Entanglement generation in periodically driven integrable quantum systems

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Outline

1. ***A class of integrable models: Spin systems and Dirac fermions***

2. ***Entanglement: Basic facts relevant to the present study.***

3. ***Periodic drive and entanglement generation***

4. ***Hasting's theorem: states generated out-of-equilibrium***

5. ***Approach to steady state: A dynamic phase transition***

6. ***Floquet Hamiltonian: Explaining the transition.***

7. ***Steady state entanglement***

8 Conclusion and future directions.

Introduction: Models and basics

A class of integrable models

Free fermionic models in d dimensions with matrix structure of the Hamiltonian

$$H = \sum_{\vec{k}} \psi^{\dagger}_{\vec{k}} \left[(g(t) - b_{\vec{k}})\tau_3 + \Delta_{\vec{k}}\tau_1 \right] \psi_{\vec{k}},$$

Two component fermion creation operator
$\psi_{\vec{k}} = (c_{\vec{k}}, c^{\dagger}_{-\vec{k}})^T$

Tuning parameter: chosen to be a periodic function of time according to a chosen protocol

Pauli matrices in Particle-hole space

H represents, for different realizations of g(t), Δ_k and b_k, Ising model in d=1, Kitaev model in d=2 , and Dirac fermions describing quasiparticles of Graphene and topological insulators (also in d=2).

Subject of this talk: Behavior of entanglement entropy of H when subjected to a periodic drive characterized by number of periods n and frequency ω.

Specific Example: Ising model in transverse field

Spin Hamiltonian

$$H = J(-\sum_{\langle ij\rangle} S_i^z S_j^z + g\sum_i S_i^x)$$

Jordan-Wigner transformation:

$$s_i^x = \left(c_i + c_i^+\right)\prod_{j<i}\left(1-2c_j^+c_j\right)$$

$$s_i^y = \left(c_i - c_i^+\right)\prod_{j<i}\left(1-2c_j^+c_j\right)$$

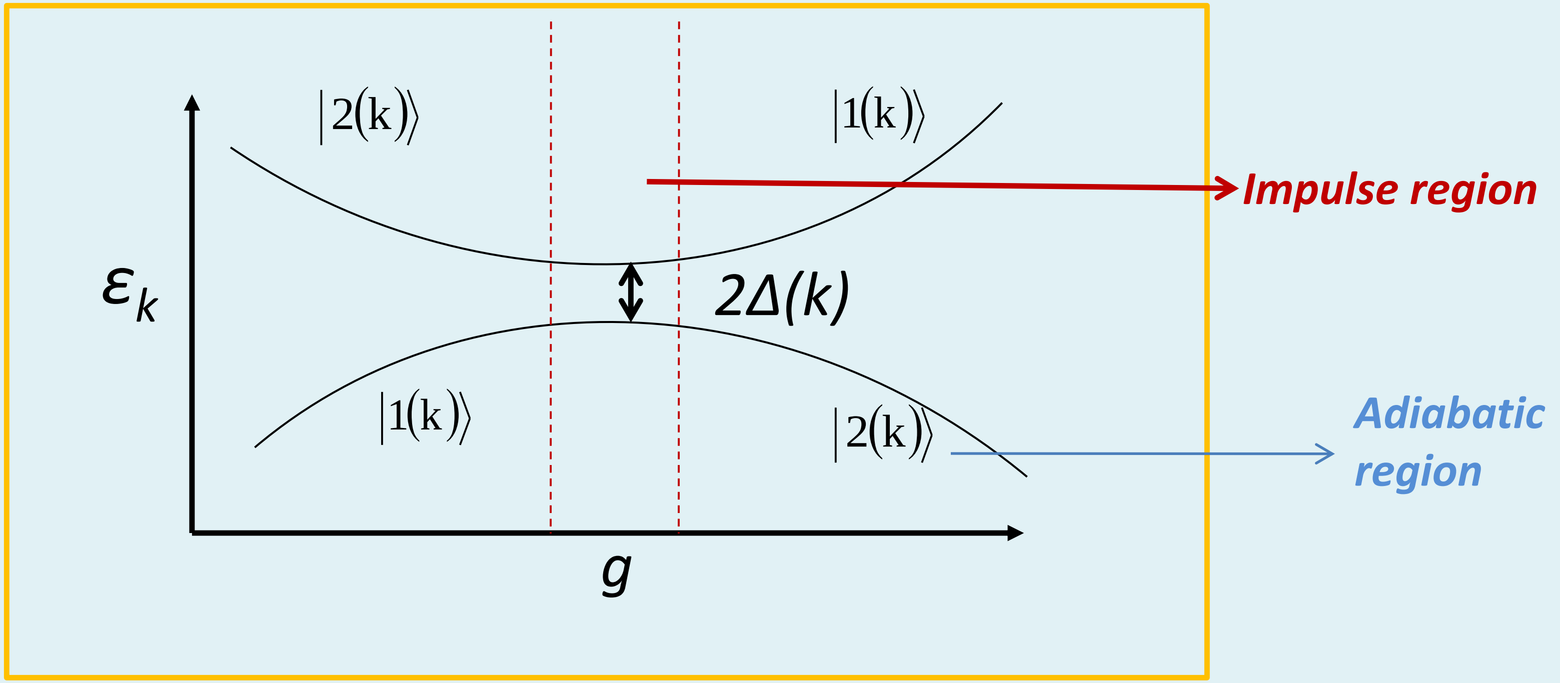
$$s_i^z = 1-2c_j^+c_j$$

Hamiltonian in term of the fermions: [J=1]

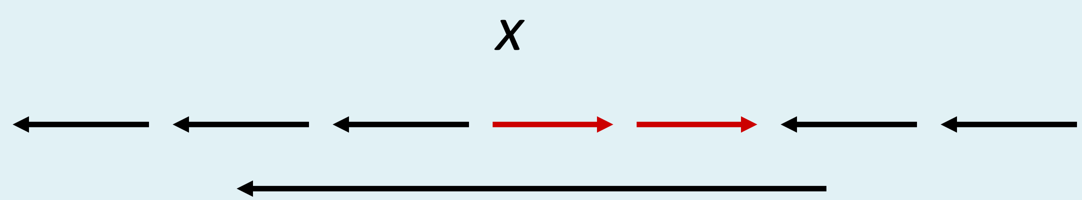
$$H = \sum_k \left\{2\left[g-\cos(ka)\right]c_k^+c_k + \sin(ka)\left[c_k^+c_{-k}^+ + c_{-k}c_k\right]\right\}$$

$$\varepsilon_k^{\pm} = \pm 2\sqrt{\left((g - \cos(k))^2 + (\sin(k))^2\right)}$$

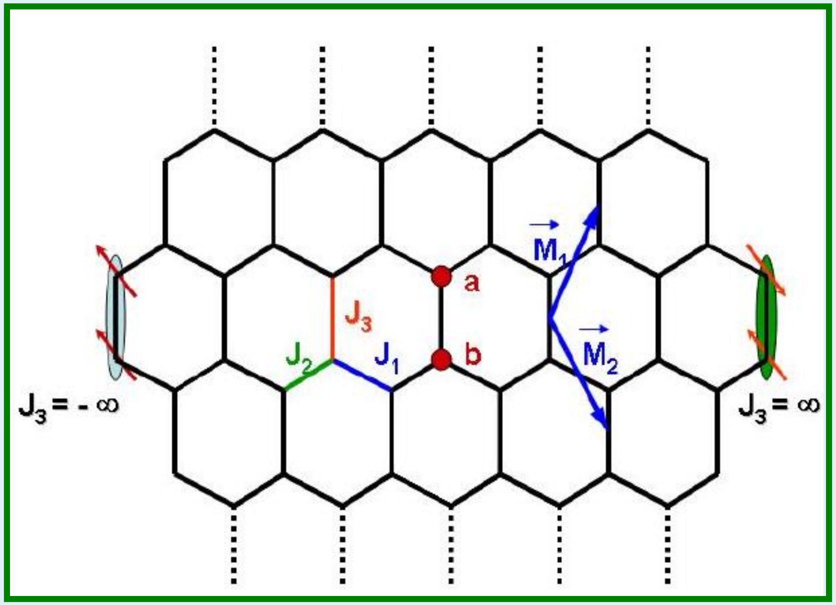
$$g = g_0 \frac{t}{\tau}$$



Defect formation occurs mostly between a finite interval near the quantum critical point.



Kitaev Model in d=2



$$H = \sum_{j+l=\text{even}} (J_1 \sigma^x_{j,l} \sigma^x_{j+1,l} + J_2 \sigma^y_{j-1,l} \sigma^y_{j,l} + J_3 \sigma^z_{j,l} \sigma^z_{j,l+1})$$

Jordan-Wigner transformation

$$H_F = i \sum_{\vec{n}} [J_1 b_{\vec{n}} a_{\vec{n}-\vec{M}_1} + J_2 b_{\vec{n}} a_{\vec{n}+\vec{M}_2} + J_3 D_{\vec{n}} b_{\vec{n}} a_{\vec{n}}],$$

a and b represents Majorana Fermions living at the end sites of the vertical bonds of the lattice.

D_n is independent of a and b and hence commutes with H_F: Special property of the Kitaev model

Ground state corresponds to D_n=1 on all links.

Solution in momentum space

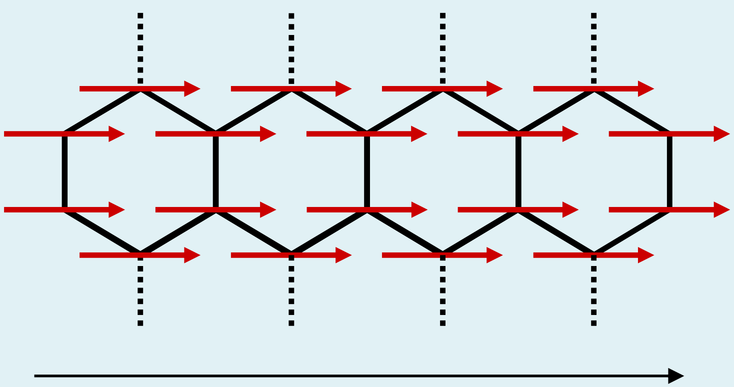
$$H_F = \sum_{\vec{k}} \psi'^{\dagger}_{\vec{k}} H'_{\vec{k}} \psi'_{\vec{k}},$$

$$H'_{\vec{k}} = 2[J_1 \sin(\vec{k}\cdot\vec{M}_1) - J_2 \sin(\vec{k}\cdot\vec{M}_2)]\sigma^1 + 2[J_3 \quad + J_1 \cos(\vec{k}\cdot\vec{M}_1) + J_2 \cos(\vec{k}\cdot\vec{M}_2)]\sigma^3$$

Off-diagonal element

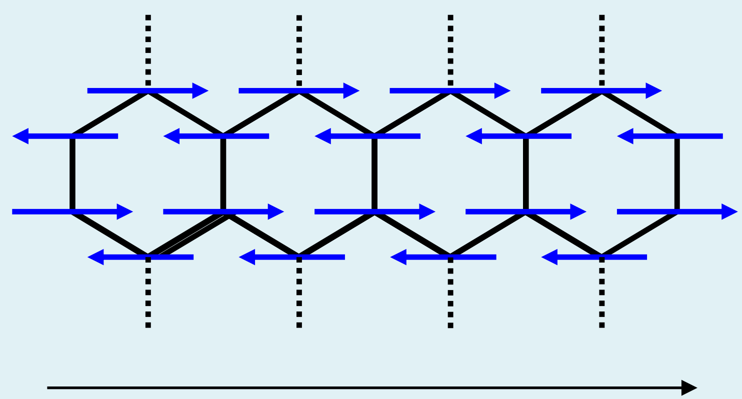
Diagonal element

$$E_{\vec{k}} = 2[\{J_1 \sin(\vec{k}\cdot\vec{M}_1) - J_2 \sin(\vec{k}\cdot\vec{M}_2)\}^2 + \{J_3 + J_1 \cos(\vec{k}\cdot\vec{M}_1) + J_2 \cos(\vec{k}\cdot\vec{M}_2)\}^2]^{1/2}$$



z

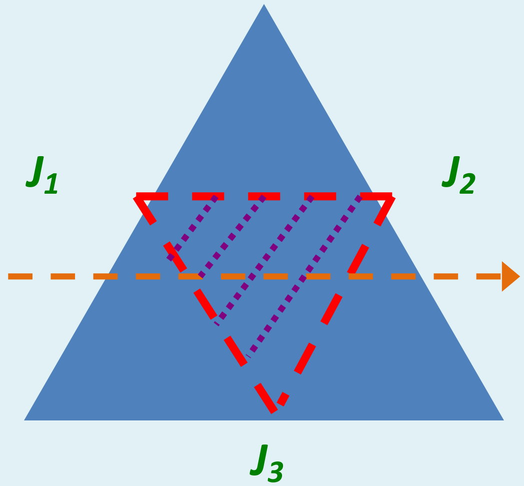
$J_3 \rightarrow -\infty$



z

$J_3 \rightarrow \infty$

Gapless phase when J_3 lies between(J_1+J_2) and |J_1-J_2|. The bands touch each other at special points in the Brillouin zone whose location depend on values of J_i s.



Quenching J_3 linearly takes the system through a critical line in parameter space and hence through the line

$$\sin(\mathbf{k}\cdot\mathbf{M}_1) = \frac{J_2}{J_1}\sin(\mathbf{k}\cdot\mathbf{M}_2)$$

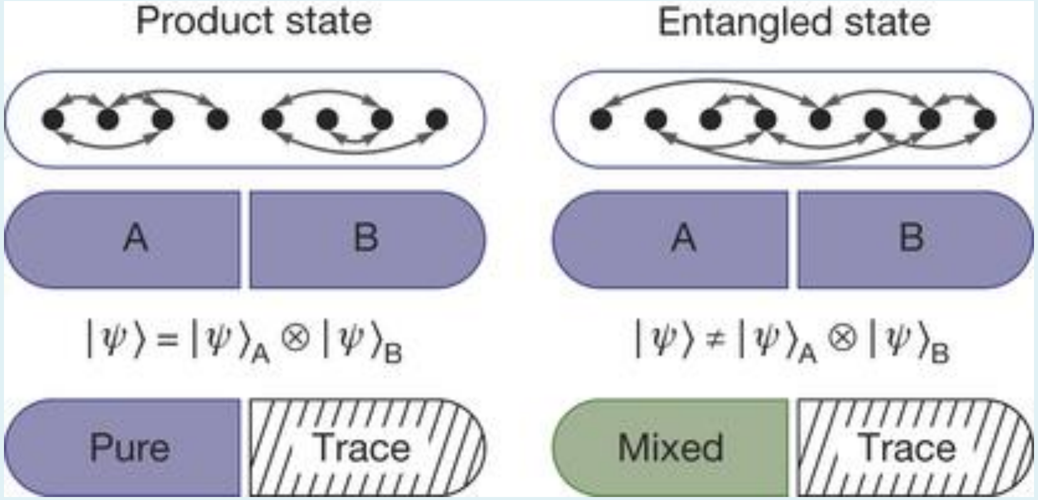
in momentum space.

In general a quench of d dimensional system can take the system through a d-m dimensional gapless surface in momentum space.

For Kitaev model: d=2, m=1

For quench through critical point: m=d

Entanglement: A few basic facts



R. Islam et al. Nature 2015.

Focus: Entanglement entropy of ground states of many-body Hamiltonian.

Reason: May lead to classification of states or phase transition which eludes the standard methods (such as Landau-Ginzburg paradigm for phase transitions)

Several measures of entanglement:

$$S = -\mathrm{Tr}[\rho \ln \rho] = \lim_{m\to 1} S^{(m)}.$$

Von-Neuman

mth Renyi entropy

$$S^{(m)} = (1-m)^{-1}\mathrm{Tr}[\rho^m]$$

Entanglement entropy for ground states of local Hamiltonian obeys area law

$$S \sim l^{d-1} \text{ for } d \geq 1.$$

Hasting's theorem

The possible violation of this occurs for gapless ground states and are usually logarithmic

$$S \sim l^{d-1} \ln l.$$

For d=1, the coefficient of the log term is the central charge of the associated CFT

The sub-leading term Γ in the entanglement entropy for gapped systems, if non-zero, indicates additional long-range component and is a signature of the ground state topology.

$$S(l) = \frac{l}{a} - \Gamma$$

2D

Question: What happens when one drives the system out of equilibrium so that the system accesses several states in the Hilbert space?

Entanglement generation

Calculation of integrable models

Divide the system of linear dimension L into a subsystem of dimension l and the rest (bath).

We intend to compute the density matrix $\rho(l)$ by tracing out the bath degrees of freedom.

For integrable models, the answer can be expressed in terms two-point correlation functions

$$
\begin{aligned}
C_{ij} &= \langle c_{\vec{i}}^{\dagger} c_{\vec{j}} \rangle_n = 2 \sum_{\vec{k} \in \mathrm{BZ}/2} |u_{\vec{k}}(t)|^2 \cos(\vec{k} \cdot (\vec{i} - \vec{j}))/L^d \\
F_{ij} &= \langle c_{\vec{i}}^{\dagger} c_{\vec{j}}^{\dagger} \rangle_n = 2 \sum_{\vec{k} \in \mathrm{BZ}/2} u_{\vec{k}}^{*}(t) v_{\vec{k}}(t) \sin(\vec{k} \cdot (\vec{i} - \vec{j}))/L^d
\end{aligned}
$$

The density matrix can be written in terms of these matrix elements (Peschel et al. 2001)

$$
\begin{aligned}
\rho_\alpha &= \frac{1}{Z} \exp(-\mathcal{H}_\alpha), \\
\mathcal{H}_\alpha &= \sum_{i=1}^{l} \epsilon_i \eta_i^{\dagger} \eta_i
\end{aligned}
$$

$$
\eta_k = \sum (g_{ki} c_i + h_{ki} c_i^{\dagger})
$$

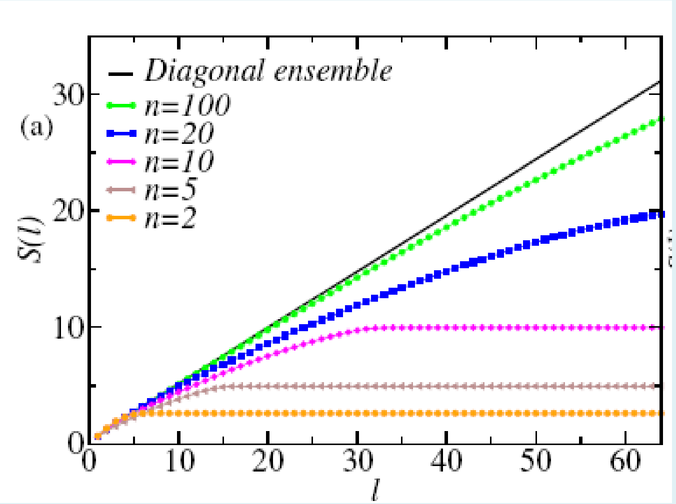
The entanglement spectrum ε_i and the functions g and h can be determined from the correlation functions C and F. Thus the correlation matrix determines $\rho(l)$ and hence S(l).

The eigenvalues p_i of the density matrix as computed from the above procedure yields

$$
S_n(l) = -\sum_{i=1}^{2l} p_i \log(p_i)
$$

Numerical computation of entanglement after n-cycles of the periodic drive.

Entanglement generation after n drive cycles



Ising model in transverse field in d=1

Periodic drive of the transverse field for n cycles with frequency ω

Protocol followed: Square pulse

$$g(t) = g_i, \text{ for } (n-1)T \leq t \leq (n-1/2)T$$

$$g(t) = g_f \text{ for } (n-1/2)T \leq t \leq nT$$

$S_n(l)$ satisfies area-law for small n in accordance with expected behavior.

However, the minimum l beyond which $S_n(l)$ satisfies area-law diverges with n.

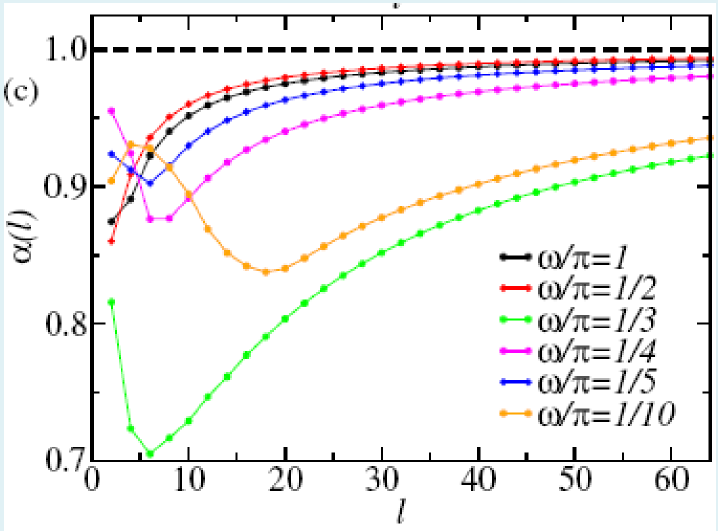
Periodic drive provides a route to realization of states with non area-law entanglement entropies.

Consequence of finite weight of the final state in a major fraction of states in the Hilbert space of the initial Hamiltonian.

Qualitatively similar to linear spread of S after quench (Huse et al) $S_n(l) \sim l^{\alpha(n,\omega)}$

$$d-1 \leq \alpha(n,\omega) \leq d.$$

How fast one approach volume law



We define an estimator α

$$\alpha(l) = \log[S_\infty(2l)/S_\infty(l)]/\log(2)$$

For d=1 α=0 indicates area law
α=1 indicates volume law

For large ω, there is a rapid convergence of α to 1 indicating approach to volume law

For small ω, α is a non-monotonic function of frequency.

The approach of α to unity may be quite slow for small ω

It may require a very large subsystem size

Thus periodic drive may be used to realize states with non-area and non-volume law enatnglement entropy for any finite subsystem.

Qualitative Criteria for a non-area law: Analog of Hasting's theorem

Hastings theorem states that ground state of any local Hamiltonian must have an area-law entanglement entropy → *Not directly applicable to driven systems since the final state is not the ground state of the driven Hamiltonian for any t*

Idea: Turn the problem around. Is it possible to obtain an Hamiltonian for which the final State after the drive is the ground state?

The state of the system after n drive periods is $\psi_k(t_f = nT)$

We seek the solution of

$$\mathcal{H}_{\vec{k}t}\psi_{\vec{k}}(t_f) = -\sqrt{\epsilon_{\vec{k}t}^2 + |\Delta_{\vec{k}t}|^2}\psi_{\vec{k}}(t_f).$$

The two component structure of the wavefunction allows us to write

$$\mathcal{H}_{\vec{k}t} = \epsilon_{\vec{k}t}\tau_3 + \Delta_{\vec{k}t}\tau^+ + \Delta^*_{\vec{k}t}\tau^-$$

Obtain solution for ε_{kt} and Δ_{kt} subject to the condition that H_{kt} approaches the system Hamiltonian in the adiabatic limit

The solutions are

$$\epsilon_{\vec{k}t} = \Delta_{\vec{k}}(|u_{\vec{k}}(t_f)|^2 - |v_{\vec{k}}(t_f)|^2)/(2|u_{\vec{k}}(t_f)||v_{\vec{k}}(t_f)|)$$

$$\Delta_{\vec{k}t} = \Delta_{\vec{k}} \exp(i(\alpha_{\vec{k}} - \beta_{\vec{k}}))$$

where u and v are the two components of the final wavefucntion.

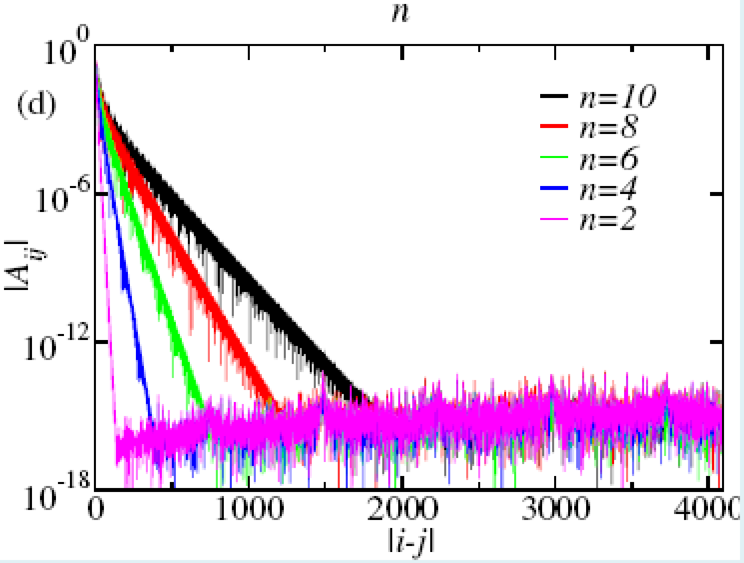
$$\alpha_{\vec{k}}(\beta_{\vec{k}}) = \mathrm{Arg}[u_{\vec{k}}(t_f)(v_{\vec{k}}(t_f))].$$

Thus in real space one obtains

$$\mathcal{H}_t = \sum_{\vec{i}\vec{j}}(A_{\vec{i}-\vec{j}} c^\dagger_{\vec{i}} c_{\vec{j}} + B_{\vec{i}-\vec{j}} c_{\vec{i}} c_{\vec{j}} + \mathrm{h.c.}),$$

From the plot of $|A_{ij}|$ as a function of distance between the sites, we find that A_{ij} decays exponentially with $|i-j|$.

$A_{ij} \sim \mathrm{Exp}[-|i-j|/R(n,\omega)]$



T=1

The length-scale R increases rapidly with n for any ω and crosses l for some finite n=n'

For n >> n', the system may have non-area law behavior since the state after n drive cycles is the ground state of an effectively long-ranged Hamiltonian H_t

→ ***Application of Hasting's theorem to driven systems .***

A dynamical transition

Approach to the steady state: dynamic transition

In the limit of infinite n, the system is known to reach a steady state which is given by the diagonal ensemble (which is same as GGE for periodically driven integrable models)

→ *One can drop cross terms while calculating any fermioinc Correlator:*

We denote the correlation function thus computed as fermionic steady state correlators.

The corresponding steady state value of the correlation matrix is $\mathcal{C}_\infty(l)$

One can then define a distance measure D which measures how close C_n is to its steady state value.

$$\mathcal{D} = \mathrm{Tr}[(\mathcal{C}_\infty(l) - \mathcal{C}_n(l))^\dagger (\mathcal{C}_\infty(l) - \mathcal{C}_n(l))]^{1/2}/(2l).$$

$$0 \leq \mathcal{D} \leq 1$$

Numerically one finds that there are two distinct dynamical regimes in these driven systems which are separated by a transition.

Regime 1

$$\mathcal{D} \sim (\omega/n)^{(d+2)/2}$$

→

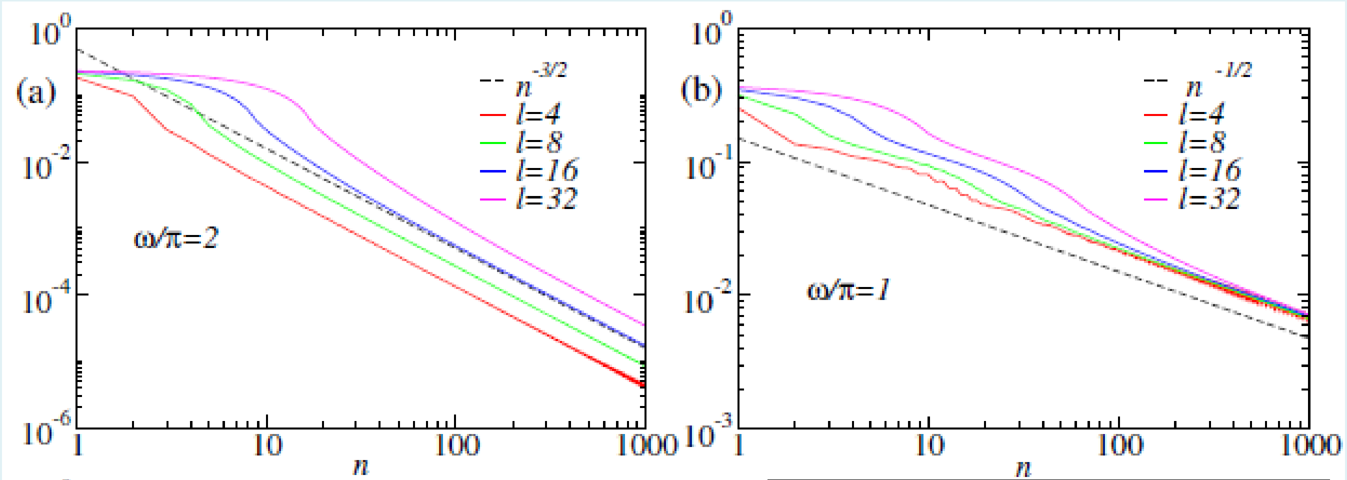
Regime 2

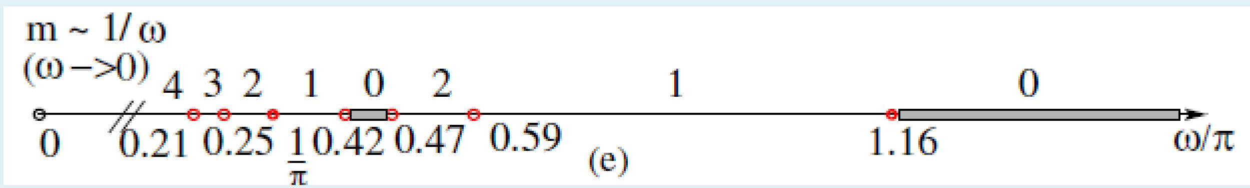
$$\mathcal{D} \sim (\omega/n)^{d/2}$$

Dynamical transition and reentrance at d=1

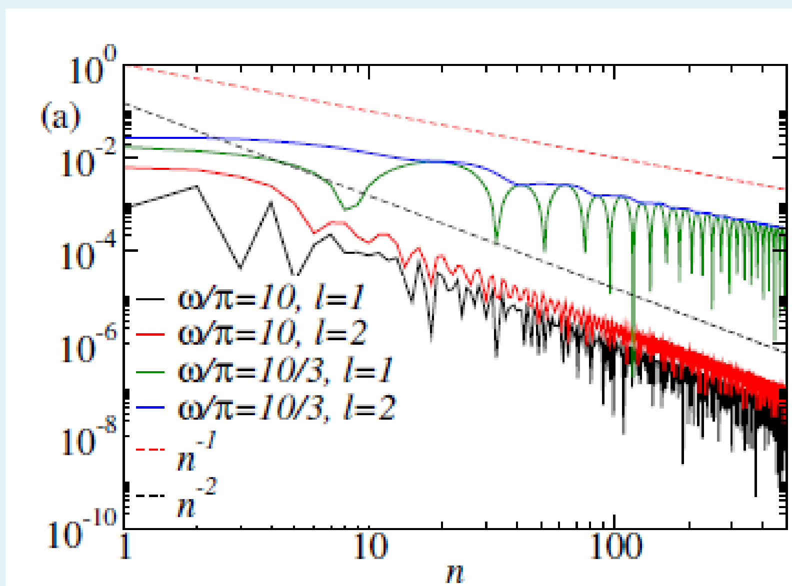
Dynamical transition and Reentrance

1D Ising





2D Kitaev



Interpretation of the transition: Evolution matrix

The unitary evolution in the presence of a periodic drive after n drive cycles leads to

$$\psi^i = \prod_{\vec{k}} \psi^i_{\vec{k}} = \prod_{\vec{k}} (u^i_{\vec{k}}, v^i_{\vec{k}})^T \quad \Longrightarrow \quad \psi^f = \prod_{\vec{k}} \psi^f_{\vec{k}} = \prod_{\vec{k}} (u^{nf}_{\vec{k}}, v^{nf}_{\vec{k}})^T.$$

The parametrization of U_k follows from its unitary nature: θ, α, and γ are real quantities

$$\psi^f_{\vec{k}} = U^n_{\vec{k}} \psi^i_{\vec{k}}, \quad \psi'_{\vec{k}} = U_{\vec{k}} \psi^i_{\vec{k}},$$

$$U_{\vec{k}} = \begin{pmatrix} \cos(\theta_{\vec{k}}) e^{i\alpha_{\vec{k}}} & \sin(\theta_{\vec{k}}) e^{i\gamma_{\vec{k}}} \\ -\sin(\theta_{\vec{k}}) e^{-i\gamma_{\vec{k}}} & \cos(\theta_{\vec{k}}) e^{-i\alpha_{\vec{k}}} \end{pmatrix}$$

One can find U_k as a function of initial and final values of the wavefunctions.

$$\sin^2(\theta_{\vec{k}}) = \Big[|u^f_{\vec{k}}|^2 v^{i2}_{\vec{k}} + |v^f_{\vec{k}}|^2 u^{i2}_{\vec{k}} - 2|u^f_{\vec{k}}||v^f_{\vec{k}}| u^i_{\vec{k}} v^i_{\vec{k}} \cos(\mu_{\vec{k}} - \mu'_{\vec{k}}) \Big] \qquad (20$$

$$\gamma_{\vec{k}} = \arctan\left(\frac{|u^f_{\vec{k}}| v^i_{\vec{k}} \sin(\mu_{\vec{k}}) + u^i_{\vec{k}} |v^f_{\vec{k}}| \sin(\mu'_{\vec{k}})}{|u^f_{\vec{k}}| v^i_{\vec{k}} \cos(\mu_{\vec{k}}) - u^i_{\vec{k}} |v^f_{\vec{k}}| \cos(\mu'_{\vec{k}})} \right)$$

$$\alpha_{\vec{k}} = \arctan\left(\frac{|u^f_{\vec{k}}| u^i_{\vec{k}} \sin(\mu_{\vec{k}}) - v^i_{\vec{k}} |v^f_{\vec{k}}| \sin(\mu'_{\vec{k}})}{|u^f_{\vec{k}}| u^i_{\vec{k}} \cos(\mu_{\vec{k}}) + |v^f_{\vec{k}}| v^i_{\vec{k}} \cos(\mu'_{\vec{k}})} \right)$$

$$u^f_{\vec{k}} = |u^f_{\vec{k}}| \exp[i\mu_{\vec{k}}] \text{ and } v^f_{\vec{k}} = |v^f_{\vec{k}}| \exp[i\mu'_{\vec{k}}].$$

For an initial state (0,1), this yields the simple result

$$\sin(\theta_{\vec{k}}) = |u_{\vec{k}\,f}|, \ \alpha_{\vec{k}} = -\mathrm{Arg}(v_{\vec{k}\,f}) \text{ and } \gamma_{\vec{k}} = \mathrm{Arg}(u_{\vec{k}\,f}).$$

Interpretation of the transition: Floquet Hamiltonian

For stroboscopic measurements at the end of n drive periods, the system Is descrbed by the Floquet Hamiltonian → $U_{\vec{k}} \; = \; e^{-iH_{\vec{k}F}T}$

For the present class of integrable models U_k is 2 by 2 matrix. Thus one may write

$$H_{\vec{k}F} = \vec{\sigma}\cdot\vec{\epsilon}_{\vec{k}}. \;\text{where}\; \vec{\epsilon}_{\vec{k}} = (\epsilon_{1k}, \epsilon_{2k}, \epsilon_{3k}).$$

$$U_{\vec{k}} \; = \; e^{-i(\vec{\sigma}\cdot\vec{n}_{\vec{k}})\phi_{\vec{k}}}, \quad n_{\vec{k}} = \frac{\vec{\epsilon}_{\vec{k}}}{|\vec{\epsilon}_{\vec{k}}|} \quad , \phi_{\vec{k}} = T|\vec{\epsilon}_{\vec{k}}|$$

One can express the Floquet Hamiltonian in terms of the parameters of U and hence in terms of the initial and final wavefunctions for each k

$$\begin{aligned}
\epsilon_{\vec{k}1} &= -|\vec{\epsilon}_{\vec{k}}|\sin(\theta_{\vec{k}})\sin(\gamma_{\vec{k}})\mathrm{Sgn}[\sin(\phi_{\vec{k}})]/D_{\vec{k}} \\
\epsilon_{\vec{k}2} &= -|\vec{\epsilon}_{\vec{k}}|\sin(\theta_{\vec{k}})\cos(\gamma_{\vec{k}})\mathrm{Sgn}[\sin(\phi_{\vec{k}})]/D_{\vec{k}} \\
\epsilon_{\vec{k}3} &= -|\vec{\epsilon}_{\vec{k}}|\cos(\theta_{\vec{k}})\sin(\alpha_{\vec{k}})\mathrm{Sgn}[\sin(\phi_{\vec{k}})]/D_{\vec{k}} \\
D_{\vec{k}} &= \sqrt{1-\cos^2(\theta_{\vec{k}})\cos^2(\alpha_{\vec{k}})} \\
|\vec{\epsilon}_{\vec{k}}| &= \arccos[\cos(\theta_{\vec{k}})\cos(\alpha_{\vec{k}})]/T
\end{aligned}$$

→ *Exact expression For the Floquet Hamiltonian*

Relation of Floquet Hamiltonian with elements of correlation matrix

$$C_{ij} = \langle c^\dagger_{\vec{i}} c_{\vec{j}} \rangle_n = 2 \sum_{\vec{k}\in \mathrm{BZ}/2} |u_{\vec{k}}(t)|^2 \cos(\vec{k}\cdot(\vec{i}-\vec{j}))/L^d \quad (2)$$

$$F_{ij} = \langle c^\dagger_{\vec{i}} c^\dagger_{\vec{j}} \rangle_n = 2 \sum_{\vec{k}\in \mathrm{BZ}/2} u^*_{\vec{k}}(t) v_{\vec{k}}(t) \sin(\vec{k}\cdot(\vec{i}-\vec{j}))/L^d$$

The elements of the correlation matrix depend on the final wavefunction

It can be expressed in terms of the initial wavefunction and the elements of the Floquet Hamiltonian after n drive cycles

$$\langle c^\dagger_{\vec{i}} c_{\vec{j}} \rangle_n = \langle c^\dagger_{\vec{i}} c_{\vec{j}} \rangle_\infty - \frac{1}{(2\pi)^d} \int_{\vec{k}\in BZ/2} d^d k \cos(\vec{k}\cdot(\vec{i}-\vec{j})) \times (1-\hat{n}^2_{\vec{k}3}) \cos(2n\phi_{\vec{k}}) \quad (7)$$

$$\langle c^\dagger_{\vec{i}} c^\dagger_{\vec{j}} \rangle_n = \langle c^\dagger_{\vec{i}} c^\dagger_{\vec{j}} \rangle_\infty + \frac{1}{(2\pi)^d} \int_{\vec{k}\in BZ/2} d^d k \sin(\vec{k}\cdot(\vec{i}-\vec{j})) \times \left[\hat{n}_{\vec{k}3}(\hat{n}_{\vec{k}1} + i\hat{n}_{\vec{k}2}) \cos(2n\phi_{\vec{k}}) + i(\hat{n}_{\vec{k}1} + i\hat{n}_{\vec{k}2}) \sin(2n\phi_{\vec{k}})\right]$$

All elements of the correlation matrix can be expressed in terms of elements of H_F

$$\langle c^\dagger_{\vec i} c_{\vec j}\rangle_n = \langle c^\dagger_{\vec i} c_{\vec j}\rangle_\infty - \frac{1}{(2\pi)^d}\int_{\vec k \in BZ/2} d^d k \cos(\vec k\cdot(\vec i - \vec j))$$
$$\times (1-\hat n^2_{\vec k 3})\cos(2n\phi_{\vec k}) \qquad (7)$$
$$\langle c^\dagger_{\vec i} c^\dagger_{\vec j}\rangle_n = \langle c^\dagger_{\vec i} c^\dagger_{\vec j}\rangle_\infty + \frac{1}{(2\pi)^d}\int_{\vec k \in BZ/2} d^d k \sin(\vec k\cdot(\vec i - \vec j))$$
$$\times\Big[\hat n_{\vec k 3}(\hat n_{\vec k 1} + i\hat n_{\vec k 2})\cos(2n\phi_{\vec k}) + i(\hat n_{\vec k 1} + i\hat n_{\vec k 2})\sin(2n\phi_{\vec k})\Big]$$

$$\begin{aligned}
\epsilon_{\vec k 1} &= -|\vec\epsilon_{\vec k}|\sin(\theta_{\vec k})\sin(\gamma_{\vec k})\mathrm{Sgn}[\sin(\phi_{\vec k})]/D_{\vec k}\\
\epsilon_{\vec k 2} &= -|\vec\epsilon_{\vec k}|\sin(\theta_{\vec k})\cos(\gamma_{\vec k})\mathrm{Sgn}[\sin(\phi_{\vec k})]/D_{\vec k}\\
\epsilon_{\vec k 3} &= -|\vec\epsilon_{\vec k}|\cos(\theta_{\vec k})\sin(\alpha_{\vec k})\mathrm{Sgn}[\sin(\phi_{\vec k})]/D_{\vec k}\\
D_{\vec k} &= \sqrt{1-\cos^2(\theta_{\vec k})\cos^2(\alpha_{\vec k})}\\
|\vec\epsilon_{\vec k}| &= \arccos[\cos(\theta_{\vec k})\cos(\alpha_{\vec k})]/T
\end{aligned}$$

For large n, the contribution to the frequency dependent part of the correlation function comes from saddle point of ϕ and hence $|\varepsilon|$

Such saddle points at occurs at k=k0 for which $d|\varepsilon|/dk_i=0$

$$\cot(\theta_{\vec k})d\alpha_{\vec k}/dk_i = -\cot(\alpha_{\vec k})d\theta_{\vec k}/dk_i$$

This condition may be satisfied for a specific k_0 not necessarily at the BZ edge or center,

$$\sin(\theta_{\vec k}) = 0 = d\alpha_{\vec k}/dk_i.$$

This occurs for k for which $n_{1k}=n_{2k}=0$.This in turn implies that U is diagonal or the off-diagonal term in H vanishes This happens at BZ edge or center

Saddle point evaluation of correlators

Within saddle point approximation appropriate for large n one may express these integrals as

$$\int f(\vec{k}) \exp(in\phi(\vec{k})) d^d k \approx \exp(in\phi(\vec{k}_0))(n|\phi"(\vec{k}_0|))^{-d/2} \times \exp(\pi i\mu/4) \left(f(\vec{k}_0) + i\frac{f''(\vec{k}_0)}{2\phi''(\vec{k}_0)}\frac{1}{n} + \mathcal{O}(1/n^2) \right) \quad (9)$$

μ=Sgn[ϕ"] and f(k) Is a smooth function of k around k_0

The function f(k) can be read off from the expression of correlation functions.

Key point: $f(k_0)$ vanishes if k_0 happens to be at the edge or center of the Brillouin zone where n_{3k}=1 →

$$\sin(\theta_{\vec{k}}) = 0 = d\alpha_{\vec{k}}/dk_i.$$

In this case, the correlation functions show a $(\omega/n)^{(d+2)/2}$ decay to its steady state value

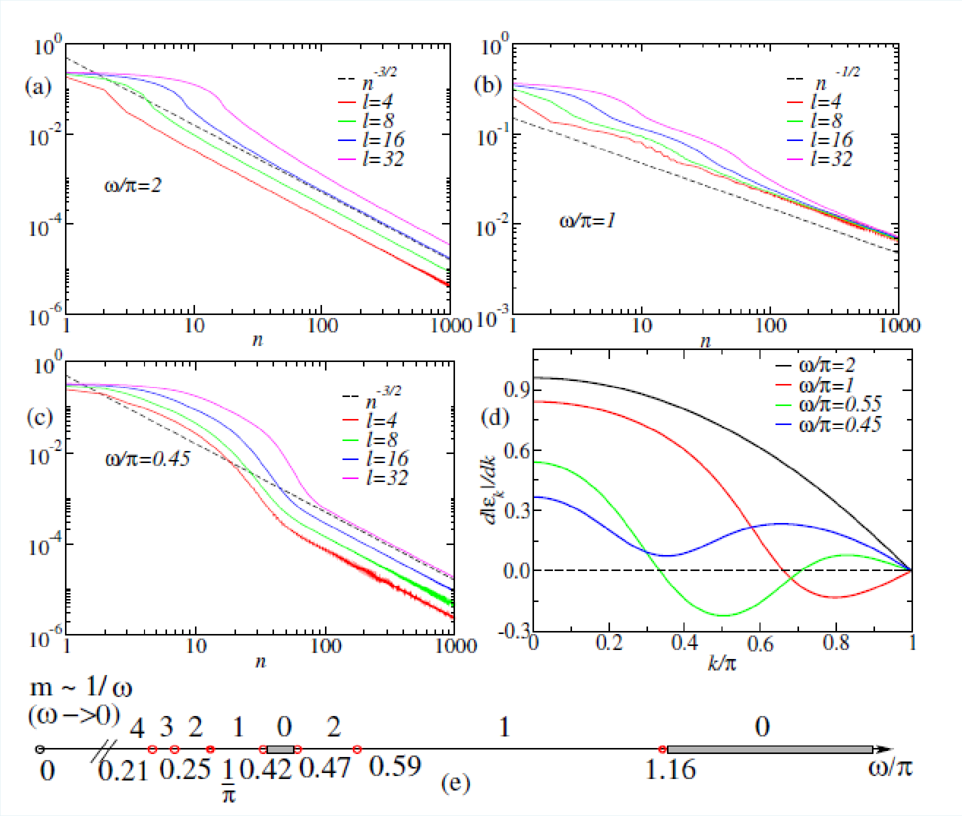
For any other position of k_0, $f(k_0)$ is finite →

$$\cot(\theta_{\vec{k}}) d\alpha_{\vec{k}}/dk_i = -\cot(\alpha_{\vec{k}}) d\theta_{\vec{k}}/dk_i$$

In this case, the correlation functions show a $(\omega/n)^{d/2}$ decay to its steady state value

The position of the saddle point depends on the drive frequency → ***Drive frequency induced dynamic transition between two regimes***

1D Ising model



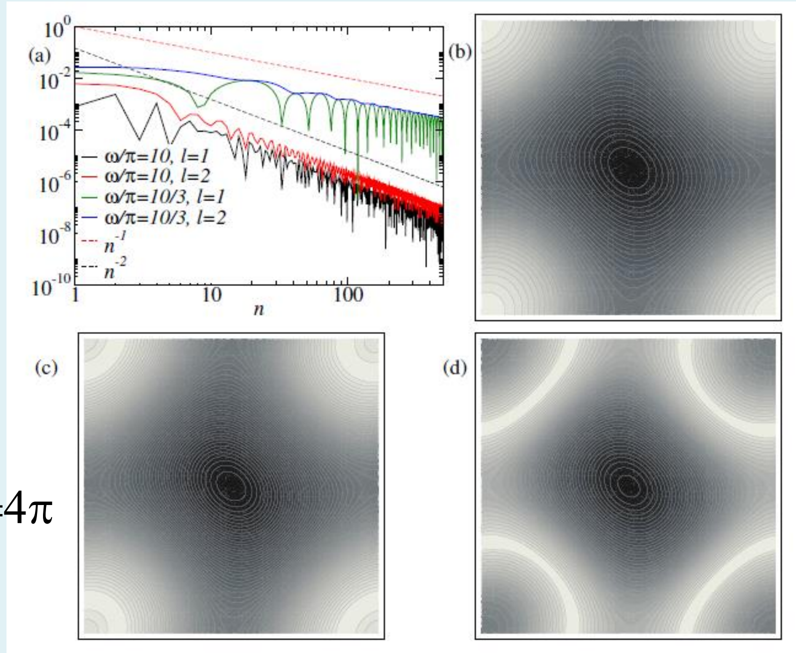
For large ω, H_F is well approximated by time average of H and has its saddle point at k=0,π

As ω is decreased additional zero of d|ε|/dk occurs at k=k_0 at ω= 1.16π leading to a transition

Upon further decrease of ω, the number of such zeroes between k=0,π may revert back to zero leding to re-entrant behavior of the phases

For small ω, the number of zeroes proliferate as 1/ω. --- no transition occurs in this regime.

2D Kitaev model



ω=10 π

ω=4π

ω=3.3 π

A single transition between the two phases at $\omega_c = 4\pi$ with $J_1=J_2=1$

Appearance of a line of zero rather than a single point in the BZ ruling out reentrance

Transition occurs due to appearance of new minima in the spectrum of the Floquet Hamiltonian

Dynamical analog of a first order transition

The number of zeroes can not change continually as a function of $1/\omega$. Thus ω_c is expected to be finite.

Such transitions can not be captured by Magnus or other $1/\omega$ expansion techniques

Reason for line of zeros: Models with special symmetry

Additional symmetry of a class of 2D model $H_{\vec{k}} = h[g_p(k_x) + \alpha_p g_p(k_y); \beta(t)]$

where the time dependent term is independent of k and the functional form of kx and ky are similar

For the Kitaev model $g_1 = \cos(k_i), \; g_2 = \sin(k_i)$

$\beta(t) = J_3(t)/J_1, \text{ and } \alpha_1 = \alpha_2 = J_2/J_1$

For such Hamiltonians, since $\beta(t)$ is independent of kx and ky, dynamics does not change this symmetry. → *U, H_F and $|\varepsilon|$ shares the same symmetry*

Such a functional form guarantees that if $\partial|\vec{\epsilon}_{\vec{k}}|/\partial k_x = 0$ *so is* $\partial|\vec{\epsilon}_{\vec{k}}|/\partial k_y$. → *Thus one has a line of zeroes in the 2D BZ*

Such models do not show re-entrant behavior since an entire line of zeroes do not generically vanish due to change in ω

A specific protocol and the phase diagram

Consider the following protocol:

$$g(t) = g_0 + g_1 \sum_{n=0}^{\infty} \delta(t - nT),$$

For this protocol, one may obtain an analytic form for the evolution operator U_k

$$U_k(T,0) = e^{-ig_1\tau_3} e^{-iT((g_0-\cos(k))\tau_3+\sin(k)\tau_1)} = \begin{pmatrix} \alpha_k & -\beta_k^* \\ \beta_k & \alpha_k^* \end{pmatrix}$$

$$\alpha_k = e^{-ig_1}(\cos(\Phi_k) - i\,\sin(\Phi_k)\hat{n}_{kz})$$

$$\beta_k = -ie^{-ig_1}\hat{n}_{kx}\sin(\Phi_k),$$

$$\epsilon_k = \sqrt{(g_0 - \cos(k))^2 + (\sin(k))^2},$$

$$\hat{n}_{kz} = (g_0 - \cos(k))/\epsilon_k, \quad \Phi_k = T\epsilon_k.$$

$$\hat{n}_{kx} = \sin(k)/\epsilon_k,$$

This allows us to obtain the Floquet spectrum as

$$\alpha_{kF} = \frac{1}{T}\arccos[\cos(\Phi_k + g_1) + (1 - \hat{n}_{kz})\sin(\Phi_k)].$$

For large g_0 one gets n_{kz} ~ 1 and thus one gets

$$\alpha_{kF} = \epsilon_k + \frac{g_1}{T} - \frac{\sin^2 k\,\sin(\Phi_k)\sin(g_1)}{2T(g_0 - \cos(k))^2|\sin(\Phi_k + g_1)|}.$$

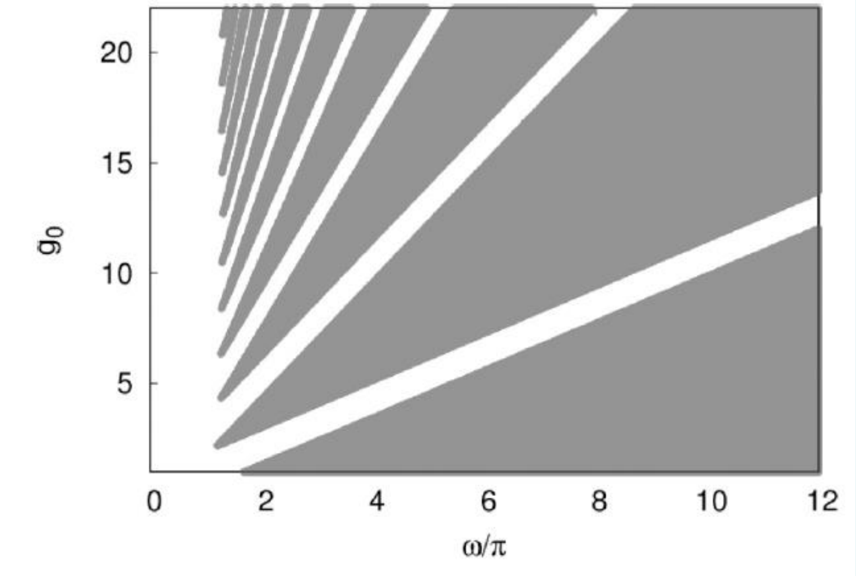
The position of new zeroes occur when

$$\frac{g_0}{\epsilon_k}\left(1-\frac{\sin^2 g_1 \sin^2 k \,\mathrm{sgn}(\sin(\Phi_k+g_1))}{2g_0^2\sin^2(\Phi_k+g_1)}\right) = \frac{\cos k \sin\Phi_k \sin g_1}{g_0^2 T|\sin(\Phi_k+g_1)|},$$

This relation can only be satisfied, for large g0, in a narrow region around the point $(\Phi_k+g_1)=m\pi$ *where* $g_0|\sin(\Phi_k+g_1)|\sim 1.$

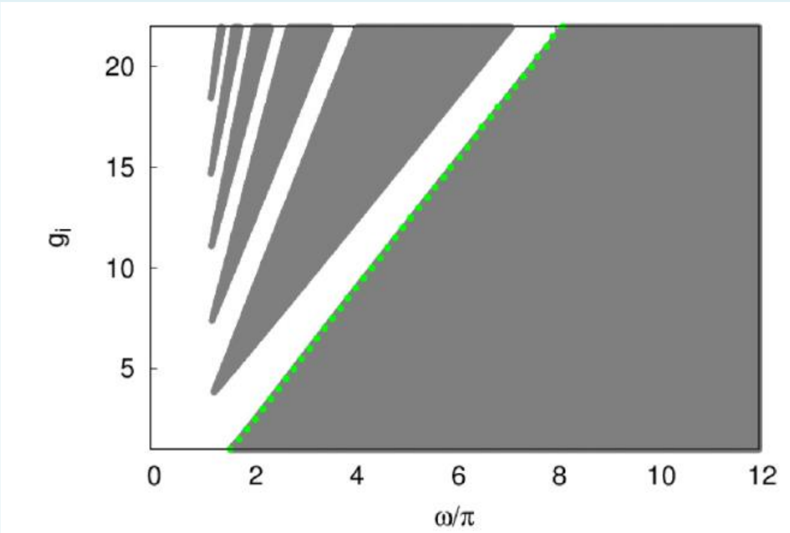
The density of reentrant regions increases with g_0 for large g_0

$1/g_0$ acts as a suitable expansion parameter for obtaining analytic results

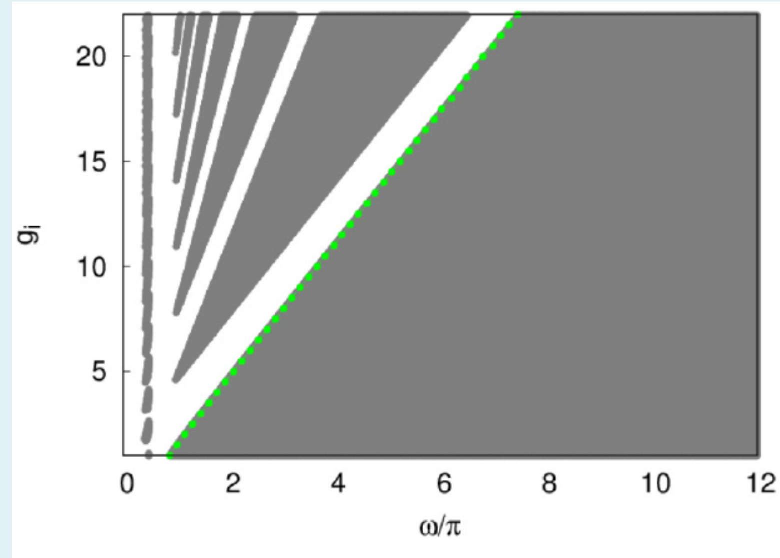


Delta function kick with g_1=1

Phase diagram for square pulse



Square pulse g_f=0



Square pulse g_f=2

$$|\epsilon_{\vec{k}}| = \arccos(M_{\vec{k}})/T$$

$$M_{\vec{k}} = \cos(\Phi_{\vec{k}i})\cos(\Phi_{\vec{k}f}) - \hat{N}_{\vec{k}i} \cdot \hat{N}_{\vec{k}f} \sin(\Phi_{\vec{k}i})\sin(\Phi_{\vec{k}f}),$$

$$\Phi_{\vec{k}i(f)} = E_{\vec{k}i(f)}T/2 \text{ with } E_{\vec{k}i(f)} = \sqrt{(g_{i(f)} - b_{\vec{k}})^2 + \Delta_{\vec{k}}^2}$$

$$\hat{N}_{\vec{k}i(f)} = \left(\frac{\Delta_{\vec{k}}}{E_{\vec{k}i(f)}}, 0, \frac{g_{i(f)} - b_{\vec{k}}}{E_{\vec{k}i(f)}}\right).$$

Floquet spectrum for square pulse protocol which leads to the phase diagrams shown above.

Conclusion and Future Directions

1. *There exist two dynamical regimes for relaxation of correlation functions in periodically driven many-body systems,*

2. *These two regimes are separated by a dynamic transition; they shall show up in any local correlations such as magnetization of the Ising model.*

3. *This transition can be thought as dynamic analog of first order phase transitions.*

4. *Periodically drive integrable models provide route to generation of states with non area-law entanglement entropy.*

5. *Recent experiments have measured second Renyi entropy for ultracold bosons; similar experiments, suitably modified, may verify some of the theoretical predictions.*

6. *Can these be generalized to non-integrable models?*

7. *Can one see effects of integrability breaking on these transitions by suitably tuning model Hamiltonian parameters?*

Diagonal ensemble

To obtain the correlation function in the steady state one needs to compute ψ_f In the limit when n approaches infinity.

$$|\psi_{\vec{k}}(t = nT)\rangle = \exp[-inH_{\vec{k}F}T]|\psi(t = 0)\rangle$$

Thus if $|1_k\rangle$ and $|2_k\rangle$ be the eigenstates of Floquet Hamiltonian, one can write

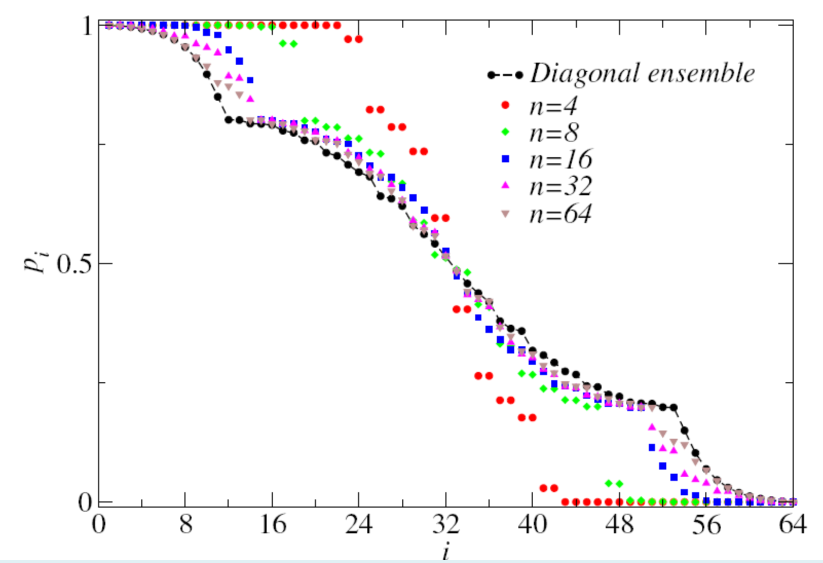
$$\langle\psi_{\vec{k}}(nT)|O_{\vec{k}}|\psi_{\vec{k}}(nT)\rangle = p_{\vec{k}}\langle 1_{\vec{k}}|O_{\vec{k}}|1_{\vec{k}}\rangle + (1 - p_{\vec{k}})\langle 2_{\vec{k}}|O_{\vec{k}}|2_{\vec{k}}\rangle$$

$$p_{\vec{k}} = |\langle 1_{\vec{k}}|\psi_{\vec{k}}(t = 0)\rangle|^2$$

In doing this we have omitted all cross terms due to rapid oscillation of phase factors that originates from the difference in Floquet energy of the two states

For small n, p_i are mostly peaked around 0 and 1 leading to intensive (area-law) entanglement entropy.

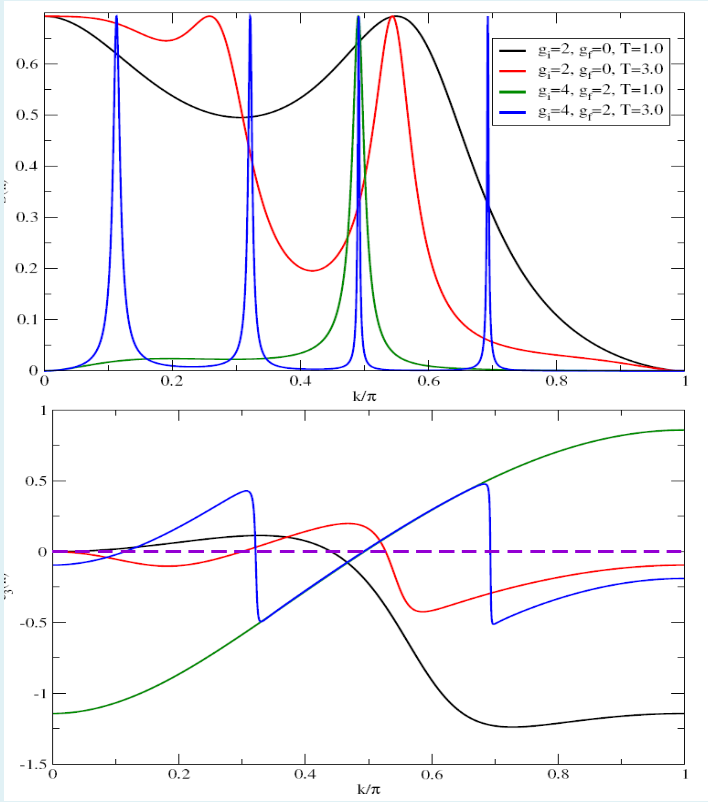
For large n, p_i s spread out with a finite density around ½ leading to extensive (volume law) entropy.

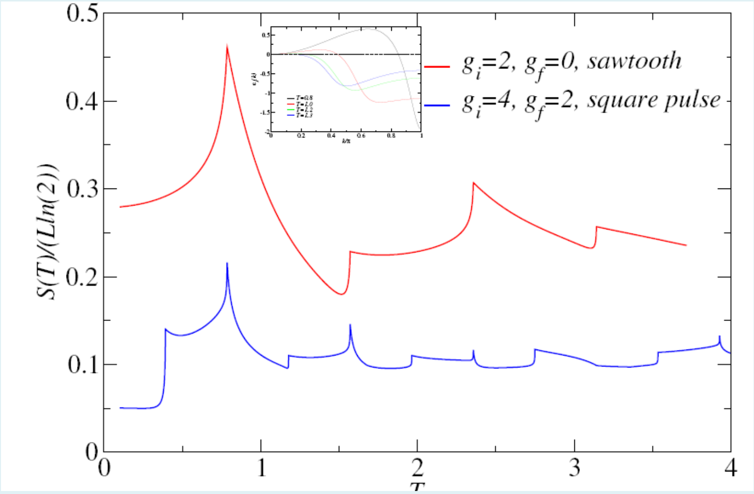


Approach to GGE with n

The steady state entanglement entropy shows a non-monotonic structure as a function of w.

$$\frac{S_{tot}}{L} = \frac{1}{\pi}\int_0^{\pi} S(k)dk$$
$$S(k) = -p(k)\log p(k) - (1-p(k))\log(1-p(k))$$





The number of peaks of S(k) change by unity when ω is varied across special values ω*.

The value of p_k is closest to ½ if ε_{3k} =0. A peak appears in S(k) when this happens.

The appearance of a new k leads to Jump in area under the curve and Hence a jump in S across ω*