Thermodynamics along individual quantum trajectories of a qubit

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Theory collaborators: Eric Lutz, Alessandro Romito, Klaus Mølmer, Andrew Jordan,

Naghiloo et al, Arxiv 2016, 2017

Where in the world is St. Louis?

Experimental research with superconducting qubits.

Quantum Measurement: Zeno effects, quantum trajectories

State smoothing and post-selection: weak values, retrodiction, optimal routes

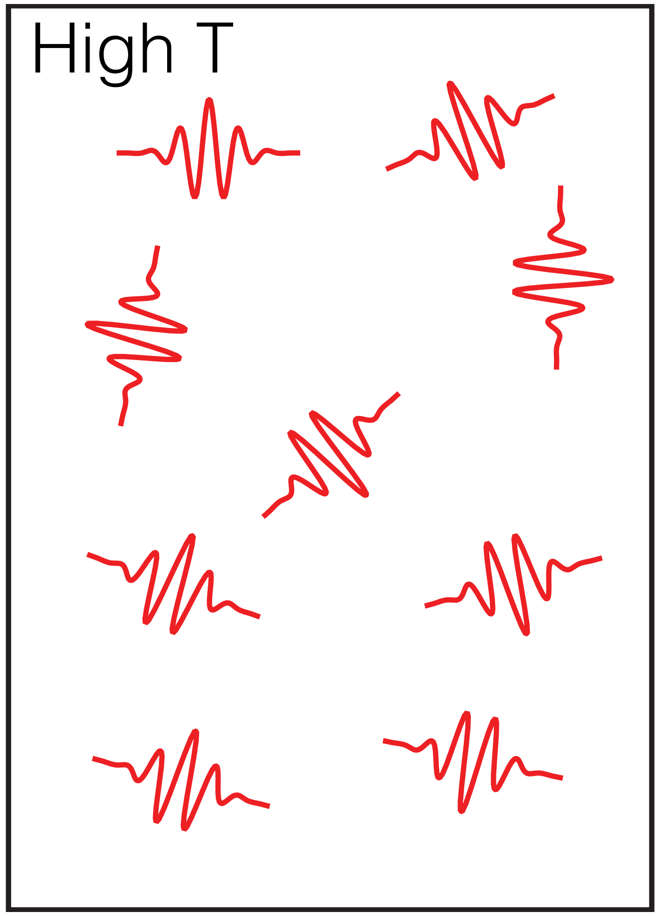
Metrology: frequency metrology, Axion dark matter search

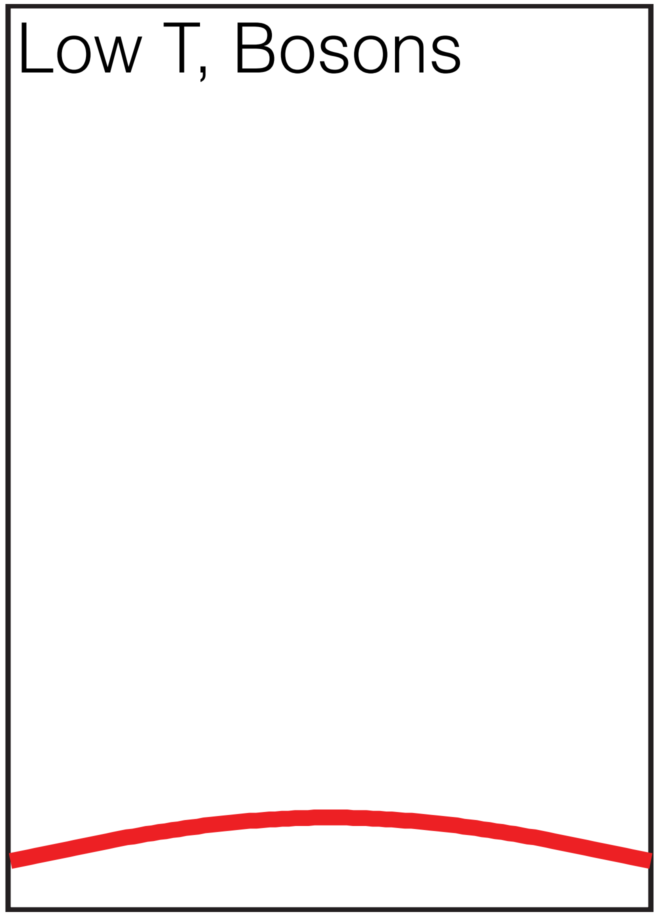
Quantum Thermodynamics: heat, work, entropy, heat engines

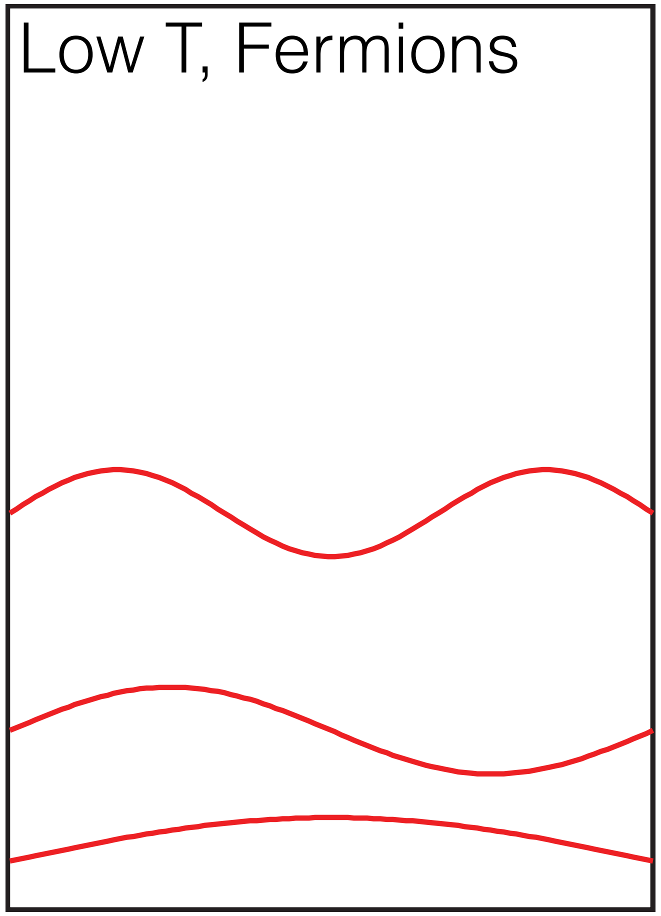
Thermodynamics…
toward the quantum regime

Quantities such as heat and work for macroscopic systems

How do we get fire to do work?



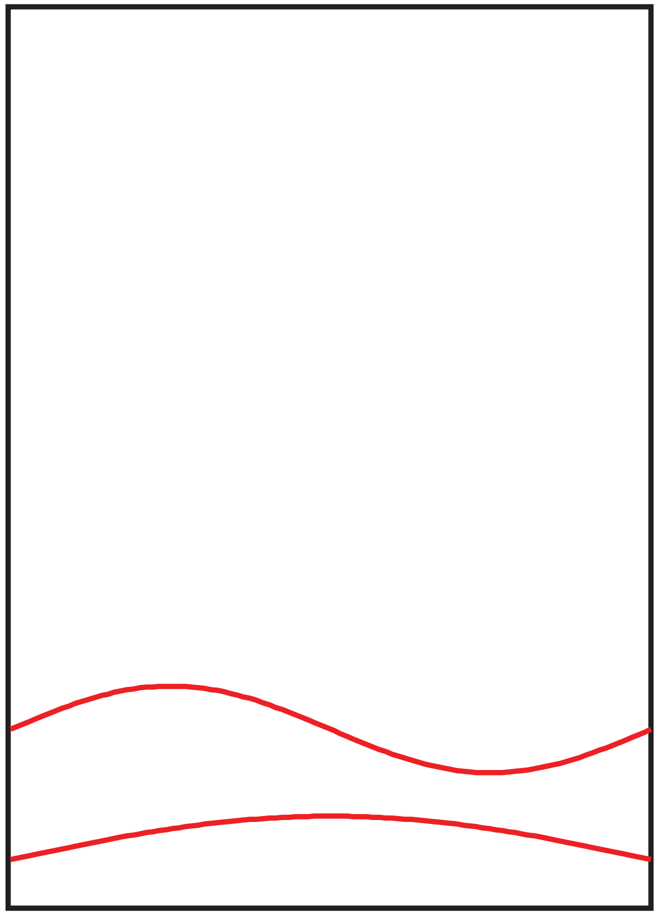




Thermodynamics…
toward the quantum regime

Quantities such as heat and work for macroscopic systems

How do we get fire to do work?



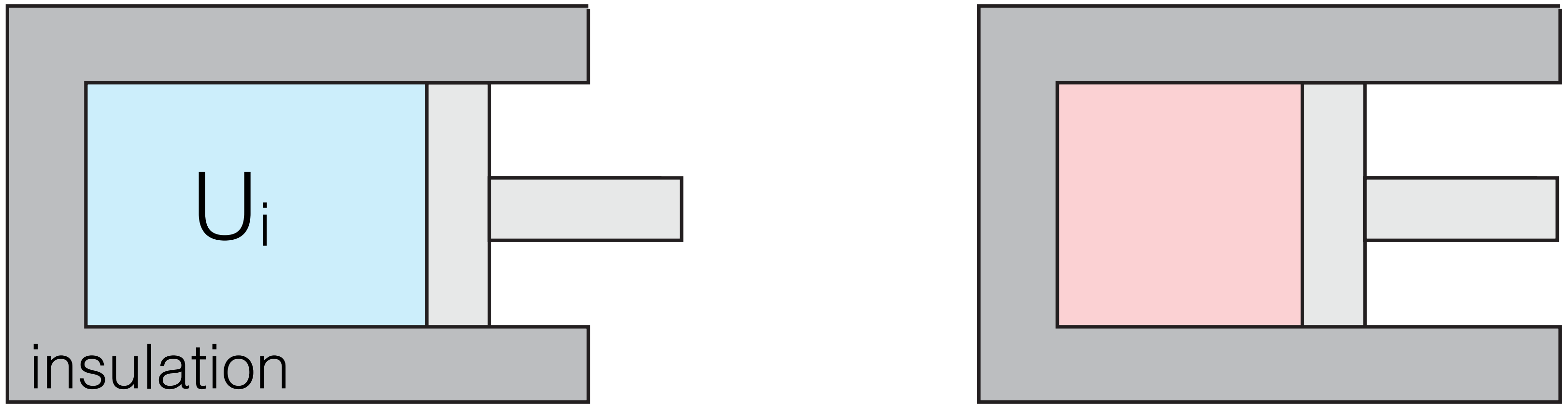
Thermodynamics for a single quantum system

$\psi = \alpha\psi_1 + \beta\psi_2$

Quantify heat and work

Classical Thermodynamics

Work is defined as change in energy of an isolated system



$W = \Delta U$

Non-isolated systems: heat is the difference

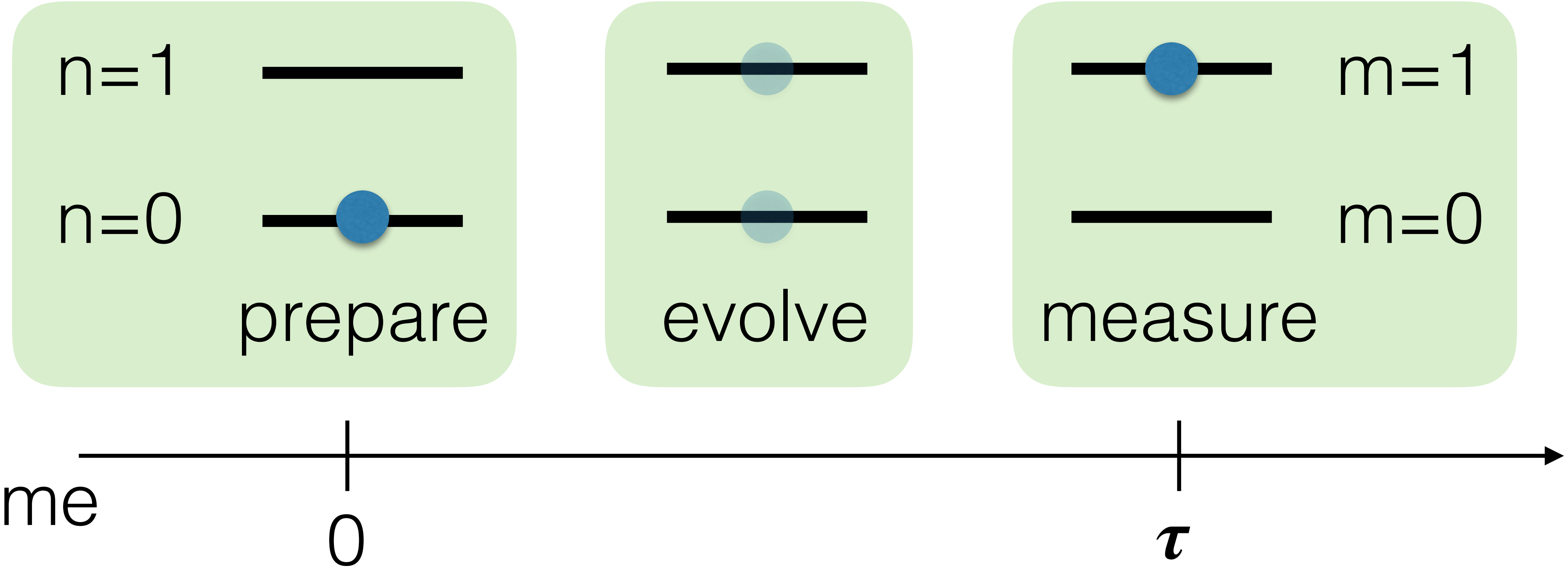
$Q = \Delta U - W$

Thermal isolation is important for distinguishing heat form work

At the quantum level…

Systems need not occupy definite states.

-Distribution of total energy change from transition probabilities



$$P(\Delta U) = \sum_{m,n} P^{\tau}_{m,n} P^{0}_{n} \delta(\Delta U - (E^{\tau}_{m} - E^{0}_{n}))$$

-Verification of fluctuation theorems for closed systems.

Batalhão PRL 2014, An Nature Phys. 2015

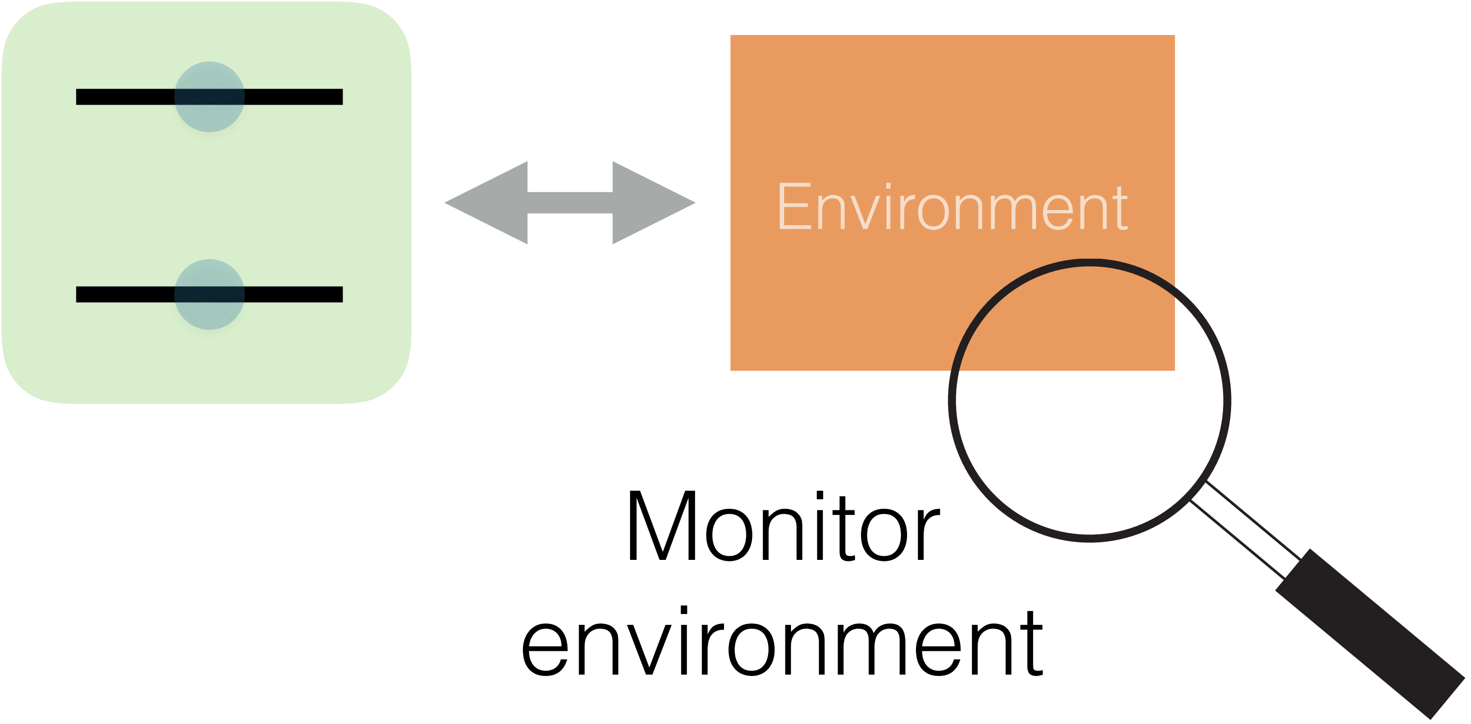
Heat or Work?

Open system: cannot in general separate heat from work

(Work is not an observable)

Talkner PRE 2007

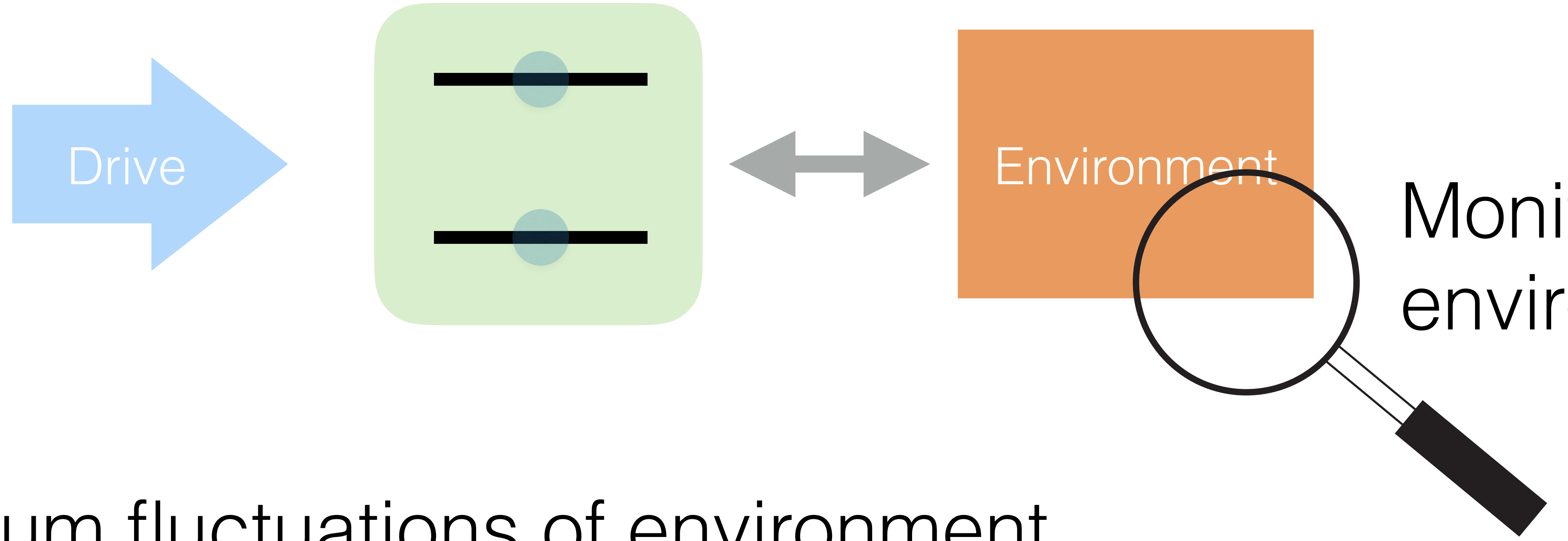
Except when the environment can be monitored



-Environment is weakly entangled with qubit.

-Measurement of environment conveys information and induces back action on qubit. (Not necessarily projection into eigenstates)

Quantum trajectories

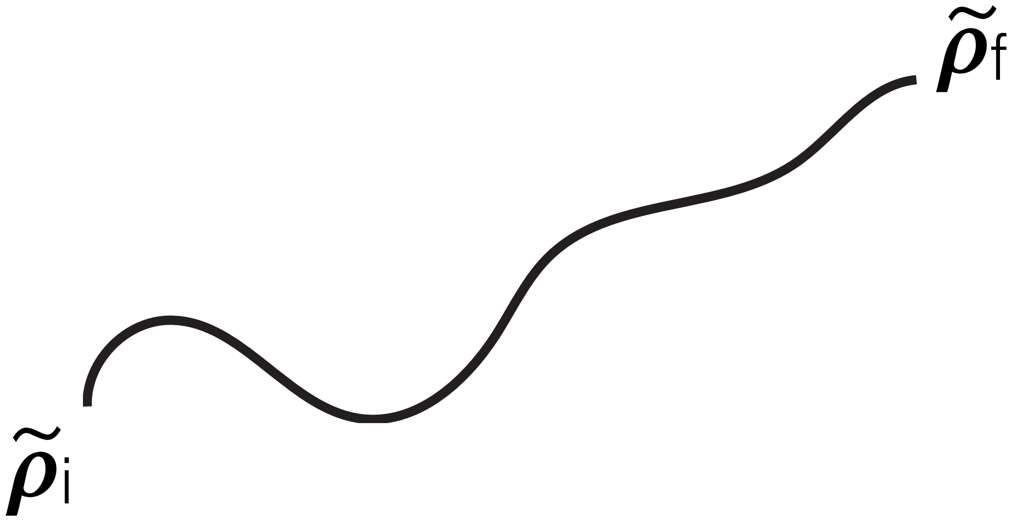


Monitor environment

-Quantum fluctuations of environment

→ stochastic evolution + unitary evolution from drive

Quantum trajectory of state $\tilde{\rho}$(t), even for a single experimental protocol.



$$d\tilde{\rho}_t = \delta\mathbb{W}[\tilde{\rho}_t]dt + \delta\mathbb{Q}[\tilde{\rho}_t]dt$$

Decompose trajectory into unitary and non-unitary components

Alonso PRL 2016

Theory

$$d\tilde{U}_t = \mathrm{tr}[H_t\tilde{\rho}_t] - \mathrm{tr}[H_{t-dt}\tilde{\rho}_{t-dt}]$$

$$= \mathrm{tr}[\tilde{\rho}_{t-dt}dH_t] + \mathrm{tr}[H_t\boxed{d\tilde{\rho}_t}]$$

Both stochastic and unitary parts

$$-\frac{i}{\hbar}\mathrm{tr}[H_t[H_t,\rho_t]dt] = 0$$

only stochastic part contributes

$$= \underbrace{\mathrm{tr}[\tilde{\rho}_{t-dt}dH_t]}_{\delta\tilde{W}_t} + \underbrace{\mathrm{tr}[H_t d\tilde{\rho}_t]}_{\delta\tilde{Q}_t}$$

Work is associated with change in Hamiltonian, heat is associated with stochastic changes in state.

Alonso PRL 2016

Goals for this research

- Experimentally determine $\delta\tilde{Q}_t$ and $\delta\tilde{W}_t$ along single quantum trajectories from stochastic and unitary components of $\tilde{\boldsymbol{\rho}}$(t).

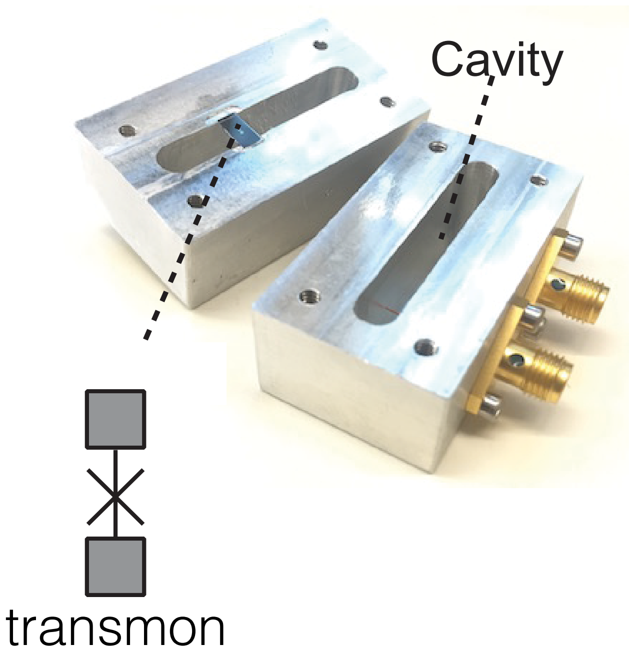
- Verify the 1st law. $$\Delta U = \int_0^\tau \delta\tilde{W} + \int_0^\tau \delta\tilde{Q}$$

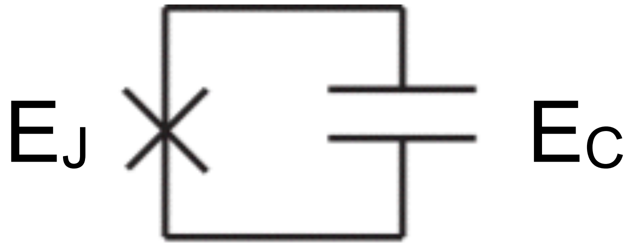
- Quantum feedback loop eliminate heat by applying additional work.

- Verify Jarzynski equality from transition probabilities.

The experimental system

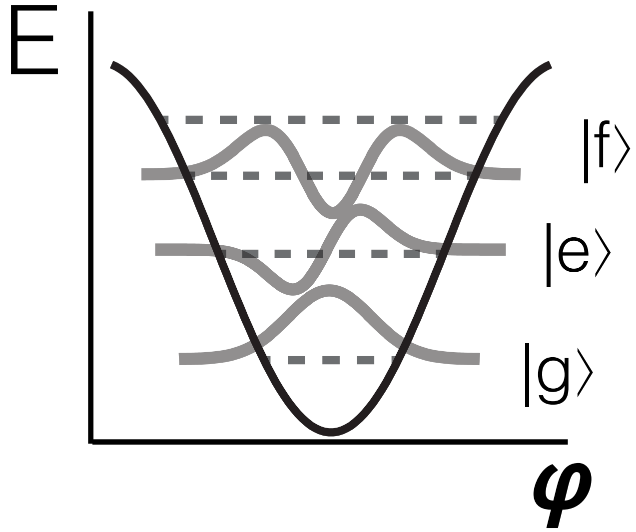
Transmon qubit resonantly coupled to a waveguide cavity

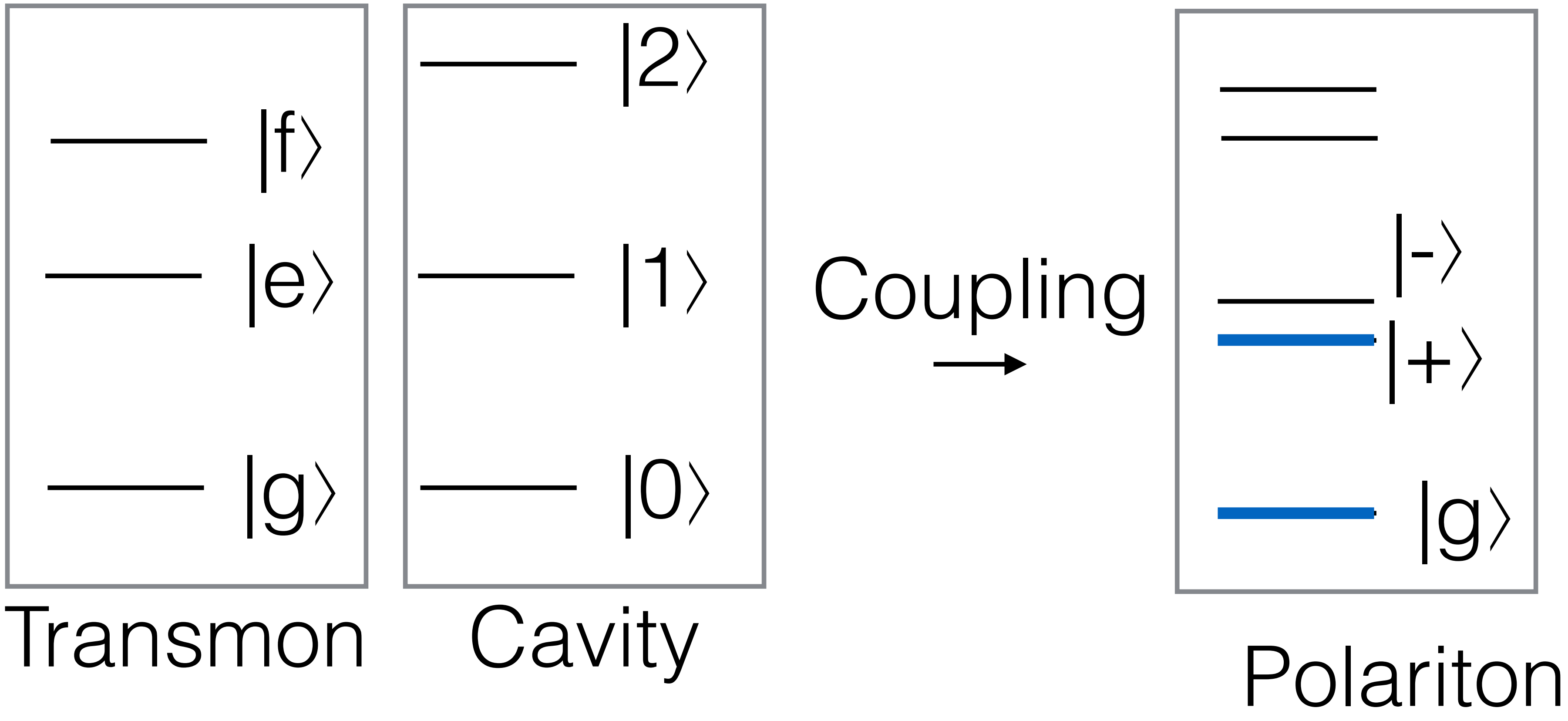




$E_J >> E_C$

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos \hat{\varphi}.$$

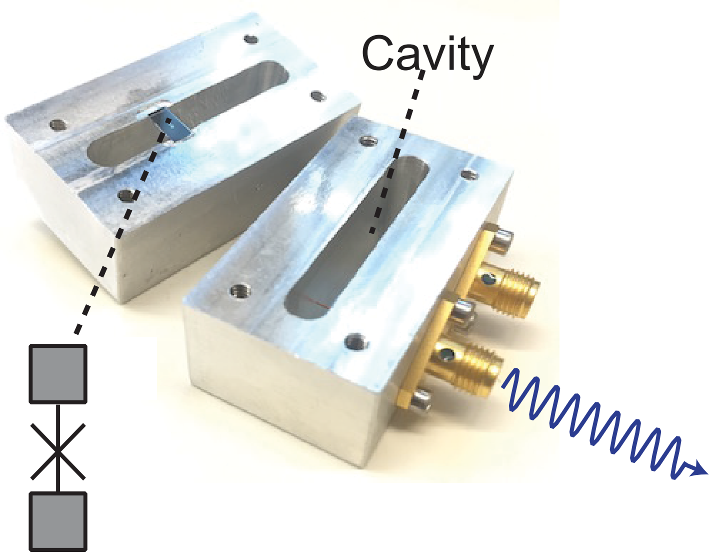


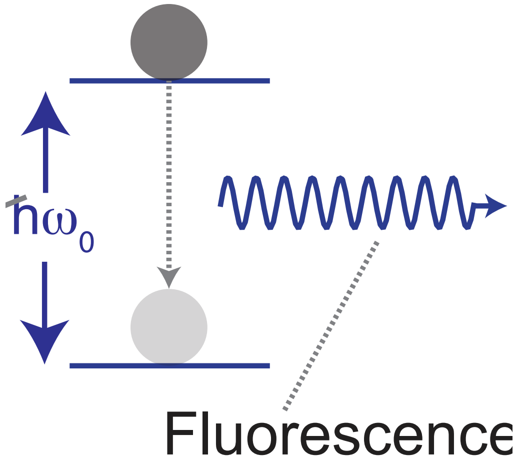


Koch PRB 2007, Paik PRL 2011

The experimental system

Transmon qubit resonantly coupled to a waveguide cavity





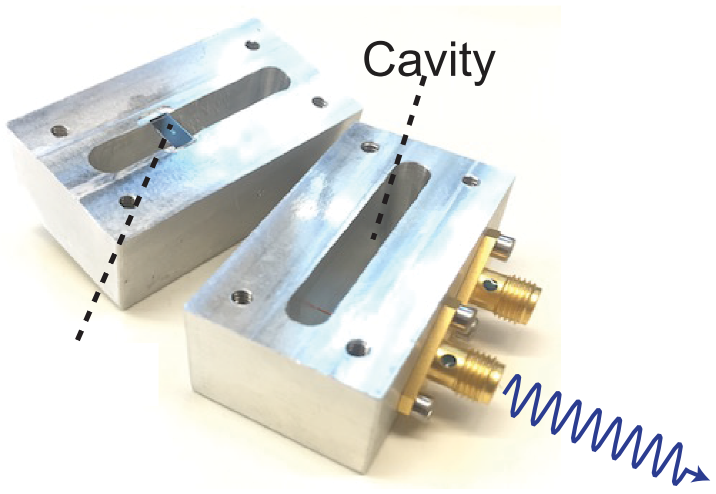
$\omega/2\pi$ = 6.541 GHz
T_1 = 590 ns

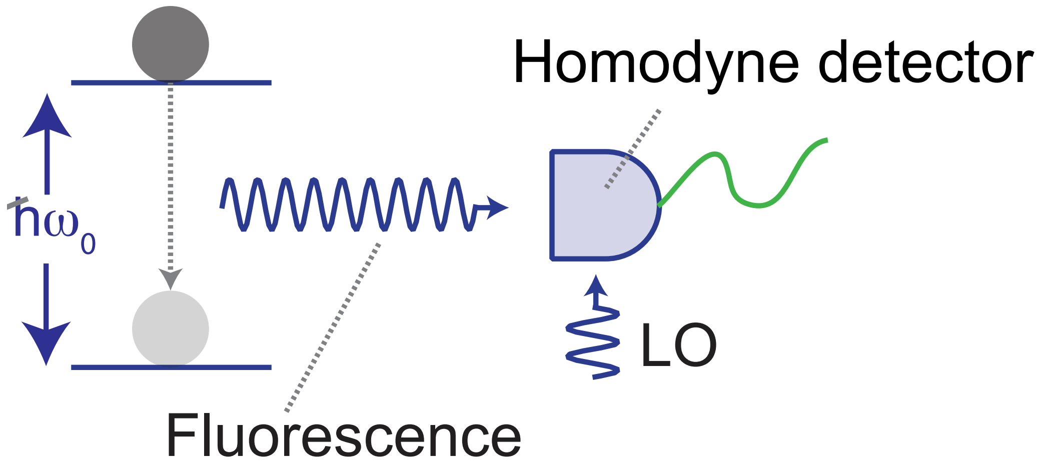
"One dimensional" atom

Murch Nature 2013

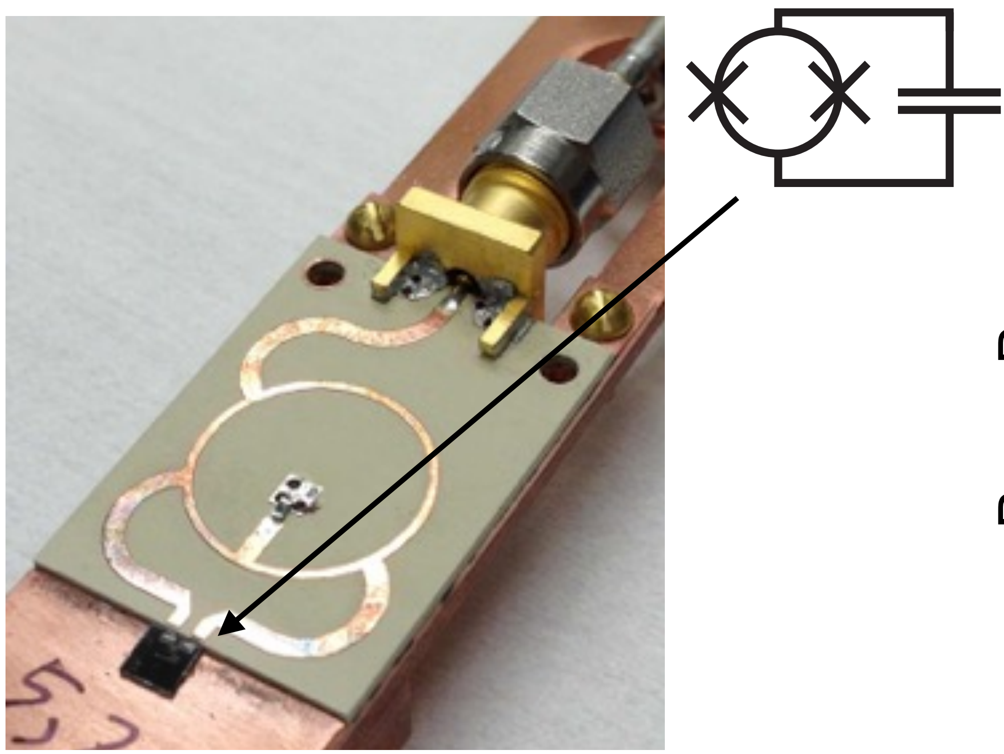
The experimental system

Transmon qubit resonantly coupled to a waveguide cavity

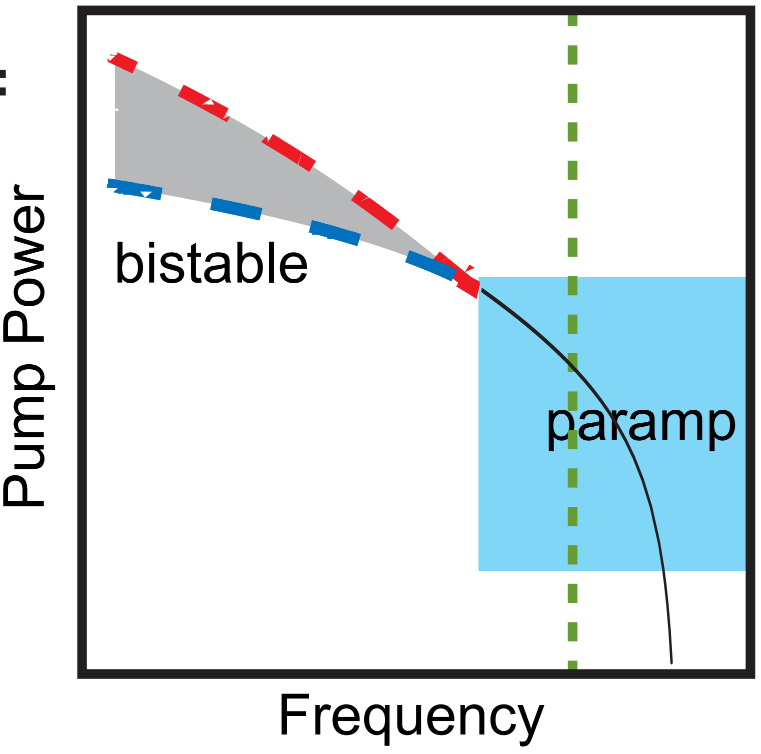




Homodyne detector: Josephson parametric amplifier



JILA, Berkeley, Yale,
Saclay, WU, MIT…

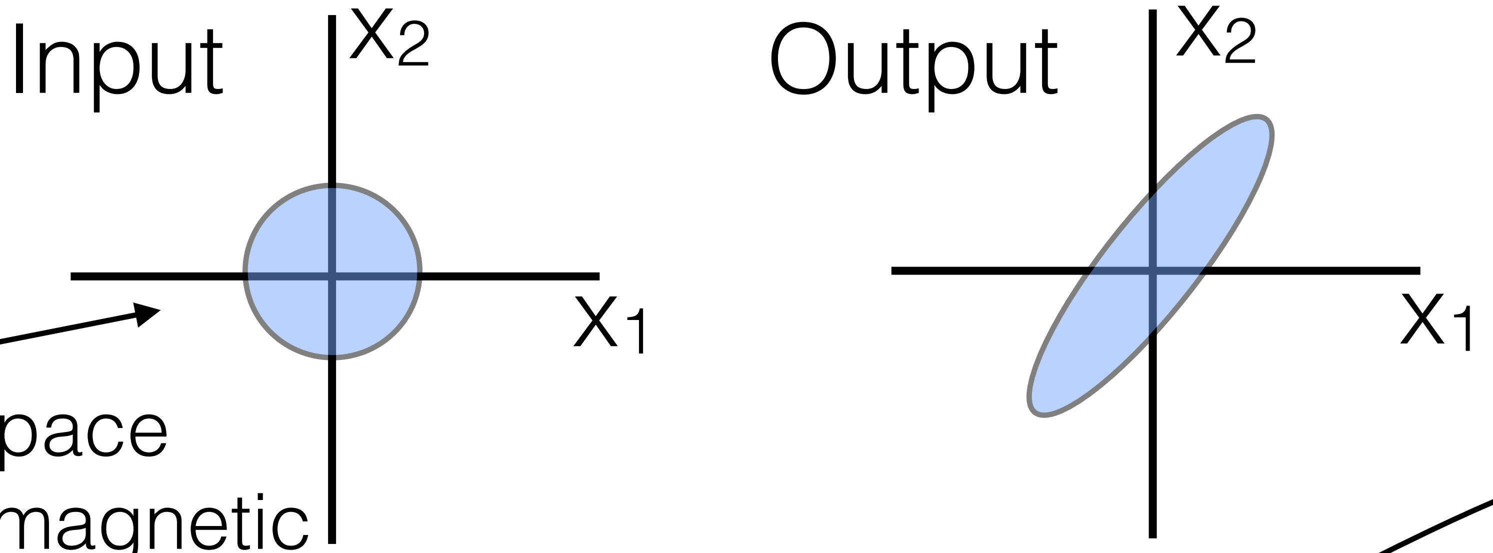


When Pump freq. = signal freq. we get “phase sensitive amplification”, (squeezing).

Ideal, noiseless homodyne detection

Murch Nature 2013

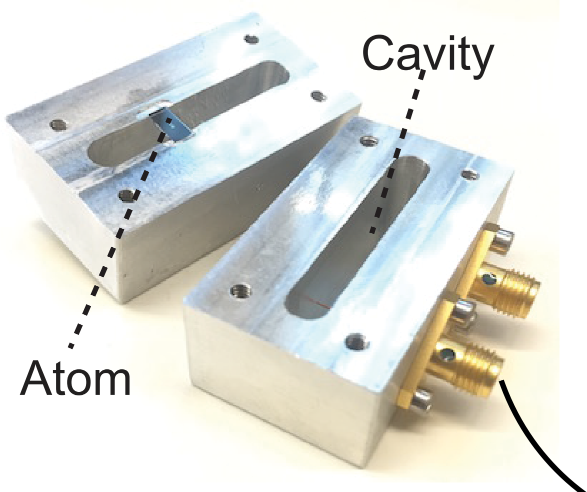
Phase sensitive amplification

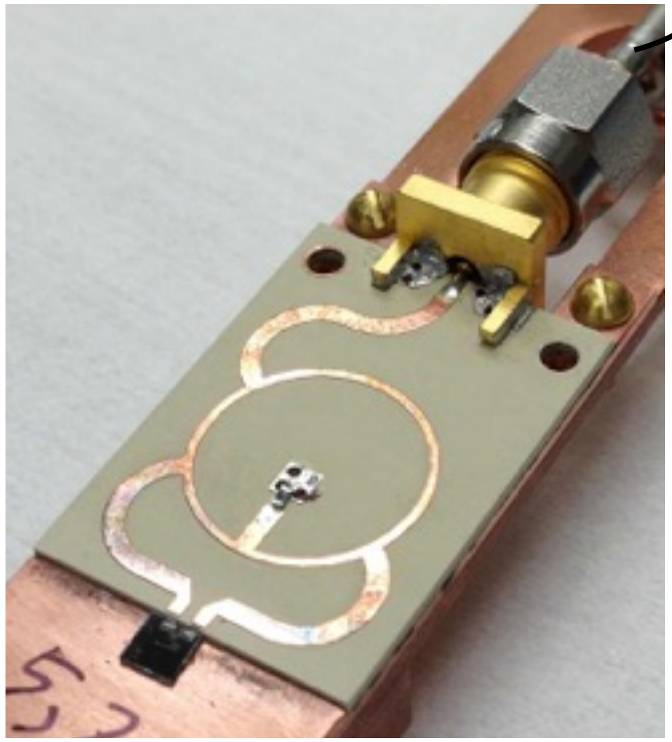


Measure an arbitrary field quadrature.

$$a^\dagger e^{i\phi} + a e^{-i\phi}$$

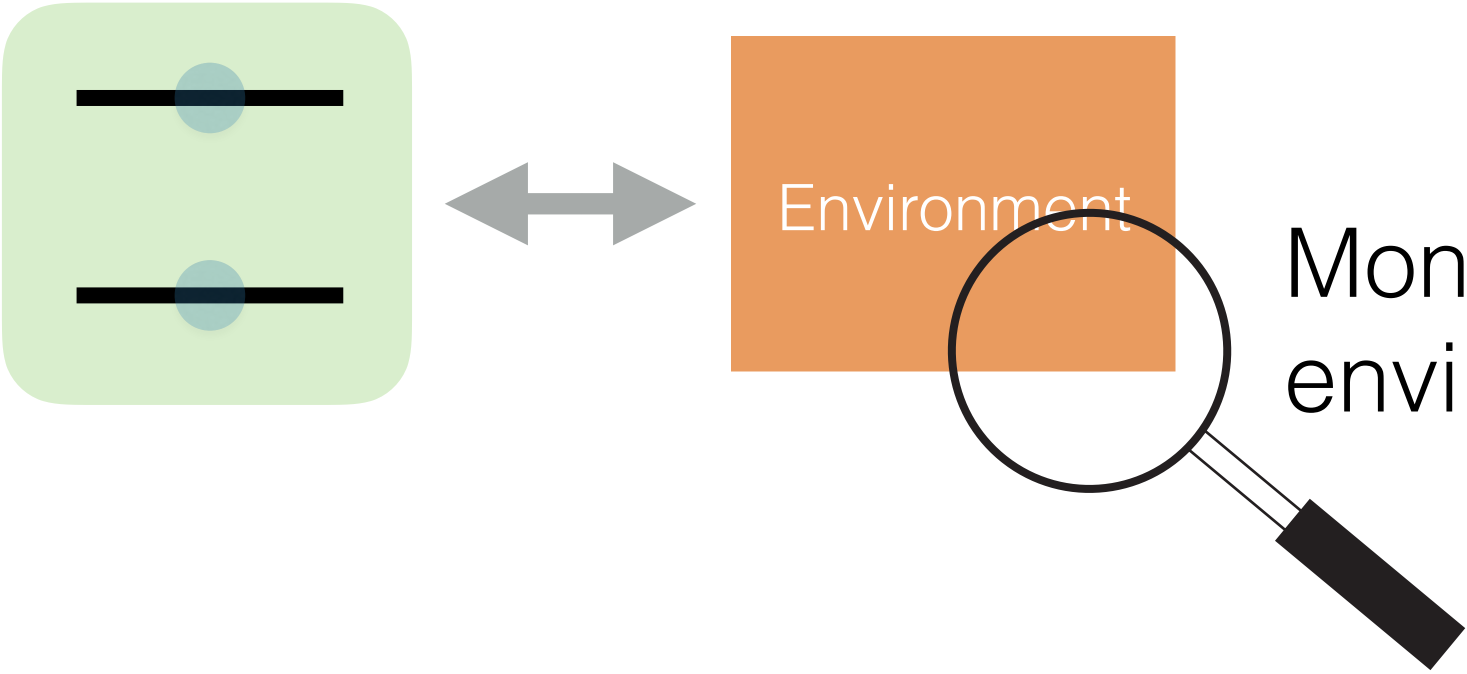
Quadrature space of the electromagnetic field





Naghiloo Nat. Com. 2013

Homodyne measurement of fluorescence



Monitor environment

Interaction Hamiltonian: $H_{\text{int}} = \gamma(a^\dagger \sigma_- + a\sigma_+)$ couples an arbitrary field quadrature, $a^\dagger e^{i\phi} + ae^{-i\phi}$ to the emitter dipole, $\sigma_- e^{i\phi} + \sigma_+ e^{-i\phi}$. If we set $\phi = 0$ then the homodyne signal is proportional to $\sigma_- + \sigma_+ = \sigma_x$.

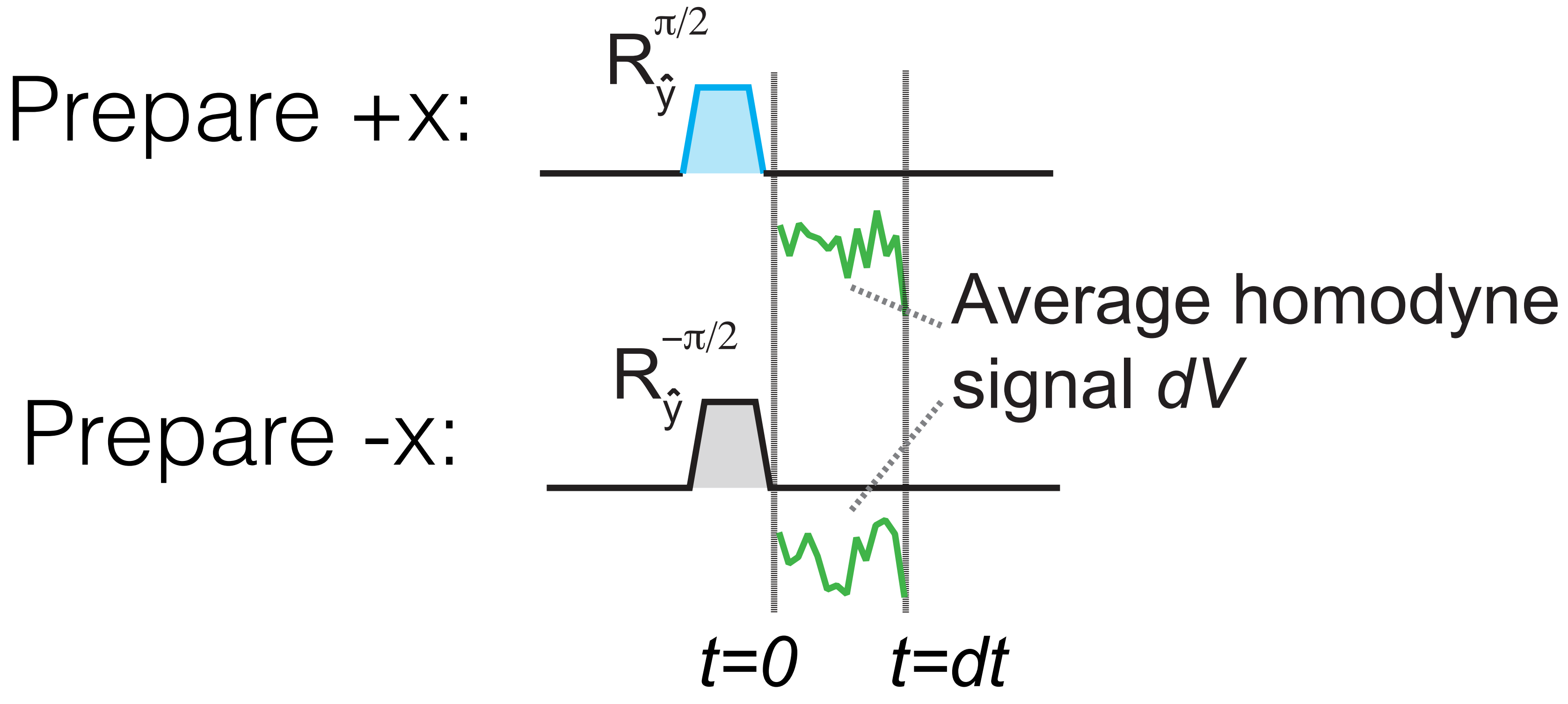
All told, our signal is:

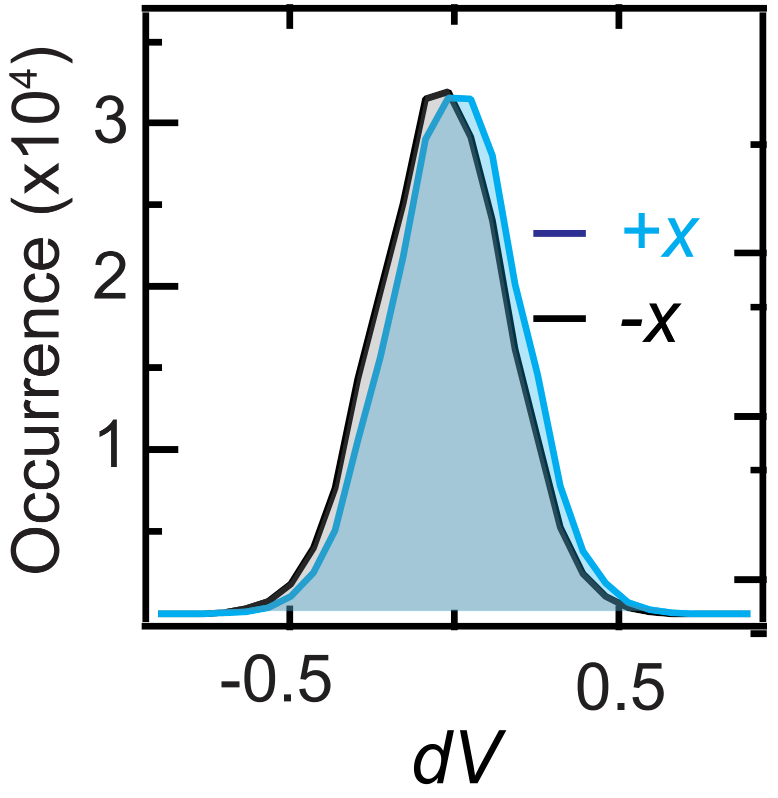
$$dV_t = \sqrt{\eta}\gamma\langle\sigma_x\rangle dt + \sqrt{\gamma}dW_t$$

proporal to ⟨σx⟩

+ zero mean white noise

A SIMPLE EXPERIMENT TO CHECK:



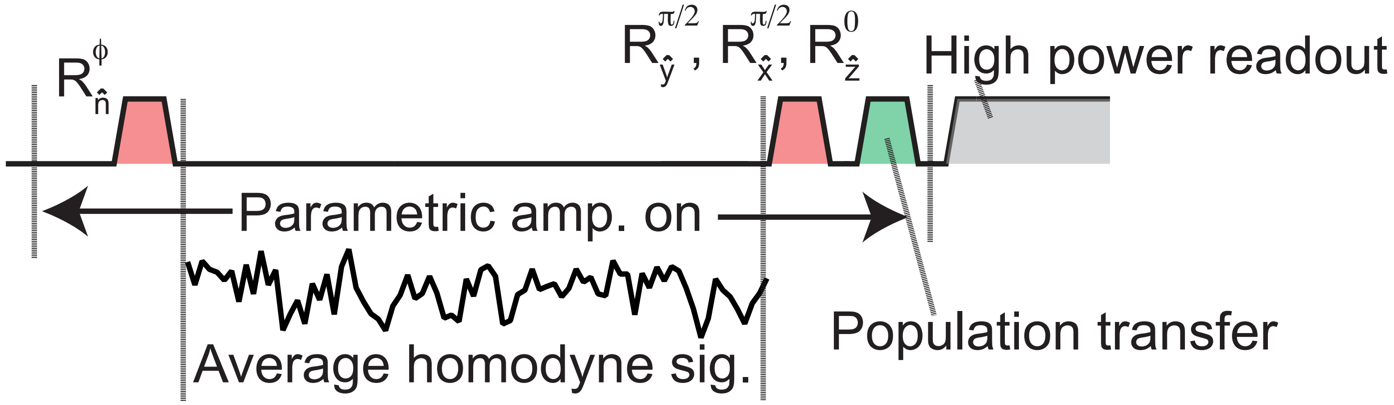


(the signal is dominated by noise)

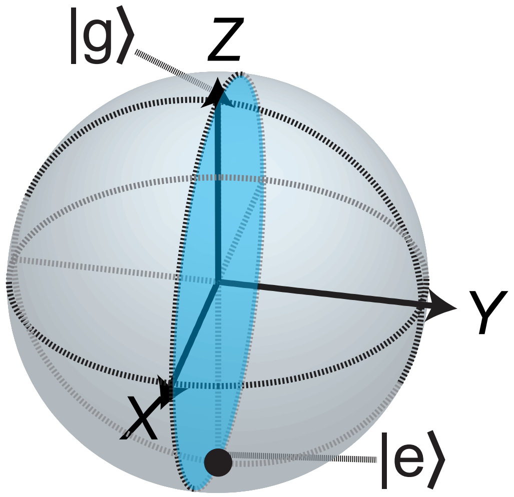
$$dV_t = \sqrt{\eta}\gamma\langle\sigma_x\rangle dt + \sqrt{\gamma}dW_t$$

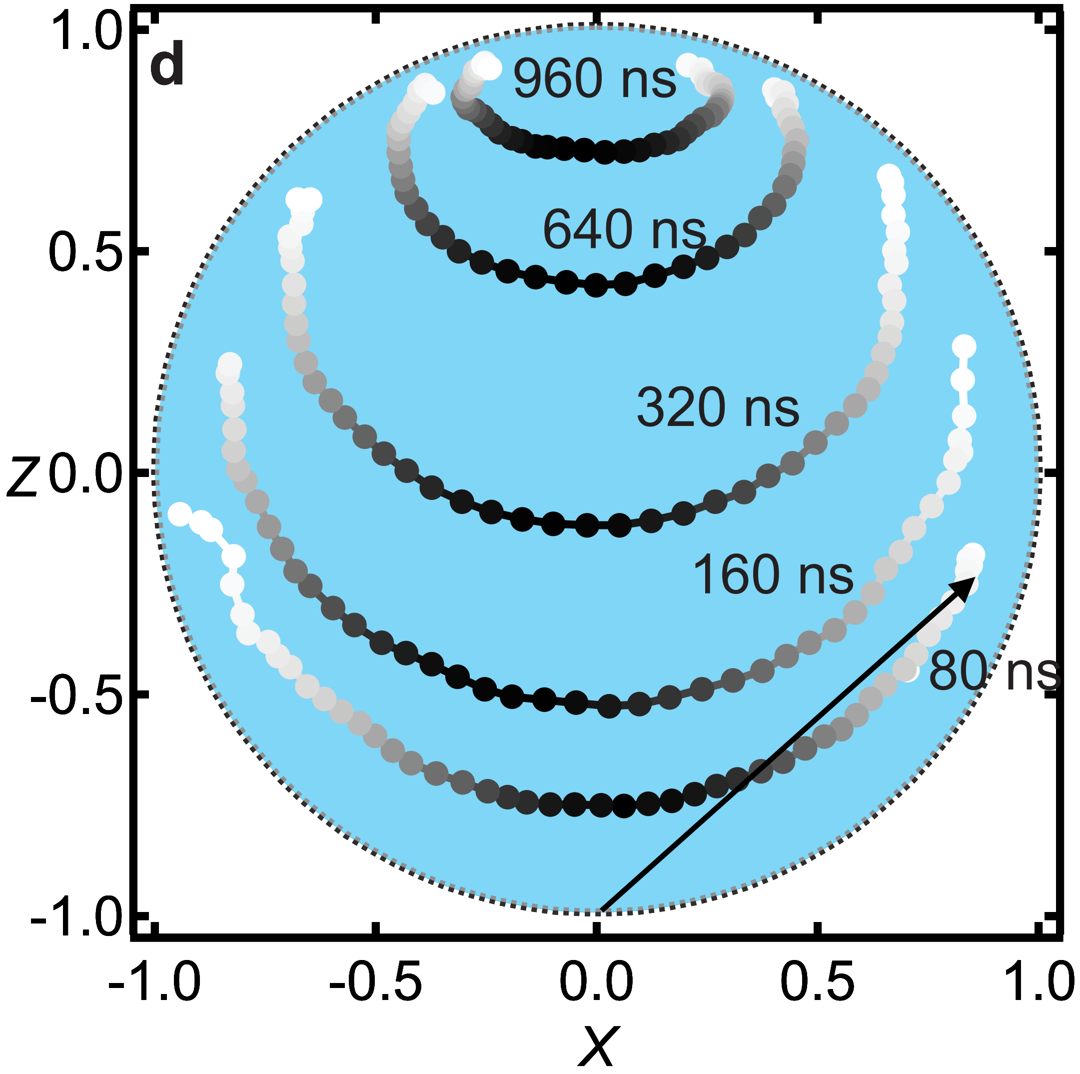
“Weak measurement”

DECAY DYNAMICS



repeat several times,average tomography conditioned on average signal





“quantum smiley”

STOCHASTIC MASTER EQUATION

Open up a textbook….

$$\mathrm{d}\rho = \gamma\mathcal{D}[\sigma_-]\rho\mathrm{d}t + \sqrt{\eta\gamma}\mathcal{H}[\sigma_-\mathrm{d}W_t]\rho$$

“dissipation” and “jump” superoperators

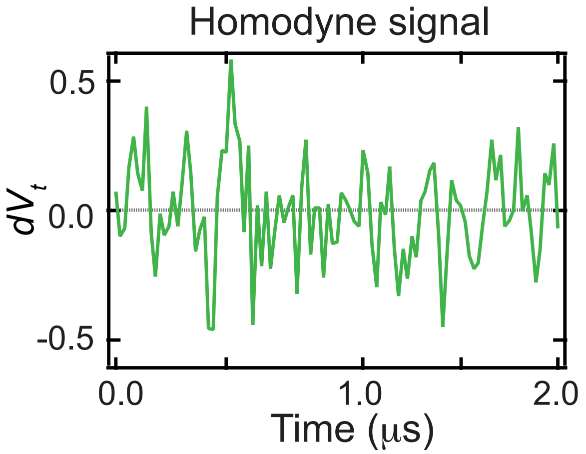
Stochastic differential equations:

$$\mathrm{d}x = -\frac{\gamma}{2}x\mathrm{d}t + \sqrt{\eta}\left(1 - z - x^2\right)\left(\mathrm{d}V_t - \gamma\sqrt{\eta}x\mathrm{d}t\right),$$

$$\mathrm{d}z = \gamma(1 - z)\mathrm{d}t + \sqrt{\eta}x(1 - z)\left(\mathrm{d}V_t - \gamma\sqrt{\eta}x\mathrm{d}t\right),$$

$$\mathrm{d}y = -\frac{\gamma}{2}y\mathrm{d}t - \sqrt{\eta}xy\left(\mathrm{d}V_t - \gamma\sqrt{\eta}x\mathrm{d}t\right).$$

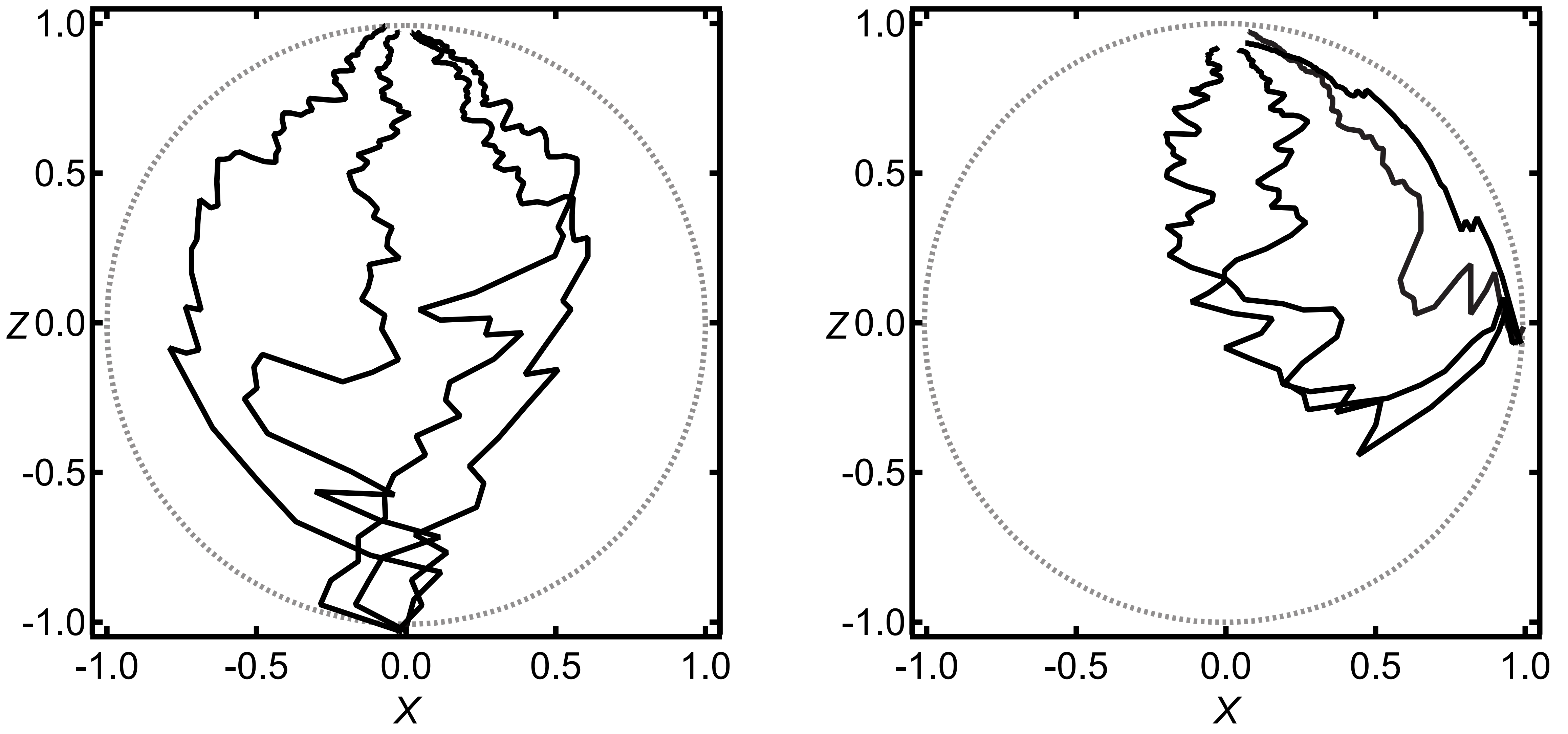
STOCHASTIC MASTER EQUATION



→

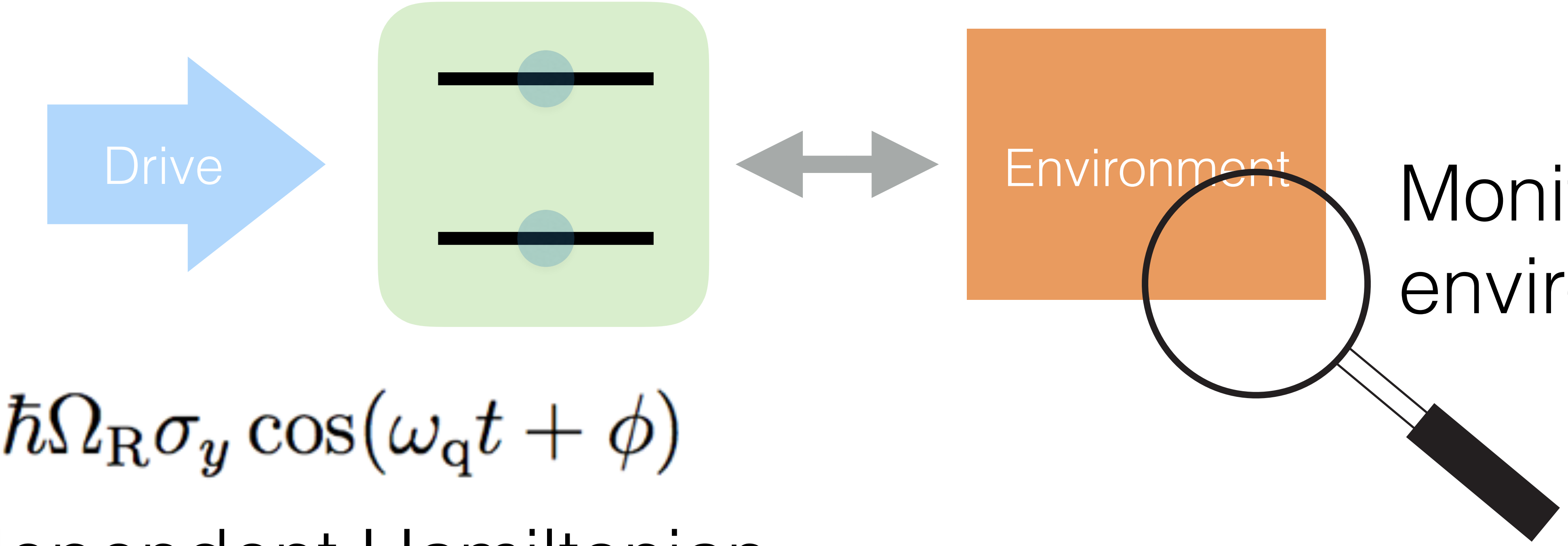
$$\begin{aligned}
\mathrm{d}x &= -\frac{\gamma}{2}x\mathrm{d}t + \sqrt{\eta}\left(1 - z - x^2\right)\left(\mathrm{d}V_t - \gamma\sqrt{\eta}x\mathrm{d}t\right),\\
\mathrm{d}z &= \gamma(1 - z)\mathrm{d}t + \sqrt{\eta}x(1 - z)\left(\mathrm{d}V_t - \gamma\sqrt{\eta}x\mathrm{d}t\right),\\
\mathrm{d}y &= -\frac{\gamma}{2}y\mathrm{d}t - \sqrt{\eta}xy\left(\mathrm{d}V_t - \gamma\sqrt{\eta}x\mathrm{d}t\right).
\end{aligned}$$

→ (x(t), y(t), z(t))



M. Naghiloo, …, KM, Nat. Comm. 2016, P. Campagne-Ibarcq et al PRX 2016

Resonance fluorescence



Monitor environment

$$H_{\mathrm{R}} = \hbar\Omega_{\mathrm{R}}\sigma_y\cos(\omega_{\mathrm{q}}t + \phi)$$

Time dependent Hamiltonian

Stochastic master equation:

$$d\tilde{\rho}_t = \underbrace{-\frac{i}{\hbar}[H_R, \tilde{\rho}_t]\,dt}_{\delta\mathbb{W}[\tilde{\rho}_t]} + \underbrace{\gamma\mathcal{D}[\sigma_-]\tilde{\rho}_t\,dt + \sqrt{\eta\gamma}\mathcal{H}[\sigma_- dX_t]\tilde{\rho}_t}_{\delta\mathbb{Q}[\tilde{\rho}_t]}$$

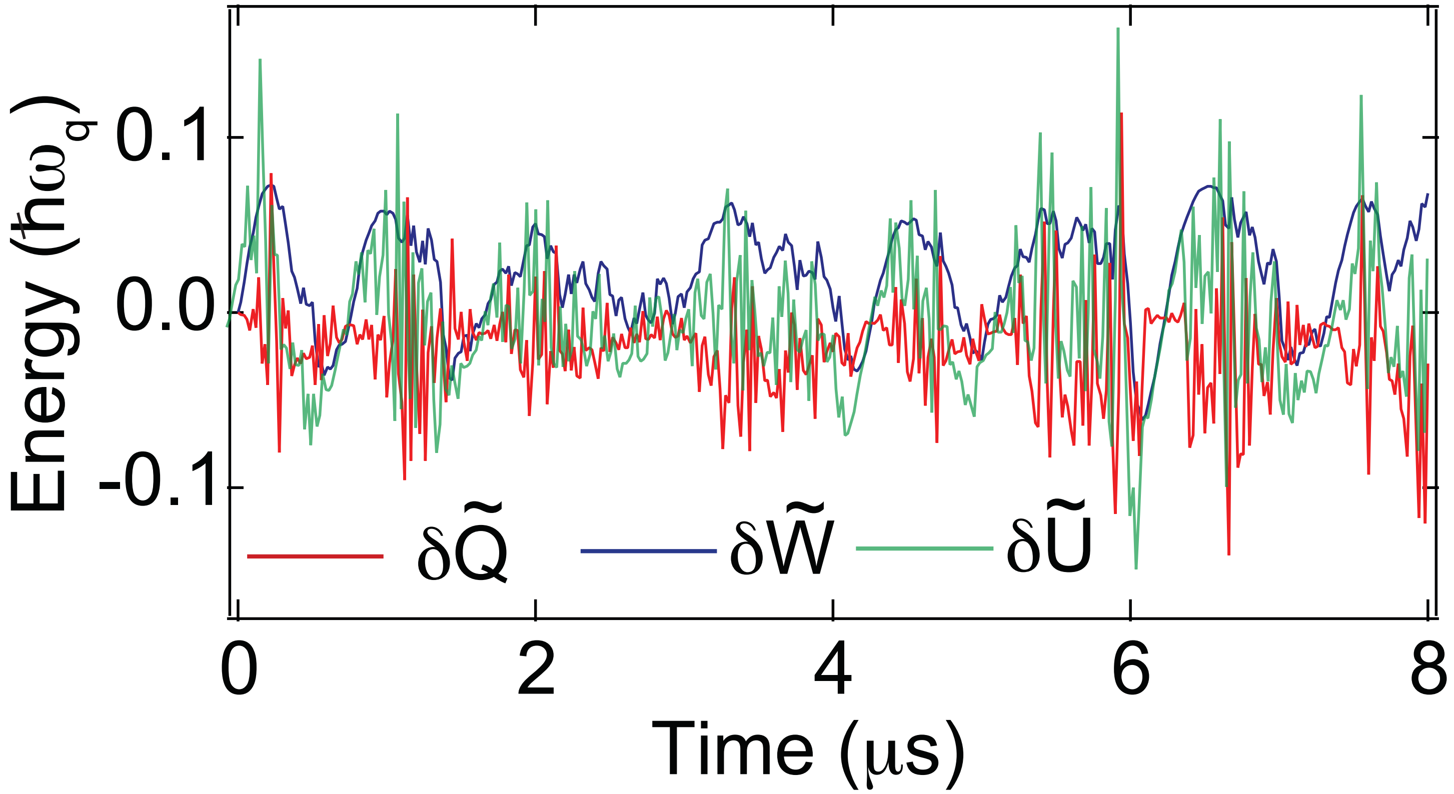
Naghiloo arXiv 2016

Heat and work

$$d\tilde{\rho}_t = \underbrace{-\frac{i}{\hbar}[H_R, \tilde{\rho}_t]\,dt}_{\delta\mathbb{W}[\tilde{\rho}_t]} + \underbrace{\gamma\mathcal{D}[\sigma_-]\tilde{\rho}_t\,dt + \sqrt{\eta\gamma}\mathcal{H}[\sigma_- dX_t]\tilde{\rho}_t}_{\delta\mathbb{Q}[\tilde{\rho}_t]}$$

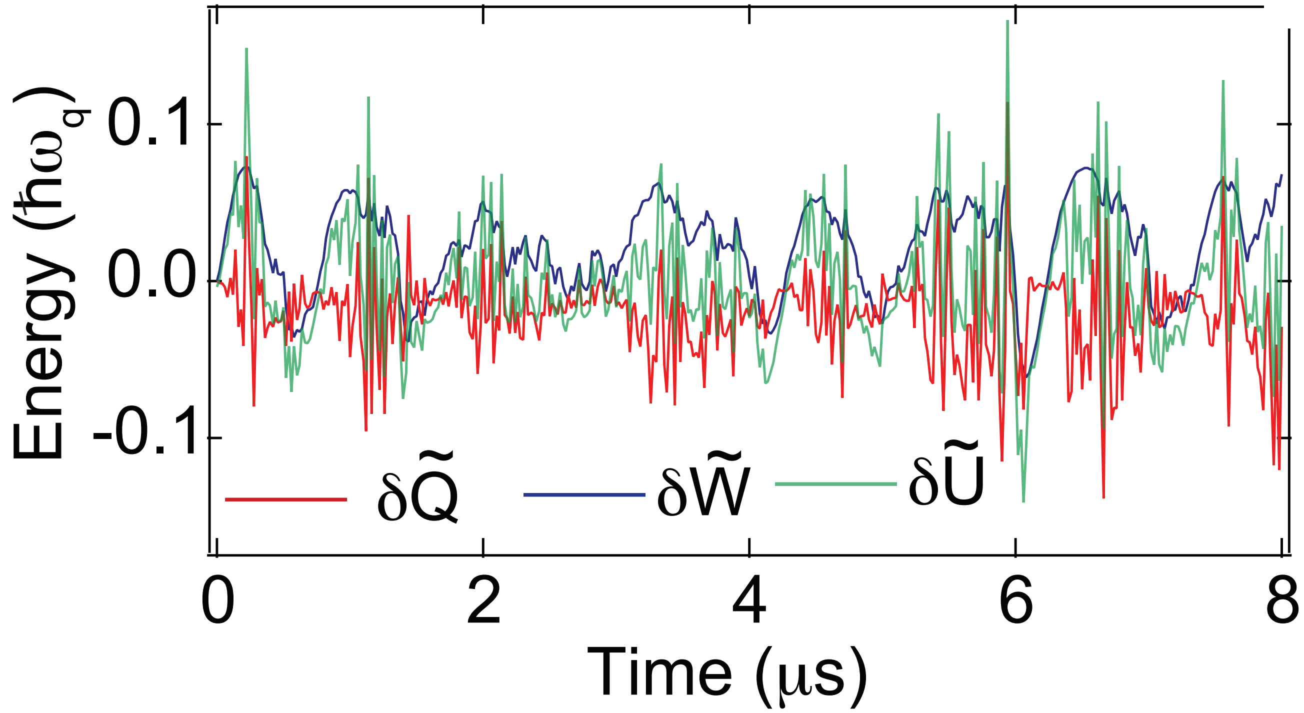
$$\delta\tilde{W} \;=\; \hbar\omega_{\mathrm{q}}\mathrm{tr}\left[\Pi_{m=1}\delta\mathbb{W}[\tilde{\rho}_t]\right] \qquad \delta\tilde{Q} \;=\; \hbar\omega_{\mathrm{q}}\mathrm{tr}\left[\Pi_{m=1}\delta\mathbb{Q}[\tilde{\rho}_t]\right]$$

Heat and work along a single quantum trajectory



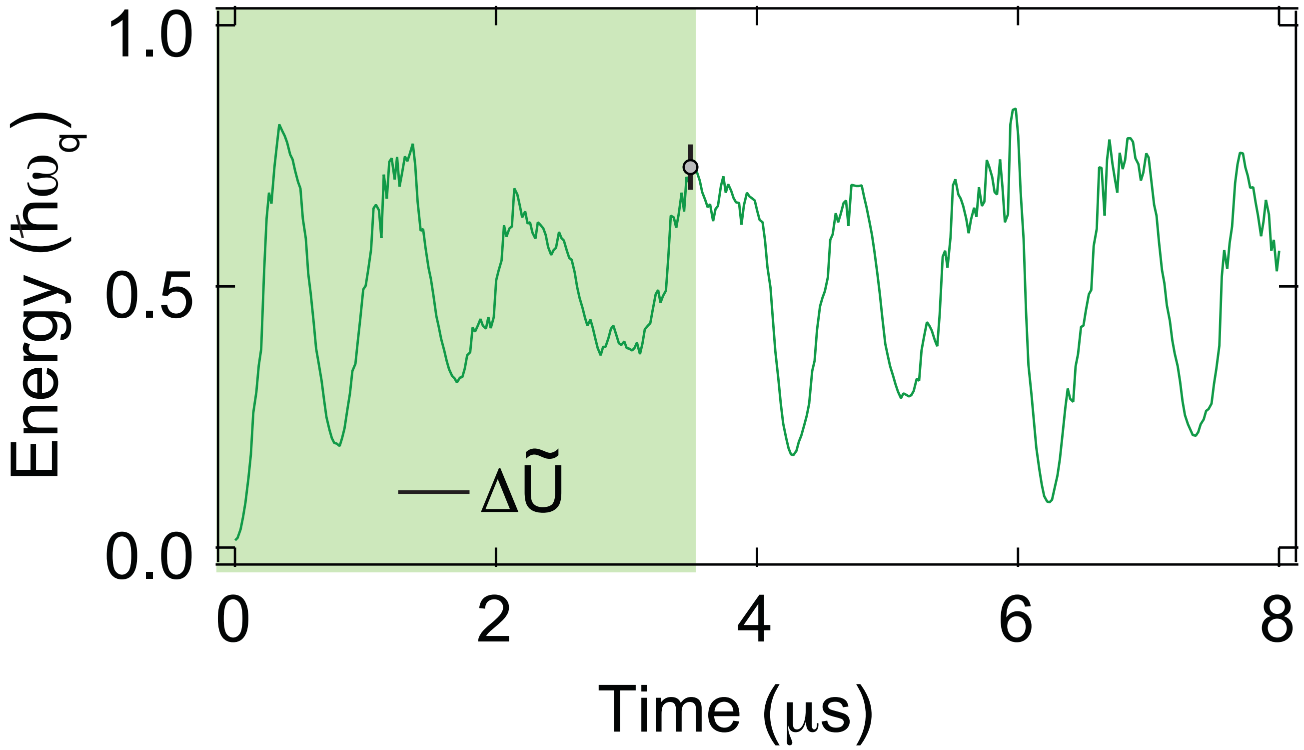
First Law of Thermodynamics

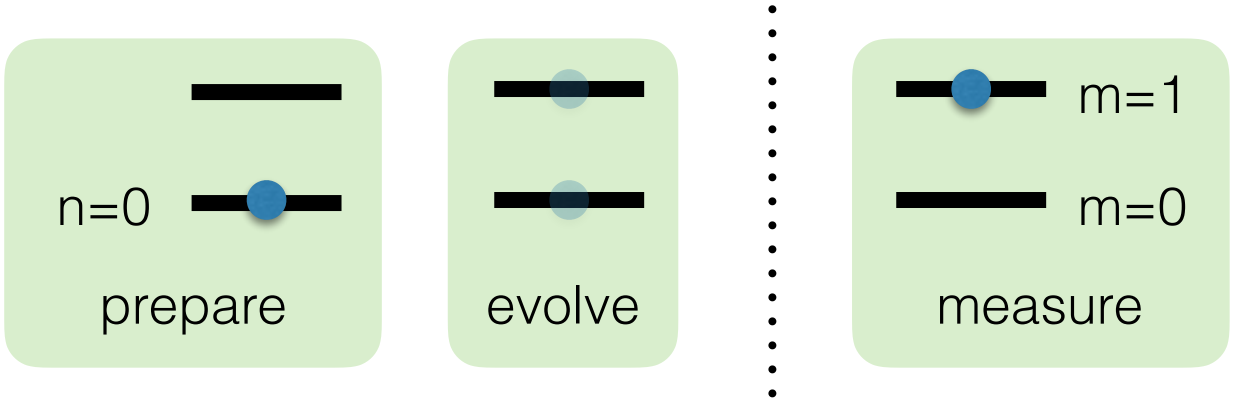
First law of thermodynamics: $\Delta U = \int_0^\tau \frac{\delta \tilde{W}}{dt}\, dt + \int_0^\tau \frac{\delta \tilde{Q}}{dt}\, dt$



Instantaneous heat and work along a single quantum trajectory.

Integrated over time.

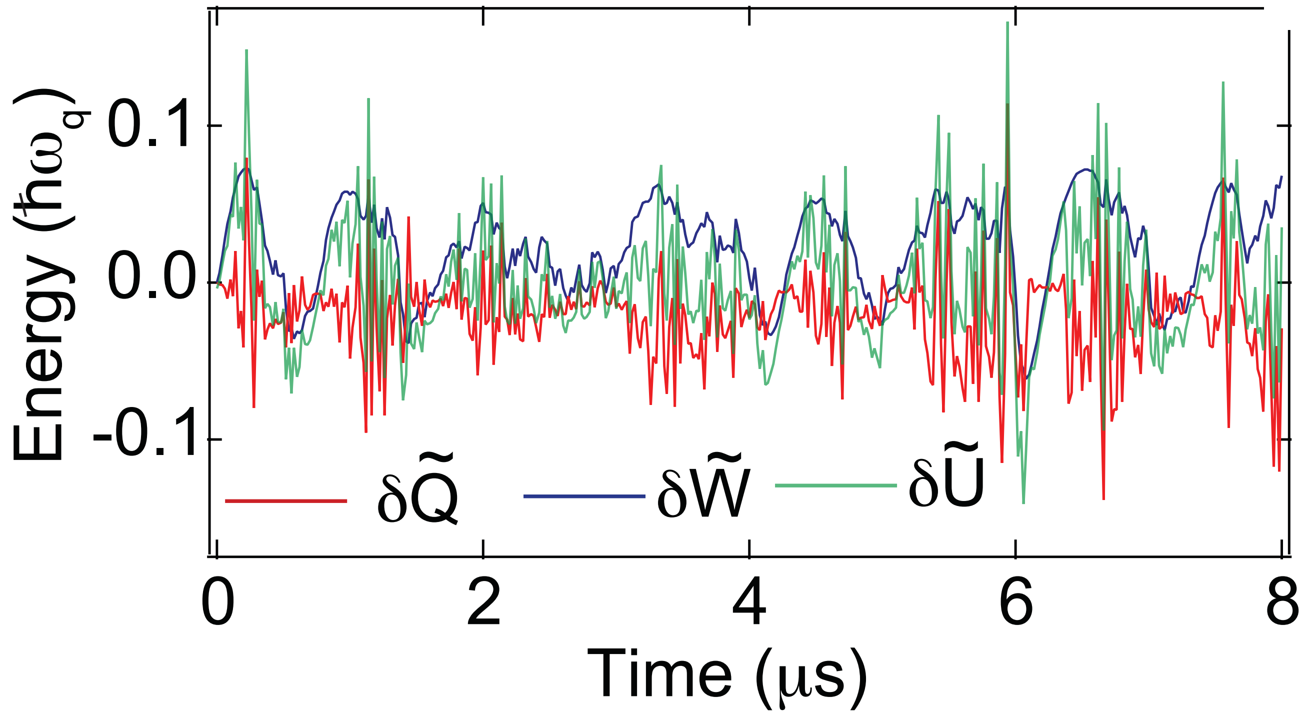




Condition on $\Delta\tilde{U}$

First Law of Thermodynamics

First law of thermodynamics: $\Delta U = \int_0^{\tau} \frac{\delta \tilde{W}}{dt}\, dt + \int_0^{\tau} \frac{\delta \tilde{Q}}{dt}\, dt$



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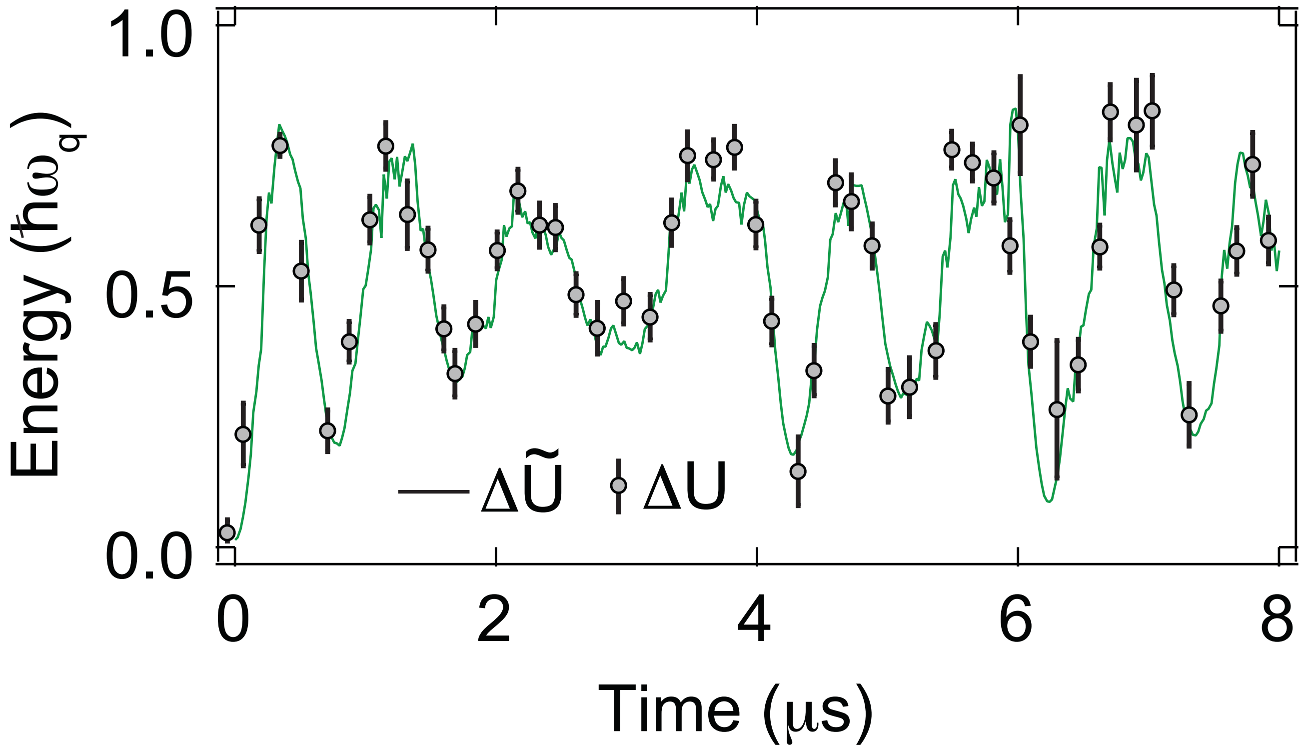
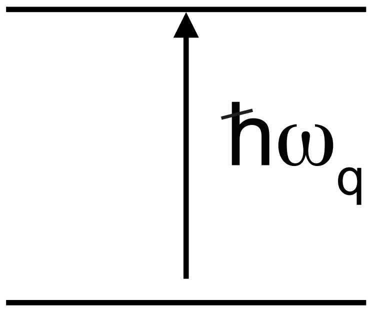


Matches the energy changes of obtained from transition probabilities.

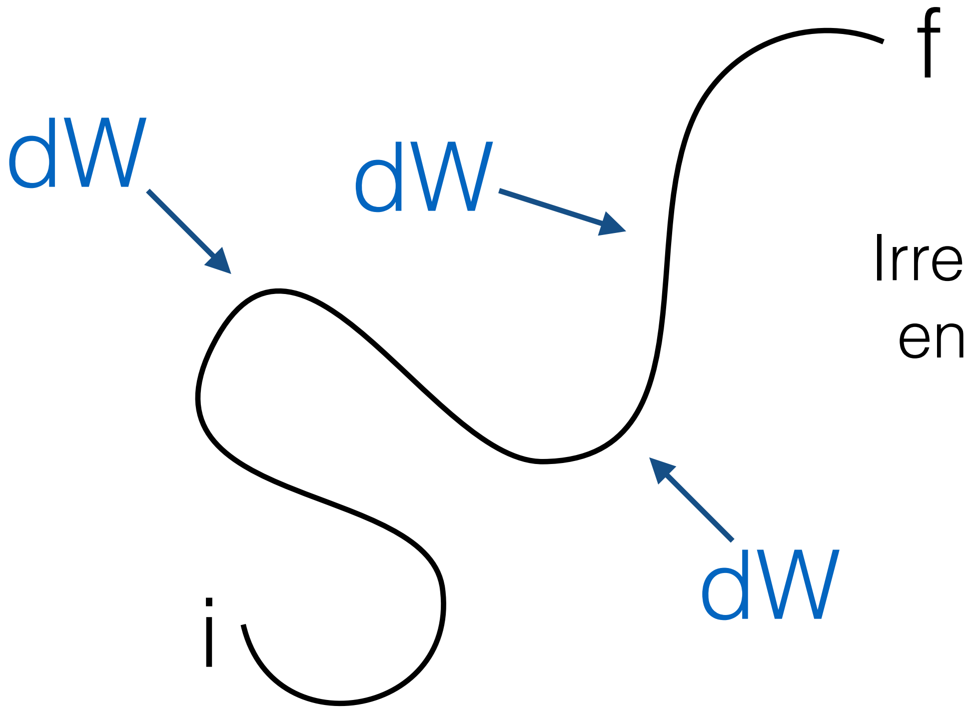


Second Law of Thermodynamics

Jarzynski equality: $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$

F = U-TS (Ability of a system to do work)

in general: ΔF≤W (equality for quasi-static)



ΔF?

Irreversible work along an ensemble of trajectories

Closed system: dQ=0

$$\langle e^{-\beta W} \rangle = \int P(W) e^{-\beta W} dW = \int P(\Delta U) e^{-\beta \Delta U} d\Delta U$$

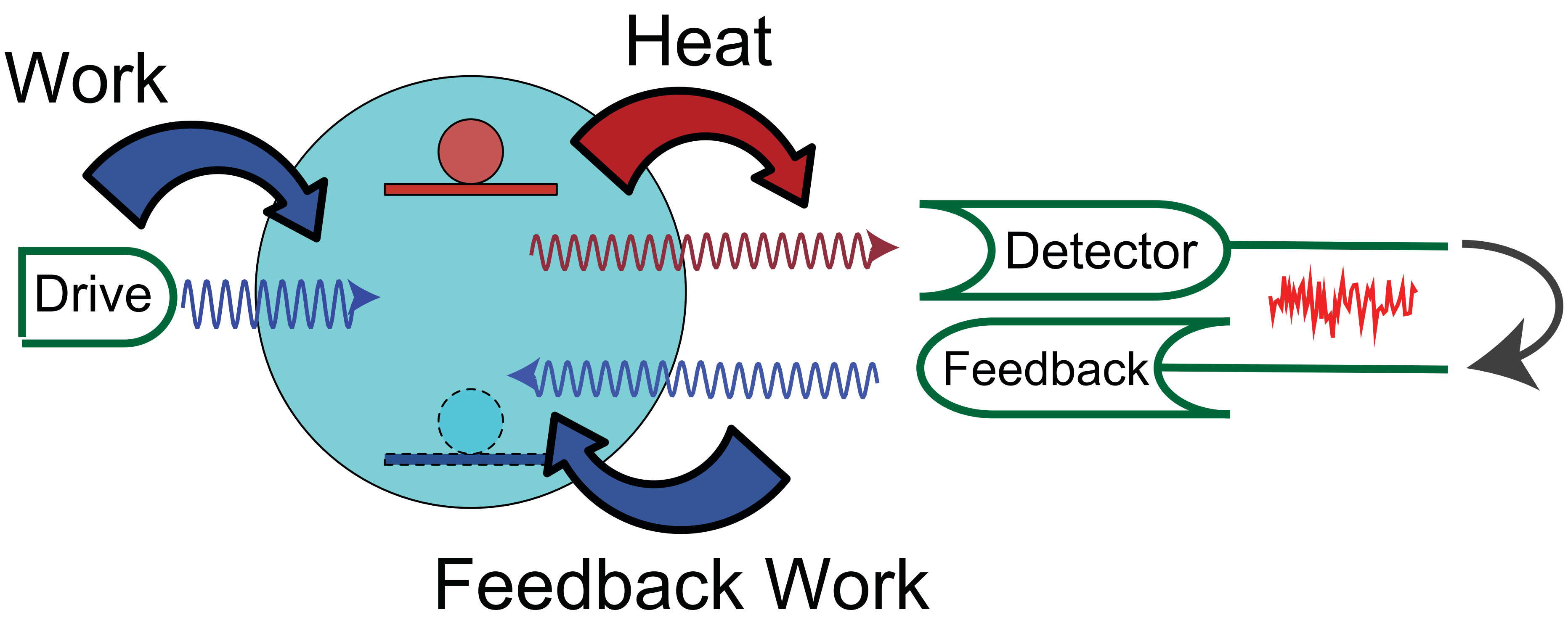
Infer work distribution from total energy change

Batalhão PRL 2014, An Nature Phys. 2015, Jarzynski PRL 1997

Quantum feedback to isolate qubit

Goal: determine work distribution from projective energy measurements (transition probability).

Use quantum feedback loop to cancel heat contributions to the transition probability.

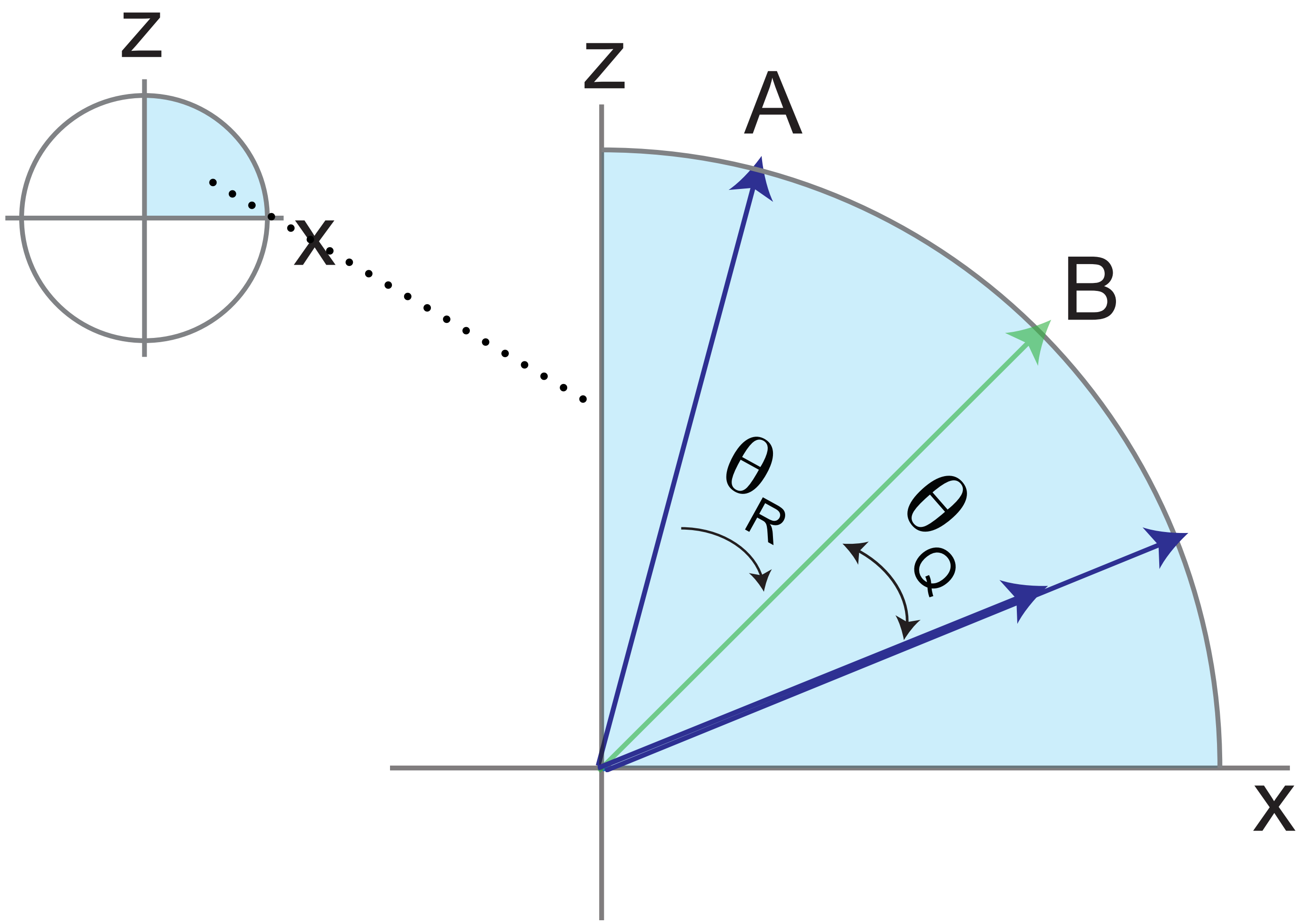


Alonso PRL 2016

Quantum feedback loop

Bloch sphere representation of qubit state

Break evolution into infinitesimal steps



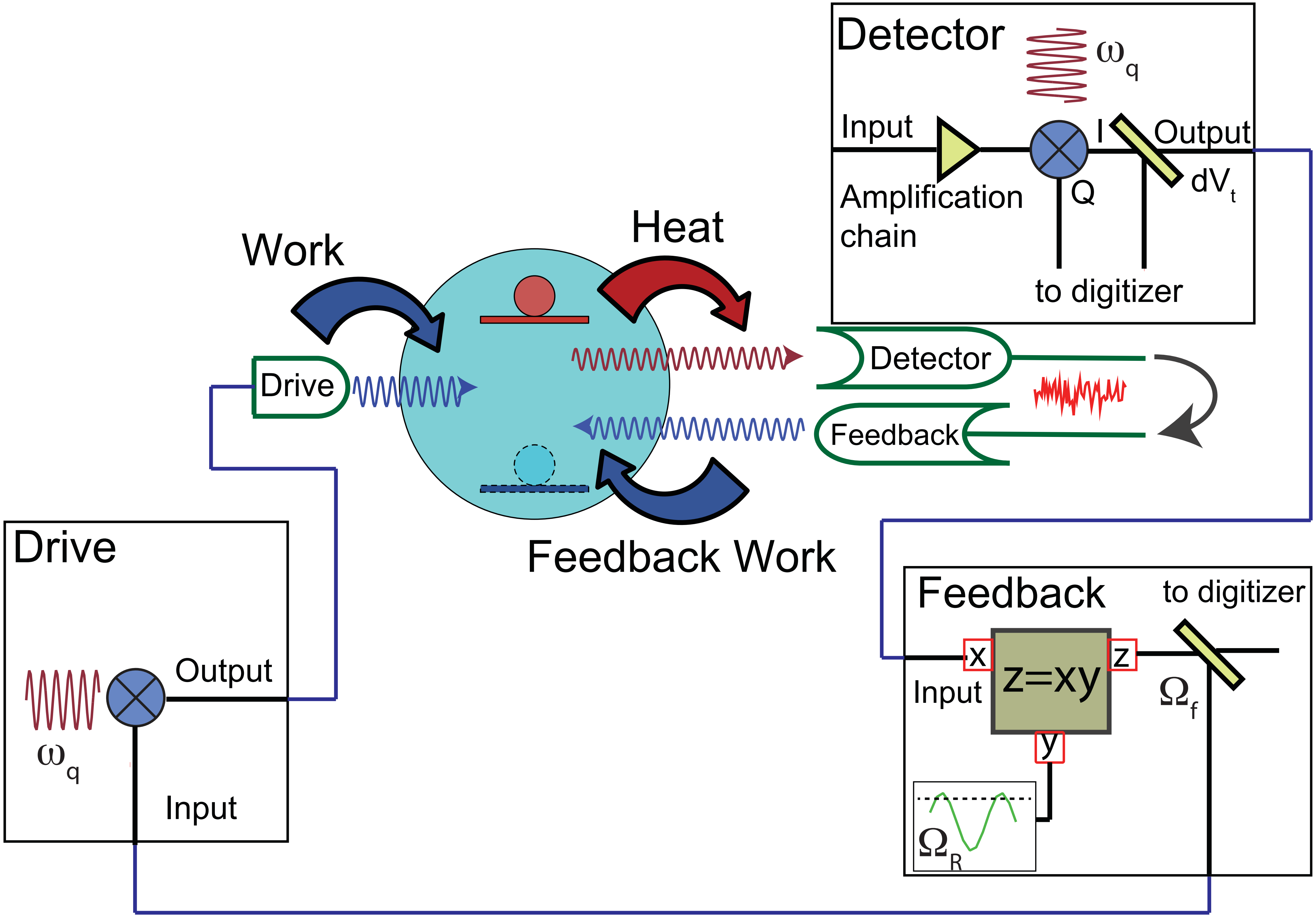
-unitary evolution due to the drive (work)

-stochastic evolution due to heat

-Inefficient detection: average over stochastic evolution

-Apply addition unitary rotation to maintain original phase

Quantum phase locked loop



Vijay, Nature 2012, Campagne-Ibarcq PRL 2016

Quantum phase locked loop

State update in Bloch components

$$dz = +\Omega x dt + \boxed{\gamma(1-z)dt + \sqrt{\eta}x(1-z)(dV_t - \gamma\sqrt{\eta}xdt)}$$

$$dx = -\Omega z dt \boxed{- \frac{\gamma}{2}xdt + \sqrt{\eta}(1-z-x^2)(dV_t - \gamma\sqrt{\eta}xdt)}$$

Cancel these terms

With unitary rotations:

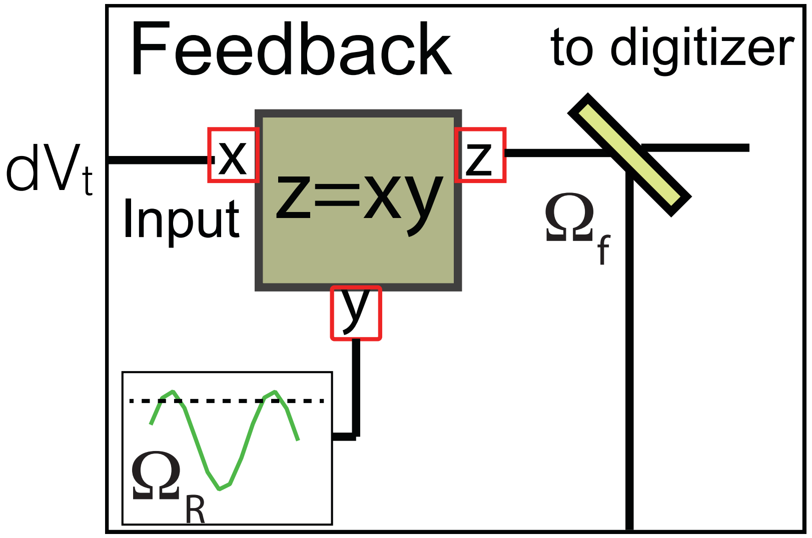
$$dz = \Omega_{\mathrm{F}} x dt, \quad dx = -\Omega_{\mathrm{F}} z dt,$$

Focus on z component

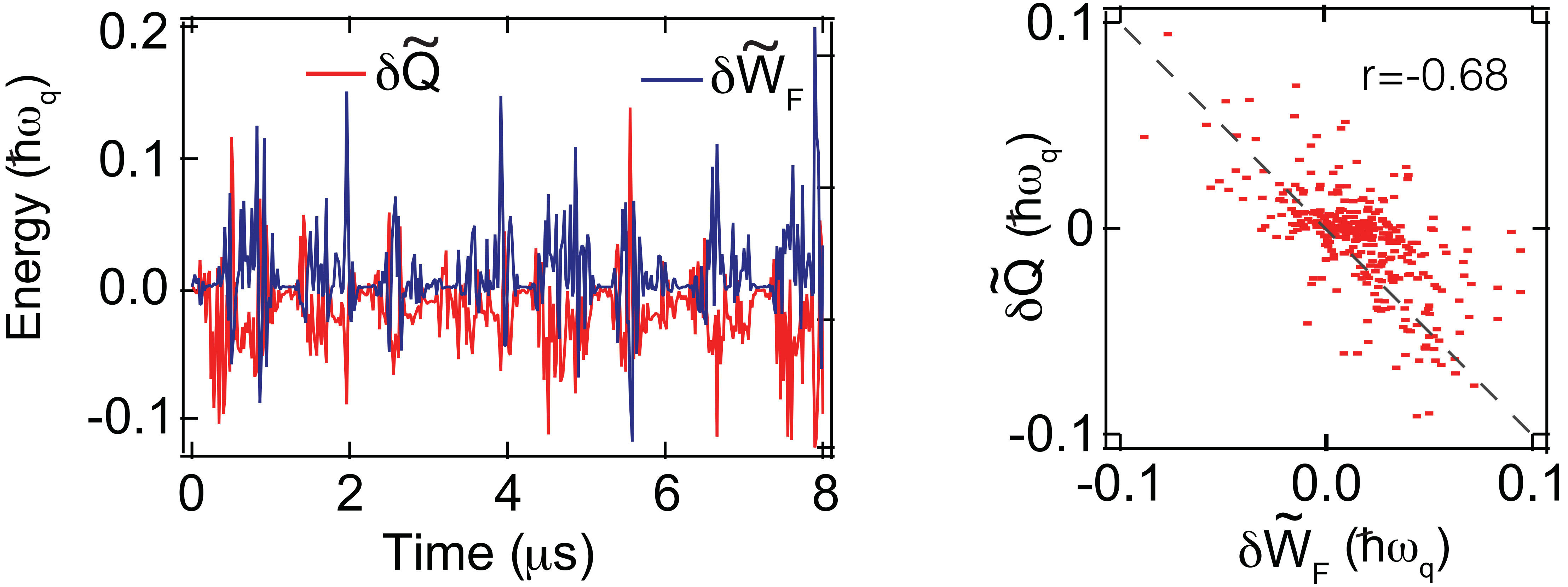
small for weak measurement

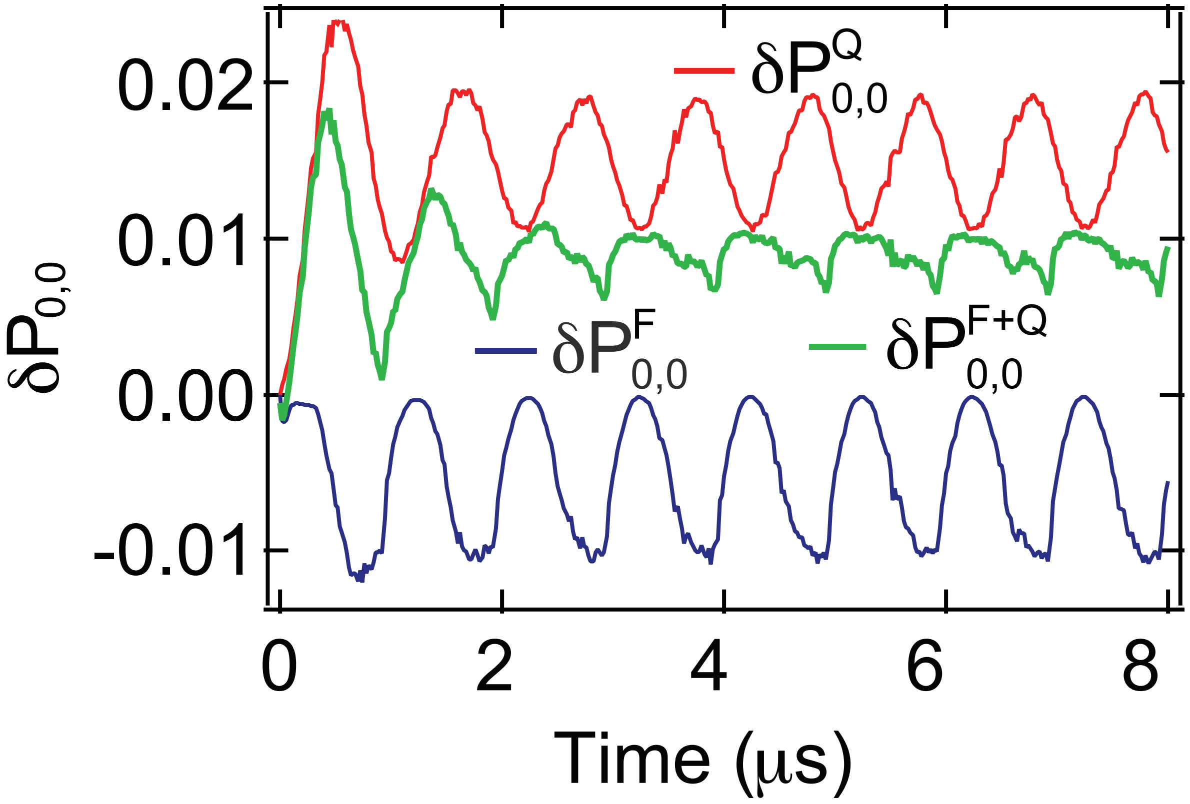
$$\Omega_{\mathrm{F}} x dt = -\sqrt{\eta}x(1-\underbrace{\boxed{z}}_{\boxed{\cos(\Omega t + \phi)}})(dV_t - \cancel{\gamma\sqrt{\eta}xdt})$$

$$\Omega_{\mathrm{F}} = \sqrt{\eta}(\cos(\Omega t + \phi) - 1)dV_t/dt$$



Isolating the qubit with quantum feedback



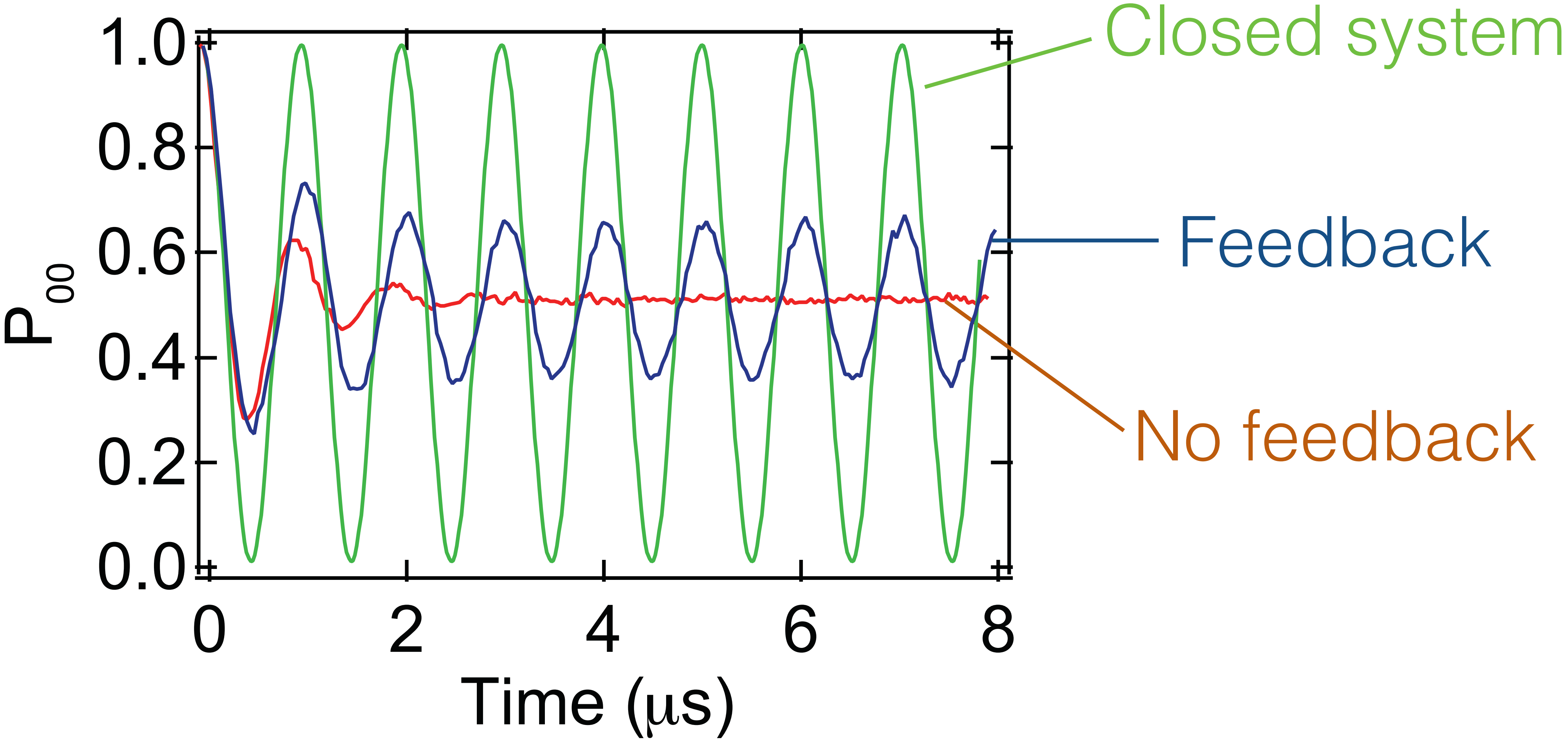


-Additional work at each time point to compensate for the exchanged heat

-Quantum efficiency (η = 0.3) means the compensation is not perfect.

Persistent Rabi oscillations

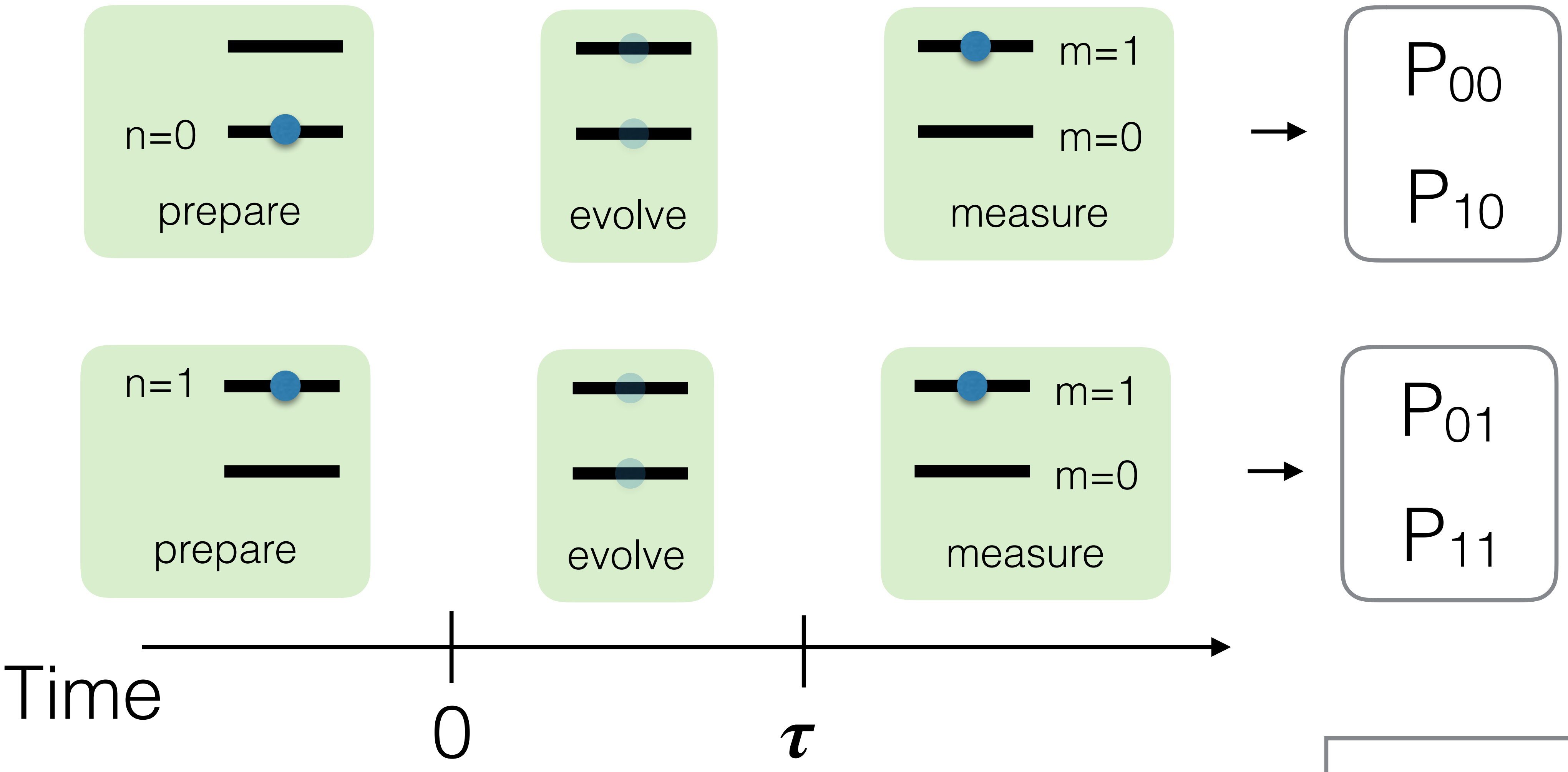
Ensemble transition probabilities:



With feedback, transition probabilities oscillate as with closed system, lower contrast.

Vijay, Nature 2012

Jarzynski Equality



n=0

prepare

evolve

m=1

m=0

measure

→ P_{00} P_{10}

n=1

prepare

evolve

m=1

m=0

measure

→ P_{01} P_{11}

Time

0

τ

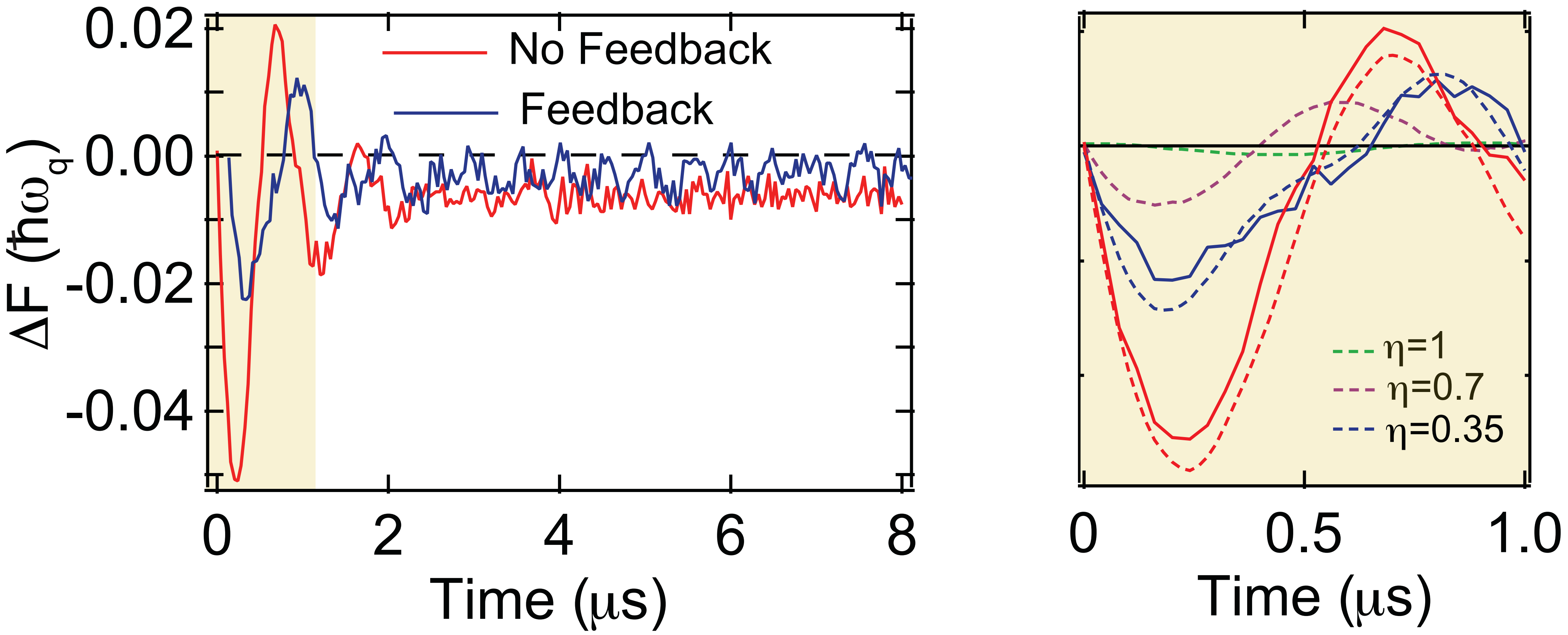
$$P(\Delta U) = \sum_{m,n} P^{\tau}_{m,n} P^{0}_{n} \delta(\Delta U - (E^{\tau}_{m} - E^{0}_{n}))$$

$$P^0_n = \frac{e^{-\beta E_n}}{e^{-\beta E_0} + e^{-\beta E_1}}$$

$$\beta/\hbar\omega_{\mathrm{q}} = 3.5$$ (P_1=3%)

$$\langle e^{-\beta W} \rangle = \int P(W) e^{-\beta W} dW = \int P(\Delta U) e^{-\beta \Delta U} d\Delta U$$

Jarzynski Equality

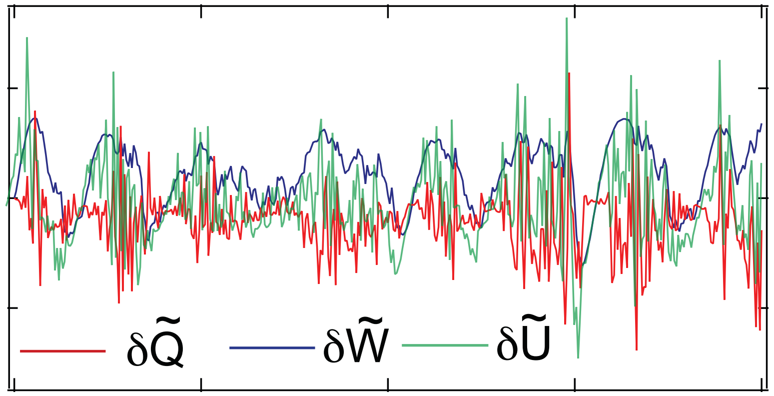


Initial and final Hamiltonians are the same so expect ΔF=0

Feedback reduces deviations by factor of 2

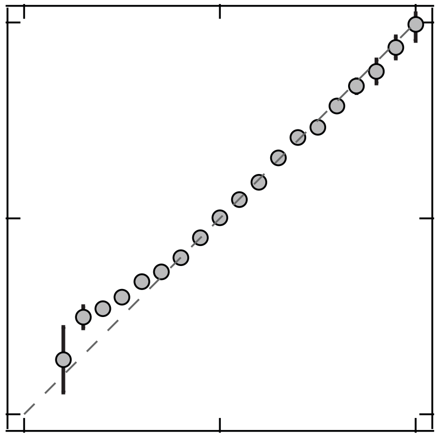
Residual deviation is due to finite quantum efficiency

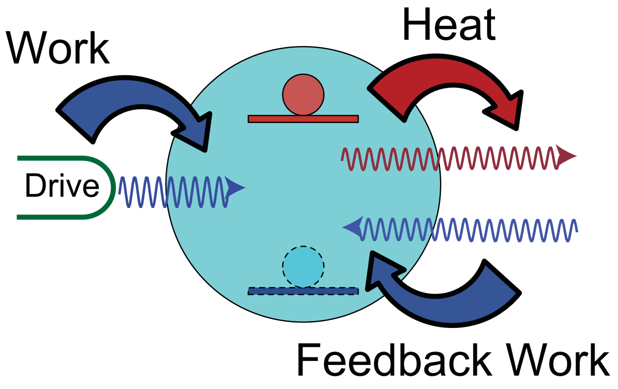
Summary so far



Identify heat and work along single quantum trajectories.

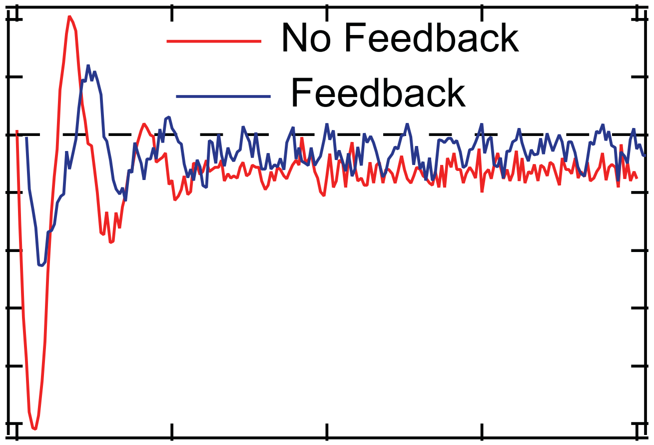
Verify that the sum of the integrated heat and work along single quantum trajectories is equal to the total energy change





Quantum feedback loop to compensate for heat with additional work.

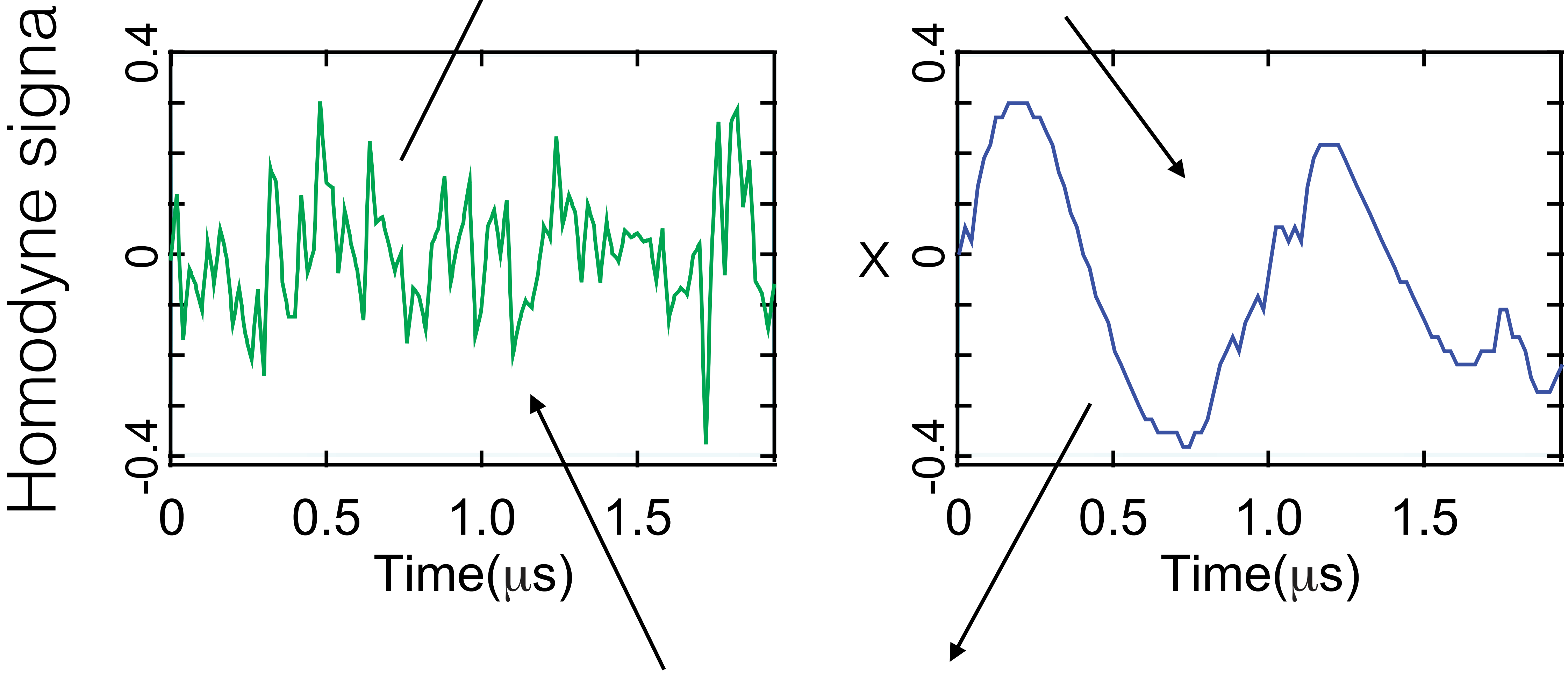
Jarzynski equality based on projective energy measurements.



Thermodynamics from the statistical mechanics angle.

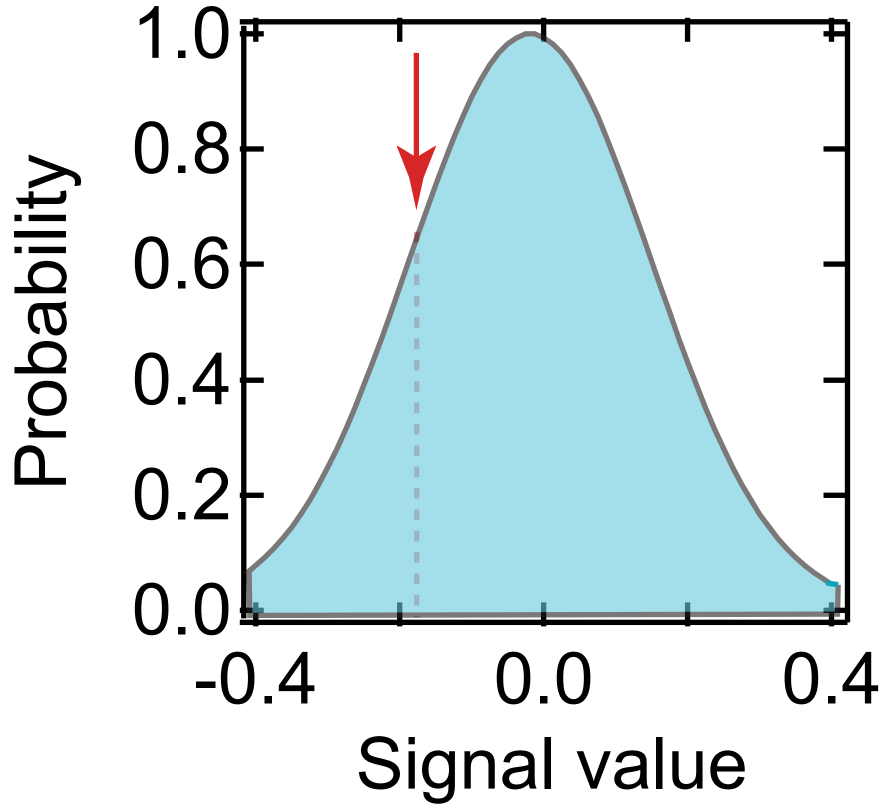
$$dz = -\Omega x dt + \gamma(1-z)dt + \sqrt{\eta}x(1-z)(dI_t - \gamma\sqrt{\eta}x dt)$$

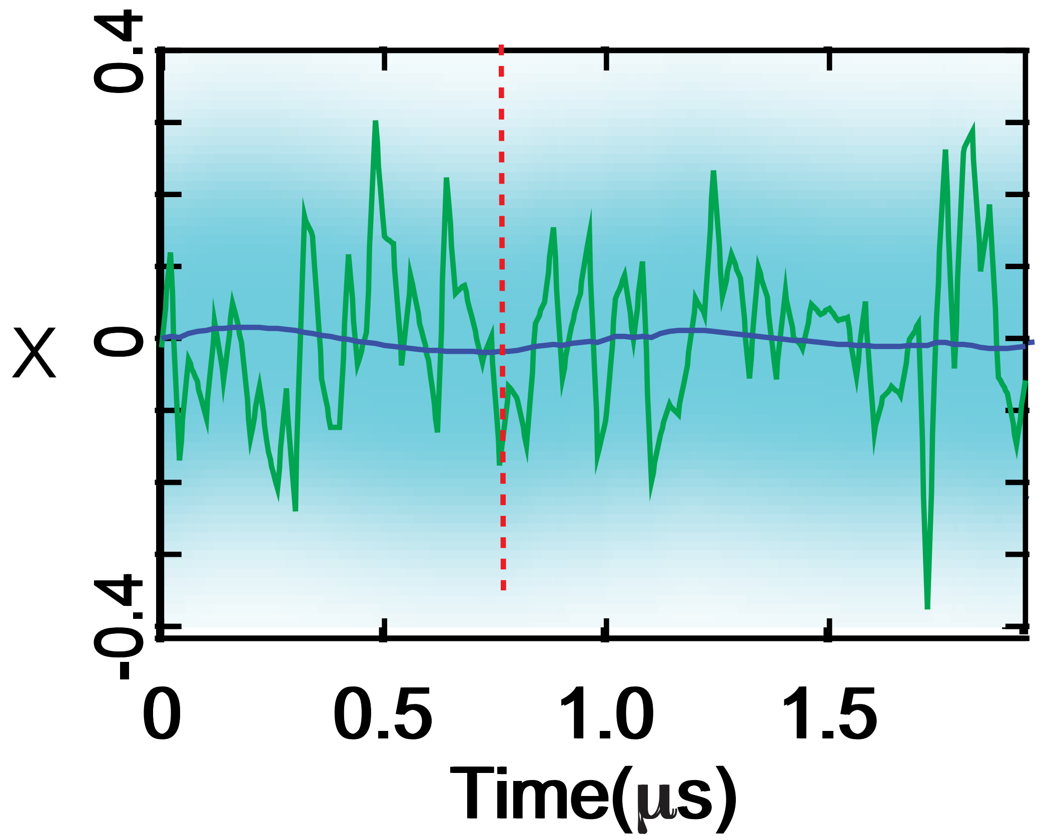
$$dx = +\Omega z dt - \frac{\gamma}{2}x dt + \sqrt{\eta}(1-z-x^2)(dI_t - \gamma\sqrt{\eta}x dt)$$



$$dV_t = \sqrt{\eta}\gamma\langle\sigma_x\rangle dt + \sqrt{\gamma}dW_t$$

Path Probability





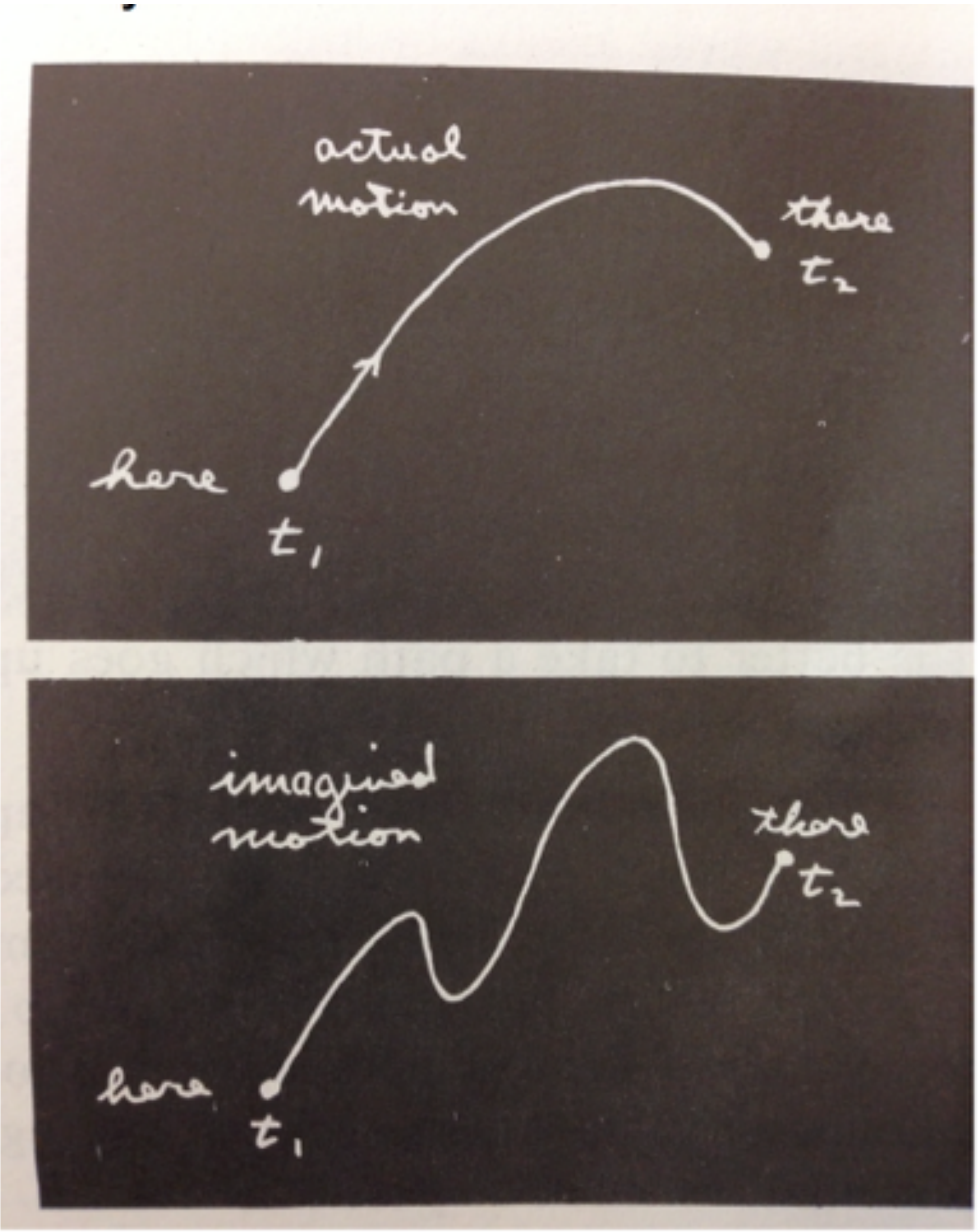
$$P_{path} = \prod_{n=0}^{N-1} \mathcal{P}(dV_t \mid x) = \prod_{n=0}^{N-1} e^{-\frac{(dI_t - \gamma\sqrt{\eta}x dt)}{2\gamma dt}}$$

A most likely path?

Borrow some ideas from Feynman:

Classical mechanics: principle of least action

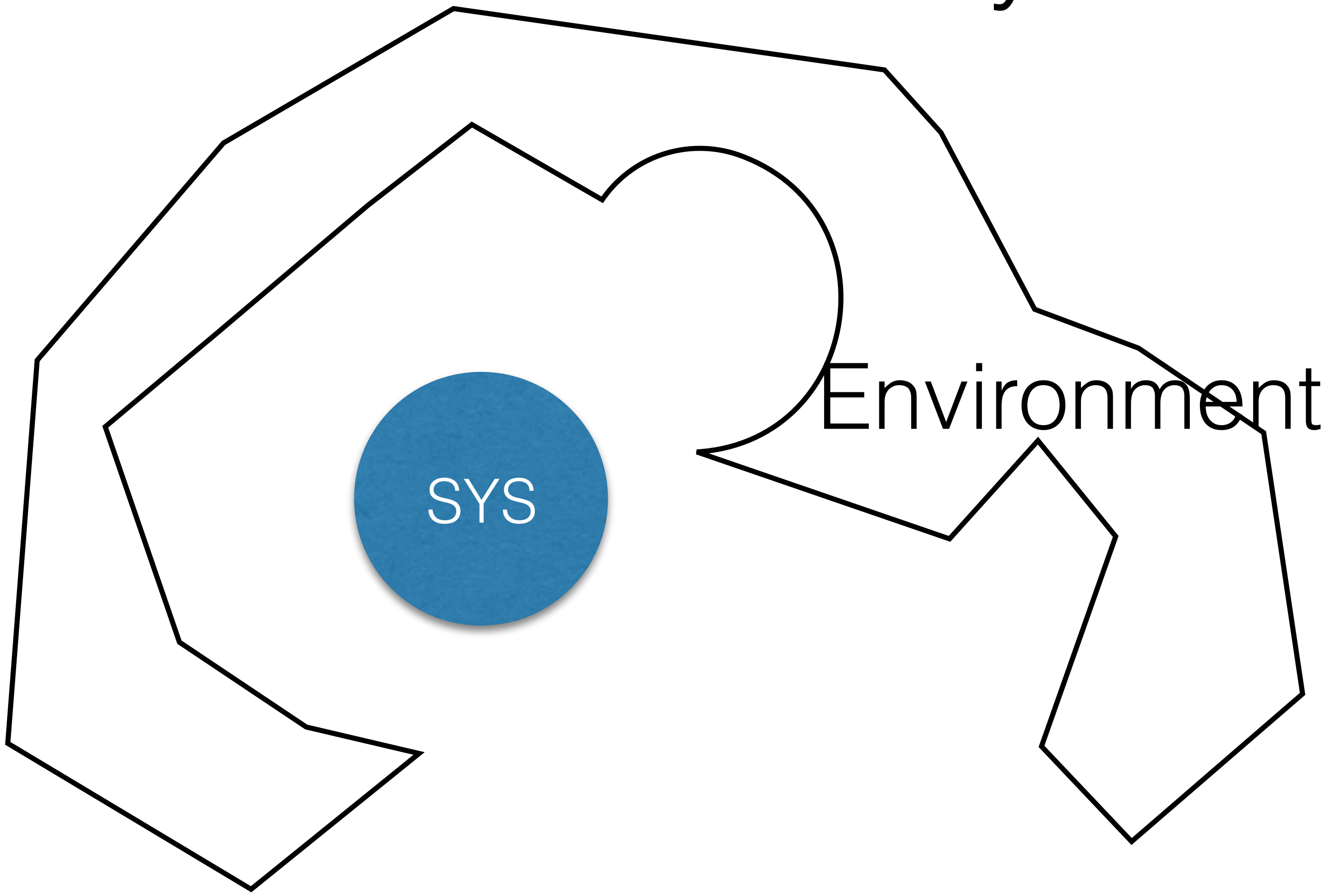
Stochastic trajectories: extremize the stochastic action (maximize the path probability)

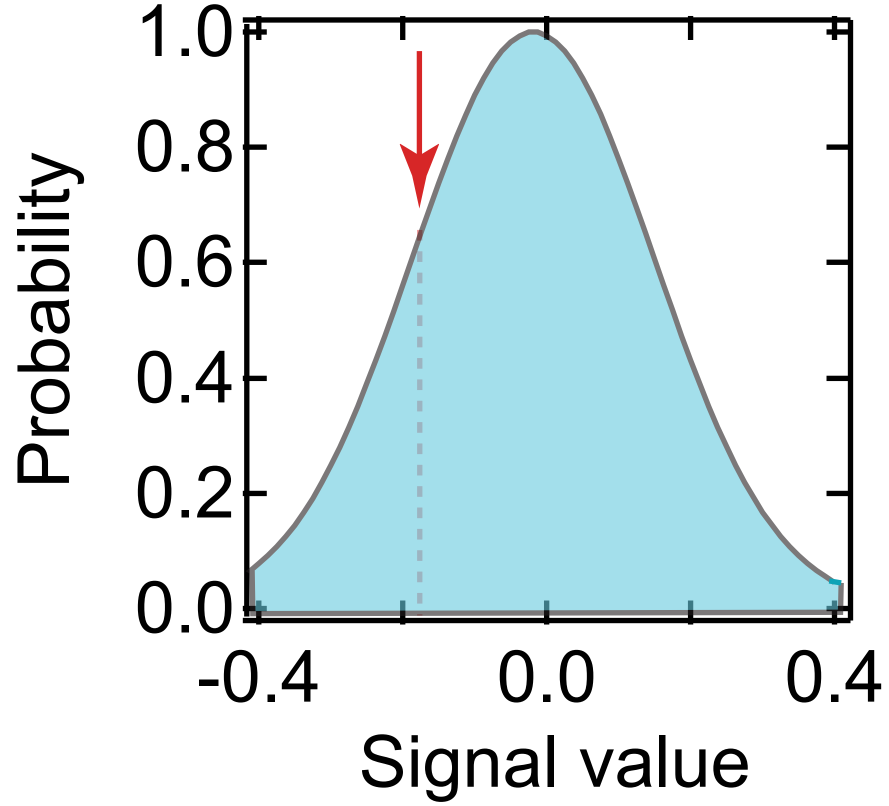


S. Weber et al. Nature (2014), A. Chantasri, PRA **88**, 042110 (2013)

M. Naghiloo et al. arXiv 2016: “Quantum caustics in resonance fluorescence trajectories”

Path Probability is related to entropy.





$\Delta S_{SYS} \sim \ln(P_{final}/P_{initial})$

ΔS_{env}?

Seifert: look at the path probability!

$\Delta S_{env} \sim \ln(P_{forward}/P_{backwards})$

Fluctuation theorem:

$$\langle e^{-\Delta Stot} \rangle = 1$$

Are measurement dynamics reversible?

Arrow of time?

J. Dressel (2016) Seifert PRL 2005

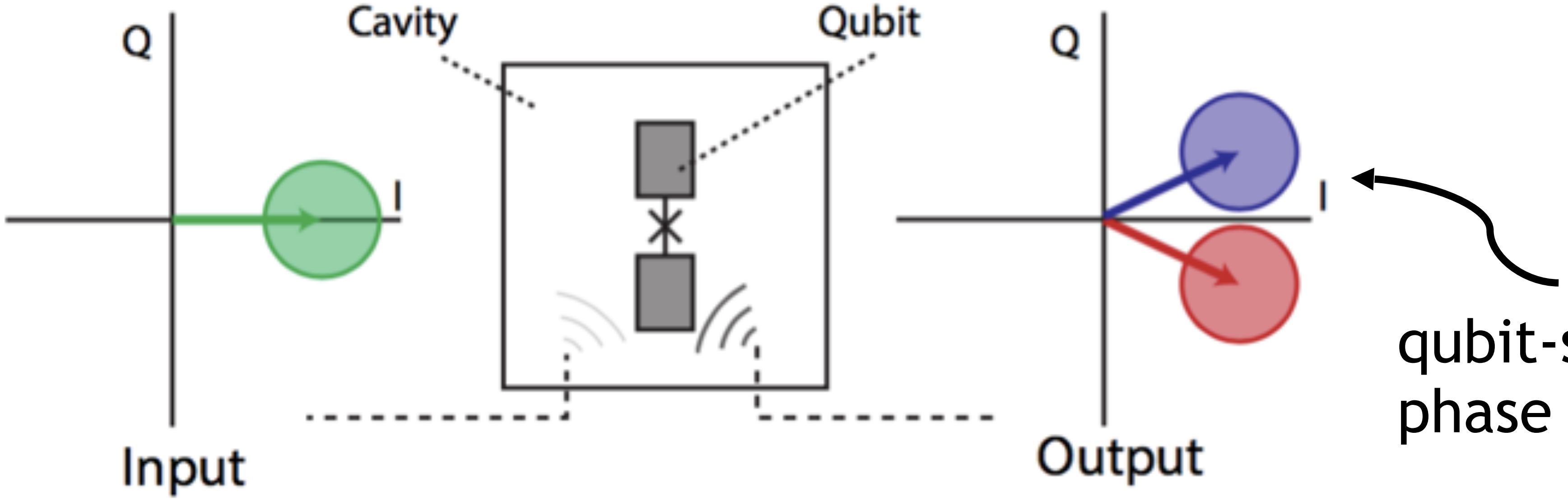
Are quantum measurements reversible?

Projective measurements: No
Weak measurements: Yes

Example: dispersive σ_z measurement.

$$\hat{H} = \underbrace{\frac{1}{2}\hbar\omega_q\hat{\sigma}_z}_{\text{qubit}} + \underbrace{\hbar\left(\omega_c + \chi\hat{\sigma}_z\right)\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right)}_{\text{cavity}}$$

Coherent cavity probe



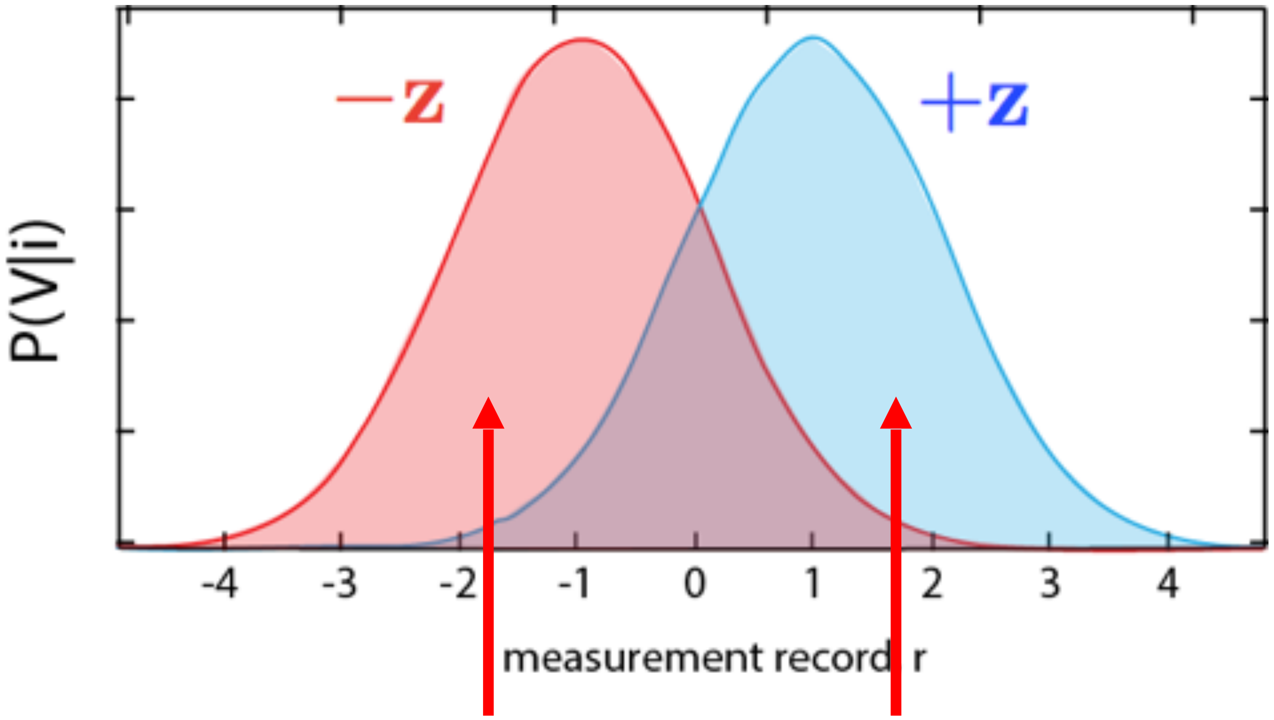
qubit-state-dependent
phase shift on the cavity probe

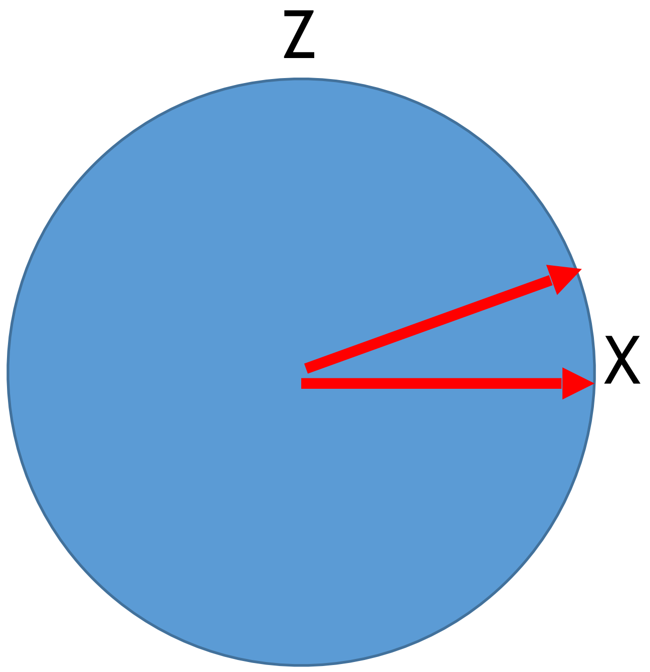
State Tracking: intuitive example

Prepare: $|+\mathbf{x}\rangle = \dfrac{|+\mathbf{z}\rangle + |-\mathbf{z}\rangle}{\sqrt{2}}$

$$P(r|+\mathbf{z}) \propto \exp[-(r-1)^2/2\tau]$$

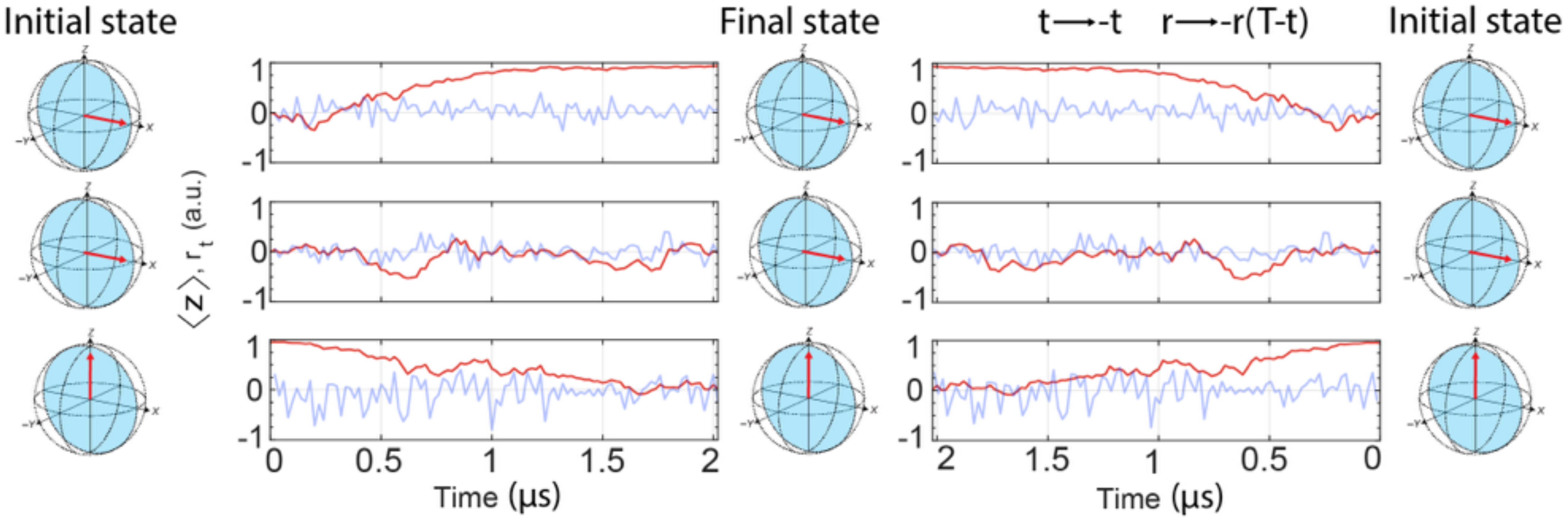
$$P(r|-\mathbf{z}) \propto \exp[-(r+1)^2/2\tau]$$





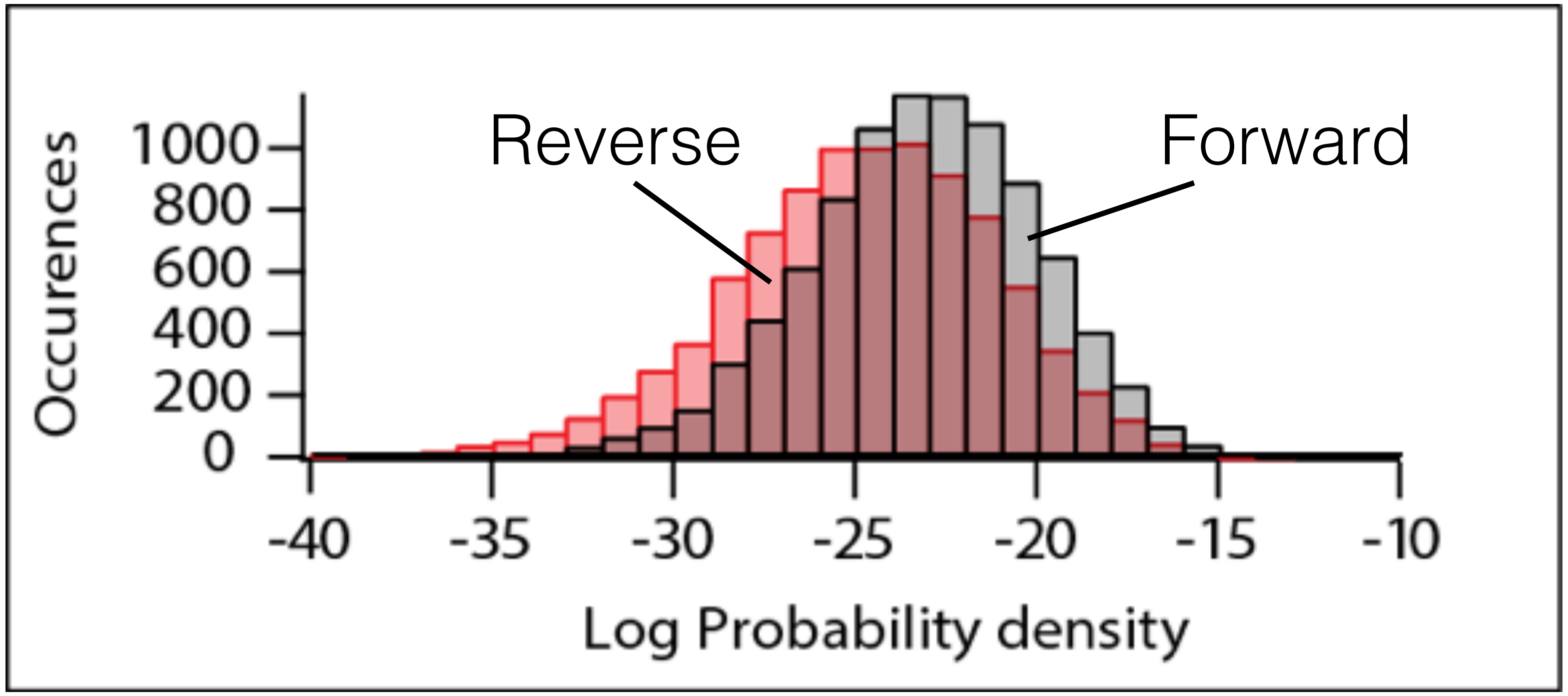
Janus sequences

A.Jordan, A. Chantasri, KWM, J. Dressel, A. Korotkov (2017)

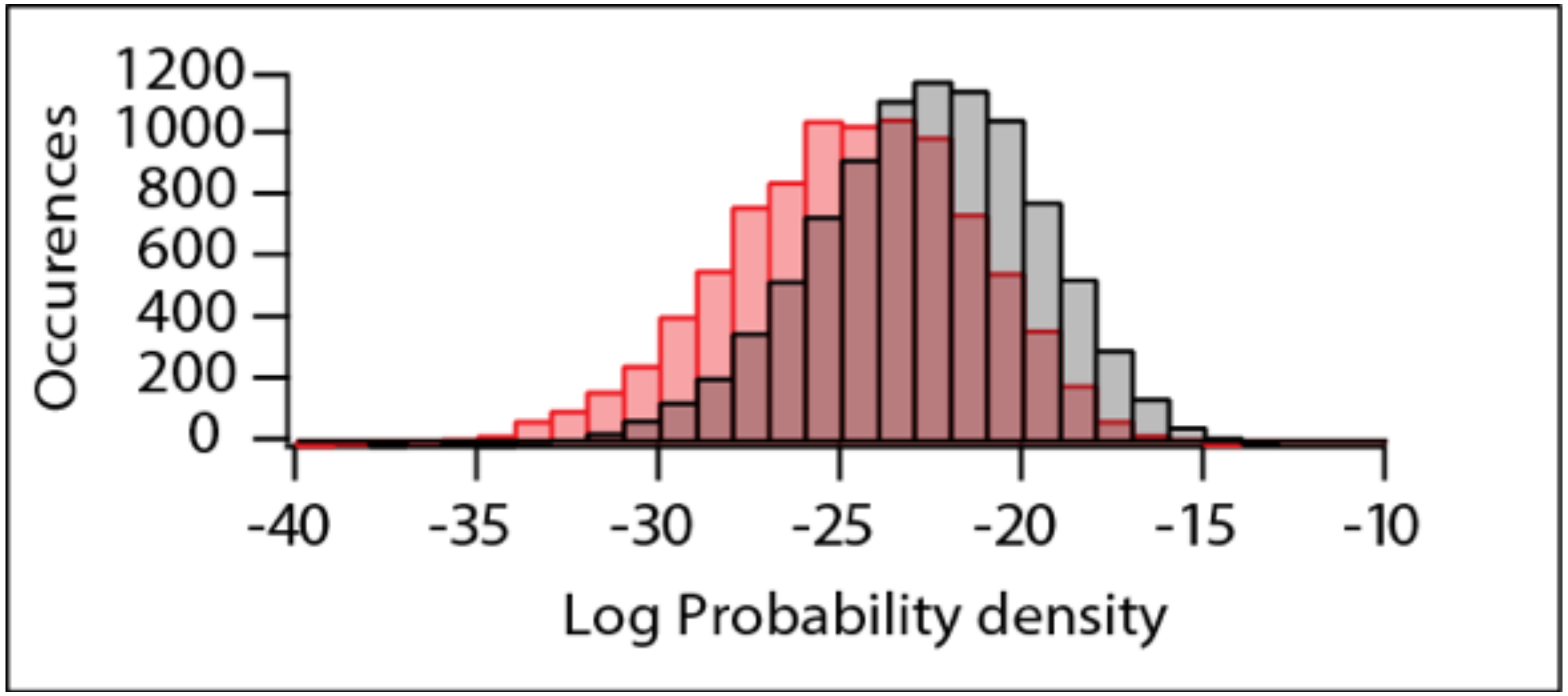


Statistical arrow of time in quantum measurement

$z_i = 0$



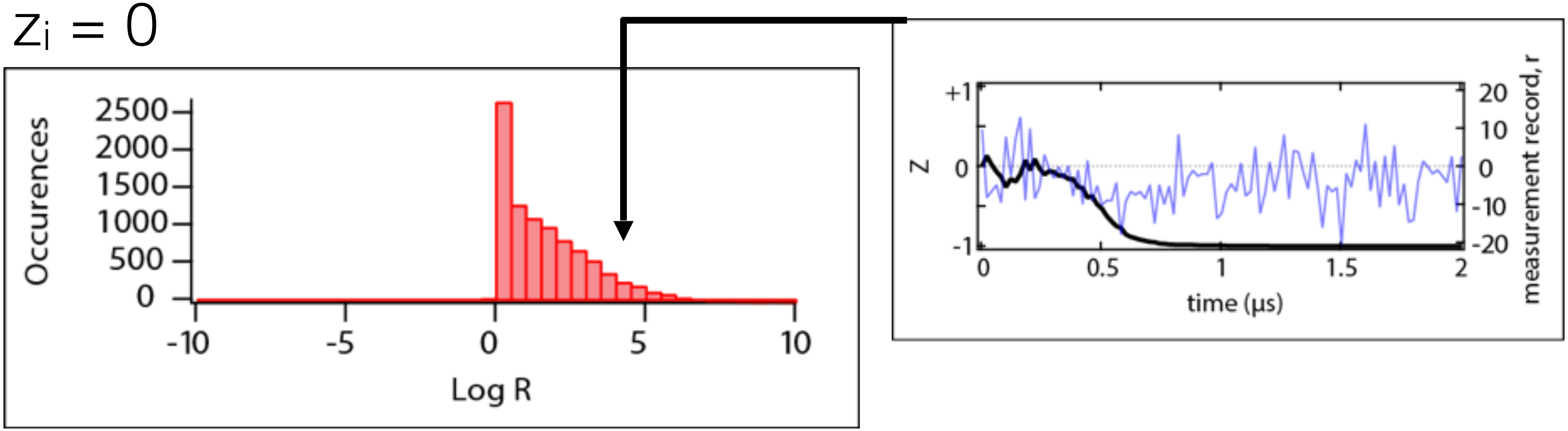
$z_i = 0.87$



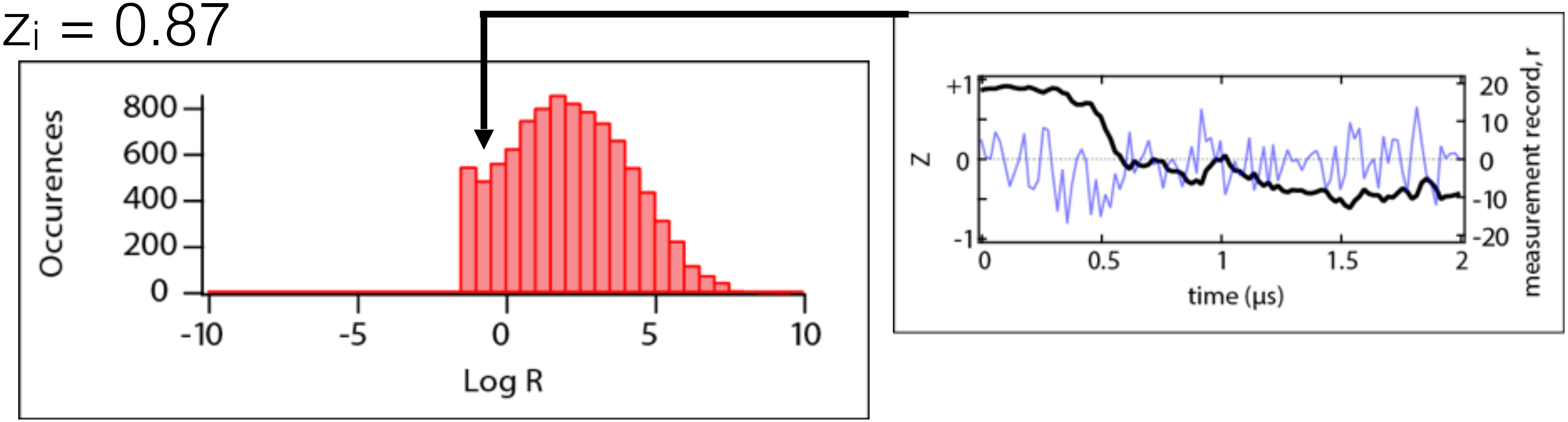
The Arrow-of-time Ratio

$$\boxed{\mathcal{R} = \frac{P_F}{P_B}}$$

$z_i = 0$



$z_i = 0.87$



seemingly “backwards in time”

What’s next?

Current case is completely classical: Probability depends only on populations.

- Include a Rabi drive: populations ⇄ coherences
- Different measurement operator (σ_-)

Fluctuation theorem, entropy ⇄ heat, temperature

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Washington University in St.Louis
imse Institute of Materials Science & Engineering

Postdoc positions available