Anti-Kibble-Zurek Behavior in Crossing the Quantum Critical Point

Anirban Dutta

IISc, India

Brief Outline

1. Kibble Zurek Scaling.

2. Motivation of this work.

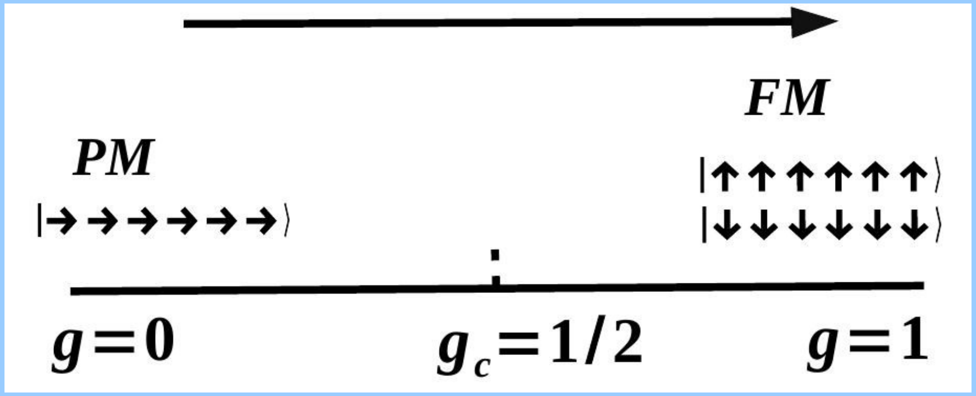
3. Anti Kibble Zurek behavior.

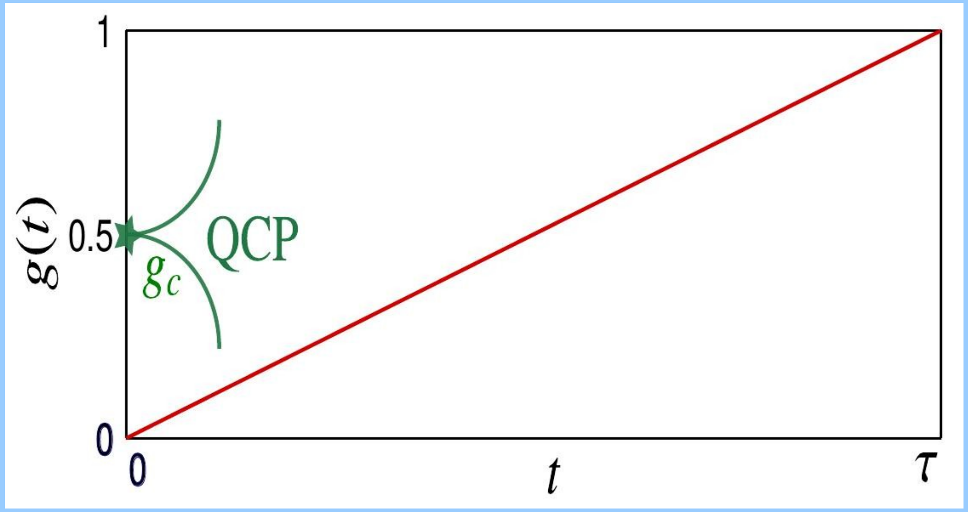
4. Conclusion.

Kibble-Zurek Scaling

Adiabatic Theorem: A quantum state prepared in the eigenstate of the Hamiltonian will remain in the instantaneous eigenstate under slow driving of the parameter of the Hamiltonian.

Adiabatic theorem breaks driving across a quantum critical point.





$$H_{TFI} = -J \sum_{i=1}^{N} \left(g \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + (1-g) \hat{\sigma}_i^x \right)$$

$$g(t) = \frac{t}{\tau}$$

Density of defect:

$$n_d = \frac{1}{2N} \sum_{n=1}^{N} \left(1 - \sigma_n^z \sigma_{n+1}^z\right)$$

Kibble-Zurek Scaling:

$$n_d \sim \tau^{-\frac{d\nu}{1+z\nu}}$$

$$n_d \sim \frac{1}{\sqrt{\tau}}$$

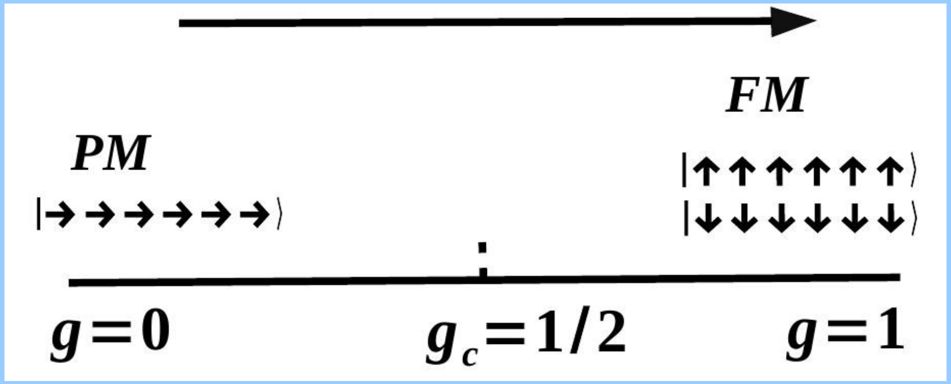
J. Dziarmaga, Adv. Phys. 59, 1063 (2010).

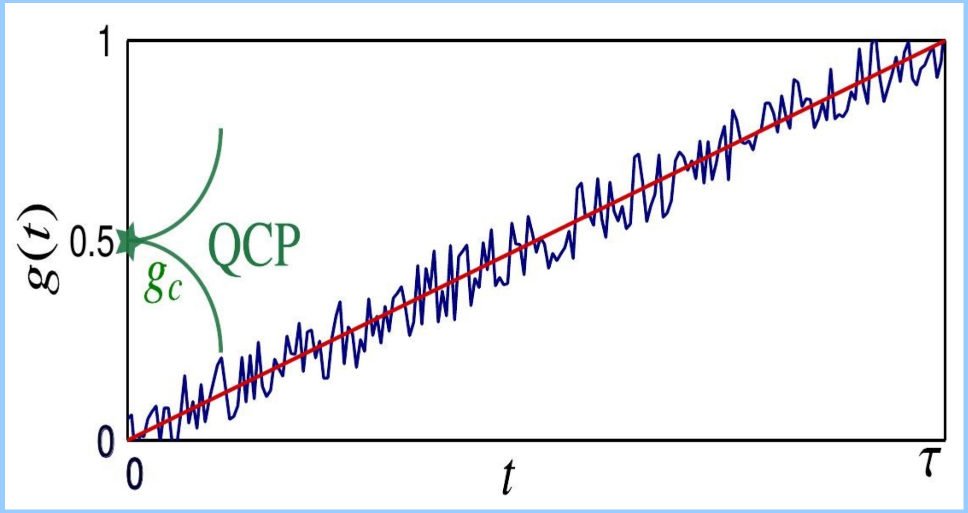
Consider the situation where the driving is not perfect:

$$g(t)=g_0(t)+\eta(t) \qquad H(t)=H_0(t)+\eta(t)V$$

Deterministic and Stochastic part of the Hamiltonian:

$$H_0=-J\sum_{i=1}^{N}\left(g_0(t)\hat{\sigma}_i^z\hat{\sigma}_{i+1}^z+(1-g_0(t))\hat{\sigma}_i^x\right) \qquad V=-J\sum_{i=1}^{N}\left(\hat{\sigma}_i^z\hat{\sigma}_{i+1}^z+\hat{\sigma}_i^x\right)$$





Gaussian white noise:

$$\langle \eta(t)\rangle=0$$

$$\langle \eta(t)\eta(t')\rangle=W^2\delta(t-t')$$

The time evolution is governed by the stochastic Schrödinger equation:

$$i\partial_t|\psi\rangle=\left(H_0(t)+\eta(t)V\right)|\psi\rangle$$

Density Matrix averaged over realizations:

$$\rho(t) = \langle \rho_{st}(t) \rangle = \langle |\psi(t)\rangle\langle\psi(t)| \rangle$$

The equation of motion for the noise averaged density matrix:

$$\frac{d}{dt}\rho(t) = -i[H_0(t), \rho(t)] + \mathscr{L}[\rho(t)]$$

A A Budini, Phys Rev A, 63, 012106
A A Budini, Phys Rev A, 64, 052110

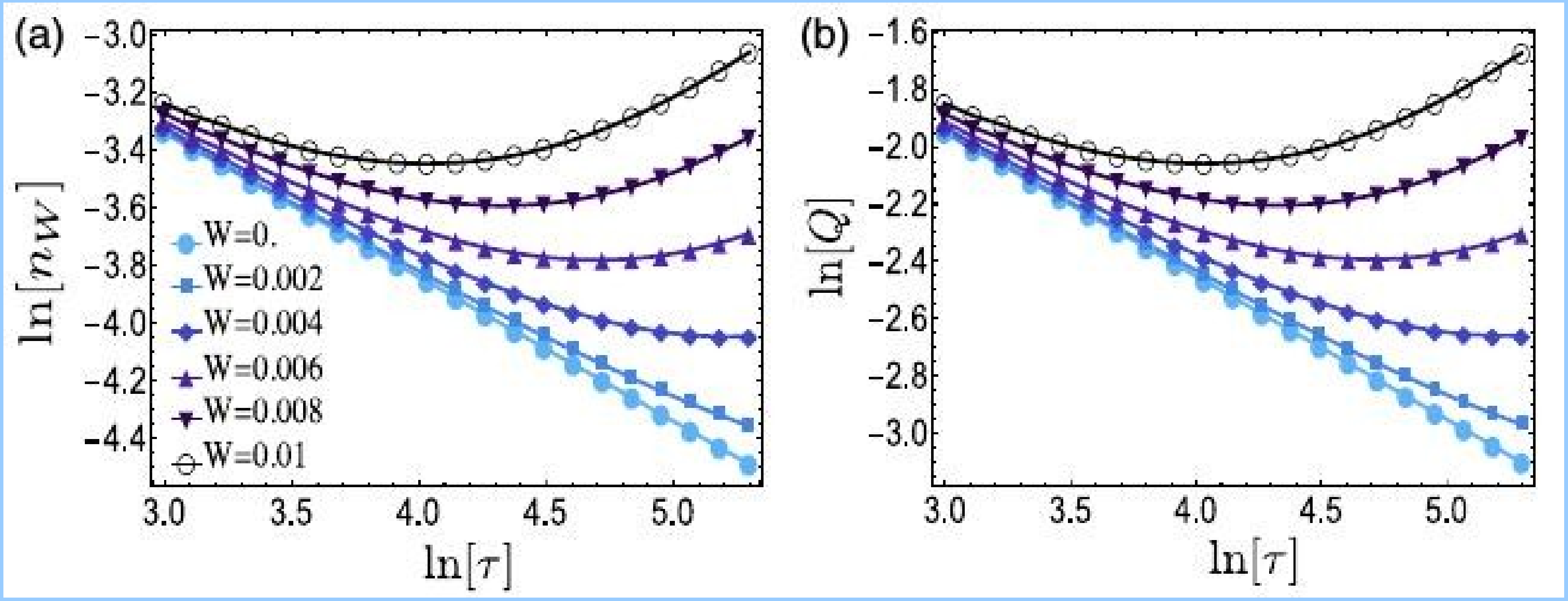
$$\mathscr{L}[\rho(t)] = -\frac{W^2}{2}[V, [V, \rho(t)]]$$

The last term can be written in standard Lindblad form:

$$\mathscr{L}[\rho(t)] = W^2\left(V\rho V^{\dagger} - \frac{1}{2}\rho V^{\dagger}V - \frac{1}{2}V^{\dagger}V\rho\right)$$

We have solved the Master equation numerically and from there calculated the noise averaged expectation value of the operator.

P. Nalbach, Smitha Vishveshwara, and Aashish A. Clerk, Phys Rev B 92, 014306



Density of excitation **Residual Energy**

$$n_d = \frac{1}{2N} \sum_{n=1}^{N} \left(1 - \sigma_n^z \sigma_{n+1}^z\right)$$

$$Q = \frac{1}{2N} \left(E(\tau) - E_{GS}(\tau)\right)$$

Density of excitation at the end of drive is well approximated as:

$$n_d \sim r\tau + c\tau^{-\beta}$$

$$\beta = \frac{d\nu}{1 + z\nu}$$

$$r \sim W^2$$

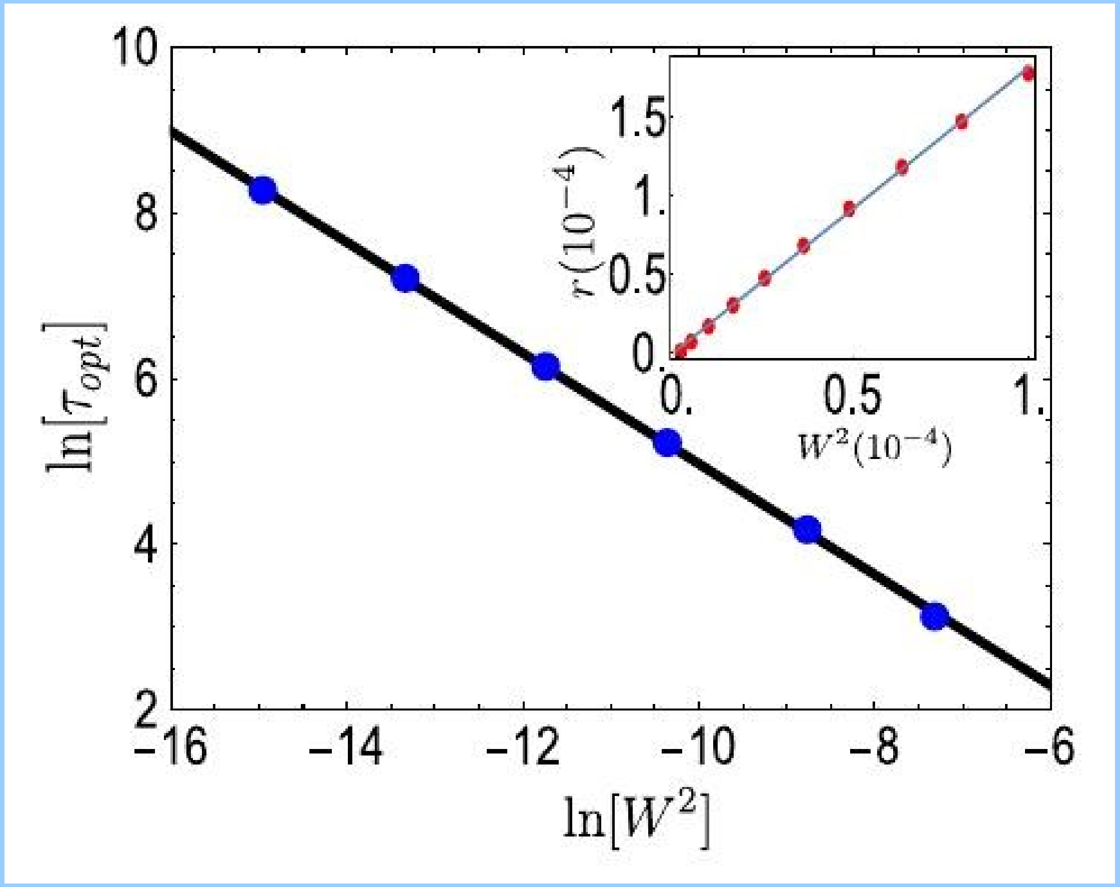
Estimation of optimal time: $\tau_{opt} \propto \left(W^2\right)^{-\frac{1}{1+\beta}}$

Verification of our prediction:

$$n_d \sim r\,\tau + c\,\tau^{-\beta}$$

$$\tau_{opt} \propto \left(W^2\right)^{-\frac{1}{1+\beta}}$$

$$r \sim W^2$$



We have provided a natural mechanism to explain the anti-Kibble-Zurek behavior in the quantum critical dynamics of a thermally isolated system driven by a noisy control field.

Our results show the limits to adiabatic strategies in quantum annealing and indicate that the optimal annealing time follows a power law as a function of the amplitude of the noise fluctuations.

Thank You

A Dutta, A Rahmani and A del Campo,
Phys. Rev. Lett. 117, 080402 (2016)