Bosonization-debosonization and the nonequilibrium Kondo problem

Carlos Bolech
University of Cincinnati

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Abstract

After critically reexamining in the previous talk the Bosonization-deBosonization (BdB) procedure for systems including 'boundaries' and subsequently introducing a Consistent BdB procedure to address shortcomings that were found in transport calculations [1], we turn our attention to the physics of quantum dots. Under the right conditions, such dots can attain the Kondo regime in which tunneling conduction is possible at low temperatures despite the Coulomb blockade. We study this physics by focusing on the two-lead Kondo model [2]. The bosonization formalism can be used to access a solvable limit of this model known as its Toulouse point. I shall show that a consistent BdB procedure yields a modified set of physical results that are in better agreement with the phenomenology of the problem. Besides its general experimental relevance, the Toulouse limit of the two-lead Kondo model is a key theoretical prototype of a strongly correlated system away from equilibrium but nevertheless admitting a closed solution.

<u>References</u>:

[1] Nayana Shah and C. J. Bolech, Phys. Rev. B 93, 085440 (2016).

[2] C. J. Bolech and Nayana Shah, Phys. Rev. B 93, 085441 (2016).

PHYSICAL REVIEW B **93**, 085440 (2016)

Consistent bosonization-debosonization. I. A resolution of the nonequilibrium transport puzzle

Nayana Shah and C. J. Bolech
Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221-0011, USA
(Received 13 August 2015; revised manuscript received 22 November 2015; published 29 February 2016)

We critically reexamine the bosonization-debosonization procedure for systems including certain types of localized features (although more general scenarios are possible). By focusing on the case of a tunneling junction out of equilibrium, we show that the conventional approach gives results that are not consistent with the exact solution of the problem even at the qualitative level. We identify inconsistencies that can adversely affect the results of all types of calculations. We subsequently show a way to avoid these and proceed consistently. The extended framework that we develop here should be widely applicable.

PHYSICAL REVIEW B **93**, 085441 (2016)

Consistent bosonization-debosonization. II. The two-lead Kondo problem and the fate of its nonequilibrium Toulouse point

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C. J. Bolech and Nayana Shah

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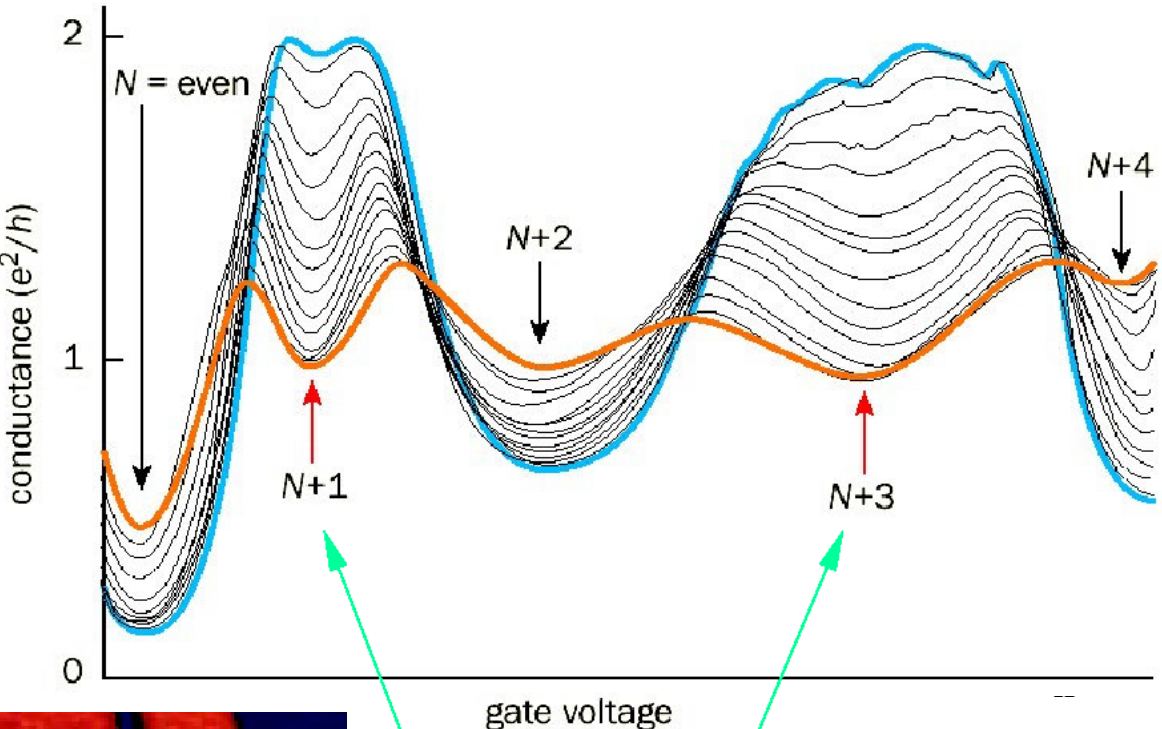
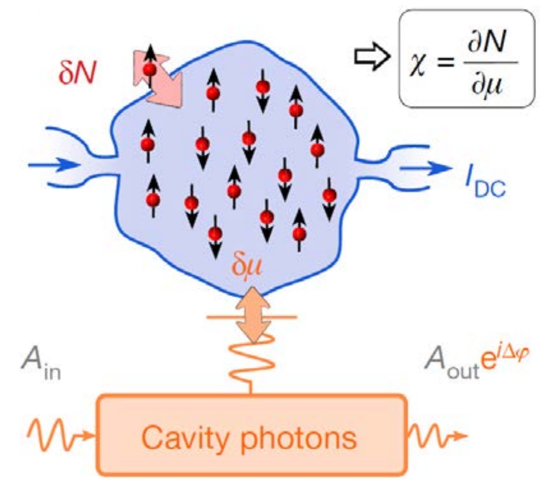
Following the development of a scheme to bosonize and debosonize consistently [N. Shah and C. J. Bolech, Phys. Rev. B **93**, 085440 (2016)], we present in detail the Toulouse-point analytic solution of the two-lead Kondo junction model. The existence and location of the solvable point is not modified, but the calculational methodology and the final expressions for observable quantities change markedly as compared to the existent results. This solvable point is one of the remarkably few exact results for nonequilibrium transport in correlated systems. It yields relatively simple analytical expressions for the current in the full range of temperature, magnetic field, and voltage. It also shows precisely, within the limitations of the Toulouse fine-tuning, how the transport evolves depending on the relative strengths of interlead and intralead Kondo exchange couplings ranging from weak to strong. Thus its improved understanding is an important stepping stone for future research.

“Modern” Times: Quantum Dots

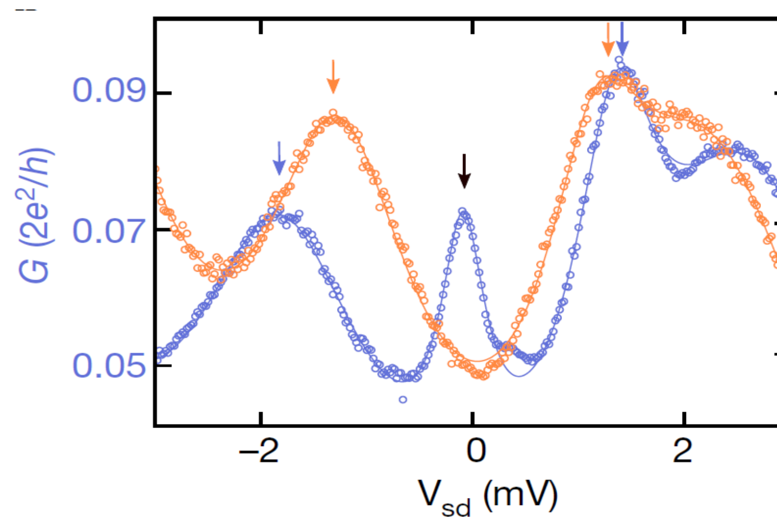


Desjardins et al.
Nature (2017)



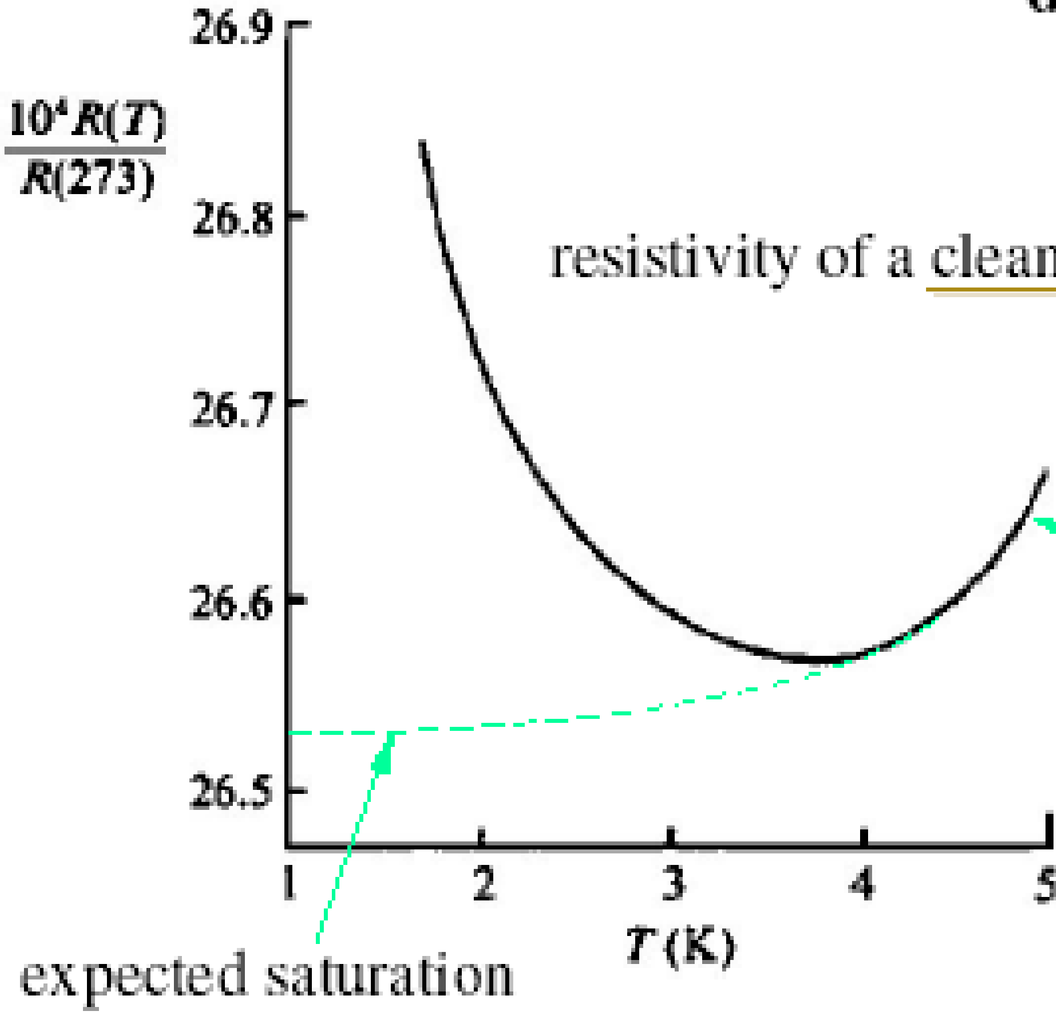
$$G \propto \ln\left(E_C/T\right)$$

van der Wiel et al.
Science (2000)



The beginning… before Kondo

de Haas *et. al.* (1934)



$\rho_{\text{phonons}} \propto T^5$

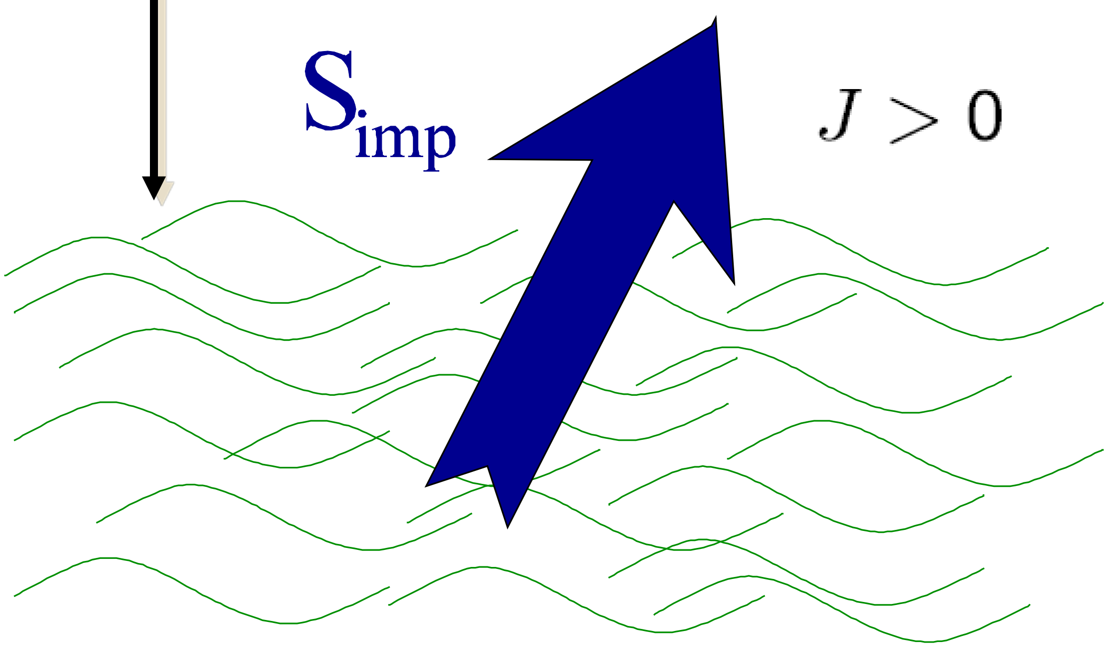
"… it is of course very interesting to investigate the influence of the purity of the metals on this minimum."

A simple-looking Model

$$H_{\mathsf{Kondo}} = \underline{H_{\mathsf{bulk}}} + J\ \vec{S}_{\mathsf{imp}} \cdot \underline{\left(\psi^\dagger_\sigma \vec{\sigma}_{\sigma\sigma'} \psi_{\sigma'}\right)}_{x=x_{\mathsf{imp}}}$$

Electrons in a metallic band modeled using Landau's Fermi-Liquid Theory

Local spin density of the band electrons



The s-d Model

- Zener (1951)
- Kasuya (1956)
- Yosida (1957)
- Kondo (1964)

Beyond Born's Approximation

$$\rho(T) = \rho_{\mathsf{bulk}}(T) + c\, n_{\mathsf{F}} J^2 + 2c\, n_{\mathsf{F}}^2 J^3 \ln \frac{D}{k_{\mathsf{B}} T}$$

Jun Kondo

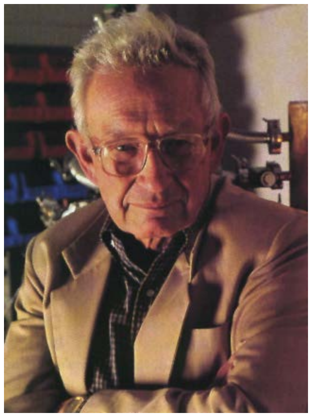
Prog. Theor. Phys. **32**, 37 (1964)

Emergent scale, below which perturbation theory breaks down

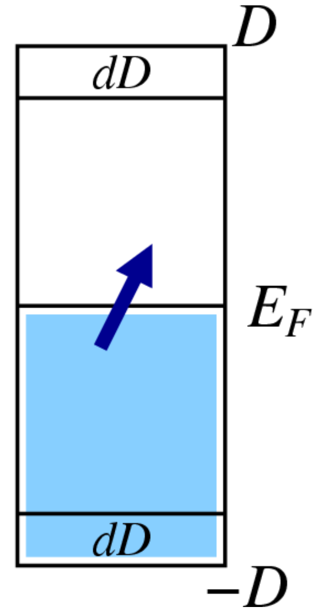
$$k_{\mathsf{B}} T_{\mathsf{K}} \approx D\, e^{-1/(2 n_{\mathsf{F}} J)}$$

Scaling and Renormalization Group

Poor Man's Scaling - Anderson (1970)

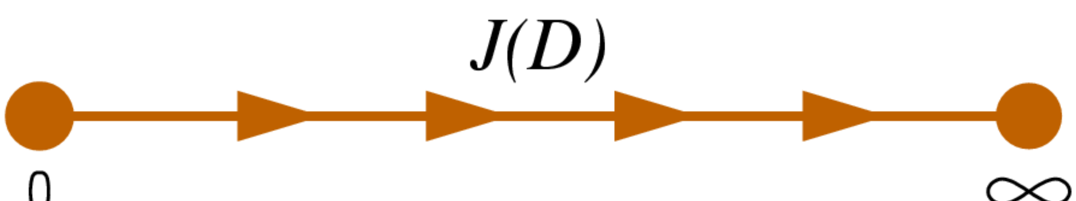


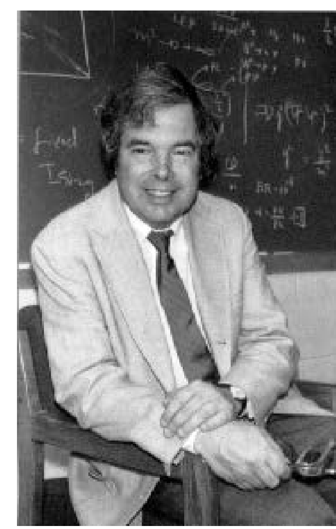
$$\frac{\partial J}{\partial(\ln D)} = -2n_{\mathrm{F}} J^2$$



Renormalization Group - Wilson (1974)

- Carried out (numerically) the R.G. program to completion for the first time. (On the Kondo model)
- Identified the fixed point reached when the energy is scaled down to zero.





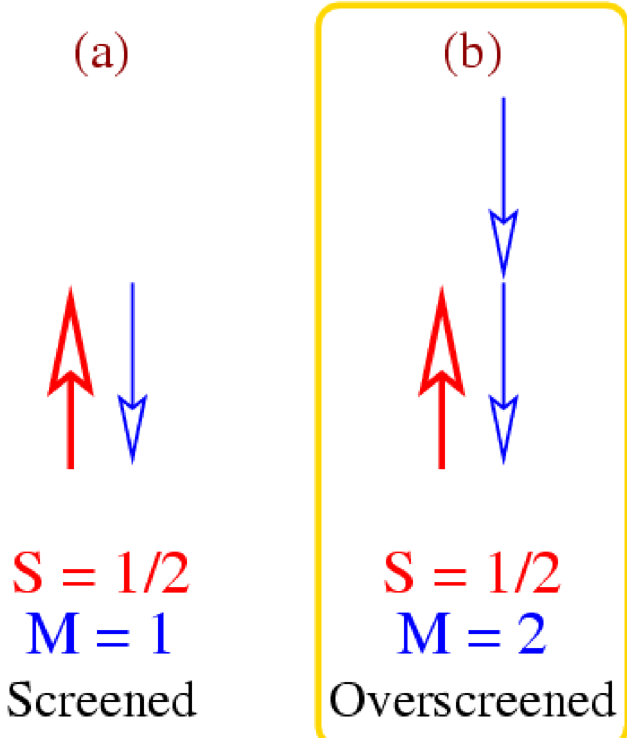
Single- to Multi-channel Kondo

"Fermi to Non-Fermi Liquid"

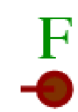
$$H_{\text{2chK}} = H_{\text{bulk}} + J\ \vec{S}_{\text{imp}} \cdot \left(\psi^{\dagger}_{\alpha\sigma}\vec{\sigma}_{\sigma\sigma'}\psi_{\alpha\sigma'}\right)_{x=x_{\text{imp}}}$$

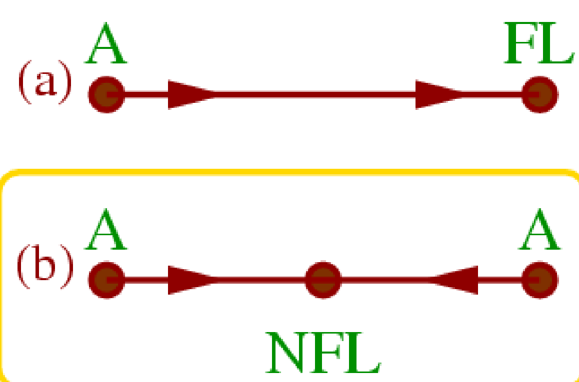
Nozières and Blandin (1980)

extra channel/flavor index



$$S^0_{\text{imp}} = k_B \ln\sqrt{2}$$
$$C_{\text{imp}} \sim T \ln T_K/T$$





Non-perturbative Approaches to Kondo

Technique	Single-channel [Fermi Liquid]	Two-channel [non-Fermi Liquid]
Numerical RG	Wilson (1974)	Cragg, Lloyd and Nozières (1980)
	Krishna-Murthy, Wilkins and Wilson (1980)	Koga and Cox (1999); Anders (2004)
Bethe Ansatz	Andrei (1980); Wiegmann (1980)	Andrei and Destri (1984)
	Wiegmann (1981); Kawakami and Okiji (1981)	CJB and Andrei (2002)
Boundary-CFT	Affleck (1990)	Affleck and Ludwig (1991)
	Fujimoto, Kawakami and Yang (1991)	Johannesson, Andrei and CJB (2003)
Bosonization	Toulouse (1969); Schlottmann (1978)	Emery and Kivelson (1992)
	Kotliar and Si (1996)	CJB and Iucci (2006)

Other methods: Monte Carlo, Large-N (e.g. NCA), …

No silver bullet, but …

- different methods illuminate different aspects
- Bosonization is the only one for non-equilibrium (also NCA)

Bosonization of Kondo Problems

$$H = H_{\text{Kondo}} + H_{\text{anisotropy}}$$

$$H_{\text{Kondo}} = H_{\text{bulk}} + J\ \vec{S}_{\text{imp}} \cdot \left(\psi^{\dagger}_{\alpha a}\vec{\sigma}_{ab}\psi_{\alpha b}\right)_{x=x_{\text{imp}}}$$

$$H_{\text{anisotropy}} = \Delta_z\ S^z_{\text{imp}}\ \left(\psi^{\dagger}_{\alpha a}\sigma^z_{ab}\psi_{\alpha b}\right)_{x=x_{\text{imp}}}$$

- Go to a bosonic basis: $\psi_{\alpha\sigma}(x) = \frac{1}{\sqrt{2\pi a}} F_{\alpha\sigma} e^{-i\phi_{\alpha\sigma}(x)}$
- Some transformations…
- The one- and two-flavor cases (and only those) map to RL models (non-int for a particular anisotropy)

One-flavor (Toulouse)

$$H_{\text{RL}} = H_0 + \sqrt{2\Delta}\left[\psi^{\dagger}_s d + d^{\dagger}\psi_s\right]_{x=x_{\text{imp}}}$$

Two-flavor (Emery and Kivelson)

$$H_{\text{mRL}} = H_0 + \sqrt{2\Gamma}\left[(\psi^{\dagger}_{sf} + \psi_{sf})(d^{\dagger} - d)\right]_{x=x_{\text{imp}}}$$

Mixed-valence (CJB and Iucci)

$$H_{\text{biRL}} = H_0 + \sqrt{2\Delta}\left[\psi^{\dagger}_{sf} d + d^{\dagger}\psi_{sf}\right]_{x=x_{\text{imp}}} + \\ + \sqrt{2\Gamma}\left[(d^{\dagger} + d)(f^{\dagger} - f)\right]$$

The two-flavor problem can be adapted to describe quantum dots attached to leads

Toulouse-limits Time-line

A. Toulouse (and later Schlottmann using bosonization)

Found an exactly solvable limit for the Kondo model.

C. R. Seances Acad. Sci., Ser. B **268**, 1200 (1969)

J. Phys. (Paris) **39**, 1486 (1978)

B. Emery and Kivelson

Extension to an exactly solvable limit for the 2-channel Kondo model.

Phys. Rev. B **46**, 10812 (1992)

   a. CJB and Iucci

   Extension to Mixed-valence (2-channel Anderson model).

   Phys. Rev. Lett. **96**, 056402 (2006); Phys. Rev. B **77**, 195113 (2008)

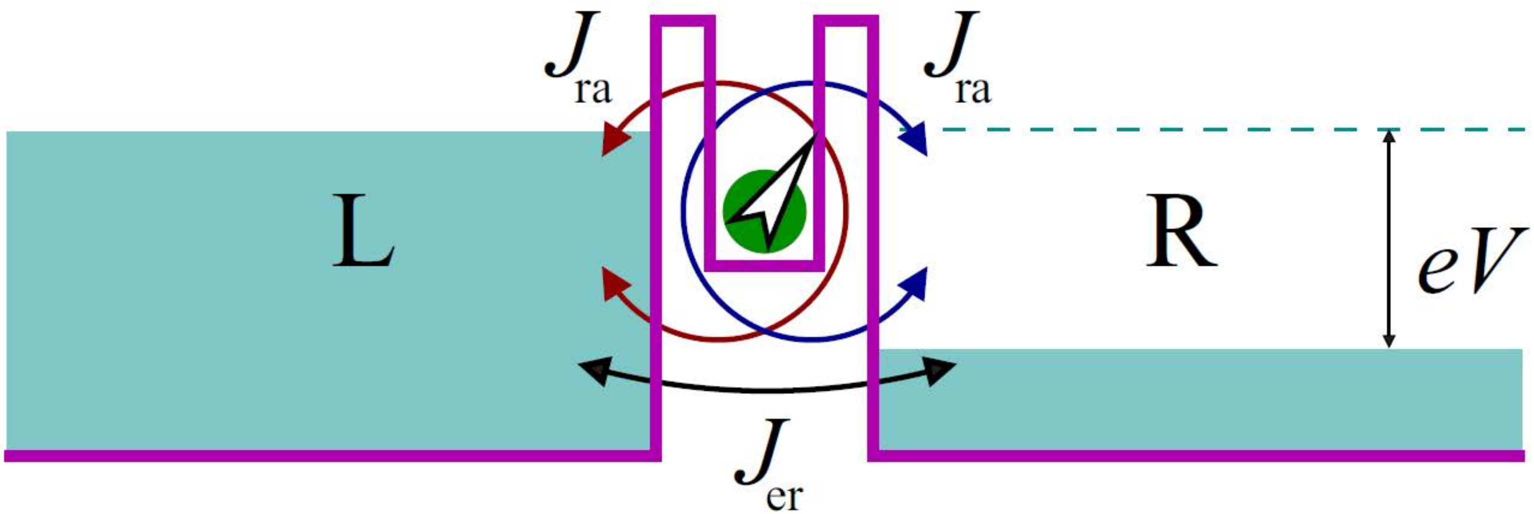
C. Schiller and Hershfield

Reworked E&K solution for the 2-lead Kondo junction model.

Phys. Rev. B **51**, 12896 (1995); Phys. Rev. B **58**, 14978 (1998)

The Two-lead Kondo Problem

$$H = \sum_{\sigma,\ell} \left(\int \mathcal{H}^0_\ell \, dx + H^z_{\mathrm{K}} + H^\perp_{\mathrm{K}} \right) + H_{\mathrm{field}}$$



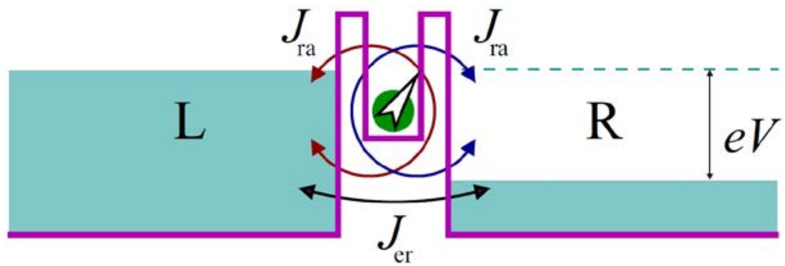
$$\mathcal{H}^0_\ell = \psi^\dagger_{\sigma\ell}(x,t)(-i v_{\mathrm{F}} \partial_x)\psi_{\sigma\ell}(x,t)$$

$$H^\perp_{\mathrm{K}} = J^\perp_{\ell\ell'} \, S^\sigma_{\mathrm{imp}} \left(\tfrac{1}{2}\psi^\dagger_{\bar{\sigma}\ell}(0,t)\psi_{\sigma\ell'}(0,t)\right)$$

$$H_{\mathrm{field}} = -h \, S^z_{\mathrm{imp}}$$

$$H^z_{\mathrm{K}} = J^z_{\ell\ell'} \, S^z_{\mathrm{imp}} \left(\frac{\sigma}{2} \psi^\dagger_{\sigma\ell}(0,t)\psi_{\sigma\ell'}(0,t) \right)$$

Revisiting the Toulouse-limit Solution



A. Map steady-state problem to a time-dependent "equilibrium" one

B. Bosonize and transform as usual, but keeping "n-twiddles"
   i. Bosonize
   ii. Change boson basis to the physical one
   iii. Carry out an EK rotation
   iv. Fine-tune the anisotropy to find non-interacting point
   v. de-Bosonize to achieve a refermionization of the problem

C. Verify the treatment of Klein factors remains unaltered

D. Find out how to treat the "n-twiddles" *consistently*

Treatment of Klein Factors

These relations are similar to the ones that arise when constructing conventional BdB dictionaries.

$$\breve{\psi}_{\sigma\ell}(x,t) = \frac{1}{\sqrt{2\pi a}} F_{\sigma\ell}(t) e^{-i\phi_{\sigma\ell}(x,t)}$$

Spinful Junction

$$F^{\dagger}_{\uparrow R} F_{\uparrow L} = F^{\dagger}_{sl} F^{\dagger}_{l}$$
$$F^{\dagger}_{\downarrow R} F_{\downarrow L} = F^{\dagger}_{l} F_{sl}$$
$$F^{\dagger}_{\uparrow L} F_{\uparrow R} = F_{l} F_{sl}$$
$$F^{\dagger}_{\downarrow L} F_{\downarrow R} = F^{\dagger}_{sl} F_{l}$$

2-ch Kondo

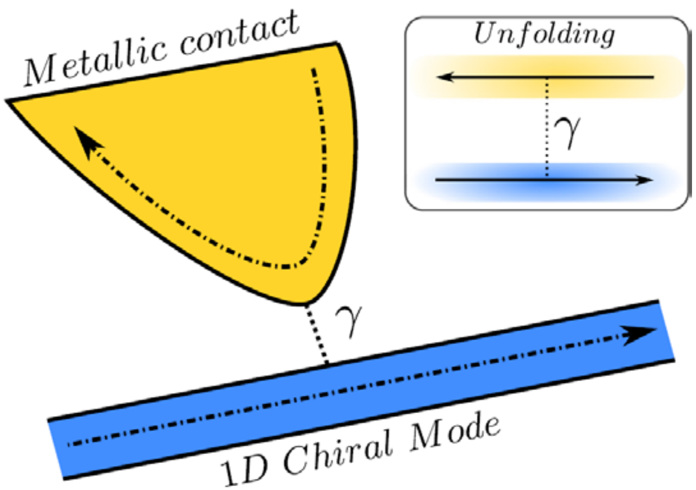
$$F^{\dagger}_{\uparrow R} F_{\downarrow R} = F^{\dagger}_{sl} F^{\dagger}_{s}$$
$$F^{\dagger}_{\uparrow L} F_{\downarrow L} = F_{sl} F^{\dagger}_{s}$$
$$F^{\dagger}_{\downarrow R} F_{\uparrow R} = F_{s} F_{sl}$$
$$F^{\dagger}_{\downarrow L} F_{\uparrow L} = F_{s} F^{\dagger}_{sl}$$

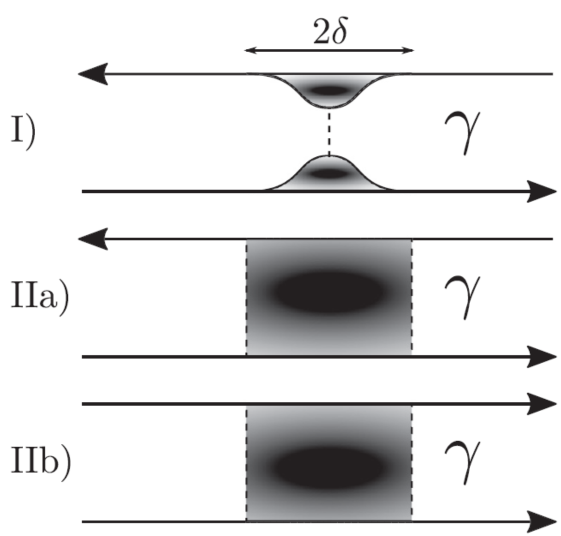
Kondo cotunneling

$$F^{\dagger}_{\uparrow R} F_{\downarrow L} = F^{\dagger}_{s} F^{\dagger}_{l}$$
$$F^{\dagger}_{\uparrow L} F_{\downarrow R} = F_{l} F^{\dagger}_{s}$$
$$F^{\dagger}_{\downarrow L} F_{\uparrow R} = F_{l} F_{s}$$
$$F^{\dagger}_{\downarrow R} F_{\uparrow L} = F_{s} F^{\dagger}_{l}$$

- Can identify relations among KF bilinears based on their defining properties
- Any one of these three sets defines the only four arbitrary phases
- The other two sets can be derived from the one used to define the phases

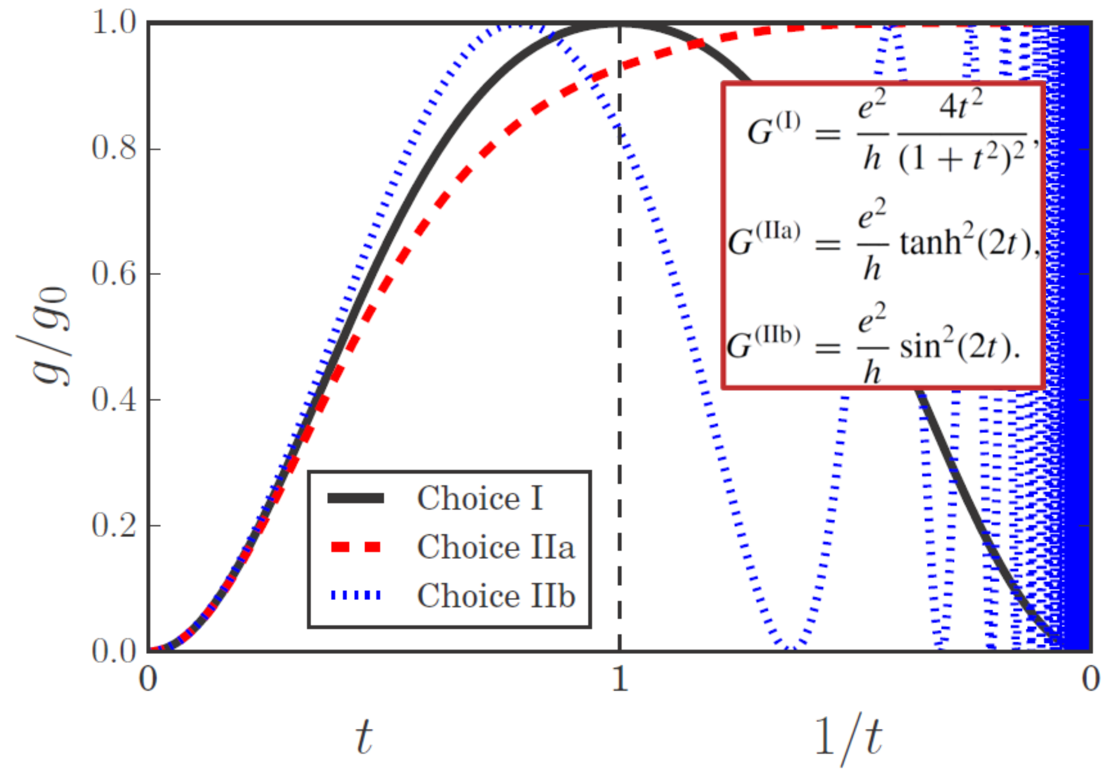
Consistent Treatment of Contacts





Filippone and Brouwer, PRB (2016)

Simple junction problem



- Different regularizations yield different results
- Standard Bosonization can be *consistent* w/ Choice I
- Only Choice I is *consistent* with unfolding
- Local-Action approach is *consistent* with Choice I
- “Consistent BdB” recovers Choice I results
- Other choices correspond to different physics and “conventional BdB” (w/spin) might blur the differences

After the BdB Procedure (Take 1)

$$\begin{aligned}\tilde{H}_{\rm K}^{\perp} = &\, J_{\rm ra}\frac{\tilde{n}_c\tilde{n}_l^{+}}{2}(d^{\dagger}\psi_{sl}(0) + \psi_{sl}^{\dagger}(0)d)\\ &+ J_{\rm ra}\frac{\tilde{n}_c\tilde{n}_l^{-}}{2}(d^{\dagger}\psi_{sl}^{\dagger}(0) + \psi_{sl}(0)d)\\ &- J_{\rm er}\frac{\tilde{n}_c\tilde{n}_{sl}^{+}}{2}(d^{\dagger}\psi_l(0) + \psi_l^{\dagger}(0)d)\\ &- J_{\rm er}\frac{\tilde{n}_c\tilde{n}_{sl}^{-}}{2}(\psi_l^{\dagger}(0)d^{\dagger} + d\psi_l(0)),\end{aligned}$$

$$J_{\rm ra} = J_{\ell\ell}^{\perp}/\sqrt{2\pi a} \quad \text{and} \quad J_{\rm er} = J_{\ell\bar{\ell}}^{\perp}/\sqrt{2\pi a}$$

Conventionally:

replacing $\tilde{n}_{\nu}^{\pm} \rightarrow 1$ everywhere

One recovers a *Majorana-hybridization* model

$$\mathcal{H}' = i\hbar v_F \sum_{\nu=c,s,f,sf} \int_{-\infty}^{\infty} \psi_{\nu}^{\dagger}(x)\, \frac{\partial}{\partial x}\psi_{\nu}(x)\,dx$$

$$+ \frac{J^{+}}{2\sqrt{2\pi a}}[\psi_{sf}^{\dagger}(0) + \psi_{sf}(0)](d^{\dagger} - d)$$

$$+ \frac{J_{\perp}^{LR}}{2\sqrt{2\pi a}}[\psi_{f}^{\dagger}(0) - \psi_{f}(0)](d^{\dagger} + d)$$

Compare with Schiller and Hershfield ⇒
Phys. Rev. B **58**, 14978 (1998)

Note: in their notation, the lead index is called "flavor" (thus *f* and *sf* are used) and there is an overall minus sign difference.

After the BdB Procedure (Take 2)

$$\tilde{H}_{\rm K}^{\perp} = J_{\rm ra}\frac{\tilde{n}_c\tilde{n}_l^{+}}{2}(d^{\dagger}\psi_{sl}(0) + \psi_{sl}^{\dagger}(0)d)$$
$$+ J_{\rm ra}\frac{\tilde{n}_c\tilde{n}_l^{-}}{2}(d^{\dagger}\psi_{sl}^{\dagger}(0) + \psi_{sl}(0)d)$$
$$- J_{\rm er}\frac{\tilde{n}_c\tilde{n}_{sl}^{+}}{2}(d^{\dagger}\psi_{l}(0) + \psi_{l}^{\dagger}(0)d)$$
$$- J_{\rm er}\frac{\tilde{n}_c\tilde{n}_{sl}^{-}}{2}(\psi_{l}^{\dagger}(0)d^{\dagger} + d\psi_{l}(0)),$$

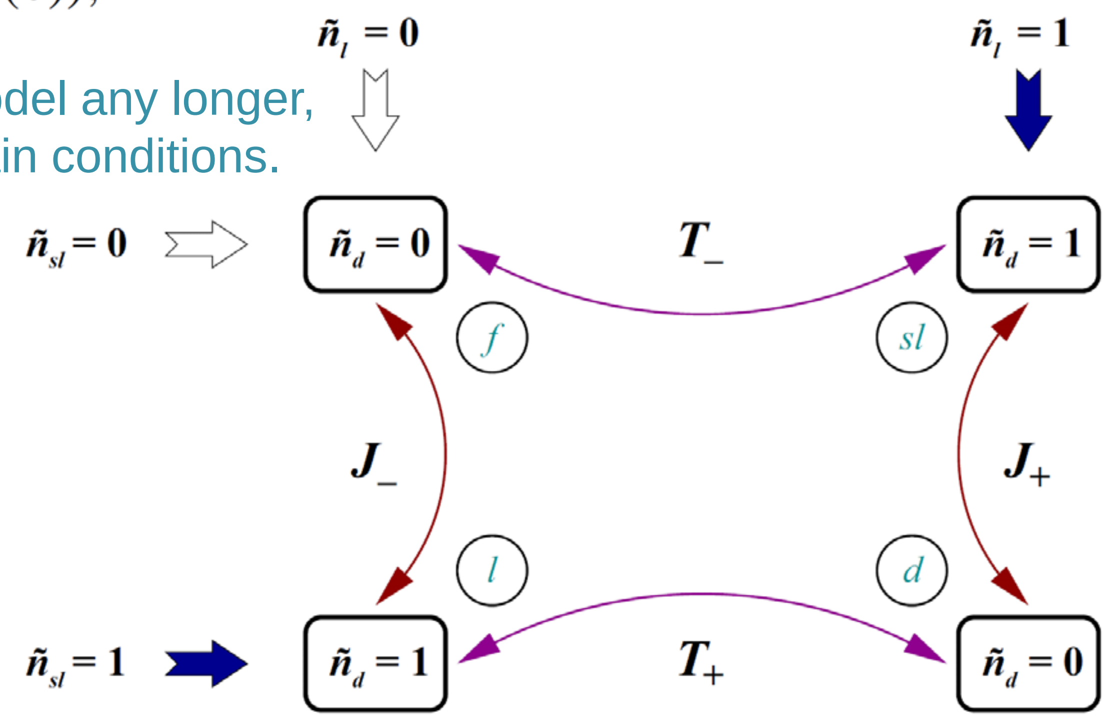
$$J_{\rm ra} = J_{\ell\ell}^{\perp}/\sqrt{2\pi a} \quad \text{and} \quad J_{\rm er} = J_{\ell\bar{\ell}}^{\perp}/\sqrt{2\pi a}$$

Consistently:
One has a *correlated-hopping* model

$$J_{\pm} = \frac{J_{\rm ra}\tilde{n}_c\tilde{n}_l^{\pm}}{4v_{\rm F}} \quad \text{and} \quad T_{\pm} = \frac{J_{\rm er}\tilde{n}_c\tilde{n}_{sl}^{\pm}}{4v_{\rm F}}$$

Note: this is not a non-interacting model any longer, but we can map it to one under certain conditions.

$$\tilde{H}_{\rm K}^{\perp} = \frac{J_{\rm ra}^{+}}{2}(\tilde{d}^{\dagger}\tilde{\psi}_{sl}(0) + \tilde{\psi}_{sl}^{\dagger}(0)\tilde{d})$$
$$+ \frac{J_{\rm ra}^{-}}{2}(\tilde{f}^{\dagger}\tilde{\psi}_{l}(0) + \tilde{\psi}_{l}^{\dagger}(0)\tilde{f})$$
$$- \frac{J_{\rm er}^{+}}{2}(\tilde{d}^{\dagger}\tilde{\psi}_{l}(0) + \tilde{\psi}_{l}^{\dagger}(0)\tilde{d})$$
$$- \frac{J_{\rm er}^{-}}{2}(\tilde{f}^{\dagger}\tilde{\psi}_{sl}(0) + \tilde{\psi}_{sl}^{\dagger}(0)\tilde{f})$$



Current at the Toulouse Point

Conventionally:

$$I = \int_0^{+\infty} \frac{J_{\text{er}}^2\left[\left(J_{\text{er}}^2 + K^2\right)\left(\omega^2 + J_{\text{ra}}^4\right) + h^2 J_{\text{ra}}^2\right]\left[s_l(\omega) - \bar{s}_l(\omega)\right]}{\omega^4 + \left[J_{\text{ra}}^4 + \left(J_{\text{er}}^2 + K^2\right)^2 - 2h^2\right]\omega^2 + \left[J_{\text{ra}}^2\left(J_{\text{er}}^2 + K^2\right) + h^2\right]^2} \frac{d\omega}{2\pi}$$

$$T_\pm \longmapsto J_{\text{er}}/2 \text{ and } J_\pm \longmapsto (J_{\text{ra}} \pm K)/2$$

$$K = (J_{\text{ra}}^{\text{L}} - J_{\text{ra}}^{\text{R}})/2$$

$$s_\nu(\omega) \equiv \tanh \tfrac{\omega - \mu_\nu}{2T_{\text{emp}}} \text{ and } \bar{s}_\nu(\omega) \equiv \tanh \tfrac{\omega + \mu_\nu}{2T_{\text{emp}}}$$

Consistently:

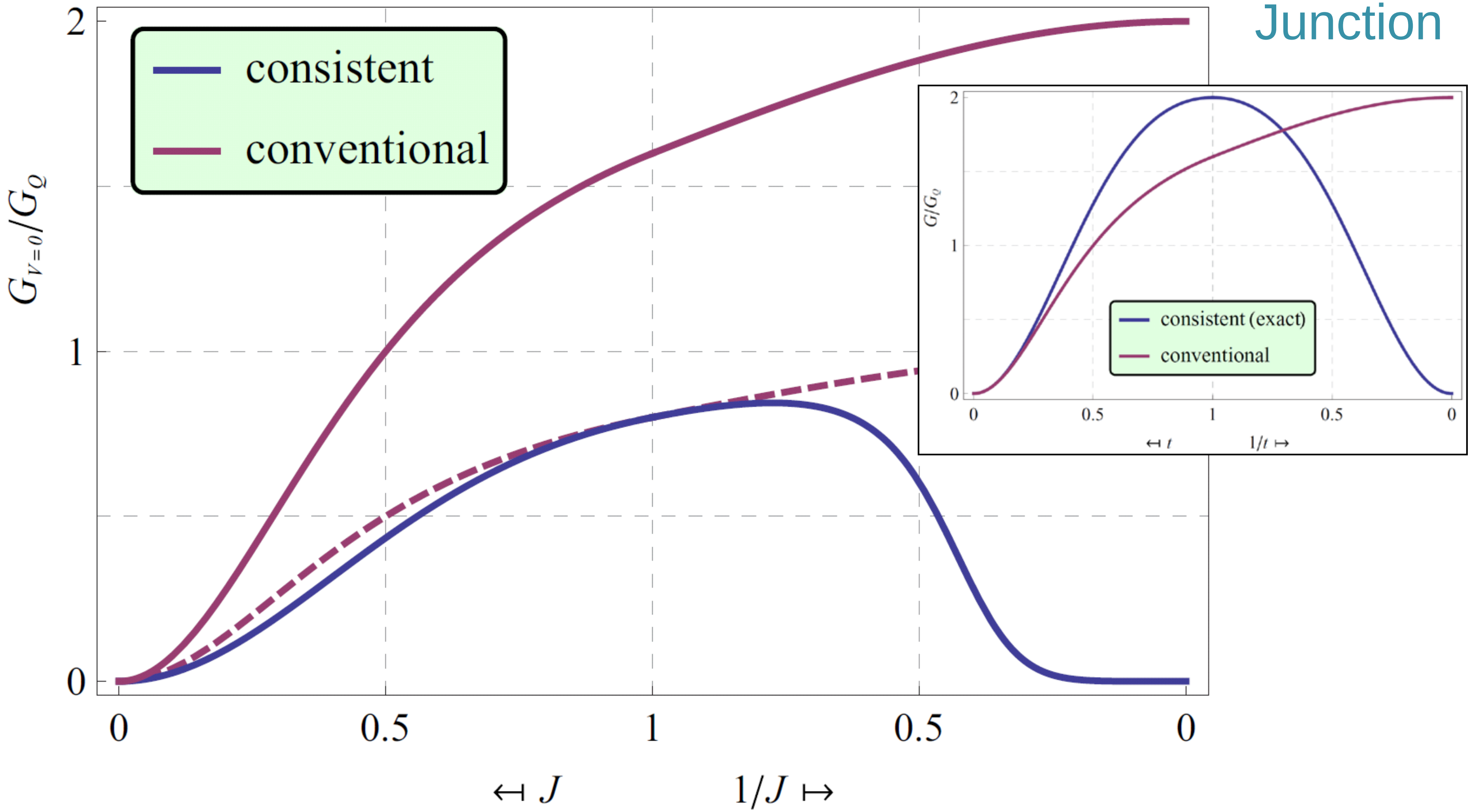
$$I = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{2\left(J_{\text{ss}}^2 - J_{\text{ds}}^2\right)(\omega + h)^2}{\left[(\omega + h)^2 - J_{\text{ds}}^2 - (eV/2)^2\right]^2 + 4J_{\text{ss}}^2(\omega + h)^2}\left[s_l(\omega) - s_{sl}(\omega)\right]$$

$$J_{\text{ss/ds}} = (J_{\text{ra}}^2 \pm J_{\text{er}}^2)/4$$

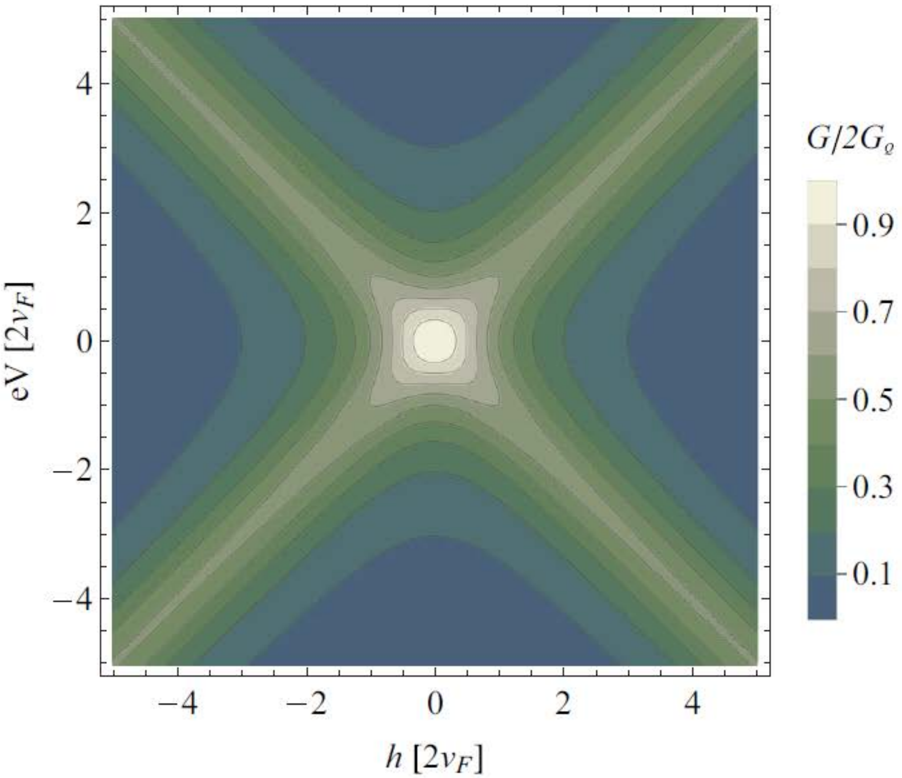
Zero-voltage Differential Conductance

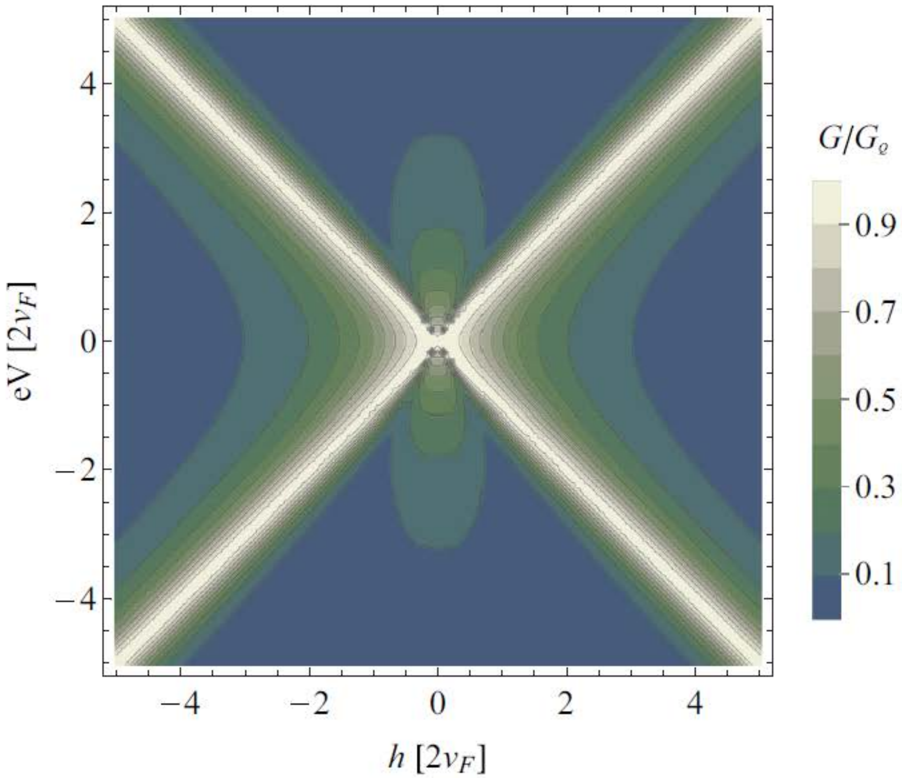
Kondo Junction

Regular Junction



Differential Conductance Maps

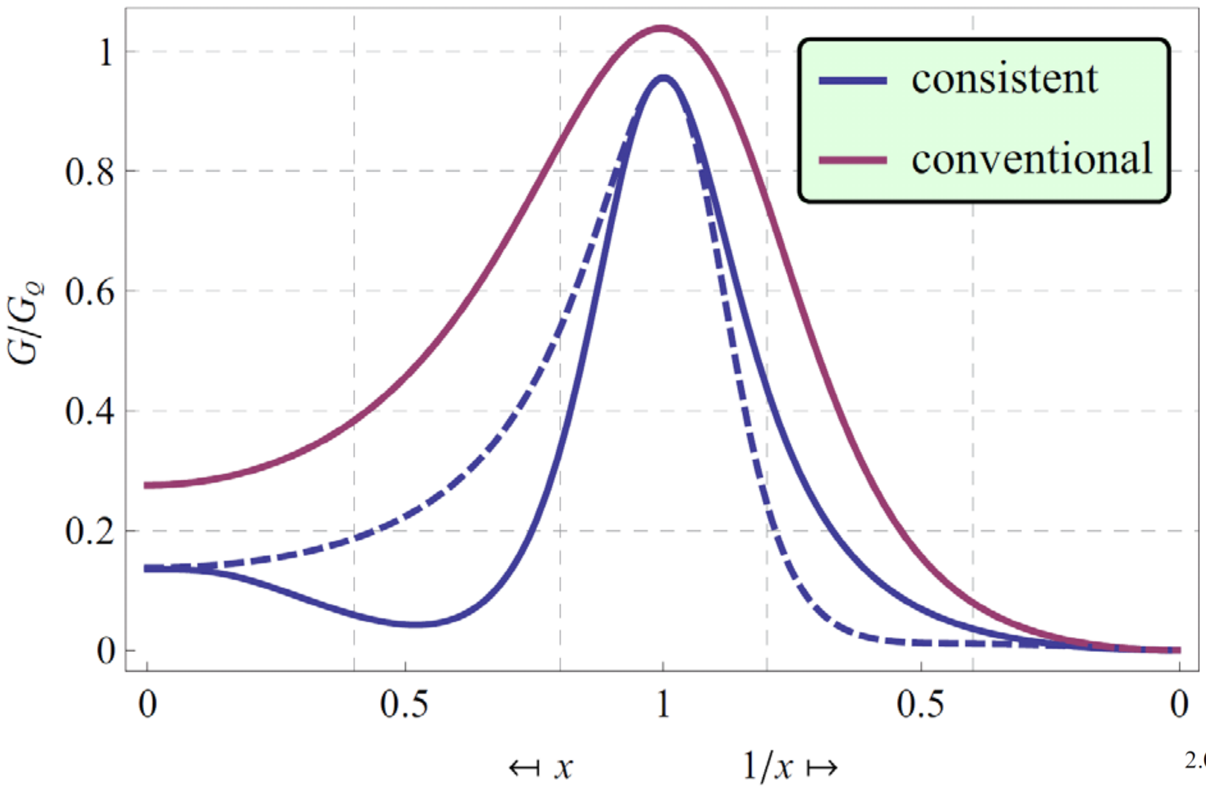




Conventionally: There is an unexpected symmetry between field and voltage.

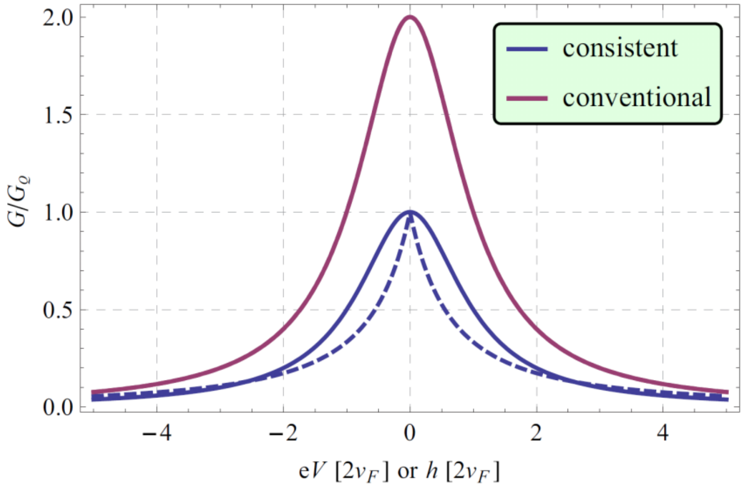
Consistently: The unaccounted symmetry is now removed.

Some dI/dV Cross Sections



Off-centered cuts

Centered cuts



Conventionally: Field and voltage are the same.

Consistently: Field (solid) or Voltage (dashed).

Talk Highlights

Key Ideas:

- ✓ We found a procedure to **Bosonize** and **Debosonize** *consistently*
- ✓ This new procedure provides valuable general guidance
  - Can be extended to many different problems
  - Calculations are more complicated but still tractable
- ✓ Many possible ramifications and open directions
  - Other junction problems (quantum-Hall edges, etc.)
  - Quantum Impurities and ac drives (cavities, etc.)
  - Back-scattering (classical impurities, disorder, etc.)

1st Example of Application: nonequilibrium Kondo problem

- ❑ We build on the previous work
- ❑ The consistent approach yields new results
- ❑ These new results are more physical
  - Different and richer behavior with field and voltage
  - Splitting of the Kondo resonance

Following the development of a scheme to bosonize and debosonize consistently [1,2], we presented in detail the Toulouse-point analytic solution of the two-lead (nonequilibrium) Kondo junction model. The existence and location of the solvable point is not modified, but the calculational methodology and the final expressions for observable quantities change markedly as compared to the previously accepted results.

THANK YOU

References:

[1,2] See Phys. Rev. B **93**, 085440 and 085441 (2016).