Keldysh Field Theory for Open Quantum Systems: Non-Markovian Dynamics Localization and Quasiprobability Distributions

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Outline

Brief Introduction to Keldysh Field Theory

Non-markovian dynamics

Non-interacting bosons coupled to bosonic bath

Localization

Effect of interactions on Markovian system dynamics

Universal quasiprobability distribution of quantum jumps

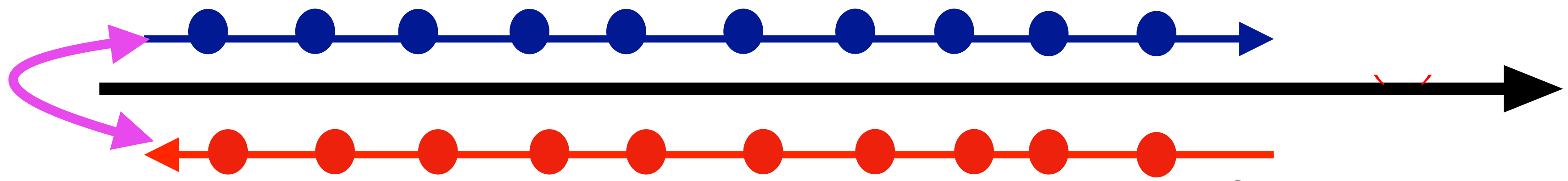
Keldysh Field Theory

A. Kamenev, Field Theory of Non Eqbm Systems

$$\hat{\rho}(t) = U(t,-\infty)\hat{\rho}(-\infty)U^{\dagger}(t,-\infty)$$

Path Integral along fwd contour $\phi_+(x,t)$ $\int D[\phi_+]e^{iS[\phi_+]}$

$\hat{\rho}(-\infty)$



$t = \infty$

$\phi_+(x,\infty) = \phi_-(x,\infty)$

Path Integral along reverse contour $\phi_-(x,t)$ $\int D[\phi_-]e^{-iS[\phi_-]}$

Boundary Term
Couples + and - fields

$$Z = \int D[\phi_+]D[\phi_-]e^{i(S[\phi_+]-S[\phi_-]+\ldots)}$$

Keeps Track of Boundary Terms

$$Z[J] = \int D[\phi_+]D[\phi_-]e^{i(S[\phi_+]-J_+\phi_+-S[\phi_-]-iJ_-\phi_-+\ldots)}$$

$$\langle \phi_\alpha \phi_\beta \rangle = \left.\frac{\partial^2 Z[J]}{\partial J_\alpha \partial J_\beta}\right|_{J=0}$$

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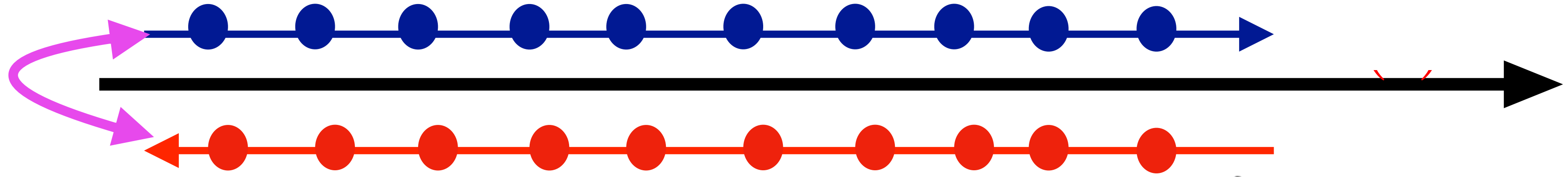


$t = \infty$

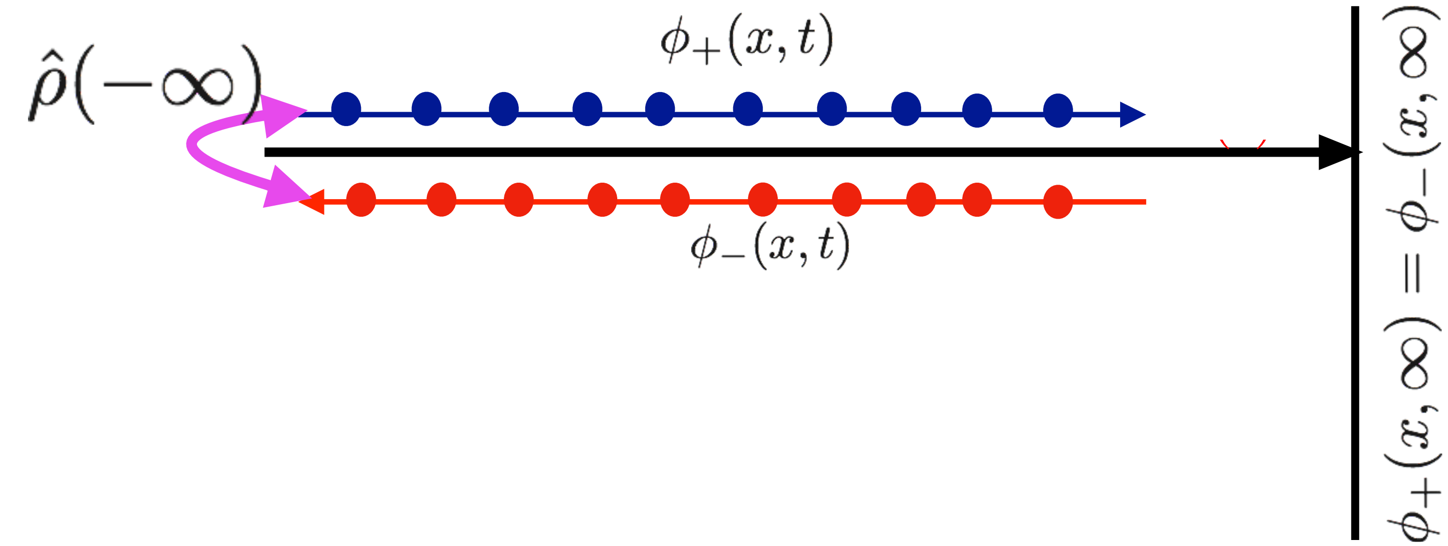
Path Integral along reverse contour $\phi_-(x,t) \int D[\phi_-]e^{-iS[\phi_-]}$

$$\phi_+(x,\infty) = \phi_-(x,\infty)$$

$$Z = \int D[\phi_+]D[\phi_-]e^{i(S[\phi_+]-S[\phi_-]+\ldots)}$$

- ✱ Density matrices natural starting point for open quantum systems
- ✱ No "adiabatic" ramp up of interactions needed —> can be used for non-eqbm dynamics
- ✱ Two field formalism makes it possible to treat effects of dissipation
- ✱ Used in quantum transport theory and to treat disordered systems

Classical and quantum fields



$$\phi_{cl} = \frac{1}{\sqrt{2}}(\phi_+ + \phi_-) \qquad \phi_q = \frac{1}{\sqrt{2}}(\phi_+ - \phi_-)$$

General Quadratic action

$$S_0 = \int d^dx \int dt \int d^dx' \int dt' [\phi^*_{cl}(x,t), \phi^*_q(x,t)] G^{-1}(x,t;x',t') \begin{bmatrix} \phi_{cl}(x',t') \\ \phi_q(x',t') \end{bmatrix}$$

$$G^{-1} = \begin{bmatrix} 0 & G_A^{-1} \\ G_R^{-1} & -\Sigma_K \end{bmatrix}$$

$S[\phi_{cl}, \phi_q=0] = 0$ holds for interacting case as well

Simple Example

$$H = \sum_k \omega_k b_k^\dagger b_k \qquad G^{-1}_{R(A)}(k) = \delta_{tt'}(i\partial_t - \omega_k \pm i0^+)$$

Knows only about propagation

For initial thermal ensemble $\Sigma_k = i0^+ n_B(\omega_k)\delta t, -\infty \delta_{t',-\infty}$

Knows about distribution fn.s

A. Kamenev, Field Theory of Non Eqbm Systems

Classical and quantum fields

$$\phi_{cl} = \frac{1}{\sqrt{2}}(\phi_+ + \phi_-) \qquad \phi_q = \frac{1}{\sqrt{2}}(\phi_+ - \phi_-)$$

Green's functions

$$G_{\alpha\beta}(x,t;x',t') = -i\langle\phi_\alpha(x,t)\phi^*(x',t')\rangle = \begin{bmatrix} G_K(x,t;x',t') & G_R(x,t;x',t') \\ G_A(x,t;x',t') & \underline{0} \end{bmatrix}$$

Upper Triangular Mat. Lower Triangular Mat.

$$G_R(t,t') \sim \Theta(t-t') \qquad G_A(t,t') \sim \Theta(t'-t)$$

"Causality" structure

$$G_R^\dagger = G_A \qquad G_R(t,t) + G_A(t,t) = 0$$

$$G_K^\dagger = -G_K$$ Anti-Hermitian

Dyson Eqn. and Self Energy

$$G_A^{-1} = G_{A0}^{-1} - \Sigma_A \qquad G_R^{-1} = G_{R0}^{-1} - \Sigma_R$$

$$G_{R(A)} = [G_{R(A)0}^{-1} - \Sigma_{R(A)}]^{-1} \qquad G_K = G_R \Sigma_K G_A$$

G_K is directly related to densities and currents in the system

A. Kamenev, Field Theory of Non Eqbm Systems

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Open Quantum Systems

How do Quantum Systems equilibriate?

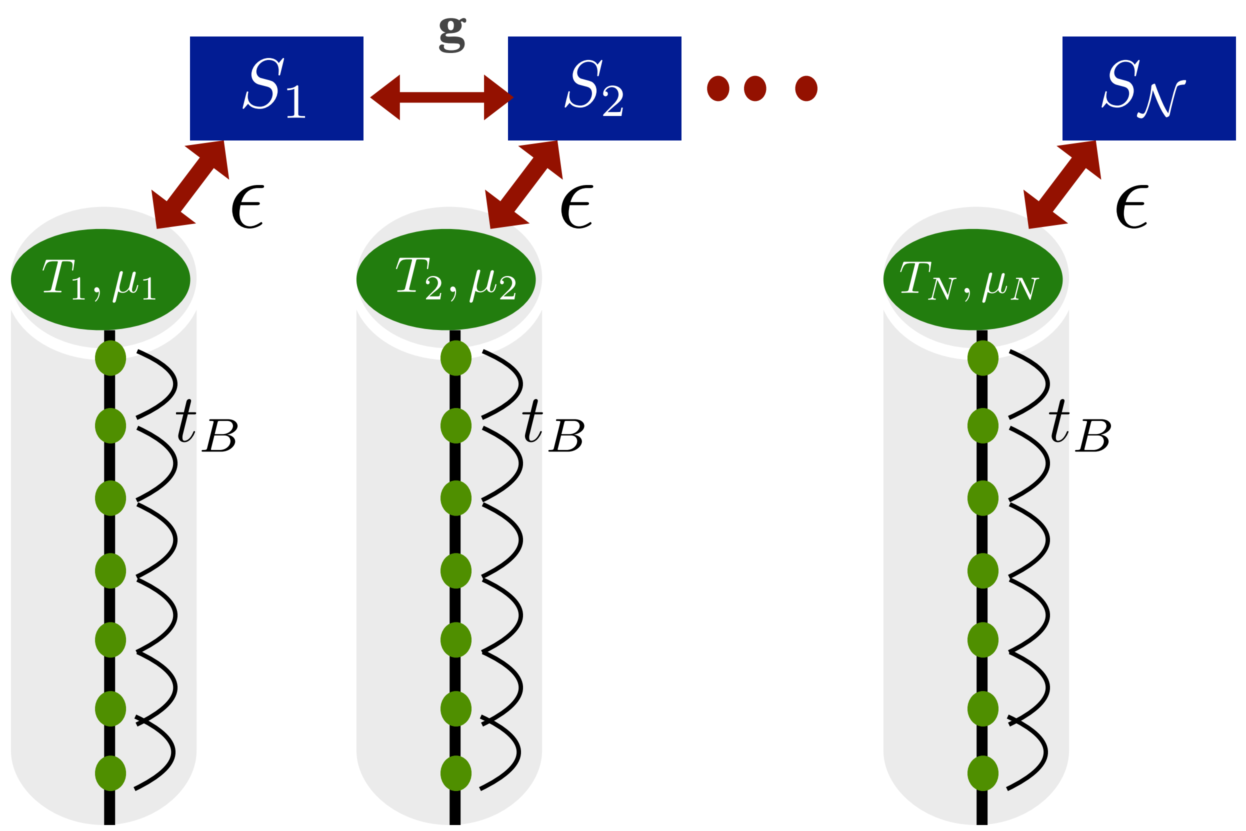
How does Statistical Mechanics emerge out of Q. Mech?

We want to understand how large many body systems behave when coupled to external baths

Experimental Realizations:

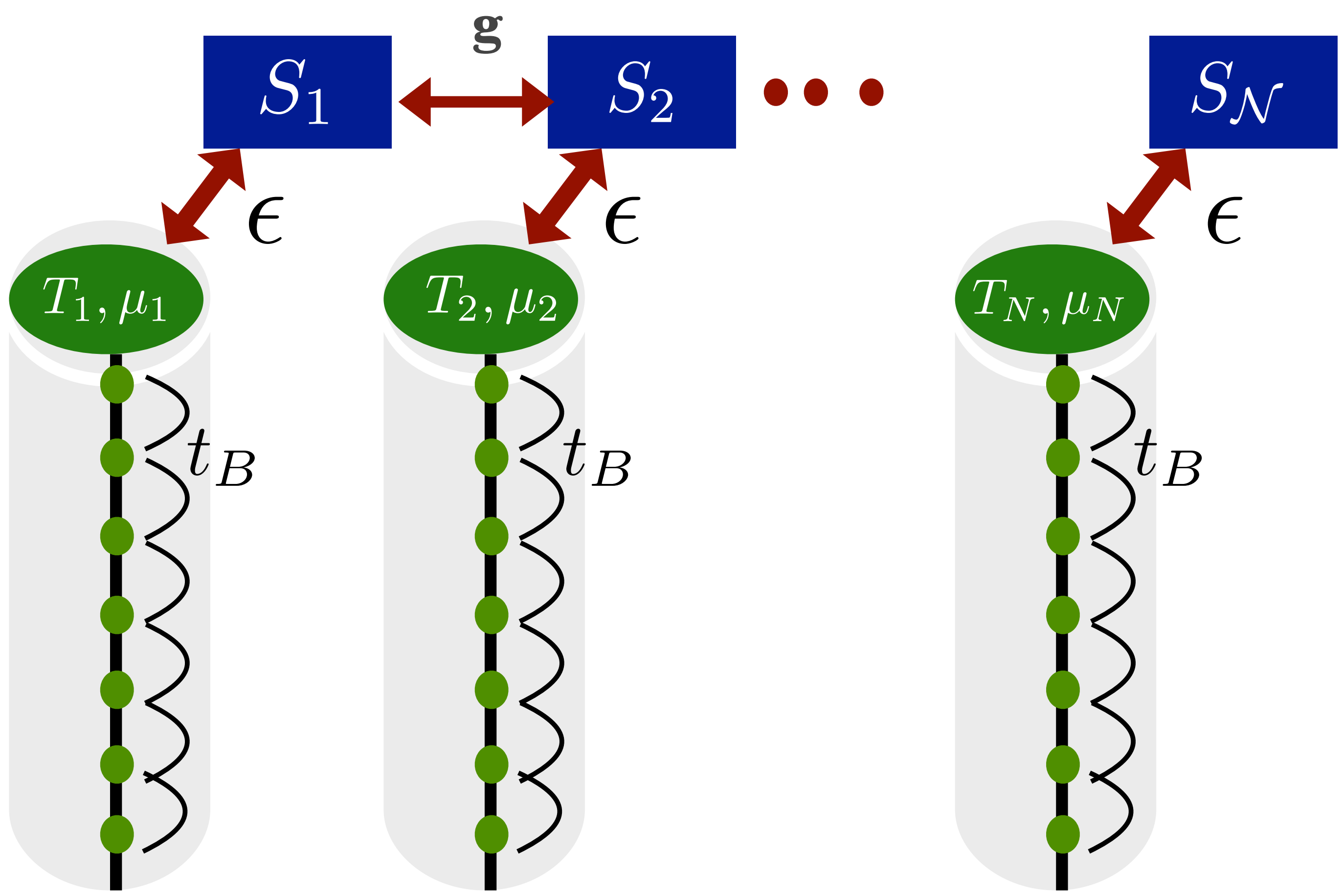
- Cavity QED
- Cold Atoms
- JJ arrays
- Optomechanics
- Quantum dot arrays

Large number of degrees of freedom connected to multiple external baths



A Simple Model

In principle, S can be optical cavity, josephson jn., quantum dot (fermionic version), etc.



$$H_s = -g \sum_r a_r^\dagger a_{r+1} + h.c.$$

$$H_b^r = -t_B \sum_\delta b_\delta^{(r)\dagger} b_{\delta+1}^{(r)} + h.c$$

$$H_{sb} = \epsilon \sum_r a_r b_1^{(r)\dagger} + a_r^\dagger b_1^{(r)}$$

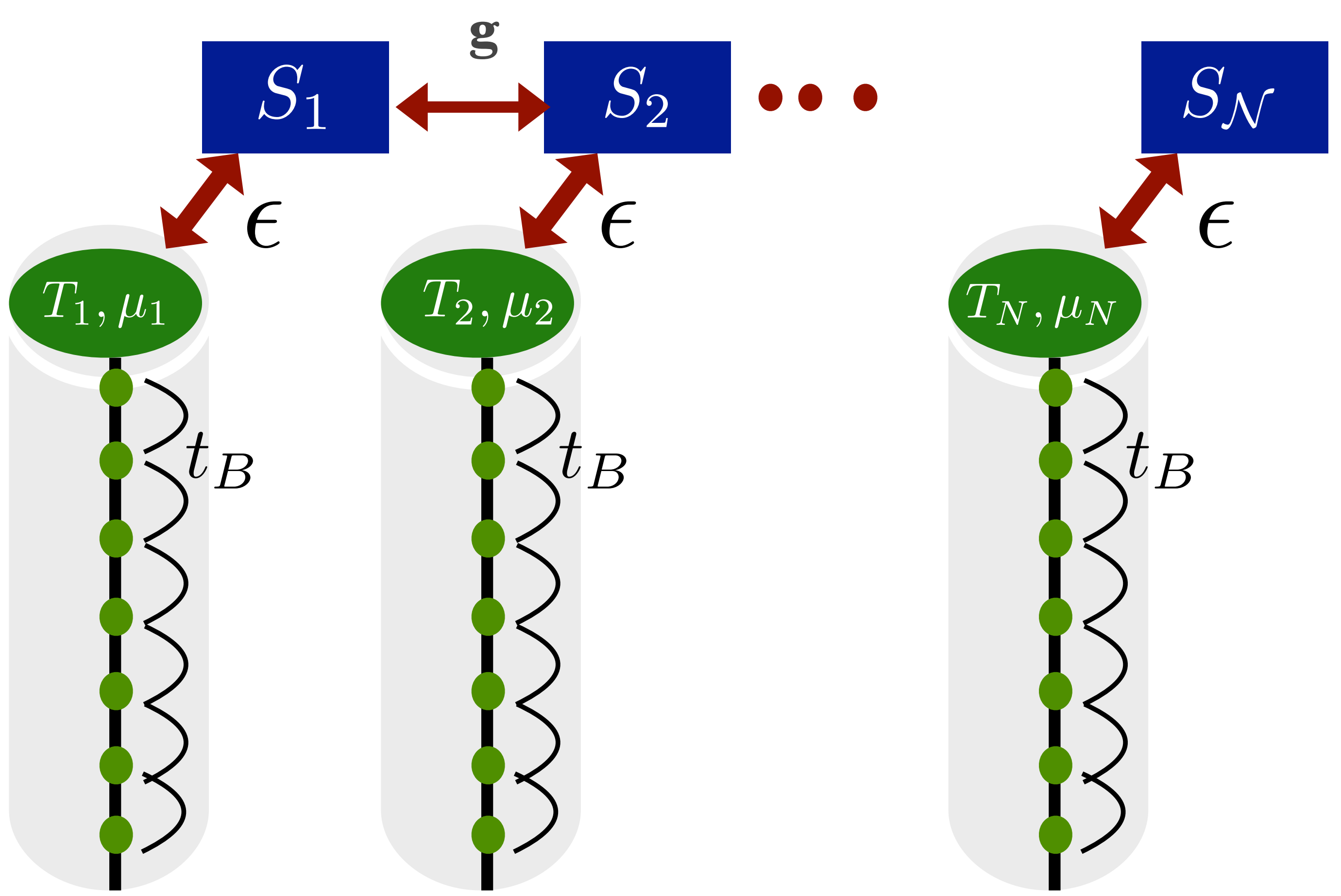
- Bosons hopping on a 1-D lattice with amplitude g
- Each site is coupled to a bath, which is a chain of non-interacting bosons in 1D with hopping strength t_B
- Each bath has its own temperature T_r and chemical potential μ_r
- Linear coupling between system and edge site of bath with coupling strength ε

$g, \varepsilon < t_B$

A. Purakayastha et. al, 2016

A Simple Model

Assumption: bath much larger than system — no feedback on bath



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$$H_{sb} = \epsilon \sum_r a_r b_1^{(r)\dagger} + a_r^\dagger b_1^{(r)}$$

Eigenbasis of bath:

$$H_b^r = \sum_\alpha \Omega_\alpha B_\alpha^{(r)\dagger} B_\alpha^{(r)}$$

$$H_{sb} = \epsilon \sum_{r\alpha} \kappa_\alpha a_r B_\alpha^{(r)\dagger} + \kappa_\alpha^* a_r^\dagger B_\alpha^{(r)}$$

wt. of eigenvector α on site 1 of bath

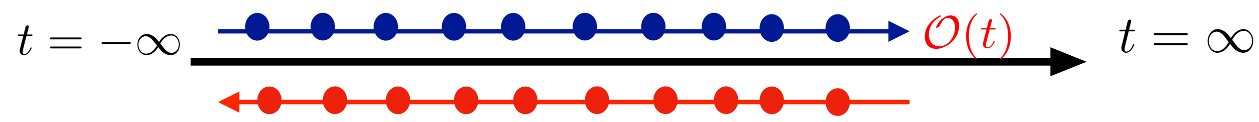
Spectral Density of bath

$$J(\omega) = \textstyle\sum_\alpha |\kappa_\alpha|^2 \delta(\omega - \Omega_\alpha)$$

controls effect of bath on system (in addition to T and μ of the bath)

Interested in steady state dynamics

Keldysh Theory for Simple Model



$$S_s = \int d\omega \sum_{rr'} [\phi^*_{cl}(r,\omega), \phi^*_q(r,\omega)] \begin{bmatrix} 0 & (\omega - i0^+)\delta_{rr'} - g\delta_{r',r\pm1} \\ (\omega + i0^+)\delta_{rr'} - g\delta_{r',r\pm1} & i0^+\rho_0(\omega) \end{bmatrix} \begin{bmatrix} \phi_{cl}(r',\omega) \\ \phi_q(r',\omega) \end{bmatrix}$$

$$S_b = \int d\omega \sum_{r\alpha} [\chi^{(r)*}_{cl}(\alpha,\omega), \chi^{(r)*}_q(\alpha,\omega)] \begin{bmatrix} 0 & \omega - \Omega_\alpha - i0^+ \\ \omega - \Omega_\alpha + i0^+ & i0^+\rho^{(r)}(\omega,\alpha) \end{bmatrix} \begin{bmatrix} \chi^{(r)}_{cl}(\alpha,\omega) \\ \chi^{(r)}_q(\alpha,\omega) \end{bmatrix}$$

$$G_b^{R(A)}(\alpha,\omega) = \frac{1}{\omega - \Omega_\alpha \pm i0^+}$$

$$G_b^K(r,\alpha,\omega) = -i2\pi \coth\left[\frac{\omega - \mu_r}{2T_r}\right]\delta(\omega - \Omega_\alpha)$$

$G^{R(A)}$ indep. of site

G^K has info on distribution fn.
Depends on site.

$$S_{sb} = \int d\omega \sum_{r\alpha} \kappa^*_\alpha [\phi^*_{cl}(r,\omega), \phi^*_q(r,\omega)]\hat{\sigma}^1 \begin{bmatrix} \chi^{(r)}_{cl}(\alpha,\omega) \\ \chi^{(r)}_q(\alpha,\omega) \end{bmatrix} + h.c.$$

☑ Quadratic in bath fields —> Integrate them out

Integrating out the bath

☑ Quadratic in bath fields —> Integrate them out

$$S = \int d\omega \sum_{rr'} \phi^\dagger(r,\omega)\hat{G}^{-1}(r,r',\omega)\phi(r',\omega) \qquad G^{-1} = \begin{bmatrix} 0 & G_A^{-1} \\ G_R^{-1} & \Sigma_K \end{bmatrix}$$

$$G_R^{-1}(r,r',\omega) = \begin{bmatrix} w-\Sigma_R(\omega) & -g & 0 & \cdot & \cdot \\ -g & w-\Sigma_R(\omega) & -g & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -g & w-\Sigma_R(\omega) & -g \\ \cdot & \cdot & \cdot & -g & w-\Sigma_R(\omega) \end{bmatrix}$$

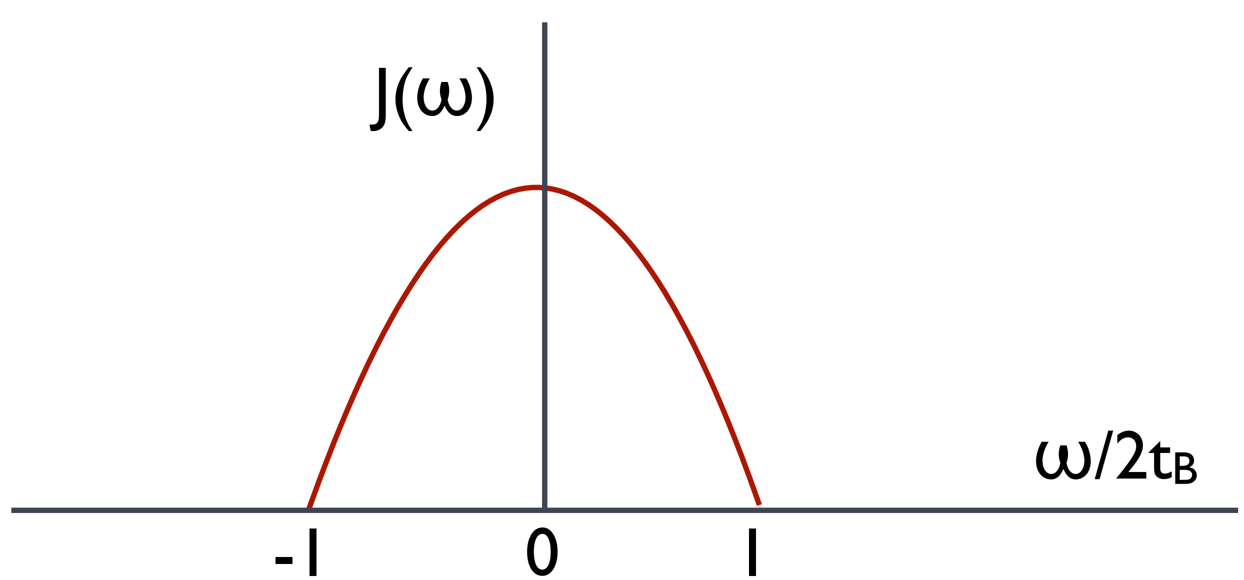
$$\Sigma_R(\omega) = \epsilon^2 \int \frac{d\omega'}{2\pi} \frac{J(\omega')}{\omega-\omega'+i0^+} \qquad \Sigma_K(r,r',\omega) = -i\delta_{rr'}\epsilon^2 J(\omega)\coth\left[\frac{\omega-\mu_r}{2T_r}\right]$$

Spectral Density of bath

$$J(\omega) = \sum_\alpha |\kappa_\alpha|^2 \delta(\omega-\Omega_\alpha)$$

$$J(\omega) = \frac{2}{t_B}\sqrt{1-\left(\frac{\omega}{2t_B}\right)^2}\,\Theta(4t_B^2-\omega^2)$$

for linear chain bath



Self Energies and Eqn. of motion

$$S = \int dt \sum_{rr'} [\phi^*_{cl}(r,t), \phi^*_q(r,t)] \begin{bmatrix} 0 & i\partial_t - \Sigma_A(t-t')\delta_{rr'} - g\delta_{r',r\pm1} \\ i\partial_t - \Sigma_R(t-t')\delta_{rr'} - g\delta_{r',r\pm1} & -\Sigma_K(t-t') \end{bmatrix} \begin{bmatrix} \phi_{cl}(r't') \\ \phi_q(r',t') \end{bmatrix}$$

Auxilliary Fields

(MSRJD in reverse)

$$\langle \zeta(r,t)\zeta^*(r',t') \rangle = -i\Sigma_K(r,t;r',t')$$

$$\int D[\zeta^*,\zeta] \int D[\phi^*_q,\phi_q] exp \Big[-i \int dt dt' \sum_{rr'} \zeta^*(r,t)\Sigma_K^{-1}(r,t;r',t')\zeta(r',t')$$

$$+ \int dt \sum_r \zeta^*(r,t)\phi_q(r,t) + \zeta(r,t)\phi^*_q(r,t) \Big]$$

Classical Saddle Point: $\left. \frac{\partial S}{\partial \phi^*_q} \right|_{\phi^*_q = \phi_q = 0} = 0$

$$i\partial_t \phi(r,t) - g[\phi(r+1,t) + \phi(r-1,t)] - \int dt' \Sigma_R(t-t')\phi(r,t') = \zeta(r,t)$$

Real part : dressed dispersion

Im part : dissipation

Complex noise

Non-Markovian dynamics: Power Law Kernels

$$\Sigma_R(t-t') = -i\epsilon^2\Theta(t-t')F.T.[J(\omega)] = -i\frac{\epsilon^2}{t_B}\Theta(t-t')\frac{J_1[2t_B(t-t')]}{t-t'} \qquad 2t_B(t-t') \gg 1$$

Purely imaginary Σ_R —> dissipative term

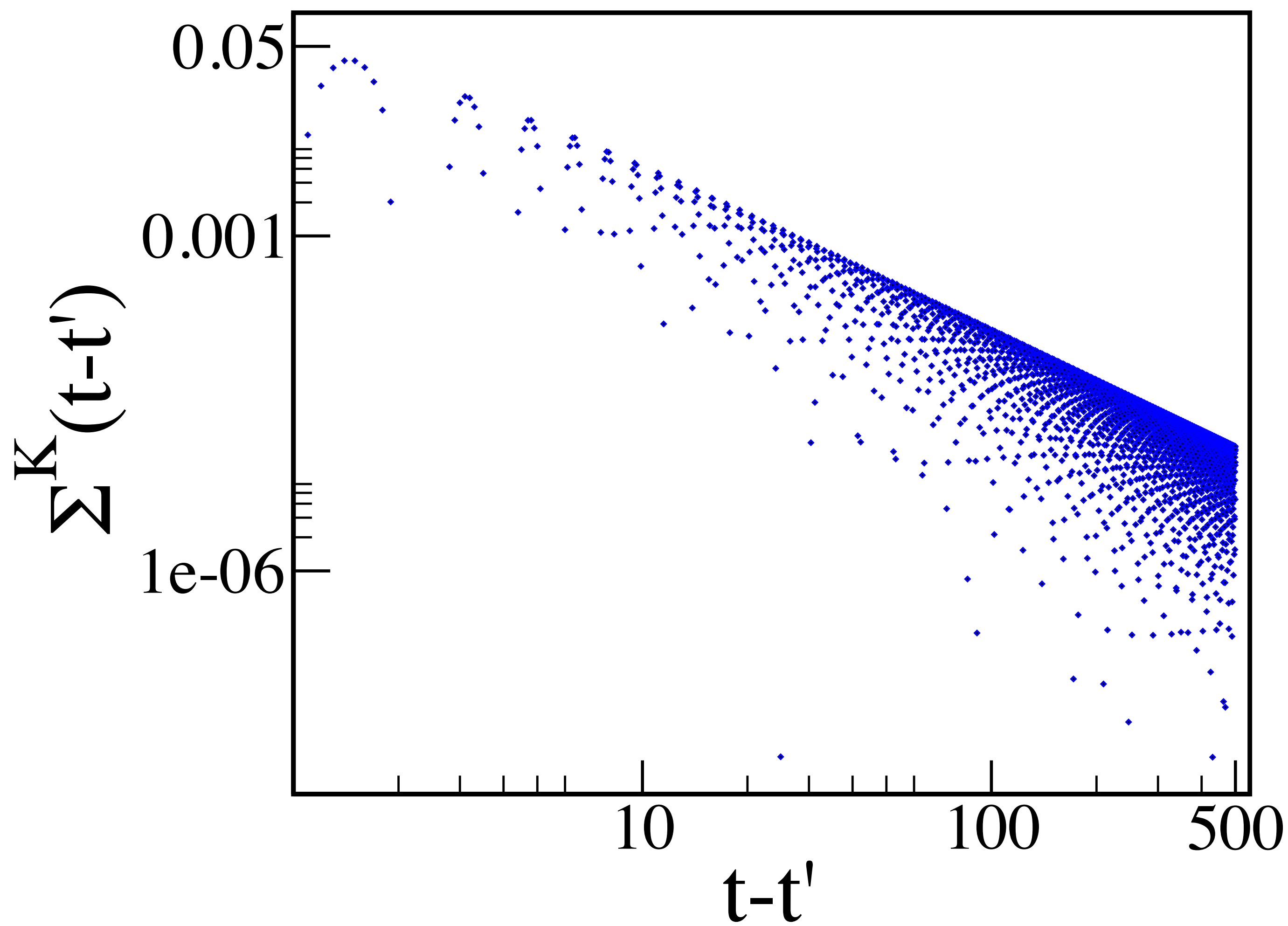
$$\sim -i\frac{\epsilon^2}{t_B^{3/2}}\Theta(t-t')\frac{\cos[2t_B(t-t')-3\pi/4]}{(t-t')^{3/2}}$$

Power law dissipative kernel

$$\Sigma_K(r, t-t') = -i\epsilon^2 F.T.\left[J(\omega)\coth\left[\frac{\omega-\mu_r}{2T_r}\right]\right]$$

$$\sim \frac{\epsilon^2(1-i)}{\sqrt{\pi}}\frac{1}{|2t_B(t-t')|^{3/2}}\left[e^{-i2t_B|t-t'|}\left[\frac{2t_B-\mu_r}{2T_r}\right] + e^{i2t_B|t-t'|}\left[\frac{-2t_B-\mu_r}{2T_r}\right]\right]$$

Power law noise kernel

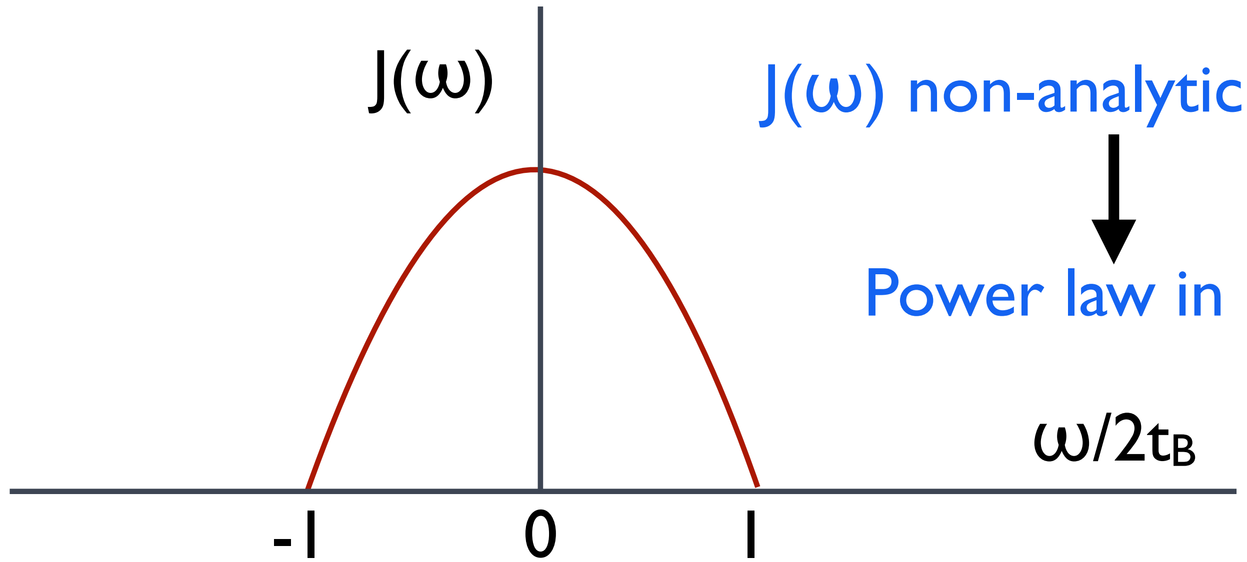


Cannot coarse grain over time to obtain Markovian dynamics

Essentially non-Markovian dynamics

Origin of power law tails

$$\Sigma_R(t-t') = -i\epsilon^2\Theta(t-t')F.T.[J(\omega)] \qquad \Sigma_K(r,t-t') = -i\epsilon^2 F.T.\left[J(\omega)\coth\left[\frac{\omega-\mu_r}{2T_r}\right]\right]$$



J(ω) non-analytic at $\omega = \pm\, 2t_B$

↓

Power law in FT

c.f. Friedel Oscillations

Bosonic Systems with no. conservation —> spectrum bounded from below

Ubiquitous non-Markovian dynamics

- Band Edges, Van Hove singularities
- Kohn Anomalies
- Kondo lattice

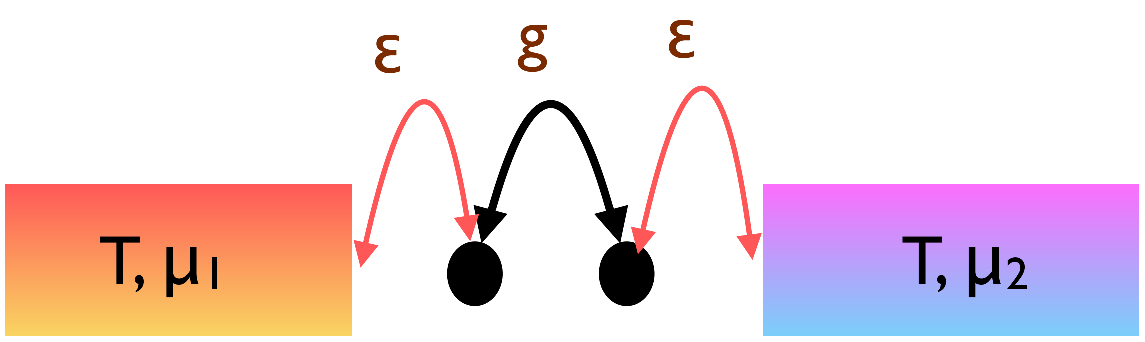
Long time behaviour sensitive to nature and location of non-analyticity in bath spectral function.

Can be used to probe singularities in the bath DOS.

Non-Markovian dynamics: Green's functions

A. Purakayastha et. al, 2016

Two site model with potential difference



$$G^R_{12}(\omega) = \frac{g}{[\omega - \Sigma^R(\omega)]^2 - g^2}$$

Poles from denom .

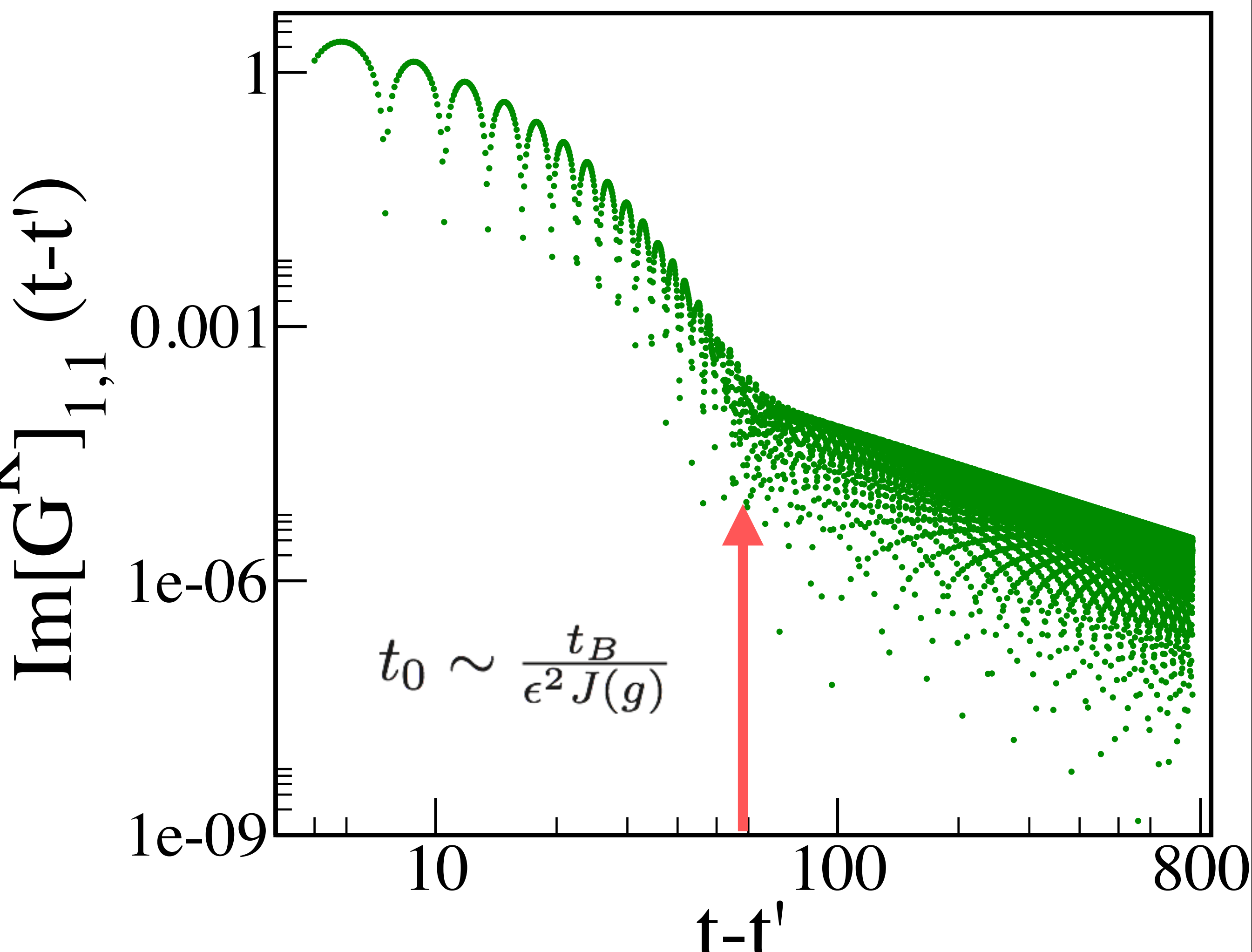
G inherits non-analyticity of ∑.

Crossover Timescale $t_0 \sim \frac{t_B}{\epsilon^2 J(g)}$

exponential decay "Markovian" part

Non-Markovian power law tail

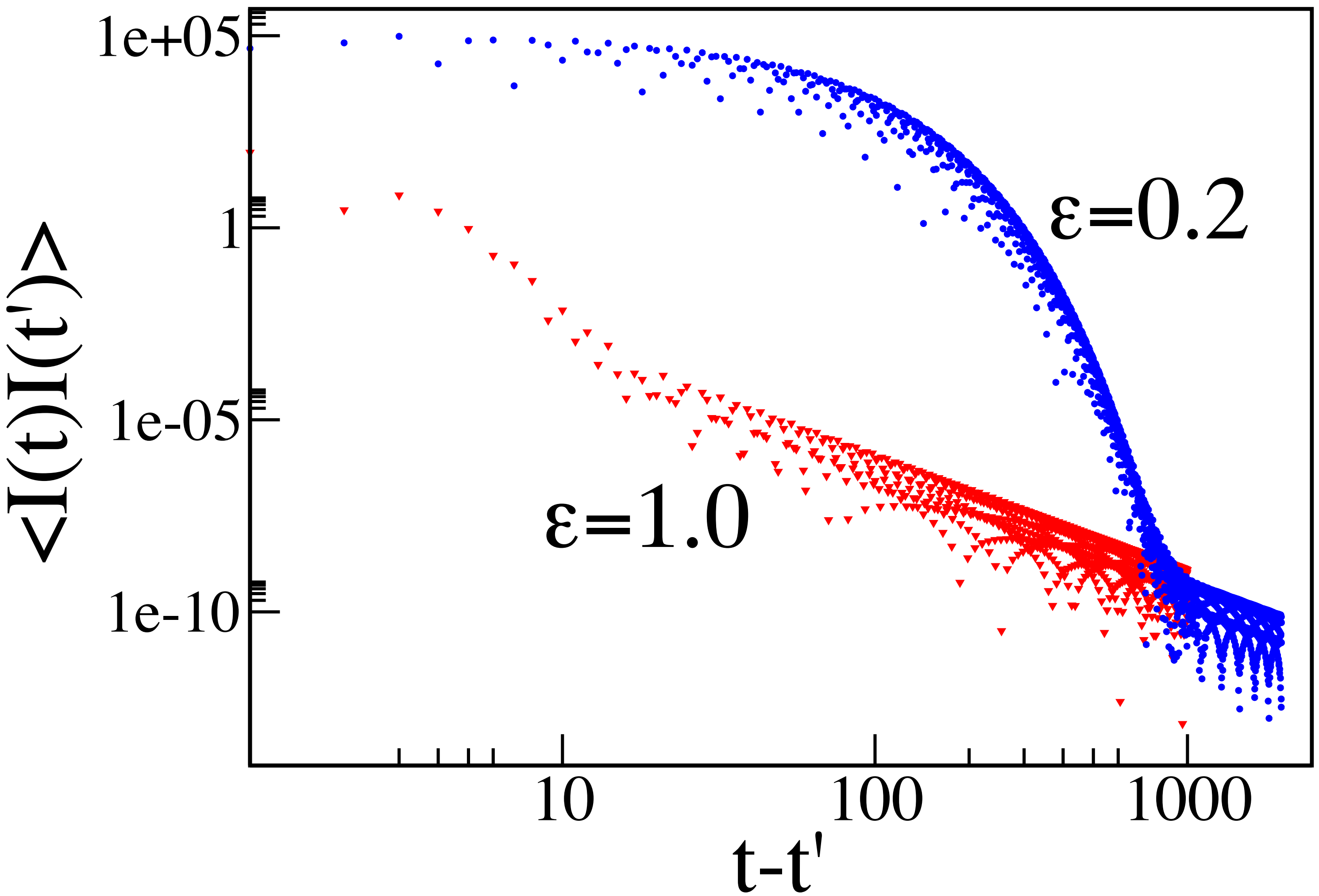
$$G^K_{12}(\omega) = G^R_{11}(\omega)\Sigma^K_{12}(\omega)G^A_{22}(\omega) + \ldots$$

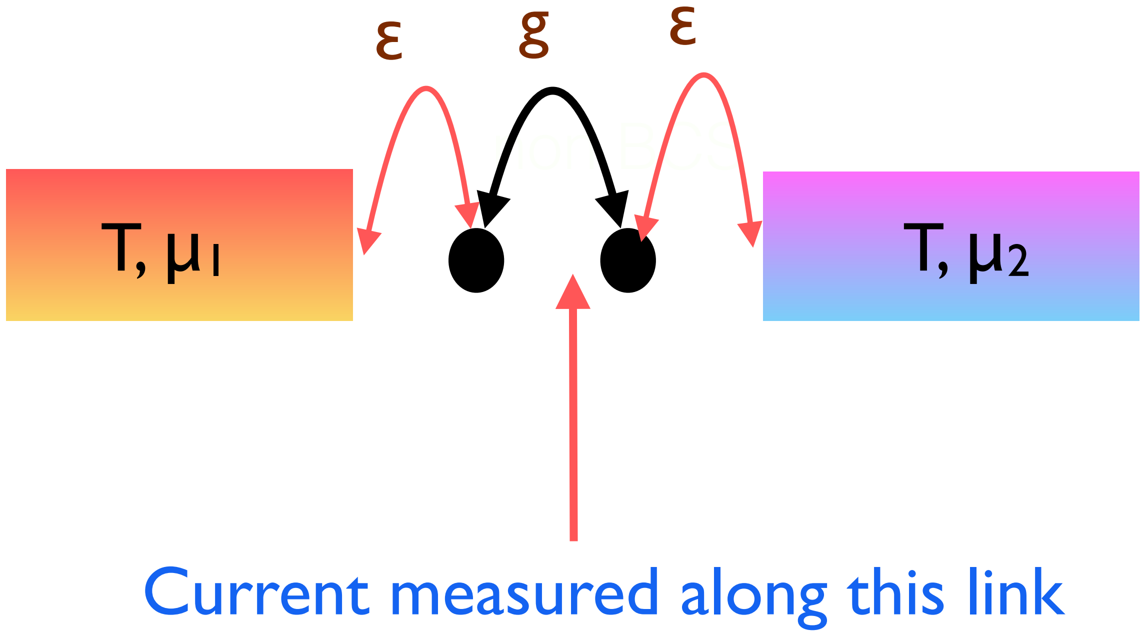


- ☑ Pole structure inherited from G^R and G^A.
- ☑ Prefactor (residue) is T dependent.
- ☑ Crossover timescale weakly temperature dependent.

Non-Markovian dynamics: Observables

- Local quantities like current do not show qualitatively different behaviour from Markovian dynamics
- Unequal time correlators like < I(t) I(t')> inherit the exponential +power law structure
- Large system bath coupling —> faster decay, but easier to see power law behaviour



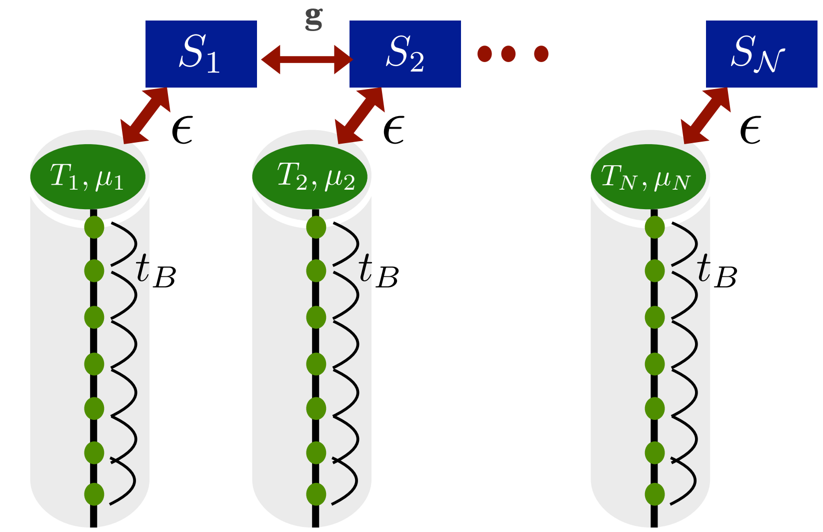


Noise measurements

Solution for the full chain

$$\Sigma_R(\omega) = \epsilon^2 \int \frac{d\omega'}{2\pi} \frac{J(\omega')}{\omega-\omega'+i0^+}$$

$$G_R^{-1}(r, r', \omega) = \begin{bmatrix} w - \Sigma_R(\omega) & -g & 0 & \cdot & \cdot \\ -g & w - \Sigma_R(\omega) & -g & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -g & w - \Sigma_R(\omega) & -g \\ \cdot & \cdot & \cdot & -g & w - \Sigma_R(\omega) \end{bmatrix}$$

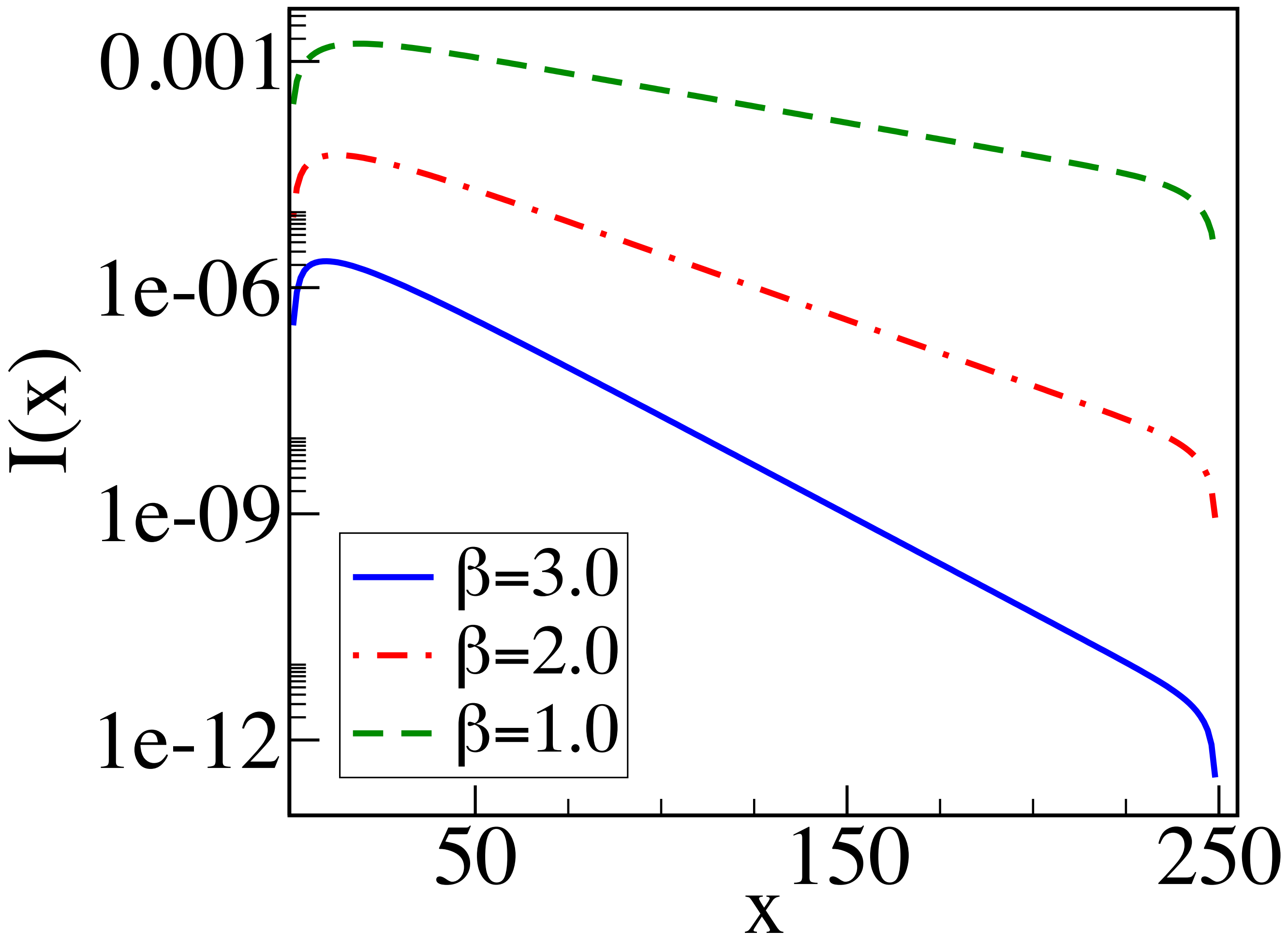


$$G^R(r, r', \omega) = (-1)^{r+r'} \frac{\cosh[(N+1-|r-r'|)\lambda] - \cosh[(N+1-r-r')\lambda]}{2\sinh\lambda \sinh[(N+1)\lambda]}$$

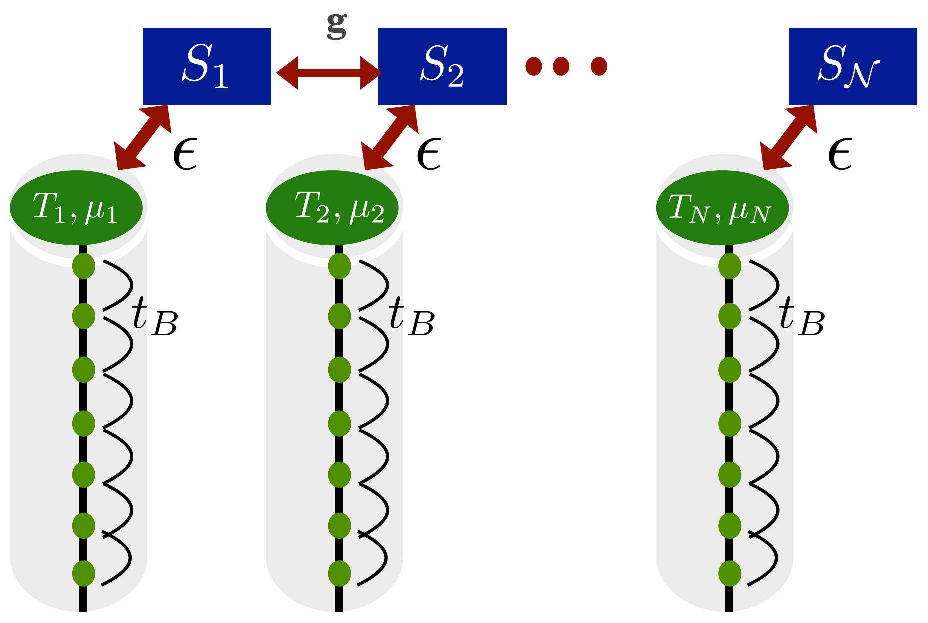
$$\cosh\lambda = \frac{\omega - \Sigma^R(\omega)}{2g}$$

$$G^K(r, r', \omega) = -i\epsilon^2 J(\omega) \sum_{l=1}^{N} G^R(r, l, \omega) \coth\left[\frac{\omega-\mu_l}{2T}\right] G^{*R}(r', l, \omega)$$

Exponential Decay of Current



Fixed T, constant gradient in μ



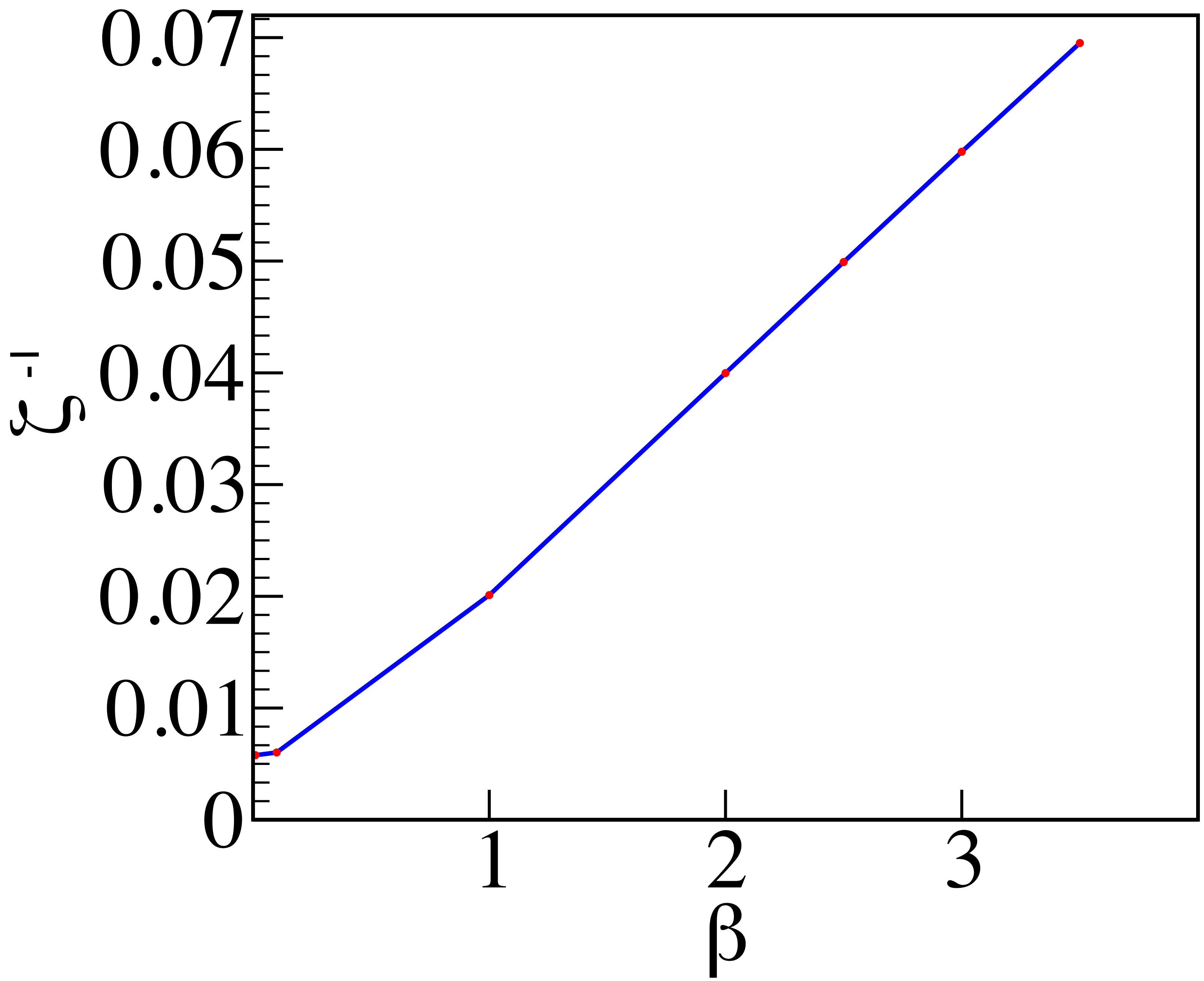
Current decays with distance

Localization length increases with temp.

Steady Current as T —> ∞

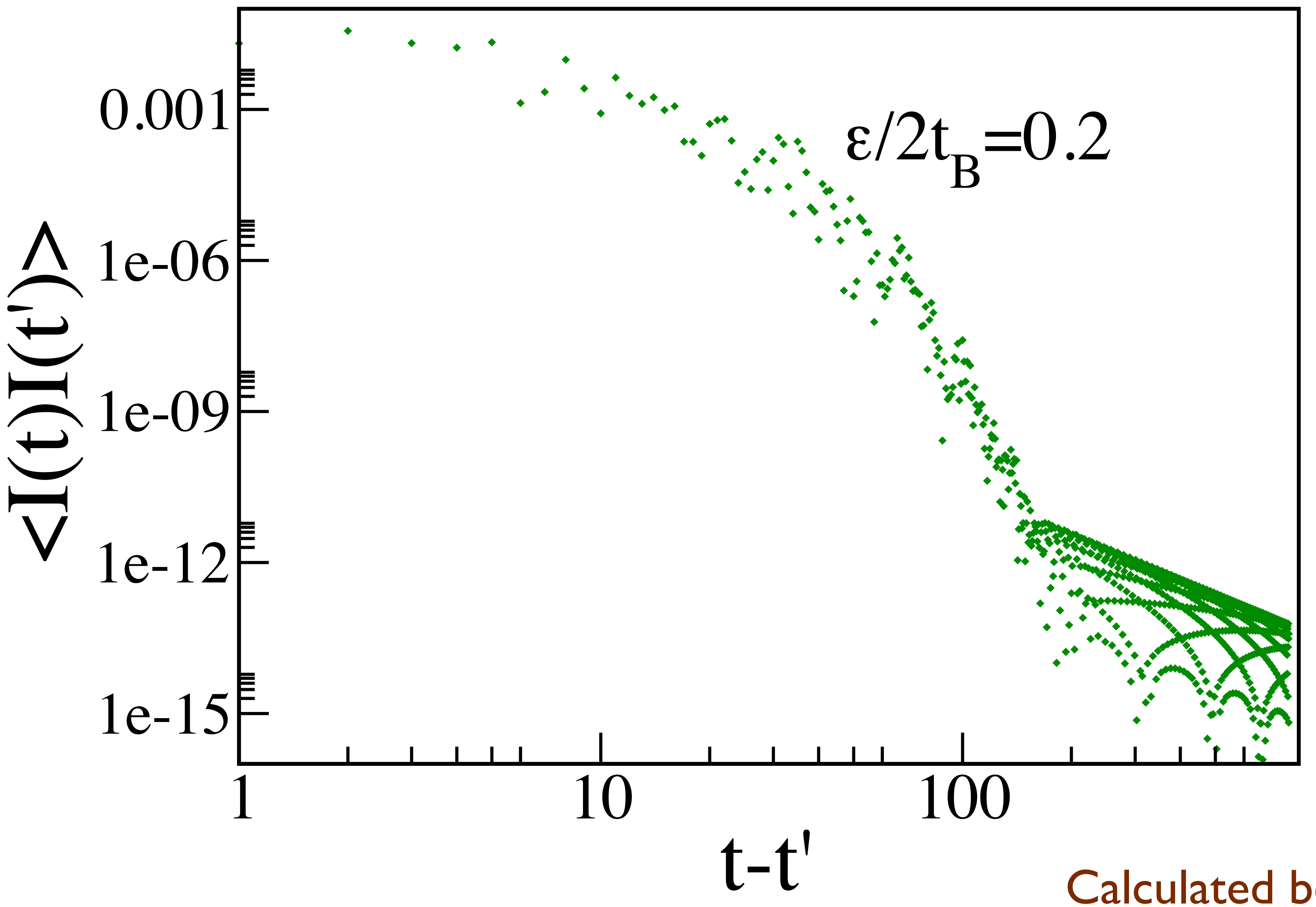
Localization Length

Localization Length scales linearly with temperature of the baths



Non-Markovian Dynamics in a chain

Current -Current correlators show long time power law behaviour



N = 50

Calculated between 16th and 18th links

Summary I

Non-analyticity in bath spectral spectral functions lead to Non-Markovian dynamics in open Quantum Systems

This shows up as power laws in Dissipation and Noise Kernels (Self Energies)

Green's functions and current current correlators show an exponential decay followed by a power law tail. Exponent controlled by nature of non-analyticity.

In a Bosonic chain coupled to individual baths, the current decays with size of system. Even in presence of non-Markovian dynamics, localization effects are seen.

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Keldysh action for interacting Bosons

$$Z = \int D[\phi_{cl}]D[\phi_q]e^{i(S_0+S_{int})}$$

Real scalar fields (coupled to Ohmic bath)

$$S_0 = \int d^dx \int dt \int d^dx' \int dt' [\phi_{cl}(x,t), \phi_q(x,t)] G^{-1}(x,t;x',t') \begin{bmatrix} \phi_{cl}(x',t') \\ \phi_q(x',t') \end{bmatrix}$$

$$G^{-1} = \begin{bmatrix} 0 & G_A^{-1} \\ G_R^{-1} & \Sigma_K \end{bmatrix}$$

$$\Sigma_K = i\delta(x-x')\delta(t-t')m_q^2$$

Keldysh Self Energy —> noise

$$G_R^{-1}(x,t;x',t') = \Theta(t-t')[\delta(x-x')\delta(t-t')(-\partial_t^2 + \nabla^2 - m_R^2 + \gamma\partial_t)]$$

Dissipation comes with opposite sign

$$G_A^{-1}(x,t;x',t') = \Theta(t'-t)[\delta(x-x')\delta(t-t')(-\partial_t^2 + \nabla^2 - m_R^2 - \gamma\partial_t)]$$

Eqn. of motion $[\partial_t^2 - \gamma\partial_t - \nabla^2 + m_R^2]\phi(x,t) = \eta(x,t)$

Langevin noise

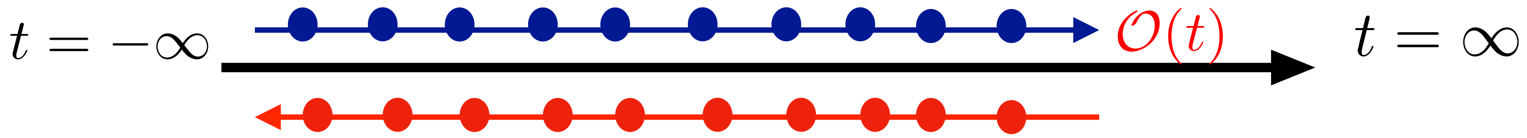
$$\langle \eta(x,t)\eta(x',t')\rangle = \delta(x-x')\delta(t-t')m_q^2$$

Keldysh action for interacting Bosons

Real scalar fields (possibly coupled to Ohmic bath)

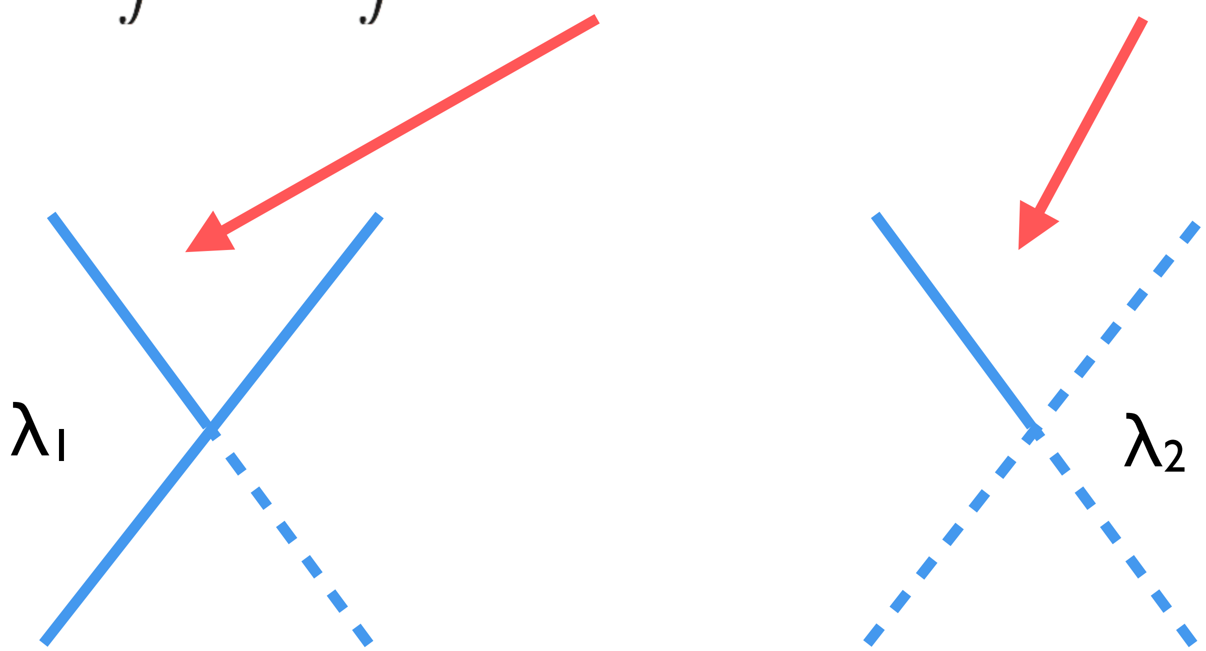
$$Z = \int D[\phi_{cl}]D[\phi_q]e^{i(S_0+S_{int})}$$

$$S_{int} = \int dt\lambda(t)\int d^dx\ [\phi_+(x,t)]^4$$

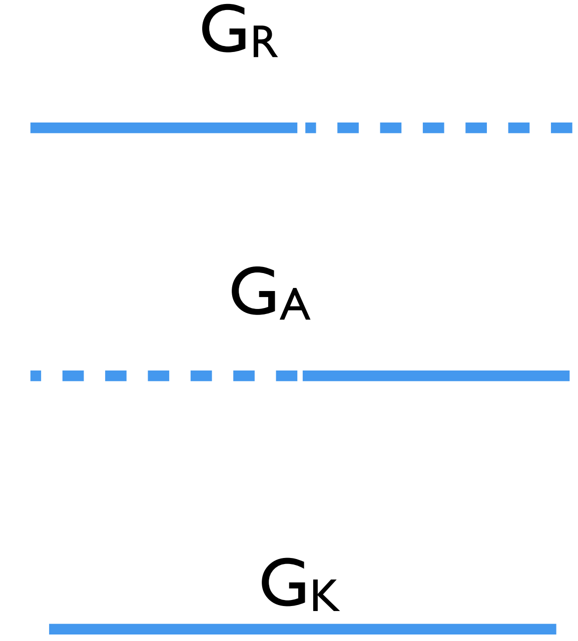


$$S_{int} = -\int dt\lambda(t)\int d^dx\ [\phi_-(x,t)]^4$$

$$S_{int} = \int dt\lambda(t)\int d^dx[\phi_{cl}^3(x,t)\phi_q(x,t) + \phi_{cl}(x,t)\phi_q^3(x,t)]$$

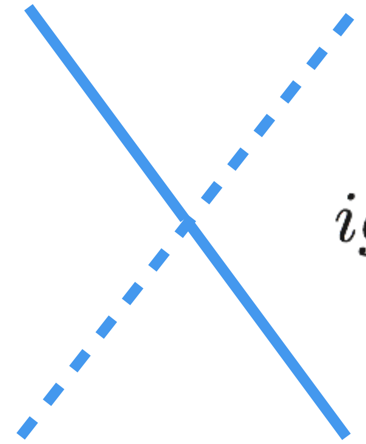


Green's Functions

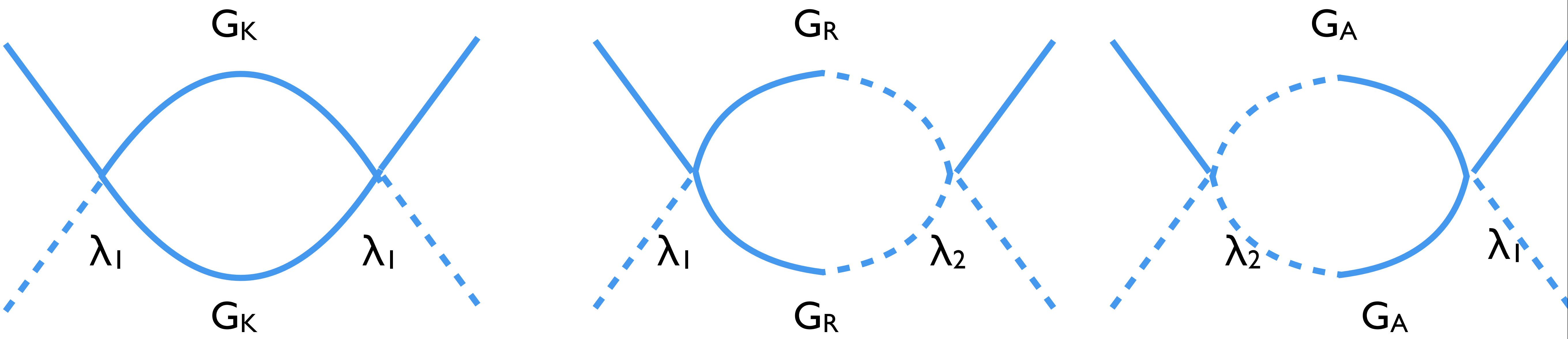


Loop Expansion for interaction vertices

Generation of new vertices



$$ig_3\phi_{cl}^2(x,t)\phi_q^2(x,t)$$



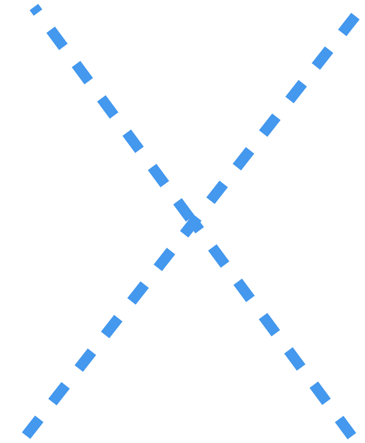
$+ \;\ldots$

Generated from microscopic couplings

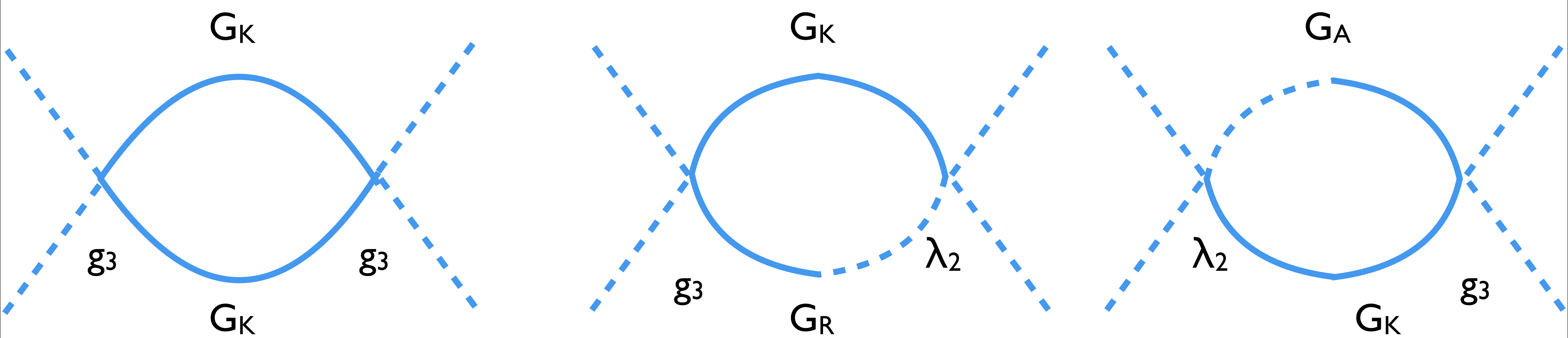
Diagrams sum to zero for T=0 adiabatic dynamics (i.e. ground state descriptions)

Loop Expansion for interaction vertices

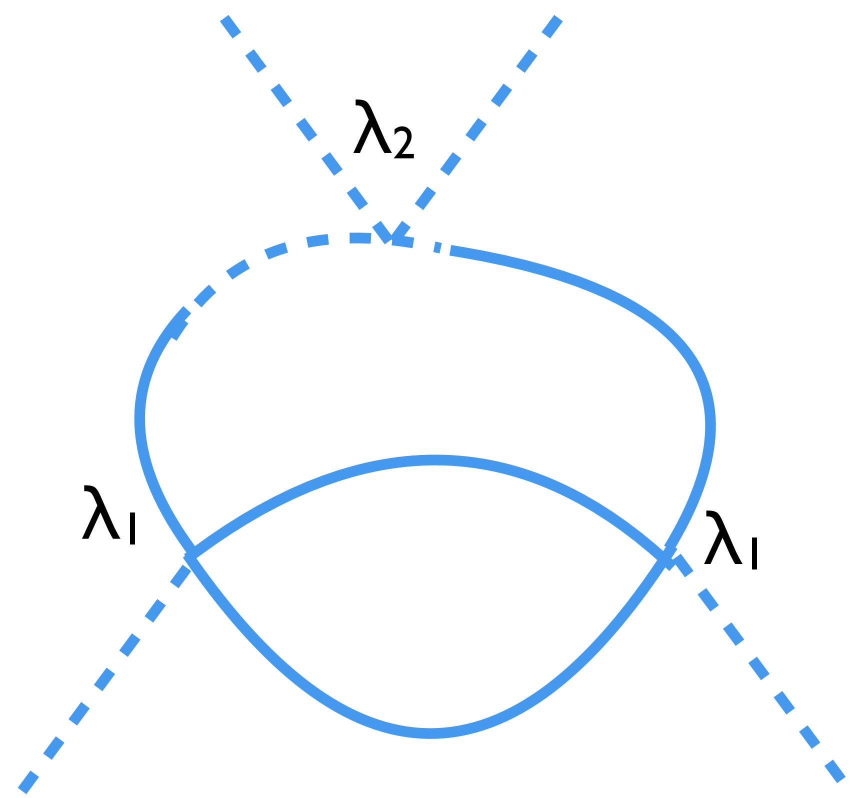
Generation of new vertices



$ig_4\phi_q^4(x,t)$

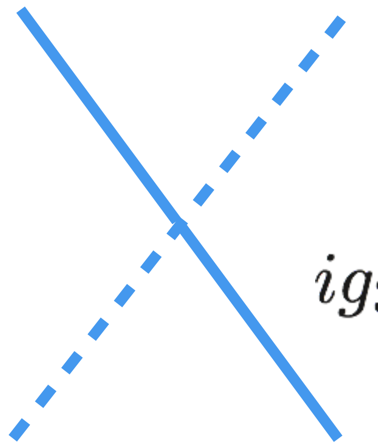


+ . . .



g_3 needs to be generated first … higher loop in microscopic couplings

Interaction vertices and Eqn. of motion



$ig_3\phi_{cl}^2(x,t)\phi_q^2(x,t)$

What does this interaction vertex represent in terms of Eq. of motion?

Even powers of ϕ_q —> Hubbard Stratanovich

$$\int D[\phi] e^{i(ig_3)\int dt \int d^dx\, \phi_{cl}^2(x,t)\phi_q^2(x,t)} = \int D[\phi] \int D[\zeta_1] e^{\int dt \int d^dx - \frac{\zeta_1^2(x,t)}{2g_3} + i\zeta_1(x,t)\phi_{cl}(x,t)\phi_q(x,t)}$$

Noisy Mass/ Frequency

Multiplicative noise

Saddle point Eqn. of motion

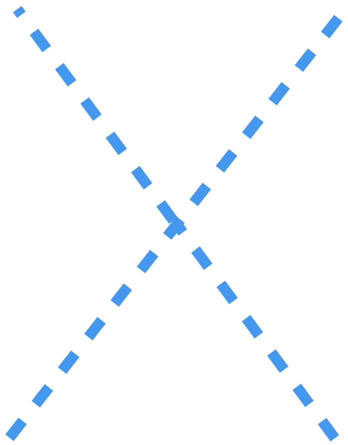
$$[\partial_t^2 + \gamma\partial_t - \nabla^2 + m_R^2]\phi(x,t) = \eta(x,t) + \zeta_1(x,t)\phi(x,t)$$

$$\langle \zeta_1(x,t)\zeta_1(x',t')\rangle = \delta(x-x')\delta(t-t')g_3$$

Delta correlated gaussian noise

Strength g_3

Interaction vertices and Eqn. of motion



$ig_4\phi_q^4(x,t)$

Hubbard Stratanovich

$$\int D[\phi]e^{i(ig_4)\int dt\int d^dx\phi_q^4(x,t)} = \int D[\phi]\int D[\zeta_2]e^{\int dt\int d^dx - \frac{\zeta_2^2(x,t)}{2g_4} + i\zeta_2(x,t)\phi_q^2(x,t)}$$

Hubbard Stratanovich once more

$$= \int D[\phi]D[\zeta]\int D[\zeta_2]\prod_{x,t}(\zeta_2^{-1/2}(x,t))e^{\int dt\int d^dx - \frac{\zeta_2^2(x,t)}{2g_4} - i\frac{\zeta^2(x,t)}{4\zeta_2(x,t)} + i\zeta(x,t)\phi_q(x,t)}$$

$$= \int D[\phi]D[\zeta]F(\zeta)e^{i\int dt\int d^dx\zeta(x,t)\phi_q(x,t)} \qquad F(\zeta) = \int_{-\infty}^{\infty} d\zeta_2\zeta_2^{-1/2}e^{-\zeta_2^2/2g_4 - i\zeta^2/4\zeta_2}$$

Saddle point Eqn. of motion

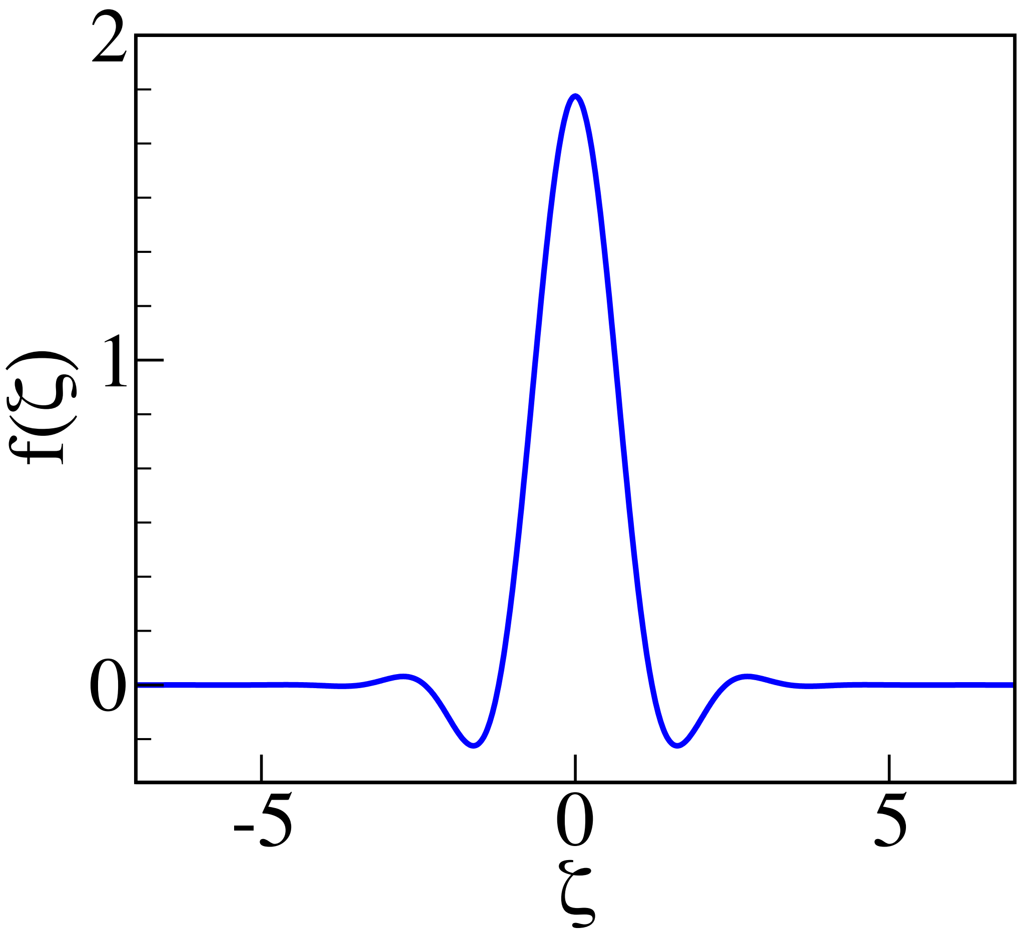
$$[\partial_t^2 + \gamma\partial_t - \nabla^2 + m_R^2]\phi(x,t) = \eta(x,t) + \zeta_1(x,t)\phi(x,t) + \zeta(x,t)$$

Source noise with non-Gaussian distribution F[ζ]

Universal Quasiprobability Distribution

$$F(\zeta) = \int_{-\infty}^{\infty} d\zeta_2 \zeta_2^{-1/2} e^{-\zeta_2^2/2g_4 - i\zeta^2/4\zeta_2}$$

$$F[\zeta] = 2g_4^{1/4}\left[\Gamma(\tfrac{5}{4})_0F_2\left[\{\},\{\tfrac{1}{2},\tfrac{3}{4}\},\frac{\zeta^4}{4g_4}\right] - \frac{\zeta^2}{\sqrt{g_4}}\Gamma[\tfrac{3}{4}]_0F_2\left[\{\},\{\tfrac{5}{4},\tfrac{3}{2}\},\frac{\zeta^4}{4g_4}\right]\right]$$



Universal Distribution at low energies

Characterized by single param g_4

E. Wigner, 1932

Distribution is negative for some values of ζ—> quasiprobability distrn.

This is a completely quantum term with no classical analogue

$$\int d\zeta F[\zeta] = \pi\sqrt{\frac{g_4}{2}}$$

$$\int d\zeta F[\zeta]\zeta = \int d\zeta F[\zeta]\zeta^2 = \int d\zeta F[\zeta]\zeta^3 = 0$$

$$\int d\zeta F[\zeta]\zeta^4 = -\frac{3\pi g_4^{3/2}}{8\sqrt{2}}$$

J. Dalibard, Y. Castin, K. Molmer, 1992
R. Dum, P. Zoller, and H. Ritsch, 1992
A. Polkovnikov, 2009

Summary II

Interacting open bosonic systems can be systematically treated within Keldysh theory

Loop Expansion generates new interaction vertices in the theory.

The new vertices are equivalent to different kind of noise in Eqn. of motion.

In addition to gaussian multiplicative noise, a “quantum” noise term is generated.

It has universal quasiprobability distribution in the low energy limit, which is not positive definite.

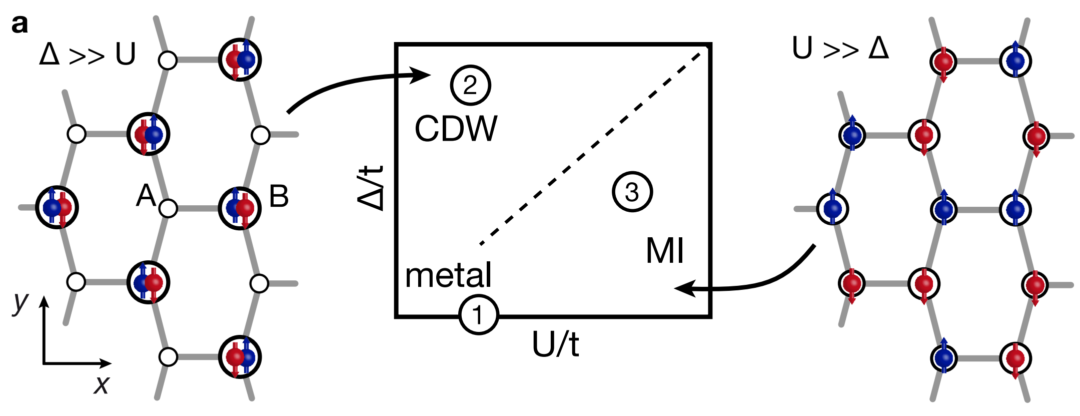
First three moments of this distribution vanish, while the fourth moment is negative.

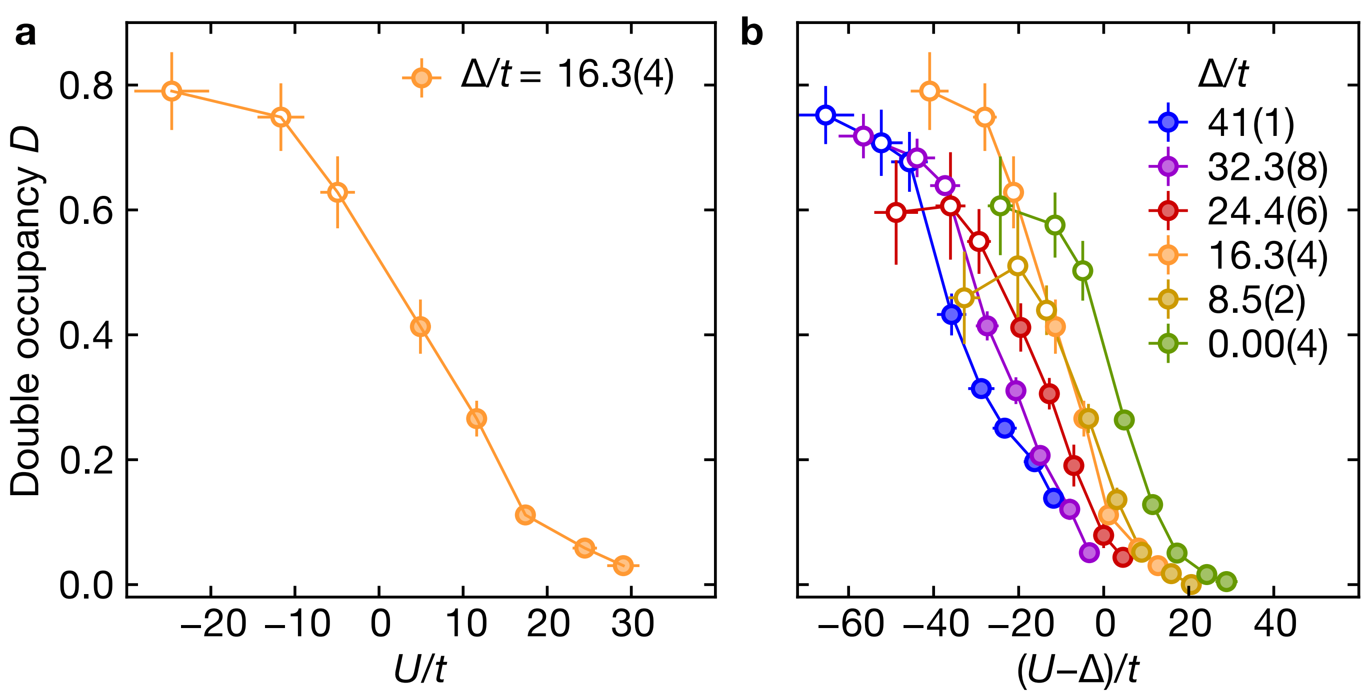
Motivation : III

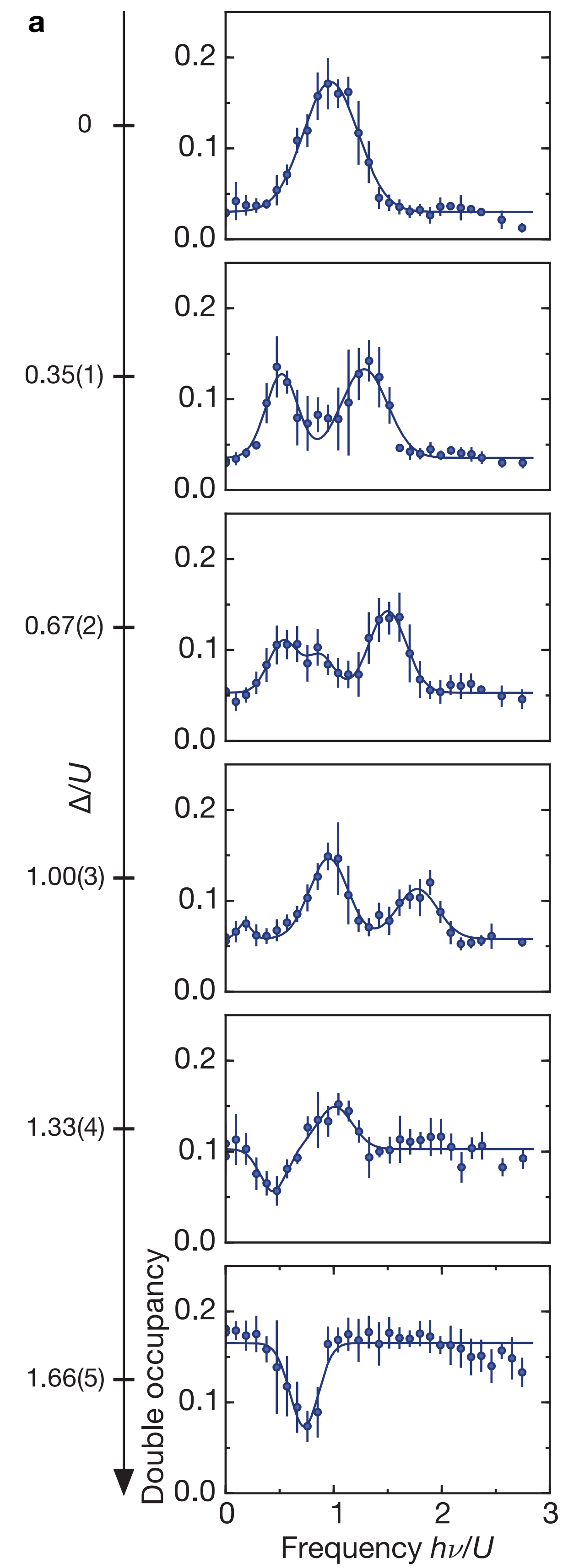
M. Messer, R. Desbuquois, T. Uehlinger, G. Jotzu, S. Huber, D. Greif, and T. Esslinger
PRL 115, 115303 (2015)

c) Cold Atom Implementation of Ionic Hubbard Model

Measurements of energy scales U+V and U-V







$$Z = \int D[\phi_{cl}]D[\phi_q]e^{i(S_0+S_{int})}$$

$$S_0 = \int d^dx \int dt \int d^dx' \int dt' [\phi_{cl}(x,t), \phi_q(x,t)] G^{-1}(x,t;x',t') \begin{bmatrix} \phi_{cl}(x',t') \\ \phi_q(x',t') \end{bmatrix}$$

$$G^{-1} = \begin{bmatrix} 0 & G_A^{-1} \\ G_R^{-1} & \Sigma_K \end{bmatrix}$$

$$G_R^{-1}(x,t;x',t') = \Theta(t-t')[\delta(x-x')\delta(t-t')(-\partial_t^2 + \nabla^2) - \Sigma_R(x,t;x',t')]$$

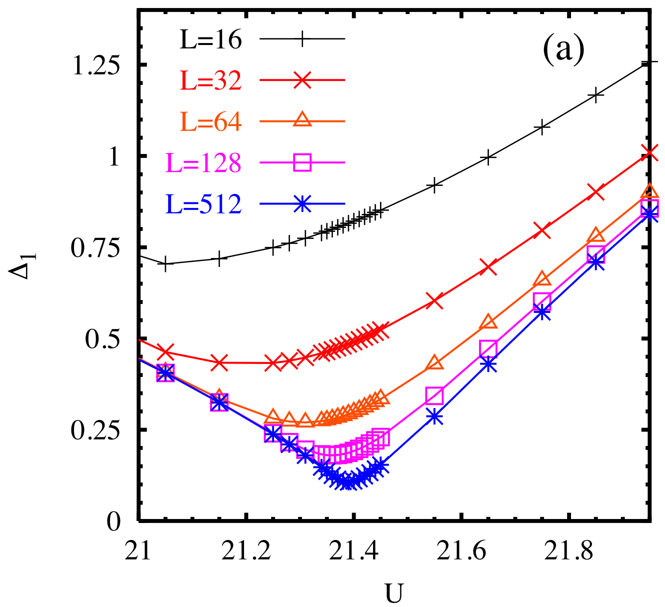
$$G_A^{-1}(x,t;x',t') = \Theta(t'-t)[\delta(x-x')\delta(t-t')(-\partial_t^2 + \nabla^2) - \Sigma_A(x,t;x',t')]$$

$$S_{int} = \frac{1}{4}\int dt \int d^dx \; g(t)[\phi_{cl}^3(x,t)\phi_q(x,t) + \phi_{cl}(x,t)\phi_q^3(x,t)]$$

What do we know?

J. Hubbard and J.B. Torrence, PRL 1981
N. Nagaosa and J. Takimoto, JPSJ 1986

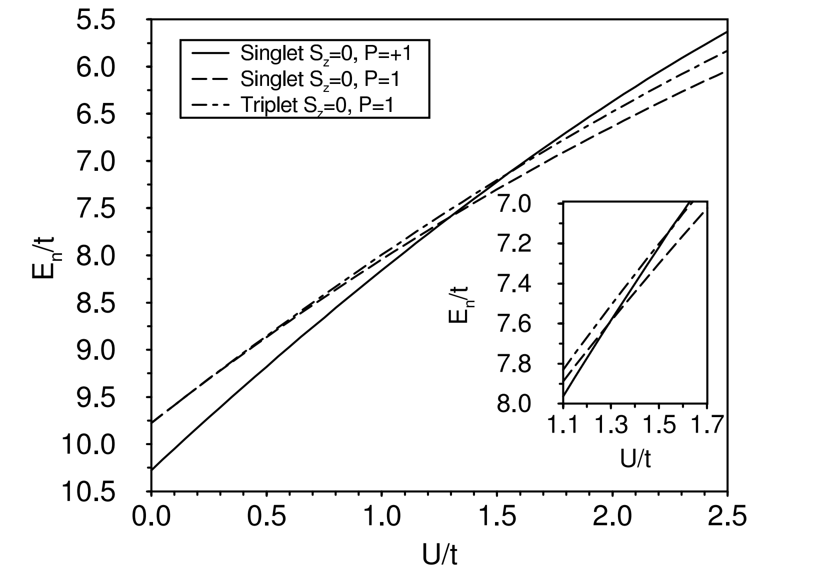
1 D Ionic Hubbard Model



I particle spectrum is gapped

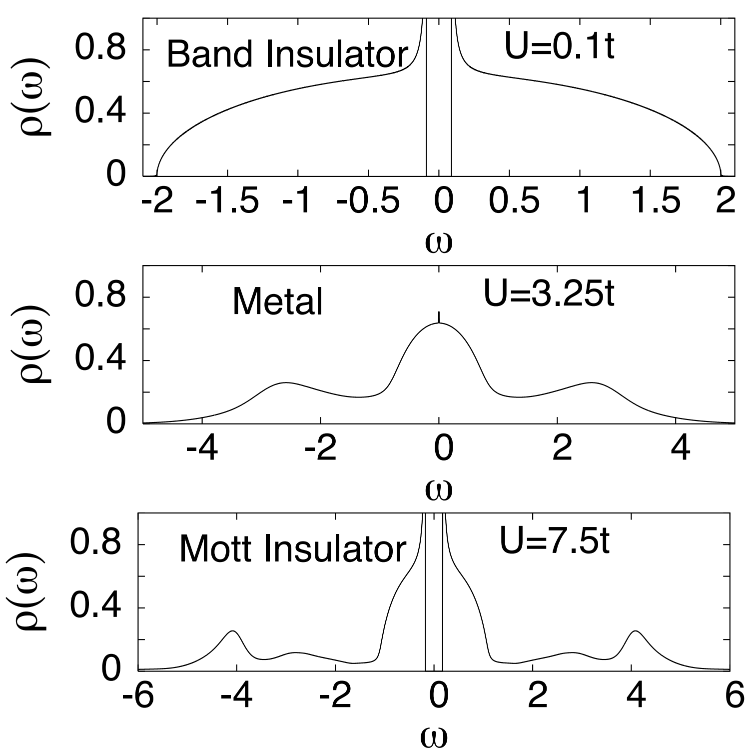
*S. R. Manmana et. al PRB **70,** 155115 (2004)*

*A P Kampf et. al. JPCM **15**, 5895 (2003)*



Level Crossing in ED.

2 D Ionic Hubbard Model

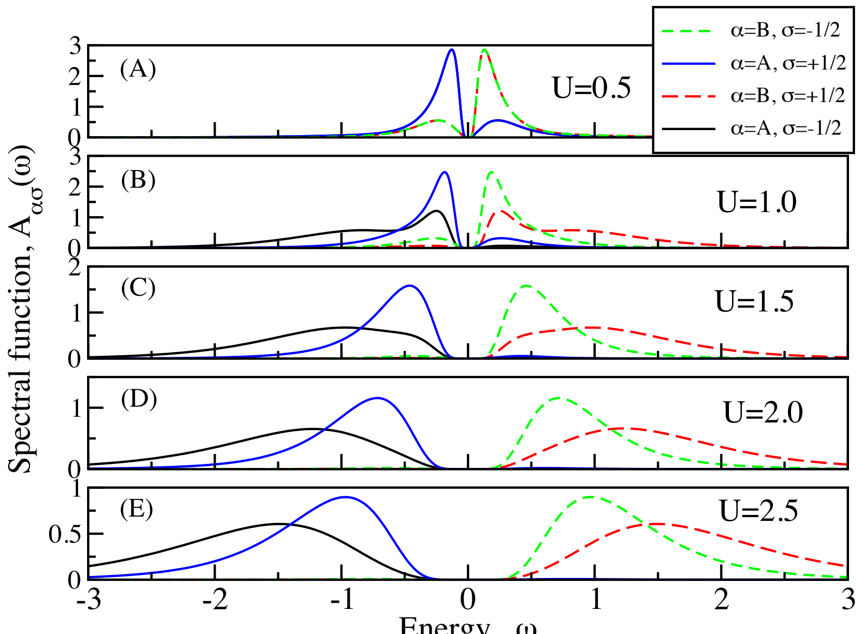


BI → Metal → MI

*A.Garg et. al PRL **97**, 046403(2006)*
*L. Craco et. al PRB **78,** 75121 (2008)*

A.Garg et. al PRL 2014
S. Bag et. al PRB 2015

K. Byczuk et. al
*PRB **79**, 121103(R) (2009)*



BI → MI