E.C.G., J. Math. Phys. 17, 821 (1976)

Lindblad G., Commun. Math. Phys. 48, 119 (1976)

what does NM mean?

...possible answers

- Lindblad ME does not hold
- the open system obeys a time-non-local ME

classical stochastic process

Markovian process

$$P_{1|n}(x_{n+1}, t_{n+1} | x_n, t_n; \dots; x_1, t_1) \equiv P_{1|1}(x_{n+1}, t_{n+1} | x_n, t_n)$$

....quantum version: very non-trivial!

quantum maps

superoperator

$\mathcal{S} : \text{operator} \rightarrow \text{operator}$

e.g: $\mathcal{H}(\rho) = -i[\hat{H}, \rho]$

$$\mathcal{L}(\rho) = -i[\hat{H}, \rho] + \sum_{\nu} \gamma_{\nu} \left(\hat{L}_{\nu} \rho \hat{L}_{\nu}^{\dagger} - \frac{1}{2} [\hat{L}_{\nu}^{\dagger} \hat{L}_{\nu}, \rho]_{+} \right)$$

so: $\dot{\rho} = \mathcal{L}(\rho) \rightarrow \rho(t) = e^{\mathcal{L}t}(\rho_0)$

quantum map

$$\rho' = \mathcal{M}(\rho)$$

P map

$$[\mathcal{M}(\rho)]^{\dagger} = \mathcal{M}(\rho) \quad \mathcal{M}(\rho) > 0 \quad \text{Tr} \{ \mathcal{M}(\rho) \} = 1$$

CP map

$$\mathcal{M} \otimes \mathbb{I}_n \quad \forall n \quad \text{is PT}$$

physical
operations

dynamical map



$$\rho(t) = \text{Tr}_R \left\{ \hat{U}_{SR,t} \rho_0 \otimes \rho_R(0) \hat{U}_{SR,t}^\dagger \right\}$$

dynamical map: $\Phi_t(\rho) = \text{Tr}_R \left\{ \hat{U}_{SR,t} \rho \otimes \rho_R(0) \hat{U}_{SR,t}^\dagger \right\}$

$$\rho(t) = \Phi_t(\rho_0)$$

S closed: $\Phi_t(\rho_0) \equiv \hat{U}_t \rho_0 \hat{U}_t^\dagger$

S open: $\Phi_t(\rho_0) = \sum_{\nu} \hat{K}_{\nu,t} \rho_0 \hat{K}_{\nu,t}^\dagger$

$$\Phi_t \text{ depends on } \begin{cases} \hat{H}_{SR} \\ \rho_R(0) \end{cases}$$

**CP map for
any t !**

when does Lindblad ME hold?

semigroup property

class of dynamical maps:

$$\Phi_t = \Phi_{t-t_1} \Phi_{t_1}$$

$$0 \leq t_1 \leq t$$

Lindbladian

$$\mathcal{L}(\rho) = -i[\hat{H}, \rho] + \sum_{\nu} \gamma_{\nu} \left(\hat{L}_{\nu} \rho \hat{L}_{\nu}^{\dagger} - \frac{1}{2} [\hat{L}_{\nu}^{\dagger} \hat{L}_{\nu}, \rho]_{+} \right)$$

Lindblad ME

$$\Phi_t = e^{\mathcal{L}t}$$



$$\dot{\rho} = \mathcal{L}(\rho)$$

note: rates γ_{ν} are positive!

CP-divisibility

inverse dynamical map: Φ_t^{-1} **may not be CP!!!**

if Φ_t^{-1} **exists** at any time:

two-time map

$$\Phi_{t,t_1} = \Phi_t \Phi_{t_1}^{-1}$$

($t \geq t_1 \geq 0$)

**may not
be CP!!!**

$$\Phi_{t,0} = \Phi_t$$

$$\Phi_{t,0} = \Phi_{t,t_1} \Phi_{t_1,0}$$

if Φ_{t,t_1} is CP: **CP-divisible** dynamics

CP-divisibility and Master Equation

$$\dot{\rho} = -i[\hat{H}(t), \rho] + \sum_{\nu} \gamma_{\nu}(t) \left(\hat{L}_{\nu}(t) \rho \hat{L}_{\nu}^{\dagger}(t) - \frac{1}{2} [\hat{L}_{\nu}^{\dagger}(t) \hat{L}_{\nu}(t), \rho]_{+} \right)$$

$$\text{CP-divisibility} : \quad \gamma_{\nu}(t) \geq 0 \quad \forall \nu$$

non-Markovianity witness

at least one rate becomes negative at some time

Rivas, A., S. F. Huelga, and M. B. Plenio, 2010, Phys. Rev. Lett. 105, 050403

Hall, M. J. W., J. D. Cresser, L. Li, and E. Andersson, 2014, Phys. Rev. A 89, 042120.

instances of CP-divisible non-Lindbladian dynamics

an atom emitting into a lossy cavity

E. Laine et al., PRL 103, 210401 (2009)

an atom emitting in front of a mirror in 1D

T. Tufarelli et al., PRA 90, 012113 (2014)

spin-boson model for some parameter ranges

G. Klos et al., PRA 86, 012115 (2012)

a qubit coupled to a spin chain at a QPT point

T. J. G. Apollaro et al., PRA 83, 032103 (2011)

..and many others

recent reviews:

Á. Rivas, S. F. Huelga, and M. B. Plenio, Rep. Prog. Phys. 77, 094001 (2014)

H.-P. Breuer, E.-M. Laine, J. Piilo, and B. Vacchini, Rev. Mod. Phys. 88, 021002 (2016)

I. de Vega and D. Alonso, Rev. Mod. Phys. 89, 015001 (2017)

spontaneous emission of an atom

model: atom RWA-coupled to a bosonic bath

$$\hat{H} = \omega_0 |e\rangle\langle e| + \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_k g_k (\hat{\sigma}_+ \hat{a}_k + \text{H.c.})$$

$$e^{-i\hat{H}t} |g\rangle |\text{vac}\rangle = |g\rangle |\text{vac}\rangle$$

$$e^{-i\hat{H}t} |e\rangle |\text{vac}\rangle = \varepsilon(t) |e\rangle |\text{vac}\rangle + \sum_k \beta_k(t) |g\rangle |\varphi_k\rangle$$

dynamical map

$$\Phi_t : \rho(0) = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix} \rightarrow \rho(t) = \begin{pmatrix} |\varepsilon(t)|^2 \rho_{ee} & \varepsilon(t) \rho_{eg} \\ \varepsilon^*(t) \rho_{ge} & \rho_{gg} + (1 - |\varepsilon(t)|^2) \rho_{ee} \end{pmatrix}$$

master equation

$$\dot{\rho} = -i \frac{S(t)}{2} [\hat{\sigma}_+ \hat{\sigma}_-, \rho] + \gamma(t) \left(\hat{\sigma}_- \rho \hat{\sigma}_+ - \frac{1}{2} [\hat{\sigma}_+ \hat{\sigma}_-, \rho]_+ \right)$$

$$S(t) = -2\text{Im} \left[\frac{\dot{\varepsilon}(t)}{\varepsilon(t)} \right] \quad \gamma(t) = -2\text{Re} \left[\frac{\dot{\varepsilon}(t)}{\varepsilon(t)} \right] \quad \boxed{\gamma(t) < 0 \Leftrightarrow \frac{d}{dt} |\varepsilon(t)|^2}$$

spontaneous emission of an atom

excitation amplitude's integro-differential equation

$$\dot{\varepsilon} = -\omega_0 \varepsilon(t) + \int_0^t dt' \sum_k |g_k|^2 e^{-i\omega_k(t-t')} \varepsilon(t')$$

$$J(\omega) = \rho(\omega) [g(\omega)]^2 \quad \text{spectral density}$$

instance: lossy cavity

$$J(\omega) = \frac{\Gamma_0}{2\pi} \frac{\lambda^2}{(\omega - \omega_0)^2 + \lambda^2} \quad \begin{array}{l} \Gamma_0 \text{ atom-cavity coupling rate} \\ \lambda \text{ cavity bandwidth} \end{array}$$

$$\varepsilon(t) = e^{-\frac{\lambda}{2}t} \left[\cosh \left(\frac{\sqrt{\lambda^2 - 2\gamma_0\lambda}}{2} t \right) + \frac{\lambda}{d} \sinh \left(\frac{\sqrt{\lambda^2 - 2\gamma_0\lambda}}{2} t \right) \right]$$

$$\Gamma_0 < \frac{\lambda}{2} \quad \text{monotonic non-exponential decay}$$

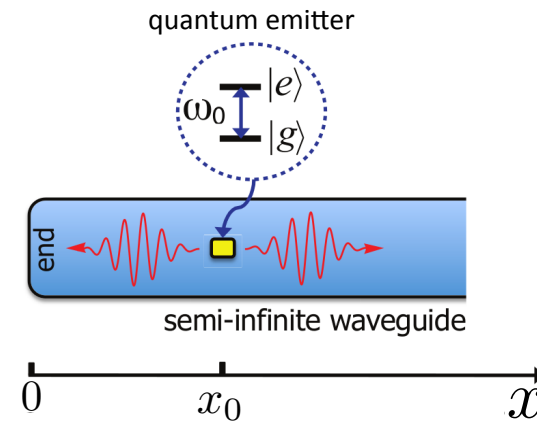
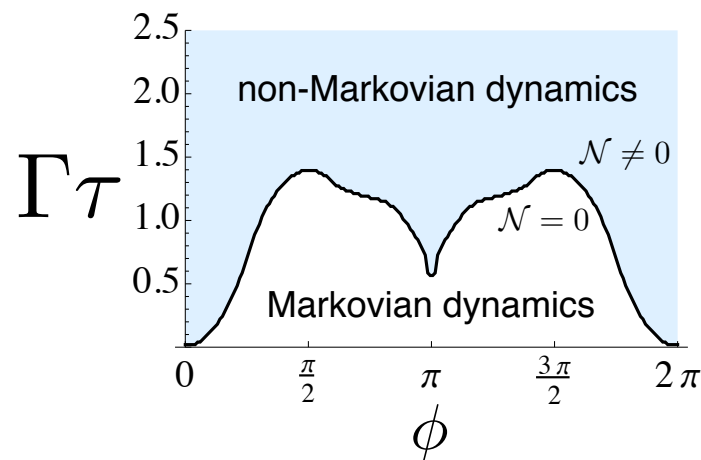
$$\Gamma_0 > \frac{\lambda}{2} \quad \text{damped oscillations}$$

spontaneous emission of an atom

atom in a semi-infinite waveguide

$$J(\omega) = \frac{\Gamma}{\pi} \sin^2 \left(\frac{\tau}{2} \omega \right)$$

$$\dot{\varepsilon}(t) = -\frac{\Gamma}{2} \varepsilon(t) + \frac{\Gamma}{2} e^{i\phi} \varepsilon(t - \tau) \theta(t - \tau)$$



T. Tufarelli, F. Ciccarello, and M. S. Kim, Phys. Rev. A 87, 013820 (2013)
T. Tufarelli, M. S. Kim, and F. Ciccarello, Phys. Rev. A 90, 012113 (2014)

a qubit undergoing single-photon scattering



$$\hat{H} = -i \int_{-\infty}^{\xi} dx \left[\hat{a}_R^\dagger(x) \partial_x \hat{a}_R(x) - \hat{a}_L^\dagger(x) \partial_x \hat{a}_L(x) \right] + \omega_0 \hat{\sigma}_+ \hat{\sigma}_- + V \int_{-\infty}^{\xi} dx \delta(x - x_0) \left[\left(\hat{a}_R^\dagger(x) + \hat{a}_L^\dagger(x) \right) \hat{\sigma}_- + \text{h.c.} \right]$$

qubit's dynamical map

$$e^{-i\hat{H}t} |g\rangle |\varphi\rangle = |g\rangle |\phi_1(t)\rangle + e(t) |e\rangle |0\rangle \quad \text{one excitation}$$

$$e^{-i\hat{H}t} |e\rangle |\varphi\rangle = |g\rangle |\chi_2(t)\rangle + |e\rangle |\psi_1(t)\rangle \quad \text{two excitations}$$

a qubit undergoing single-photon scattering



$$\hat{H} = -i \int_{-\infty}^{\xi} dx \left[\hat{a}_R^\dagger(x) \partial_x \hat{a}_R(x) - \hat{a}_L^\dagger(x) \partial_x \hat{a}_L(x) \right] + \omega_0 \hat{\sigma}_+ \hat{\sigma}_- + V \int_{-\infty}^{\xi} dx \delta(x - x_0) \left[\left(\hat{a}_R^\dagger(x) + \hat{a}_L^\dagger(x) \right) \hat{\sigma}_- + \text{h.c.} \right]$$

qubit's dynamical map

$$e^{-i\hat{H}t} |g\rangle |\varphi\rangle = |g\rangle |\phi_1(t)\rangle + e(t) |e\rangle |0\rangle \quad \text{one excitation}$$

$$e^{-i\hat{H}t} |e\rangle |\varphi\rangle = |g\rangle |\chi_2(t)\rangle + |e\rangle |\psi_1(t)\rangle \quad \text{two excitations}$$

$$\Phi_t : \rho(0) = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix} \rightarrow \rho(t) = \begin{pmatrix} p_e(t) - \Delta(t) \rho_{gg} & c(t) \rho_{eg} \\ c^*(t) \rho_{eg}^* & 1 - p_e(t) + \Delta(t) \rho_{gg} \end{pmatrix}$$

$$p_g(t) = |e(t)|^2, \quad p_e(t) = \|\psi_1(t)\|^2, \quad c(t) = \langle \phi_1(t) | \psi_1(t) \rangle$$

$$\Delta(t) = p_e(t) - p_g(t)$$

dynamical map

Bloch sphere picture

$$\mathbf{r} = (2 \operatorname{Re} \rho_{ge}, 2 \operatorname{Im} \rho_{ge}, \rho_{gg} - \rho_{ee})$$

$$\mathbf{r}(t) = \mathbf{M}_t \mathbf{r}_0 + \mathbf{v}(t)$$

$$\mathbf{M}_t = \begin{pmatrix} |c(t)| \mathbf{R}_{\theta(t)} & \mathbf{0}^T \\ \mathbf{0} & \Delta(t) \end{pmatrix} \quad \begin{aligned} \mathbf{v}(t) &= (0, 0, 1 - p_e(t) - p_g(t))^T \\ \theta(t) &= \arg[c(t)] \end{aligned}$$

for spontaneous emission:

$$\mathbf{M}_t = \begin{pmatrix} |\varepsilon(t)| \mathbf{R}_{\theta(t)} & \mathbf{0}^T \\ \mathbf{0} & |\varepsilon(t)|^2 \end{pmatrix} \quad \theta(t) = \arg[\varepsilon(t)]$$

qubit master equation

$$\dot{\rho} = -i[\hat{H}(t), \rho] + \gamma_{-}(t)\mathcal{L}_{-}(\rho) + \gamma_{+}(t)\mathcal{L}_{+}(\rho) + \gamma_z(t)\mathcal{L}_z(\rho)$$

$$\begin{aligned}\hat{H}(t) &= -2\text{Im}[\dot{c}/c]\hat{\sigma}_{+}\hat{\sigma}_{-} \\ \mathcal{L}_{\nu}(\rho) &= \hat{L}_{\nu}\rho\hat{L}_{\nu}^{\dagger} - \frac{1}{2}[\hat{L}_{\nu}^{\dagger}\hat{L}_{\nu}, \rho]_{+} \quad \hat{L}_{\pm} = \hat{\sigma}_{\pm} \quad \hat{L}_z = \hat{\sigma}_z\end{aligned}$$

time-dependent rates

$$\gamma_{+} = \frac{p_e\dot{p}_g - p_g\dot{p}_e}{\Delta} \quad \gamma_{-} = -\frac{\dot{\Delta}}{\Delta} - \gamma_{+} \quad \gamma_z = -\frac{\gamma_{+}(t) + \gamma_{-}(t)}{4} - \text{Re}\frac{\dot{c}(t)}{2c(t)}$$

$$\Delta(t) = p_e(t) - p_g(t)$$

explicit computation of DM

$$e^{-i\hat{H}t}|g\rangle|\varphi\rangle = |g\rangle|\phi_1(t)\rangle + e(t)|e\rangle|0\rangle$$

$$e^{-i\hat{H}t}|e\rangle|\varphi\rangle = |g\rangle|\chi_2(t)\rangle + |e\rangle|\psi_1(t)\rangle$$

time evolution of the two
elementary scattering processes



$$|\phi_1(t)\rangle, e(t), |\psi_1(t)\rangle$$



$$p_g(t)=|e(t)|^2, p_e(t)=\|\psi_1(t)\|^2, c(t)=\langle\phi_1(t)|\psi_1(t)\rangle$$

CP-divisibility

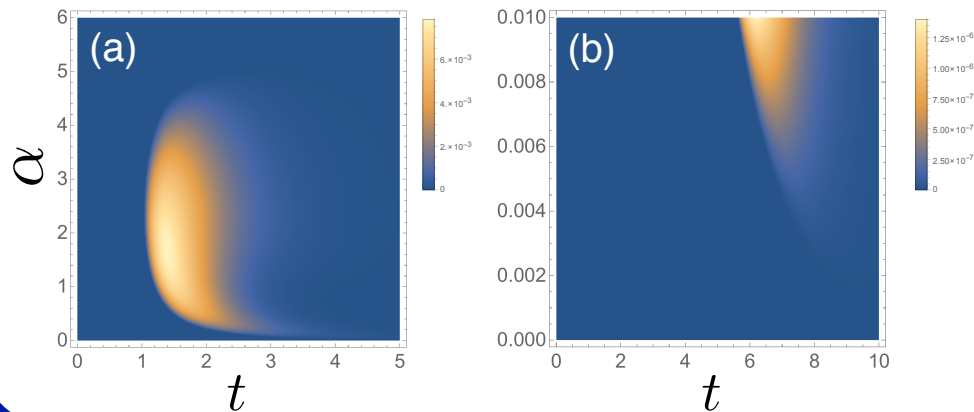
$$\gamma_+ + \gamma_- = -\frac{\dot{\Delta}}{\Delta} \quad \Delta(t) = p_e(t) - p_g(t)$$

sufficient condition for
violation of CP-divisibility:

$$\Delta(t) < 0$$

infinite waveguide

$$\Delta(t) = e^{-(\alpha+1)t} \frac{8\alpha e^{\frac{(\alpha+1)t}{2}} + (\alpha-5)(\alpha+1)e^{\alpha t} + 4(1-\alpha)}{\alpha^2 - 1} \quad \alpha = \frac{\Delta k}{\Gamma}$$



NM behavior for

$$\Delta k \sim \Gamma$$

non-Markovianity measure

$$\mathbf{r}(t) = \mathbf{M}_t \mathbf{r}_0 + \mathbf{v}(t) \quad \mathbf{M}_t = \begin{pmatrix} |c(t)| \mathbf{R}_{\theta(t)} & \mathbf{0}^T \\ \mathbf{0} & \Delta(t) \end{pmatrix}$$

$|\det \mathbf{M}_t|$ = volume of accessible states

geometric measure of non-Markovianity

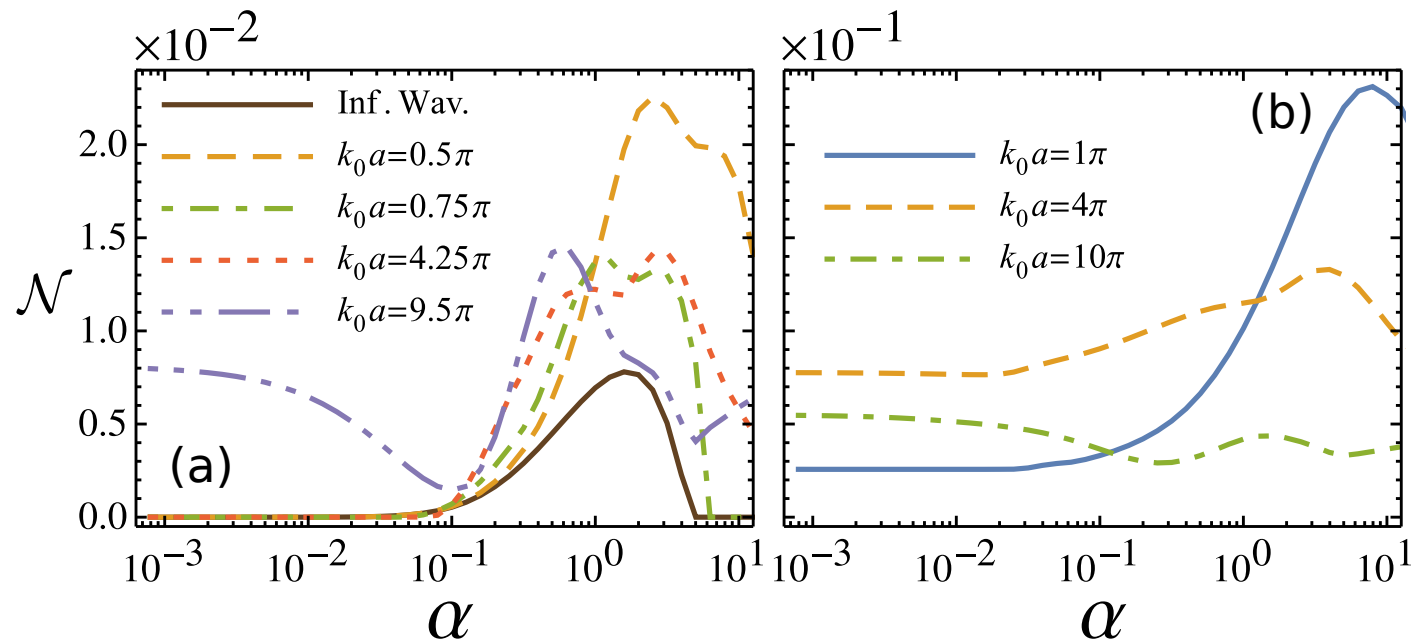
Lindbladian dynamics: $\frac{d}{dt} |\det \mathbf{M}_t| < 0$

$$\mathcal{N} = \int_{\partial_t |\det \mathbf{M}_t| > 0} dt \frac{d}{dt} |\det \mathbf{M}_t|$$

$\frac{d}{dt} |\det \mathbf{M}_t| > 0 \Rightarrow$ rates become negative

S. Lorenzo, F. Plastina, and M. Paternostro. PRA 88, 020102(R) (2013)

non-Markovianity measure



$$\alpha = \frac{\Delta k}{\Gamma}$$

Y.-L. L., Fang, F. Ciccarello H. U. Baranger, arXiv:1707.05946

conclusions

we worked out the dynamical map of a qubit
undergoing single-photon scattering in a waveguide

to this aim we worked out the scattering time evolution

qubit open dynamics has features not occurring with
spontaneous emission

we applied recently developed measures and criteria
of quantum non-Markovianity

two fundamental sources of NM behavior:
(1) wavepacket finiteness, (2) mirror feedback



thank you!