Generating RVB states via Dicke subradiance

arXiv:1609.04853

R. Ganesh

L. Theerthagiri, G. Baskaran

The Institute of Mathematical Sciences, Chennai

Dicke's thought experiment

Prerequisite: two level atoms or 'spins' which couple to photons. An unexcited spin ($|\downarrow\rangle$) can absorb a photon to become excited ($|\uparrow\rangle$) and vice versa.

Consider an $\uparrow$ spin which will emit a photon with a certain rate. If we bring a second unexcited spin ($\downarrow$) close to it, will this change the emission properties?

Naive expectation: It will not as the second spin does not emit.

R. H. Dicke, Physical Review 1954

Quantum effects: The wavefunction for both spins is

$$
\begin{aligned}
|\uparrow\downarrow\rangle &= \frac{1}{\sqrt{2}}\left[\frac{\{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\}}{\sqrt{2}} + \frac{\{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\}}{\sqrt{2}}\right] \\
&= \frac{1}{\sqrt{2}}\Big[\underbrace{|s\rangle}_{\text{dark}} + \underbrace{|t_0\rangle}_{\text{bright}}\Big]
\end{aligned}
$$

- A photon will now only be emitted with probability half
- Dicke showed that the average rate of emission does not change as the triplet radiates at twice the rate
- Building on this quantum picture, he identified states of N spins with the highest rate of emission – 'superradiance'
- Note that observation of emitted photon will collapse the spin wavefunction to $|t_{-1}\rangle$

R. H. Dicke, Physical Review 1954

The Dicke model

Minimal model for spins interacting with photons:

- Spins are placed within a cavity – quantised photon modes
- Only one mode is near resonance – ignore others
- $\underbrace{\lambda_{deB}}_{\text{size of spin}} \ll \underbrace{a}_{\text{spin-spin spacing}} \ll \underbrace{\lambda_{ph}}_{\text{photon wavelength}}$
  - Spins do not interact directly with one another
  - They couple to the photon mode as if they were all at the same point

Dicke Hamiltonian:

$$\begin{aligned} H &= \omega_0 a^\dagger a + B \sum_{i=1}^{N} \hat{S}_i^z + g \sum_{i=1}^{N} \left[\hat{S}_i^- a^\dagger + \hat{S}_i^+ a \right] \\ &= \omega_0 a^\dagger a + B \hat{S}_{tot}^z + g \left[\hat{S}_{tot}^- a^\dagger + \hat{S}_{tot}^+ a \right]. \end{aligned}$$

The Dicke model – states

$$H = \omega_0 a^\dagger a + B\hat{S}^z_{tot} + g\left[\hat{S}^-_{tot} a^\dagger + \hat{S}^+_{tot} a\right].$$

$$|S_{tot}, m_{tot}, n_{ph}\rangle \xrightarrow{H} |S_{tot}, m_{tot}+1, n_{ph}-1\rangle$$
$$|S_{tot}, m_{tot}, n_{ph}\rangle \xrightarrow{H} |S_{tot}, m_{tot}-1, n_{ph}+1\rangle$$

Conserved quantities: S_{tot}, $m_{tot} + n_{ph}$

'Superradiance': In a system of N spin-1/2 spins, the rate of photon emission highest from $|S_{tot} = N/2, m_{tot} = 0\rangle$.

- seen in BECs, magnons, cavity QED experiments, etc.

'Subradiance'[1]: No emission from any state with $S_{tot} = 0$.
- long lived states with potential for use as quantum memories

[1] Scully, PRL 2015 and others

Realising Dicke's thought experiment

Realised only in 2014[1]: $N = 2$ spins in a cavity. Measured quantity: density matrix of outgoing photon

(Inverse) time scales in a Dicke model experiment

- $\tilde{g}$: rate of spin-photon coupling
- κ: rate of loss of photon from cavity
- spin-spin interaction, non-radiative loss, etc.

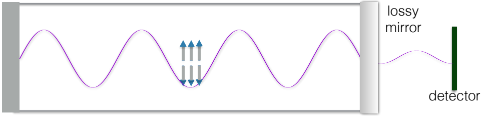
To realise Dicke's thought experiment, we need:
$\kappa \gg \tilde{g} \gg$ all other rates – 'lossy cavity limit'

If the spins emit photon(s), they are not reabsorbed. Rather, they 'immediately' leave the cavity. They can then be measured outside as they exit the cavity.

---

[1] J. A. Mlynek et. al., Nature Communications 2014

Proposal to generate RVB states



Dicke's thought experiment as a Stern-Gerlach measurement:

- Initialise N spins in a direct product state
- Measure number of photons emitted – timescale: $\tilde{g}^{-1}$
- Claim: null observation collapses spin wavefunction onto an RVB state
- Further claim: *almost* every outcome of this measurement leads to an RVB state

Two spin example:

- Initial state: $|\uparrow\downarrow\rangle$
- Possible outcome for photon measurement: 0 or 1
- 0-measurement collapses wavefunction onto singlet

Detecting null emission

Non-detection of photons is also a measurement in the quantum mechanical sense!

No detection $\leftrightarrow$ no photon was emitted by the spins

Possible issues:

- Non-radiative decay of spins
- Photon leaves cavity through other mirror
- Initialisation in a direct product state may have errors

Current experiments already have the above issues under control[1]

- Technology for number detection at the single photon level
- High precision to avoid false readouts of null measurement

Possible test for RVB character – if a photon is pumped in, the RVB state cannot absorb it

[1] J. A. Mlynek et. al., Nature Communications 2014

Radiation 'trapping'

$$H = \omega_0 a^\dagger a + B\hat{S}^z_{tot} + g\left[\hat{S}^-_{tot} a^\dagger + \hat{S}^+_{tot} a\right].$$

Note: a state of N (even) spin-1/2 spins can have $S_{tot} = 0, 1, \ldots, N/2$.

Consider an initial state with one $\uparrow$ spin and all others $\downarrow$.

$$|\psi_{init.}\rangle = |\uparrow\downarrow\cdots\downarrow\rangle$$

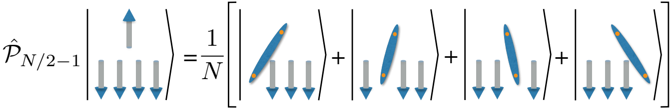
$$m_{tot} = -N/2 + 1 \quad \longrightarrow \quad S_{tot} = N/2 - 1, N/2$$

$$|\psi_{init.}\rangle = a_{N/2}|N/2, -N/2+1\rangle + a_{N/2-1}|N/2-1, -N/2+1\rangle$$

- $|N/2, -N/2+1\rangle$: 'bright' state - can emit one photon and decay to $|N/2, -N/2\rangle$
- $|N/2-1, -N/2+1\rangle$: 'dark' state - cannot reduce m_{tot} any further $\implies$ cannot emit

‘Radiation trapping’

- Probability of emission: $1/N$ – small for large N
- Bright component: $\frac{1}{\sqrt{N-1}}\left[|\uparrow\downarrow\cdots\downarrow\rangle + |\downarrow\uparrow\cdots\downarrow\rangle + \ldots |\downarrow\downarrow\cdots\uparrow\rangle\right]$
- Positive photon observation: spins collapse onto $|\downarrow\cdots\downarrow\rangle$
- Null observation collapses spins onto



– a ‘resonating valence bond’ state

- An excitation is ‘trapped’ – it is continually absorbed and reemitted coherently – does not escape outside the cavity
- Phenomenon of trapping known since the 1960’s – however, RVB character has not been pointed before
- ’Weak’ RVB – one valence bond shared among N spins

Tavis and Cummings, Physical Review 1968

Resonating Valence Bond state

- A linear combination of singlet arrangements – typically highly entangled wavefunction
- First proposed by Pauling for benzene
- Proposed ground states for 'frustrated' magnets: first suggested in the triangular lattice antiferromagnet[1]
- 'Spin liquid' wavefunction: first example of topological order[2]
- Precursor to high-T_c superconductivity – 'mean-field' momentum space description [3]
- No 'clean' examples (except in organic chemistry) – e. g., S. Nascimbène et. al., PRL 2012 – a cold atoms realisation with 4 spins
- Required: RVB realisations that are amenable to doping, manipulations, etc.

---

[1] Anderson, Mat. Res. Bull. 1973

[2] Kivelson, Rokhsar and Sethna, Physical Review B 1987

[3] Anderson, Science 1987, Baskaran Zou Anderson, Solid State Comm. 1987

Strong RVB

Consider an initial state with $N/2$ spins excited and $N/2$ unexcited

$$|\Psi_{initial}\rangle = |\underbrace{\uparrow \ldots \uparrow}_{N/2}\underbrace{\downarrow \ldots \downarrow}_{N/2}\rangle = \sum_{S=0}^{N/2} a_S |S_{tot} = S, m_{tot} = 0\rangle$$

$$m_{tot} = 0 \longrightarrow S_{tot} = \begin{cases} 0 & \text{dark} \\ 1 & \text{emits 1 photon} \rightarrow \text{m}_{\text{tot}} = -1 \\ 2 & \text{emits 2 photons} \rightarrow \text{m}_{\text{tot}} = -2 \\ 3 & \text{emits 3 photons} \rightarrow \text{m}_{\text{tot}} = -2 \\ \vdots & \vdots \\ N/2 & \text{emits N/2 photons} \rightarrow \text{m}_{\text{tot}} = -\text{N}/2 \end{cases}$$

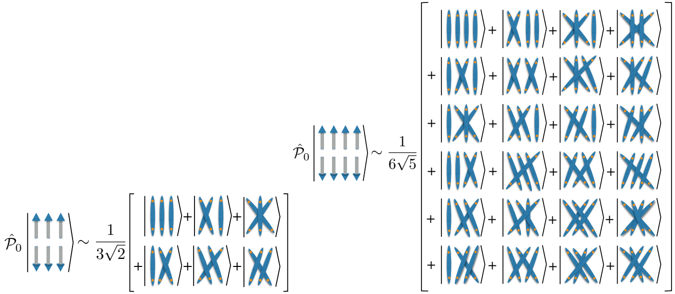
Probability of observing p photons is $|a_p|^2$

Claim: Null observation collapses spins onto a ‘strong RVB’ state

RVB construction

Rule for constructing the strong RVB state

- Arrange $\uparrow$ spins in top row and $\downarrow$ spins in bottom row
- Place dimers (singlets) connecting each site in top row to a site in bottom row
- Superpose all such configurations *symmetrically* and *in-phase*



Note the symmetry under in-row permutations.

Proof of RVB Nature of dark state

Key ingredient: symmetry under in-row permutations

- Lemma 1: A state of M spins that is symmetric under any permutation of the M constituent spins must necessarily have $S_{tot} = M/2$, i.e., it has the maximal total angular momentum quantum number. Conversely, if a state of M spins has $S_{tot} = M/2$, it is symmetric under any permutation of the constituent spins.
- Lemma 2: For the system with M spins, a quantum state with $S_{tot} = M/2$ is uniquely defined by its m_{tot} quantum number, i.e., a state given by $|S_{tot} = M/2, m\rangle$ is unique.
- Lemma 3: For the system with M spins, the projection operator onto the subspace with fixed S_{tot} quantum number is symmetric under permutations. This can be seen by explicit construction. We have

$$\hat{P}_{S=\Sigma} = \prod_{S\neq\Sigma} \frac{\hat{S}_{tot}^2 - S(S+1)}{\Sigma(\Sigma+1) - S(S+1)}. \quad (1)$$

Proof of RVB Nature of dark state

- The dark state obtained from null emission is given by

$$|\Psi_{dark}\rangle \sim \hat{P}_{S=0}|\Psi_{initial}\rangle$$

- This state is symmetric under in-row permutations $\rightarrow$

$$|\Psi_{dark}\rangle = \sum_{\lambda=0}^{N/2} C_\lambda |S_{tot} = N/4, m_{tot} = N/4 - \lambda\rangle_t \otimes \\ |S_{tot} = N/4, m_{tot} = \lambda - N/4\rangle_b.$$

- The coefficients C_λ is simply the Clebsch Gordan coefficient $C\{j_1 = N/4, m_1 = N/4 - \lambda; j_2 = N/4, m_2 = N/4 + \lambda; j_3 = 0\}$

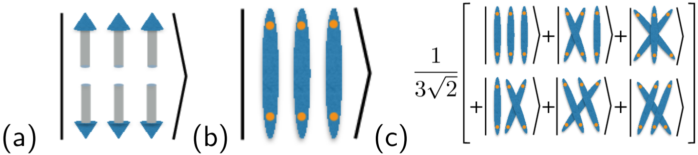
Proof of RVB Nature of dark state

- The RVB state also has in-row permutation symmetry $\rightarrow$ if decomposed into row wavefunctions, each row-wavefunction must have $S_{tot} = N/4$
- We explicitly expand the wavefunction in the form

$$|\Psi_{RVB}\rangle = \sum_{\lambda=0}^{N/2} D_\lambda |S_{tot} = N/4, m_{tot} = N/4 - \lambda\rangle_t \otimes |S_{tot} = N/4, m_{tot} = \lambda - N/4\rangle_b$$

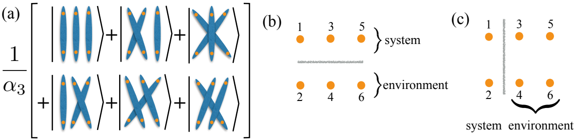
- By explicit evaluation and regrouping of terms, we see that $D_\lambda = C_\lambda$, i.e., the proposed RVB state and the dark state are identical!
- Probability for null emission: $\langle\Psi_{initial}|\Psi_{RVB}\rangle|^2 = |D_{\lambda=0}|^2 = (N/2+1)^{-1} \sim N^{-1}$
- Probability for creating an RVB state of 20 spins $\sim 10\% \rightarrow$ 90% of the runs must be discarded

Entanglement in RVB state



Definition: an entangled state is one that cannot be written as a direct product wavefunction

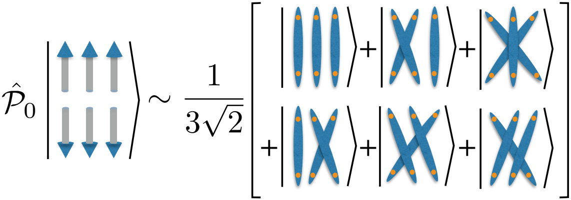
A fixed valence bond configuration: cannot be written as a direct product of spin wavefunctions. However, it can be written as a direct product of dimers $\rightarrow$ not entangled in this sense



Entanglement entropy from row-wise decomposition
$= \log(1 + N/2)$, grows with N

Summary

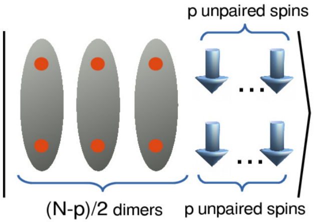
- Initialise N spins in a direct product state within a lossy cavity
- Measure number of photons emitted
- Null observation collapses wavefunction onto an RVB state
- Route to synthesize entangled pure states of 20 or more constituents



Collapse by positive photon observation

If we observe $p \neq 0$ photons, this collapses the spin wavefunction onto a 'doped' RVB state

$$|\Psi_{RVB}\rangle \sim \sum_{P(t)} \sum_{P(b)}$$



- Unpaired spins are analogues of 'spinons' (unpaired electrons) from the RVB theory of high-T$_c$ superconductivity
- The number of spinons is always even!
- Spinons are formed in pairs – Cooper pairing

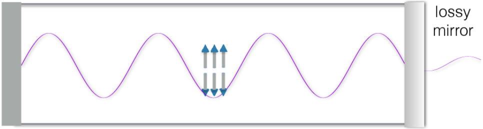
Collapse by positive photon observation

$$|\Psi_{initial}\rangle = |\underbrace{\uparrow \ldots \uparrow}_{N/2}\underbrace{\downarrow \ldots \downarrow}_{N/2}\rangle = \sum_{S=0}^{N/2} a_S |S_{tot} = S, m_{tot} = 0\rangle$$

$m_{tot} = 0 \implies$

$$S_{tot} = \left\{ \begin{array}{c|c|c} 0 & \text{dark} & \text{undoped RVB} \\ 1 & \text{emits 1 photon} & 2 \text{ spinons} \\ 2 & \text{emits 2 photons} & 4 \text{ spinons} \\ 3 & \text{emits 3 photons} & 6 \text{ spinons} \\ \vdots & \vdots & \vdots \\ N/2-1 & \text{emits N/2}-1 \text{ photons} & N-2 \text{ spinons} \\ N/2 & \text{emits N/2 photons} & \text{Non}-\text{RVB}: |\downarrow \ldots \downarrow\rangle. \end{array} \right.$$

Incipient superconductivity and RVB states



If we do not measure the photon number, we obtain a mixed state of spins

$$\hat{\rho}_{spin} = \mathrm{Tr}_{ph.}\hat{\rho} = \sum_{p=0}^{N/2} \alpha_p |\Psi_p\rangle\langle\Psi_p| \qquad (2)$$

– a state with fluctuating (even) spinon number – incipient superconductivity $\sim$ finite size system with superconducting correlations

For large N, a small spin-spin interaction may suffice to generate coherence between different number sectors $\implies$ signatures of superconductivity!