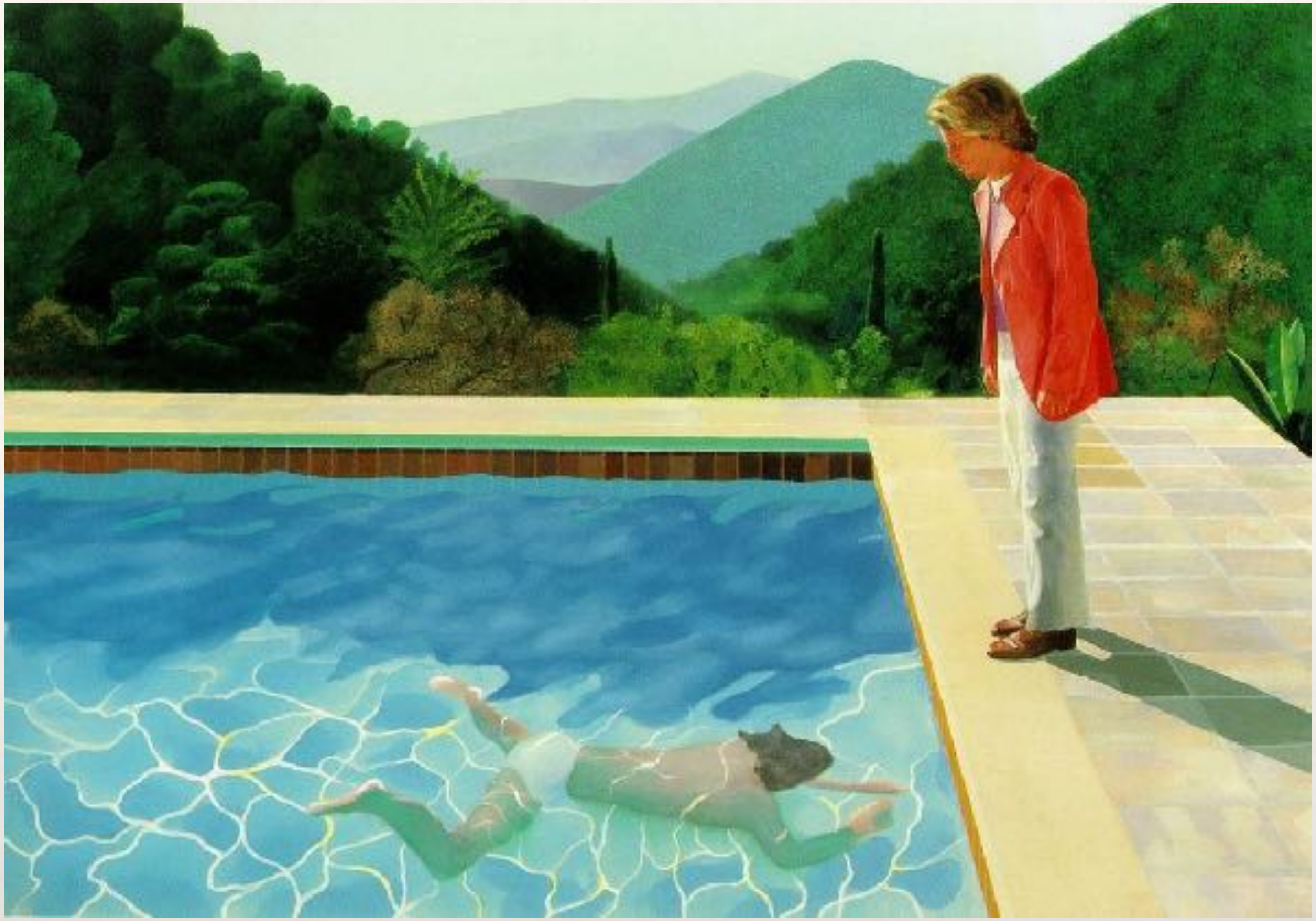
Portrait of an Artist (Pool with Two Figures).
David Hockey
(1972)



Emergence of singularities from decoherence in a Josephson junction

Aaron Goldberg, Asma Al-Qasimi and **Duncan O'Dell**

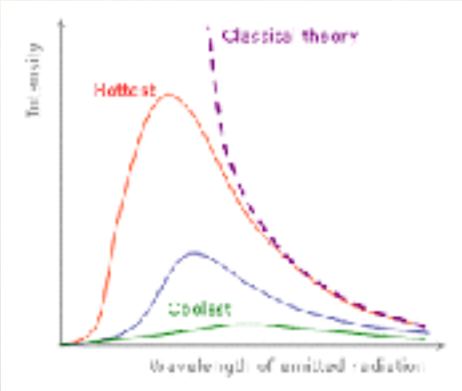
McMaster University, Canada

ICTS, Bangalore, 2 August 2017

Paradox ?

Quantum mechanics resolves many singularities of classical world:

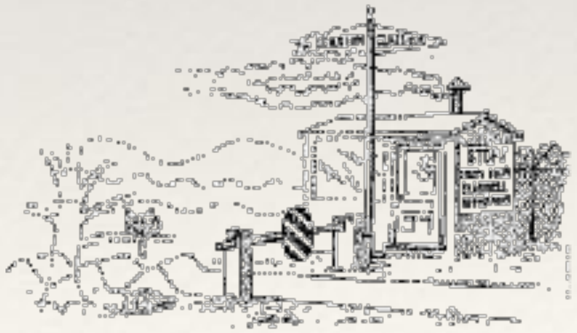
- caustics
- ultraviolet catastrophe
- black hole singularities of classical GR (+information paradox)



Decoherence returns us to a classical world:

- Zurek, Decoherence and the Transition from Quantum to Classical, Physics Today (1991)
- Brune et al, Observing the progressive decoherence of a meter in QM, Phys. Rev. Lett. (1996)
- Myatt et al, Decoherence of quantum superpositions through coupling to engineered reservoirs, Nature (2000)
- ……

Do the singularities also return?



The border between quantum and classical.

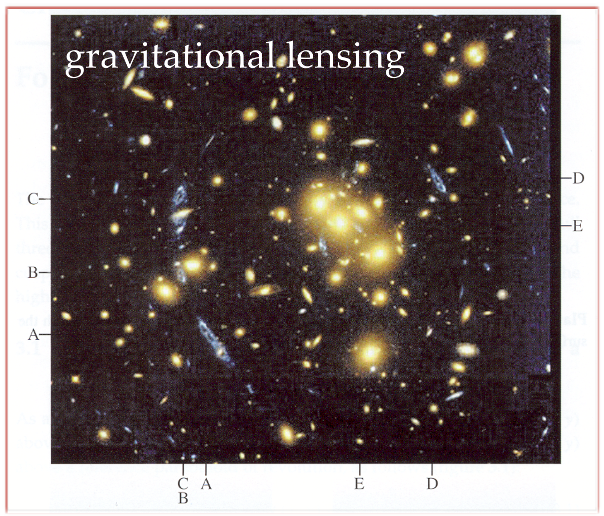
Outline

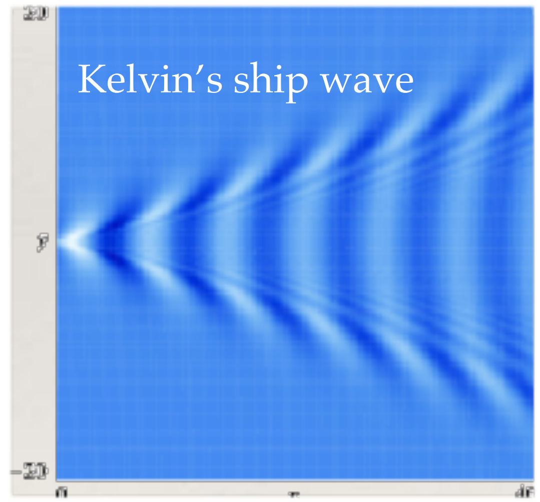
1. Ray singularities: caustics & catastrophes

2. Wave singularities: quantum catastrophes in Bose-Einstein condensates

3. Effect of decoherence on a quantum catastrophe: continuous weak measurement

Natural focusing: Caustics

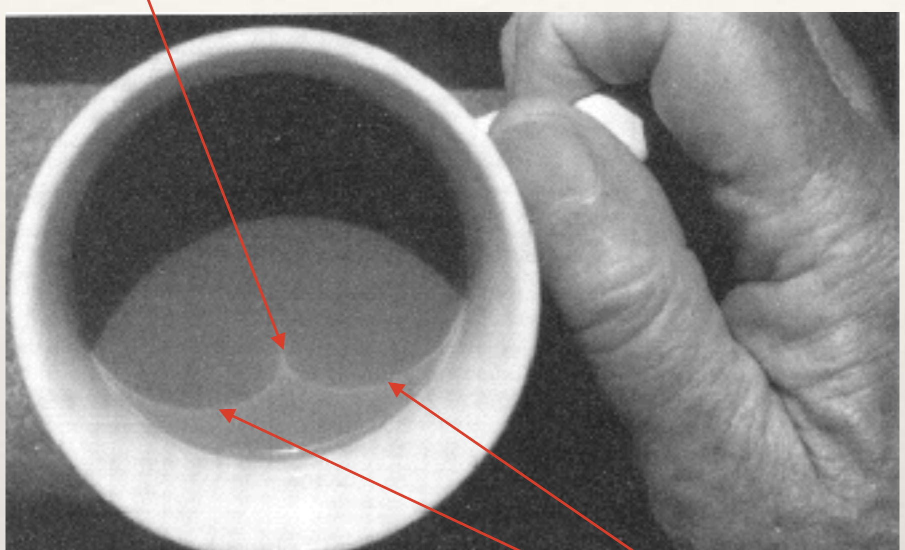




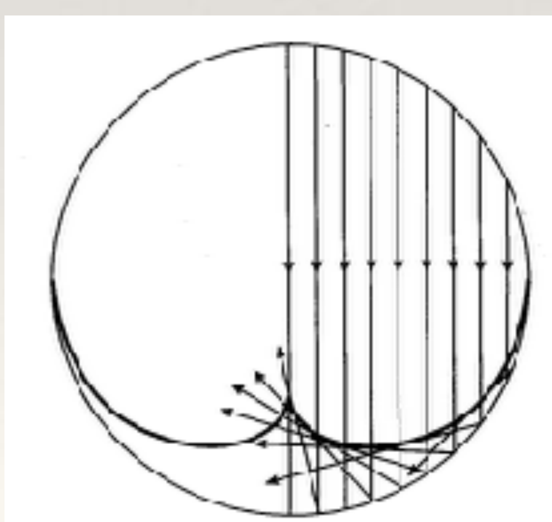
+twinkling of starlight (Berry 1977), wave packet dynamics at the Anderson transition (Lemarie 2010),…..all are singular in the ray theory

Cusp caustic

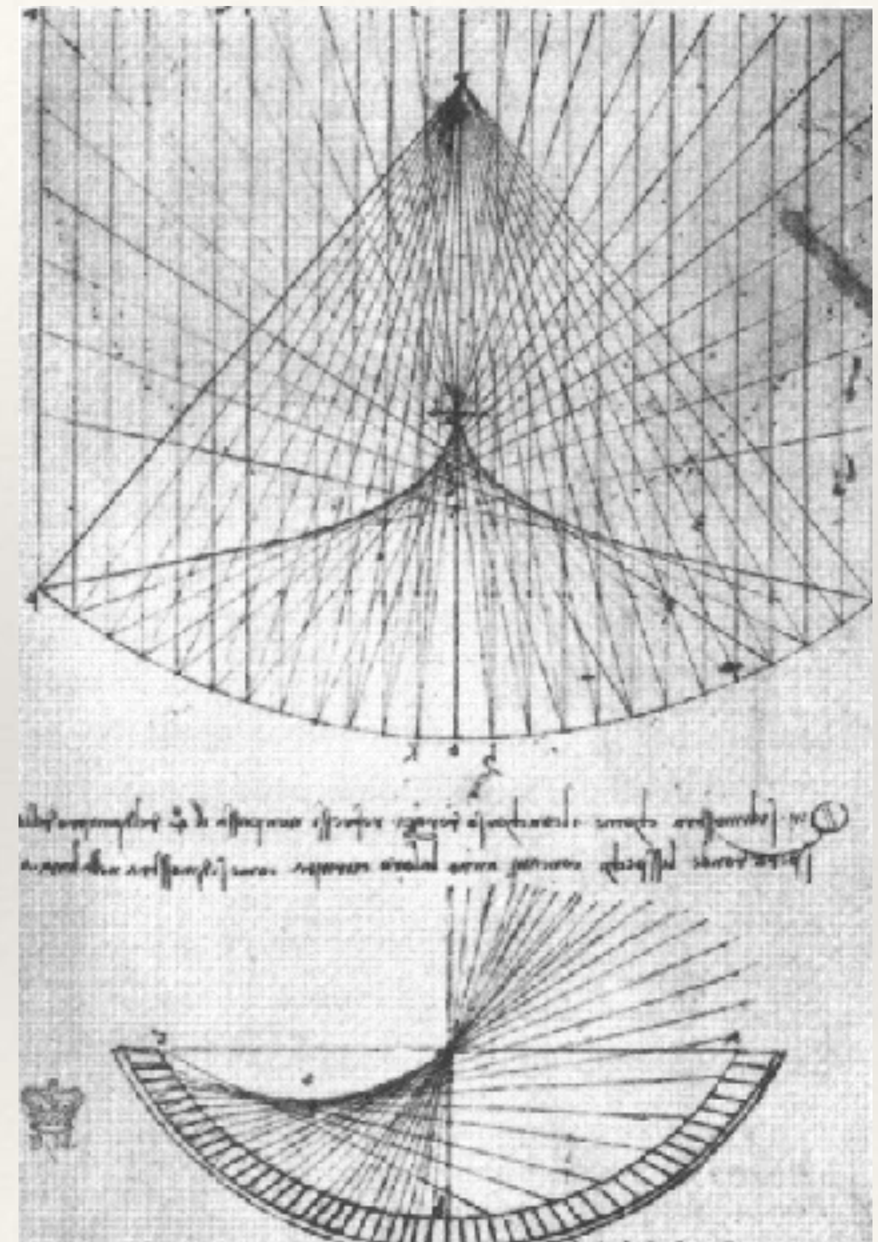
cusp point



fold lines

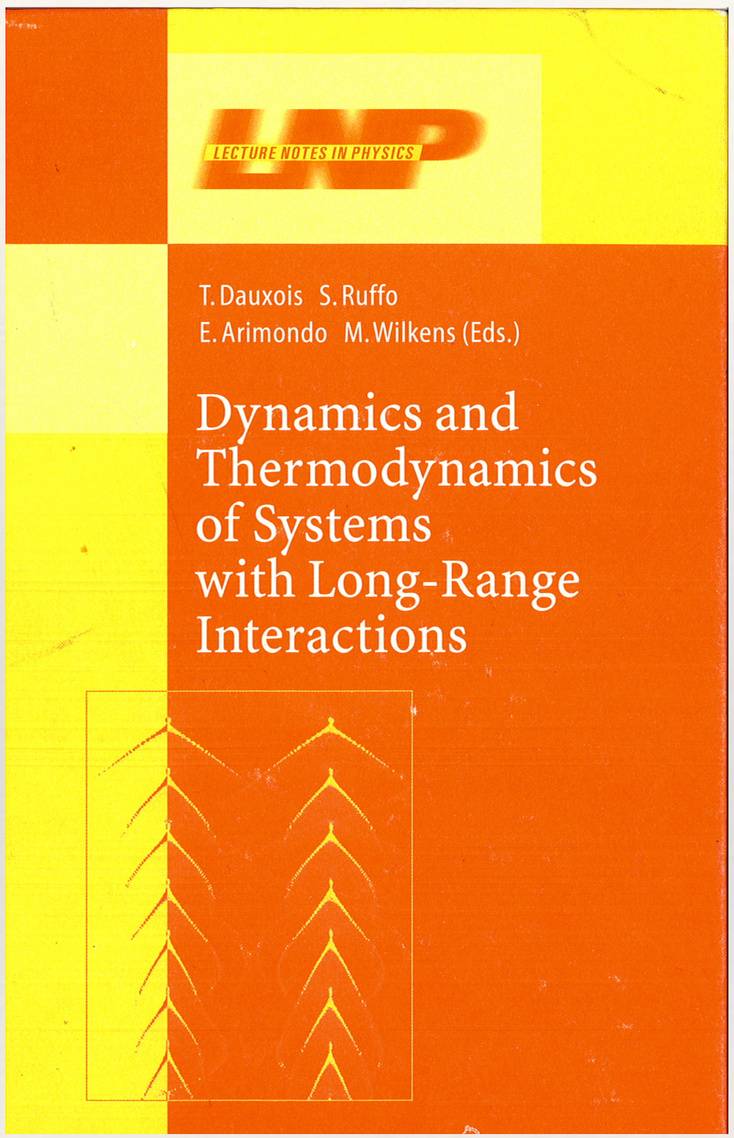


- Structurally stable
- Generic
- Singular…light intensity diverges (in geometric theory)

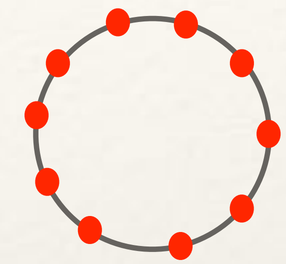


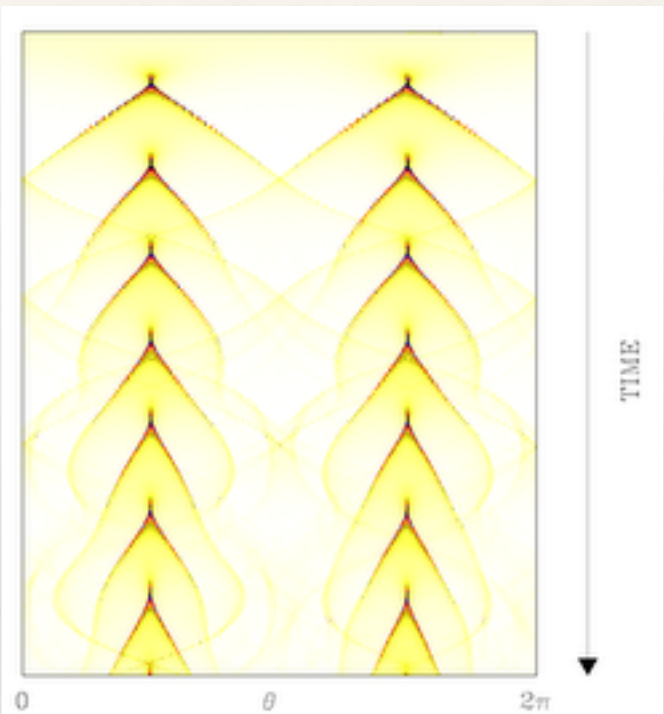
Leonardo de Vinci c. 1508

Dynamics of N particles on a ring



$$\sum_i \frac{p_i^2}{2} + \frac{\epsilon}{2N}\sum_{i,j}[1-\cos(\theta_i-\theta_j)]$$





Particle density as a function of time. Initial density on ring at *t=0* is uniform. Interaction is *repulsive.*

Julien Barre´, Freddy Bouchet, Thierry Dauxois, and Stefano Ruffo, Phys. Rev. Lett. **89**, 110 (2002). **T. Dauxois**, **V. Latora**, **A. Rapisarda**, **S. Ruffo**, **A. Torcini**, "The Hamiltonian Mean Field Model: from Dynamics to Statistical Mechanics and back" in: **T. Dauxois**, **S. Ruffo**, **E. Arimondo**, **M. Wilkens** (Eds.), Dynamics and Thermodynamics of Systems with Long-Range Interactions, Lecture Notes in Physics 602, Springer (2002).

Gravity-driven catastrophes

The Large Scale Structure of the Universe I. General Properties. One- and Two-Dimensional Models

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and

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(*Received August* 11, 1981)

Evolution of initially smooth perturbations in a cold self-gravitating medium in a Friedmann Universe gives rise to the formation of singularities in the distribution of density in a manner similar to that of catastrophe theory. Using the approximate nonlinear theory of gravitational instability the objects formed were found to possess a very oblate shape—a "pancake" (Zeldovich, 1970). This result is shown in this paper to be a general feature of the evolution of so-called Lagrangian systems (Arnold, 1980). Pancakes are one of the several kinds of generic singularity formed at the nonlinear stage of evolution of such a system. Some of the others are a cusp, a beak-to-beak, a swallow-tail. In this paper we present the full list of singularities for the one- and two-dimensional cases (Figures 1–9). The three dimensional singularities will be discussed in Part II of the paper. We discuss the geometrical and some dynamical properties of each kind of singularity. We give also asymptotic laws for the growth of the density near each kind of singularity. This list of singularities gives the elements from which the large scale structure of the Universe is constructed.

Our scheme naturally explains the existence of the flattened superclusters, as well as the rich clusters of galaxies by themselves and their chains. It is useful to note that the process of formation of smaller objects (galaxies) must be considered against the background of structures described in this paper.

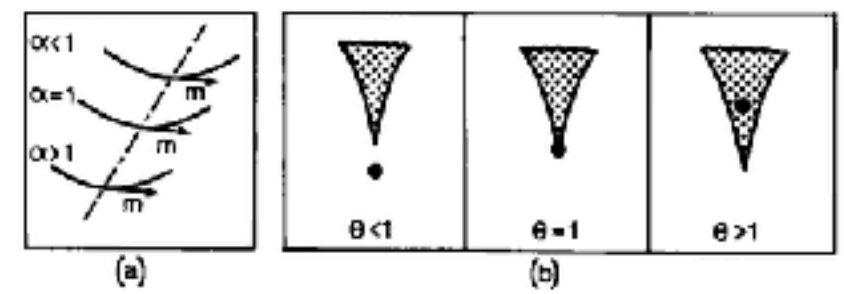


FIGURE 3 An edge of a pancake ("cusp" type A_3). The same designations as in Figure 2. The dash-dot line is formed by the edge points realized at different times.

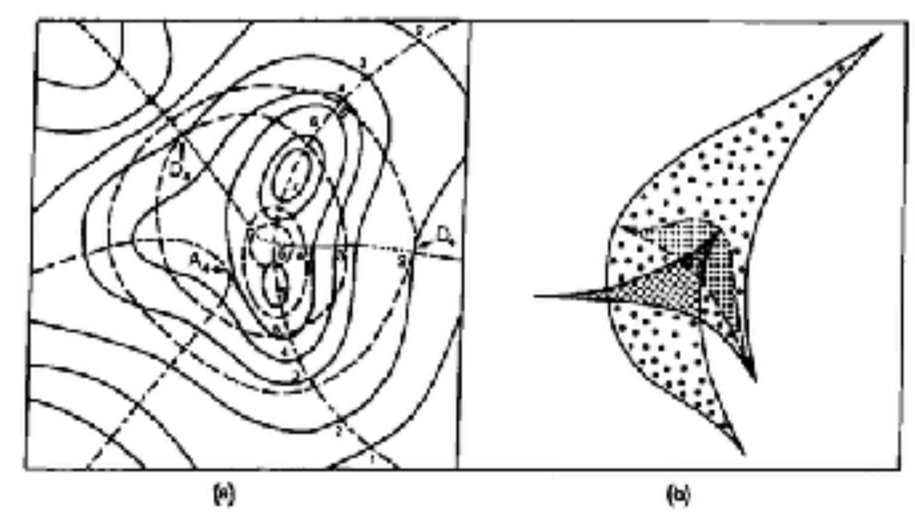
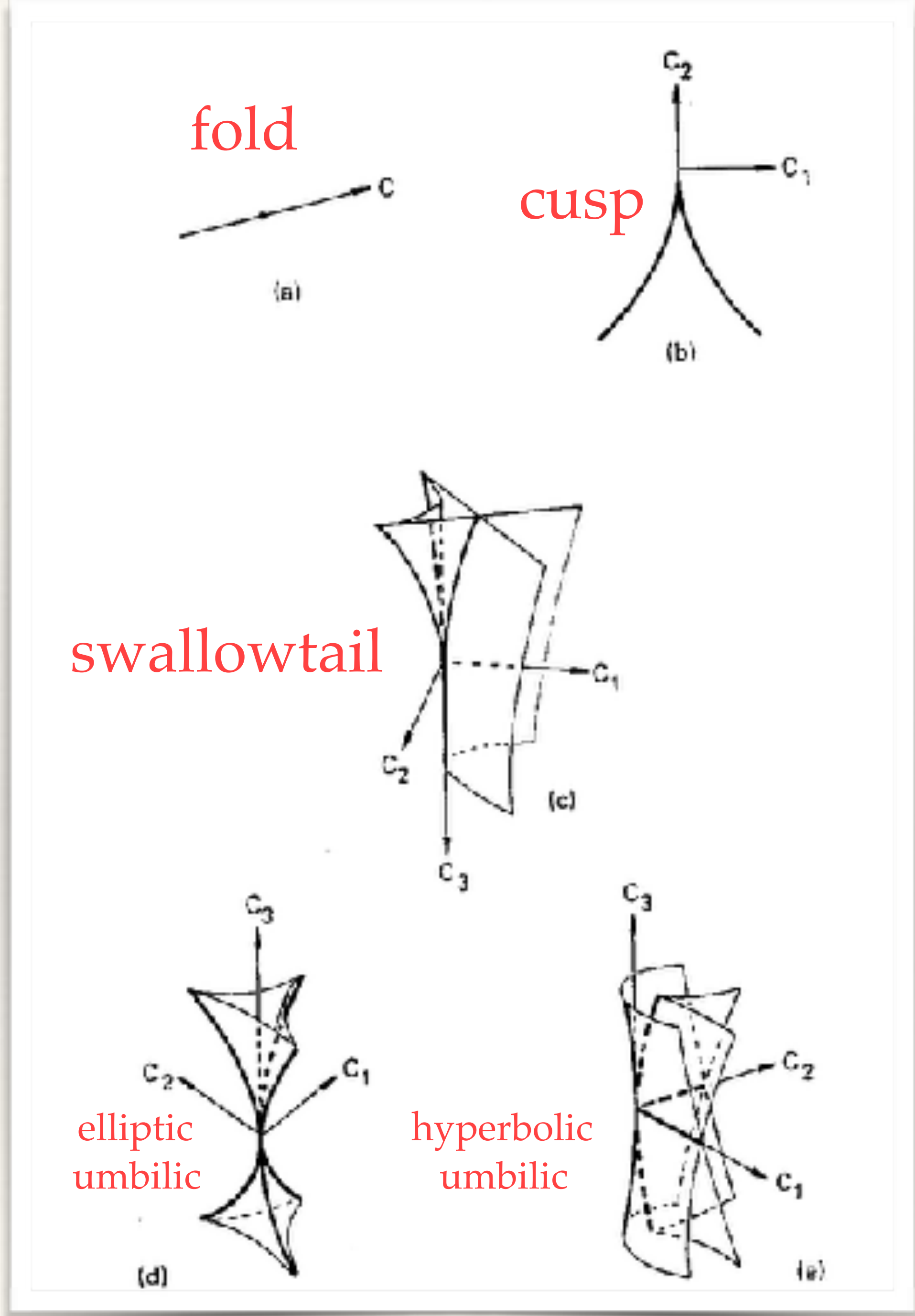


FIGURE 11 An example of the real structure arising in numerical simulations with statistical initial conditions. The designations are the same as in previous figures. (b) represents the Eulerian plane at the time of realization of one of two singularities of type D_4 ("purse").

Geophys. Astrophys. Fluid Dynamics **20**, 111 (1982)

Structurally stable catastrophes with K≤3



Berry & Upstill, Prog. Opt. **18**, 257 (1980)

Structurally stable caustics and their generating functions with K≤4

name	codimension K	**$\phi(s;C)$** [generating function]
Fold	1	$s^3/3+Cs$
Cusp	2	$s^4/4+C_2s^2/2+C_1s$
Swallowtail	3	$s^5/5+C_3s^3/3+C_2s^2/2+C_1s$
Elliptic umbilic	3	$s_1^3-3s_1s_2^2- C_3(s_1^2+s_2^2)-C_2s_2-C_1s_1$
Hyperbolic umbilic	3	$s_1^3+s_2^3- C_3s_1s_2-C_2s_2-C_1s_1$
Butterfly	4	$s^6/6+C_4s^4/4+C_3s^3/3+C_2s^2/2+C_1s$
Parabolic umbilic	4	$s_1^4+s_1s_2^2+C_4s_2^2+C_3s_1^2+C_2s_2+C_1s_1$

R. Thom *Structural Stability and Morphogenesis* (Benjamin, 1975); **V.I. Arnol'd***,* Russ. Math. Survs. *30 (5) (1975) p.1*

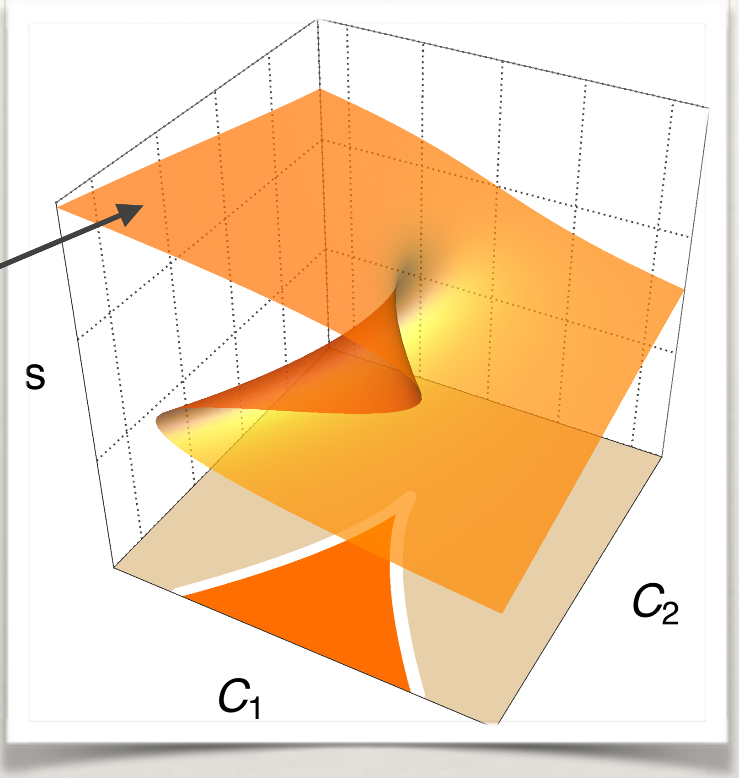
Catastrophe theory describes the **singularities of gradient maps** $\frac{\partial \phi}{\partial s_i} = 0$

Example: the cusp

$$\phi(s;C) = s^4/4 + C_2 s^2/2 + C_1 s$$

Ray equation (Fermat's principle) :

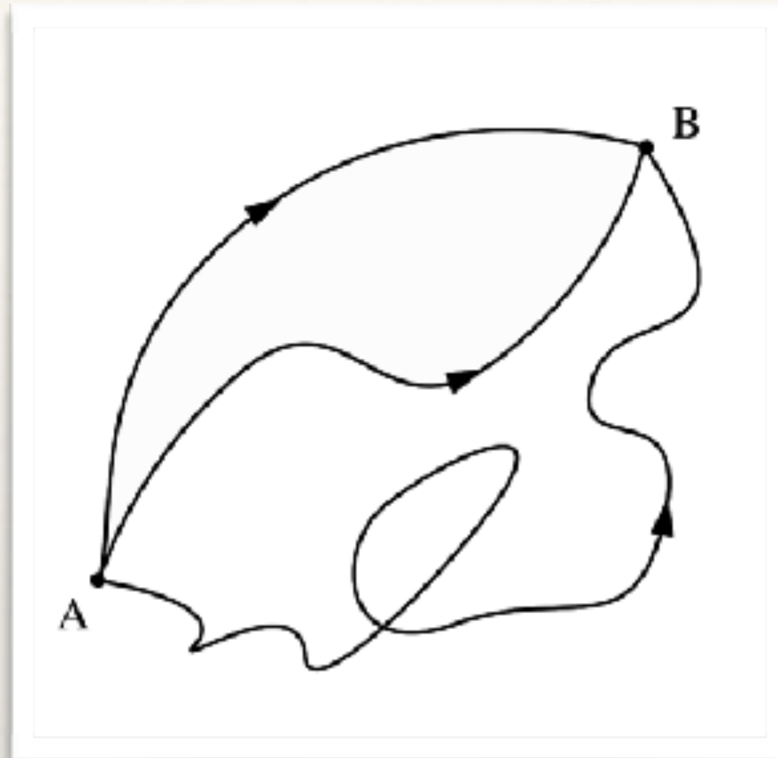
$$\frac{\partial \phi}{\partial s} = s^3 + C_2 s + C_1 = 0$$

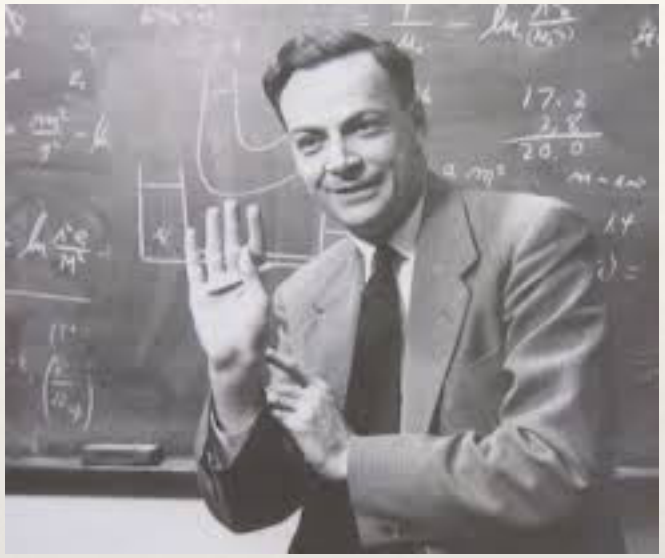


Caustic equation : $$\frac{\partial^2 \phi}{\partial s^2} = 3s^2 + C_2 = 0$$

Eliminate **s** : $$C_1 = \pm\sqrt{\frac{4}{27}}(-C_2)^{3/2}$$

Wave theory: Feynman path integral





Richard Feynman

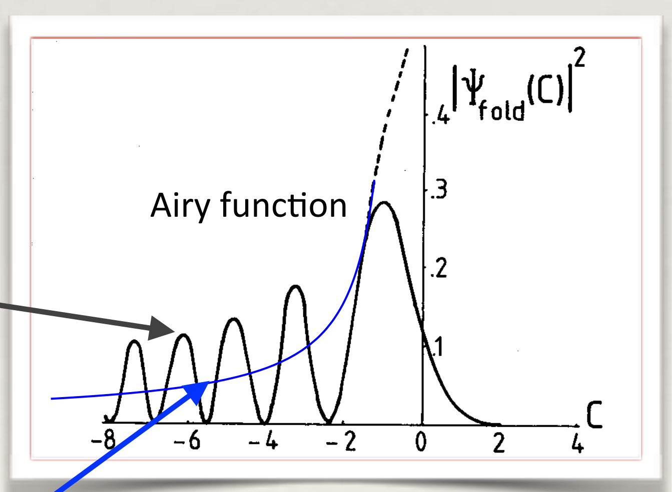
$$\Psi(B) = \mathcal{N} \sum_{\text{paths } j} e^{iS_j/\hbar}$$

Wave catastrophes

$$\Psi_{\text{fold}}(C) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(s^3/3 + Cs)}\, ds$$
$$= \sqrt{2\pi}\,\mathrm{Ai}[C]$$

G.B. Airy, Trans. Camb. Phil. Soc. **6**, 379 (1838)

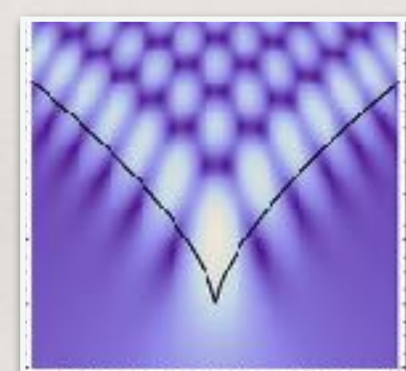
WKB theory



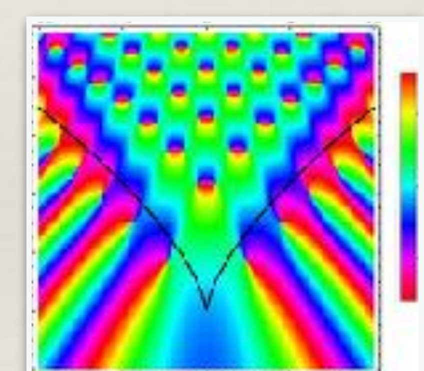
ray theory

$$\Psi_{\text{cusp}}(C_1, C_2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(s^4/4 + C_2 s^2/2 + C_1 s)} ds$$

T. Pearcey, Phil. Mag. **37**, 311 (1946)



amplitude



phase

Singularities are removed by interference

Scaling exponents

Catastrophe	Arnold Index β	Berry Indices σ_j	Berry Index γ
Fold	1/6	*2/3*	2/3
Cusp	1/4	*3/4, 1/2*	5/4
Swallowtail	3/10	*4/5, 3/5, 2/5*	9/5
Elliptic umbilic	1/3	*2/3, 2/3, 1/3*	5/3
Hyperbolic umbilic	1/3	*2/3, 2/3, 1/3*	5/3
Butterfly	1/3	*5/6, 2/3, 1/2, 1/3*	7/3
Parabolic umbilic	3/8	*5/8, 3/4, 1/2, 1/4*	17/8

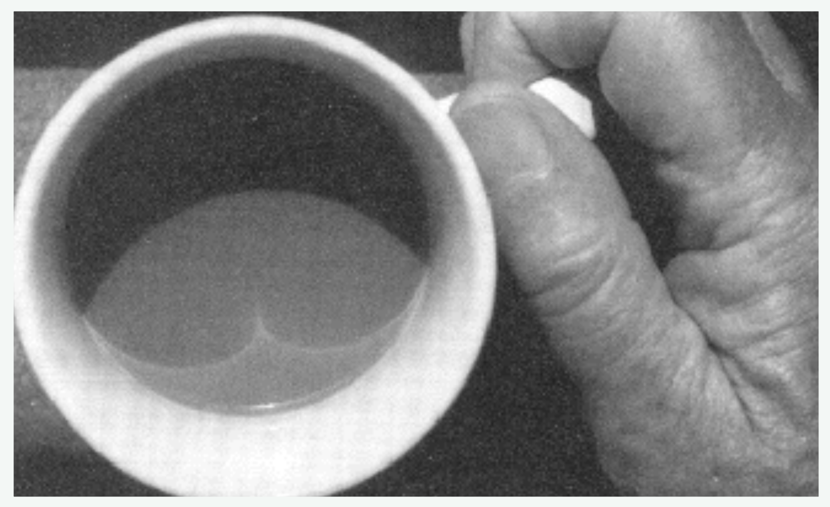
$$\psi(C_j; k) = \left(\frac{k}{k_0}\right)^{\beta} \psi\left[(k/k_0)^{\sigma_j} C_j; k_0\right]$$

Quantum catastrophes

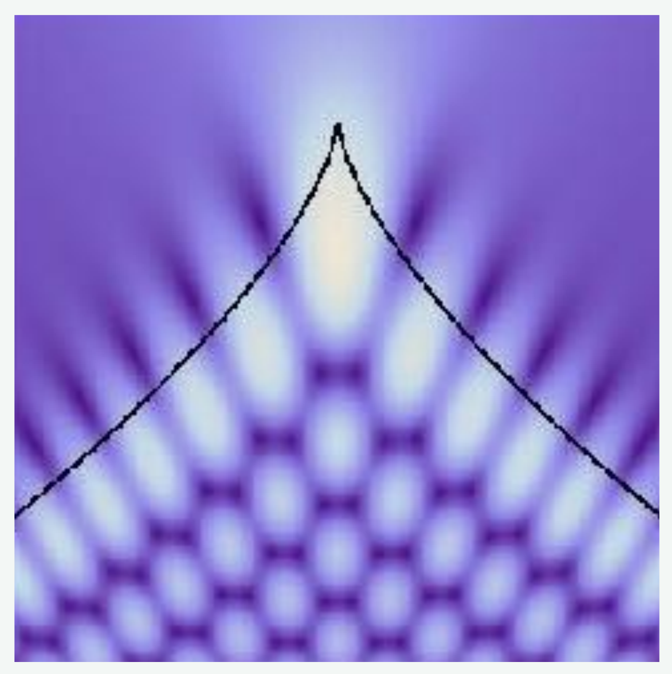
Can classical waves suffer singularities that are only cured by 2nd-quantization?

- U. Leonhardt, Nature **415**, 406 (2002)
- M.V. Berry, Nonlinearity **21**, T19 (2008)
- D. O’D, Phys. Rev. Lett. **109**, 150406 (2012)

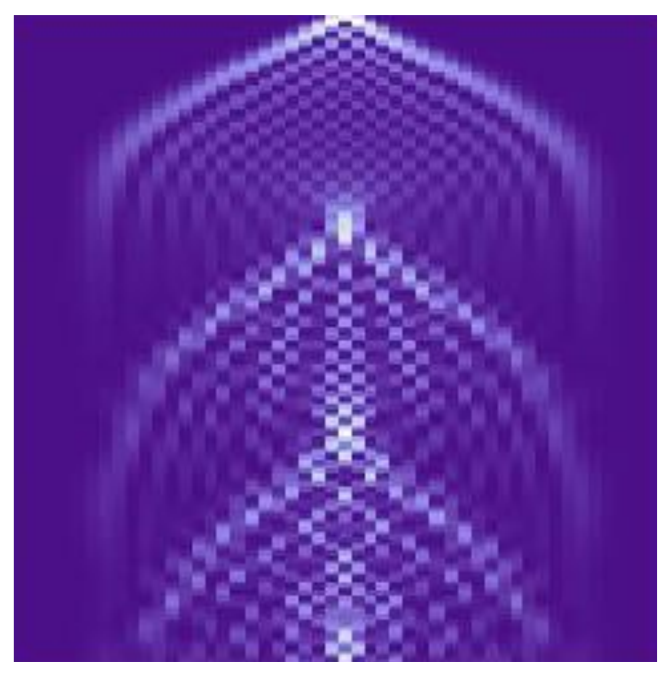
geometric catastrophe



classical wave catastrophe



quantum catastrophe



Fock space

Two-mode quantum field

$\hat{a}$ $\qquad [\hat{a}, \hat{a}^\dagger] = 1$

$\hat{b}$ $\qquad [\hat{b}, \hat{b}^\dagger] = 1$

bosonic commutation relations

Jordan-Schwinger mapping to angular momentum:

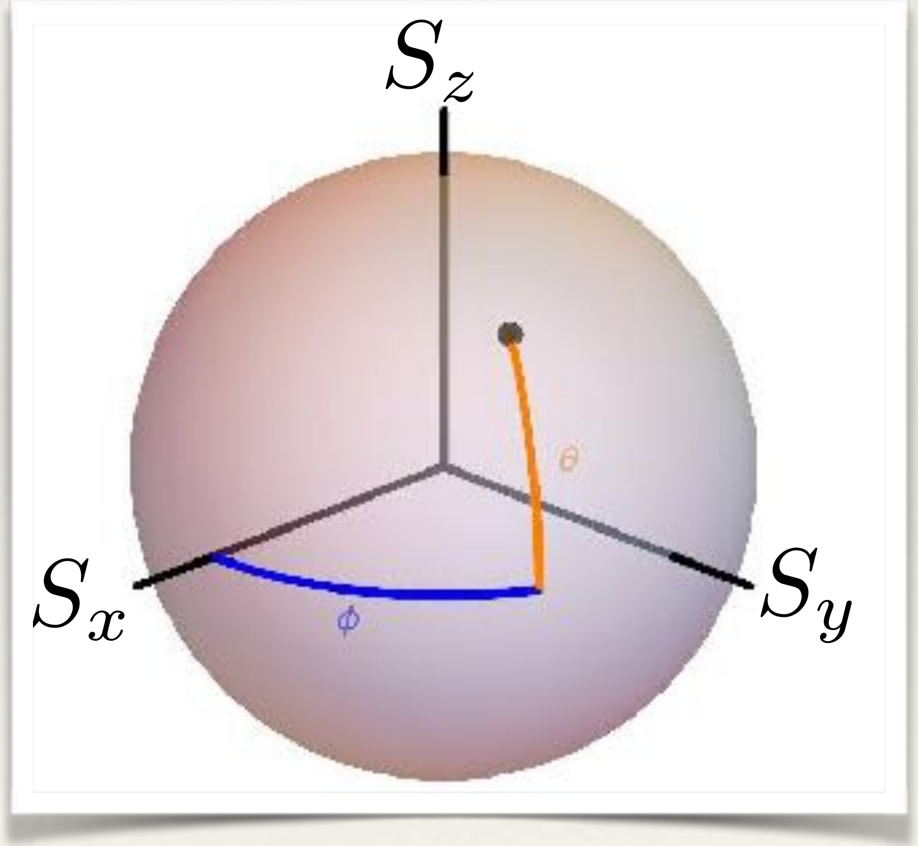
Giant spin of length S = N/2

$$\hat{S}^2 = \frac{\hat{N}}{2}\left(\frac{\hat{N}}{2} + 1\right) \qquad \hat{N} = \hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}$$

$$\hat{S}_x \equiv (\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a})/2$$

$$\hat{S}_y \equiv (\hat{a}^\dagger \hat{b} - \hat{b}^\dagger \hat{a})/2i$$

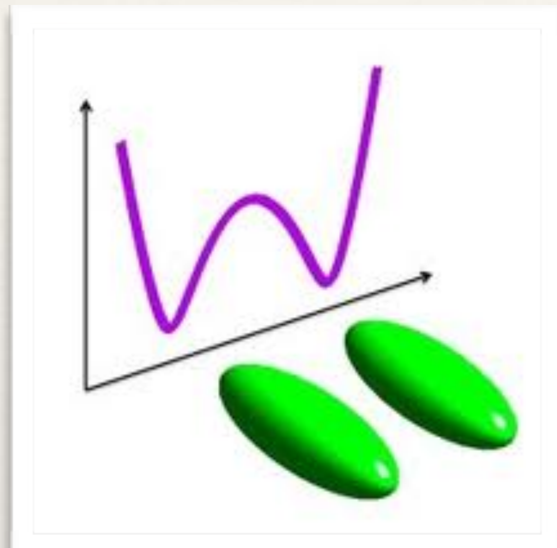
$$\hat{S}_z \equiv (\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b})/2$$



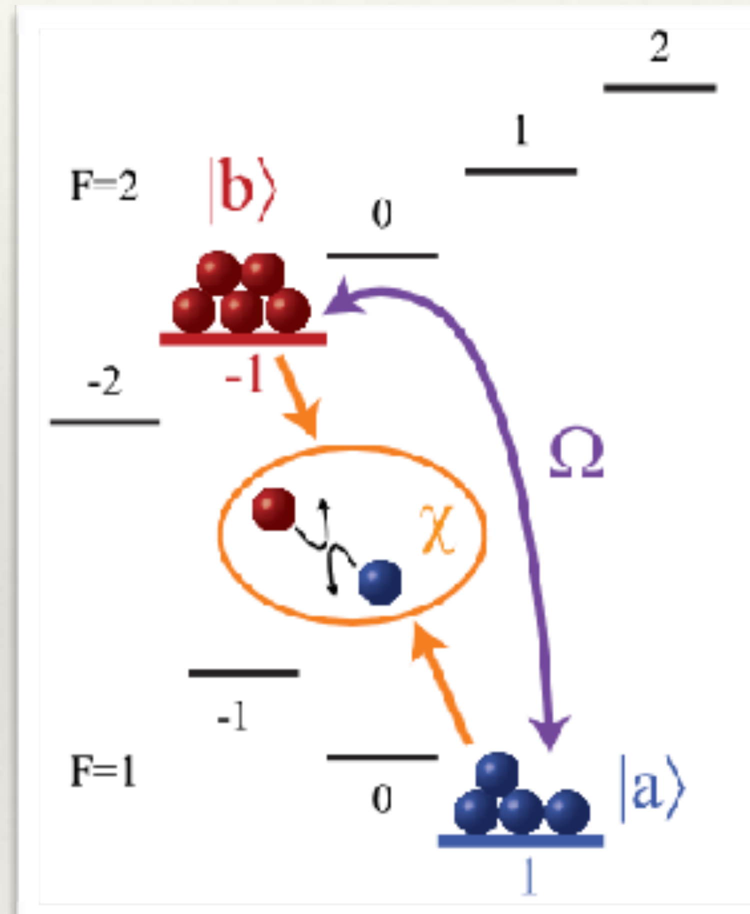
Generalized Bloch/Poincaré sphere

Example 1: BEC in a double well potential

Two external states: double well trap OR **Two internal states**



Oberthaler (Heidelberg) 2005
Steinhauer (Technion) 2007
Schmiedmayer (Vienna) 2007
Hinds (Imperial College) 2010
Thywissen (Toronto) 2011
Roati (Florence) 2015
Fattori (Florence) 2016



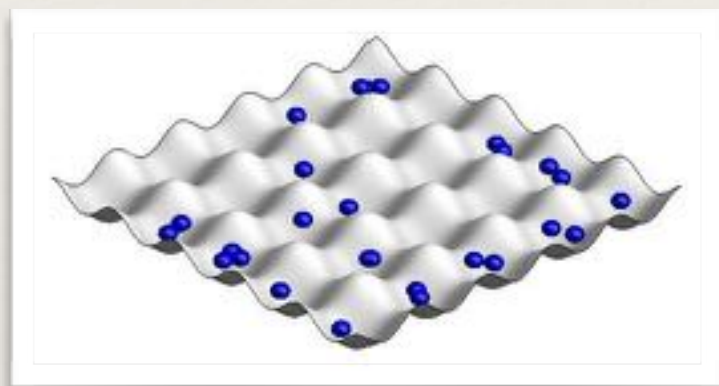
Zibold et al (2010)

+ continuous measurement of number difference ➡ open quantum system

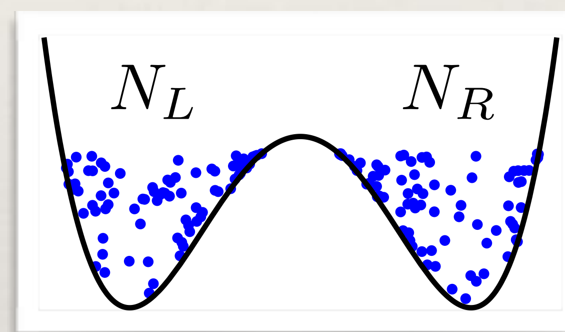
Two-mode Bose-Hubbard model

Bose-Hubbard model:
(tight-binding lattice)

$$\hat{H} = -J \sum_{\langle i,j \rangle} \underbrace{\hat{a}_i^\dagger \hat{a}_j}_{\text{hopping}} + \frac{U}{2} \sum_i \underbrace{\hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i}_{\text{on-site interactions}}$$



Reduce to two sites:



$$\hat{H} = -J(\hat{a}_l^\dagger \hat{a}_r + \hat{a}_r^\dagger \hat{a}_l) + \frac{U}{4}(\hat{a}_l^\dagger \hat{a}_l - \hat{a}_r^\dagger \hat{a}_r)^2 + \text{constant terms}$$

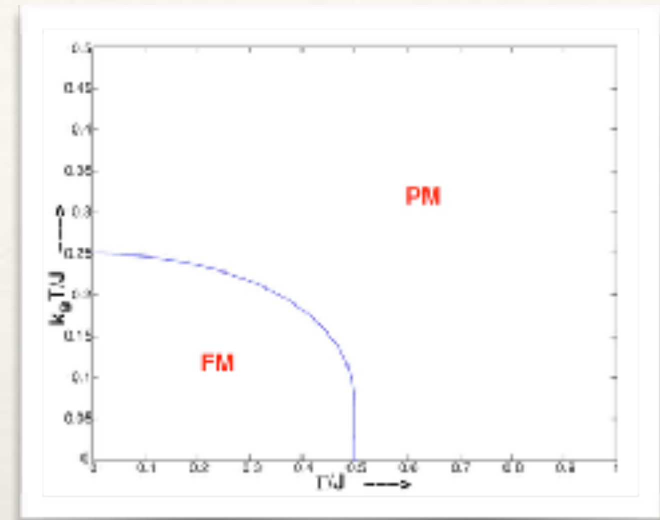
In Josephson junction language: $\hat{H} = -\frac{E_J}{N}(\hat{a}_l^\dagger \hat{a}_r + \hat{a}_r^\dagger \hat{a}_l) + \frac{E_c}{2}(\hat{a}_l^\dagger \hat{a}_l - \hat{a}_r^\dagger \hat{a}_r)^2$

In spin language: $\hat{H} = -2J\hat{S}_x + U\hat{S}_z^2$ $\qquad \hat{S}_x = \sum_i \hat{s}_x^{(i)} \qquad \hat{S}_z = \sum_i \hat{s}_z^{(i)}$

Example 2: Transverse field Ising model

Ising model with **infinite-range** interactions:

$$H = -\frac{J}{N} \sum_{i<j} S_i^z S_j^z - \Gamma \sum_i S_i^x$$



A. Das, K. Sengupta, D. Sen, and B. K. Chakrabarti Phys. Rev. B **74**, 144423 (2006)

Put: $S^z_{\text{tot}} = \sum_i S_i^z \qquad S^x_{\text{tot}} = \sum_i S_i^x$

$$\Rightarrow \quad H = -\frac{J}{2N} \left(S^z_{\text{tot}}\right)^2 - \Gamma S^x_{\text{tot}} + \text{constant}$$

Lipkin-Meshkov-Glick model (1965)

Classical motion: Josephson oscillations

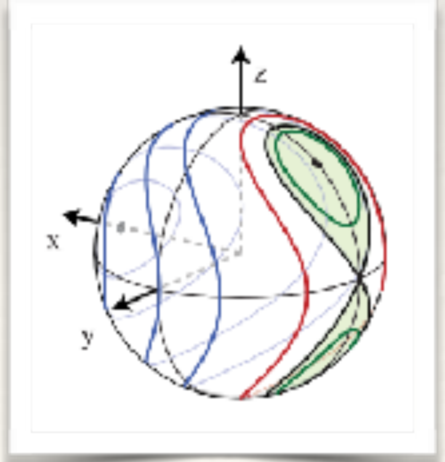
Classical limit: $$H = \frac{E_c}{2}n^2 - E_J\sqrt{1-4n^2/N^2}\cos\phi$$

(pendulum with ang. momentum dependent length)

where: $$n \equiv \frac{1}{2}(n_l - n_r) \quad , \quad \phi \equiv \phi_l - \phi_r$$

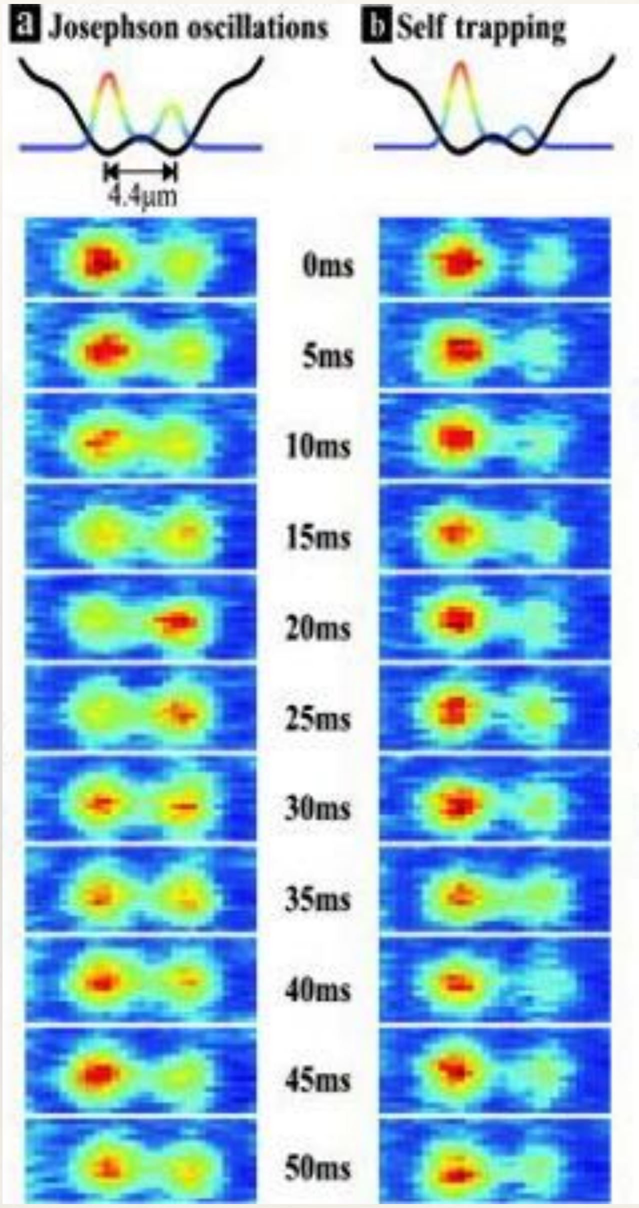
number difference **phase difference**

$$\left.\begin{aligned} \dot{\phi} &= \frac{E_c}{\hbar}n + \frac{4n/N^2}{\sqrt{1-4n^2/N^2}}\frac{E_J}{\hbar}\cos\phi && \text{phase difference} \\ \dot{n} &= -\frac{E_J}{\hbar}\sqrt{1-4n^2/N^2}\sin\phi && \text{number difference} \end{aligned}\right\} \text{Josephson's equations}$$



$$n = S_z$$

$\phi =$ azimuthal angle



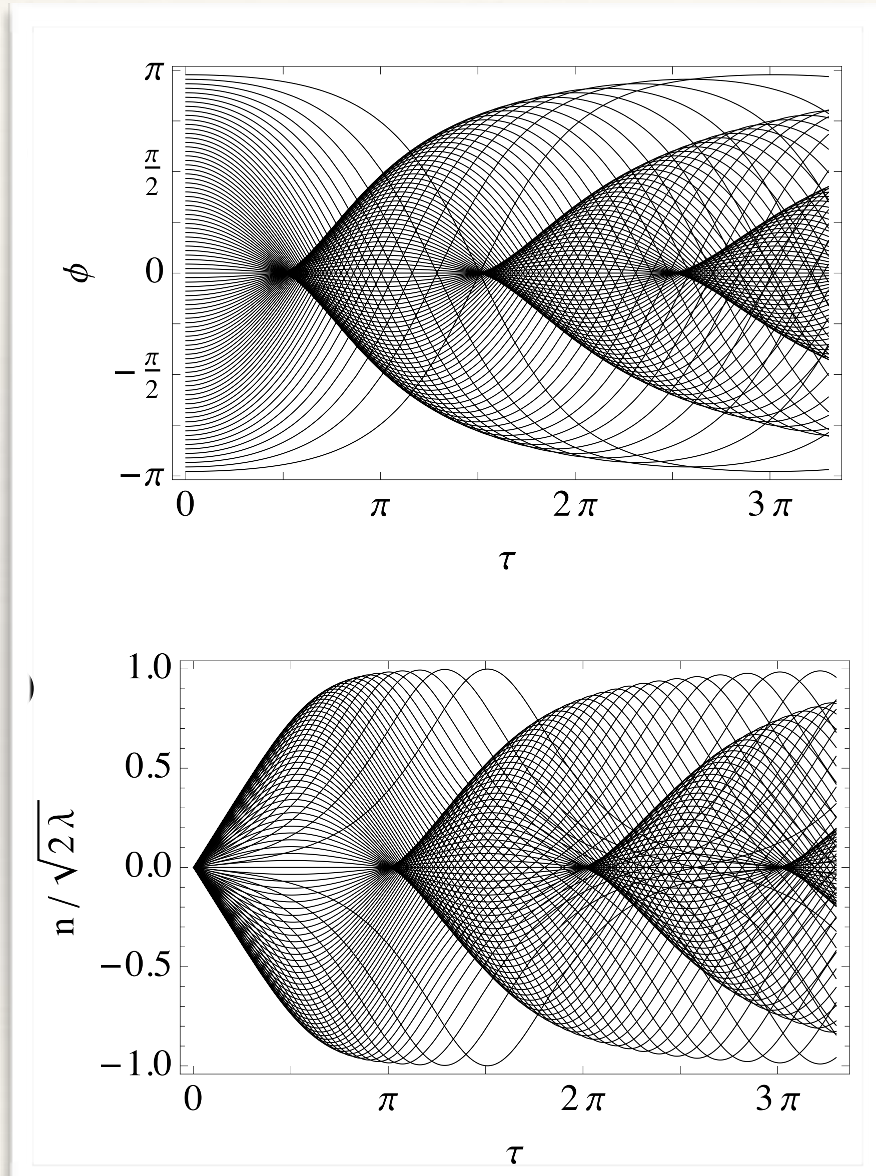
Albiez et al, PRL **95** 010402 (2005)

Classical field dynamics following a quench

Sudden connection of two independent BECs (initial state = Fock state)

phase difference

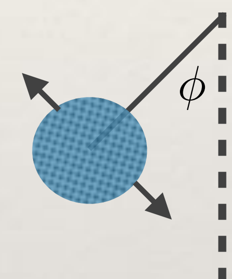
number difference



time

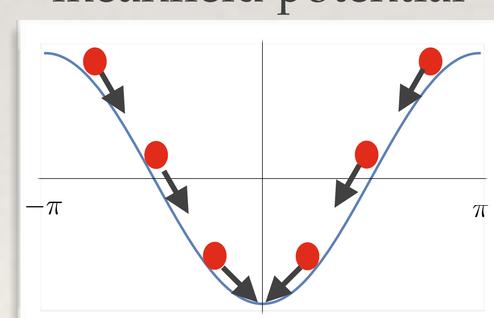
$$H = \frac{E_c}{2} n^2 - E_J \sqrt{1 - 4n^2/N^2} \cos\phi$$

$$\approx \frac{E_c}{2} n^2 - E_J \cos\phi$$



pendulum

meanfield potential

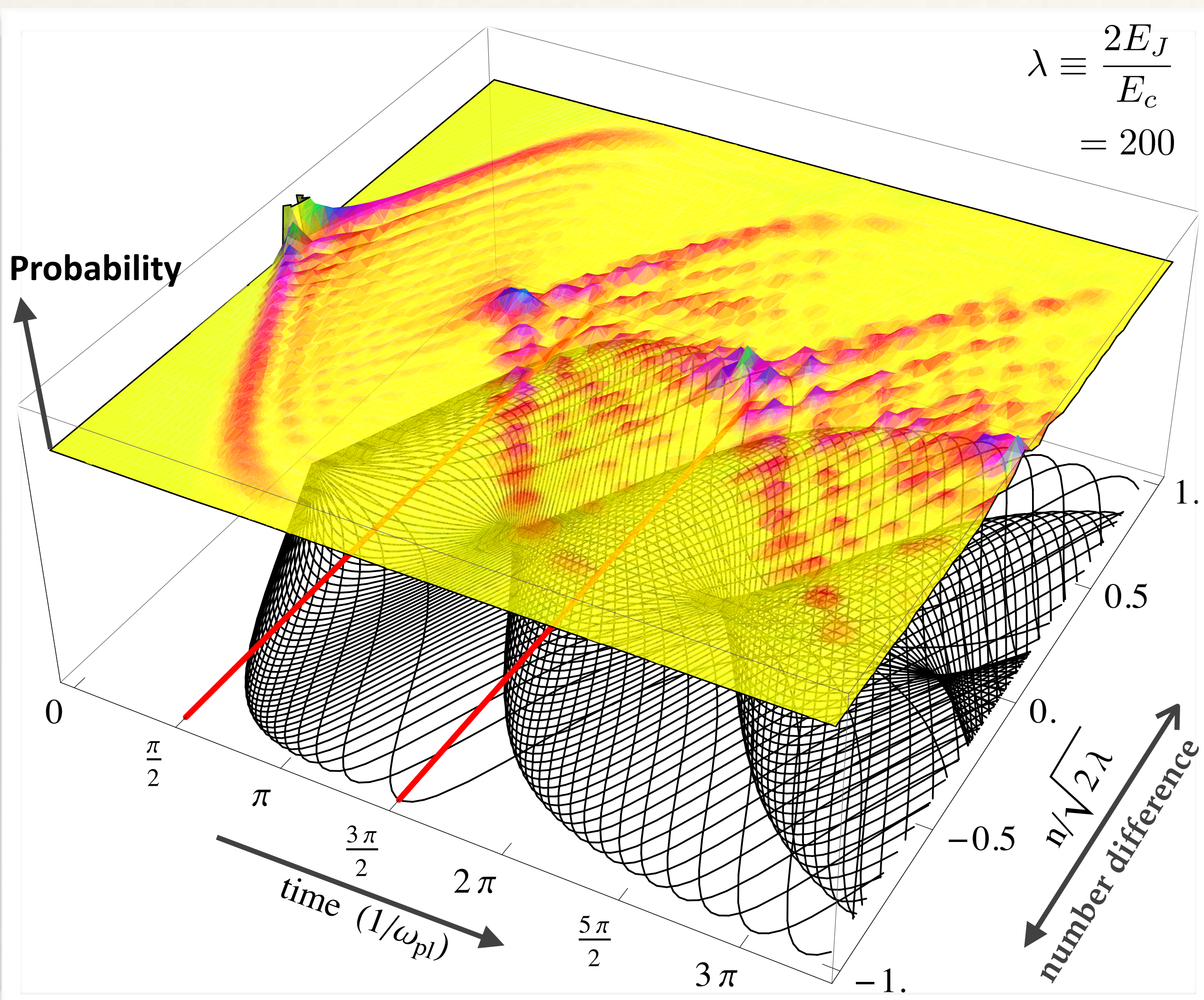


phase difference

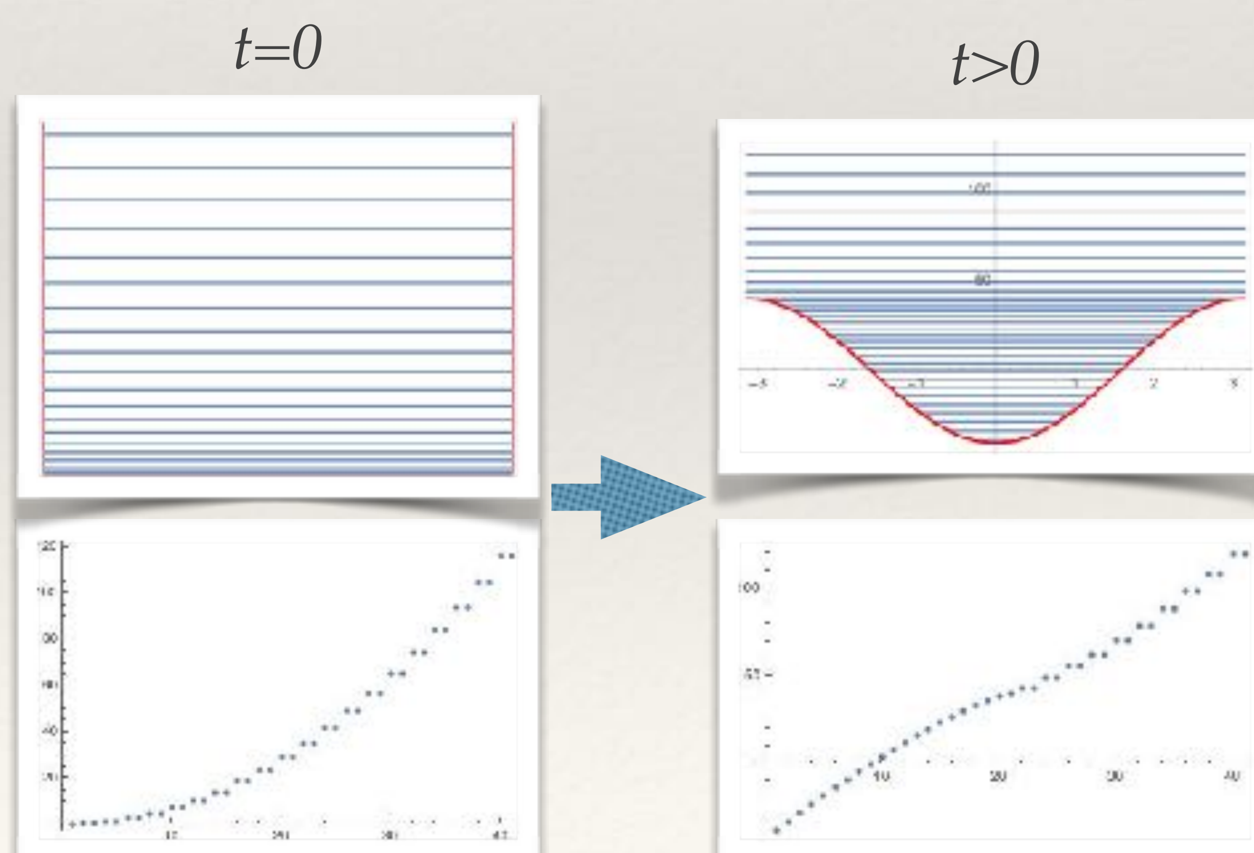
quantum: $[\hat{\phi}, \hat{n}] \approx i$ $\qquad \lambda \equiv 2E_J/E_c$

Catastrophes in Fock space following a quench

D. O'D., Phys. Rev. Lett. **109**, 150406 (2012)

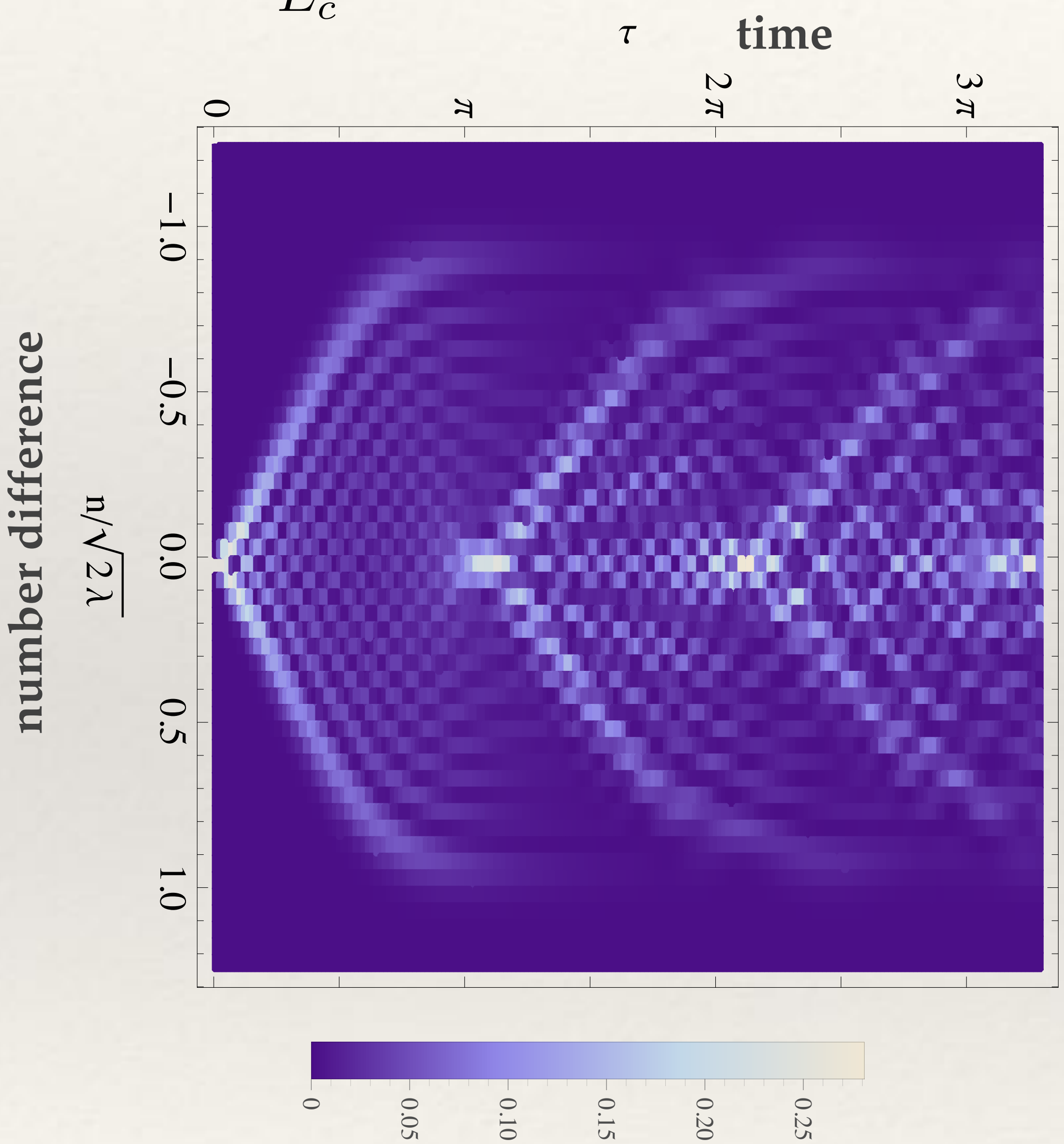


- Two **independent** BECs suddenly coupled via tunnelling junction
- Initial state is Fock state |n> = 0 (very quantum)
- Revivals at $t_n = 2\pi n/\omega_{\text{plas}}$ where $\omega_{\text{plas}} = \sqrt{E_c E_J}/\hbar$
- Revivals are catastrophes!



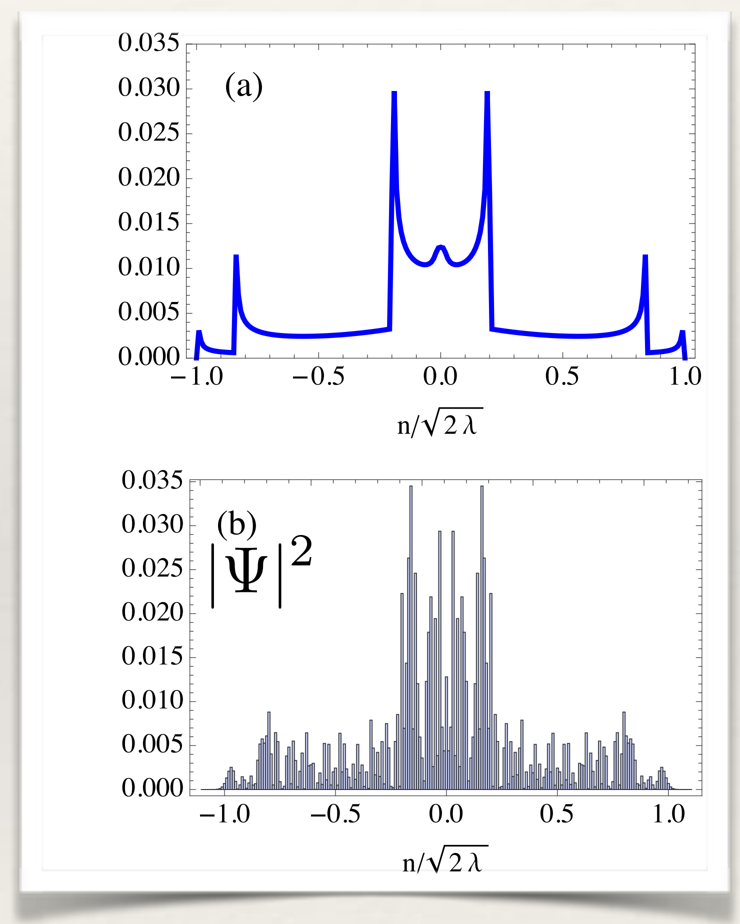
Effect of quantization

$$\lambda \equiv \frac{2E_J}{E_c} = 200$$



$$\lambda \equiv \frac{2E_J}{E_c} = 5000$$

$$\tau = 3.3\pi$$



Classical field
(Josephson equations)

Quantum field
(Bose-Hubbard theory)

Cusps in Fock space: meanfield singularities are regularized by second-quantization

Quantum catastrophes

Dynamical diffraction

Proc. Indian Acad. Sci. A2 406–412 (1936)

The diffraction of light by high frequency sound waves: Part I

C V RAMAN
and
N S NAGENDRA NATH
(Department of Physics, Indian Institute of Science, Bangalore)

Received 28 September 1935

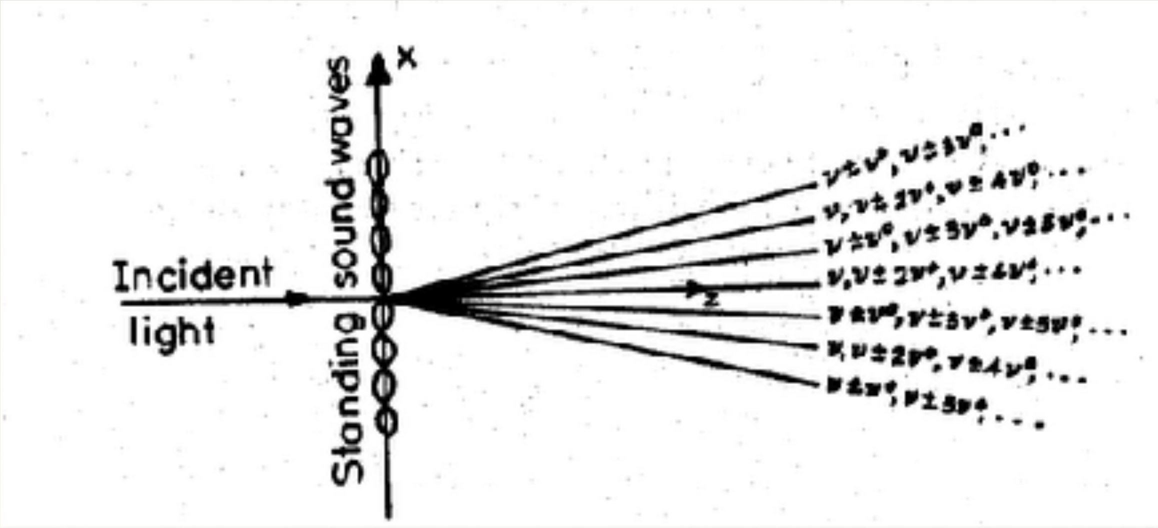
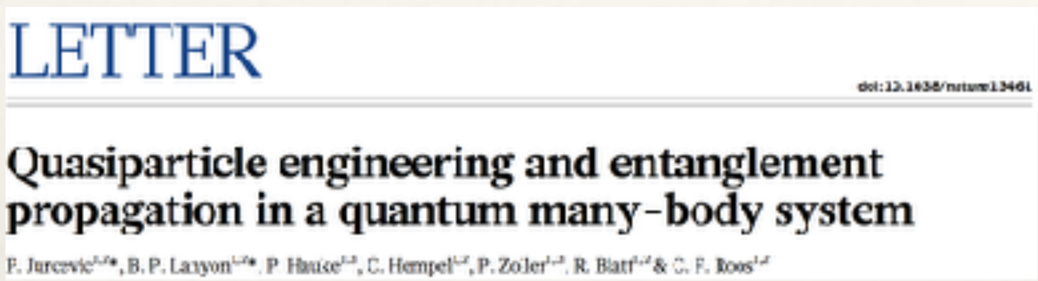


Figure 1

Relative intensities of the various components in the diffraction spectra.
(For tables, see Watson's *Bessel Functions* and *Report of the British Association*, 1915).

Light cones as catastrophes

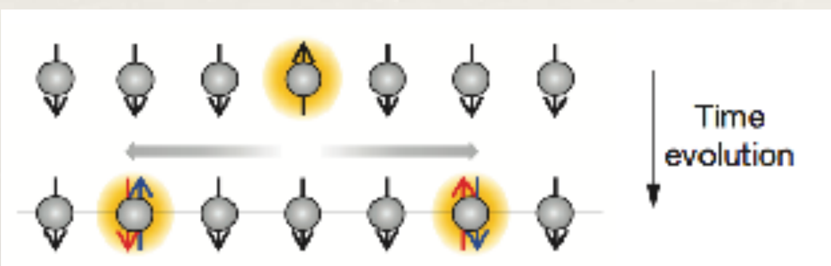
Spin chain

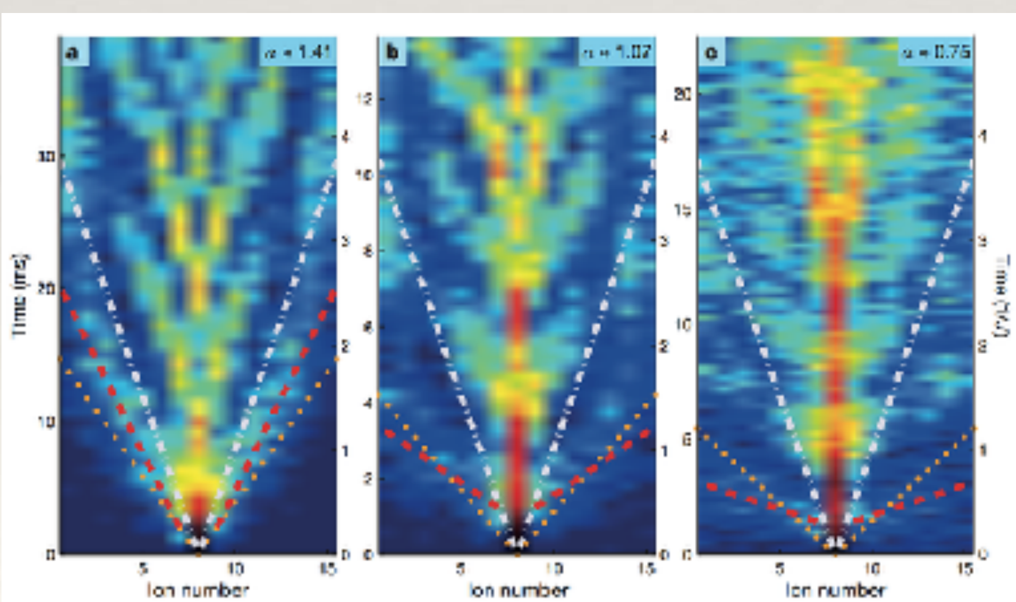

LETTER

doi:10.1038/nature13461

Quasiparticle engineering and entanglement propagation in a quantum many-body system

F. Jurcevic, B. P. Lanyon, P. Hauke, C. Hempel, P. Zoller, R. Blatt & C. F. Roos





Measured magnetization $\langle \sigma_i^z(t) \rangle$

Bose Hubbard model


LETTER

Light-cone-like spreading of correlations in a quantum many-body system

Marc Cheneau, Peter Barmettler, Dario Poletti, Manuel Endres, Peter Schauß, Takeshi Fukuhara, Christian Gross, Immanuel Bloch, Corinna Kollath & Stefan Kuhr

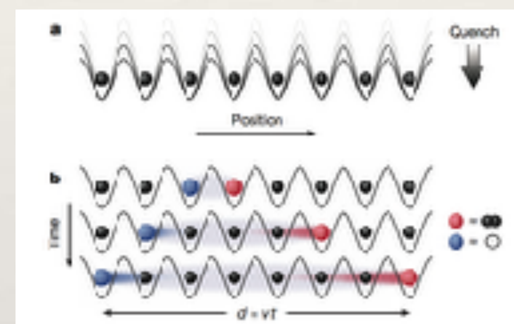


Figure 1 | Spreading of correlations in a quenched atomic Mott insulator.

$$C_d(t) = \langle \hat{s}_j(t)\hat{s}_{j+d}(t)\rangle - \langle \hat{s}_j(t)\rangle\langle \hat{s}_{j+d}(t)\rangle$$

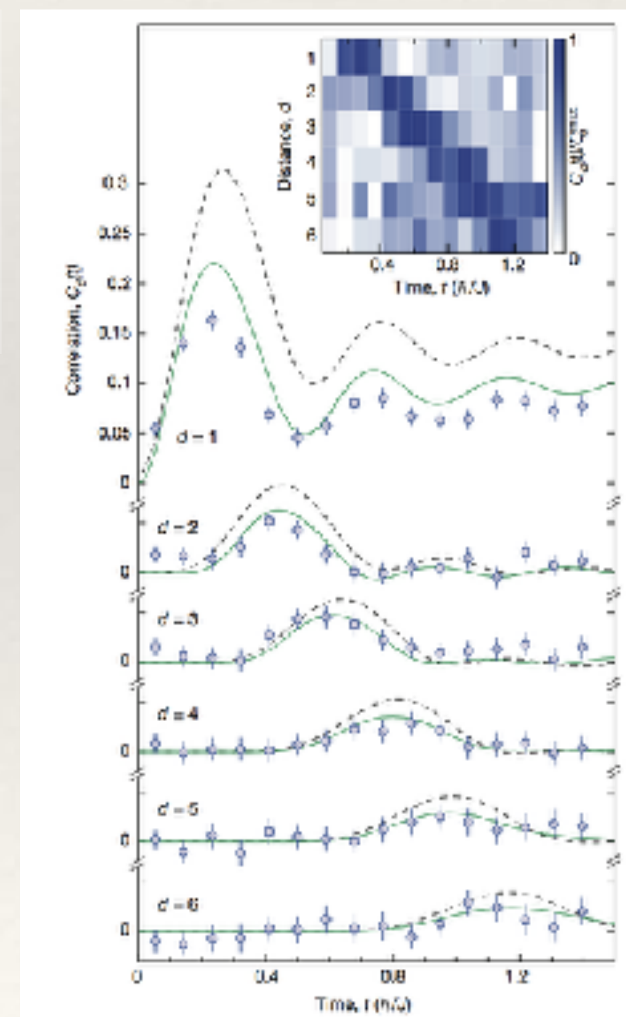
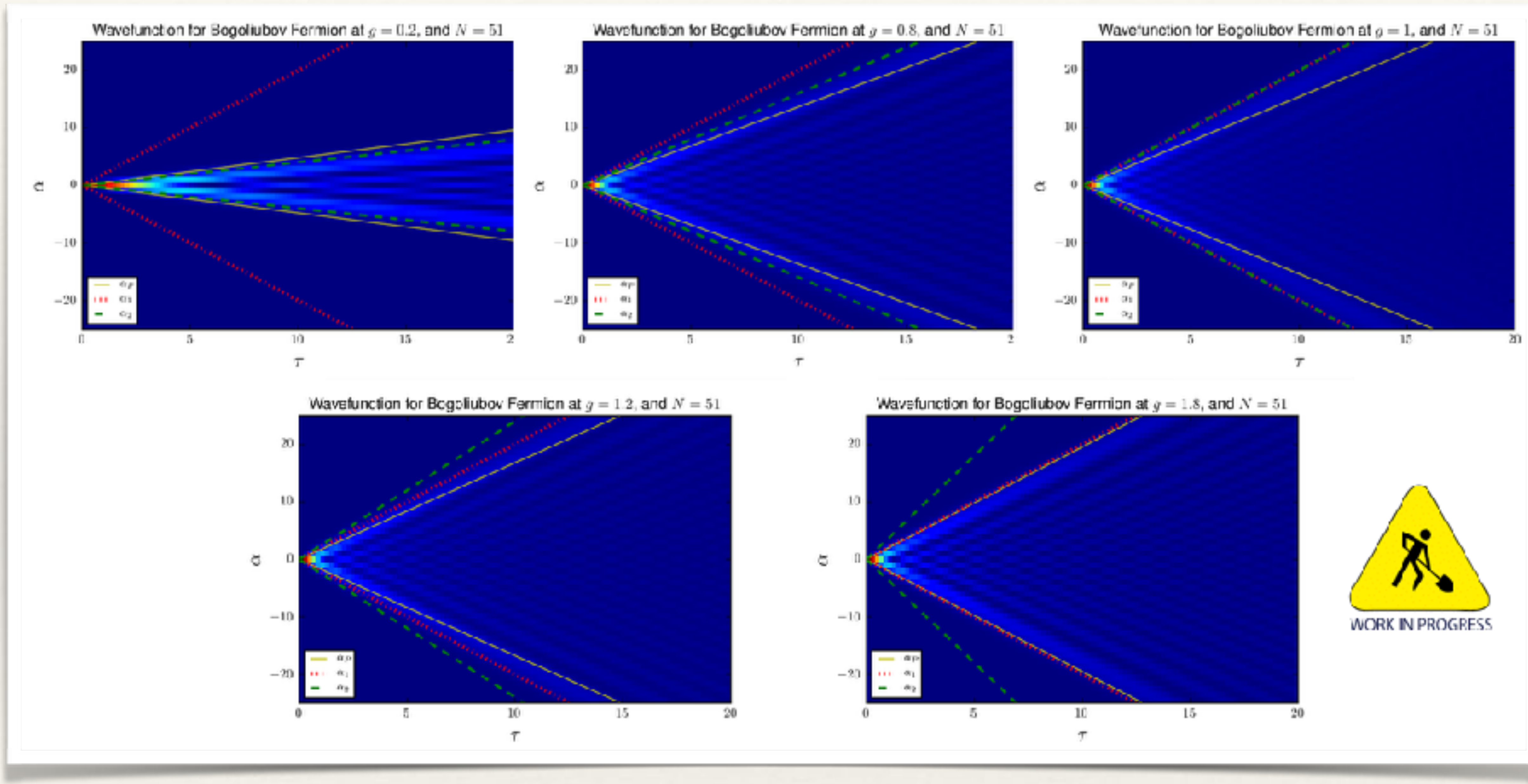


Figure 2 | Time evolution of the two-point parity correlations. After the

TFIM following a local quench



$\alpha = x_\alpha / a \qquad -N/2 \leq \alpha \leq N/2 \qquad \tau = Jt \qquad g = h/J$

Weak continuous measurement

Weak continuous measurement of number difference:

$$\frac{\partial \hat{\rho}}{\partial \tau} = i\left[\hat{S}_x, \hat{\rho}\right] - i\frac{\Lambda}{N}\left[\hat{S}_z^2, \hat{\rho}\right] - D\frac{\Lambda}{N}\left[\hat{S}_z, \left[\hat{S}_z, \hat{\rho}\right]\right]$$

(Master equation in Lindblad form)

$$\Lambda = \frac{E_c N^2}{4E_J}$$

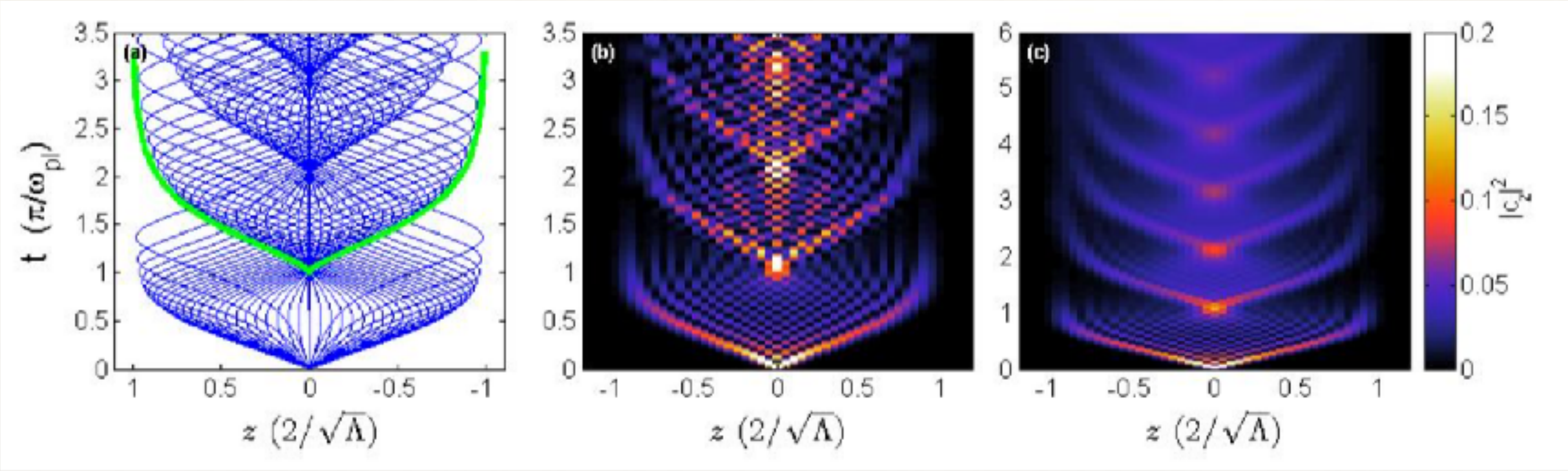
D= measurement interaction strength

J. Ruostekoski and D. F. Walls, Phys. Rev. A 58, R50 (1998).

J. F. Corney and G. J. Milburn, Phys. Rev. A 58, 2399 (1998).

Decoherence of a quantum catastrophe

classical no decoherence	quantum no decoherence	quantum with decoherence
$D = 0$	$D = 0$	$D = 0.1$



fractional number difference

$\Lambda = 25 \qquad N = 100$

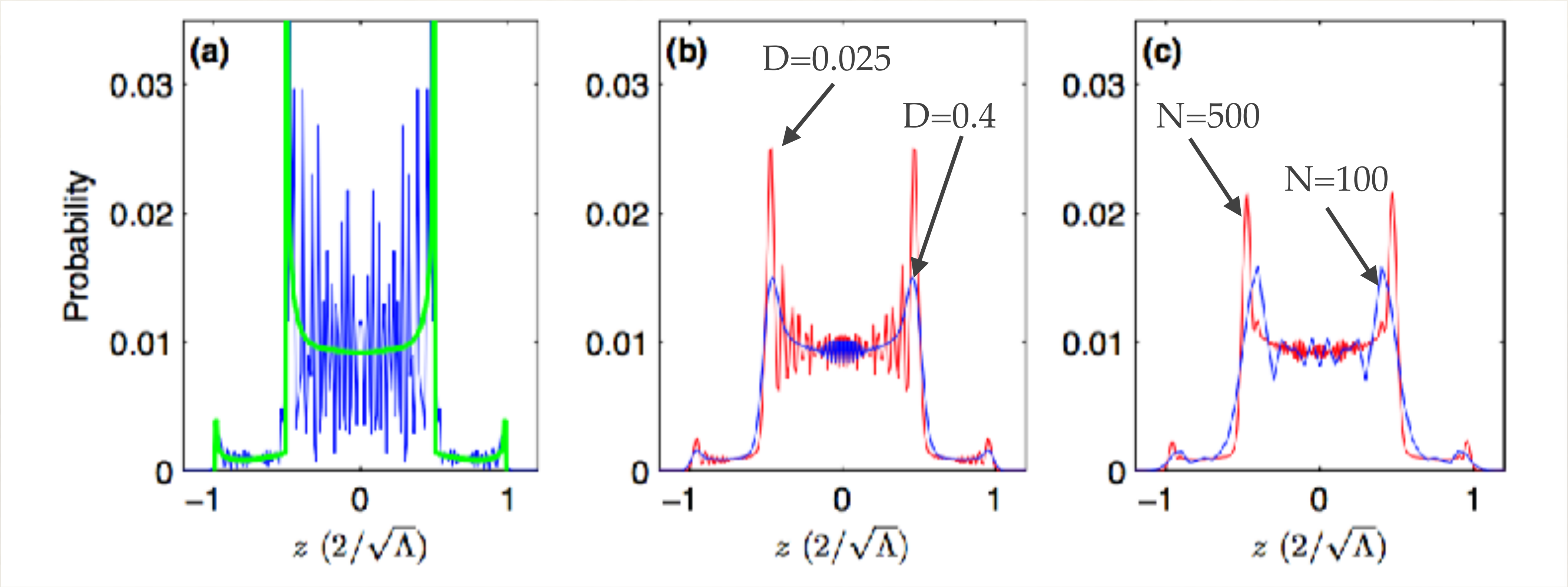
At long times: $\langle q \,|\hat{\rho}_{\mathrm{steady}}|\, z \rangle = \dfrac{\delta_{q,z}}{N+1}$

Time slice through number-difference probability density

classical vs quantum no decoherence
(N=400, D=0)

Effect of varying measurement strength
(N=400)

Effect of varying number of atoms
(D=0.1)



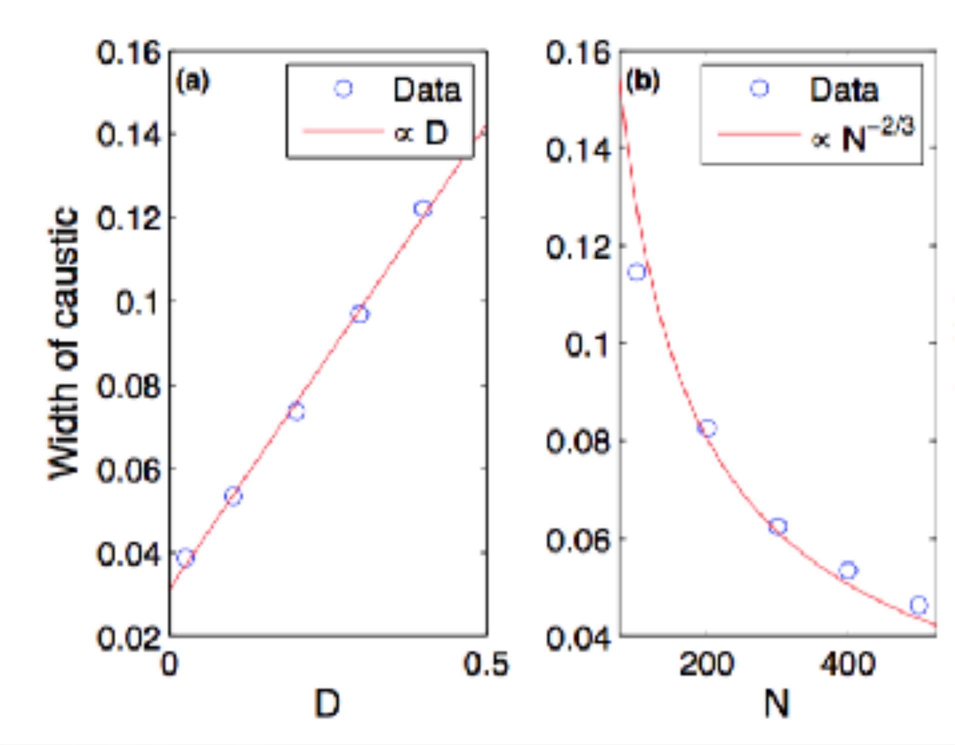
fractional number difference

time slice at: $t = \dfrac{1.5\pi}{\omega_{\mathrm{pl}}}$ $\Lambda = 25$

"Paradox" resolved: effect of quantum noise

Effect on caustic width of:

varying D (N=400) | varying N (D=0.1)



- Width of caustics increases linearly with D
- Width of caustics decreases as $N^{-2/3}$
- Master equation can be mapped onto Fokker-Planck equation which accounts for increase in width (diffusion).
- Classicality would be achieved as N→∞, D→0, but D×N=finite.
- Number measurement effectively breaks symmetry (couples to odd eigenstates), similar to adding a stochastic tilt term to Hamiltonian

time slice at: $t = \dfrac{1.5\pi}{\omega_{\rm pl}}$ $\quad \Lambda = 25$

Summary

- Catastrophes occur **generically** in classical and quantum dynamics
- Cusp is the generic singularity of a **two-mode field** in Fock space
- Three levels of structure: geometric, classical wave, quantum
- A quantum catastrophe is regularized by discreteness of field excitations (quanta)
- Continuous number difference measurements kill interference but simultaneously broaden caustics by quantum noise
- Can still reach classical limit (singular) but no paradox as it requires $N \rightarrow \infty$