

Dissipative Quantum Phase Transitions in Interacting Light-Matter Systems

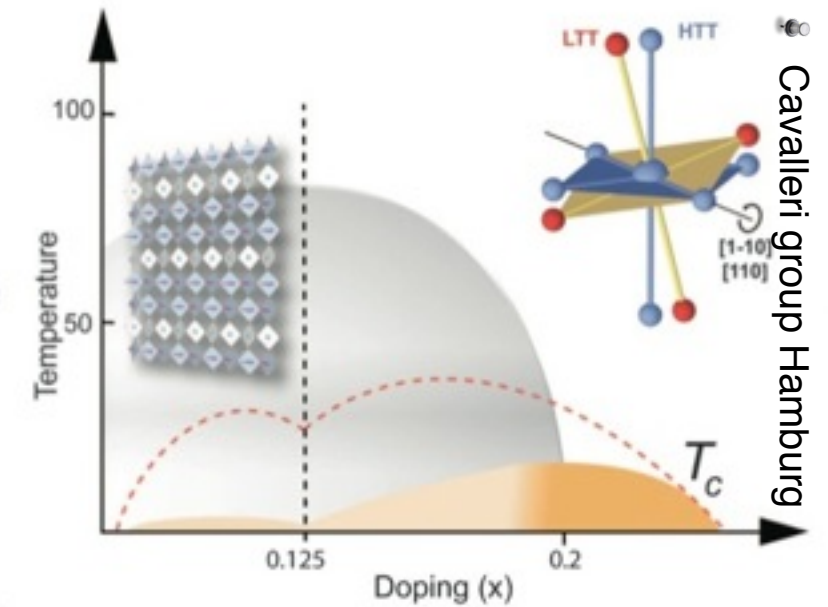
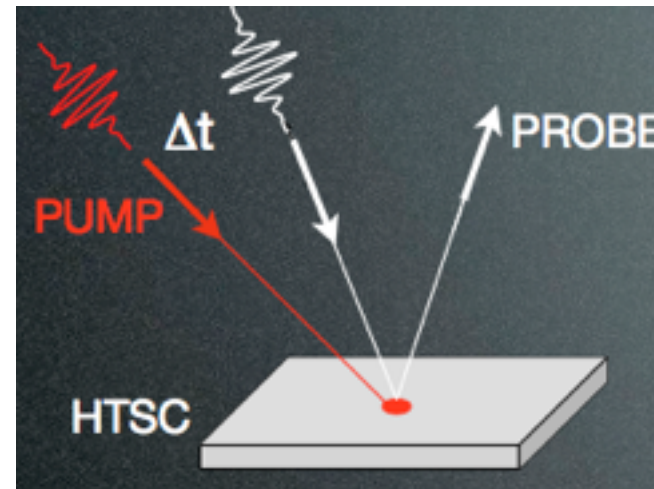
Marco Schiro'
CNRS & IPhT CEA Saclay

Light and Matter Out of Equilibrium



Condensed Matter

Light used to probe phases or (more recently) to “induce” phases of matter

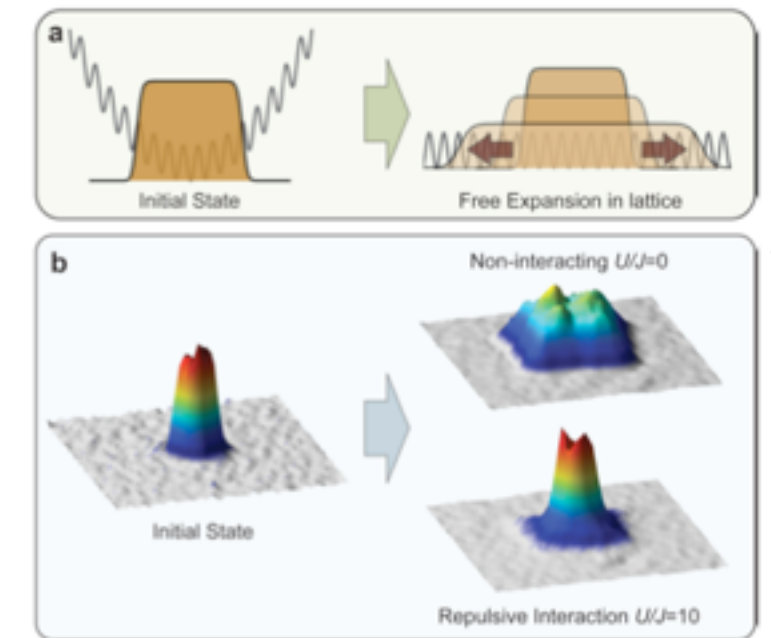
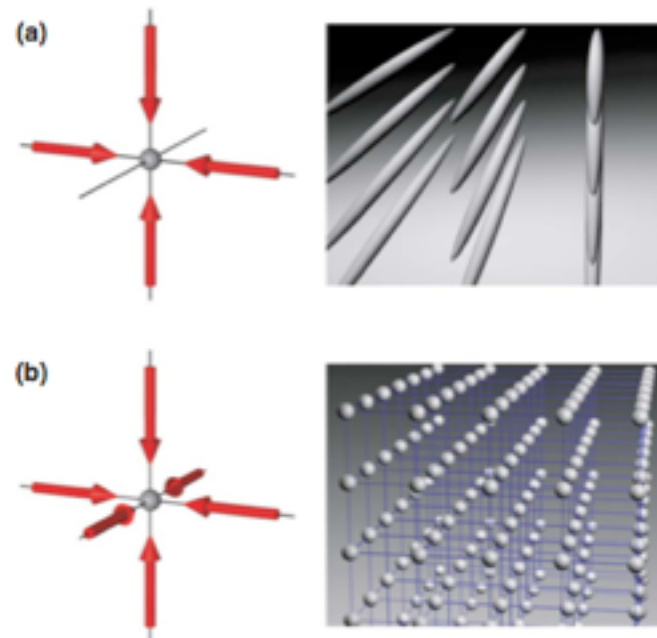


Cavalleri group Hamburg



Ultra-Cold Atoms

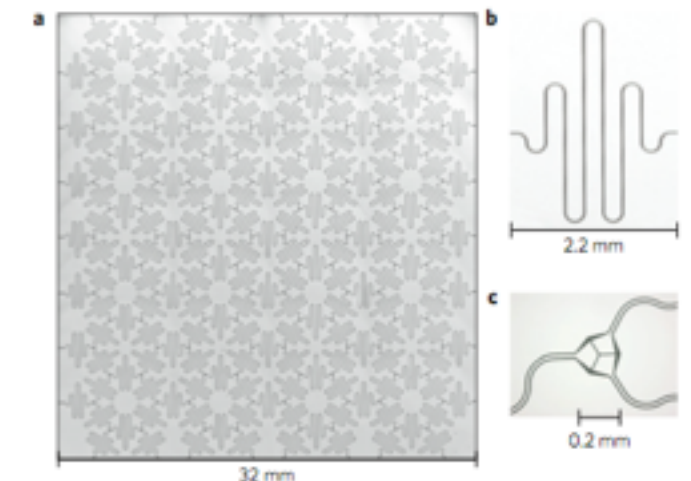
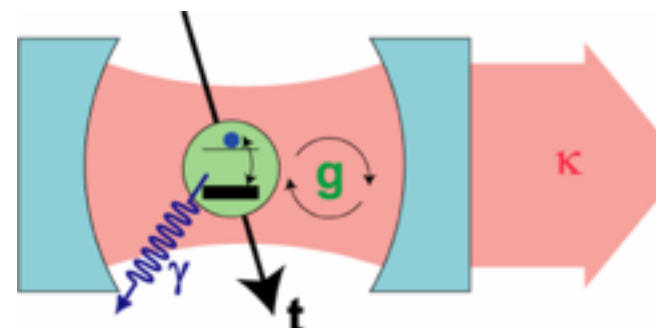
Light used to trap (optical lattices) to probe or to excite atoms



Bloch group, Munich

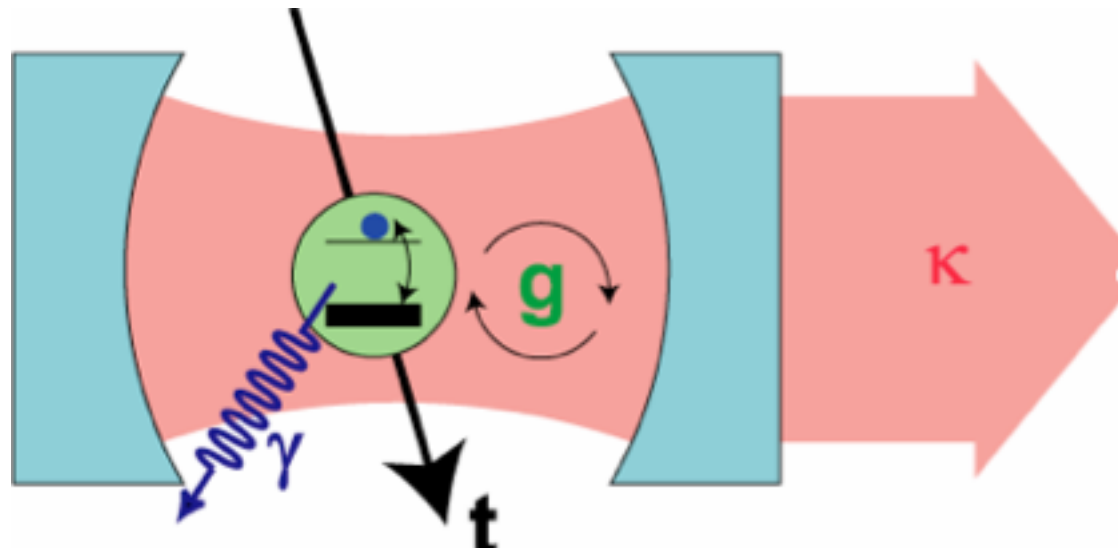


Quantum Many Body Physics with “Light” (!?)



Houck group, Princeton

Cavity Quantum ElectroDynamics

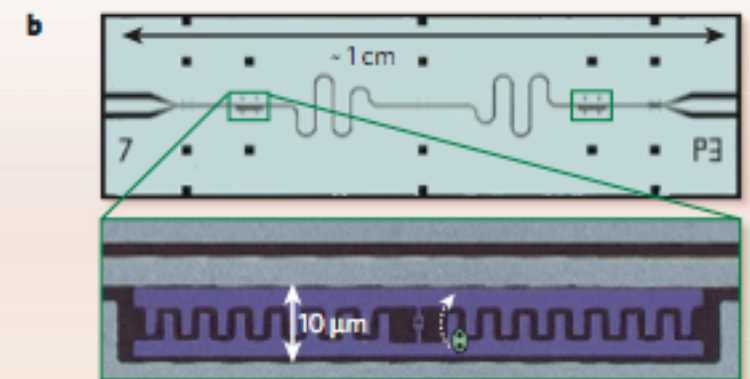
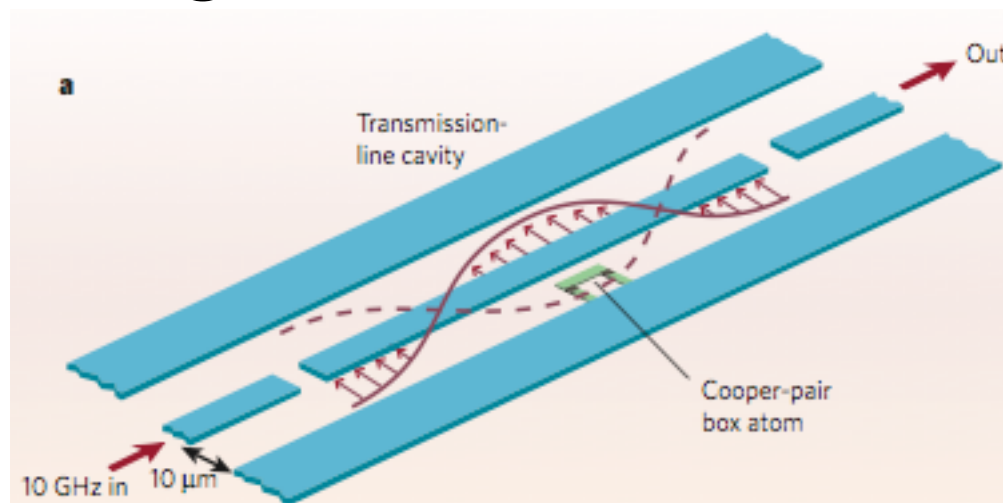


J. M. Raimond, M. Brune and S. Haroche,
Rev. Mod. Phys. **73**, 565 (2001)

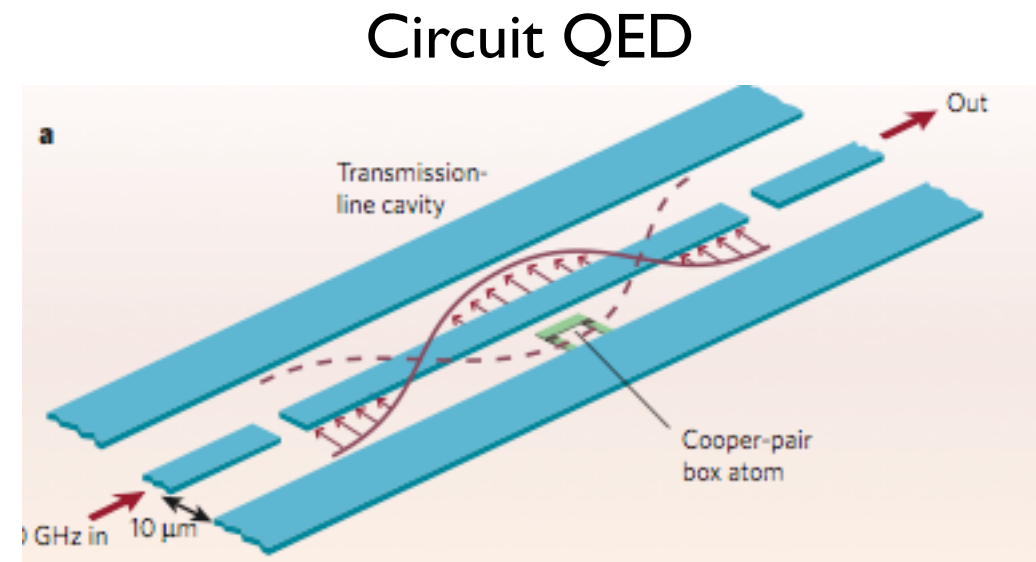
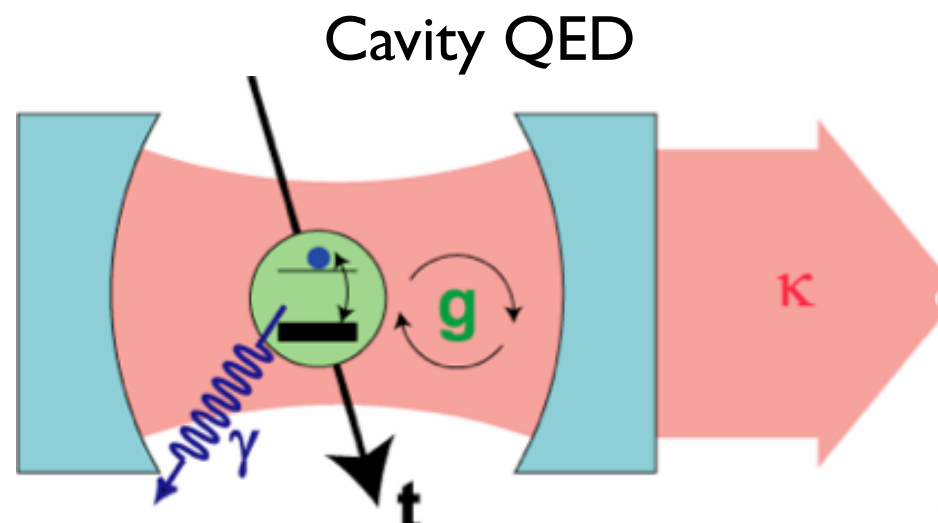
- Trapping photons and atoms into cavities!
- Light-Matter coupling (Dipole)

- Optical/Microwave 3D cavities supporting discrete modes
- Alkali or Rydberg Atoms trapped into the cavity
- A “Solid-State” Analog: Circuit QED

A. Blais et al, Phys. Rev. A
69 062320 (2004)



Energy Scales in CQED



📌 Photon Frequency/Atom splitting/Drive ω_r, ω_q

📌 Light-Matter Coupling (Vacuum Rabi Splitting) $g \ll \omega_r, \omega_q$

📌 Photon Losses and Atomic Decay Rates (Dissipation) κ, γ

$$\partial_t \rho = -i[H, \rho] + \mathcal{D}[\rho] \quad \mathcal{D}[\rho] = \frac{1}{2} (a\rho a^\dagger - \{a^\dagger a, \rho\})$$

Strong-Coupling Regime of cQED

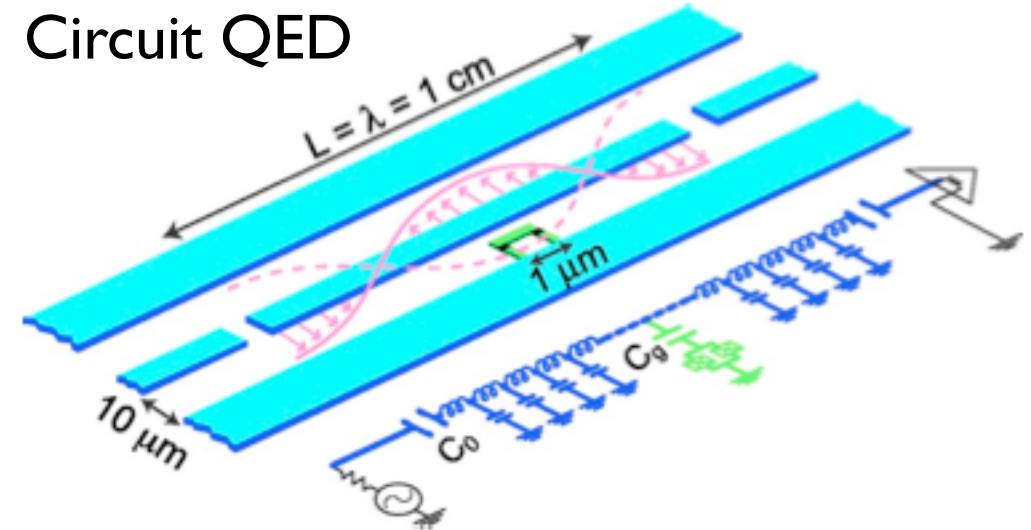
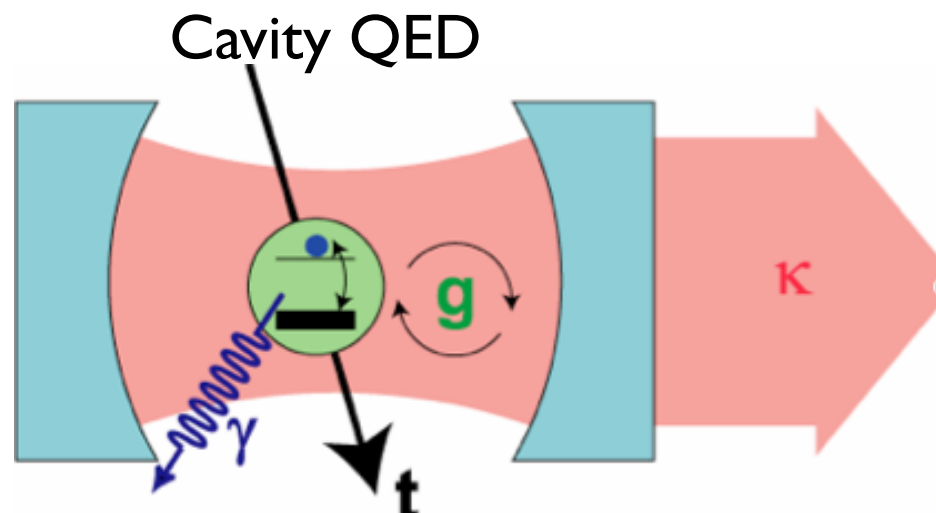
$$g \gg \kappa, \gamma$$

📌 Typical Circuit QED:

$$\omega_r \sim 5 \text{ GHz}$$

$$g \sim 0.1 - 0.5 \text{ GHz} \quad \kappa, \gamma \sim 500 \text{ kHz}$$

Quantum Models of Light and Matter



The Rabi Model

$$H_{Rabi} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger + a) (\sigma^+ + \sigma^-)$$

Harmonic
Mode
(Photon)

Anharmonic
Mode
(Atom)

Linear coupling
(dipole)



I. Rabi (1936)



Often in Rotating Wave Approximation (requires $g \ll \min(\omega_r, \omega_q)$)

$$H_{Rabi} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger \sigma^- + a \sigma^+) + g (a^\dagger \sigma^+ + a \sigma^-)$$

~~The Jaynes-Cumming Model~~

Jaynes, Cumming (1963)

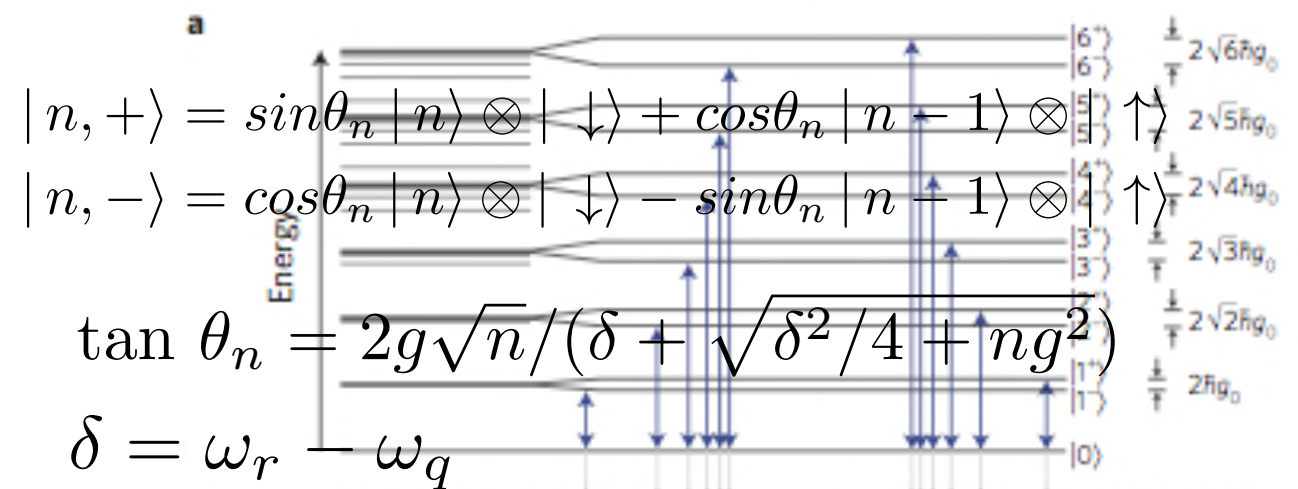
Atom-Induced Photon Non Linearity

$$H_{JC} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger \sigma^- + \text{h.c.})$$

Elementary Light-Matter Excitations: Polaritons

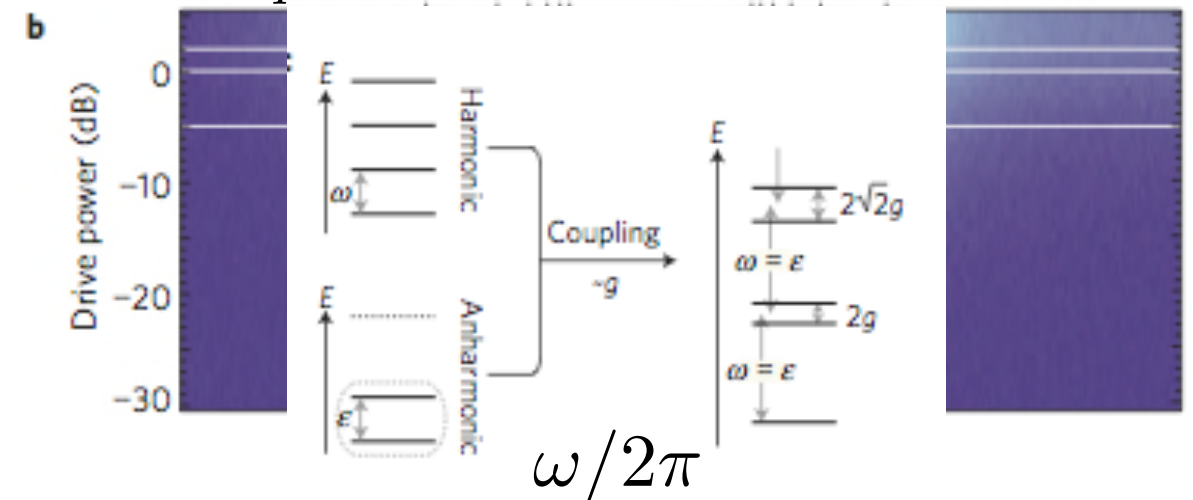
$$N_{pol} = a^\dagger a + \sigma^+ \sigma^-$$

$$[H_{JC}, N_{pol}] = 0$$



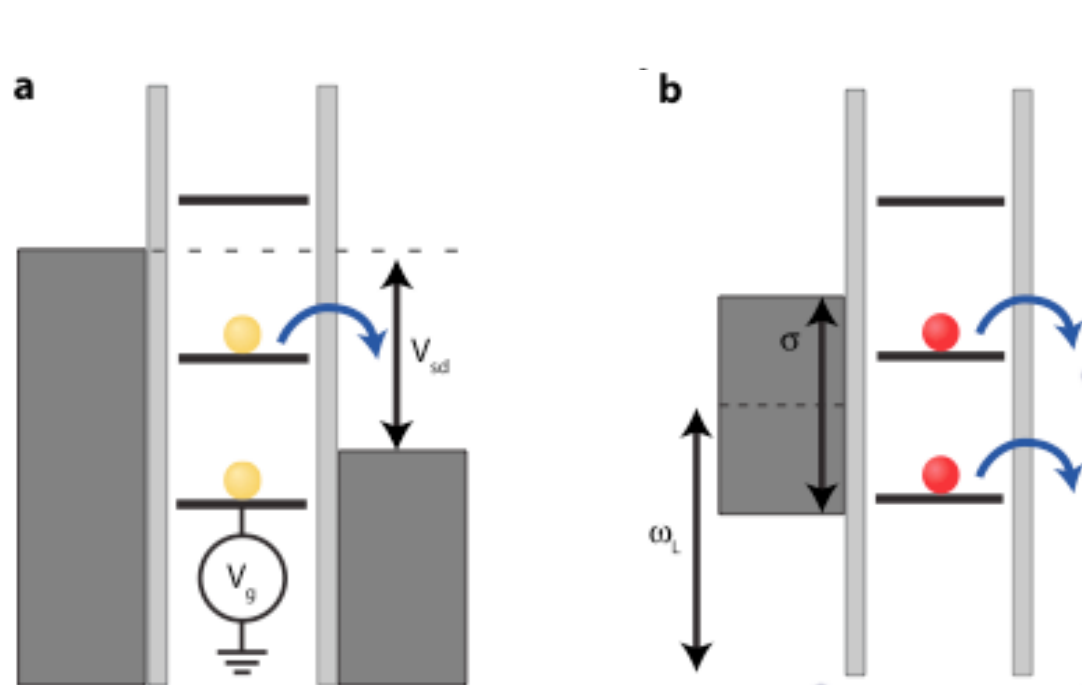
Weakly Anharmonic Spectrum

$$E_{n\pm} = \omega_r n + \frac{\delta}{2} \pm \sqrt{\delta^2/4 + ng^2}$$

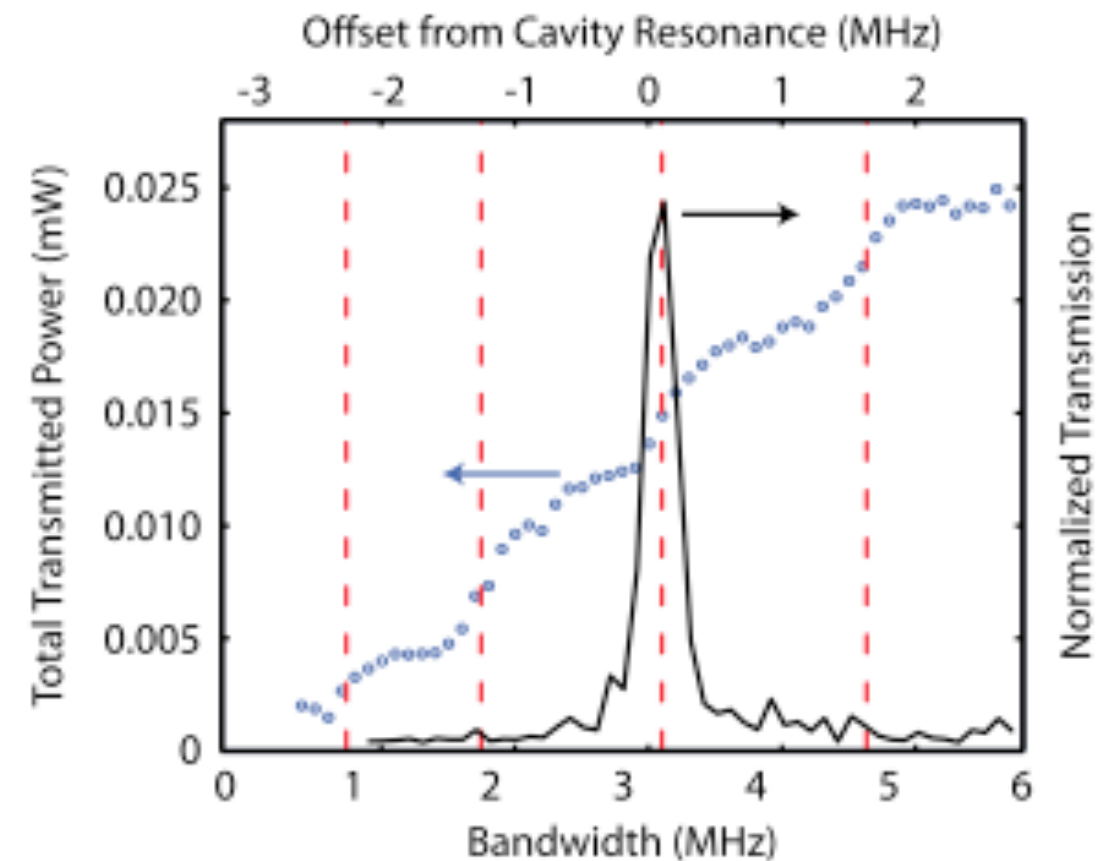


Exp Realization with Circuit QED **under driving**

Photon-Blockaded Transport



A.J Hoffmann et al, Phys. Rev. Lett **107**, (2011)



Steps in the Photon Transmission through a driven cavity

Strongly Dispersive Regime: $g \ll |\omega_r - \omega_q|$

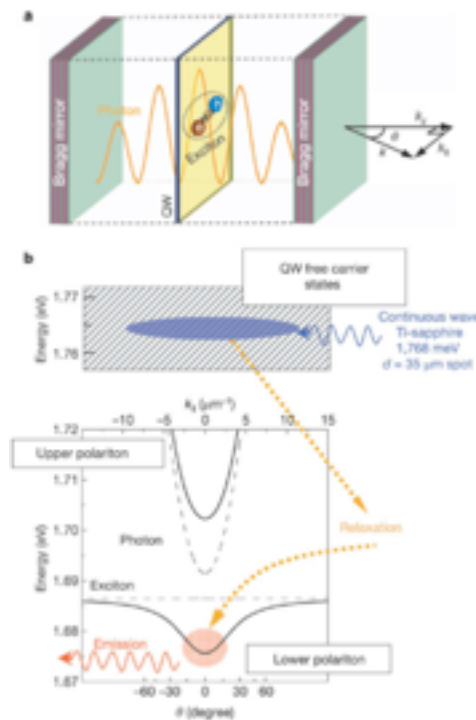
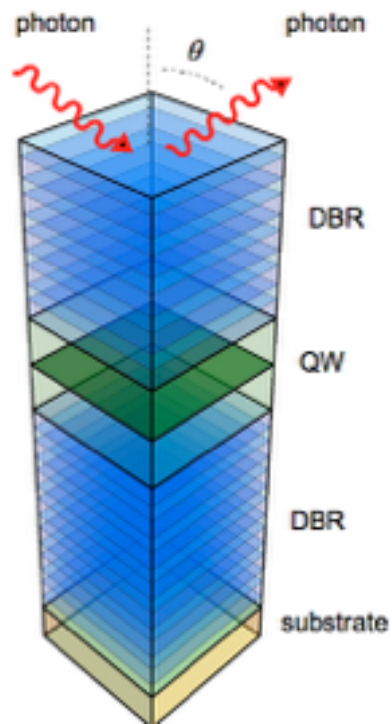
Effective Photon
Hamiltonian

$$H_{JC} \sim \tilde{\omega}_r a^\dagger a + U_{eff} (a^\dagger a)^2$$

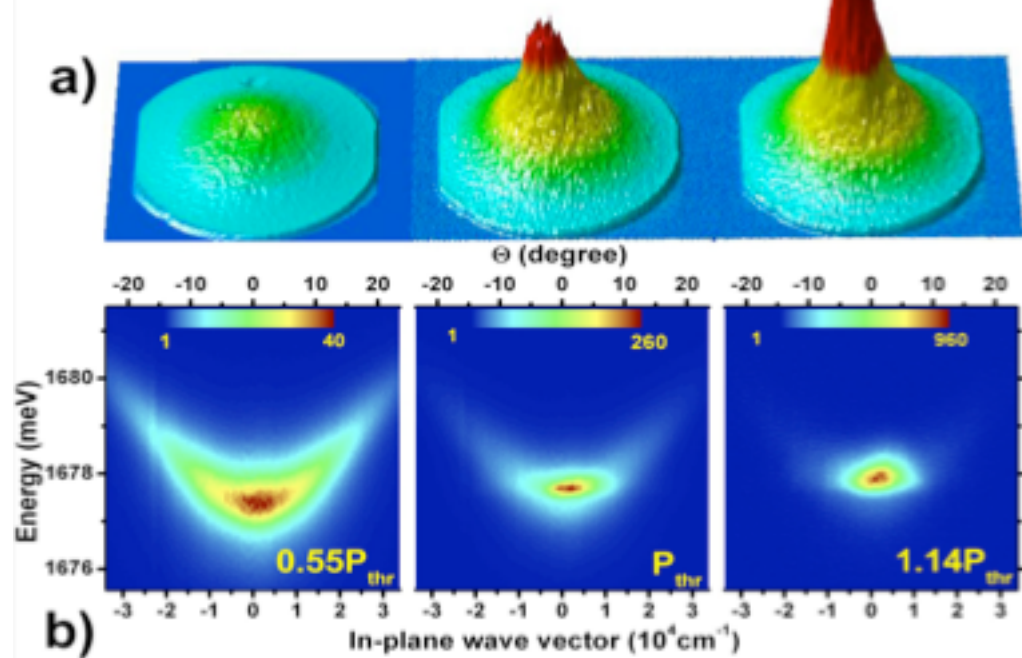
$$U_{eff} \sim g^4 / |\omega_r - \omega_q|^3$$

Platforms for Many Body Physics with Light (I)

☑ Exciton-Polariton Condensate



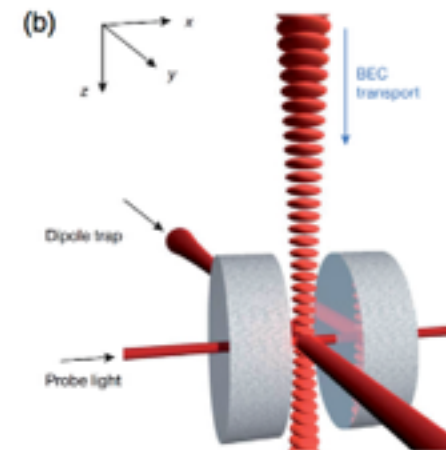
📍 Driven-Dissipative Superfluid



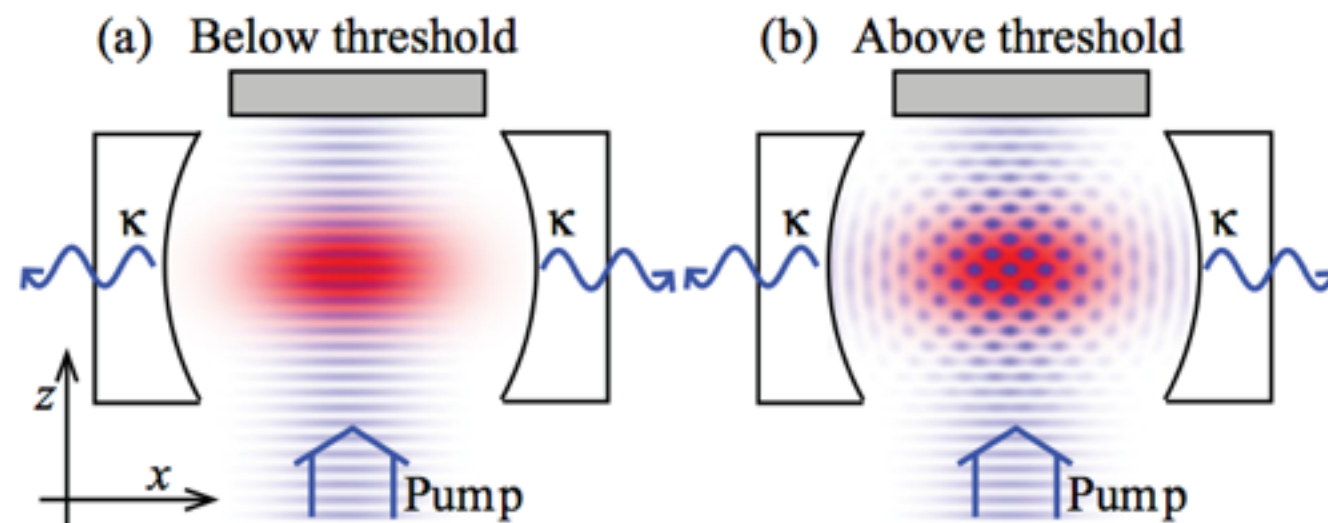
Exp: Paris (J. Bloch), EPFL, Stanford, ...

📍 I. Carusotto, C. Ciuti, RMP 85 (2013)

☑ Cold-Atoms in Optical Cavities



📍 Dissipative Dicke Transition

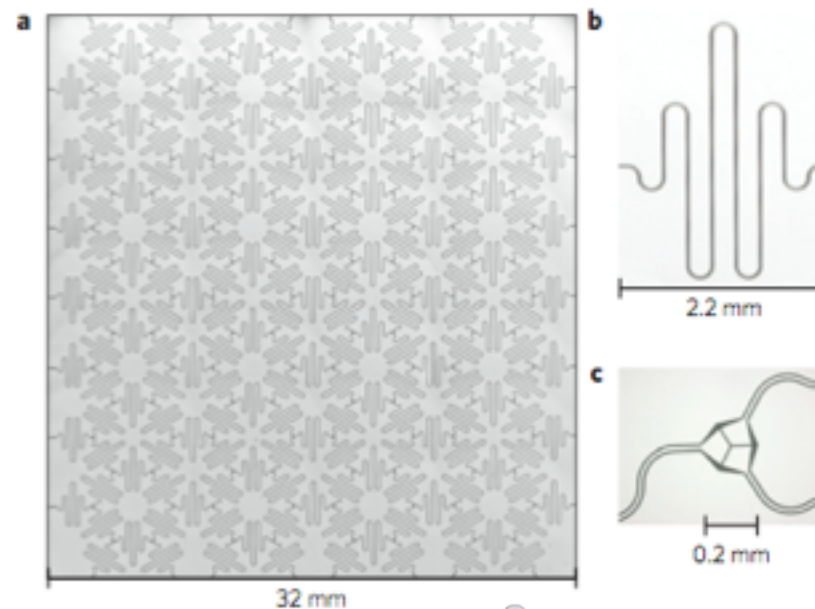
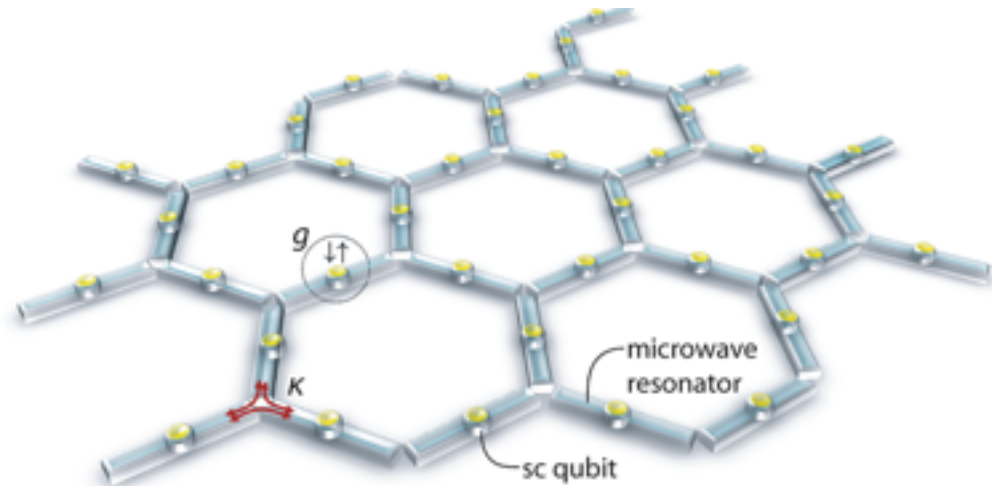


Exp: ETH, Hamburg, ...

📍 H. Ritsch, P. Domokos, F. Brennecke, T. Esslinger, RMP 85 (2013)

Platforms for Many Body Physics with Light(II)

☑ Circuit QED Arrays



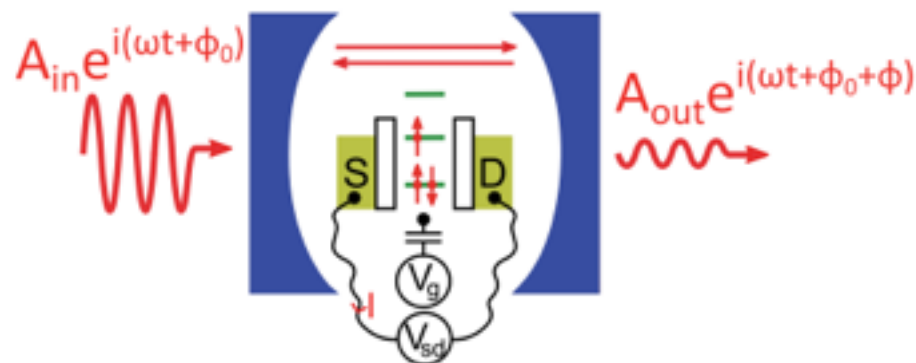
📍 Experiments in Princeton, Chicago, Paris,...

📍 Dissipative Quantum Phase Transitions

- 📍 J. Koch, A. Houck and H. E. Tureci Nat. Phys. 8 292 (2012)
- 📍 Le Hur, Henriët, Petrescu, Plekhanov, Roux, MS, CRAS (2015)

- 📍 J. Raftery et al, PRX (2014)
- 📍 M. Fitzpatrick et al, PRX (2017)
- 📍 J.M. Fink et al, (2017)

☑ Electronic NanoCircuits Coupled To Resonators (Impurity Physics with Light and Matter)

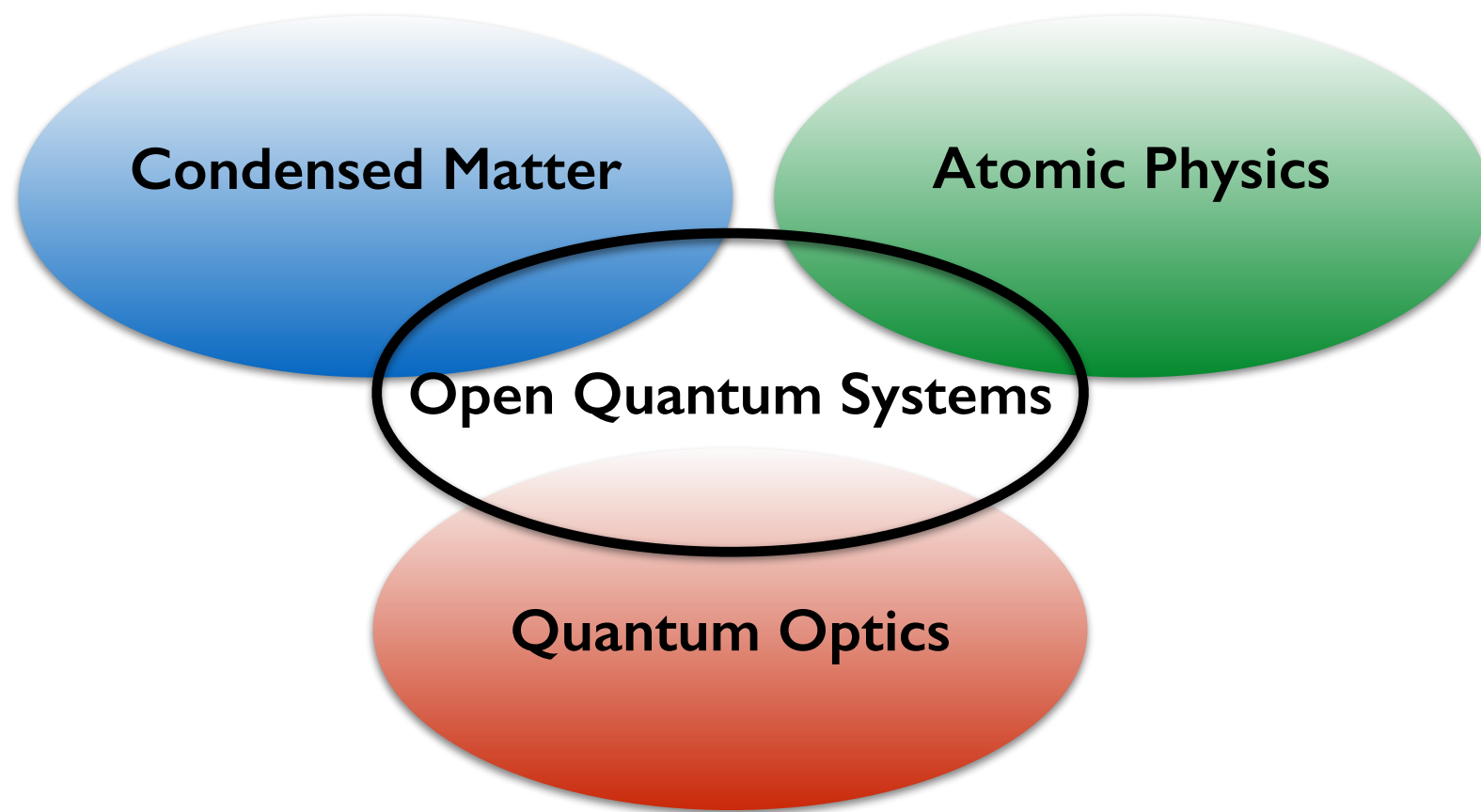


Platforms for Spin-Boson Physics: Ongoing Experiments @ Grenoble (Roch), JQI (Manucharyan), Waterloo (Lupascu)

M. Delbecq et al PRL 107 256804 (2011)

📍 Experiments at ENS Paris, Princeton,...

Open Quantum Many Body Systems



Open Quantum Systems

17 Jul, 2017 - 04 Aug, 2017

Venue: Madhava lecture hall, ICTS, Bangalore, India

There have been major recent breakthroughs, both experimental and theoretical, in the field of Open Quantum Systems. The aim of this program is to bring together leaders in the Open Quantum Systems community from a range of subfields, such as Mesoscopic Quantum Optics (Quantum Dot Circuit-QED systems), Cavity Optomechanics, Cavity-QED systems and many more. This field of hybrid quantum systems sits perfectly at the interface of condensed matter physics, quantum optics and non-equilibrium physics. Despite remarkable progress, there are still a large number of challenging and important experimental and theoretical questions thereby creating an immediate need for the program

- 📌 Interplay between coherent and dissipative evolution
- 📌 Dissipative Quantum Phase Transitions? Universality Out of Equilibrium?
- 📌 How Bath Correlations affect system properties?
Markovian vs Non-Markovian Evolution

Outline

-  Dicke Superradiance and the role of Markovian/Non-Markovian Dissipation
-  Driven-Dissipative Quantum Phase Transitions in circuit QED Lattices

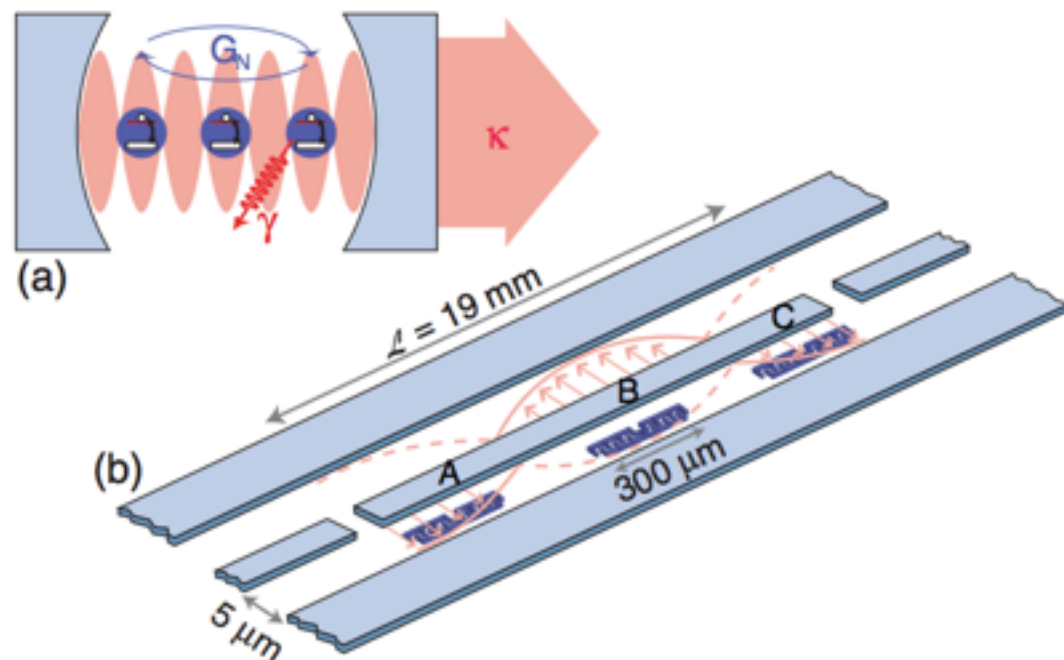
Dicke Superradiance and the role of Dissipation

Dicke Model of Light and Matter

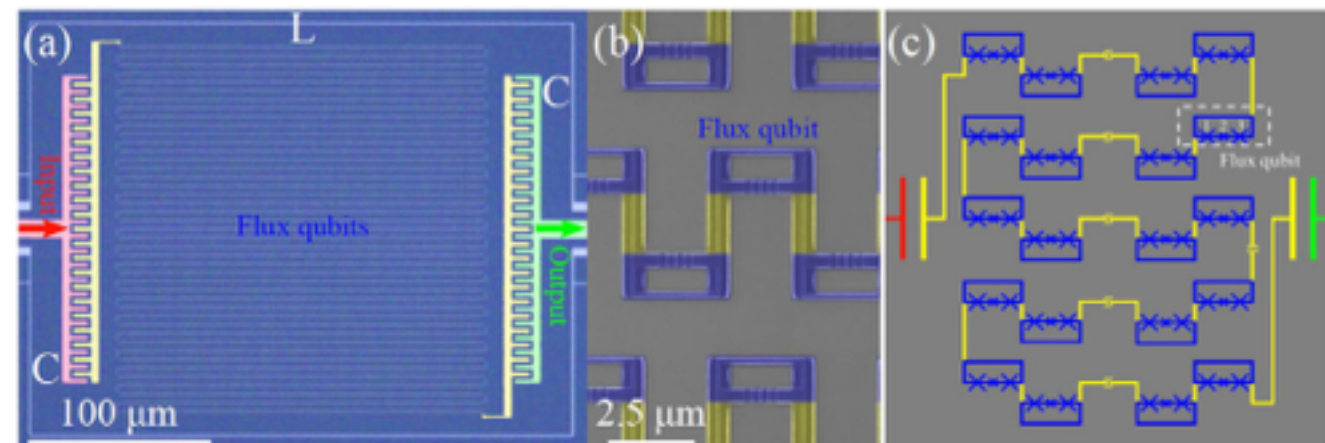
R. Dicke, Hepp&Lieb,...,Emary&Brandes

📌 Dicke Model: photon mode coupled to N spin 1/2

$$H_D = \omega_0 a^\dagger a + \omega_q \sum_i S_i^z + \frac{\lambda}{\sqrt{N}} \sum_i (a + a^\dagger) S_i^x$$



📌 Fink et al, PRL (2009)



📌 Kakuyanagi et al, PRL (2009)

📌 Z2 Symmetry

$$a \leftrightarrow -a \quad S_i^x \leftrightarrow -S_i^x$$

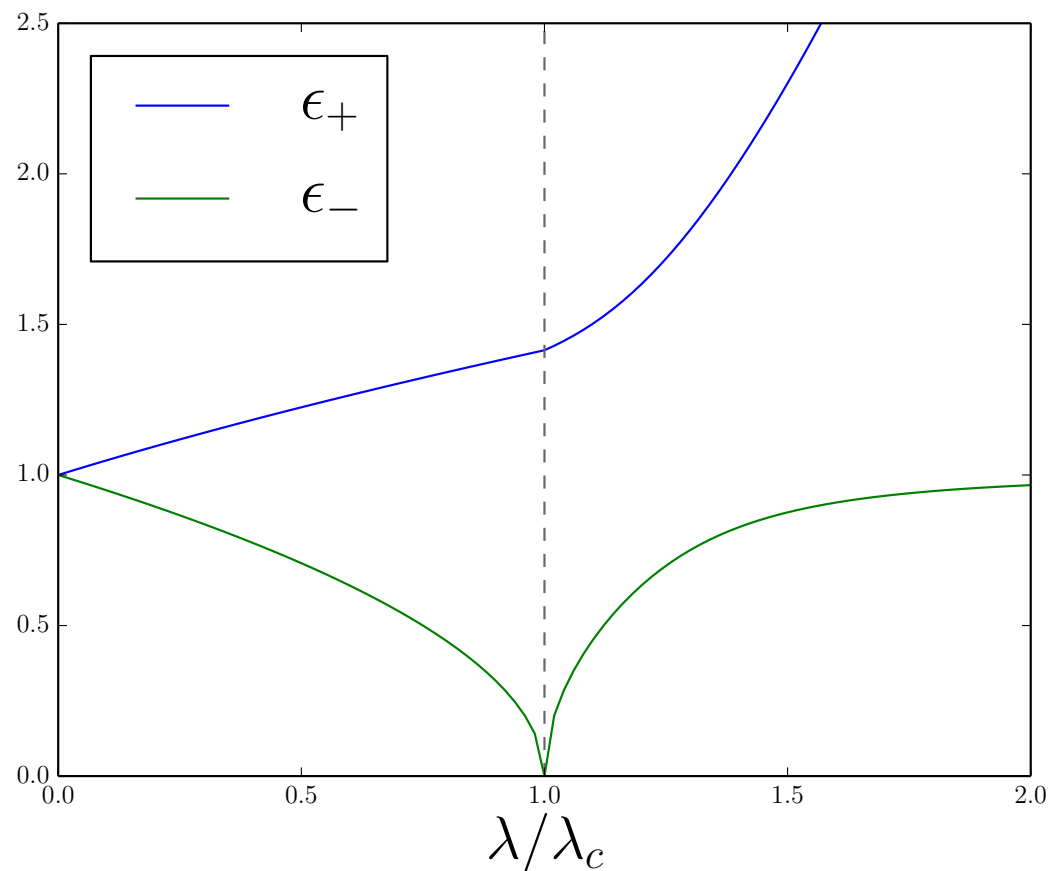
Cavity-mediated spin interaction, leading to “superradiant” phase
above critical coupling $\lambda_c = \sqrt{\omega_0 \omega_q / 4}$

Equilibrium Superradiance Phase Transition

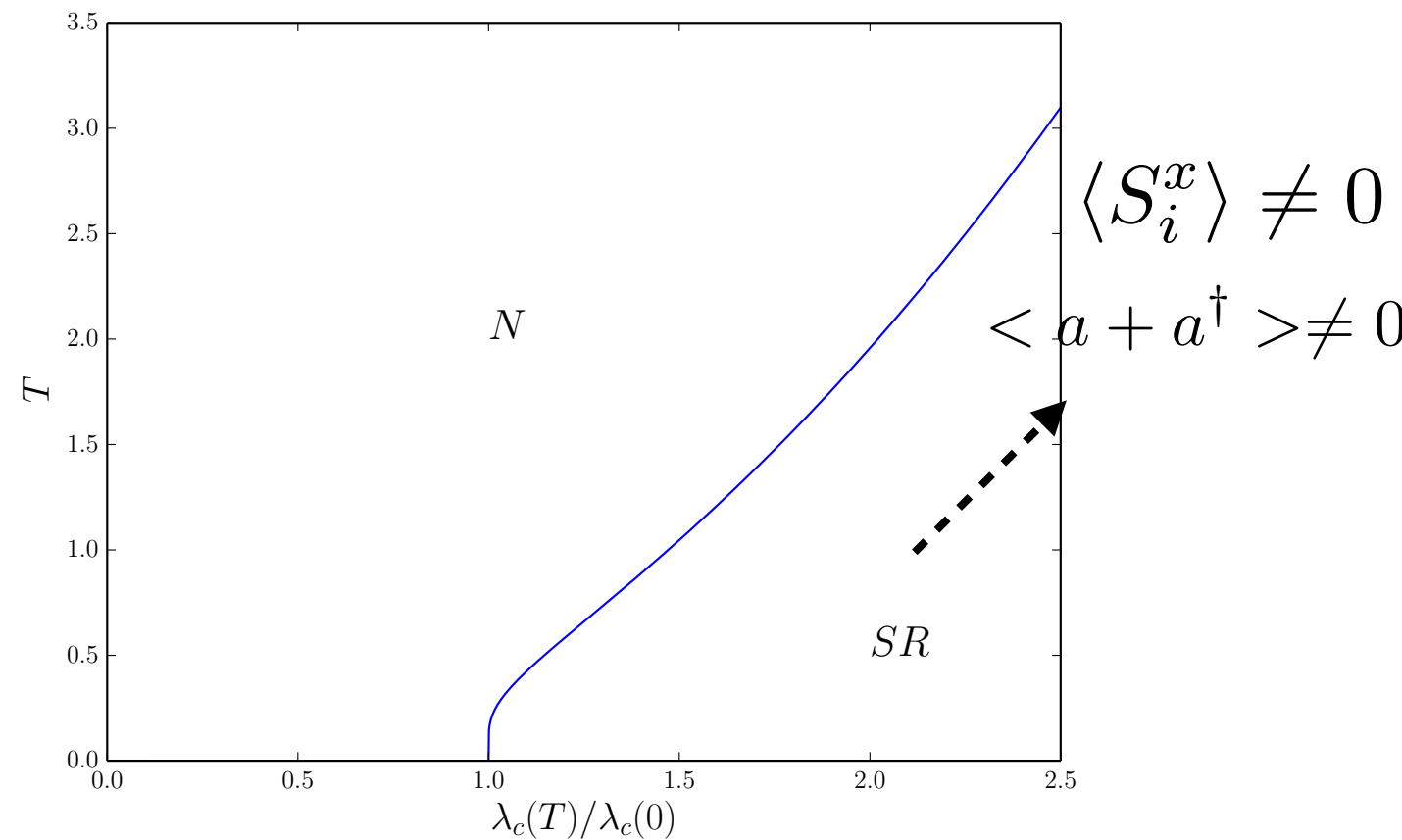
$$H_D = \omega_0 a^\dagger a + \omega_q \sum_i S_i^z + \frac{\lambda}{\sqrt{N}} \sum_i (a + a^\dagger) S_i^x$$



Excitation Spectrum



Finite-Temperature Phase Diagram



Effect of Markovian Losses

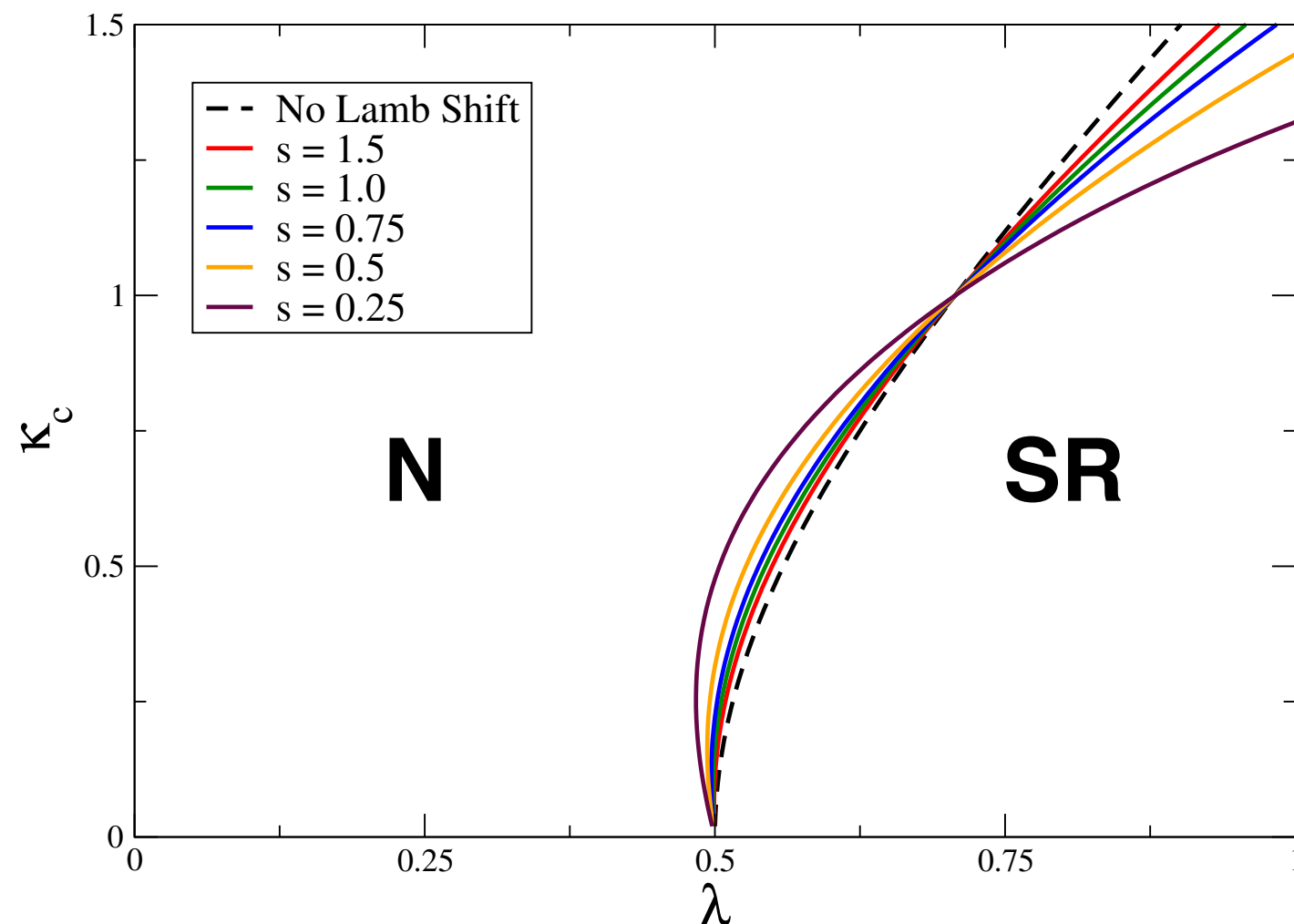
Carmichael, Nagy, Oztop, Keeling, Dalla Torre, Kulkarni, Piazza, Strack, ..

📌 Lindblad bath for photons

$$\partial_t \rho = -i[\tilde{H}_D, \rho] + \mathcal{L}(\rho)$$

$$\mathcal{L}(\rho) = \kappa (2a\rho a^\dagger - \{a^\dagger a, \rho\})$$

📌 Dissipative “Quantum” Phase Transition



$$\lambda_c^M(\kappa) = \sqrt{(\omega_0^2 + \kappa^2) \omega_q / 4\omega_0}$$

📌 Dissipation induces
“Thermal-like” behavior
(Effective Temperature)

Similar results with Atom Decay or Dephasing.
Interesting counterexample: Kirton&Keeling
PRL(2017)

❑ Is this generic for Dissipative Quantum Phase Transitions in Open Systems?

Non-Markovian Open Dicke Model

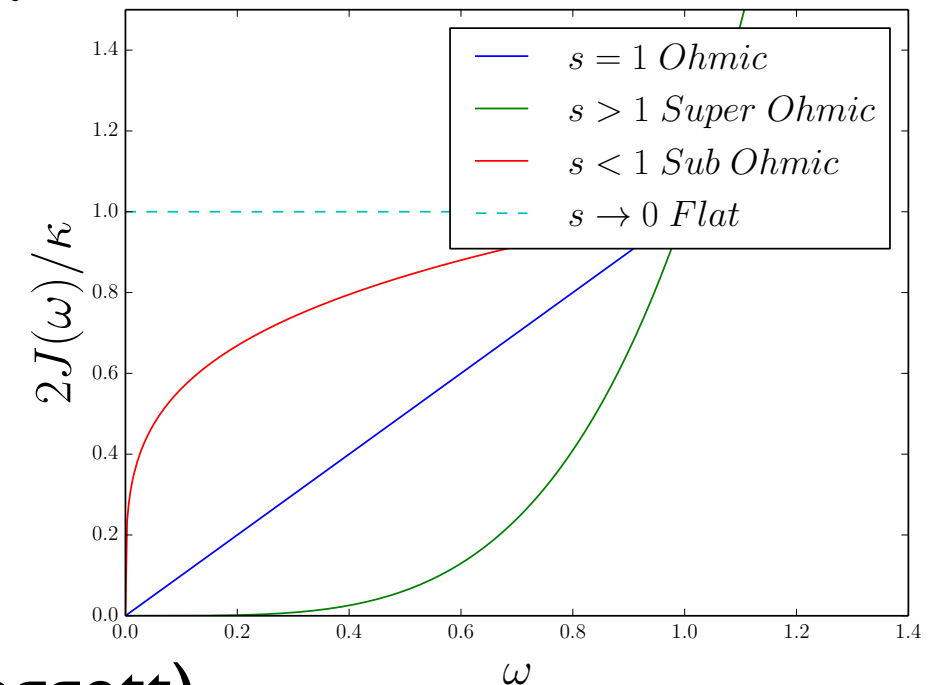
Quantum Bath for photons

See also: Nagy&Domokos (2015-2016)

$$H = H_D + \sum_k \omega_k c_k^\dagger c_k + (a + a^\dagger) \sum_k g_k (c_k + c_k^\dagger)$$

Bath Spectral Function

$$J(\omega) = \pi \sum_k g_k^2 \delta(\omega - \omega_k) \equiv 0.5\kappa (\omega/\omega_c)^s$$



Example: Single-Spin in a bosonic bath (Caldeira-Leggett)

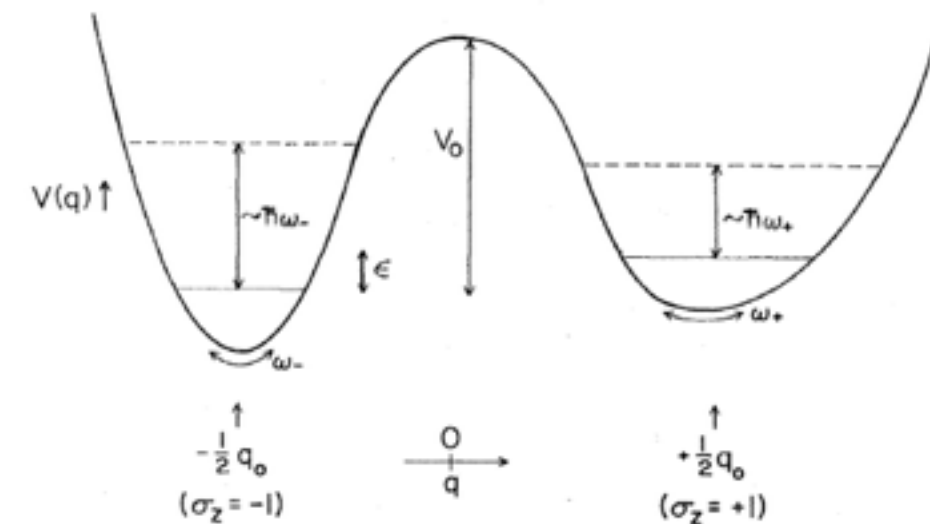
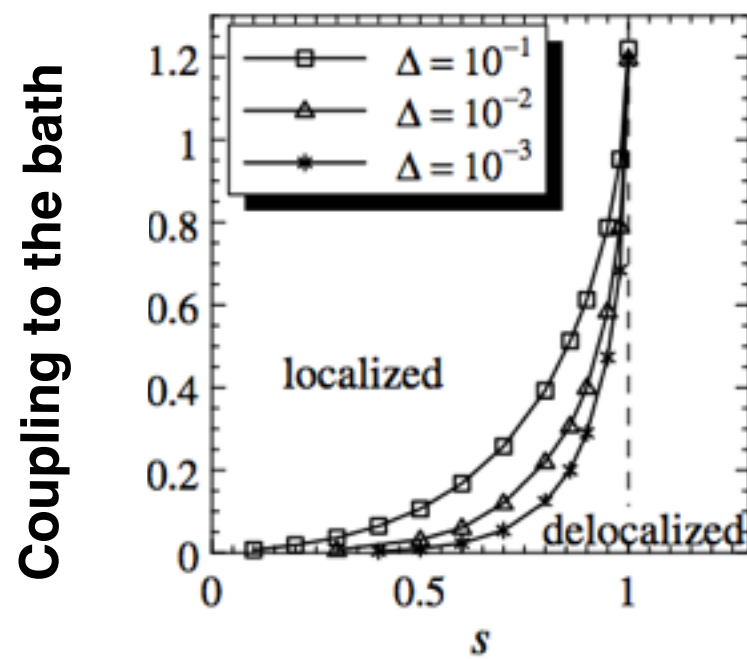


FIG. 1. A double-well system in the “two-state” limit.

Exact (“Mean Field”) Solution For $N \rightarrow \infty$

📌 For $N \rightarrow \infty$ “exact” solution via Holstein-Primakoff

$$S^+ = b^\dagger \sqrt{N - b^\dagger b} \quad S^z = b^\dagger b - N/2$$

$$H_D = \omega_q b^\dagger b + \omega_0 a^\dagger a + \lambda (a + a^\dagger) (b + b^\dagger)$$

$$H = H_D + \sum_k \omega_k c_k^\dagger c_k + (a + a^\dagger) \sum_k g_k (c_k + c_k^\dagger)$$

📌 Signature of dissipative QPT: **Susceptibility to coherent drive**

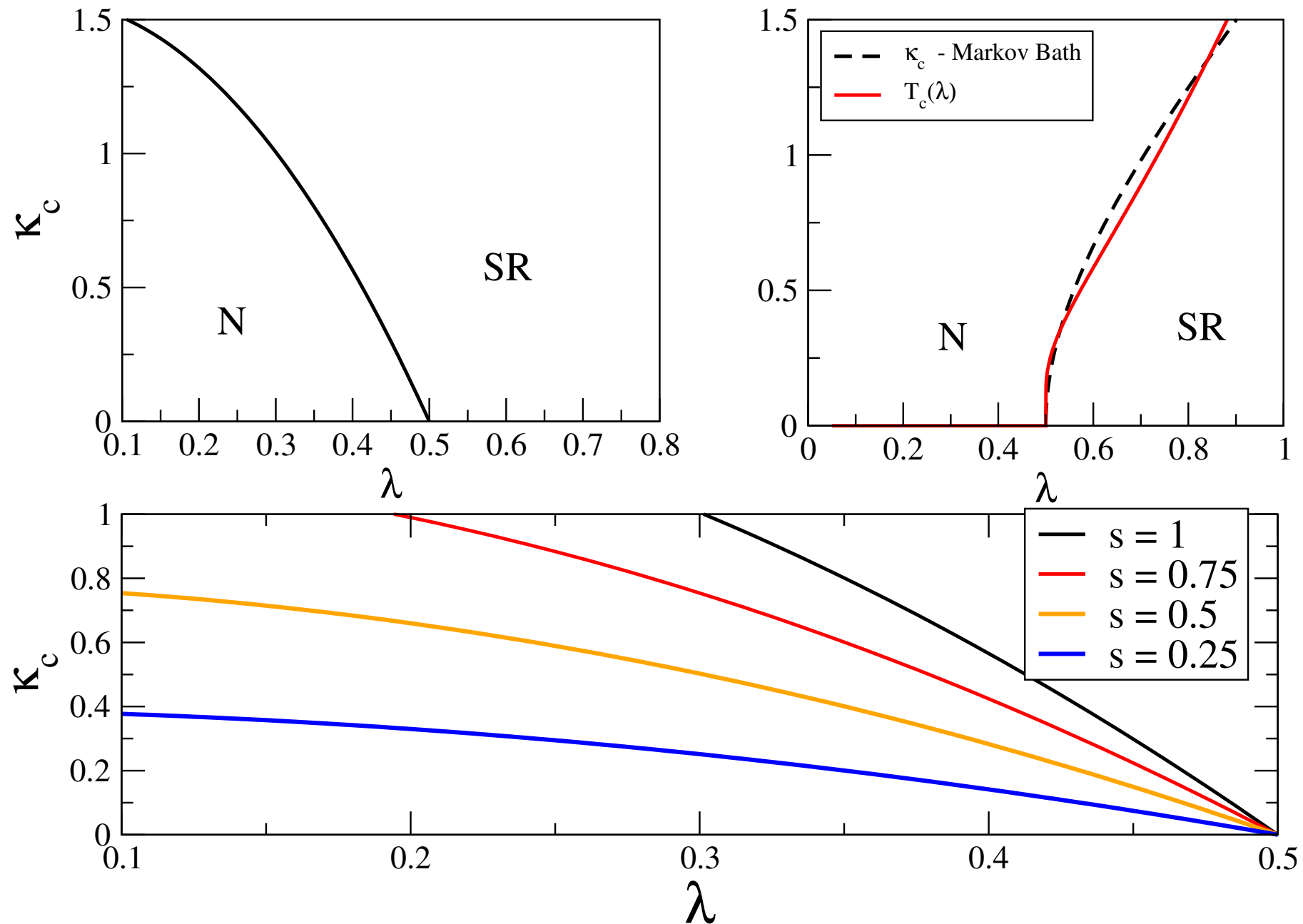
$$\chi_a^R(\omega) = -i \int_0^\infty dt e^{-i\omega t} \langle [x(t), x(0)] \rangle \quad \chi^R(\omega = 0) \sim \frac{1}{(\lambda_c^\infty(\kappa) - \lambda)^\delta}$$

$$\chi_a^R(\omega) = \frac{2\omega_0}{\omega^2 - \omega_0^2 - 2\omega_0 (\Delta^R(\omega) + \Sigma_\lambda^R(\omega))}$$

“Bath” Self-Energy

“Light-Matter” Self-Energy

Mean Field Dissipative Phase Diagram



Superradiance induced as the coupling to the bath is increased!

Mapping to Effective “Large S” Spin-Boson

- Effective Model for the N-Spins? Integrate Out Photons!

$$\mathcal{S}_{eff} = \frac{1}{N} \sum_{ij} \int d\tau d\tau' S_i^x(\tau) J_{eff}(\tau - \tau') S_j^x(\tau') + S_{loc}$$



$$H_{eff} = \omega_q \sum_i S_i^z + \sum_k g_k \left(b_k^\dagger + b_k \right) \sum_i S_i^x + \sum_k \omega_k b_k^\dagger b_k$$

- Large-S spin boson model, with effective dissipation

$$J_{eff}(\omega) = 2\pi\alpha_{eff}\omega_c(\omega/\omega_c)^s$$

$$\alpha_{eff} = \frac{\lambda^2 \kappa}{\pi\omega_c \tilde{\omega}_0(\kappa)}$$

- Z2 phase transition (“localization”) as the effective coupling to the bath is increased!

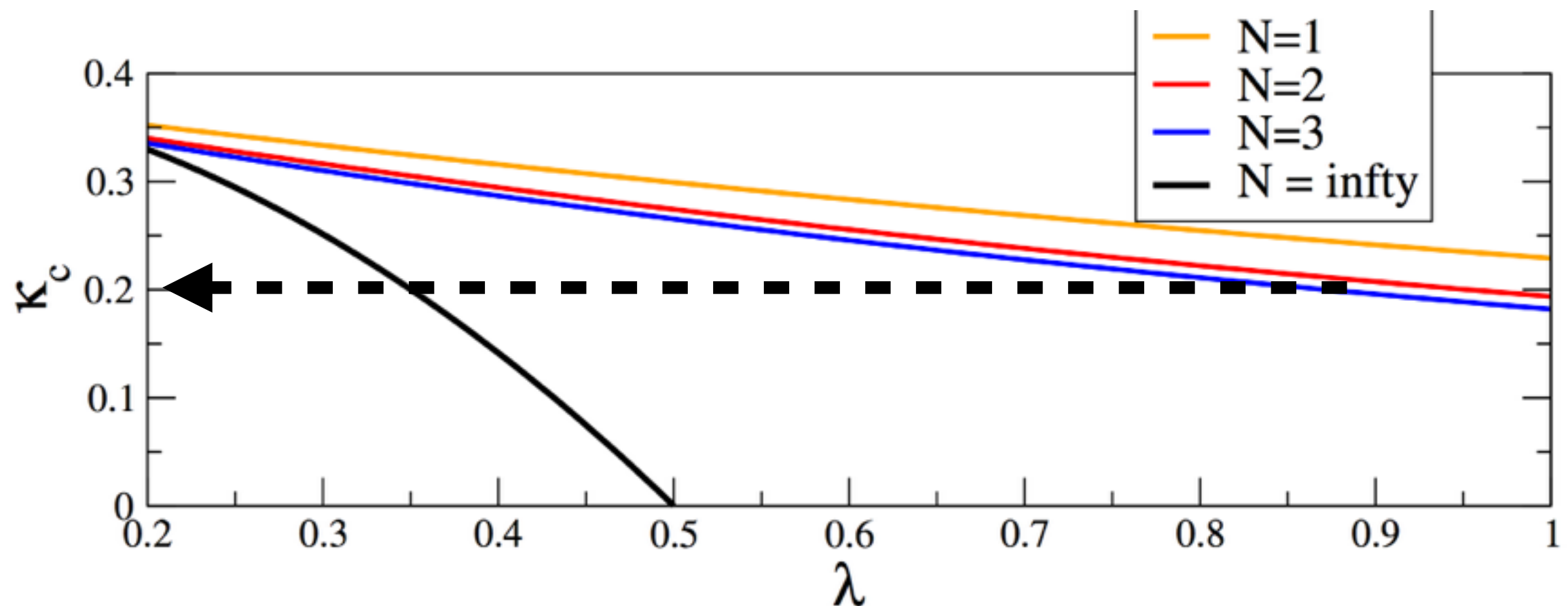
Dissipative Quantum Phase Transition at finite N

📌 Can we solve the large-S spin boson problem? Variational Ansatz!

$$|\Psi\rangle = \sum_{M=-S}^S C_M |S, M\rangle \otimes |\phi_M\rangle$$

Cfr A.Chin et al PRL (2011)
for S=1/2 spin-boson

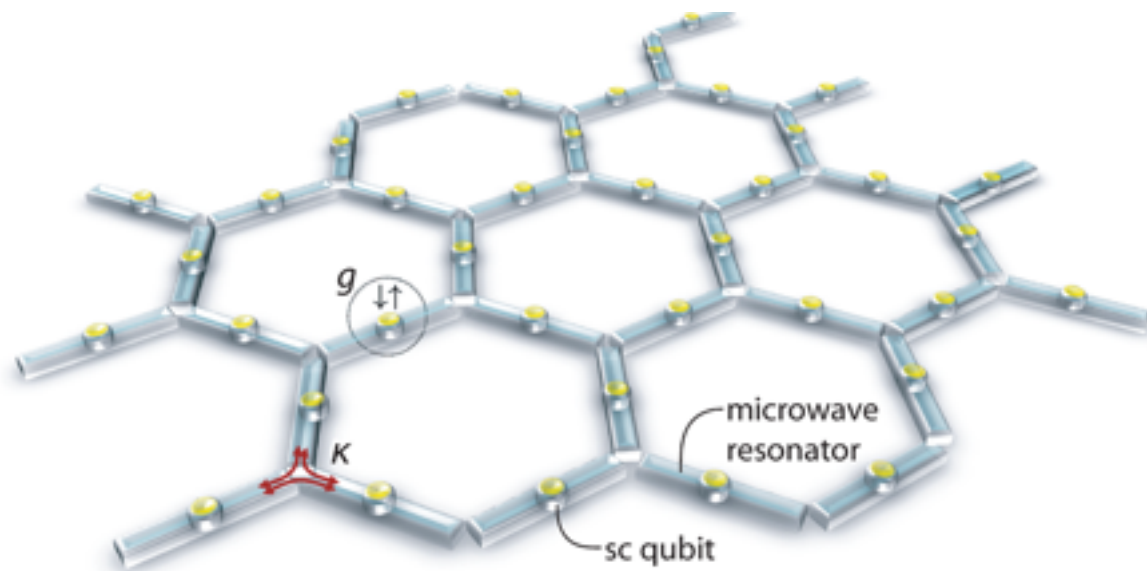
$$|\phi_M\rangle = e^{-\sum_k f_{kM} (b_k^\dagger - b_k)}$$



📌 Genuine non-Markovian Effect! No phase transition at finite N with Lindblad bath!

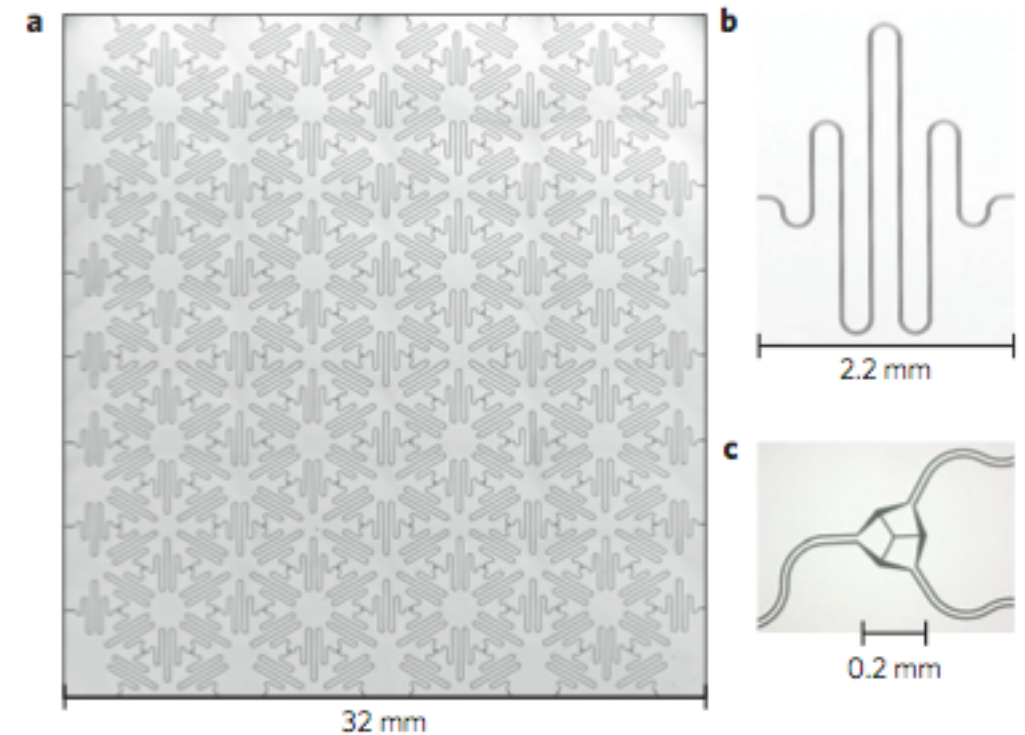
Dissipative Phase Transition in Driven Lattice Bosons

Coupled Arrays of CQED Units



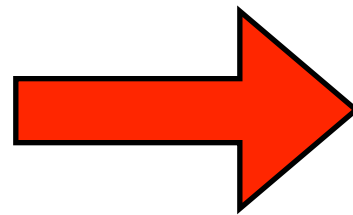
Jaynes-Cumming Lattice Model

$$H_{JCL} = \sum_{\langle ij \rangle} J_{ij} a_i^\dagger a_j + \sum_i H_{JC}^i$$

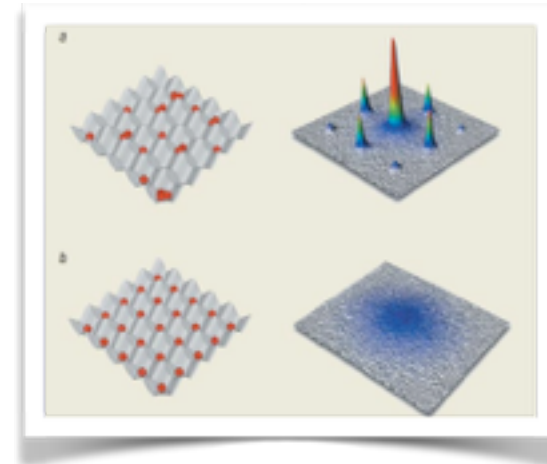
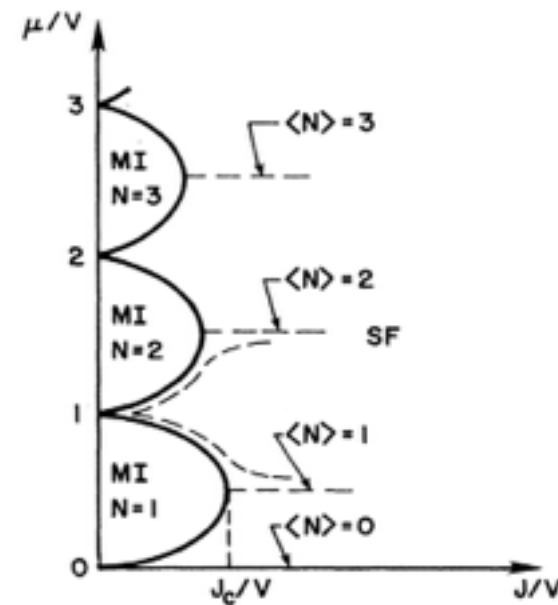


Effective Photon-Photon Interactions

Hopping between adjacent resonators/cavities



Delocalized bosonic particles with an effective onsite interaction...

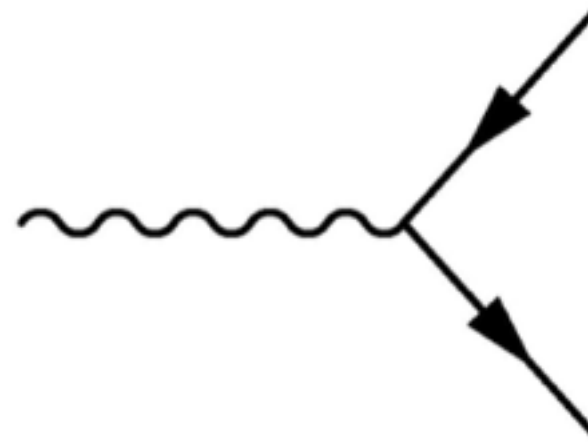


Same physics of bosonic massive particles?

The importance of being a Photon

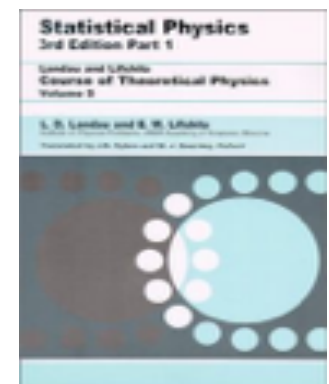
- 📌 Unlike massive particles Photons are not conserved in Nature

Think of QED!



- 📌 Photons have zero chemical potential!

$$\mu_{ph} = \frac{\partial E}{\partial N} = 0$$



- 📌 The density of a Photon Gas is set by drive-dissipation or interaction with matter!

Challenge: How to Engineer Finite-Density
Phases of Correlated Photons?

Two possible directions

☐ Increase Light-Matter Coupling: effective driving

 Counter-Rotating Terms $H_{int} = H_{JC} + g (a^\dagger \sigma^+ + \sigma^- a)$

$g \sim \omega_r, \omega_q$ 'Ultra-Strong' Coupling Regime

 M. Schiro, C. Joshi, M. Bordyuh, R. Fazio, J. Keeling and H. Tureci, PRL **116** 143603 (2016)

 M. Schiro, M. Bordyuh, B. Oztop and H. Tureci, J. Phys. B: Opt. At. Mol. Phys. **46** 224021 (2013)

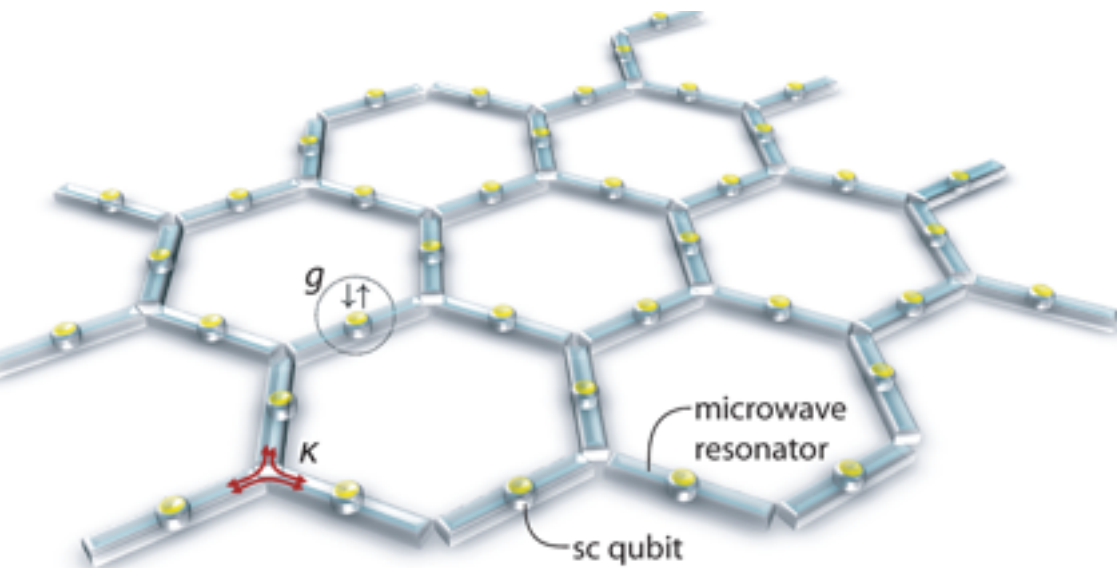
 M. Schiro, M. Bordyuh, B. Oztop, H. Tureci, PRL **109** 053601 (2012)

☒ Balancing Drive and Dissipation

 Dissipative Phase Transition in Driven Bose-Hubbard Lattices

 O. Scarlatella et al (in preparation, 2017)

Driven-Dissipative Bose-Hubbard Lattice



Master Equation

$$\partial_t \rho = -i[H, \rho] + \mathcal{D}[\rho]$$

Local Hamiltonian:

$$H_{loc} = \omega_0 a_i^\dagger a_i + \frac{U}{2} n_i^2$$

Single-particle Losses

$$\mathcal{D}[\rho] = \kappa \sum_i a_i \rho a_i^\dagger - \frac{1}{2} \left\{ a_i^\dagger a_i, \rho \right\}$$

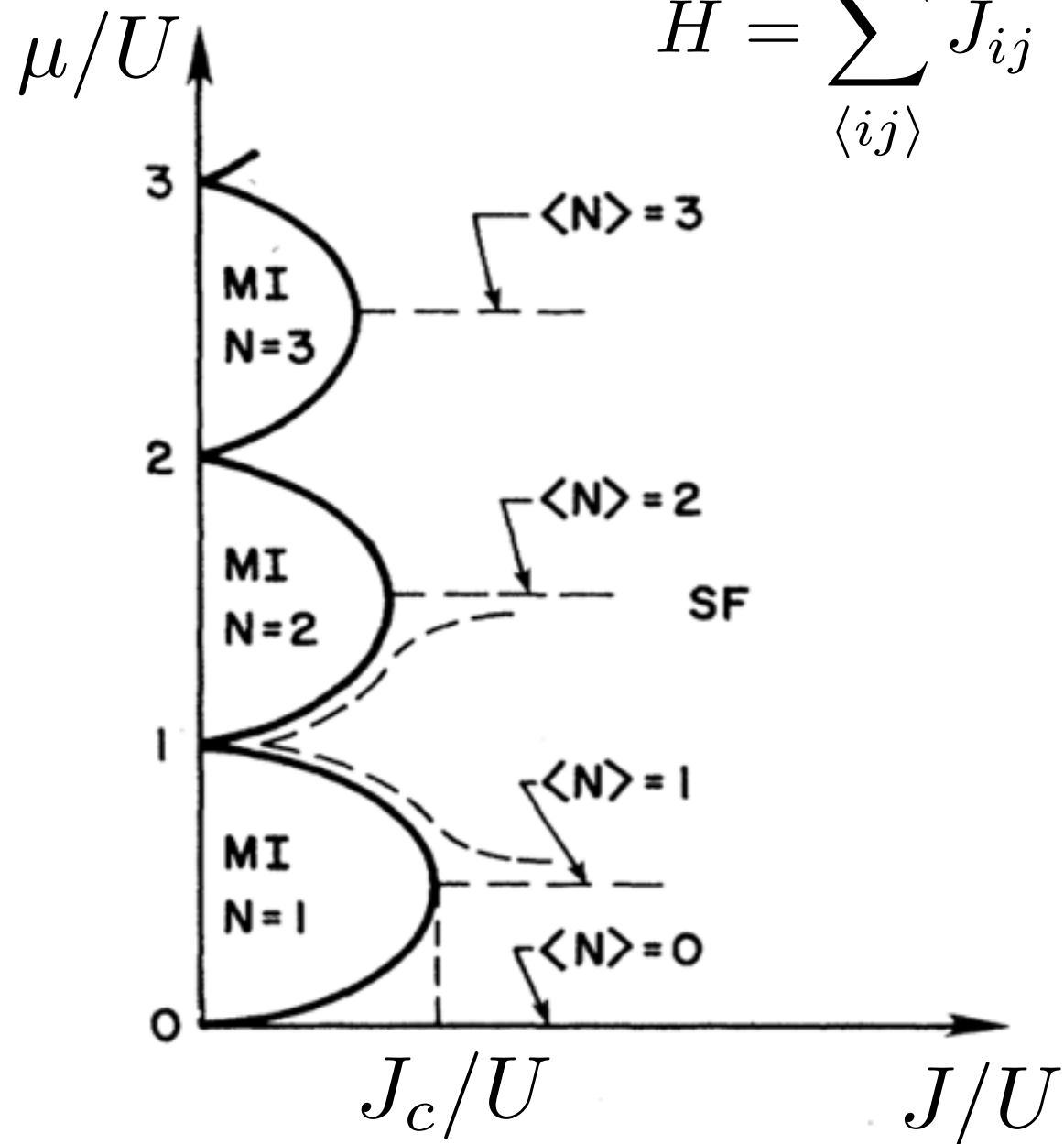
Questions:

Non-Equilibrium Stationary States? Mott Insulators of Photons? Dynamics?
Dissipative/Dynamical Transitions?

Equilibrium Phase Transitions of Lattice Bosons

📌 M. Fisher et al, PRB(1989)

$$H = \sum_{\langle ij \rangle} J_{ij} (a_i^\dagger a_j + hc) + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$



📌 $J/U \ll 1$

Mott Insulating Phase:
Gapped, Incompressible, Incoherent

📌 $J/U \gg 1$

Superfluid Phase: Gapless,
Compressible, Coherent (breaks $U(1)$)

📌 Equilibrium Phase Transition (critical theory) well understood

$$\mathcal{S}_{eff} = \int dk d\tau \psi^*(k, \tau) (k^2 + \partial_\tau^2 + g \partial_\tau + r) \psi(k, \tau) + u \int dx d\tau |\psi(x, \tau)|^4$$

📌 What happens in the driven-dissipative case?

📌 Crucial difference - $U(1)$ symmetry but no conservation law - Mott Insulator?

How to drive Bosons out of equilibrium?



Coherent Drive?

$$H_{drive}^{coh} = \sum_i \left(a_i^\dagger e^{i\Omega t} + hc \right)$$



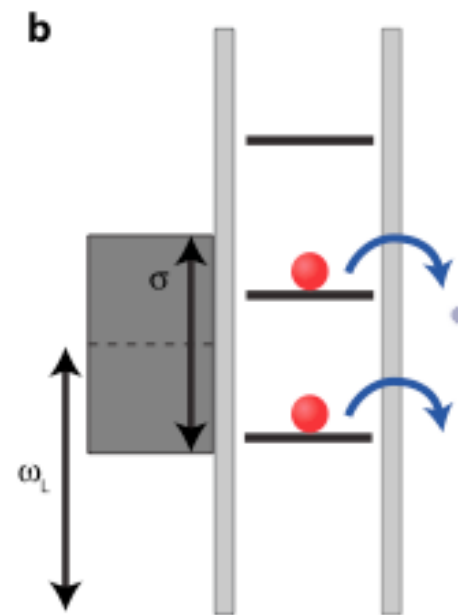
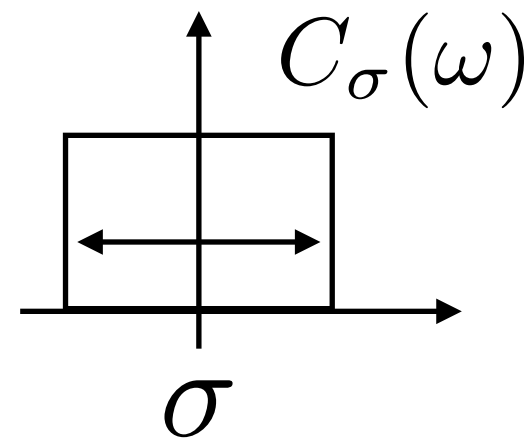
U(1) explicitly broken in the unitary part :(



Incoherent (Stochastic) Drive

$$H_{drive}^{incoh} = \sum_i \left(\eta(t) a_i^\dagger + \eta^*(t) a_i \right)$$

$$\langle \eta^*(t) \eta(t') \rangle = f C_\sigma(t - t')$$



Master Equation Treatment:

$$\mathcal{D}[\rho] = \kappa \mathcal{D}_\infty^{out} + f (\mathcal{D}_\sigma^{in} + \mathcal{D}_\sigma^{out})$$



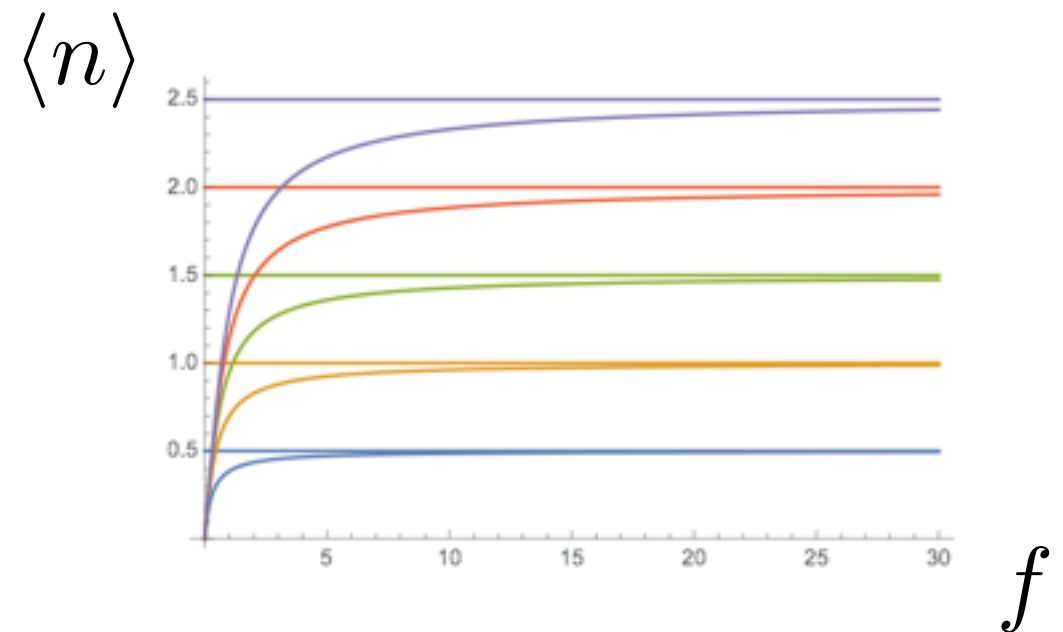
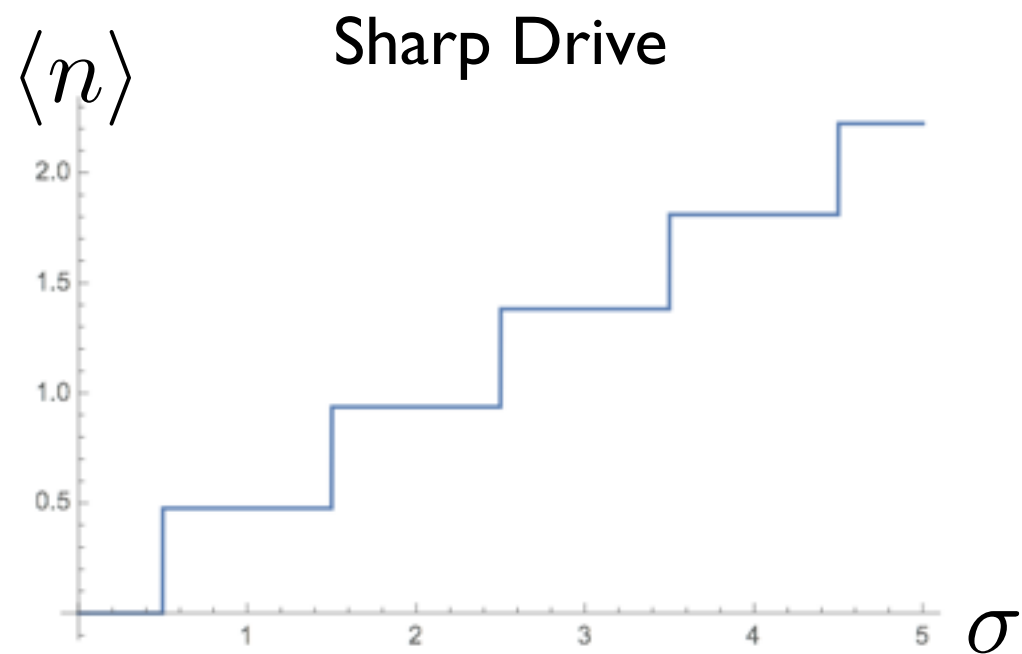
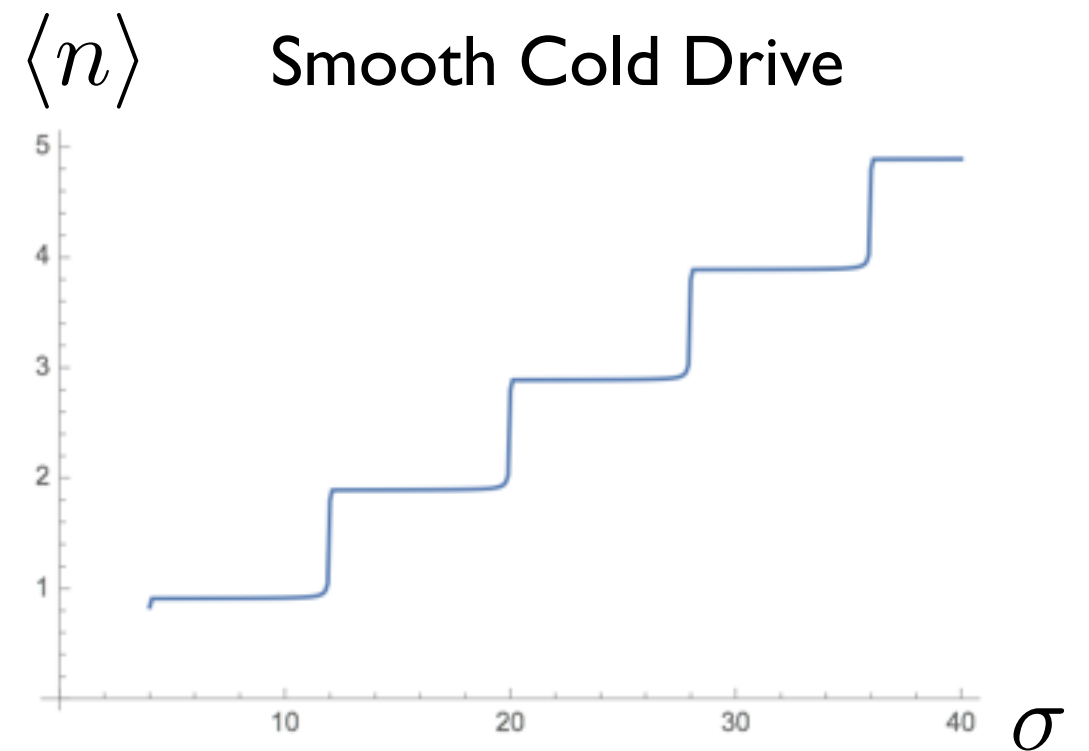
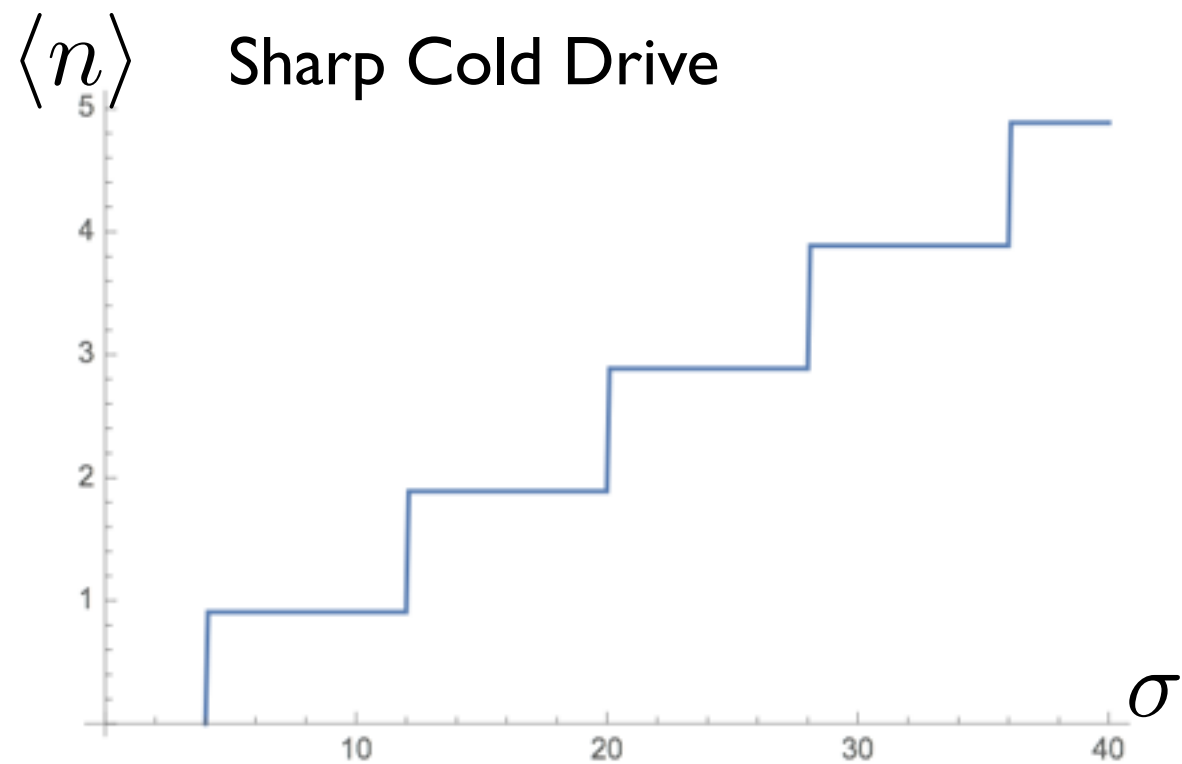
Incoherent “Cold” Drive (from qbit pop inversion)

J. Lebreuilly et al, arXiv:1704.01106

$$\mathcal{D}[\rho] = \kappa \mathcal{D}_\infty^{out} + f (\mathcal{D}_\sigma^{in} + \cancel{\mathcal{D}_\sigma^{out}})$$

Single site BH problem ($J=0$): Photon Blockade

$$U = 8 \omega_0 = 4 f = 0.5 \kappa = 0.051 \quad N_{max} = 15$$



$$\rho_{st} \sim \text{diag}(p_0, p_1, p_2, \dots)$$

 Incoherent mixture with tunable weights

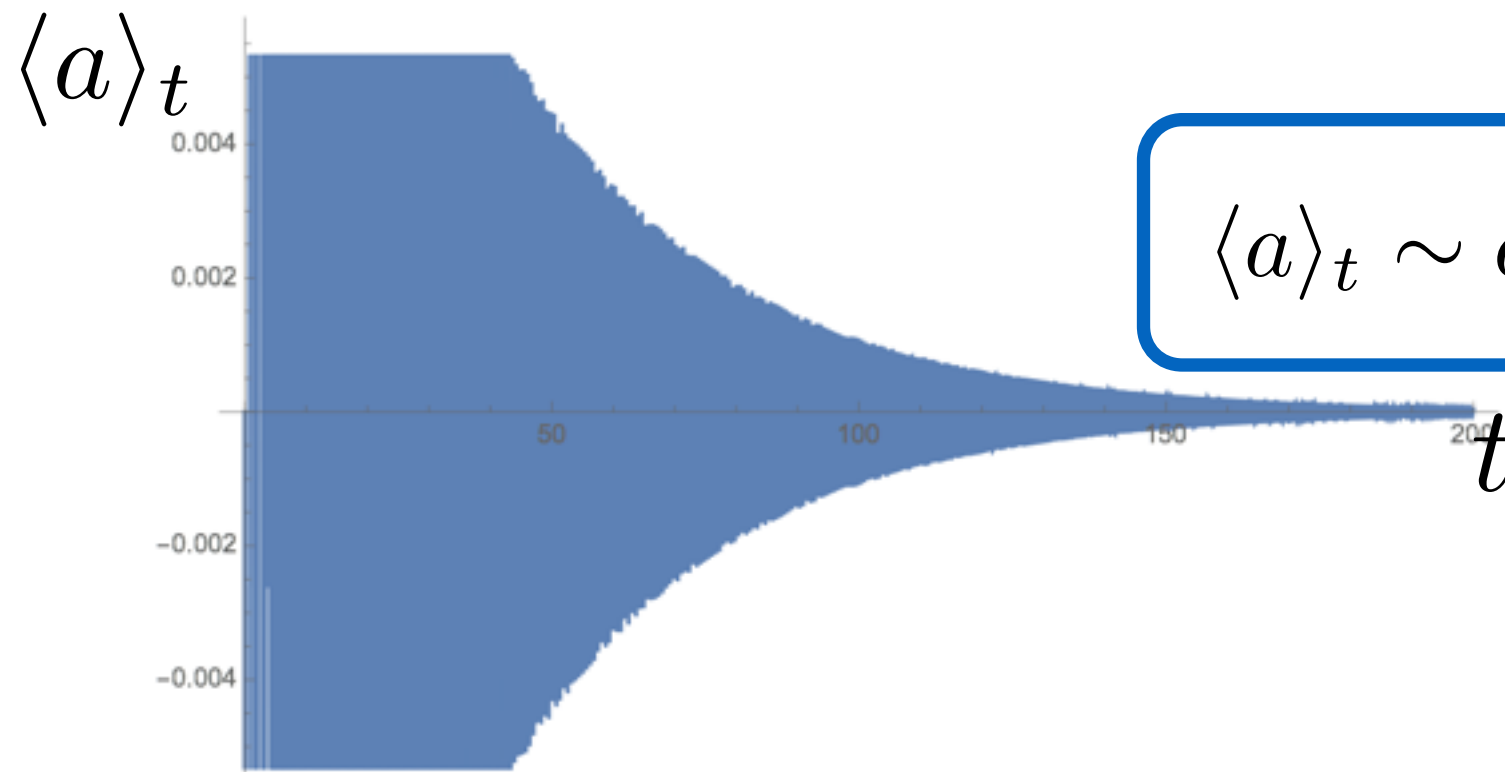
Dynamical Transition of driven-dissipative Mott Insulator

📌 Time dependent Mean-Field Decoupling

$$J < J_c$$

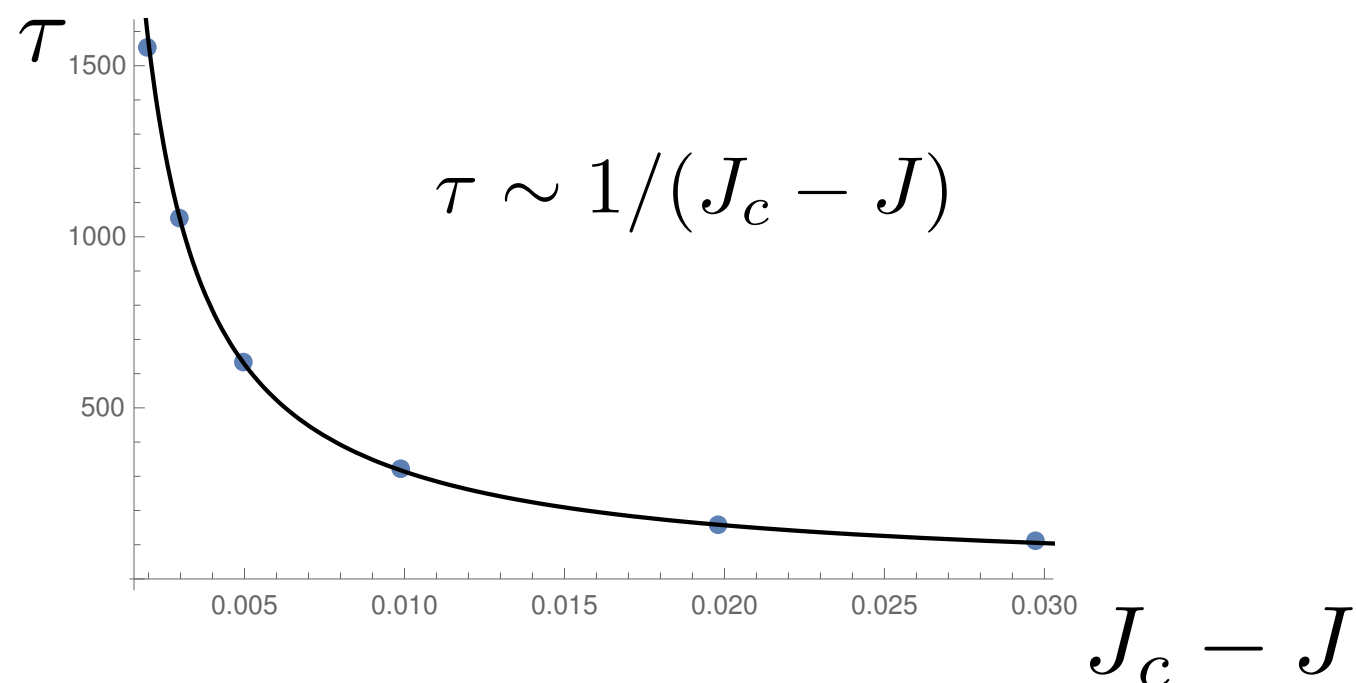
$$\partial_t \rho = -i[H_{eff}, \rho] + \mathcal{D}[\rho]$$

$$H_{eff}(t) = H_{loc} + (\Psi^*(t)a + hc)$$

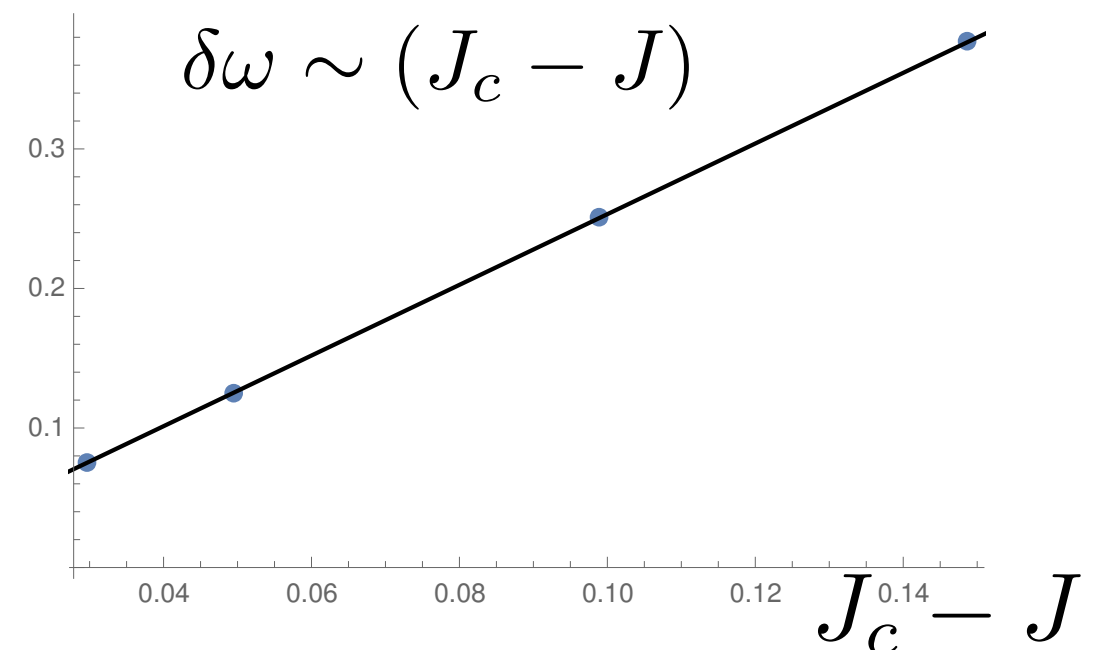


$$\langle a \rangle_t \sim e^{i\Omega_* t} e^{i\delta\omega t} e^{-t/\tau}$$

📌 Diverging Relaxation Time:

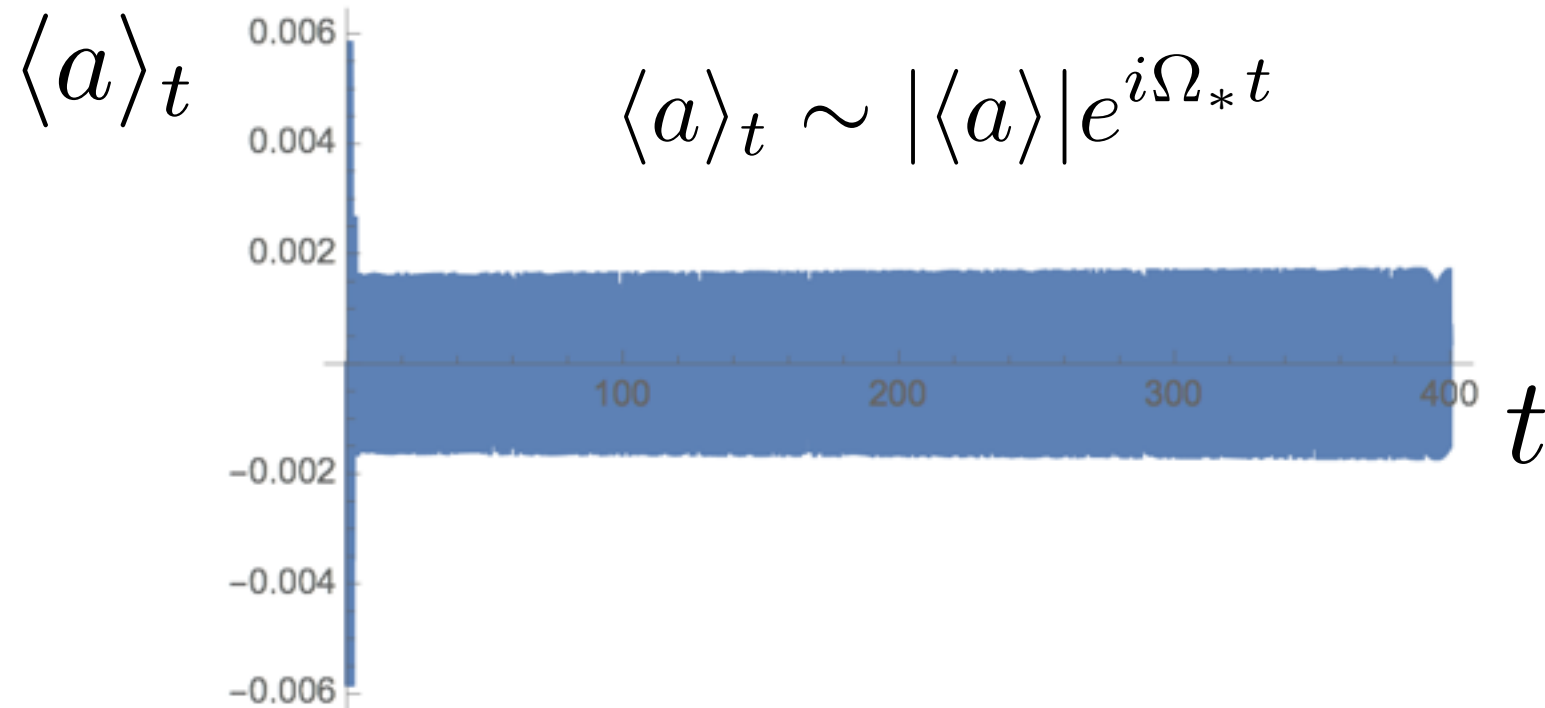


📌 Frequency Softening:

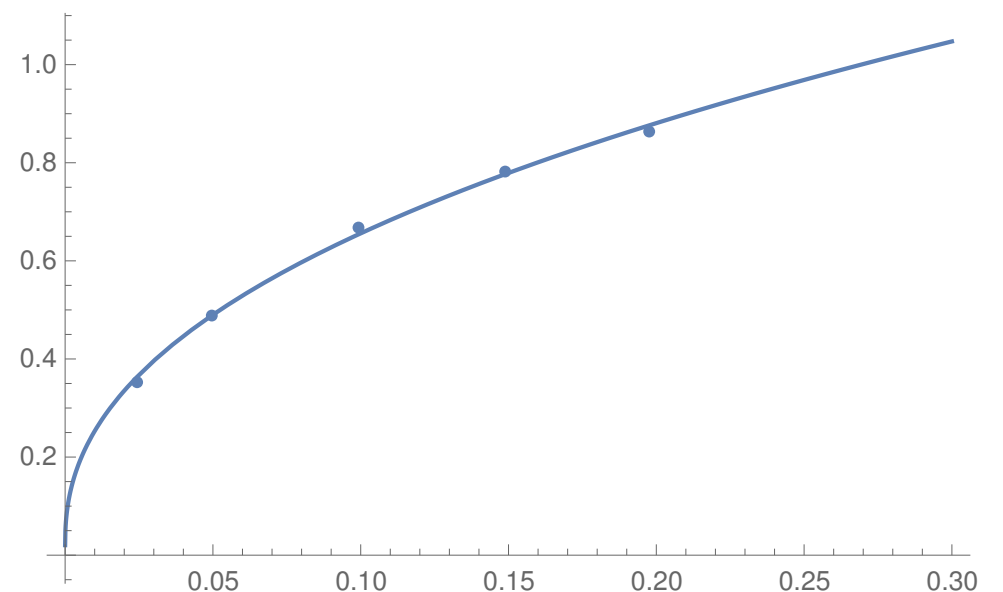
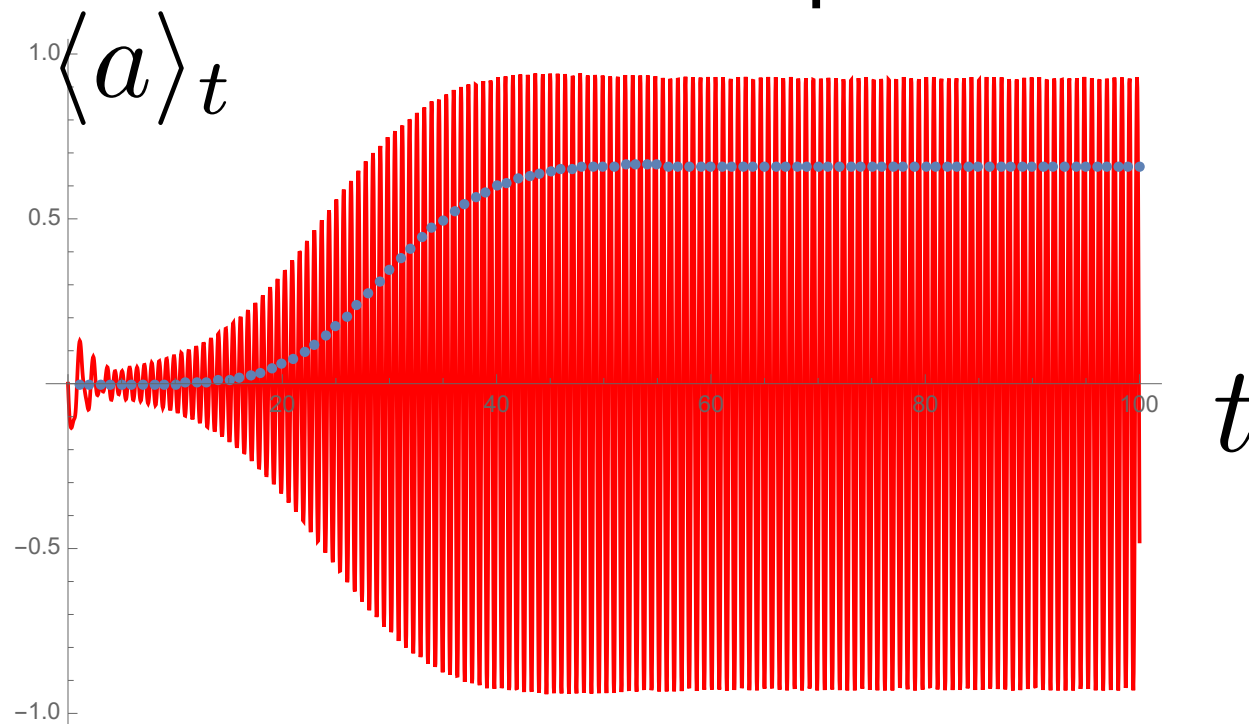


Dynamics in the U(1) Broken Symmetry Phase

At the dynamical transition ($J=J_c$) limit cycle with fast oscillations

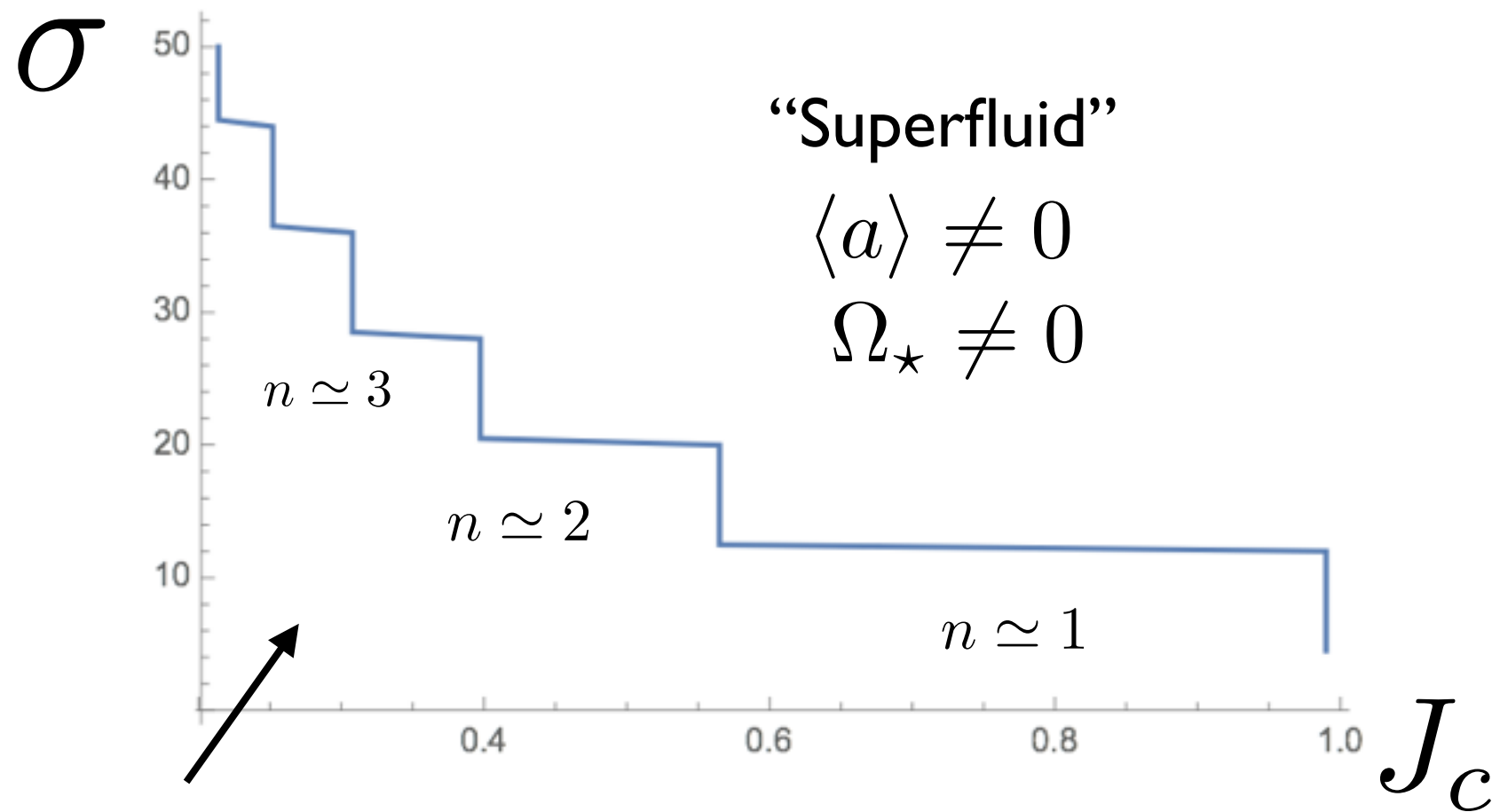


Above Critical Point: Amplification & Saturation



Limit Cycle phase is not fixed a priori, still it can be gauged away by unitary transformation into a rotating frame $U = \exp \left(-i\Omega_* t \sum_i a_i^\dagger a_i \right)$

Driven “SF-MI” Phase Boundary (Preliminary)



“Mott” $\langle a \rangle = 0$

- ❑ Open Questions: Universality Class of the Phase Transition?
Nature of the phases (Response Functions)?

Keldysh Field Theory of Interacting Lattice Bosons

📌 Effective Keldysh Action

$$Z = \int \prod_i D\Psi_i D\Psi_i^* e^{i\mathcal{S}_{eff}[\Psi_i, \Psi_i^*]}$$

$$\mathcal{S}_{eff} = \int_C dt \sum_{ij} \Psi_i^*(t) J_{ij}^{-1} \Psi_j(t) + \sum_i \Gamma[\Psi_i^*, \Psi_i]$$

📌 Generating Functional of Local Dynamical Correlations

$$\Gamma[\Psi^*, \Psi] = \log \langle T_C e^{i \int_C dt (\Psi^*(t) a + a^\dagger \Psi(t))} \rangle_{loc}$$

📌 Expand the action in power of the field (Universality/RG?)

$$\mathcal{S}_{eff} = \sum_{ij} \int_C dt dt' \Psi_i^*(t) \chi_2(t-t') \Psi_j(t') + \sum_i \int_C \chi_4(t_1, \dots, t_4) \Psi_i^*(t_1) \Psi_i^*(t_2) \Psi_i(t_3) \Psi_i(t_4)$$

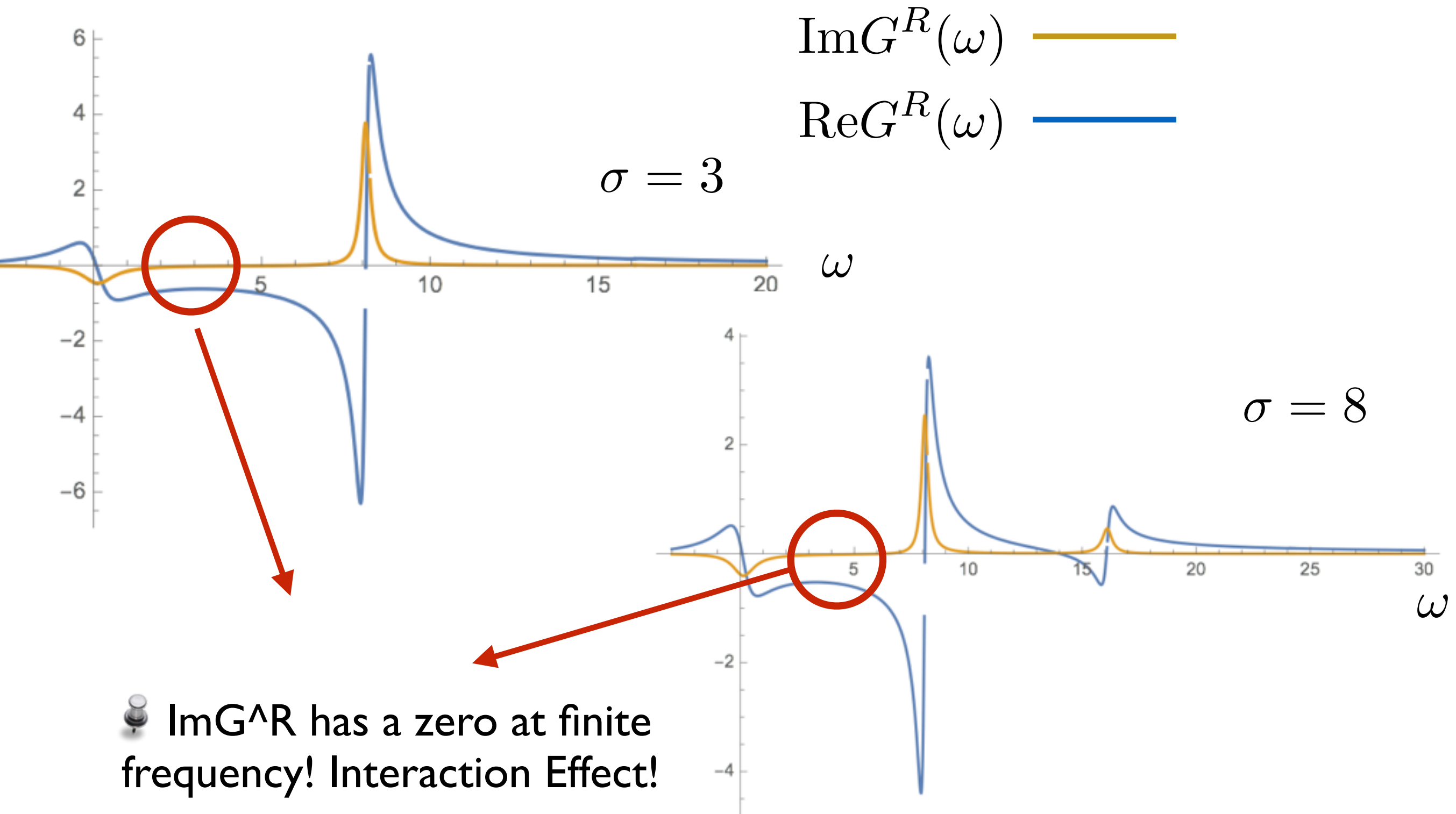

$$\langle T_C a(t) a^\dagger(t') \rangle_{loc}$$


$$\langle T_C a(t_1) a^\dagger(t_2) a(t_3) a^\dagger(t_4) \rangle_{loc}$$

📌 Key Ingredient: Green's functions of a single-site interacting driven dissipative problem

More on Single Site Bose-Hubbard: Green's Function

$$\omega_0 = 0.1 \quad f = 0.1 \quad \kappa = 0.05 \quad U = 8 \quad \langle n \rangle \sim 1 \quad N_{max} = 15$$



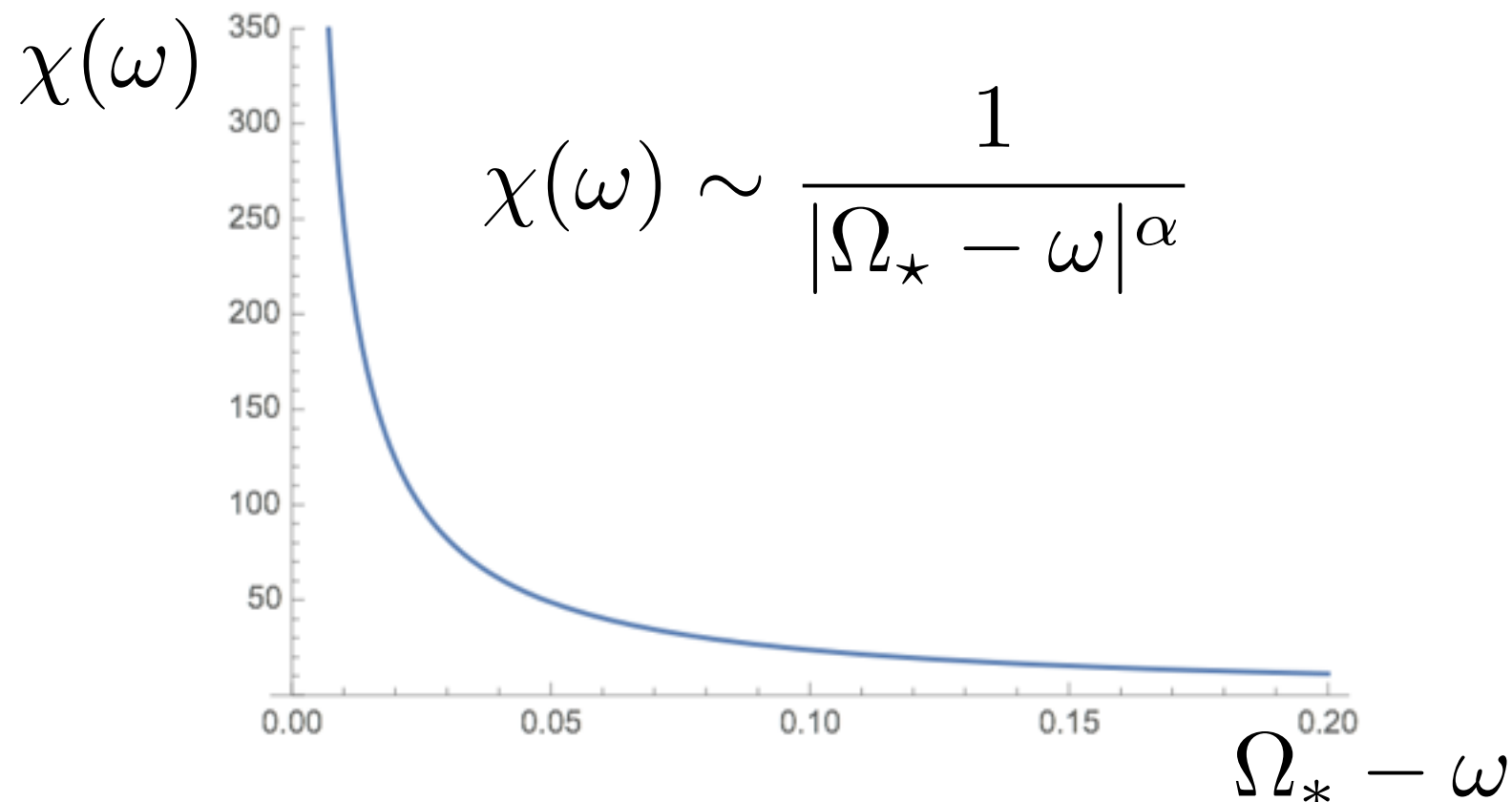
Normal Phase Instability of the Lattice

- 📌 RPA Susceptibility of order parameter

$$\chi(\omega) = \frac{1}{1/J + G_{loc}^R(\omega)}$$

- 📌 Equilibrium: Zero Frequency pole at $J=J_c$

- 📌 Non-equilibrium: Instability pushed at finite frequency!



Spectrum and Noise of Driven-Dissipative Mott Insulator

📌 Expand Retarded Green's function for:

$$q \rightarrow 0 \quad \omega - \Omega_* \rightarrow 0$$

$$(\mathcal{G}^R)^{-1}_{q,\omega} \sim r + (\xi_s q)^2 + \xi_d (\omega - \Omega_*)$$

📌 Static and Dynamic correlation lengths

$$\xi_s \sim (J_c - J)^{-1/2} \quad \xi_d \sim (J_c - J) / \partial_\omega G^R(\Omega_*)$$

📌 Important: ξ_d is a complex number!

📌 Keldysh action of Order Parameter Fluctuations $\Psi(t) \sim e^{i\Omega_* t} \delta\Psi(t)$

$$\mathcal{S} = \int dt dr \Psi_c^* (r + \xi_d \partial_t + \xi_s \nabla^2) \Psi_q + G^K(\Omega_*) \Psi_q^*(t) \Psi_q(t) + \mathcal{S}_{nl}$$

📌 Classical Equation of Motion (SaddlePoint) confirms the time-dependent mean field result

$$\Psi(t) \sim e^{i\Omega_* t} e^{i\delta\omega t} e^{-t/\tau}$$

Conclusions&Outlooks

☒ Dissipation-induced Dicke Superradiance by non-Markovian Bath

- ☐ Finite-N non-perturbative effects at small dissipation? Add non-equilibrium drive to non-Markov Bath? Incoherently driven Superradiance? Role of additional Markovian Losses?

☒ Dissipative Phase Transitions in Driven Bose Hubbard Lattice

- ☒ Driven-Dissipative “Mott” Insulator dynamically unstable at finite frequency
- ☒ Nonequilibrium Superfluid as limit cycle
- ☐ Universality of SF-MI Dissipative Quantum Transition

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