An open quantum system generalization of a 1D quasiperiodic system with a single-particle mobility edge

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<u>Collaboration:</u>

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S. Sanyal (ICTS-TIFR)

arXiv:1707.03749
arXiv:1702.05228

<u>Also:</u> Phys. Rev. A 94, 052134, (2016)
Phys. Rev. A 93, 062114, (2016)

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Purkayastha, Dhar, Kulkarni (**arXiv:1707.03749**)

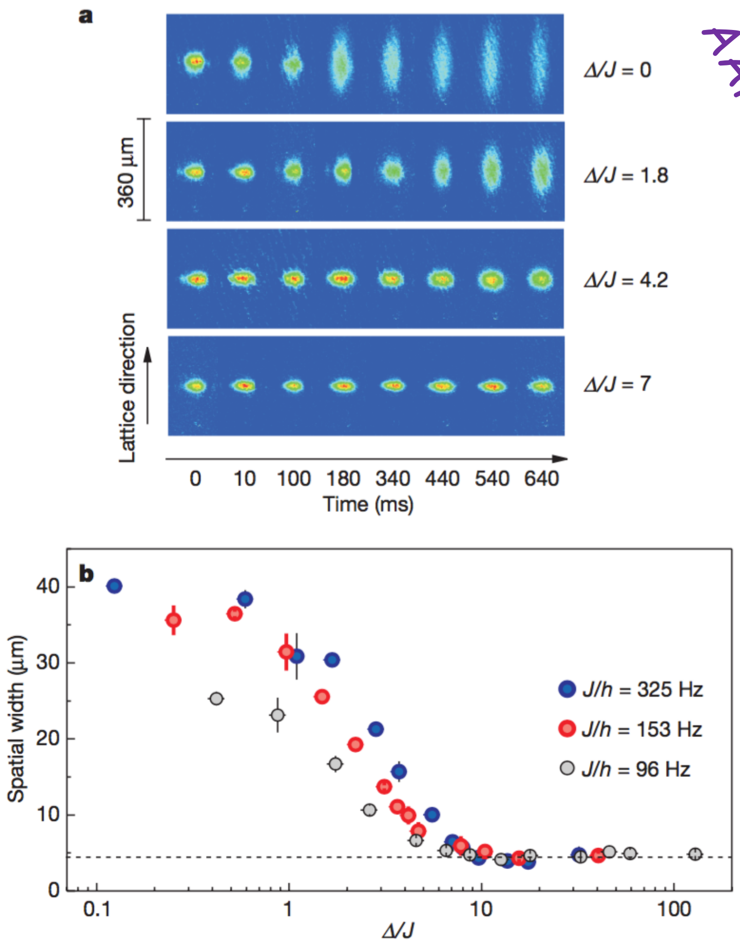
- Motivation and Experiments
- Non-equilibrium setup and methods
- Probes of the phase diagram
- Closed System Properties (for e.g., Anomalous yet Brownian)
- Results of the traditional AAH **arXiv:1702.05228, (2017)** Purkayastha, Sanyal, Dhar, **Kulkarni**
  (also arXiv:1703.05844, Verma, Mulatier, Znidaric)
- Conclusions and Outlook

$$\mathcal{H}_S = \sum_{r=1}^{N-1} (\hat{a}_r^\dagger \hat{a}_{r+1} + h.c) + \sum_{r=1}^{N} \frac{2\lambda \cos(2\pi b r + \phi)}{1 - \alpha \cos(2\pi b r + \phi)} \hat{a}_r^\dagger \hat{a}_r$$

Ganeshan et al, PRL (2015)

- *Generalized Aubry-Andre-Harper Model*
- *Couple it to two baths*

Motivation and Experiments



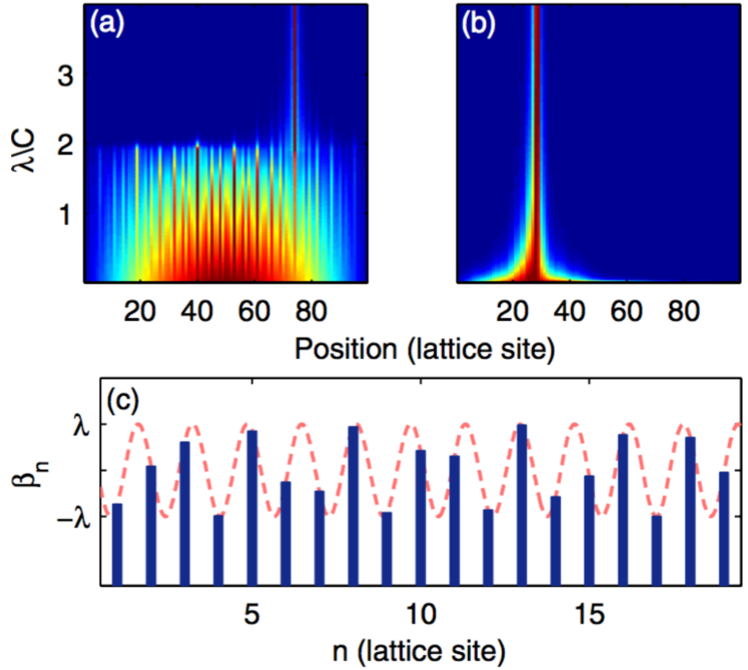
AAH in cold atoms

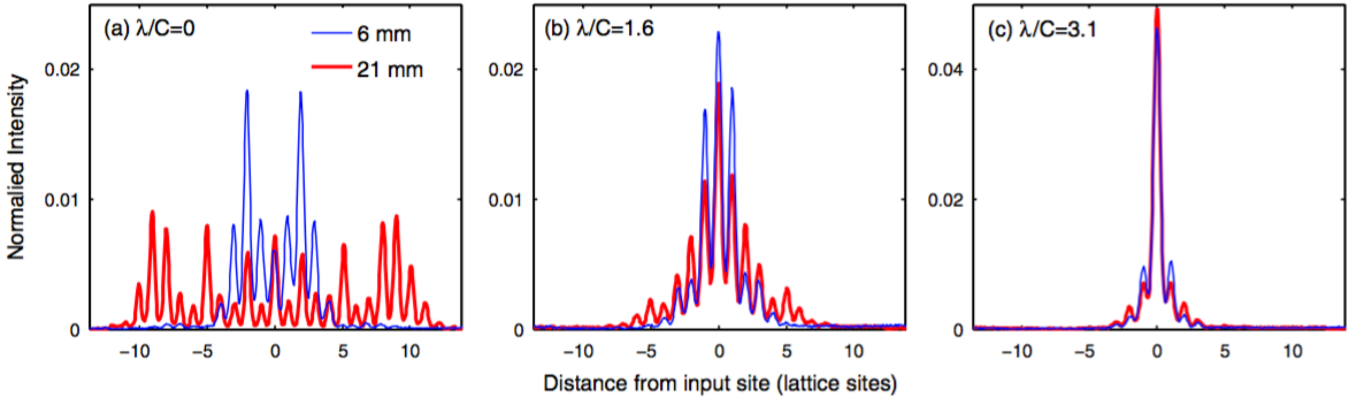
Anderson localization of a non-interacting Bose–Einstein condensate

Giacomo Roati[1,2], Chiara D'Errico[1,2], Leonardo Fallani[1,2], Marco Fattori[1,2,3], Chiara Fort[1,2], Matteo Zaccanti[1,2], Giovanni Modugno[1,2], Michele Modugno[1,4,5] & Massimo Inguscio[1,2]

Nature Letters 2008

AAH in waveguides



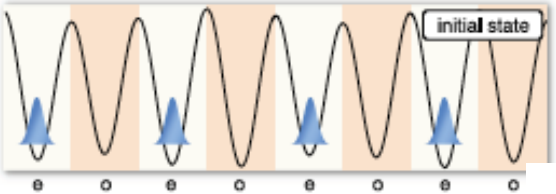


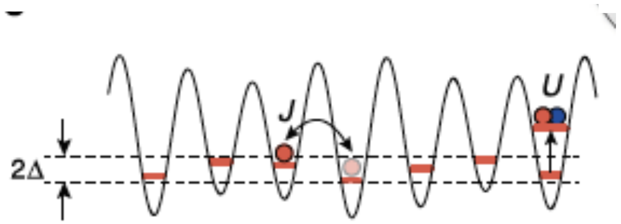
PRL 103, 013901 (2009) PHYSICAL REVIEW LETTERS week ending 3 JULY 2009

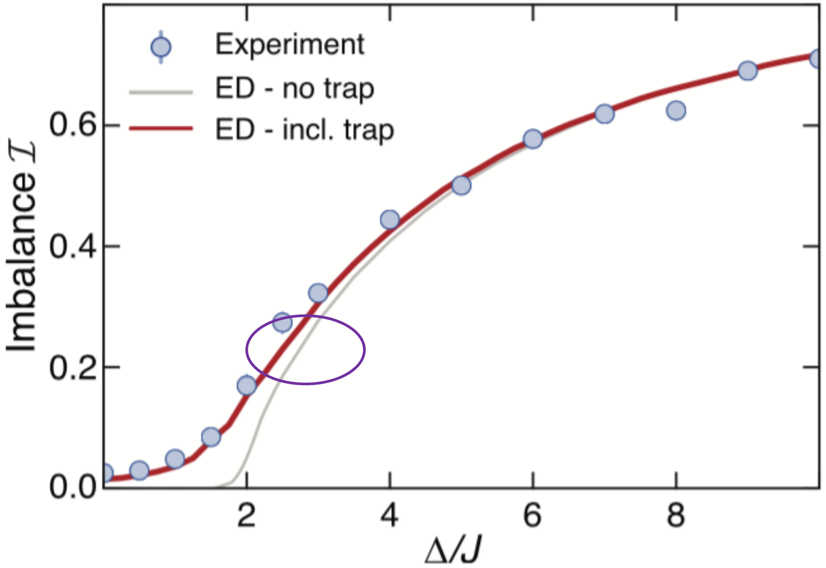
Observation of a Localization Transition in Quasiperiodic Photonic Lattices

Y. Lahini,[1,*] R. Pugatch,[1] F. Pozzi,[2] M. Sorel,[2] R. Morandotti,[3] N. Davidson,[1] and Y. Silberberg[1]

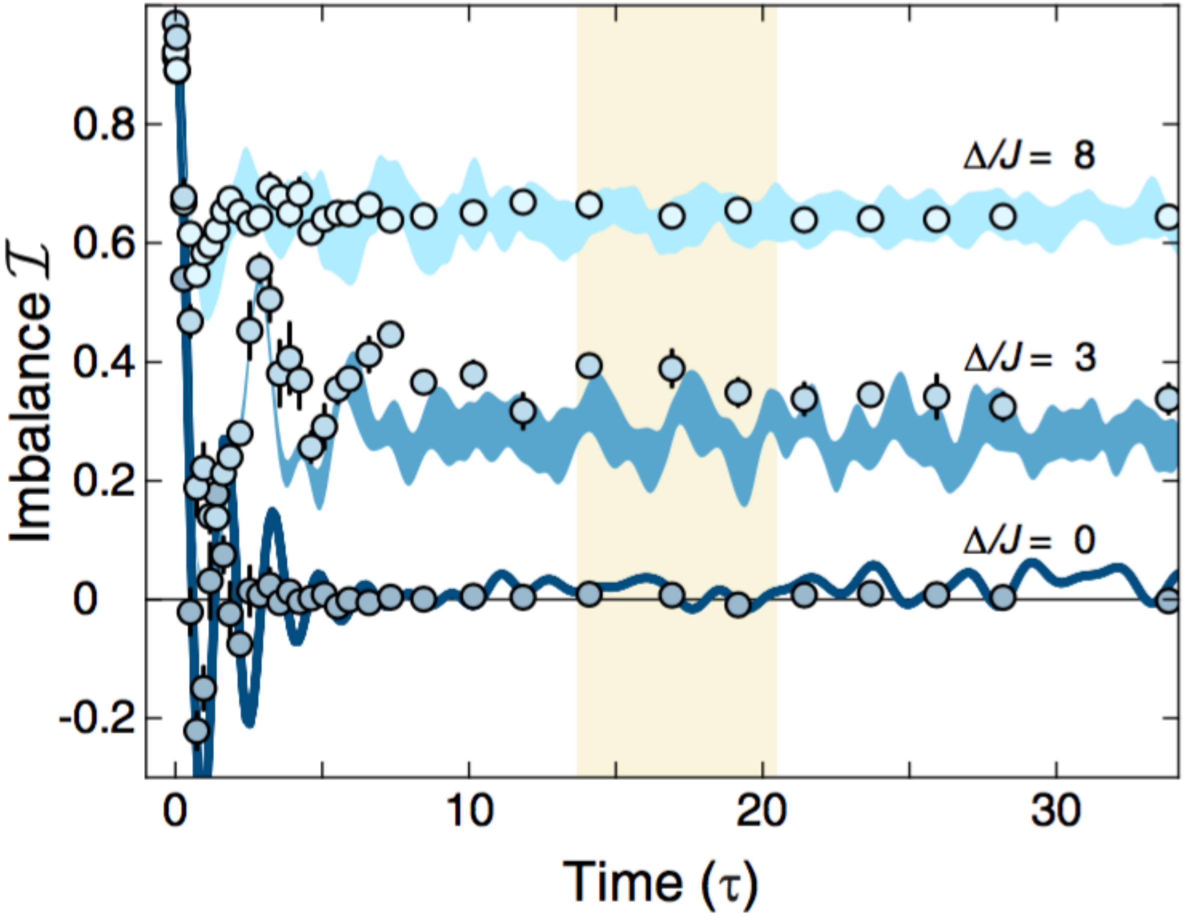
Motivation and Experiments







Non-interacting AAH



Science 349, 842 (2015)
and other experiments from Bloch group
(including controlled open system)

Deformations of the Aubry-Andre-Harper Model

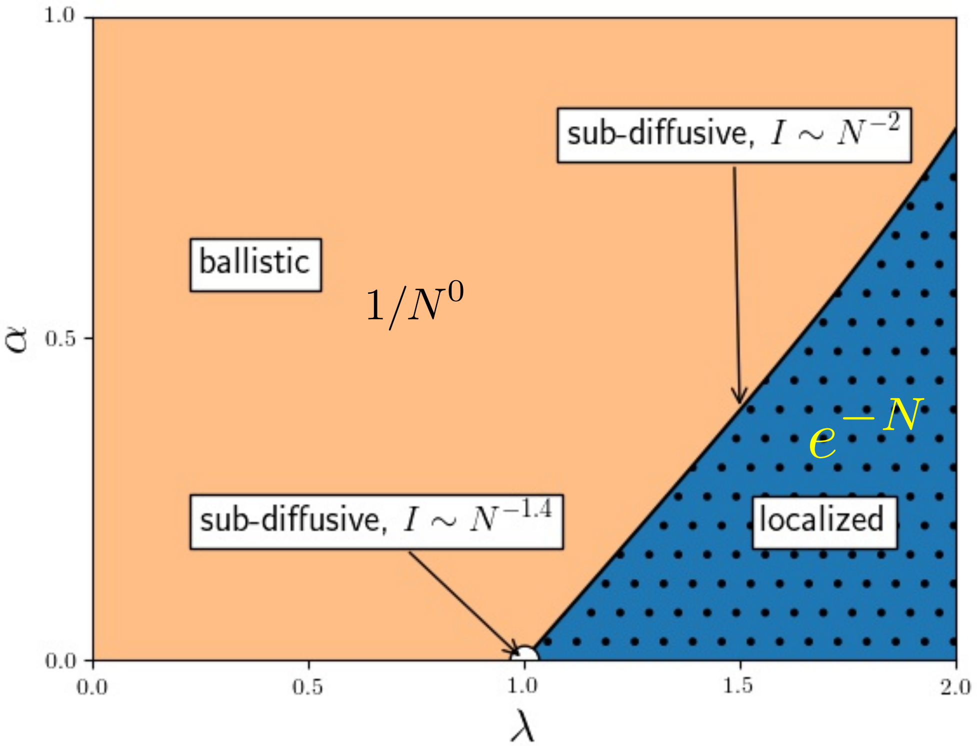
Purkayastha, Dhar, Kulkarni (arXiv:1707.03749)

$$\mathcal{H}_S = \sum_{r=1}^{N-1} (\hat{a}_r^\dagger \hat{a}_{r+1} + h.c) + \sum_{r=1}^{N} \frac{2\lambda \cos(2\pi b r + \phi)}{1 - \alpha \cos(2\pi b r + \phi)} \hat{a}_r^\dagger \hat{a}_r$$

GAAH

$$\alpha E = 2\mathrm{sgn}(\lambda)(a - \mid \lambda \mid)$$

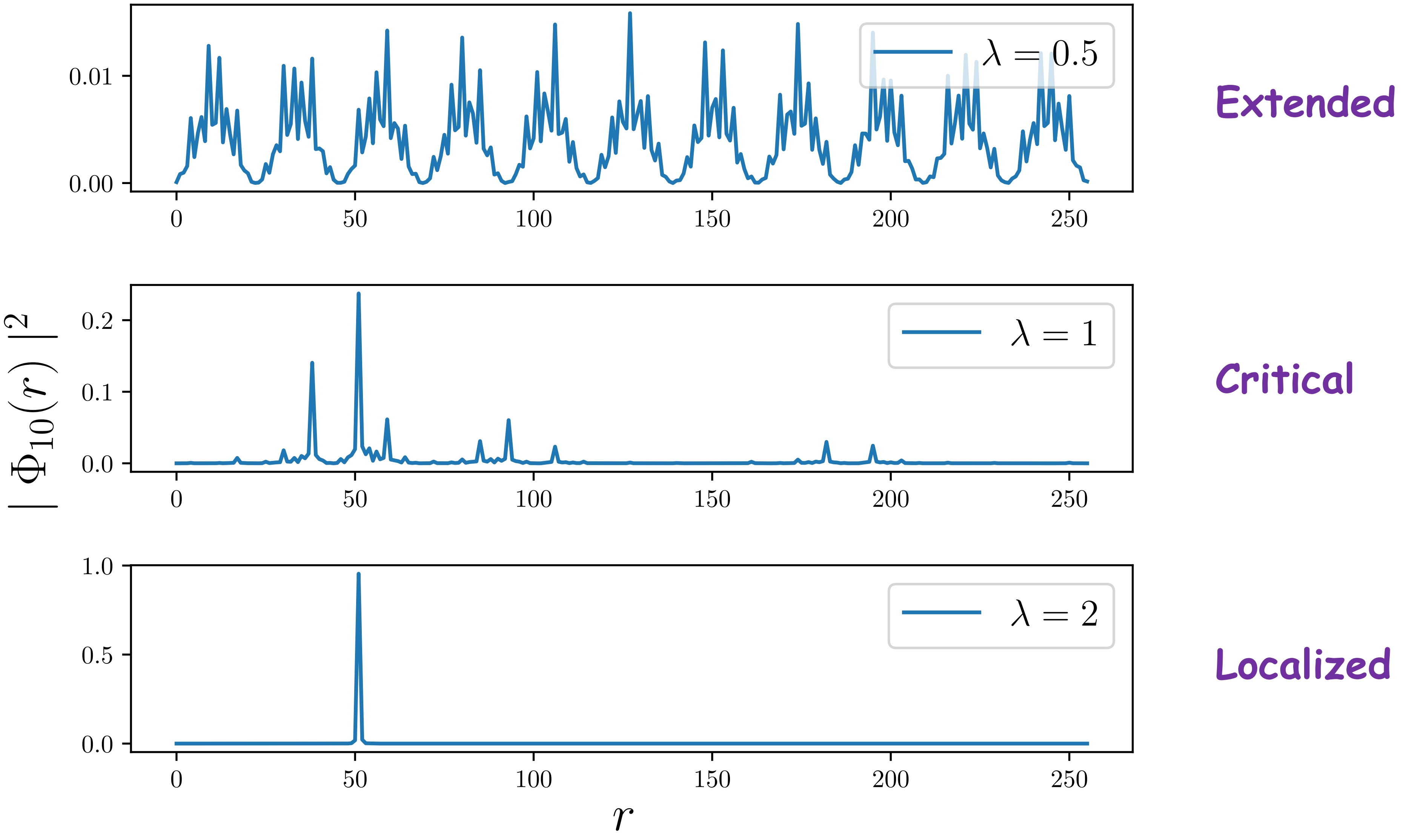
mobility edge



<u>Quantities Computed</u>

- Imbalance, both dynamics and steady state $\mathcal{I} = \frac{N_e - N_0}{N_e + N_o}$
- NESS Current and scaling with system size
- P(x, t), moments

Examples of Typical Eigenfunctions



Our non-equilibrium setup and methods

$$\mathcal{H}_S = \sum_{r=1}^{N-1} (\hat{a}_r^\dagger \hat{a}_{r+1} + h.c) + \sum_{r=1}^{N} \frac{2\lambda \cos(2\pi b r + \phi)}{1 - \alpha \cos(2\pi b r + \phi)} \hat{a}_r^\dagger \hat{a}_r$$

Hamiltonian

$$\hat{\mathcal{H}}_B^{(p)} = t_B \left(\sum_{s=1}^{\infty} \hat{b}_s^{(p)\dagger} \hat{b}_{s+1}^{(p)} + h.c.\right)$$

Bath

$$\hat{\mathcal{H}}_{SB} = \gamma(\hat{a}_1^\dagger \hat{b}_1^{(1)} + \hat{a}_N^\dagger \hat{b}_1^{(N)} + h.c.)$$

System-Bath Coupling

- Each bath (left and right) has its own temperature and chemical potential
- Our method valid for arbitrary system-bath coupling (unlike Lindblad)
- Our method valid for arbitrary inter-system coupling (unlike Local-Lindblad)

Quantities of Interest

$$I \ = \ \int_{-2t_B}^{2t_B} \ \frac{d\omega}{2\pi} J_1(\omega) J_N(\omega) \mid G_{1N}(\omega) \mid^2 \ \ [\ n_F^{(1)}(\omega) - n_F^{(N)}(\omega)],$$

$$\langle \hat{n}_r \rangle \ = \ \int_{-2t_B}^{2t_B} \frac{d\omega}{2\pi} \Big[\mid G_{r1}(\omega) \mid^2 n_F^{(1)}(\omega) + \mid G_{rN}(\omega) \mid^2 n_F^{(N)}(\omega) \Big]$$

where

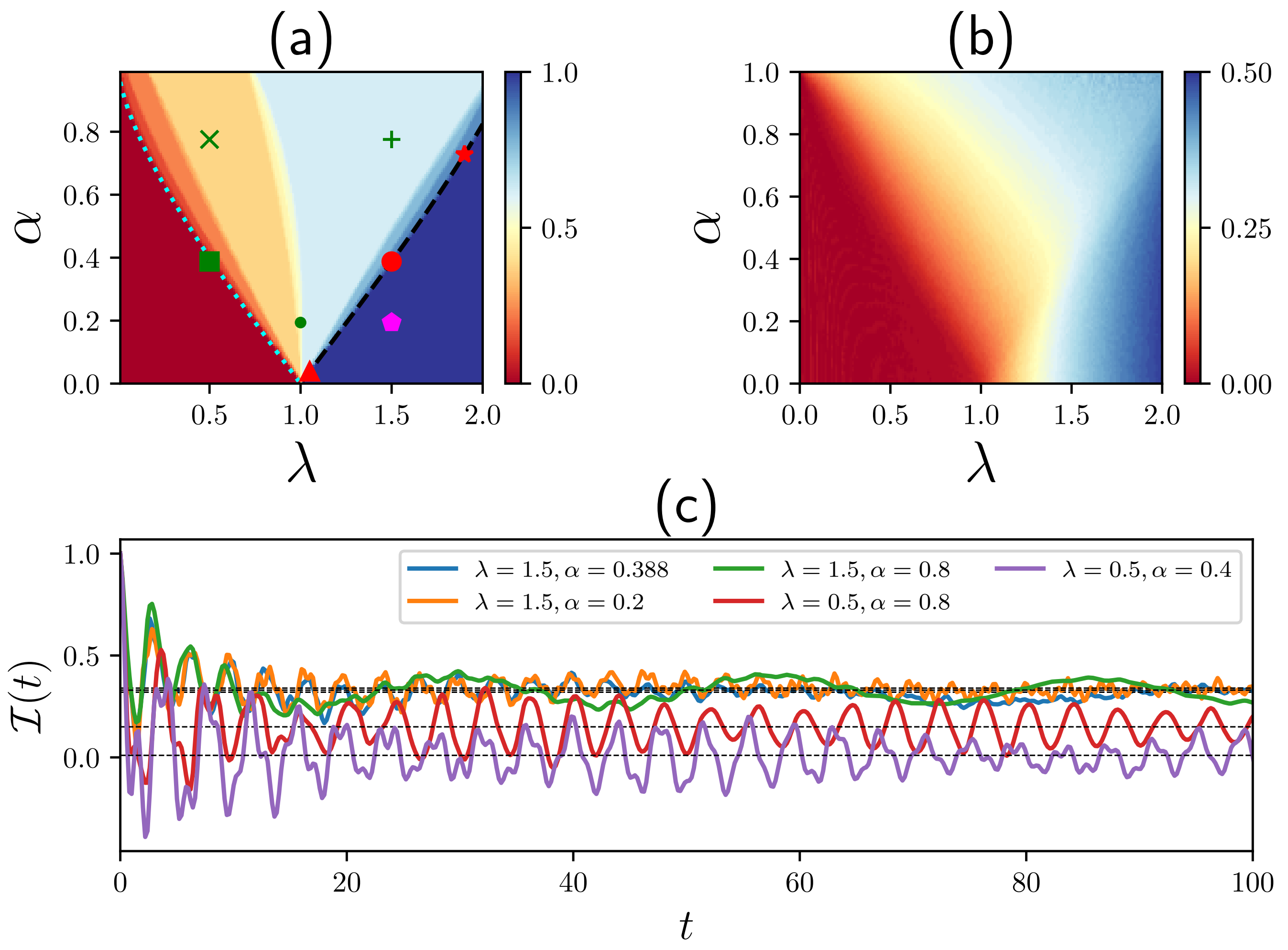
$$\mathbf{G}(\omega) \ = \ \Big[\omega \mathbf{I} - \mathbf{H}_S - \mathbf{\Sigma}^{(1)}(\omega) - \mathbf{\Sigma}^{(N)}(\omega)\Big]^{-1}$$

$$\mathbf{\Sigma}_{11}^{(1)}(\omega) = \mathbf{\Sigma}_{NN}^{(N)}(\omega) \ = \frac{\gamma^2 \omega}{2t_B^2} + \frac{i}{2} J(\omega) \qquad J(\omega) = \frac{2\gamma^2}{t_B} \sqrt{1 - \left(\frac{\omega}{2t_B}\right)^2}$$

Generalized Aubry-Andre-Harper

Purkayastha, Dhar, **Kulkarni** (arXiv:1707.03749)

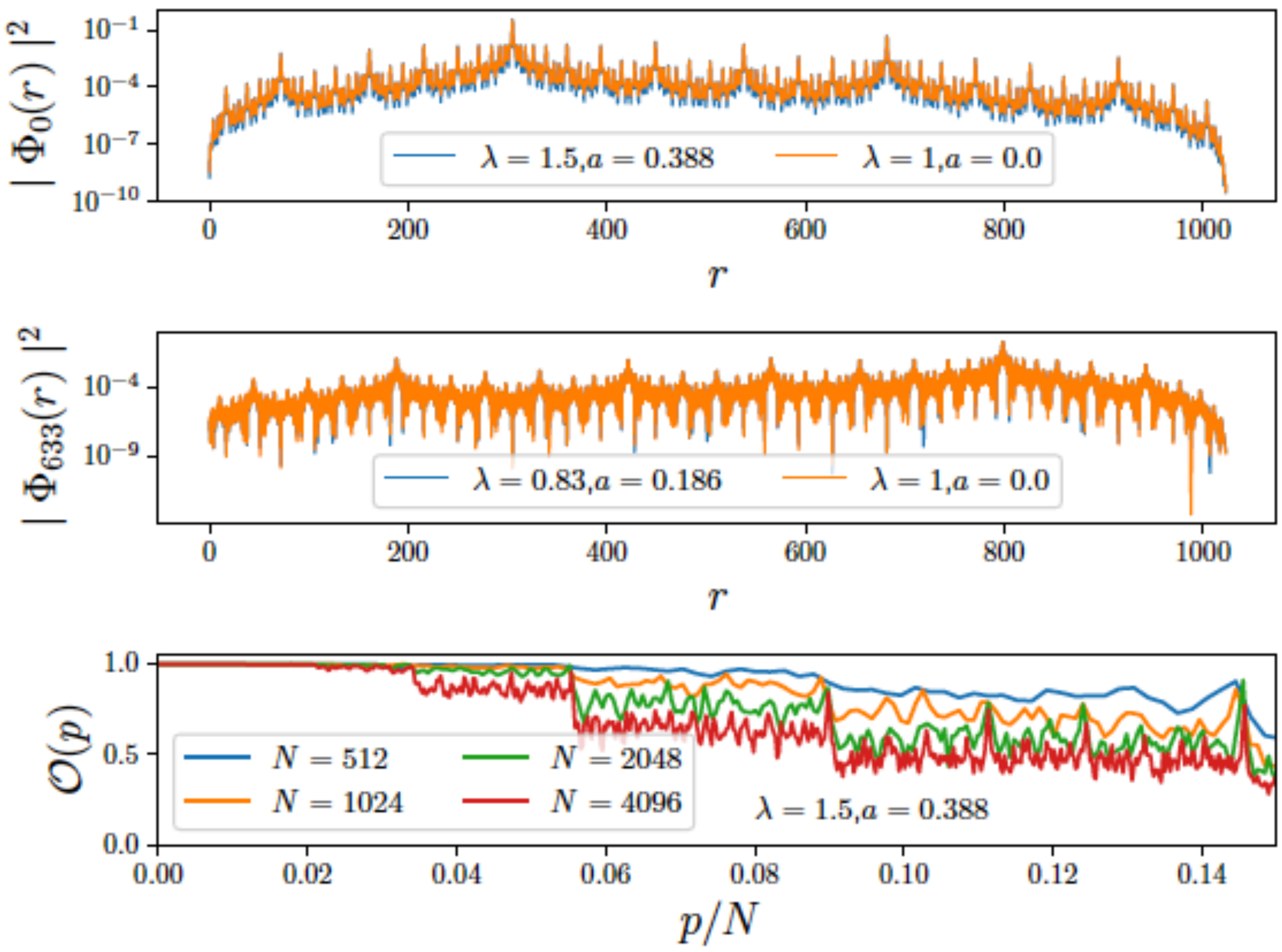
Fraction of localized eigenstates



Imbalance in steady state

Imbalance Dynamics

Wave function matching between GAAH and AAH



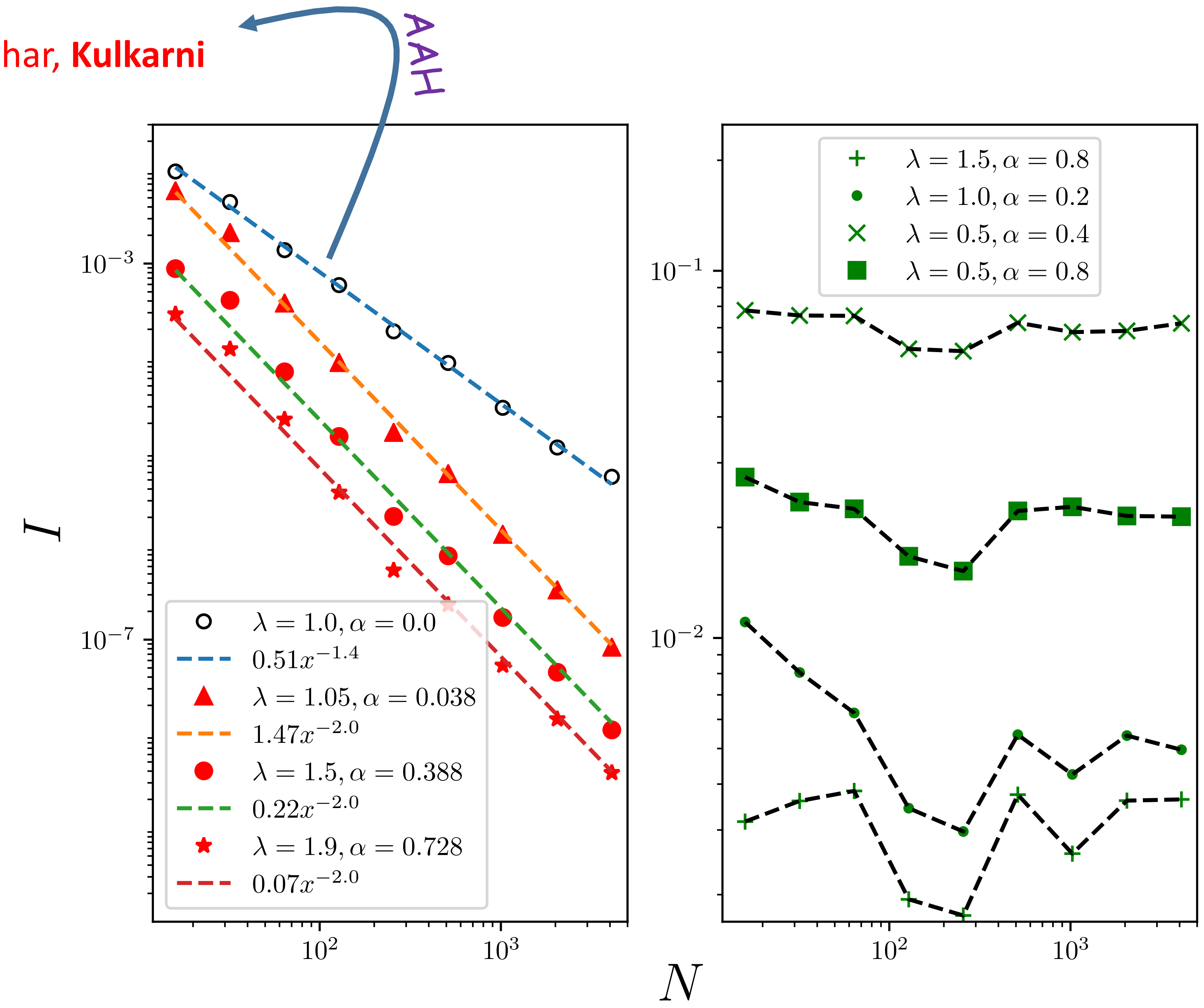
Pth state of GAAH (near self-dual point) matches with the pth state of critical AAH

GAAH Current Scaling with system size

Purkayastha, Dhar, **Kulkarni** (arXiv:1707.03749)

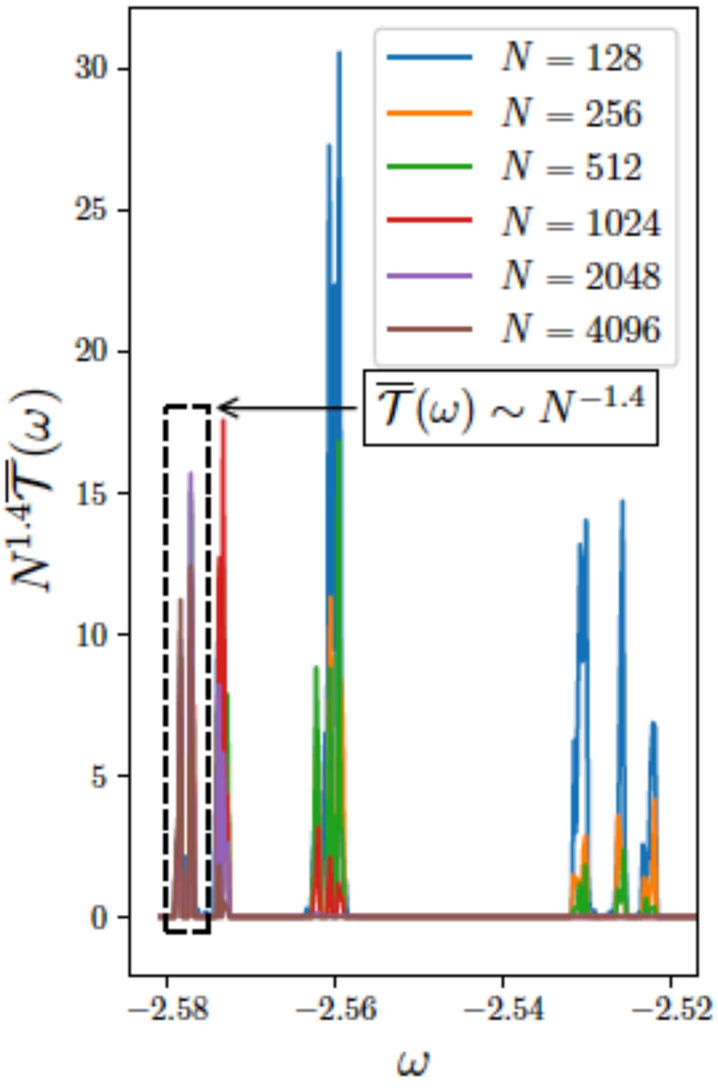
arXiv:1702.05228, (2017) Purkayastha, Sanyal, Dhar, **Kulkarni**

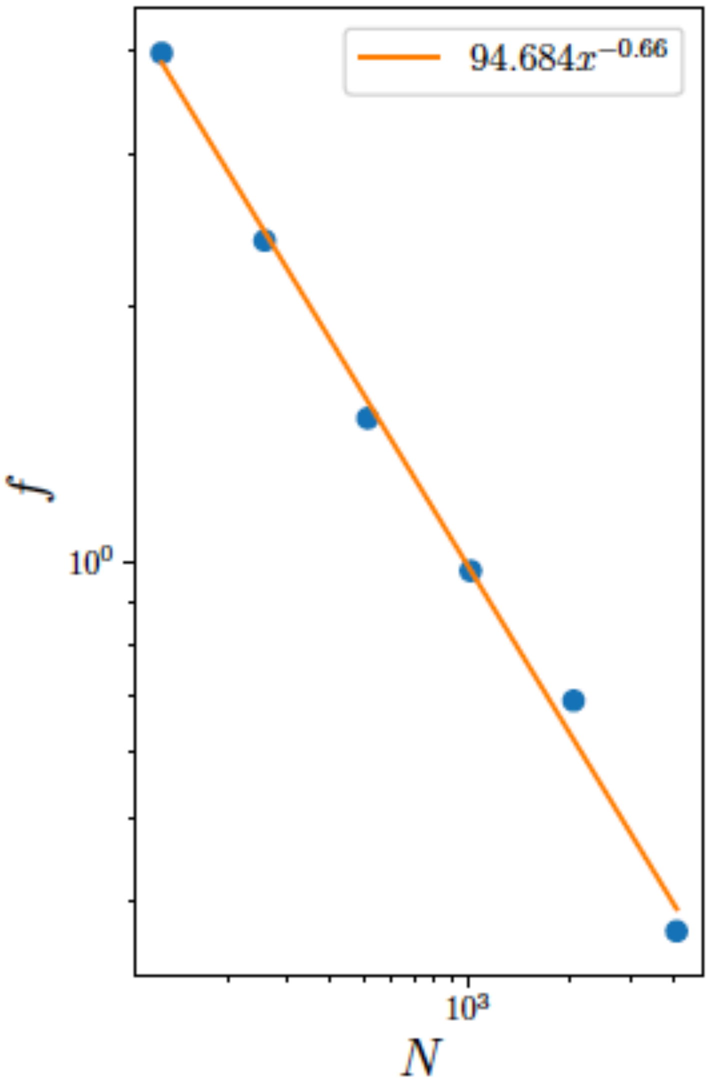
(also arXiv:1703.05844, Verma, Mulatier, Znidaric



- AAH showed 1.4
- GAAH shows 2.0
- Scaling exponent on the line is remarkably different than the AAH critical point

Reason for $1/N^2$ scaling in GAAH





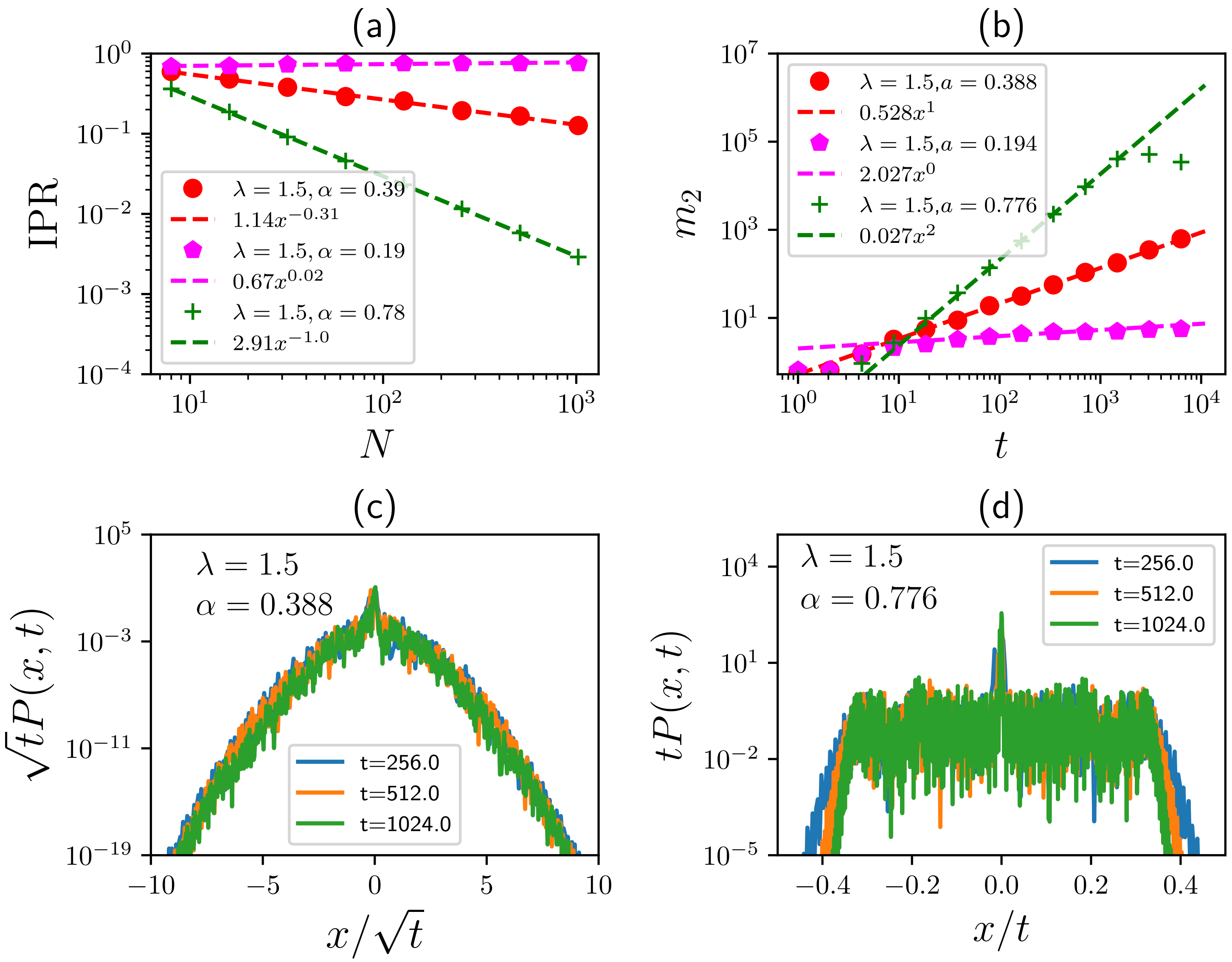
$$\overline{\mathcal{T}}(\omega) = \left[\int_{\omega-\delta E/2}^{\omega+\delta E/2} \mathcal{T}(\omega) d\omega\right] / \delta E$$

$$f = N_{max} \Delta E$$

$$f \sim N^{-0.6\pm0.1}$$

Purkayastha, Dhar, **Kulkarni** (arXiv:1707.03749)

Closed System Properties of GAAH



Experiments in a different context:

PNAS 2009, "Anamolous yet Brownian"

Nature Materials 2012, "When Brownian diffusion is not Gaussian"

Phase Co-existence
Prelimnary experiments on a similar Model (I. Bloch, private communication)

Closed System Properties

GAAH Model is Anomalous yet Brownian for all irrational numbers

Anomalous yet Brownian

Experiment, PNAS, 2009

Bo Wang[a], Stephen M. Anthony[b], Sung Chul Bae[a], and Steve Granick[a,b,c,d,1]

Departments of [a]Materials Science and Engineering, [c]Chemical and Biomolecular Engineering, [b]Chemistry, and [d]Physics, University of Illinois a
Urbana–Champaign, Urbana, IL 61801

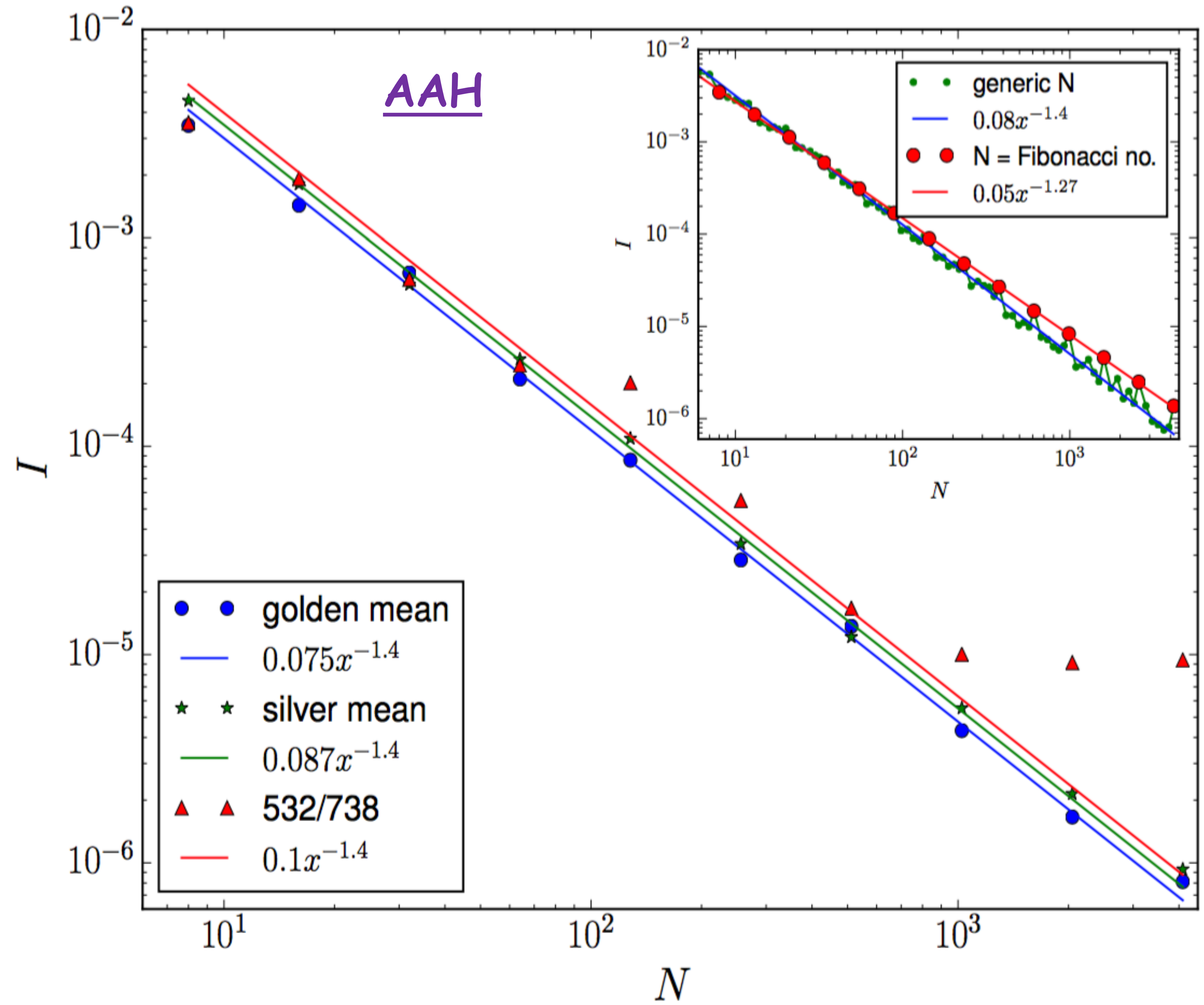
When Brownian diffusion is not Gaussian

Experiment, Nature Materials, 2012

Bo Wang, James Kuo, Sung Chul Bae and Steve Granick

It is commonly presumed that the random displacements that particles undergo as a result of the thermal jiggling of the environment follow a normal, or Gaussian, distribution. Here we reason, and support with experimental examples, that non-Gaussian diffusion in soft materials is more prevalent than expected.

We will now discuss traditional AAH model, i.e, $\alpha = 0$ →

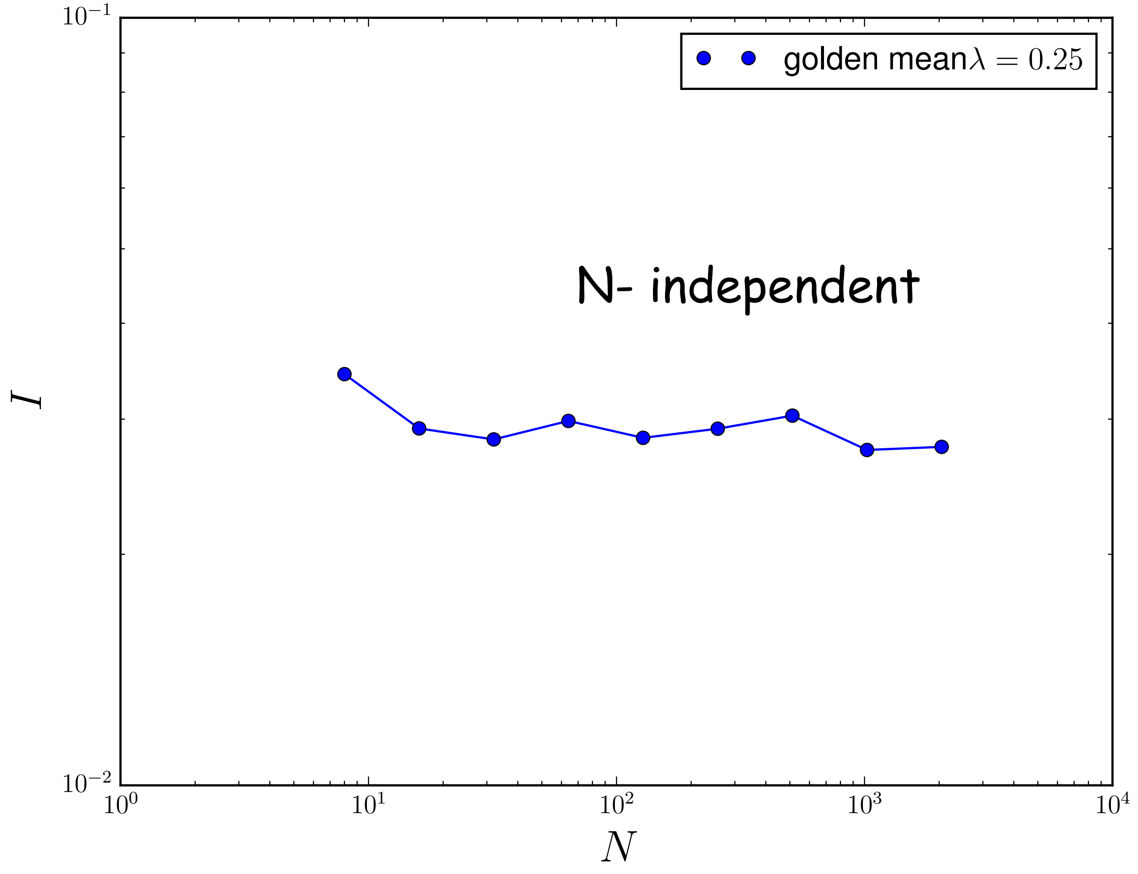


arXiv:1702.05228, (2017)

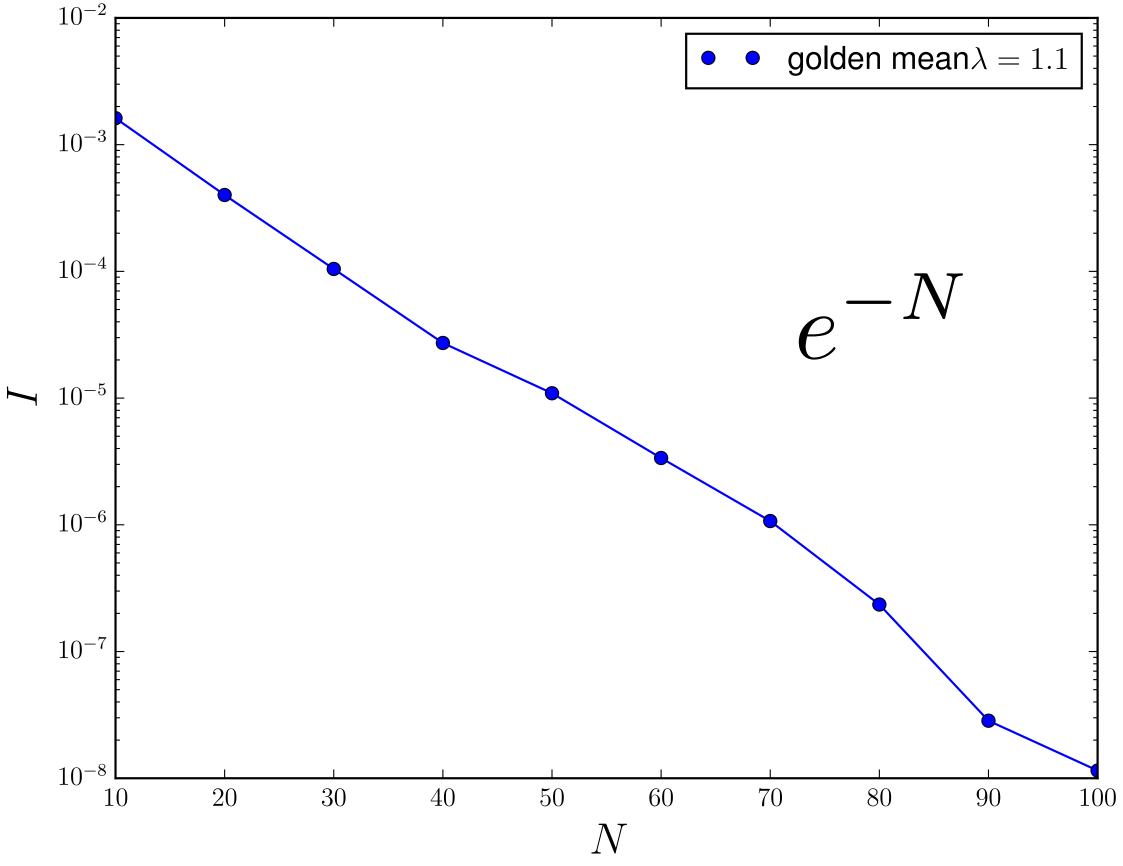
- **Current Scaling with system Size at critical point**
- Sub-diffusive for all irrational number and experimental incommensurate lattice
- Experimental numbers shows eventual flattening
- Different scaling at Fibonacci

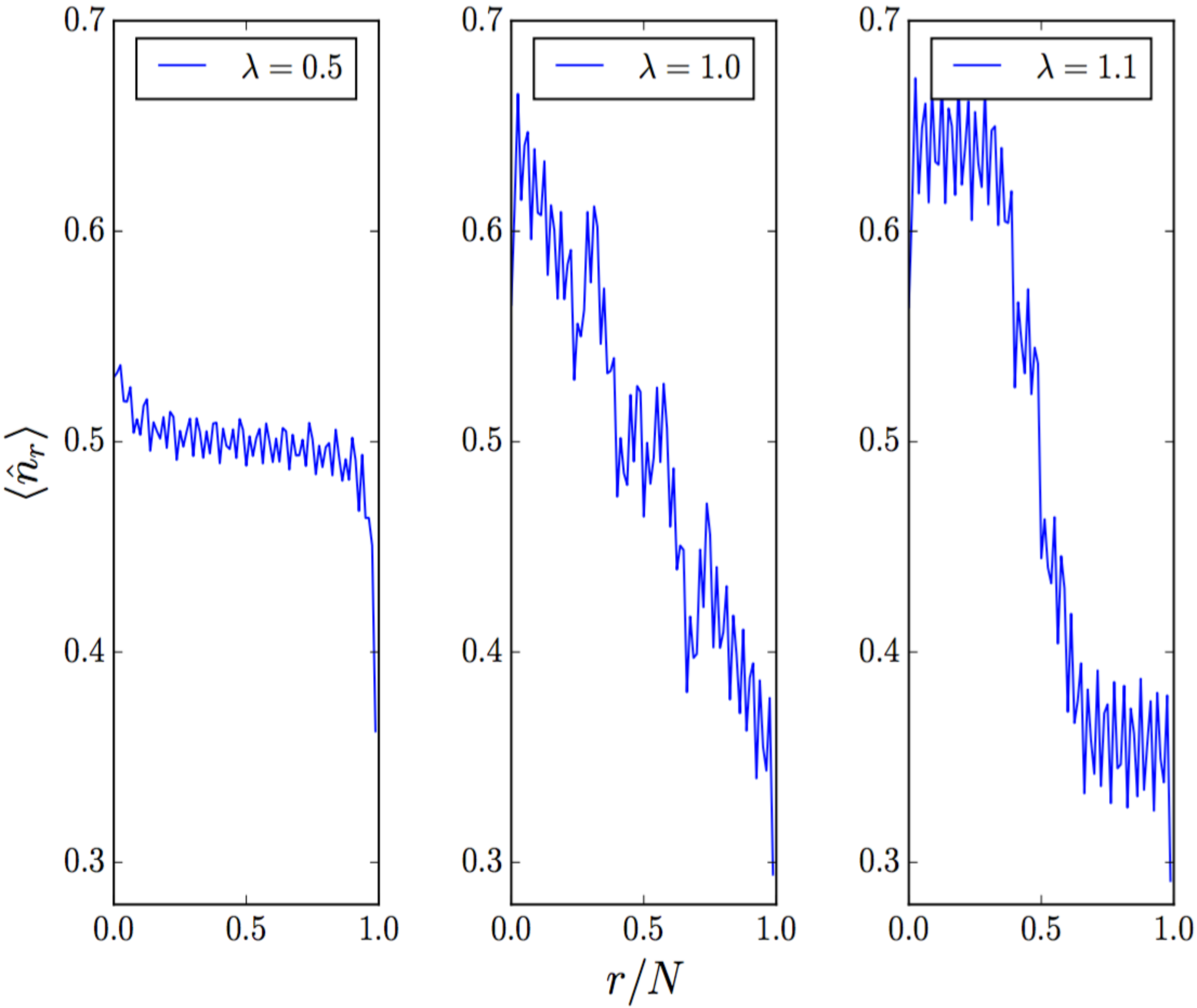
AAH

De-localized/Ballistic Regime



Localized Regime





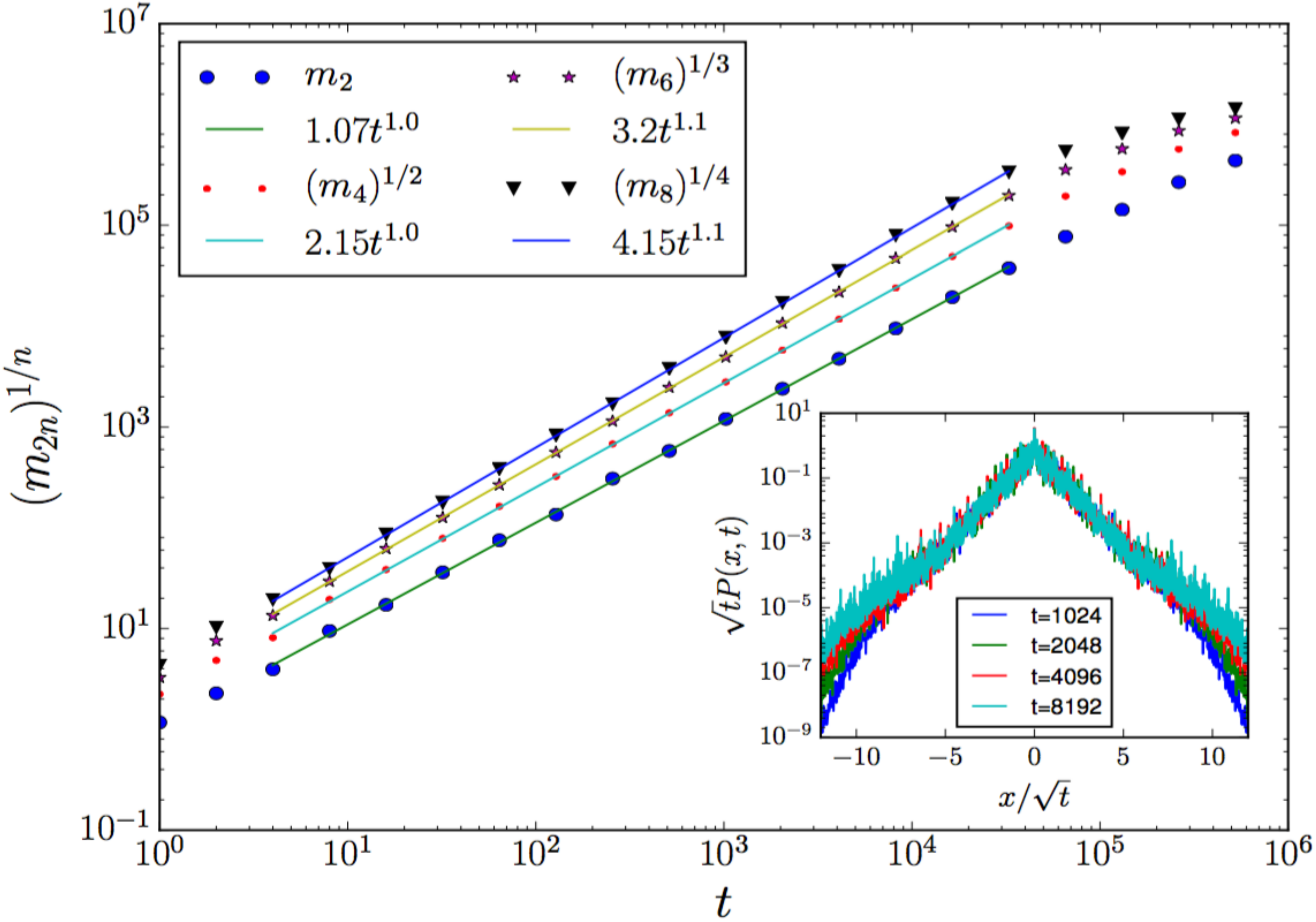
AAH

arXiv:1702.05228, (2017)

Purkayastha, Sanyal, Dhar, **Kulkarni**

NESS particle density profile for the three regimes

- Remarkably different behavior in the regimes
- NESS particle spatial density profile provides a novel real-space experimentally measurable probe of the localized, critical and de-localized phases.



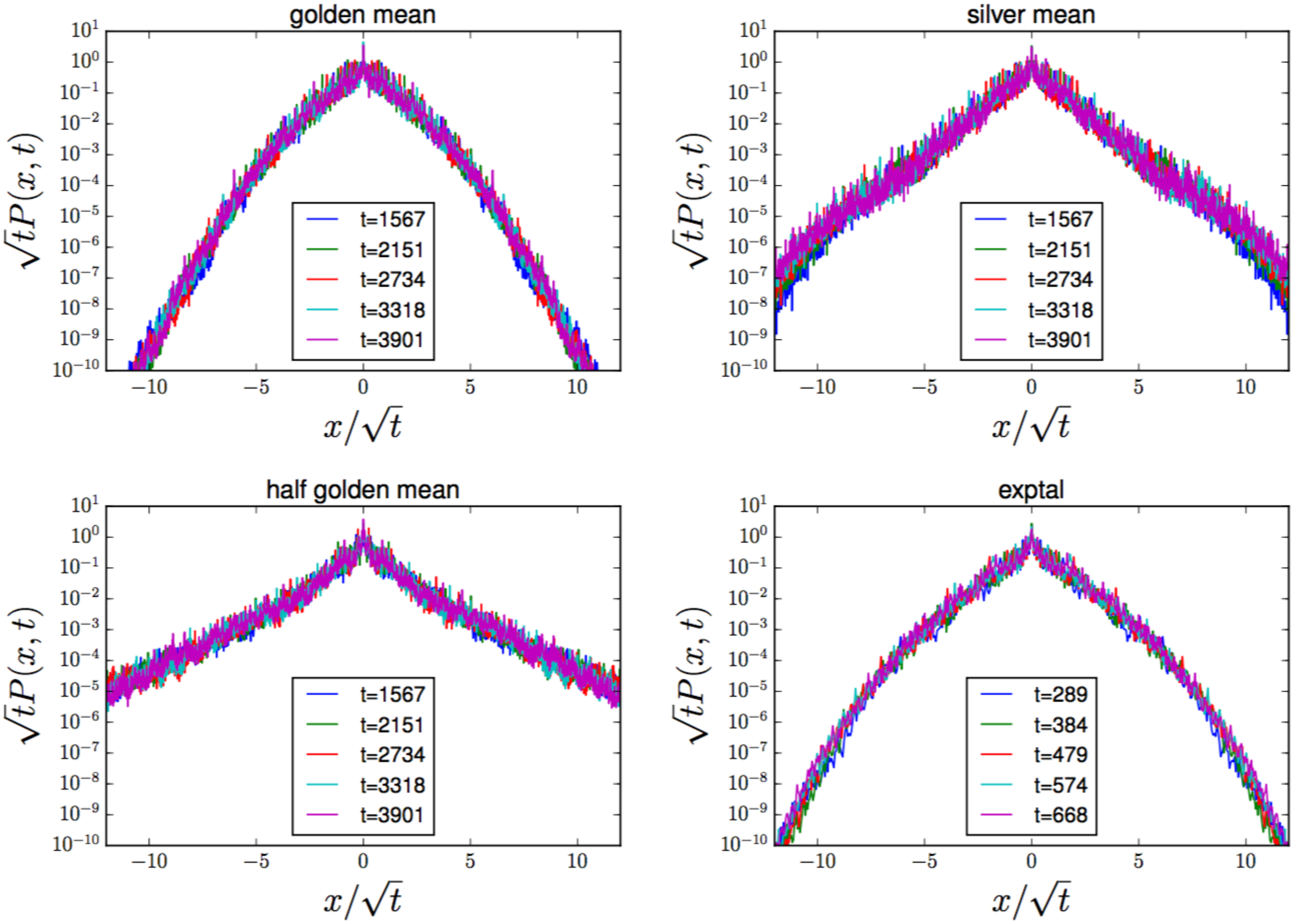
AAH

Closed System Properties

AAH

arXiv:1702.05228, (2017)

Purkayastha, Sanyal, Dhar, **Kulkarni**



All Mostly follow:

$$P(x,t) = (1/\sqrt{t}) f_1(x/\sqrt{t})$$

We find that Kurtosis

$$K = m_4/(m_2)^2 > 3$$

K >3 means non-Gaussian

Kurtosis is time-dependent signifying multi-scaling

Conclusions

arXiv:1707.03749

arXiv:1702.05228

- Interesting phase diagram GAAH model.

- Open quantum system set-up of the GAAH model captures
rich and interesting physics which are missed by the standard closed system description.

- NESS particle spatial density profile provides a novel real-space
experimentally measurable probe of the localized, critical and de-localized phases.

- Sub-diffusive nature of critical line cannot be obtained from closed system
calculations.

- Phys. Rev. A 93, 062114, (2016), Purkayastha, Dhar, **Kulkarni** (extensive analysis of
Local Lindblad, Eigenbasis lindblad, Redfied, brute-force numercs, NEGF for
ordered systems, both Steady State and Time Dynamics)