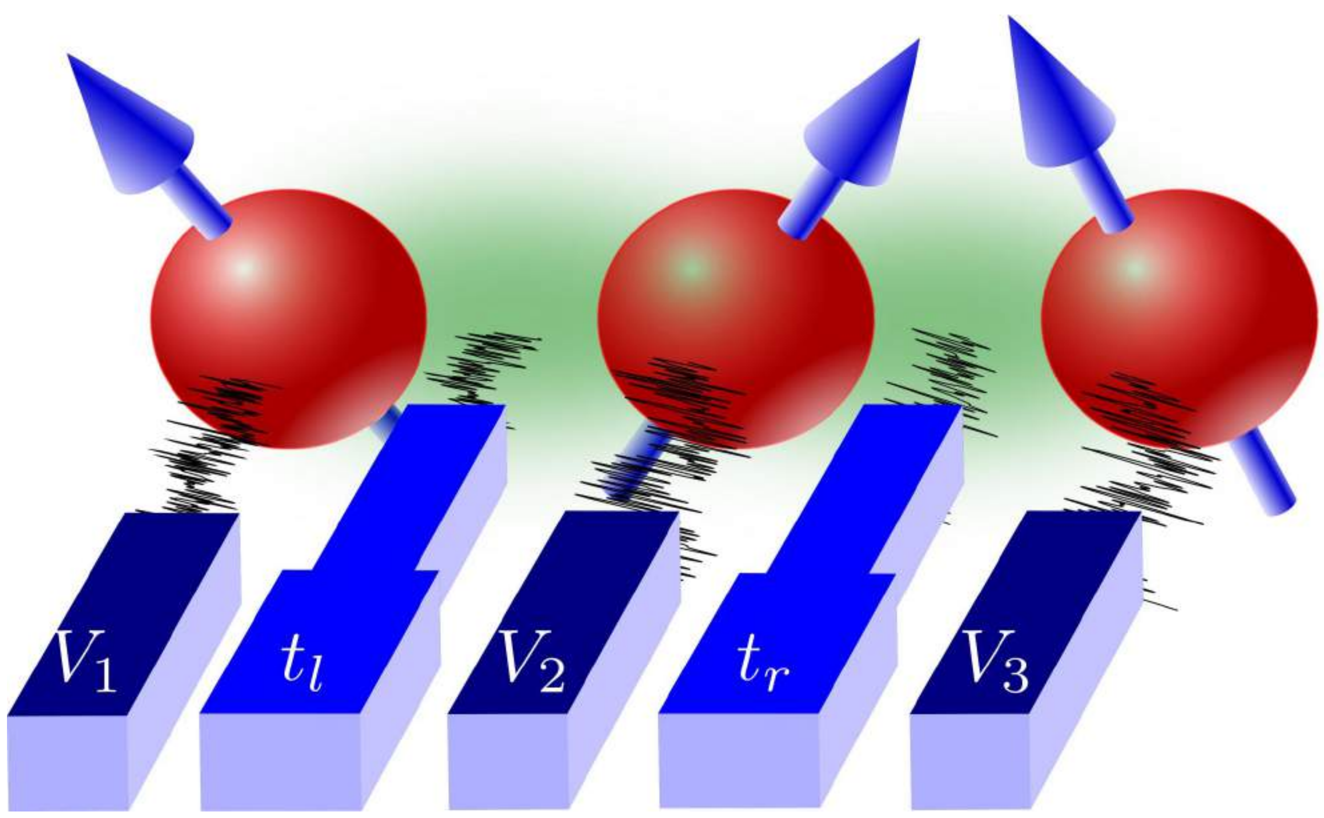
ICTS program on Open Quantum Systems
ICTS Bangalore, India
2017-07-27/28

Spin Qubits Theory



Guido Burkard

Department of Physics
University of Konstanz, Germany

Universität Konstanz

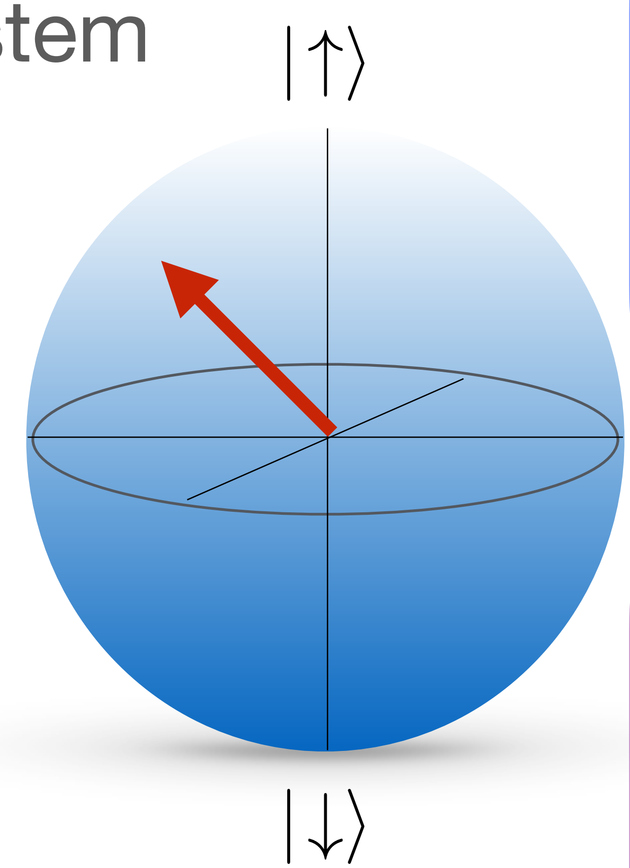
Single spins as controllable open quantum systems

- spin of subatomic particle (electron, atomic nucleus)
- spin 1/2: "ideal" quantum bits (qubit) $|0\rangle = |\uparrow\rangle \qquad |1\rangle = |\downarrow\rangle$
- "smallest" quantum system
- localized electron spins in semiconductor quantum dots

- spin is an open quantum system

fluctuating electric fields

fluctuating magnetic fields



phonons

other spins

?

Spin Qubits

lecture 1

Introduction into Spin Qubits

lecture 2

Multi-spin qubits
Coupling Spins to Electric Fields
Spin and Valley 1: Exchange

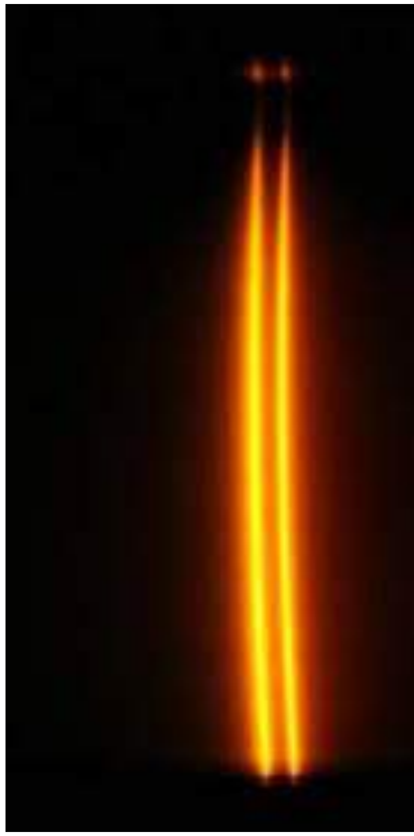
lecture 3

Spin and Valley 2: Spin Relaxation
Defect Spins

$\hbar$

classical spin

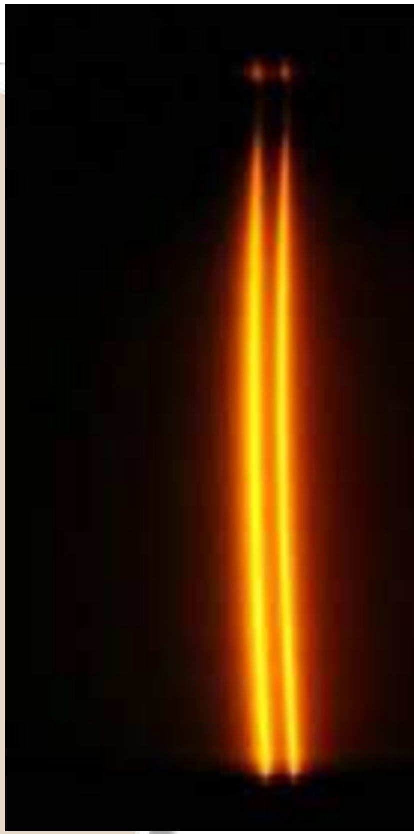
Spin



Spin

Wolfgang Pauli [Z. Phys. **31**, 373 (1924)]:

"...According to this standpoint, the doublet structure of the alkali [atom] spectra as well as the violation of Larmor's theorem originates from peculiar, non-classical two-valuedness of the quantum theoretical properties of the outer electron."



qubit

spin

Spin

- two-valued degree of freedom of the electron: angular momentum (spin) 1/2

- proposed by Ralph Kronig in Jan 1925 (then at Columbia U.)

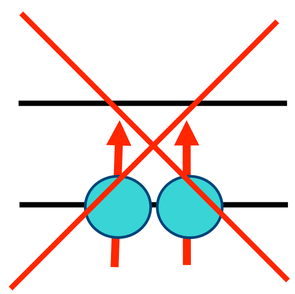
- strong reservations from Heisenberg and Pauli

- Sept 1925: Uhlenbeck and Goudsmit discovered the spin of the electron

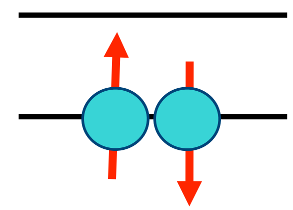
Ralph Kronig
(1904–1995)

Pauli exclusion principle

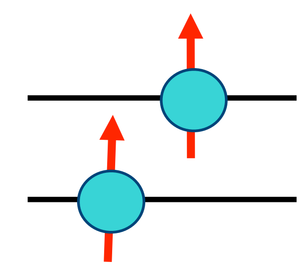
Two Fermions (e.g. electrons) cannot occupy the same quantum state.



impossible



OK



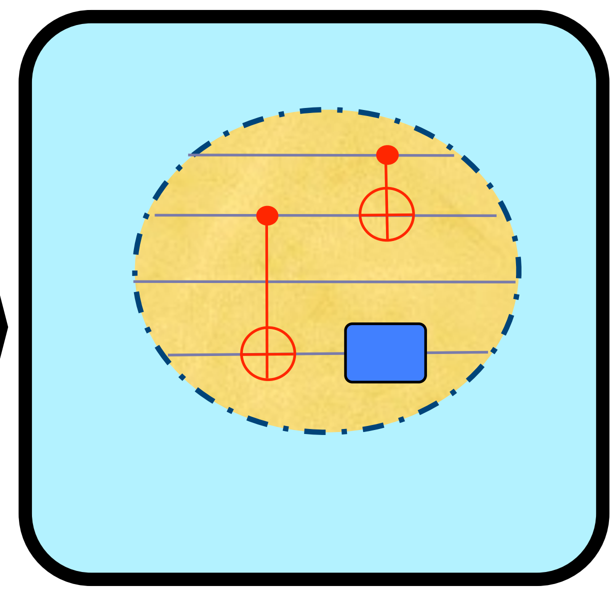
OK

- explanation of atomic spectra, periodic table of elements

- electron spin as quantum bit:
  - read out (via charge detection)
  - interaction ("exchange")
  - preparation (initialization)
  - measurement of entanglement

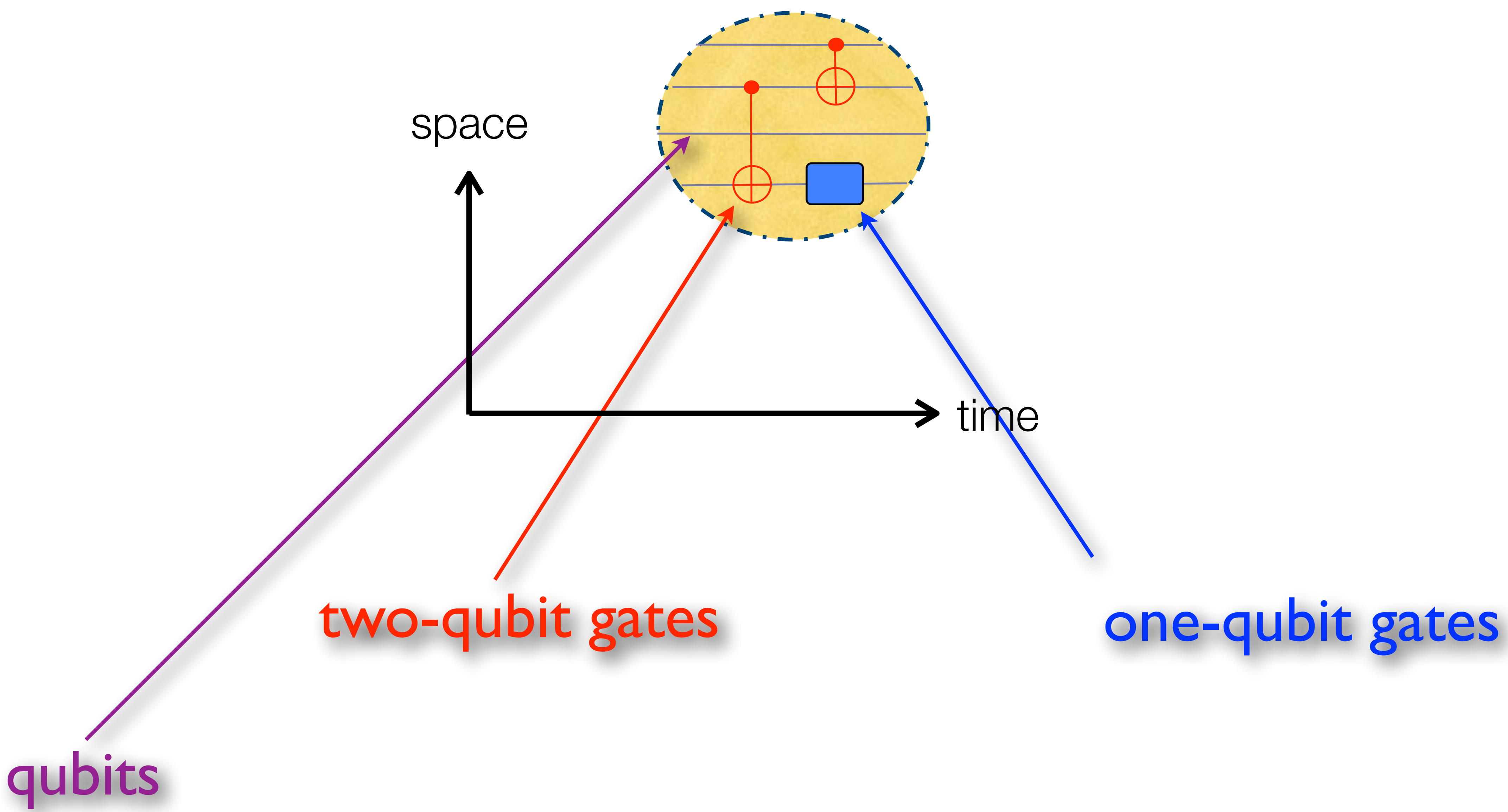
Quantum Computing

1026882660480128156892824817815720568787394176065852715431389395665038064576935989761768690856366535654167724773070835866676998080686541
69943845729583



690907165006927969581076266028000213496541133535635055793802020211742558957148628167790072236787328384488979092561312944282124232404841273038712574219

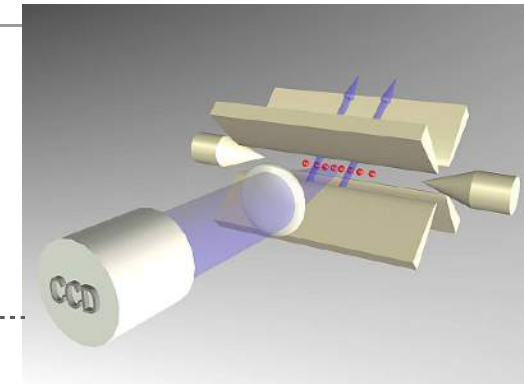
Quantum Computing



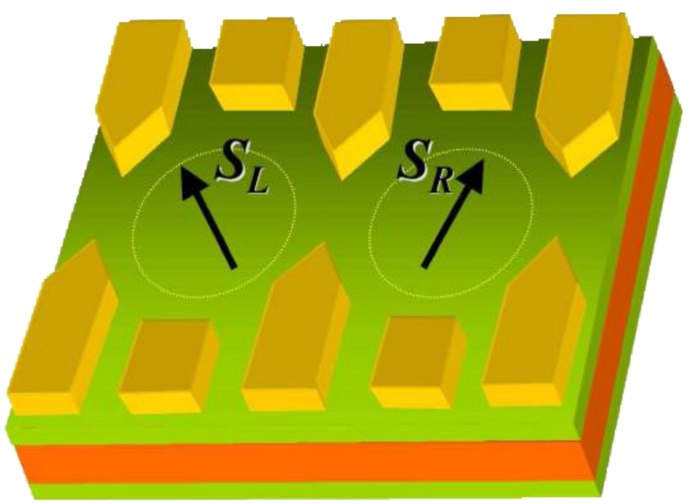
Arbitrary quantum computations can **in principle** be built from these elementary operations.

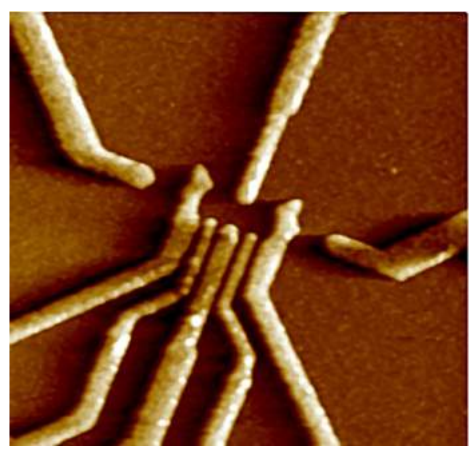
Quantum Bits (Qubits)

- atoms in ion traps
- superconducting circuits
- electron spins in solids



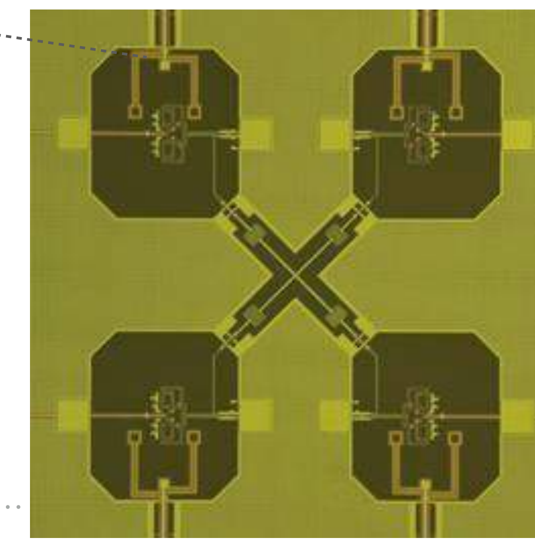
Univ. Innsbruck, NIST, …





TU Delft, Harvard Univ., ETH Zürich , Univ. Copenhagen,….

U Stuttgart, U Chicago, U Ulm, TU Delft, Harvard.,,…



TU Delft, Google, Yale,, NIST, ETHZ…..

intel®

Google

Microsoft

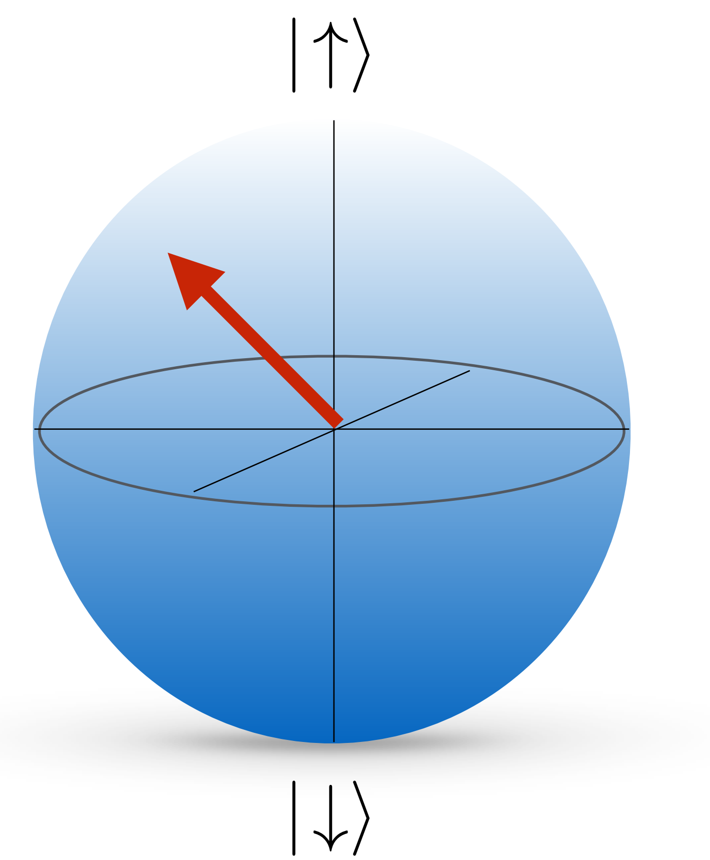
Spin Qubits

- typically spin 1/2 of subatomic particle (electron, atomic nucleus)
- quantum bit (qubit) Loss & DiVincenzo, PRA 1998

$$|0\rangle = |\uparrow\rangle \qquad |1\rangle = |\downarrow\rangle$$

- electron spins in semiconductor quantum dots
- long coherence, robust against charge noise

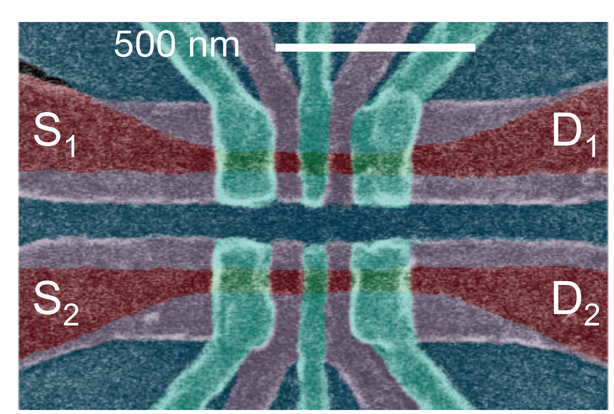
Hanson *et al.*, RMP 2007



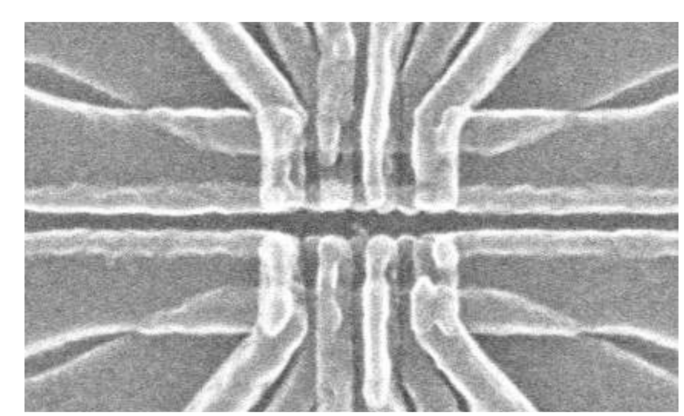
Tarucha group
Ito *et al.*, arXiv:1604.04426 5 QD charge control
Noiri *et al.*, APL 2016 3 QD 3 single spin control

Dzurak group
Veldhorst *et al.*, Nature 2015 2 QD 2 spin qubit gate

Eriksson group / Guo group
Kim *et al.*, Nature 2014 2 QD 3 spin hybrid qubit
Cao *et al.*, PRL 2016 2 QD 5 spin hybrid qubit

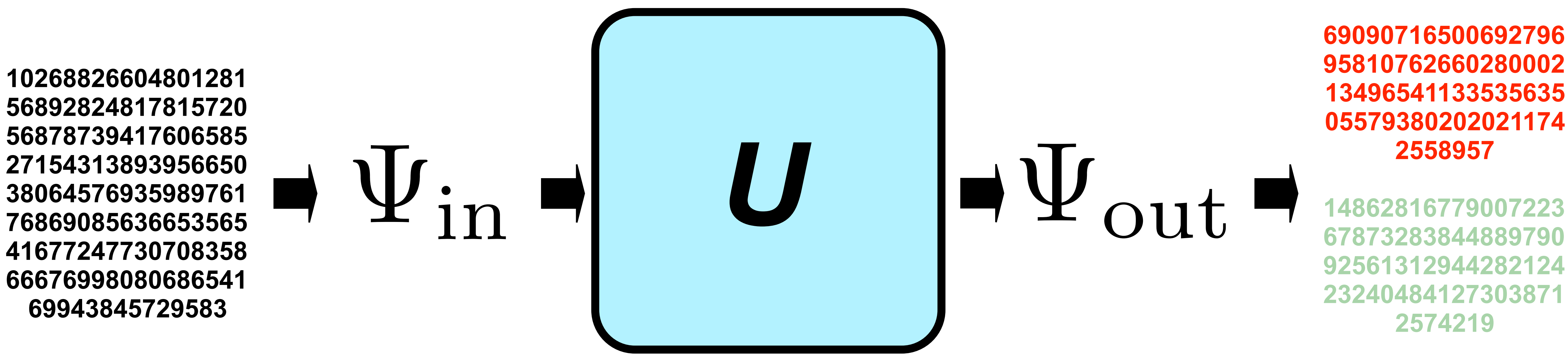


D. M. Zajac *et al.*, APL 2015



SiGe six quantum dot device (Petta group, Princeton)

Quantum Computing

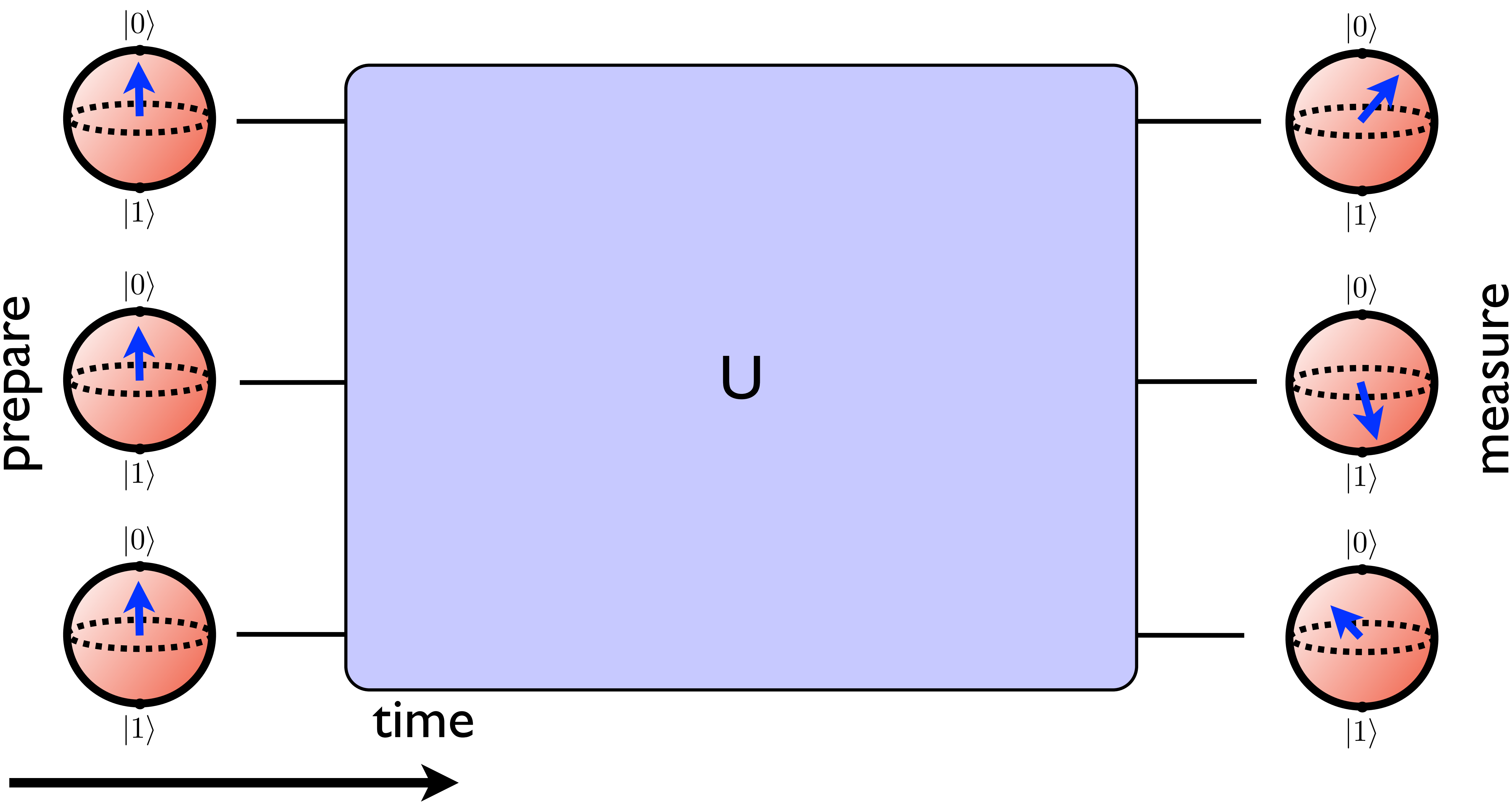


- quantum time evolution

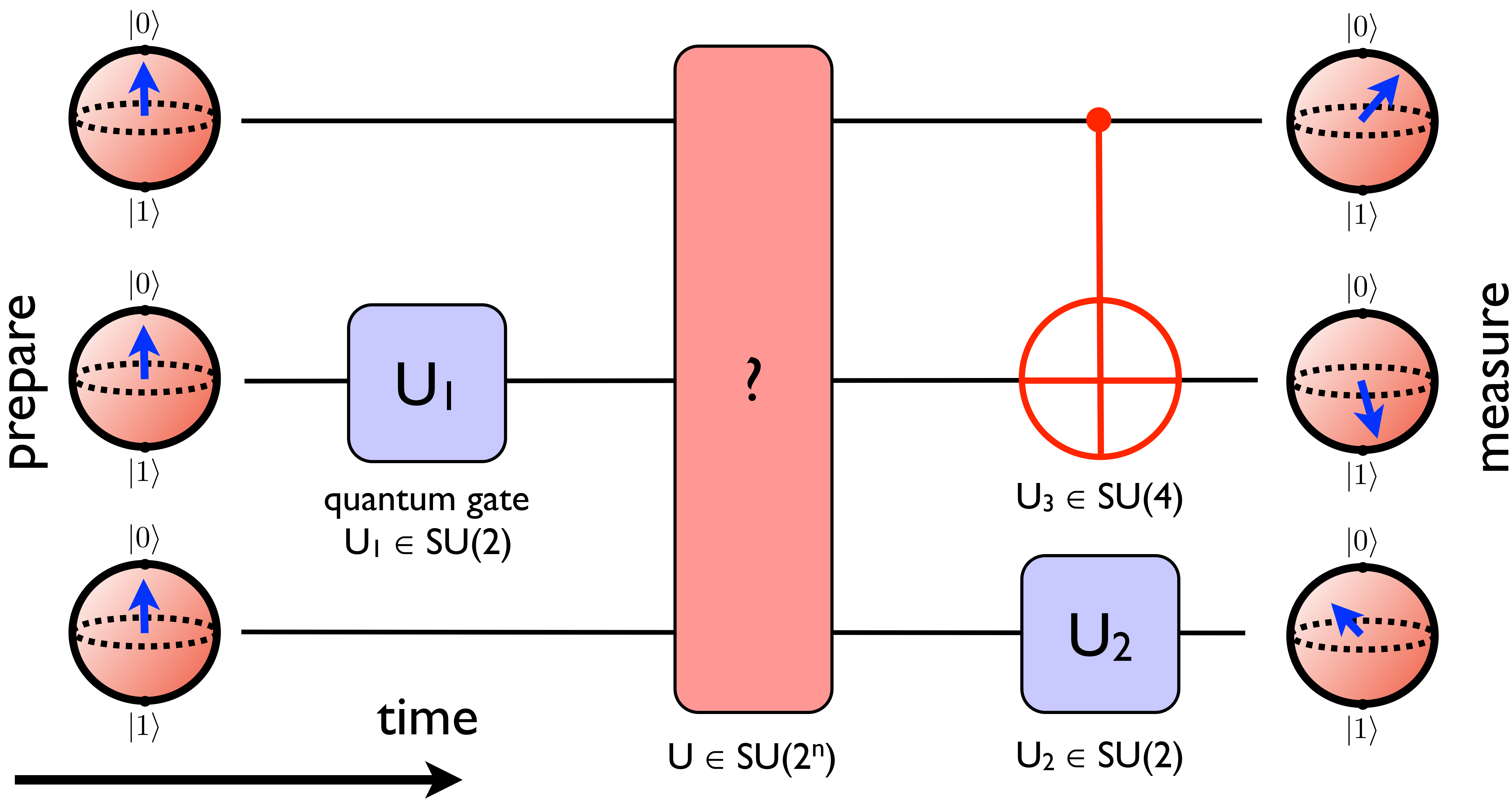
$$\Psi_{\mathrm{out}} = U\Psi_{\mathrm{in}}$$

$$U = \mathrm{T}\exp\left(-\frac{i}{\hbar}\int_0^T H(t)dt\right)$$

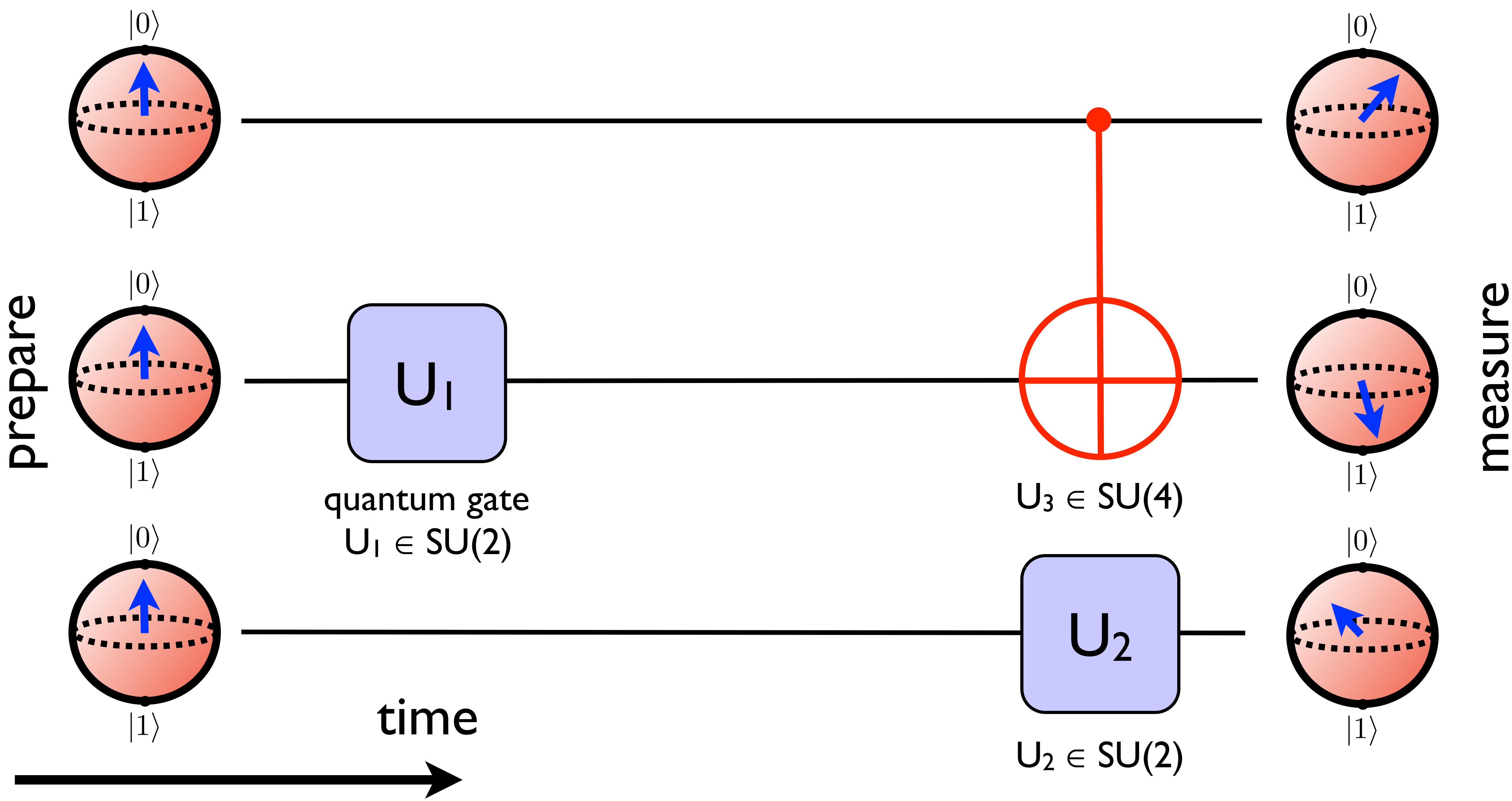
Quantum Gates



Quantum Gates



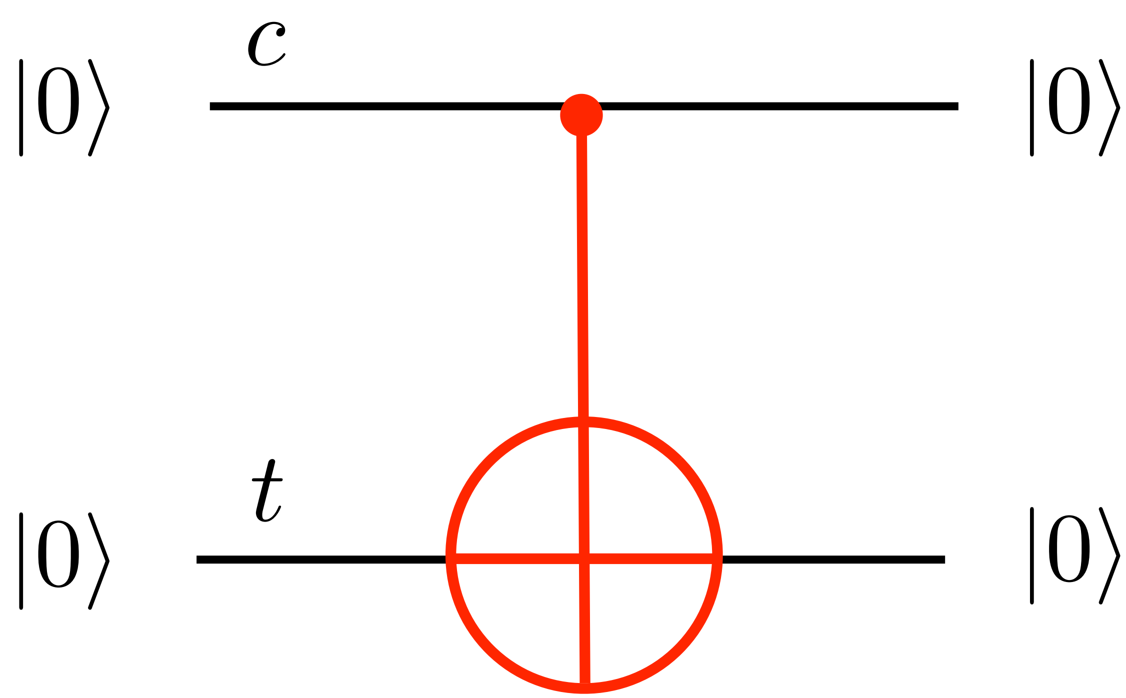
Quantum Gates



A universal set of quantum gates: {CNOT} ∪ SU(2)

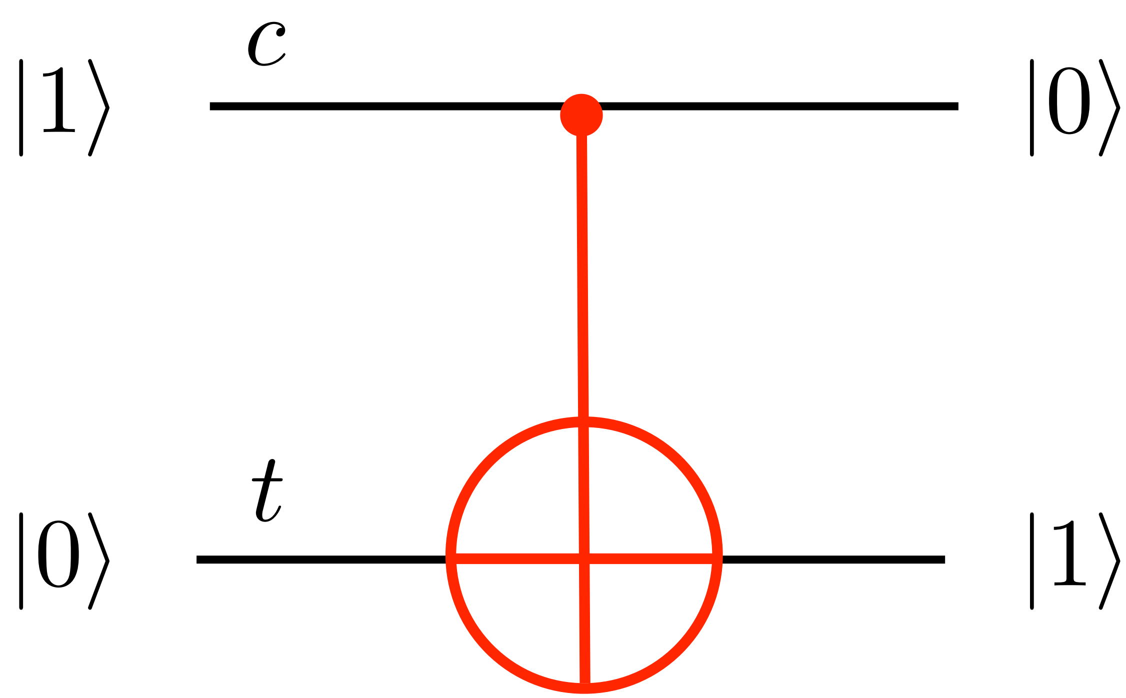
DiVincenzo, Phys. Rev. A (1995), Barenco *et al.*, Phys Rev. A (1995)

Controlled NOT gate (CNOT gate)



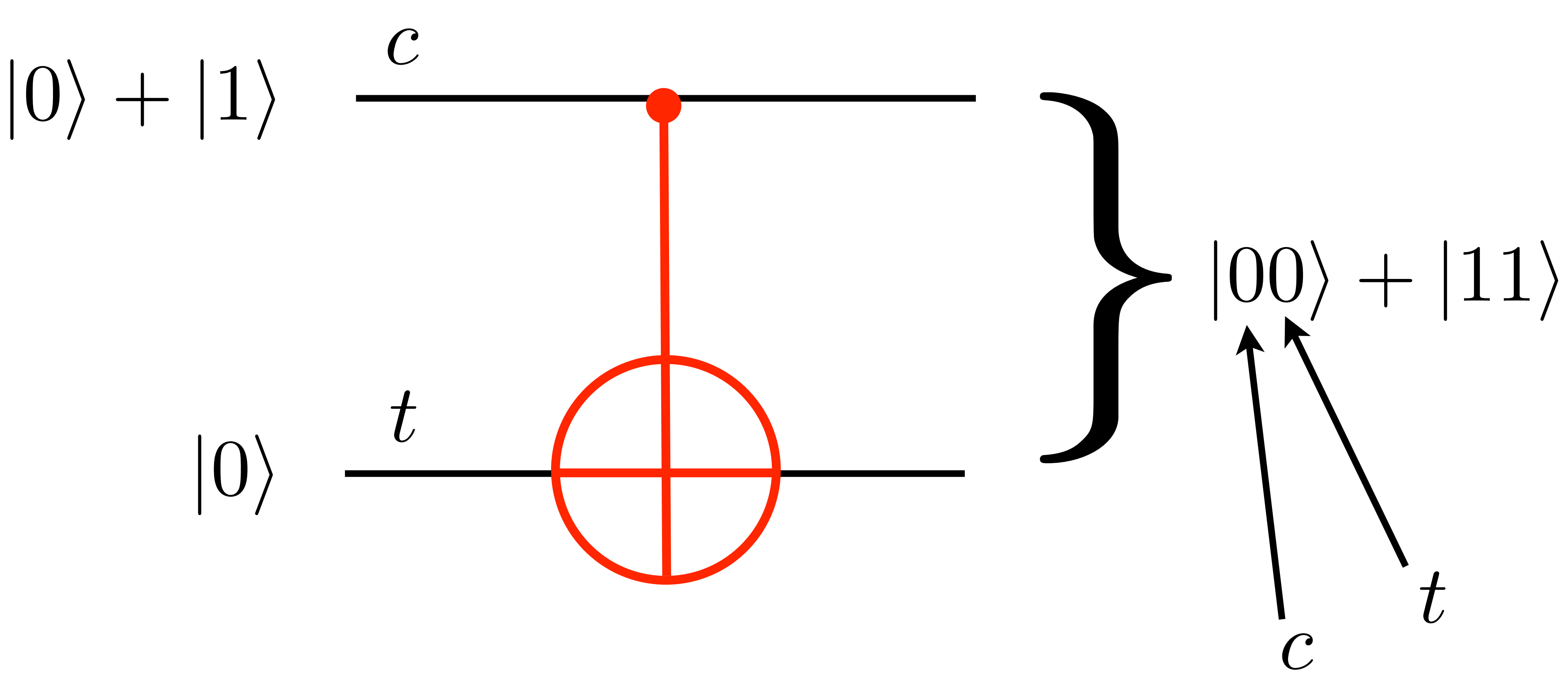
$$U_{\mathrm{CNOT}} : |c, t\rangle \longrightarrow |c, c \oplus t\rangle$$

Controlled NOT gate (CNOT gate)



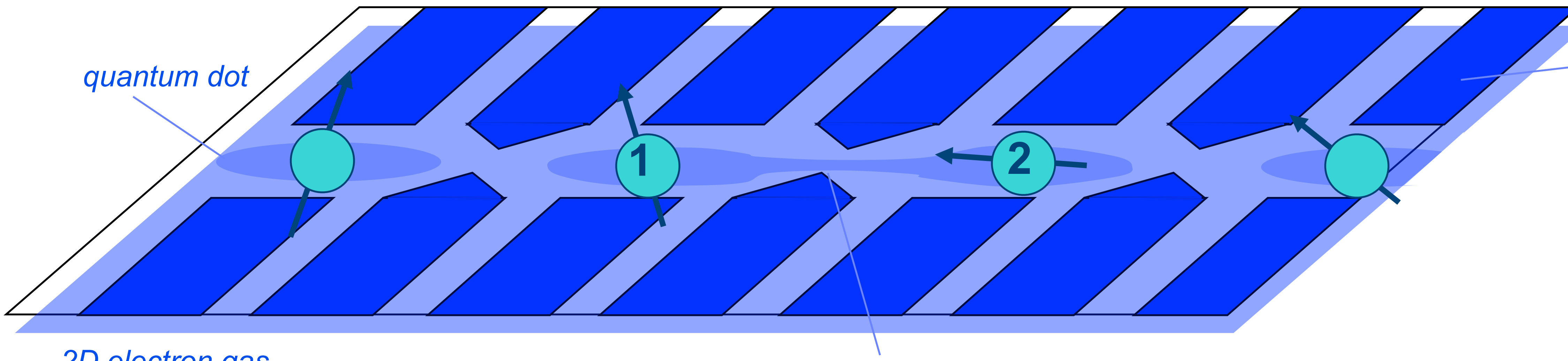
$$U_{\mathrm{CNOT}} : |c, t\rangle \longrightarrow |c, c \oplus t\rangle$$

Controlled NOT gate (CNOT gate)



$$U_{\mathrm{CNOT}} : |c, t\rangle \longrightarrow |c, c \oplus t\rangle$$

Spin Qubits



$|0\rangle$ = ↑

$|1\rangle$ = ↓

Loss & DiVincenzo, 1998

$$H = \sum_{\langle i,j \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i g_i(t)\mu_B \mathbf{B}_i(t) \cdot \mathbf{S}_i$$

exchange coupling

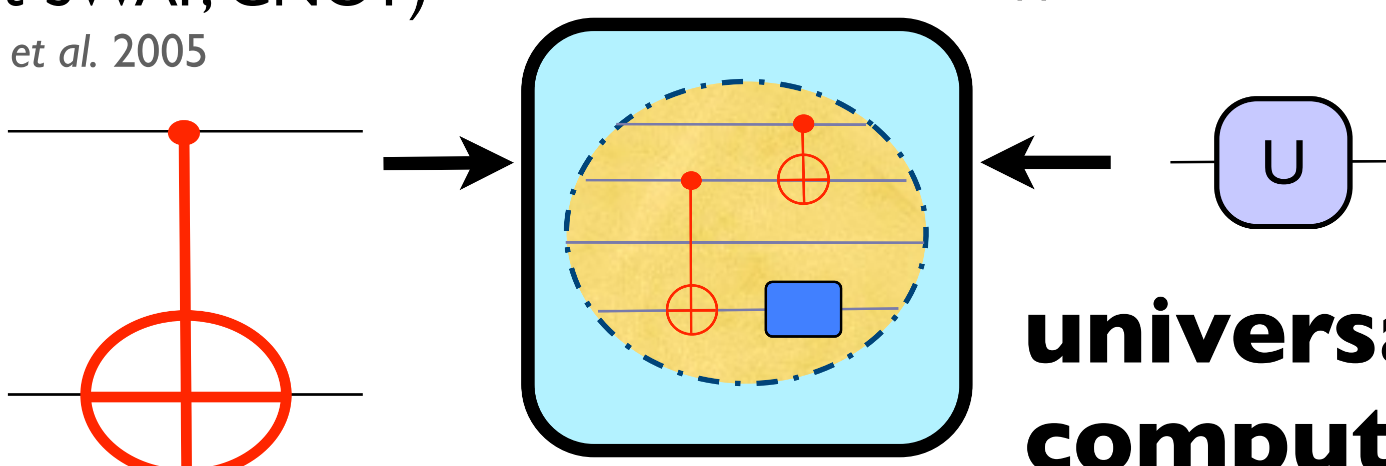
local Zeeman effect

two-qubit gates (sqrt-SWAP, CNOT)

Petta *et al.* 2005

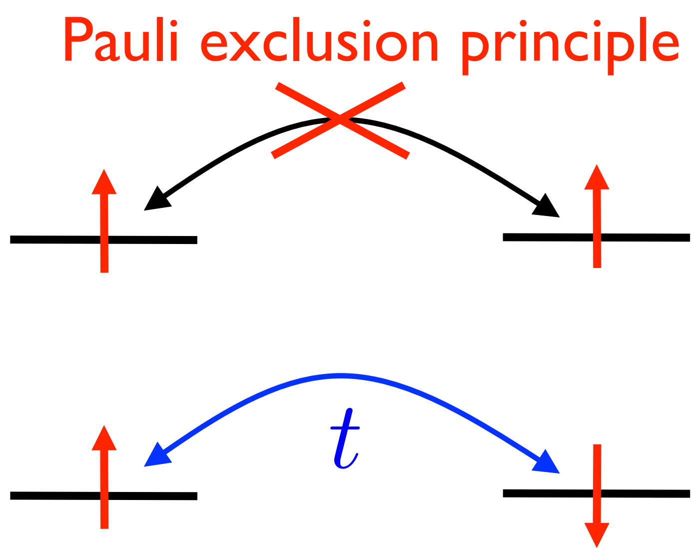
one-qubit gates, SU(2)

Koppens *et al.* 2006



universal quantum computation

Exchange coupling



spin triplet
$s = 1$

spin singlet
$s = 0$

$$E_{\text{triplet}} = E_0$$

$$E_{\text{singlet}} = E_0 - 4t^2/U$$

$$J = E_{\text{triplet}} - E_{\text{singlet}} = -4t^2/U$$

$$H = -JP_{\text{singlet}} + E_0$$

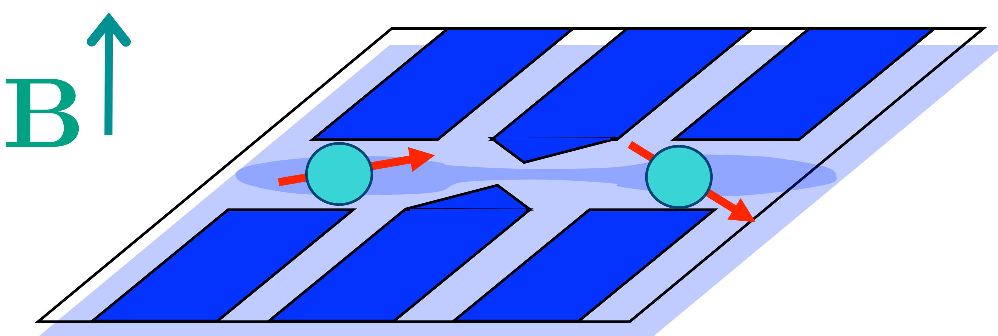
$S^2 = s(s+1)$

$$P_{\text{singlet}} = |S\rangle\langle S| = \frac{1}{2}(2 - S^2) = 1 - \frac{1}{2}(S_1^2 - S_2^2 - 2\mathbf{S}_1 \cdot \mathbf{S}_2) = \frac{1}{4} - \mathbf{S}_1 \cdot \mathbf{S}_2$$

$$= P_{\text{as}}$$

$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2 + \cancel{\text{const.}}$$

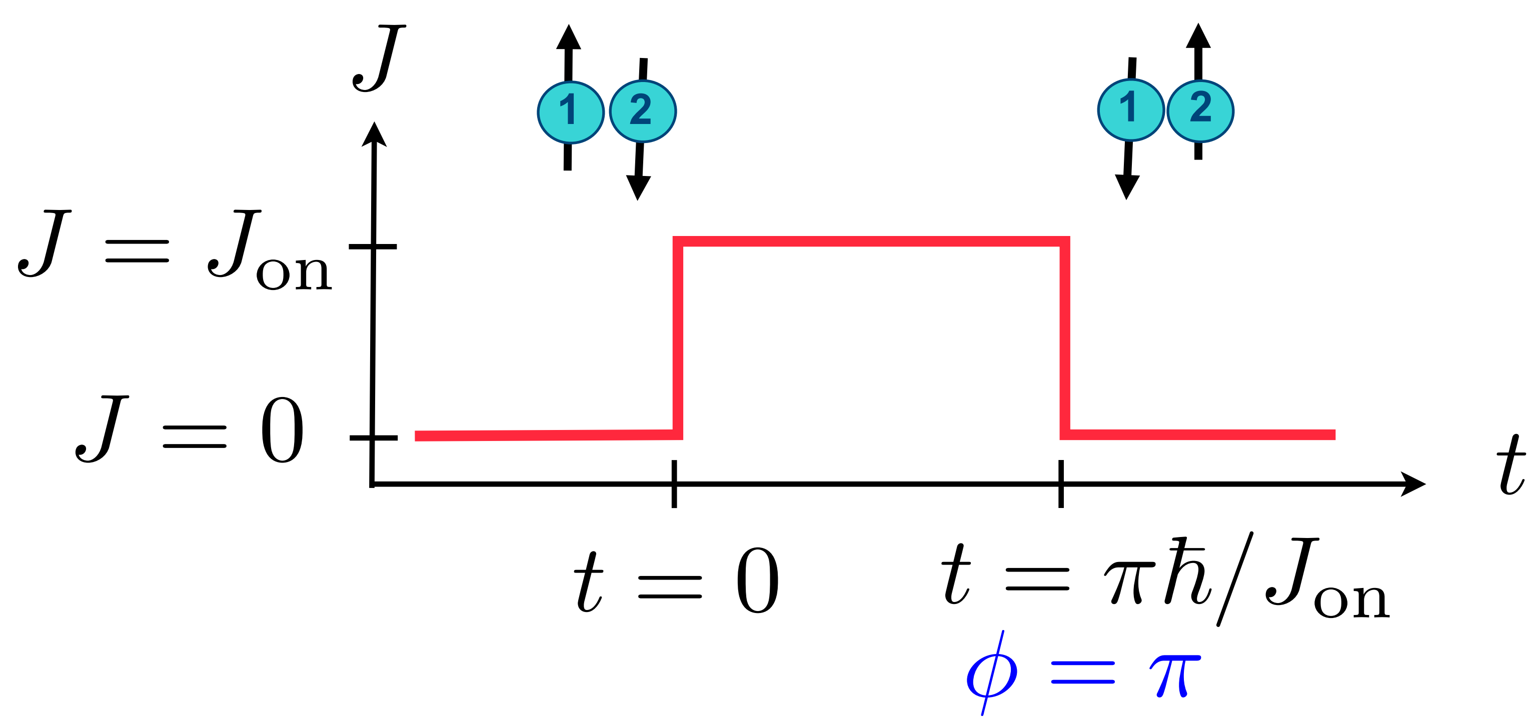
Spin-spin interactions



$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2 = -JP_{\mathrm{as}}$$

$$U(\phi) = e^{-\frac{i}{\hbar}\int_0^t dt' H(t')} = e^{-i\phi P_{\mathrm{as}}} = \mathbb{1} + (e^{i\phi} - 1)P_{\mathrm{as}}$$

$\phi = Jt/\hbar$

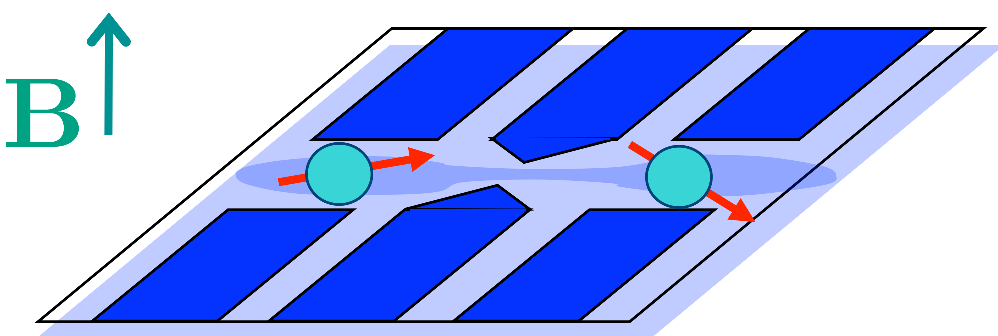


SWAP

$= 1 - 2P_{\mathrm{as}}$

Loss & DiVincenzo, Phys. Rev. A (1998).
Petta *et al.*, *Science* (2005).

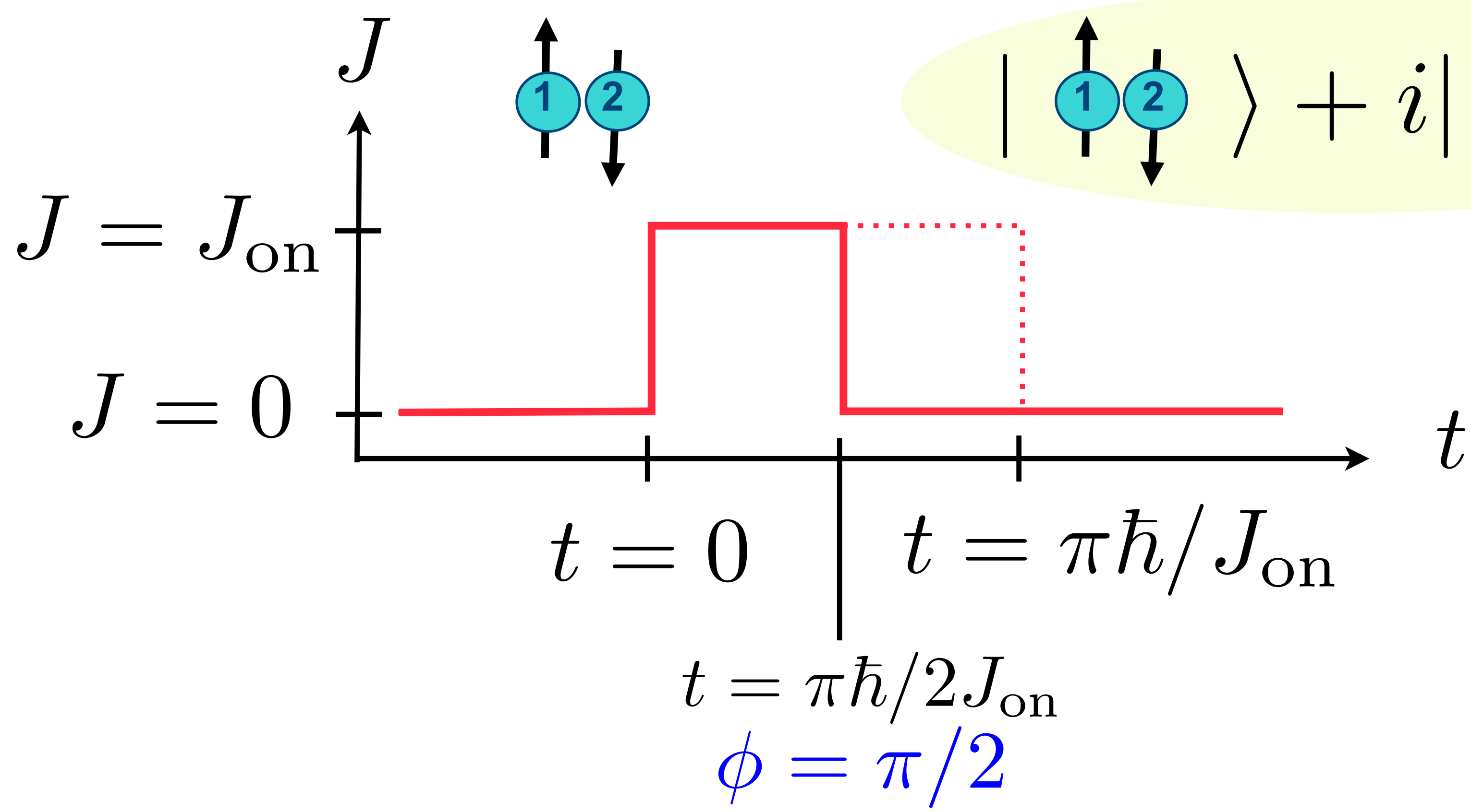
Spin-spin interactions



$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2 = -JP_{\mathrm{as}}$$

$$U(\phi) = e^{-i\int_0^t dt'\,H(t')} = \mathbb{1} + \left(e^{i\phi} - 1\right)P_{\mathrm{as}}$$

$\phi = Jt/\hbar$



$$|\uparrow_1\downarrow_2\rangle + i|\downarrow_1\uparrow_2\rangle$$

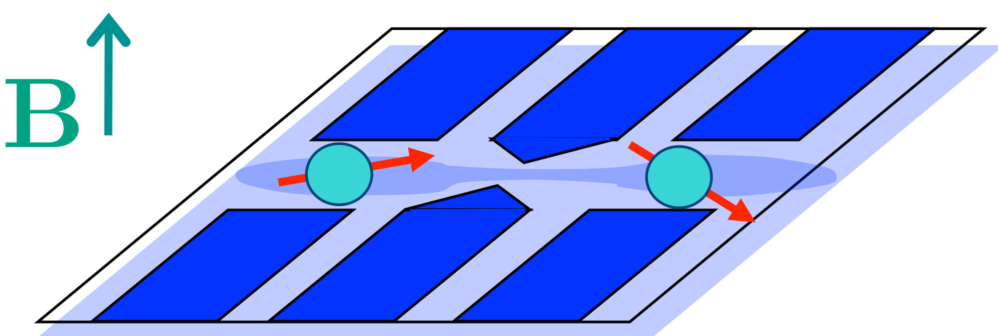
$$\sqrt{\mathrm{SWAP}}$$

$$= \frac{1+i}{2}\mathbb{1} + \frac{1-i}{2}\mathrm{SWAP}$$

entangled state
EPR pair

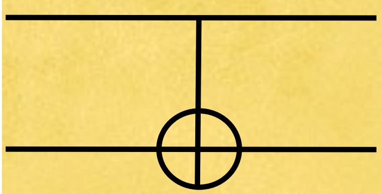
Loss & DiVincenzo, Phys. Rev. A (1998).
Petta *et al.*, *Science* (2005).

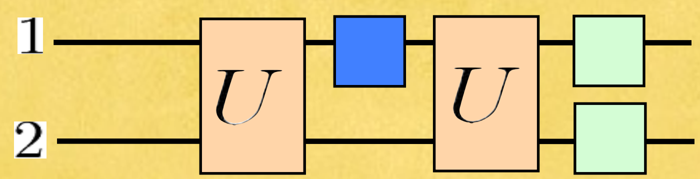
Spin-spin interactions

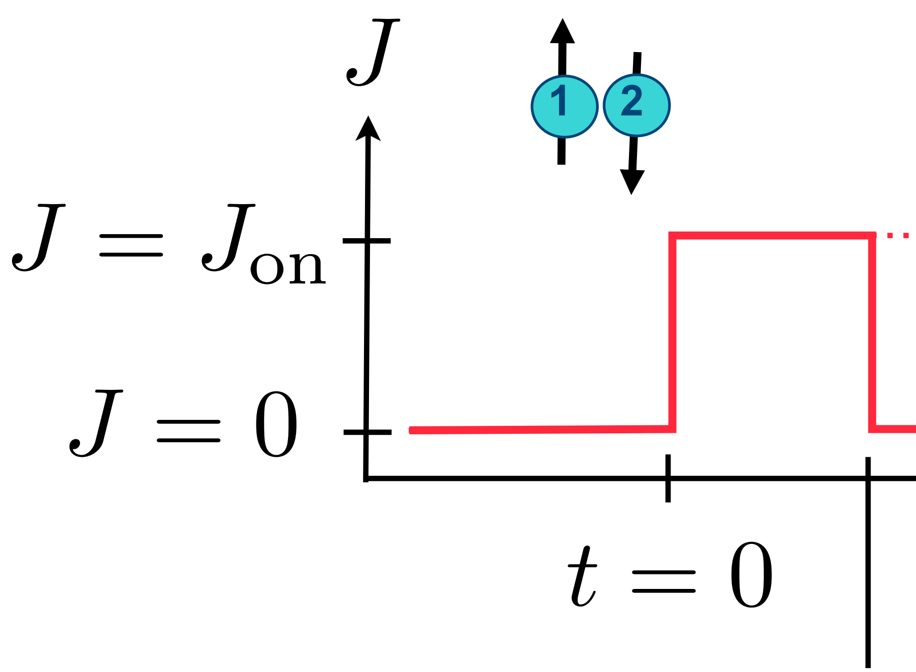


$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2 = -JP_{\text{as}}$$

$$U_{CNOT} = e^{-i\pi/2} e^{-i\pi S_2^y/2} e^{i\pi S_1^z/2} e^{-i\pi S_2^z/2} U(t) e^{i\pi S_1^z} U(t) e^{i\pi S_2^y/2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

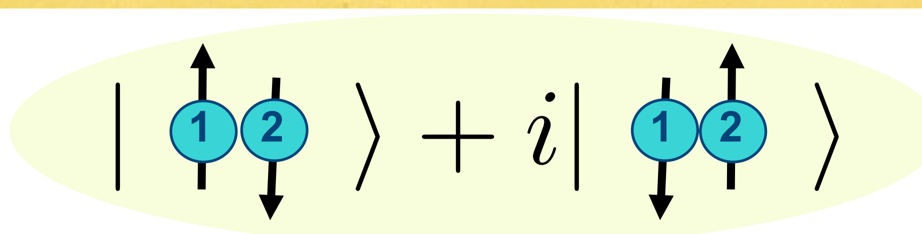






$t = \pi\hbar/2J_{\text{on}}$

$\phi = \pi/2$



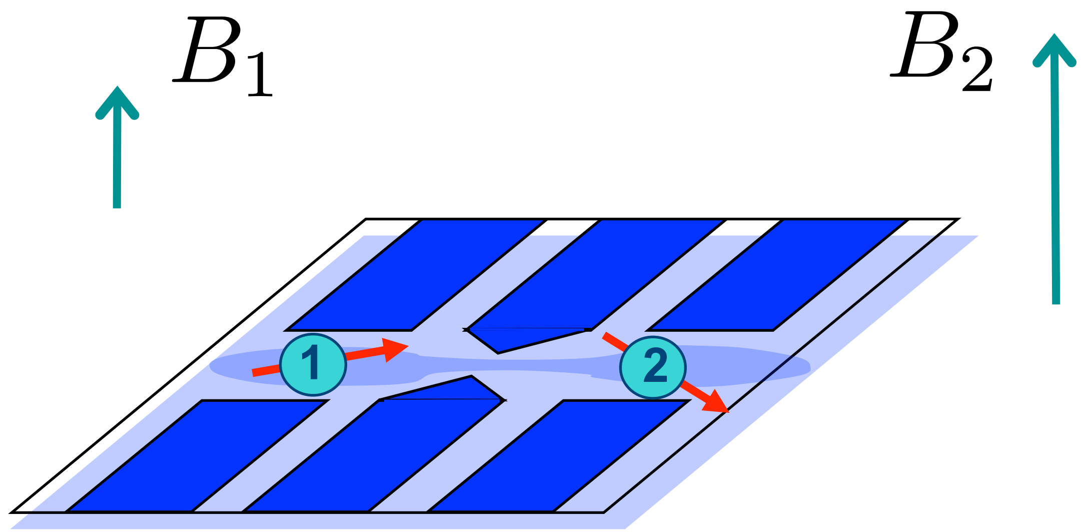
$$\sqrt{\text{SWAP}}$$

$$= \frac{1+i}{2}\mathbb{1} + \frac{1-i}{2}\text{SWAP}$$

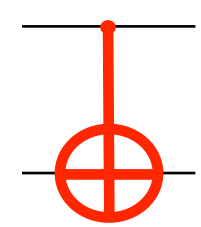
entangled state
EPR pair

Loss & DiVincenzo, Phys. Rev. A (1998).
Petta *et al.*, *Science* (2005).

Parallel pulsing

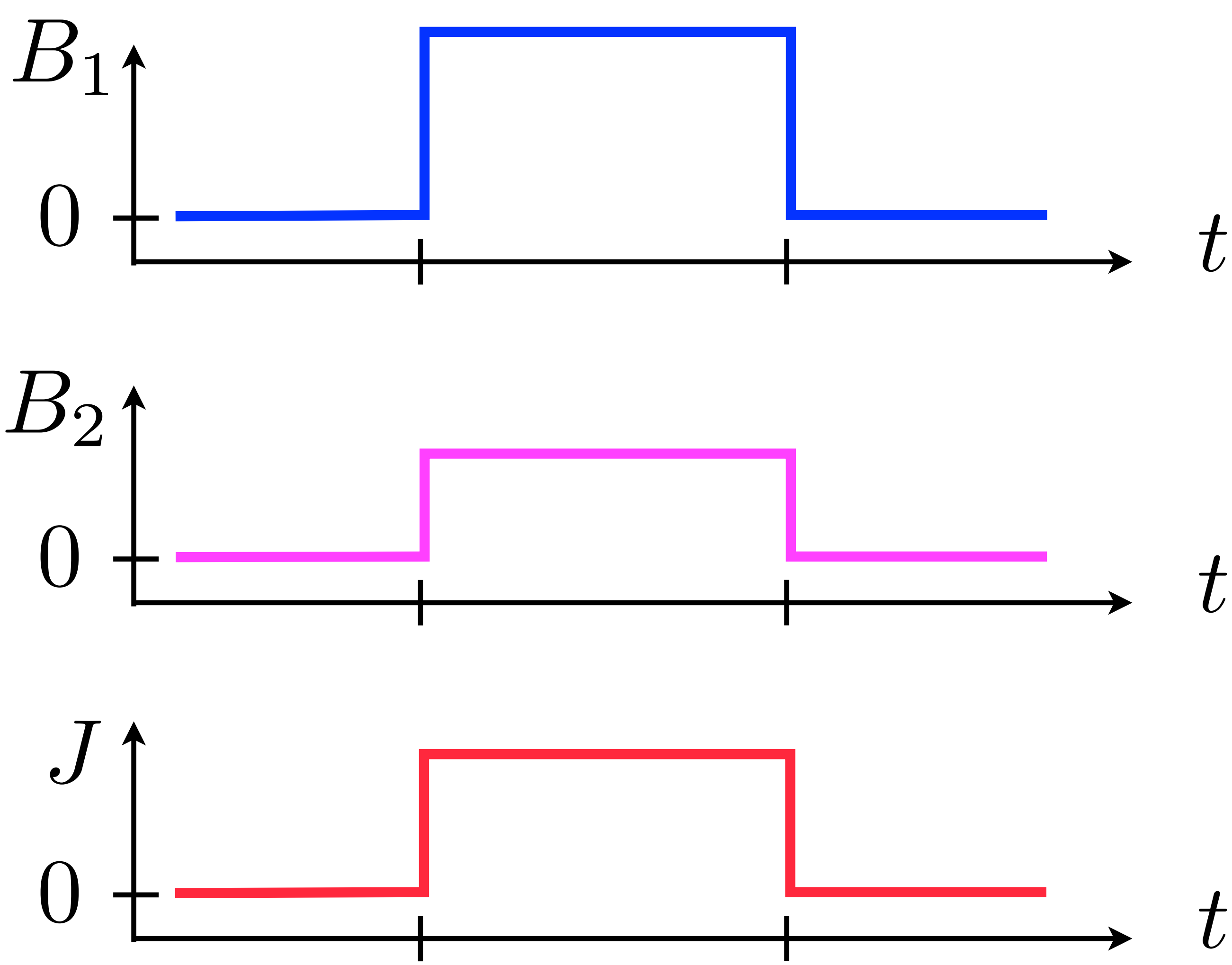


$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2 + B_1 S_1^z + B_2 S_2^z$$



$$B_1 = \frac{J}{2}\left(1 + \sqrt{3}\right)$$

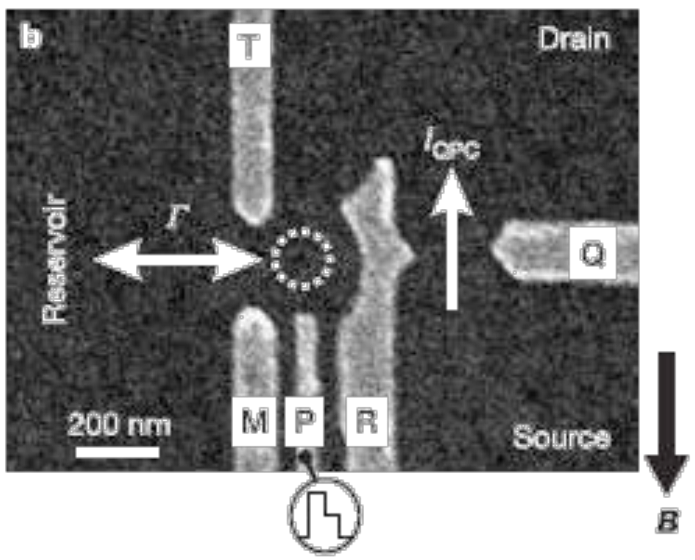
$$B_2 = \frac{J}{2}\left(1 - \sqrt{3}\right)$$



$$t_{\mathrm{op}} \sim \hbar / J$$

GB, Loss, DiVincenzo & Smolin, Phys. Rev. B (1999).

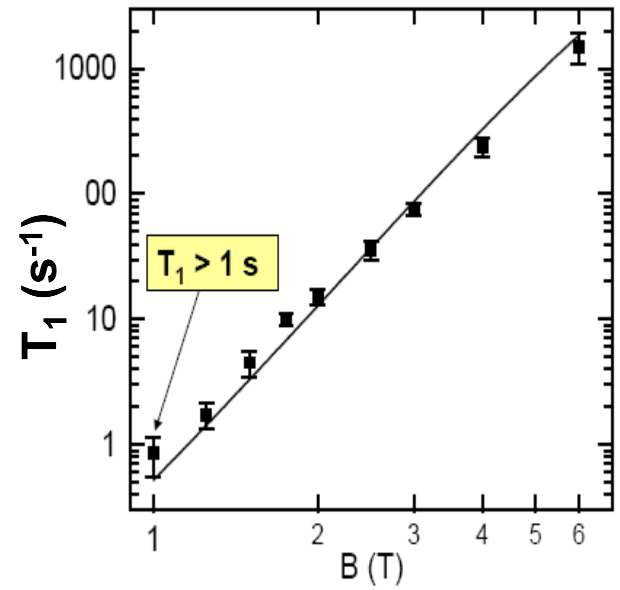
Early experimental breakthroughs in GaAs



single spin read-out, T_1

Elzerman *et al.*, PRB (2003);

--- Nature (2004)



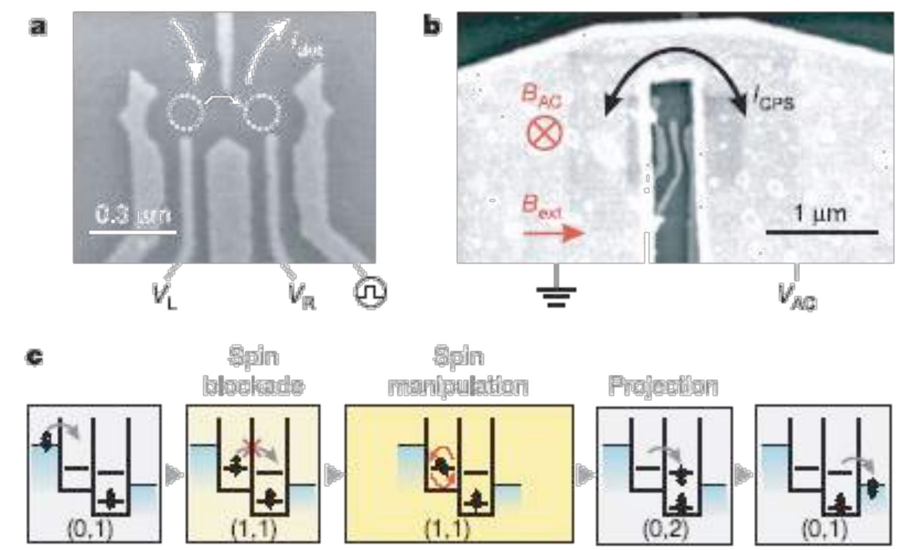
long T_1 at low fields

Amasha *et al.* PRL (2008)

single-spin ESR

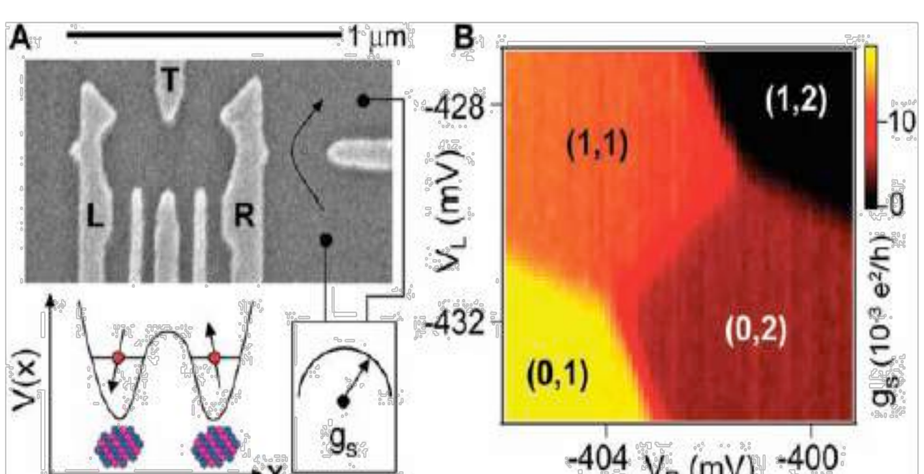
Koppens *et al.*, Nature (2006).

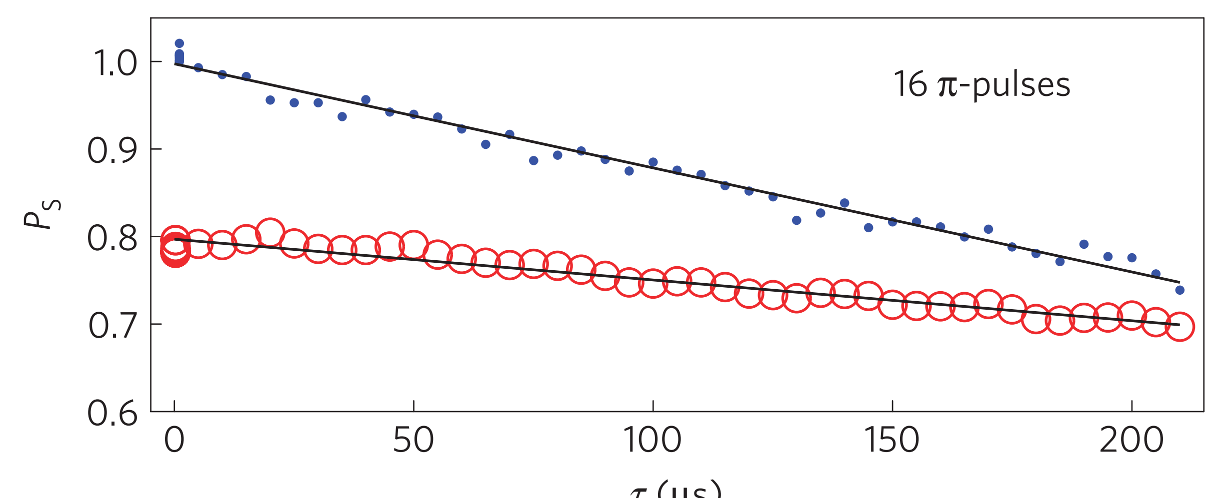
Nowack et al., Science (2007).



sqrt-swap, spin echo, T_2^*

Petta *et al.*, Science (2005).

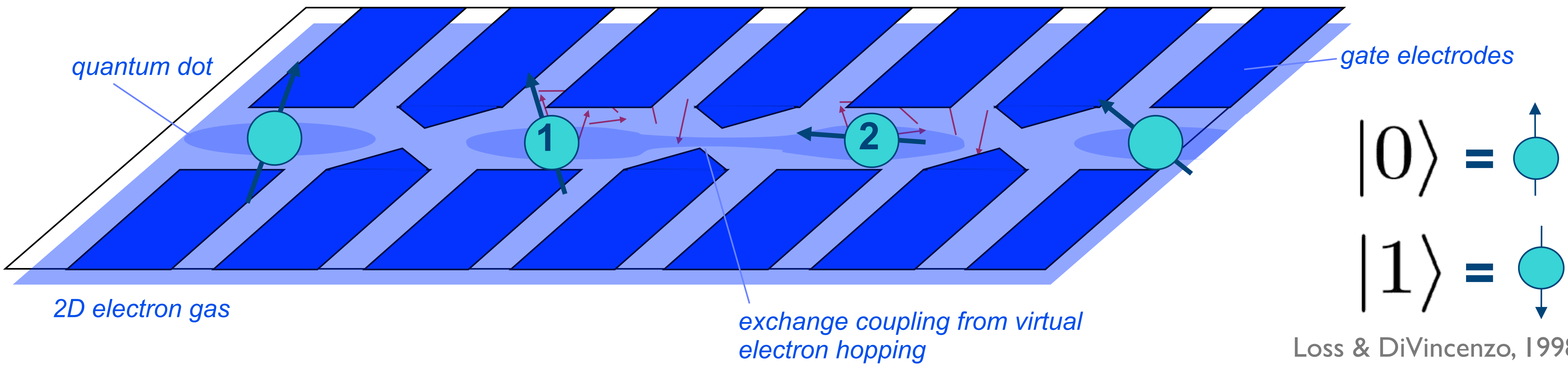




spin coherence time exceeding 200 μs using CPMG pulses

Bluhm *et al.*, Nature Phys. (2010).

What could possibly go wrong..?



$$H = \sum_{\langle i,j \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i g_i(t)\mu_B \mathbf{B}_i(t) \cdot \mathbf{S}_i$$

exchange coupling

local Zeeman effect → **multi-spin qubits**

$$+ \sum_i \mathbf{S}_i \sum_{k}^{N} A_{ik} \mathbf{I}_k$$

nuclear spins → **materials**

T_2^*

Burkard, Loss, DiVincenzo 1999
Coish & Loss 2004, 2005
Petta *et al.* 2005

Spin qubits are open quantum systems

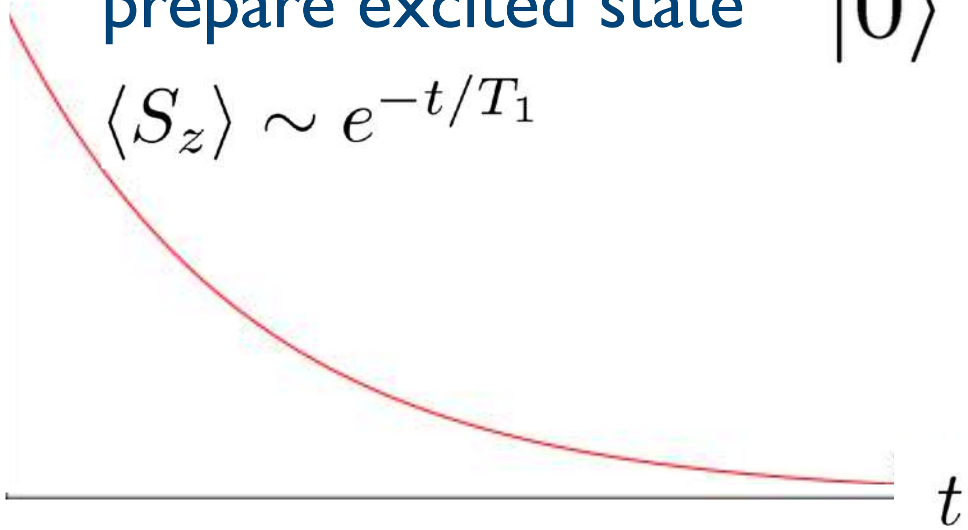
spin in a magnetic field B

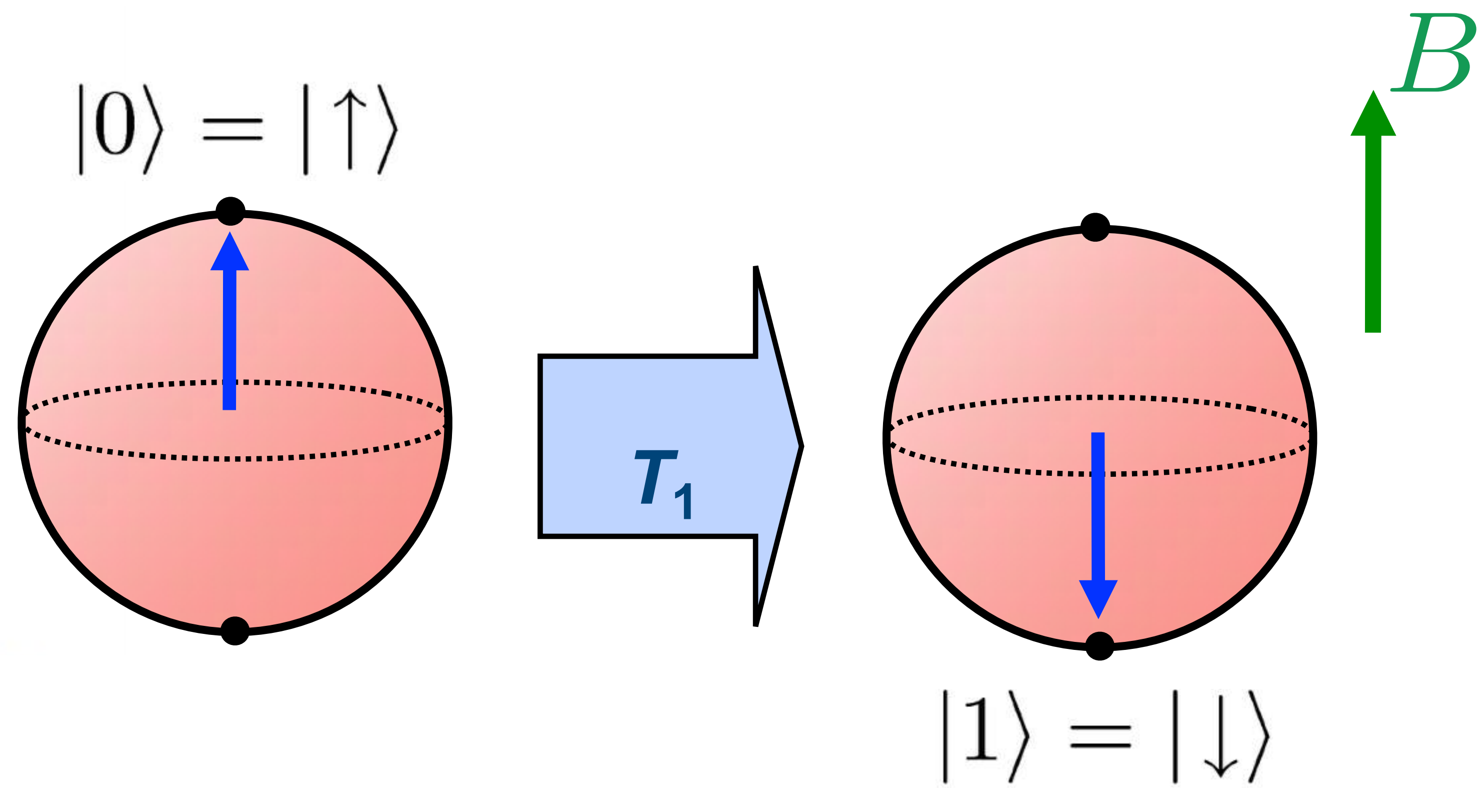
(zero temperature T=0, g-factor g>0)

relaxation T_1

prepare excited state $|0\rangle$

$\langle S_z \rangle \sim e^{-t/T_1}$



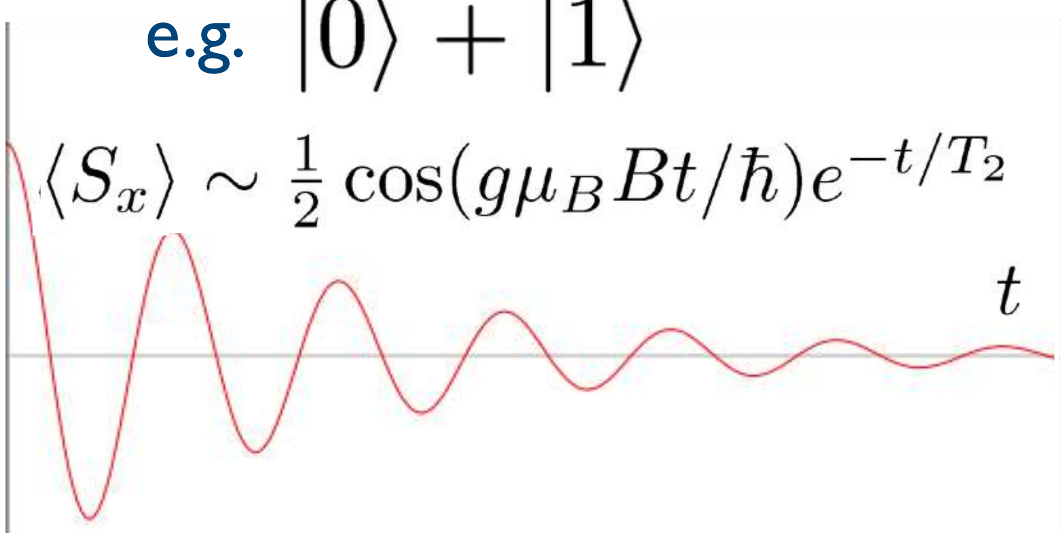


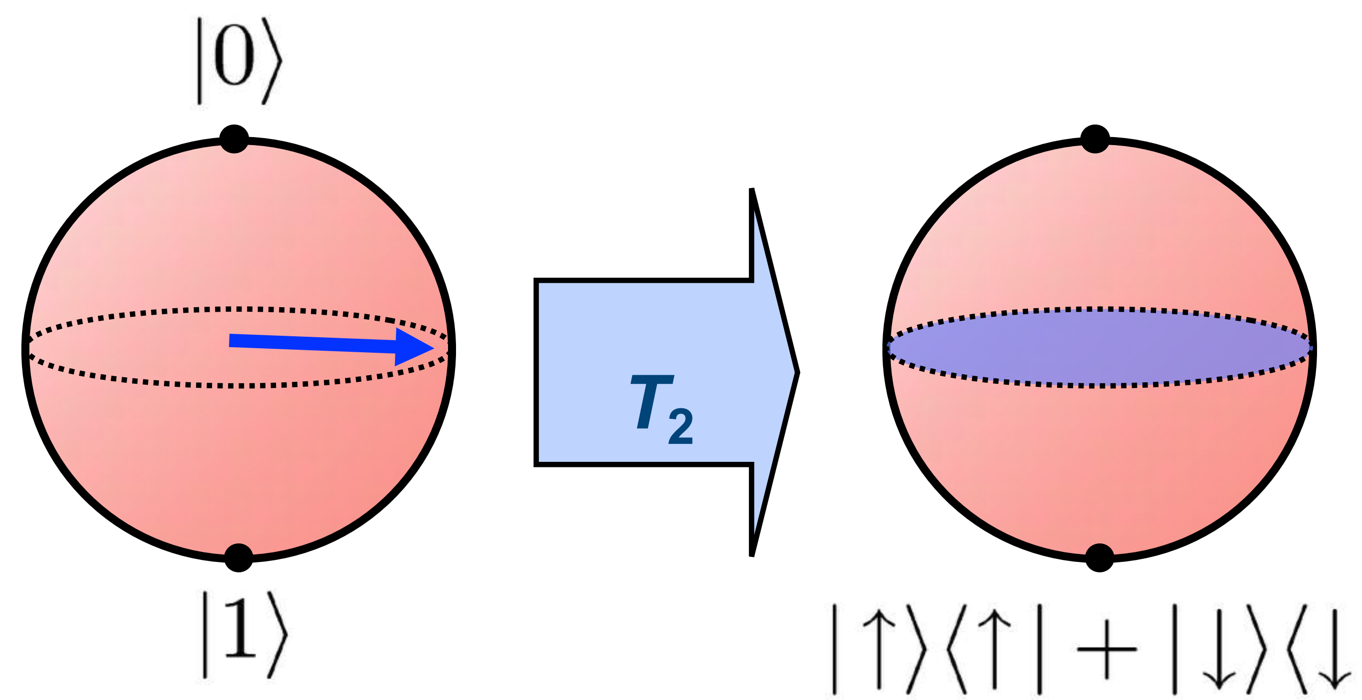
decoherence T_2

prepare transverse state,

e.g. $|0\rangle + |1\rangle$

$\langle S_x \rangle \sim \frac{1}{2} \cos(g\mu_B Bt/\hbar) e^{-t/T_2}$





Spin qubits are open quantum systems

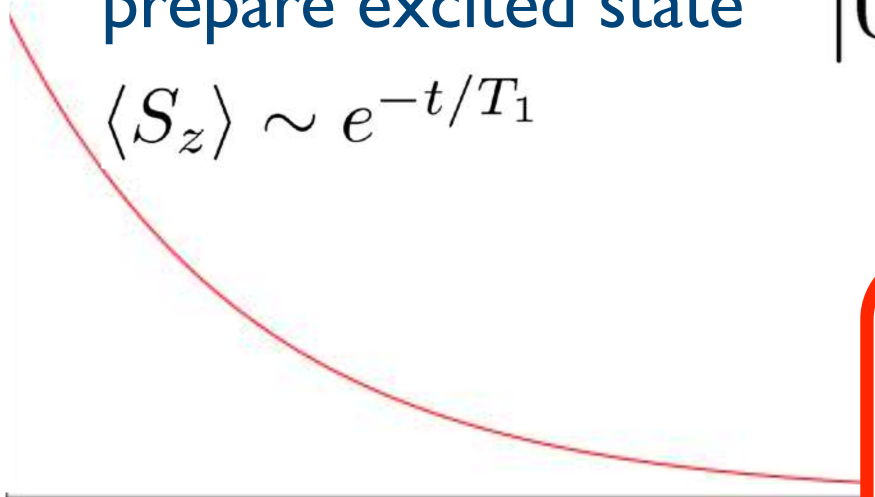
spin in a magnetic field *B*

(zero temperature *T=0*, g-factor *g>0*)

relaxation T_1

prepare excited state $|0\rangle$

$\langle S_z\rangle \sim e^{-t/T_1}$

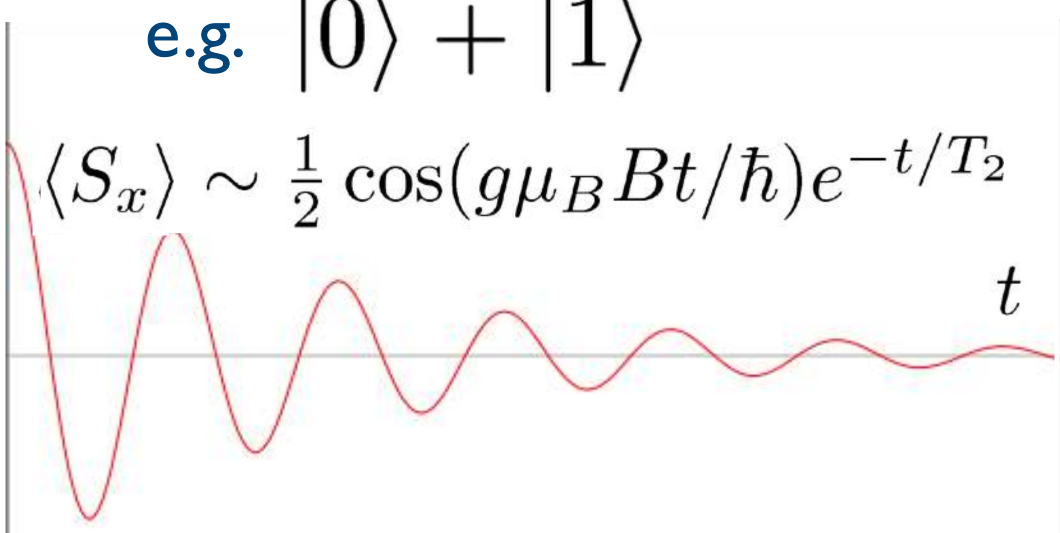


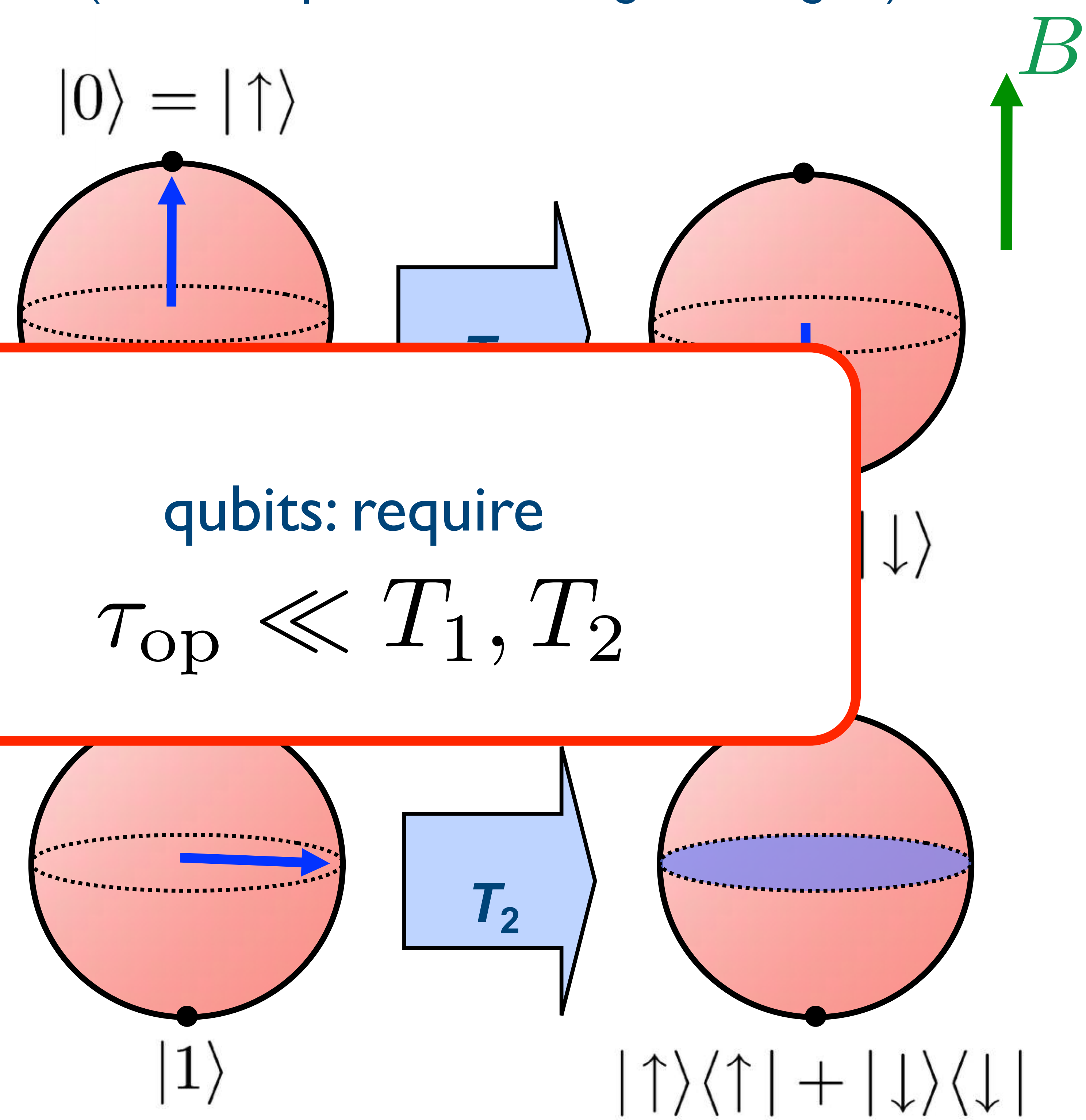
decoherence T_2

prepare transverse state,

e.g. $|0\rangle + |1\rangle$

$\langle S_x\rangle \sim \frac{1}{2}\cos(g\mu_B Bt/\hbar)e^{-t/T_2}$





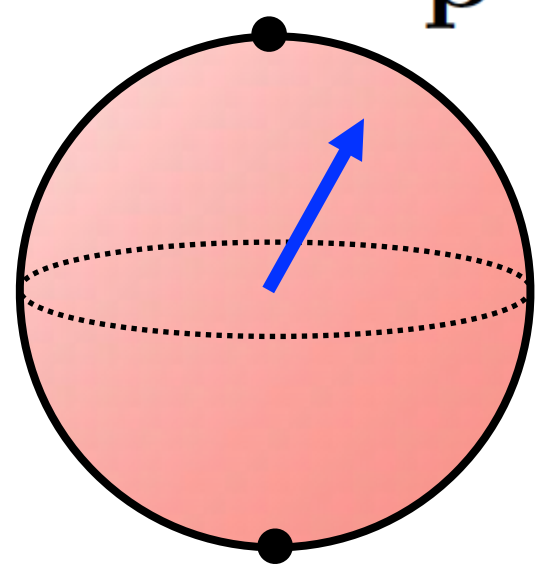
qubits: require

$$\tau_{\mathrm{op}} \ll T_1, T_2$$

Single spin in the Markovian limit: Bloch equations

Bloch vector

$$\mathbf{p} = \mathrm{Tr}(\boldsymbol{\sigma}\rho_S)$$



Bloch equation

$$\dot{\mathbf{p}} = \boldsymbol{\omega} \times \mathbf{p} - R\mathbf{p} + \mathbf{p}_0$$

$$\boldsymbol{\omega} = (0, 0, \omega)$$

Relaxation matrix in secular approximation
(up to frequency renormalization)

$$\tilde{R} = \begin{pmatrix} T_2^{-1} & 0 & 0 \\ 0 & T_2^{-1} & 0 \\ 0 & 0 & T_1^{-1} \end{pmatrix}$$

Exponential decay of transverse (x,y) components

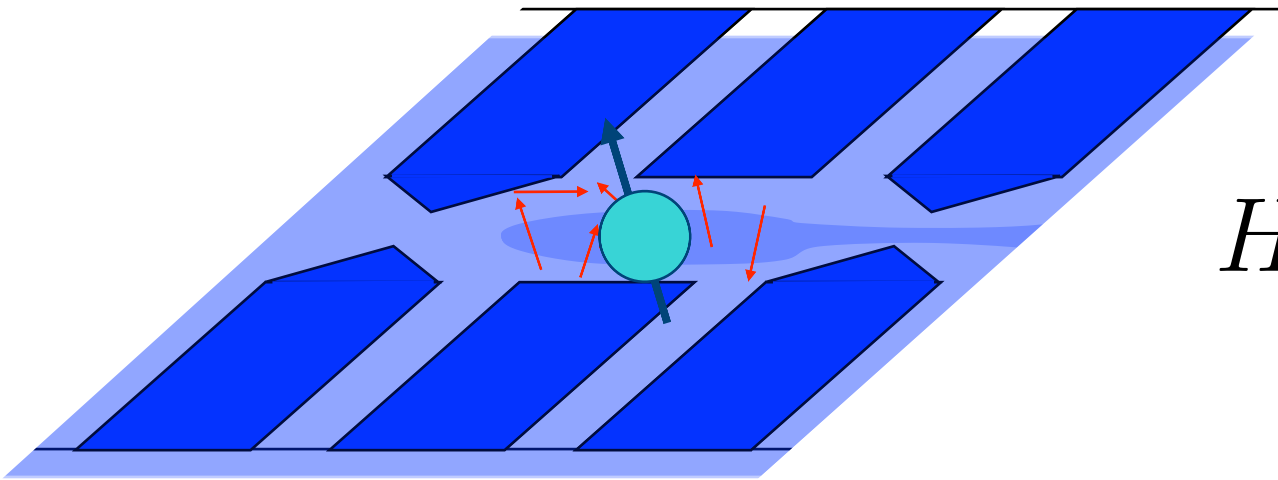
Exponential decay of longitudinal z component

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}$$

review article:

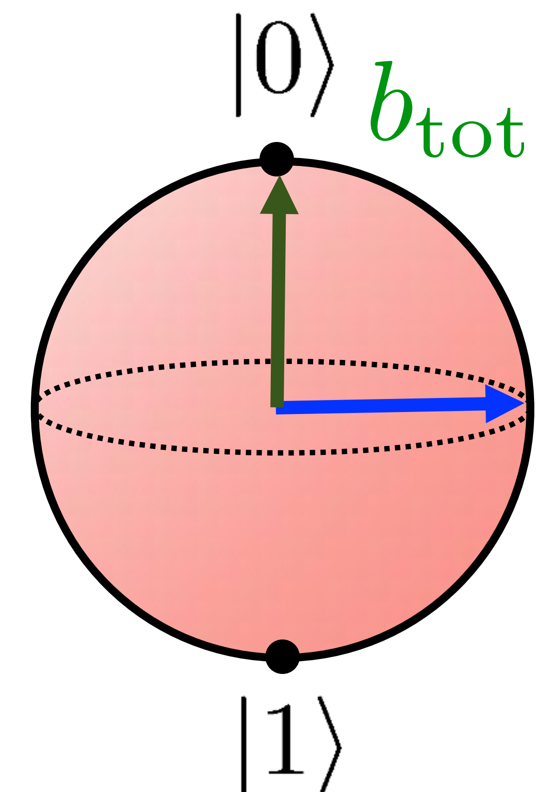
L. Chirolli and G. Burkard, Adv. Phys. 57, 225 (2008)

Nuclear spin induced dephasing



quantum operator

$$H = S_z \left(b + \hat{b}_z^{\mathrm{n}} \right) \qquad \hat{b}_z^n = \sum_k A_k \hat{I}_k^z$$

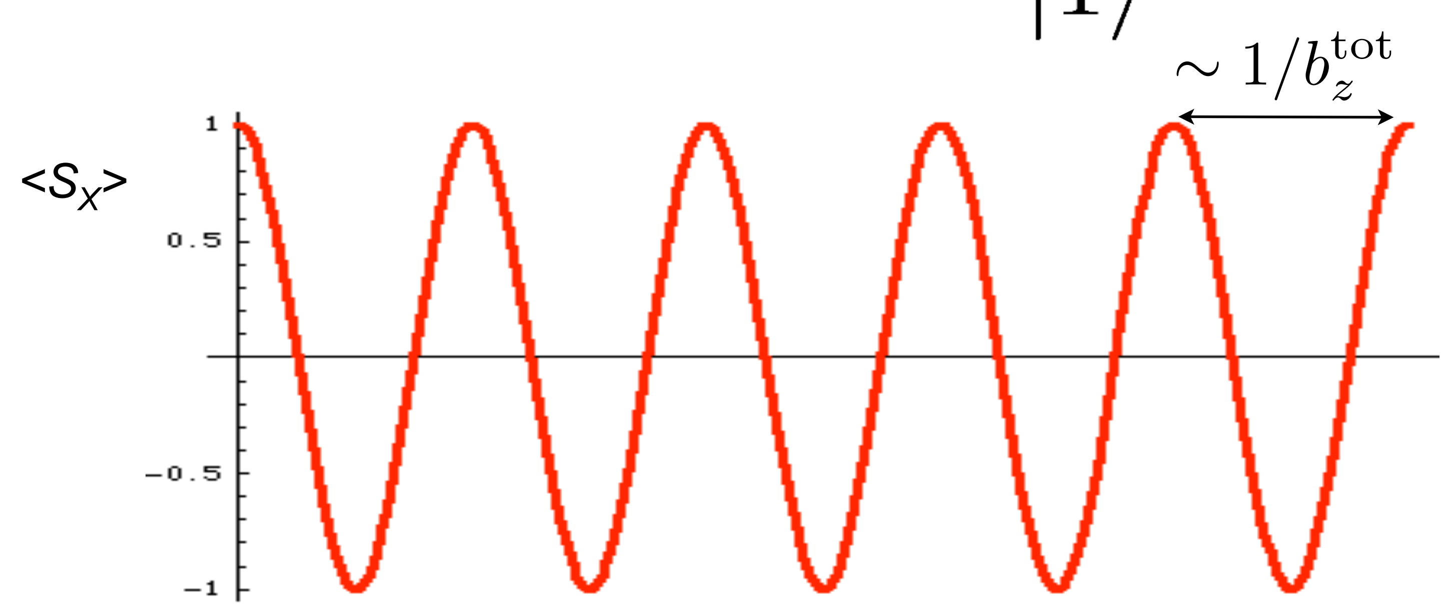


initial state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\psi^{\mathrm{n}}\rangle$

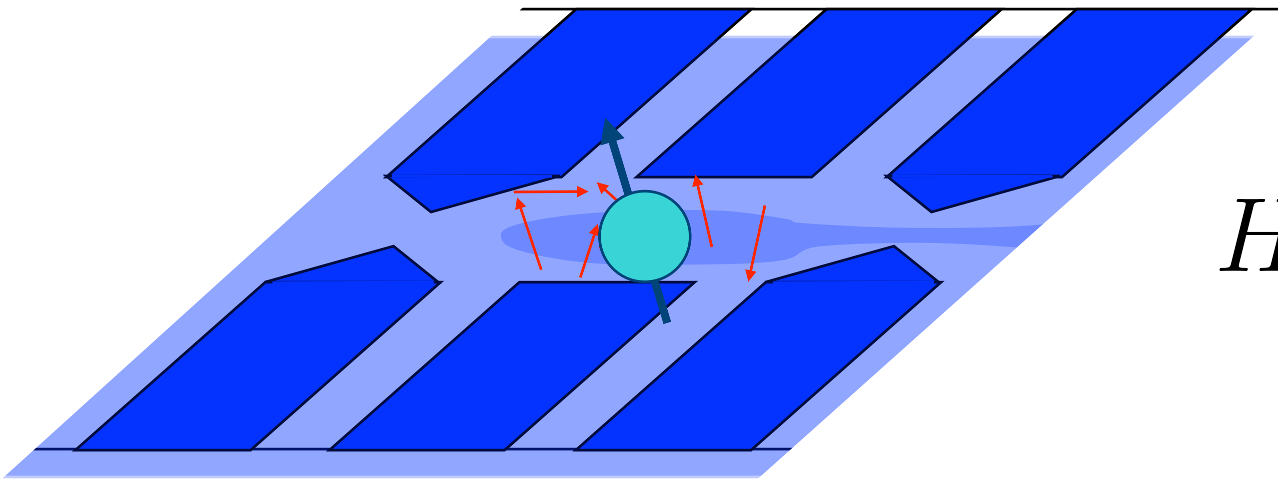
time evolution $|\psi(t)\rangle = e^{-itH}|\psi(0)\rangle$

case 0) no nuclear spins $\hat{b}_z^n = 0$

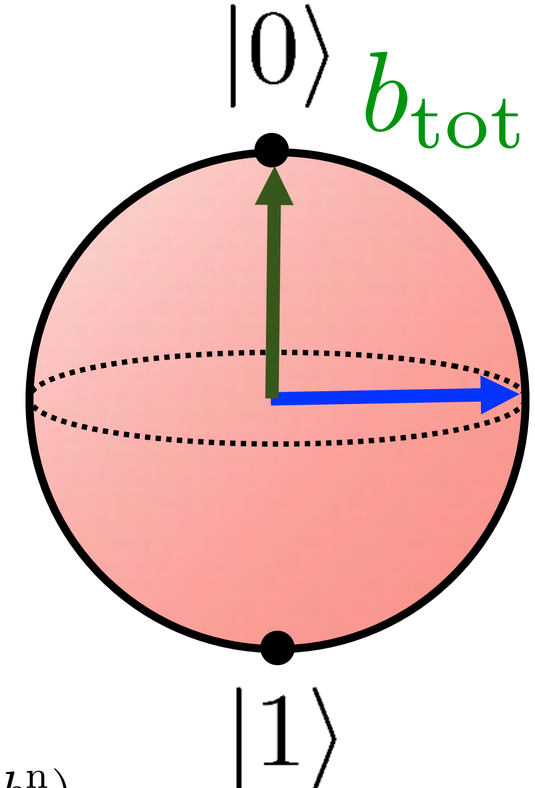
case 1) eigenstate $\hat{b}_z^{\mathrm{n}}|\psi^{\mathrm{n}}\rangle = b_z^{\mathrm{n}}|\psi^{\mathrm{n}}\rangle$



Nuclear spin induced dephasing



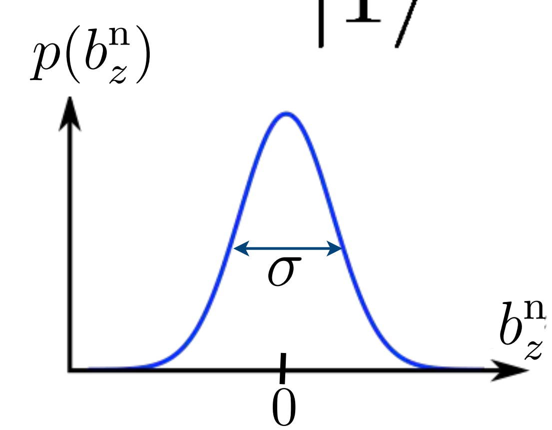
$$H = S_z\left(b + \hat{b}_z^{\mathrm{n}}\right)$$



case 2) unpolarized state (equilibrium state for $k_B T \gg g_n \mu_n B$) $\rho^{\mathrm{n}} \propto \mathbb{1}$

$$p(b_z^{\mathrm{n}}) = \mathrm{Tr}\left[\delta(\hat{b}_z^{\mathrm{n}} - b_z^{\mathrm{n}})\rho^{\mathrm{n}}\right] \approx (2\pi\sigma^2)^{-1/2}\exp(-(b_z^{\mathrm{n}})^2/2\sigma^2)$$

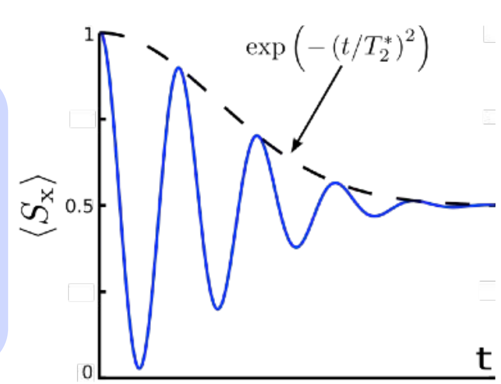
$\sigma \approx A/\sqrt{N}$



$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1 & \exp\left(-itb/\hbar - \frac{1}{2\hbar^2}\sigma^2 t^2\right) \\ \exp\left(itb/\hbar - \frac{1}{2\hbar^2}\sigma^2 t^2\right) & 1 \end{pmatrix}$$

$$\langle S_x\rangle = \mathrm{Tr}\left[S_x\rho(t)\right] = \cos(bt)e^{-(t/\tau)^2/2}$$

$$\tau = \hbar/\sigma \approx \hbar\sqrt{N}/A$$



GaAs quantum dot:

$A \approx 90\,\mu\mathrm{eV}$

$N \approx 10^6$

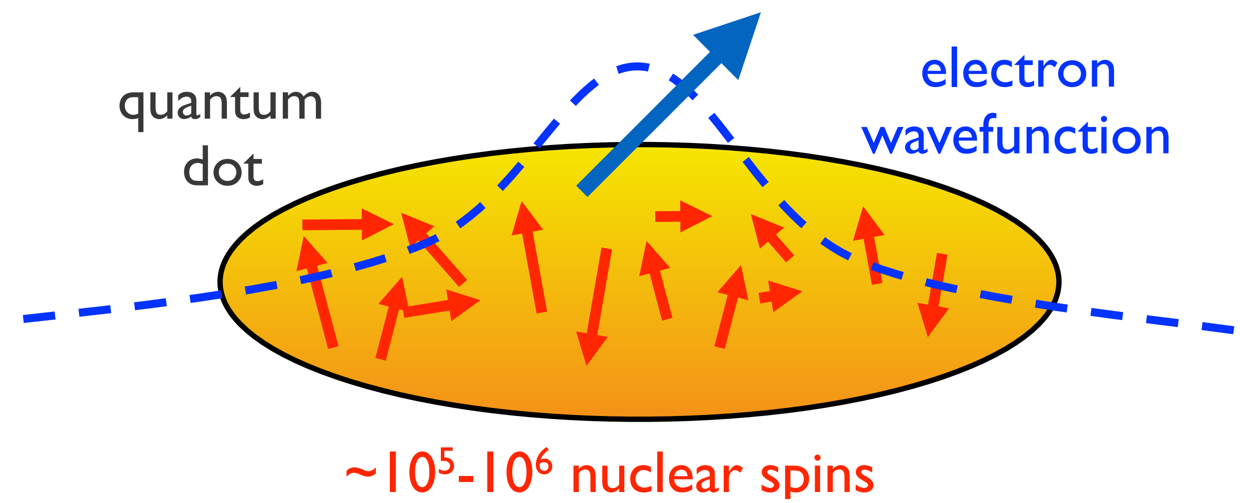
$\tau \approx 10\,\mathrm{ns}$

for more, see Coish & Loss, PRB 2004

Nuclear spin induced electron spin dephasing

- nuclear spins unavoidable in GaAs & other III-V materials
- coherence time of mobile electrons in GaAs ~100 ns

  J. Kikkawa and D. Awschalom, Phys. Rev. Lett. 80, 4313 (1998)

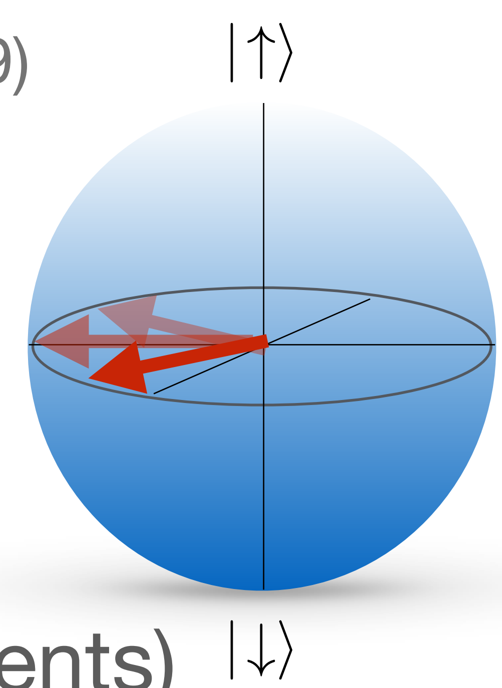


- nuclear spins will affect spin qubits in quantum dots (QDs)

  G. Burkard, D. Loss, and D. P. DiVincenzo, Phys. Rev. B 59, 2070 (1999)

- limit coherence time of e^- spin in GaAs QD to ~10 ns

  J. Petta *et al.*, Science 309, 2180 (2005)



strategies:

- control or harness nuclear spins (magnetic field gradients)
- get rid of nuclear spins: new materials

Preparation of the nuclear field

Spin blockade

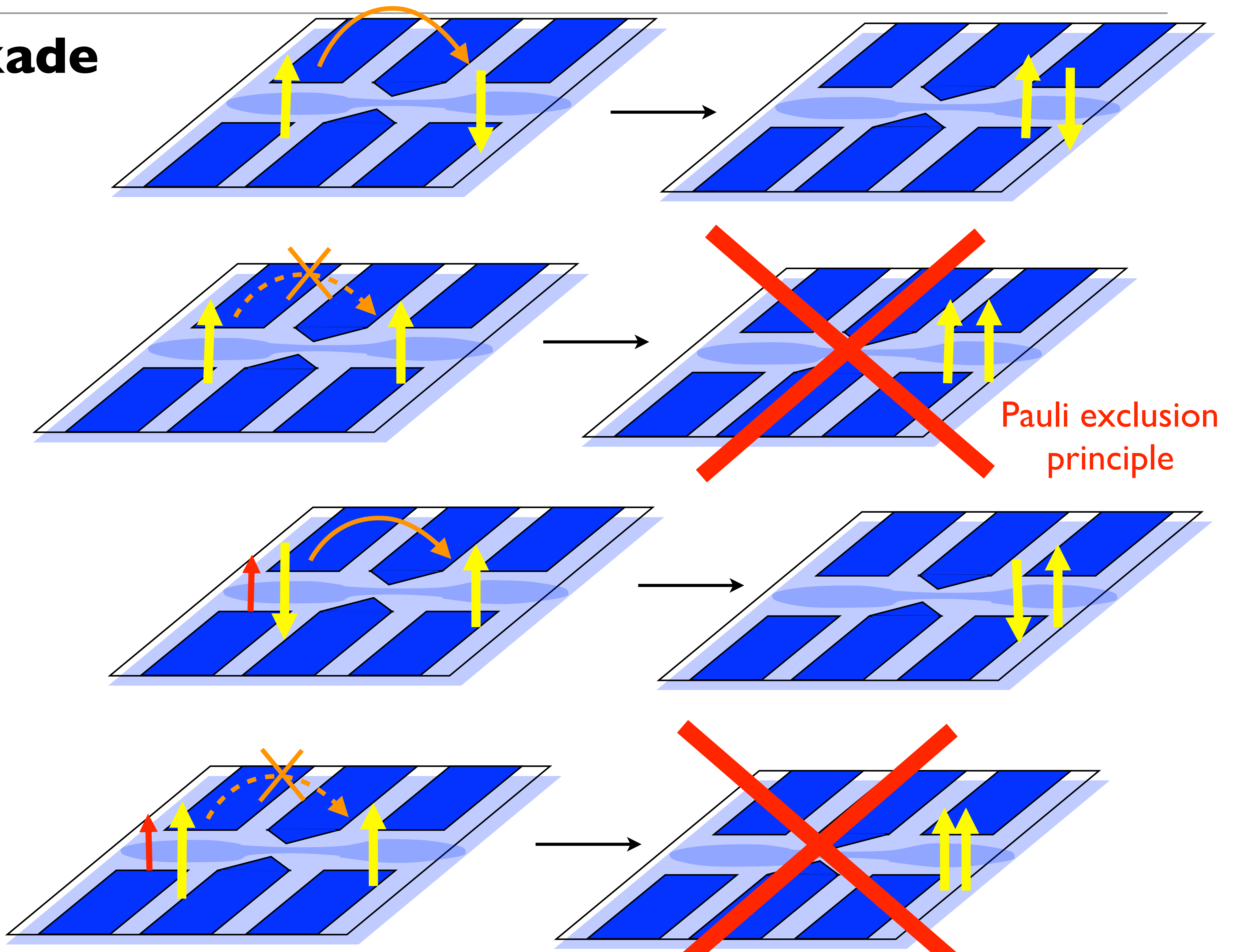
spin singlet

spin triplet

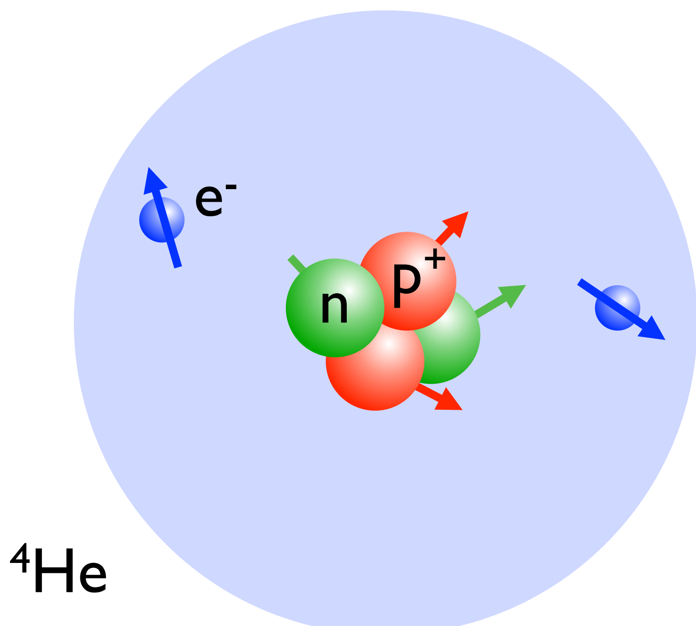
spin triplet
+ nuclear spin

QM measurement
of nuclear spin

= preparation
of nuclear spin



Nuclear physics in 2 minutes



^{4}He atom

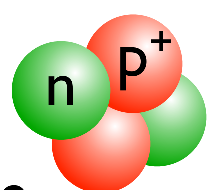
Z=2 protons p^+ (spin 1/2)

Z=2 electrons e^- (spin 1/2)

N=2 neutrons n (spin 1/2)

atomic mass: M=Z+N=4

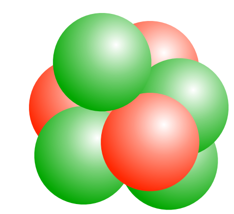
Nuclear physics in 2 minutes



^{4}He
99.9999%

^{4}He nucleus
Z=2 protons p$^+$ (spin 1/2)
N=2 neutrons n (spin 1/2)
atomic mass: M=Z+N=4

Lithium (Z=3)



^{7}Li
92%

- even proton (neutron) numbers Z (N) more stable than odd Z (N)
- Z odd: odd-odd stable isotopes very rare, typically odd-even
- even (odd) Z implies (even) odd M for most abundant stable isotopes
- lower spin nuclei typically more stable
- **even Z: zero nuclear spin most likely**
- **odd Z: nonzero nuclear spin** (odd number of spin-1/2 nucleons)

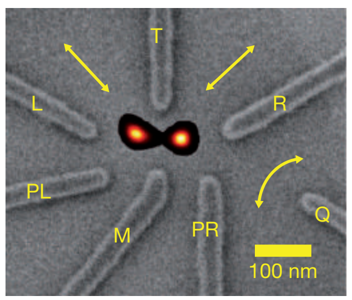
Dilute nuclear-spin materials

	II	III	IV	V	VI
		boron 5 **B** 0%	carbon 6 **C** 99%	nitrogen 7 **N** 0%	oxygen 8 **O** 99.9%
		aluminium 13 **Al** 0%	silicon 14 **Si** 95%	phosphorus 15 **P** 0%	sulfur 16 **S** 99.2%
molybdenum 42 **Mo** 73%	zinc 30 **Zn** 96%	gallium 31 **Ga** 0%	germanium 32 **Ge** 92%	arsenic 33 **As** 0%	selenium 34 **Se** 92%
tungsten 74 **W** 86%	cadmium 48 **Cd** 75%	indium 49 **In** 0%	tin 50 **Sn** 85%	antimony 51 **Sb** 0%	tellurium 52 **Te** 92%

fraction of naturally occurring nuclear spin-free isotopes (I=0)

Dilute nuclear-spin materials

95% ^{28}Si, ^{30}Si (I=0)
5% ^{29}Si (I=1/2)



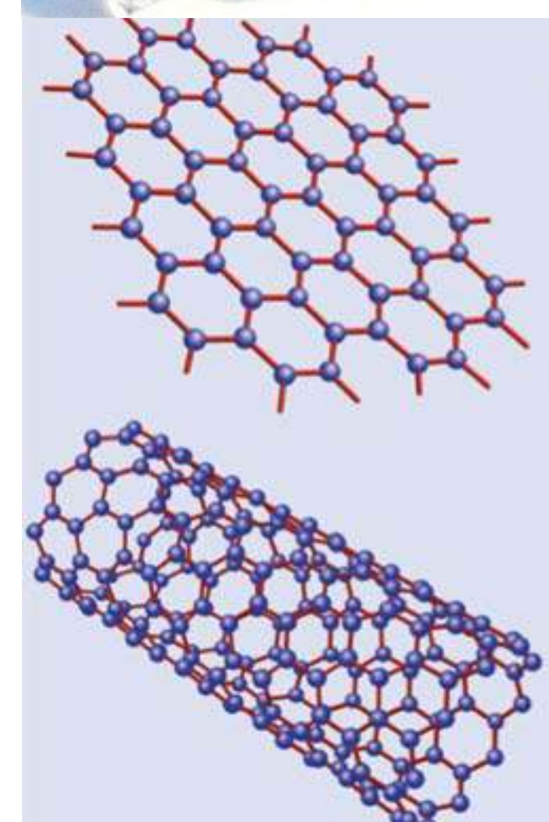
Maune *et al.*, Nature 2012

fraction of naturally occurring nuclear spin-free isotopes (I=0)

	II	III	IV	V	VI
		boron 5 **B** 0%	carbon 6 **C** 99%	nitrogen 7 **N** 0%	oxygen 8 **O** 99.9%
		aluminium 13 **Al** 0%	silicon 14 **Si** 95%	phosphorus 15 **P** 0%	sulfur 16 **S** 99.2%
molybdenum 42 **Mo** 73%	zinc 30 **Zn** 96%	gallium 31 **Ga** 0%	germanium 32 **Ge** 92%	arsenic 33 **As** 0%	selenium 34 **Se** 92%
tungsten 74 **W** 86%	cadmium 48 **Cd** 75%	indium 49 **In** 0%	tin 50 **Sn** 85%	antimony 51 **Sb** 0%	tellurium 52 **Te** 92%

99% ^{12}C (I=0)
1% ^{13}C (I=1/2)

source: Wikipedia

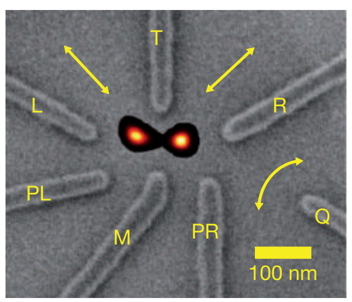


Castro Neto et al., Rev. Mod. Phys, 2009

Dilute nuclear-spin materials

Wikipedia

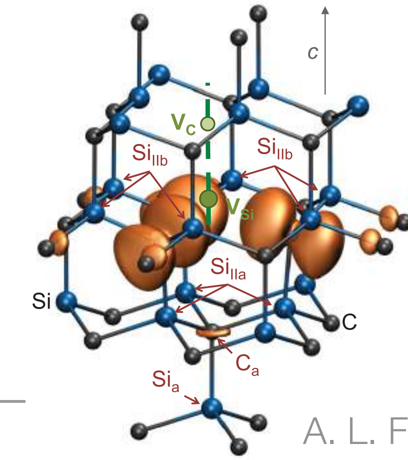
95% ^{28}Si, ^{30}Si (I=0)
5% ^{29}Si (I=1/2)



Maune *et al.*, Nature 2012

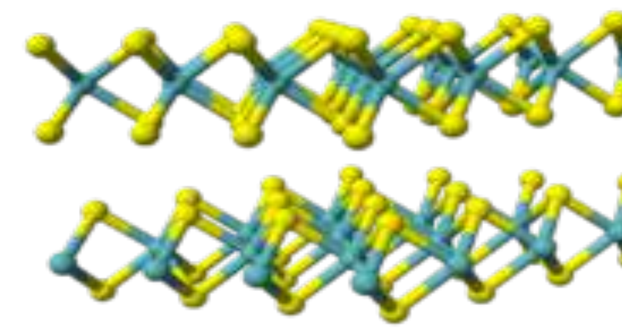
fraction of naturally occurring nuclear spin-free isotopes (I=0)

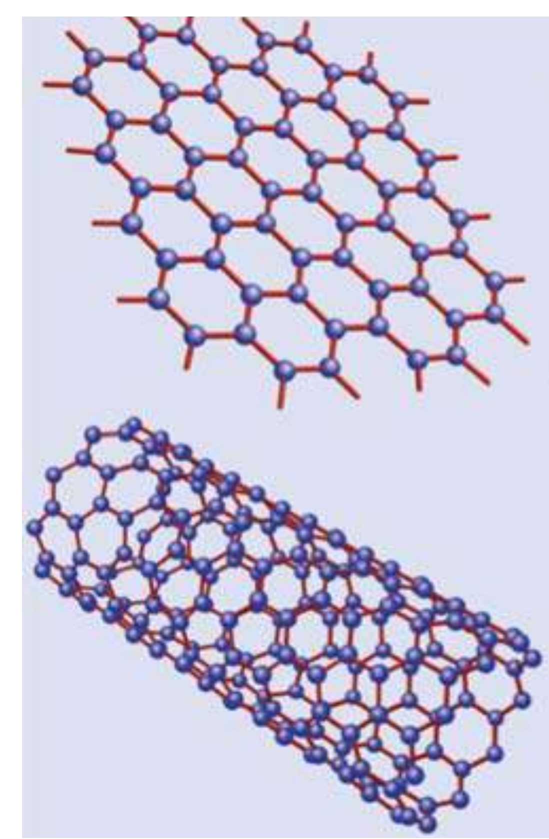
	II	III	IV	V	VI
		boron 5 **B** 0%	carbon 6 **C** 99%	nitrogen 7 **N** 0%	oxygen 8 **O** 99.9%
		aluminium 13 **Al** 0%	silicon 14 **Si** 95%	phosphorus 15 **P** 0%	sulfur 16 **S** 99.2%
molybdenum 42 **Mo** 73%	zinc 30 **Zn** 96%	gallium 31 **Ga** 0%	germanium 32 **Ge** 92%	arsenic 33 **As** 0%	selenium 34 **Se** 92%
tungsten 74 **W** 86%	cadmium 48 **Cd** 75%	indium 49 **In** 0%	tin 50 **Sn** 85%	antimony 51 **Sb** 0%	tellurium 52 **Te** 92%



A. L. Falk *et al.*, PRL (2015)

99% ^{12}C (I=0)
1% ^{13}C (I=1/2)

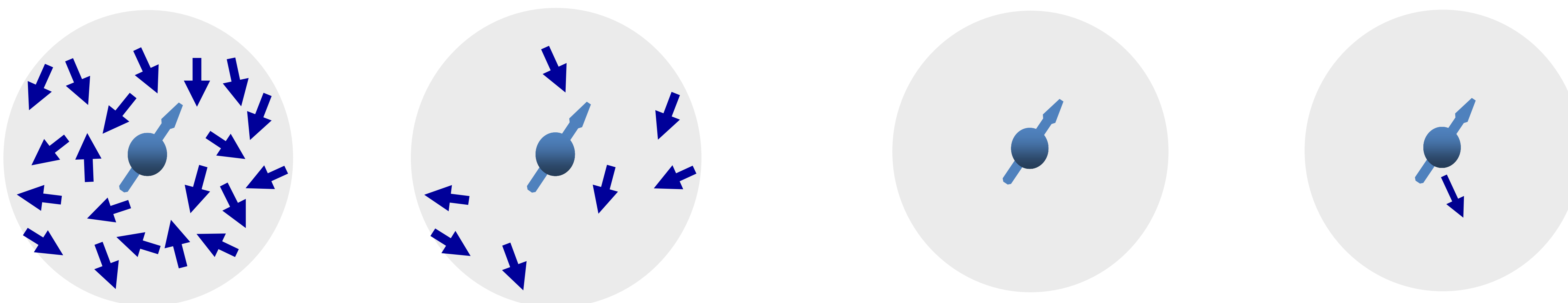




Castro Neto et al., Rev. Mod. Phys, 2009

Experimental breakthrough: Silicon Spin Qubits

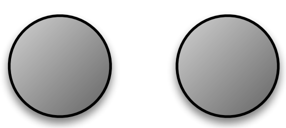
GaAs	Si	^{28}Si	^{31}P:^{28}Si

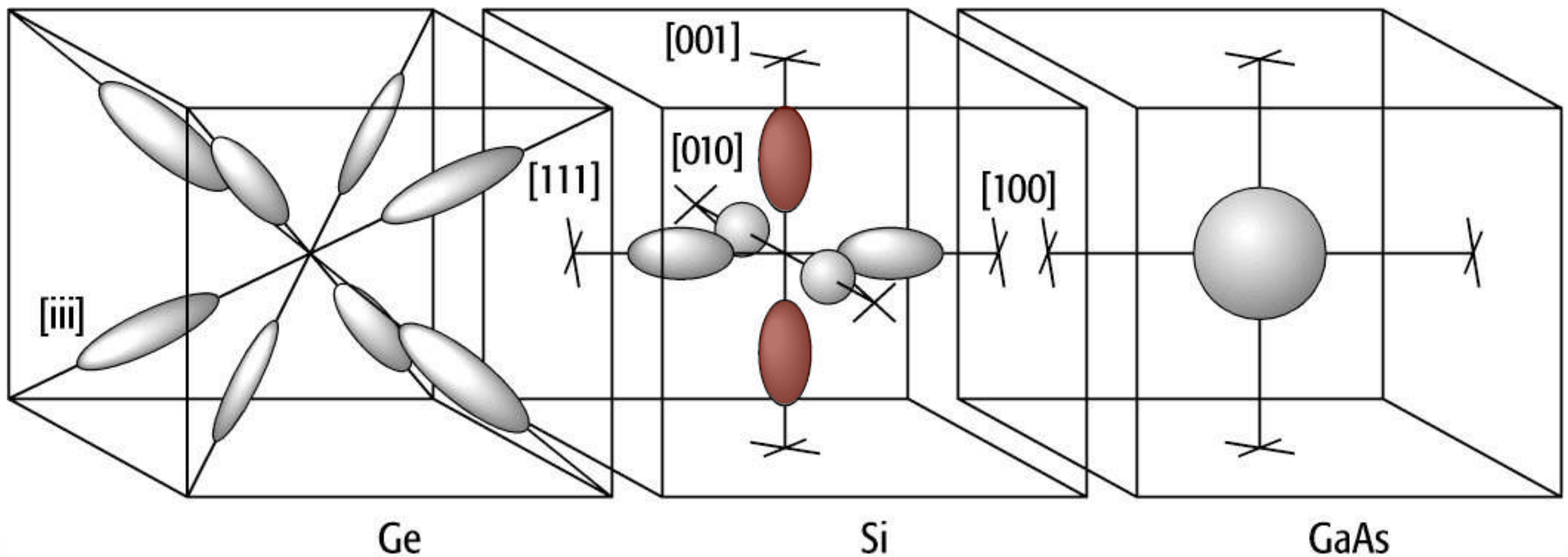


$T_2^* \sim 10$ ns	$T_2^* \sim 1$ μs	$T_2^* \sim 100$-250 μs	$T_2^* \sim 0.6$ s
$T_2^{DD} \sim 0.2$ ms	$T_2^{DD} \sim 0.5$ ms	$T_2^{DD} \sim 28$-500 ms	$T_2^{DD} \sim 35$ s
Petta et al, Science 2005	Kawakami, Scarlino, et al, Nature Nano 2014	Veldhorst, et al, Nature Nano 2014	Muhonen et al, Nature Nano 2014

adapted from L. Vandersypen (TU Delft)

Valley degeneracy (more in lecture 2)





$$|\uparrow K\rangle, |\downarrow K\rangle, |\uparrow K'\rangle, |\downarrow K'\rangle$$

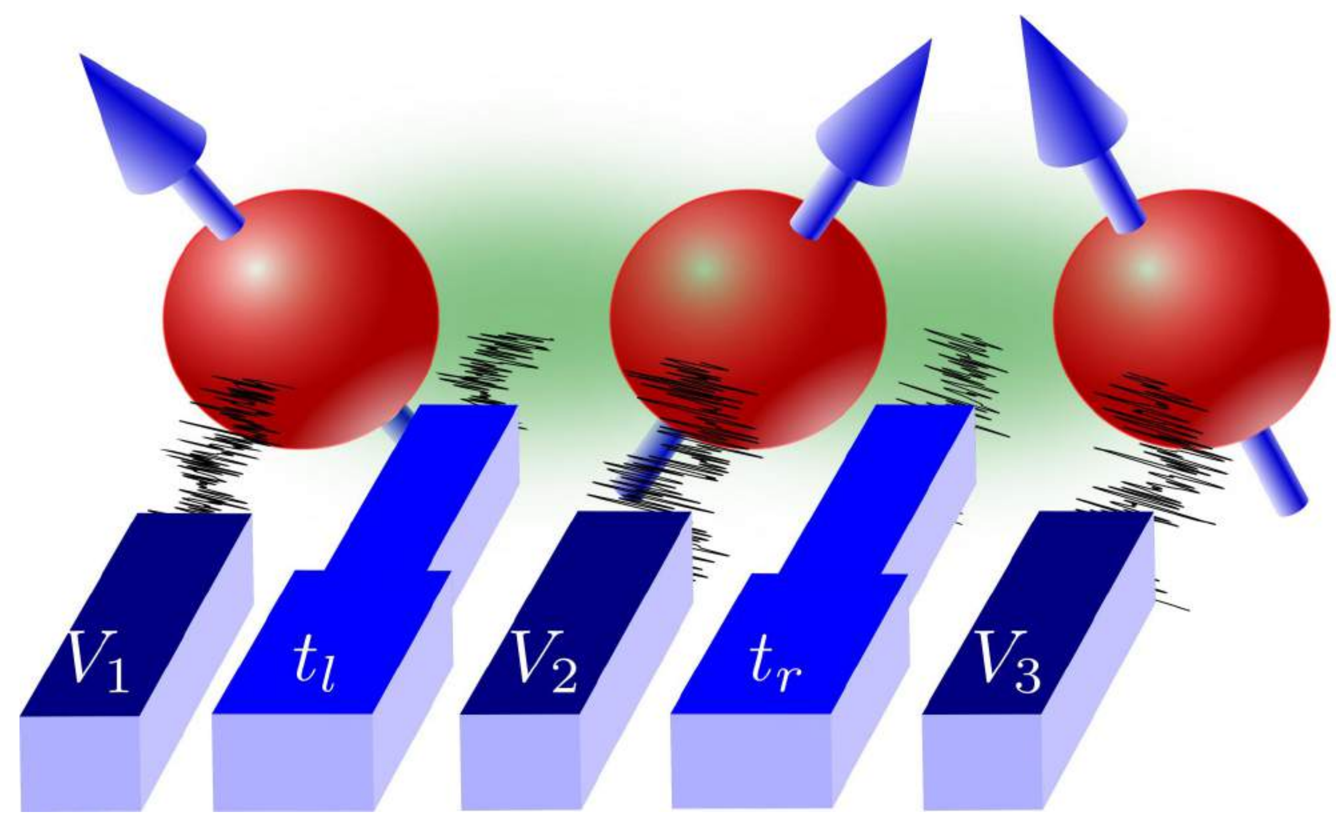
$$|\uparrow\rangle, |\downarrow\rangle$$

$$H = \frac{J}{8}\Big((\mathbf{S}_1 \cdot \mathbf{S}_2)(\tau_1 \cdot \tau_2) + \mathbf{S}_1 \cdot \mathbf{S}_2 + \tau_1 \cdot \tau_2\Big)$$

$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2$$

ICTS program on Open Quantum Systems
ICTS Bangalore, India
2017-07-27/28

Spin Qubits Theory



Guido Burkard

Department of Physics
University of Konstanz, Germany

lecture 2

Universität Konstanz

Spin Qubits: summary of lecture 1

Spin, Qubits, Spin Qubits

Exchange coupling

Decoherence

Nuclear spins (magnetic noise):
very long coherence in dilute nuclear spin materials

Materials issues

Spin Qubits

lecture 1

Introduction into Spin Qubits

lecture 2

Multi-spin qubits
Coupling Spins to Electric Fields
Spin and Valley 1: Exchange

lecture 3

Spin and Valley 2: Spin Relaxation
Defect Spins

Electric Dipole of an Electron

- coupling of EM fields to electron spin $H = -\boldsymbol{\mu} \cdot \mathbf{B} - \mathbf{d} \cdot \mathbf{E}$
- permanent magnetic dipole moment $\boldsymbol{\mu} = -g\mu_B \mathbf{S}$
- permanent electric dipole moment $\mathbf{d} = \dfrac{d_e}{\hbar/2}\mathbf{S}$

$g\mu_B \sim 10^{-4} eV/T$ $\quad |d_e| < 10^{-30} e\,\mathrm{m}$ ACME collab., Science 2014

- here: orbital electric dipole $\mathbf{d} = e\mathbf{r}$
- coupling between position and spin?
- 1 electron: spin-orbit coupling $H_{\mathrm{SOC}} \sim (e/2m^2c^2)\mathbf{S} \cdot (\mathbf{E} \times \mathbf{p})$
  or inhomogeneous B-field $H_{\nabla \mathrm{B}} \sim -e\mathbf{r} \cdot \mathbf{E} - g\mu_B \mathbf{r} \cdot \nabla(\mathbf{B} \cdot \mathbf{S})$

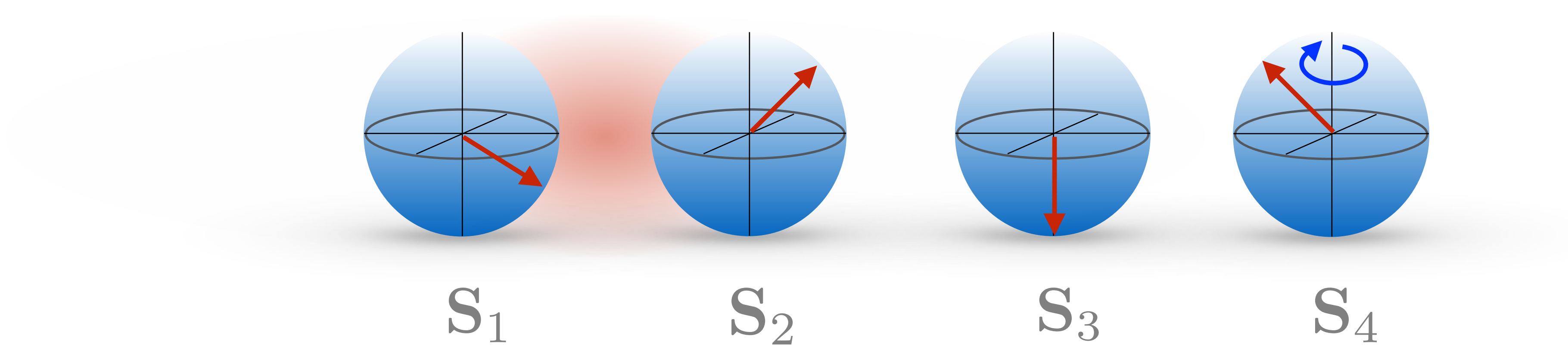
  >1 electrons: Fermi statistics $H_{\mathrm{Heisenberg}} \sim J(\mathbf{E})\mathbf{S}_1 \cdot \mathbf{S}_2$

Spin 1/2 Qubits

- exchange coupling: fast 2-qubit gates (0.1-1 ns) Petta *et al.*, Science 2005
- 1-qubit gates: slower and harder (10-100 ns) Koppens *et al.*, Nature 2006

$$H = \sum_{\langle i,j \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i g_i(t)\mu_B \mathbf{B}_i(t) \cdot \mathbf{S}_i$$

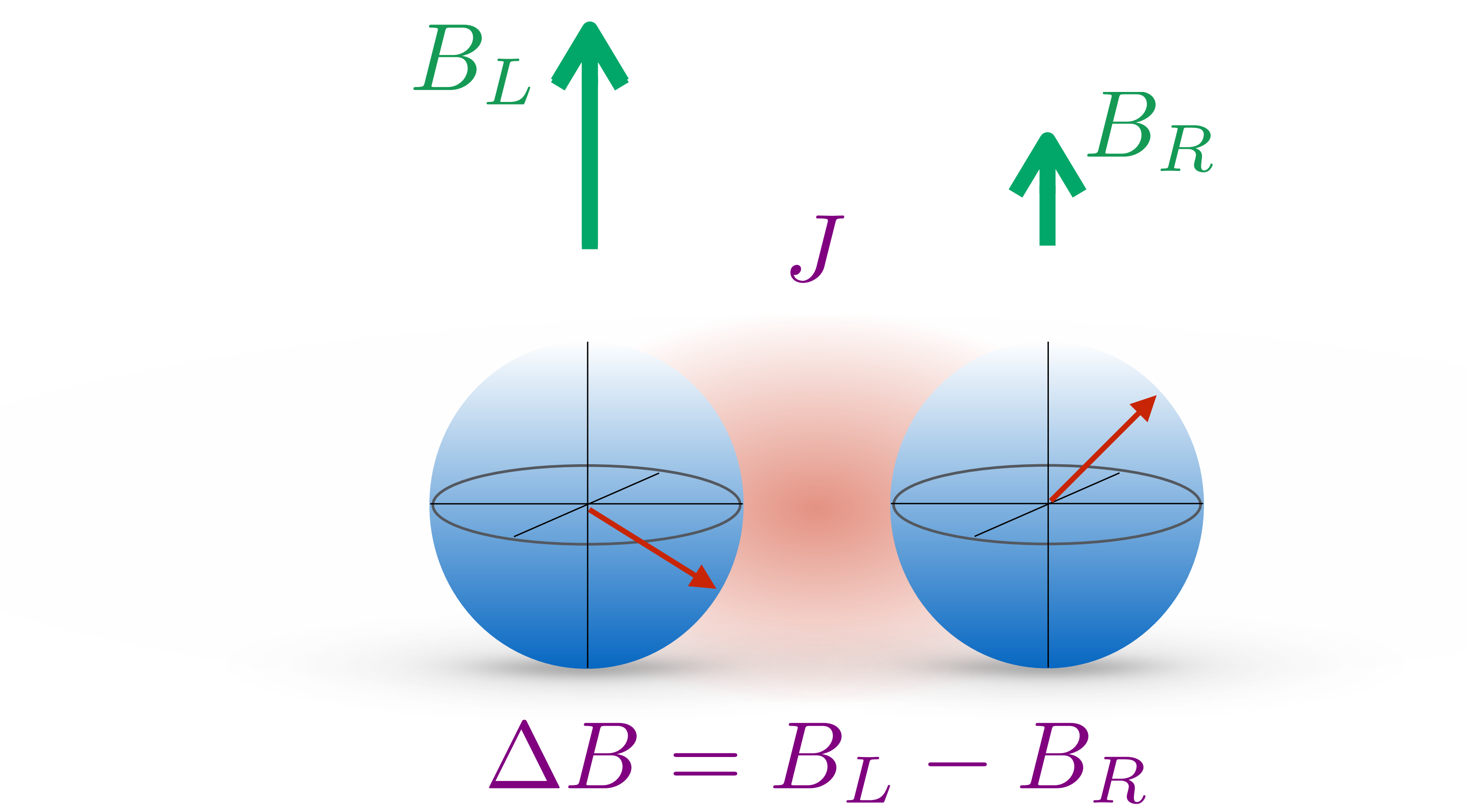
exchange coupling electron spin resonance

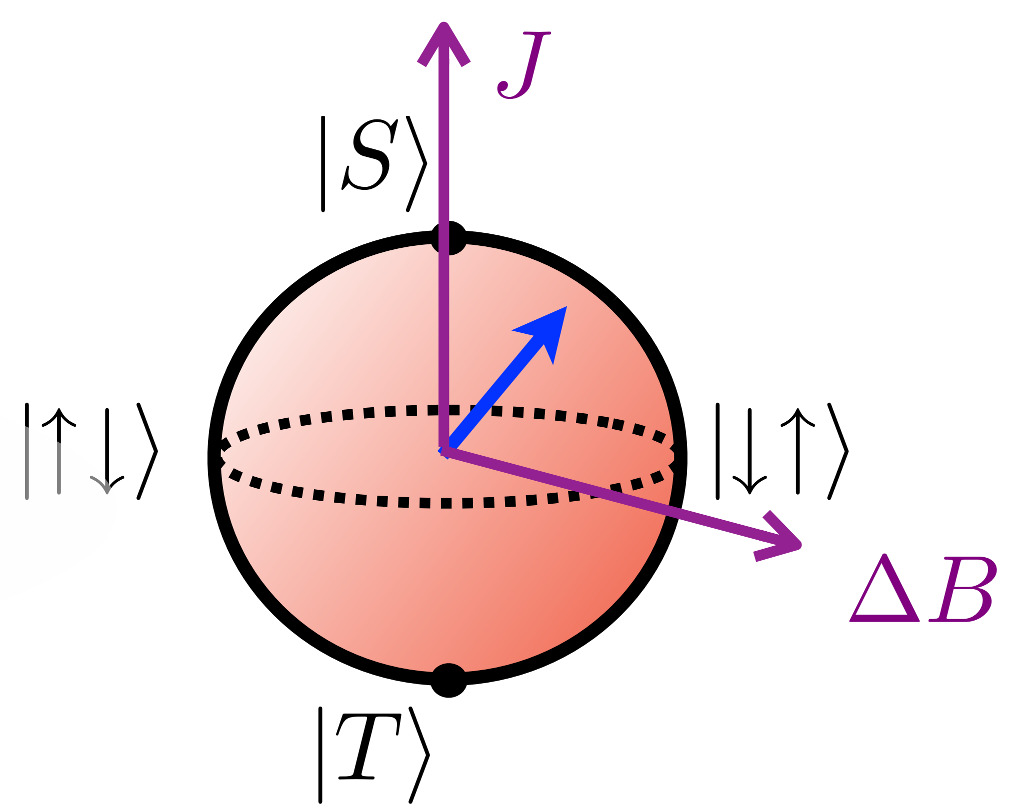


- increase single-qubit gate efficiency: EDSR Nowack *et al.*, Science 2007
  using spin-orbit coupling, nuclear spins, magnetic field gradient
- electric dipole leads to increased sensitivity to charge noise

Two-spin (singlet-triplet) qubits

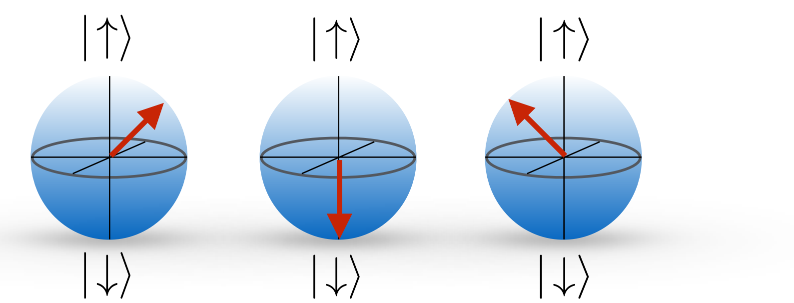
qubit	realization	spin	$\|0\rangle$	$\|1\rangle$
S-T_0	2 QDs, 2 electrons	$S=0,\,1$ $S_z=0$	$\|\uparrow\downarrow\rangle - \|\downarrow\uparrow\rangle$	$\|\uparrow\downarrow\rangle + \|\downarrow\uparrow\rangle$
S-T_+	2 QDs, 2 electrons	$S=0, S_z=0$ $S=1, S_z=1$	$\|\uparrow\downarrow\rangle - \|\downarrow\uparrow\rangle$	$\|\uparrow\uparrow\rangle$



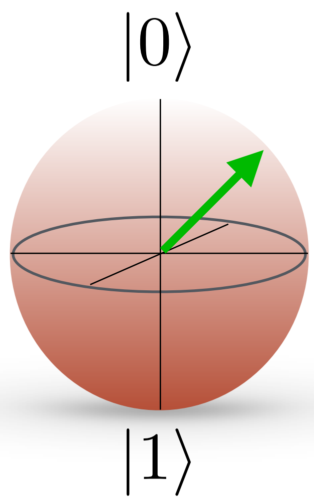


All-electric control: “Exchange only” qubits

- use three spins 1/2 to represent one qubit



$$H_{1/2} \otimes H_{1/2} \otimes H_{1/2} = H_{1/2} \oplus H_{1/2} \oplus H_{3/2}$$



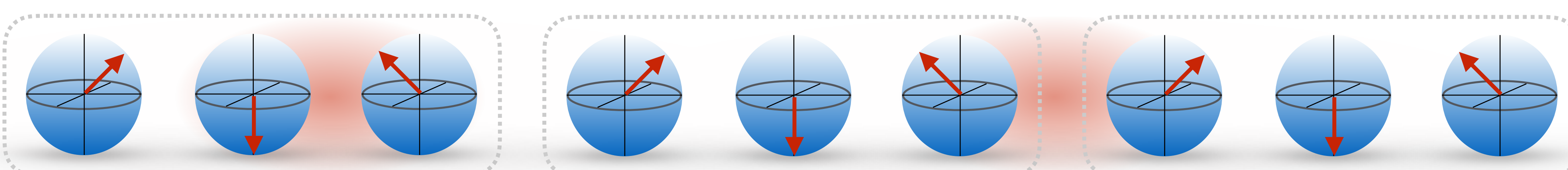
$$|0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle\right)$$

$$S = S_z = 1/2$$

$$|1\rangle = \frac{1}{\sqrt{6}}\left(2|\uparrow\downarrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle\right)$$

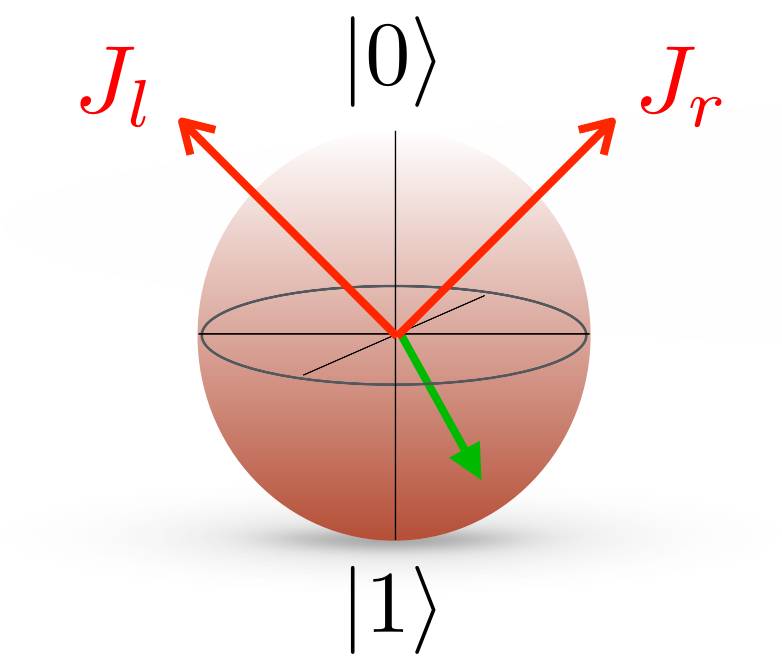
- exchange alone can navigate this subspace (many qubits)

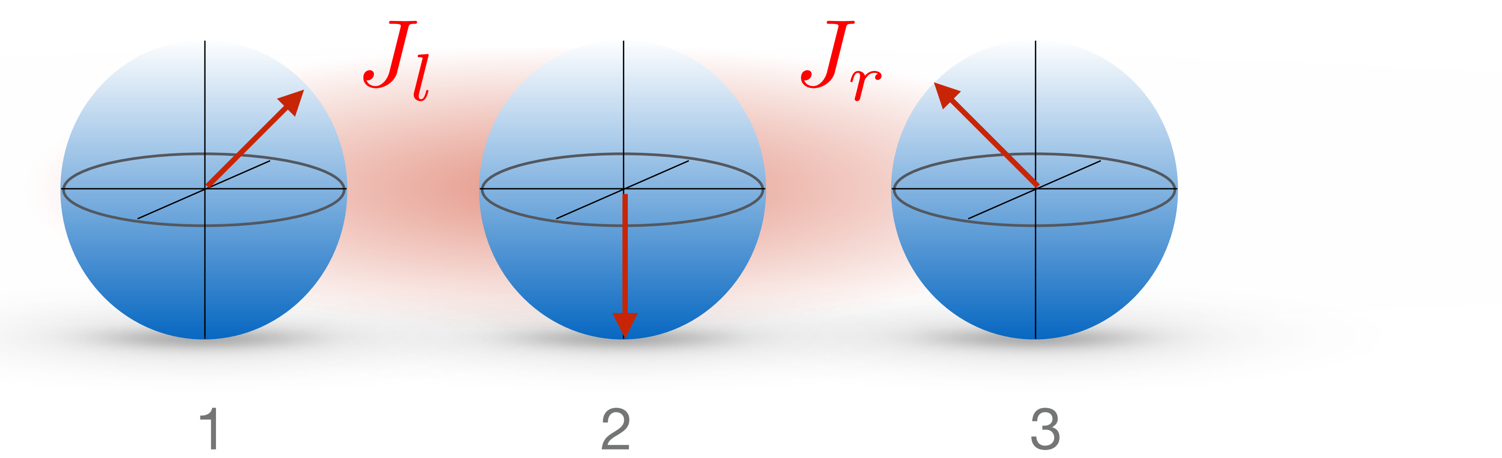
$$H = \sum_{\langle i,j\rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i g_i(t)\mu_B \mathbf{B}_i(t) \cdot \mathbf{S}_i$$

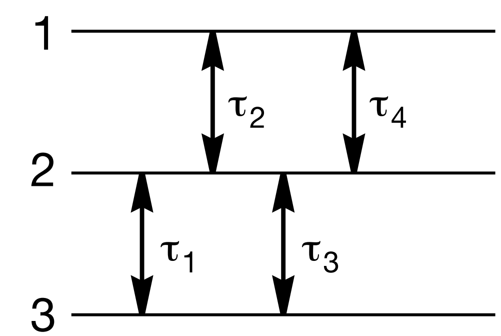


Bacon, Kempe, Lidar, and Whaley, Phys. Rev. Lett. **85**, 1758 (2000)
DiVincenzo, Bacon, Kempe, Burkard, and Whaley, Nature **408**, 339 (2000)

One-qubit gates



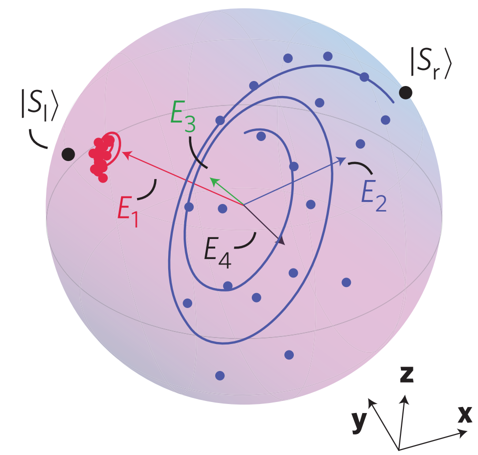




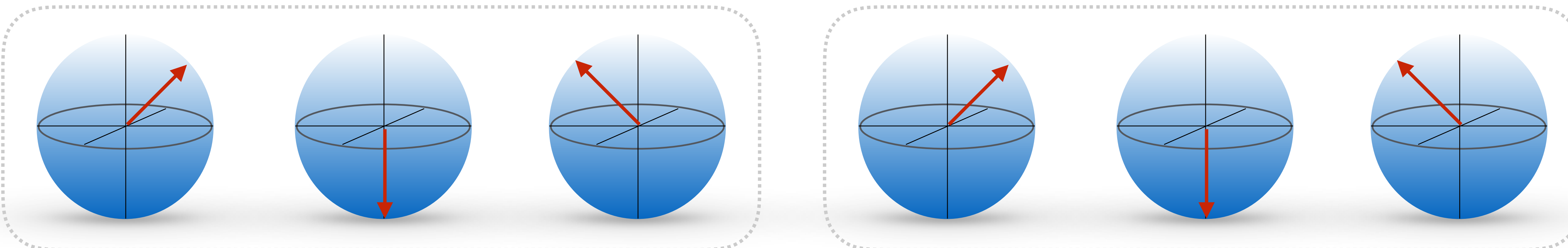
- exchange couplings rotate qubit about two non-collinear axes J_l and J_r

- generate arbitrary qubit rotations with exchange pulse sequences
  DiVincenzo, Bacon, Kempe, Burkard, Whaley, Nature **408**, 339 (2000)

- experimental realization
  Laird *et al.*, PRB **82**, 075403 (2010)
  Gaudreau *et al.*, Nature Phys. **8**, 54 (2012)
  Medford *et al.*, Nature Nano. **8**, 654 (2013)
  Eng *et al.*, Sciences Adv. e1500214 (2015)

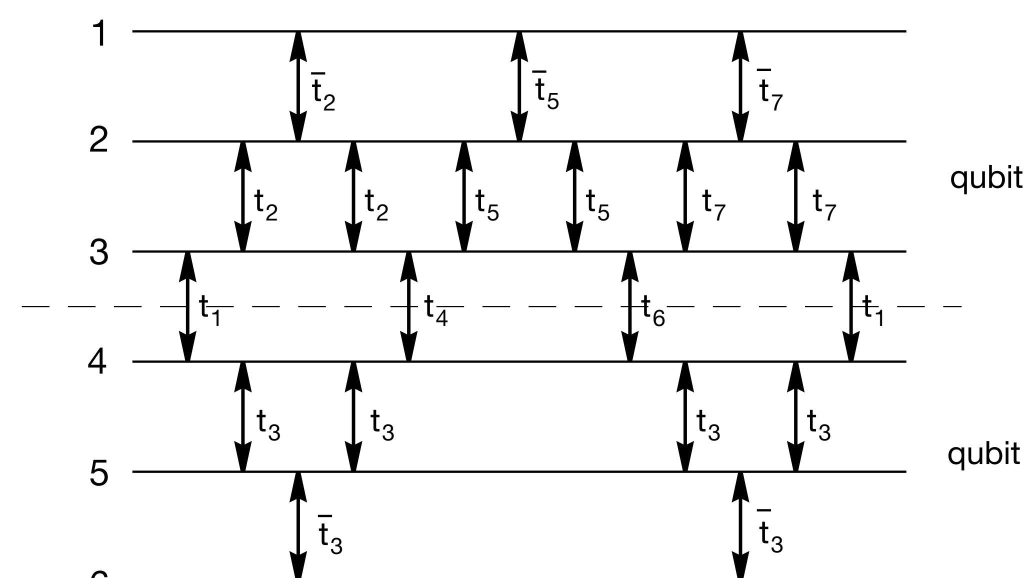


Two-qubit gates: Controlled NOT



1 2 3 4 5 6

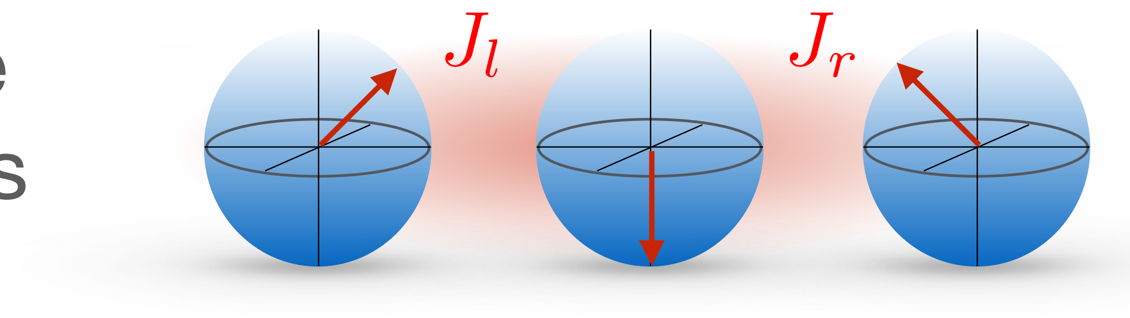
- numerical solution for CNOT
  19 pulses in series in 13 time steps
  DiVincenzo *et al.*, Nature **408**, 339 (2000)
- exact analytical solution
  (roots of 96th degree polynomial)
  Kawano *et al.*, Q. Inf. Proc. (2005)
- 8-parallel-pulse subsystem solution
  DiVincenzo *et al.*, Nature **408**, 339 (2000)
- subsystem: 22 serial pulses (13 steps)
  Fong & Wandzura, QI & QC (2011); Zeuch & Bonesteel, PRA (2016)



t_1=0.410899(2)
t_2=0.207110(20)
t_3=0.2775258(12)
t_4=0.640505(8)
t_5=0.414720(10)
t_6=0.147654(12)
t_7=0.813126(12)
$\tan(\pi t_i)\tan(\pi \bar{t}_i) = -2$

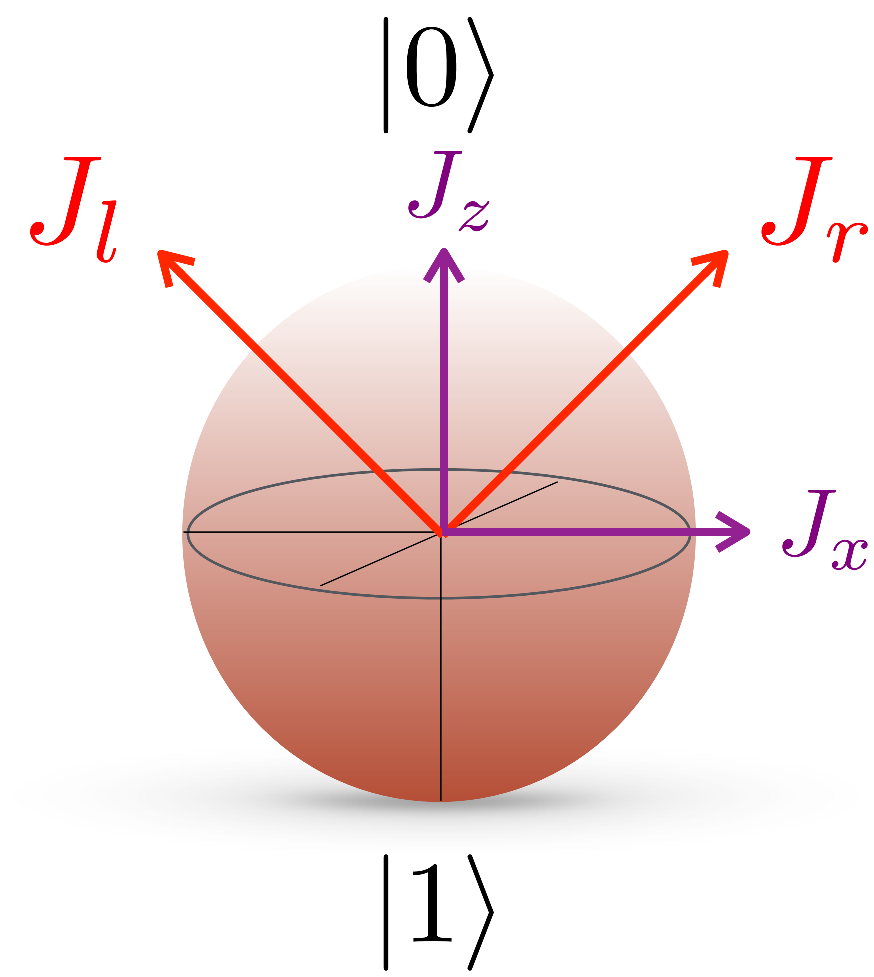
Exchange-only, RX, hybrid, and AEON qubits

- exchange-only: $J_l=0$, $J_r=0$ in idle state
  (low-bandwidth) pulse J_l or J_r for gates
  robust against charge noise (off state)



- hybrid qubit: 3 electrons in 2 quantum dots
  Shi *et al.*, PRL (2012); Kim *et al.*, Nature (2014)

- resonant exchange (RX): J_l, J_r always "on"
  radio-frequency pulse for gates: J_x or J_z
  Medford *et al.*, PRL (2013); Taylor *et al.*, PRL (2013)
  Doherty and Wardrop, PRL (2013)



- asymmetric RX (ARX) qubit
  M. Russ and G. Burkard, PRB (2015)

- always-on exchange-only (AEON) qubit
  Y.-P. Shim and C. Tahan, PRB (2016)

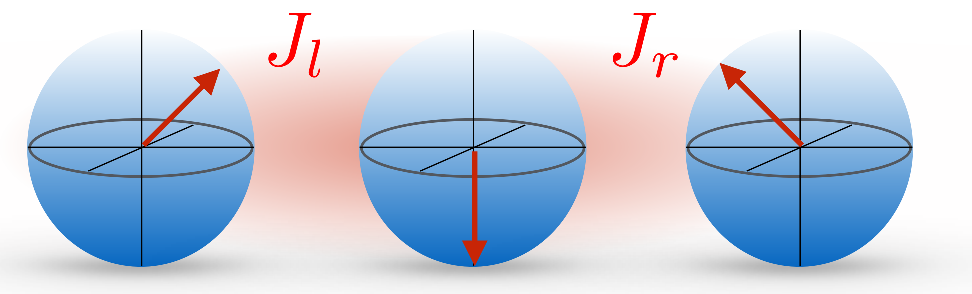
$$|0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{6}}(2|\uparrow\downarrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle)$$

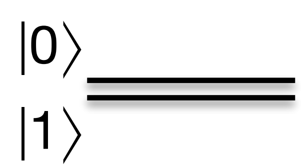
Exchange-only, RX, hybrid, and AEON qubits

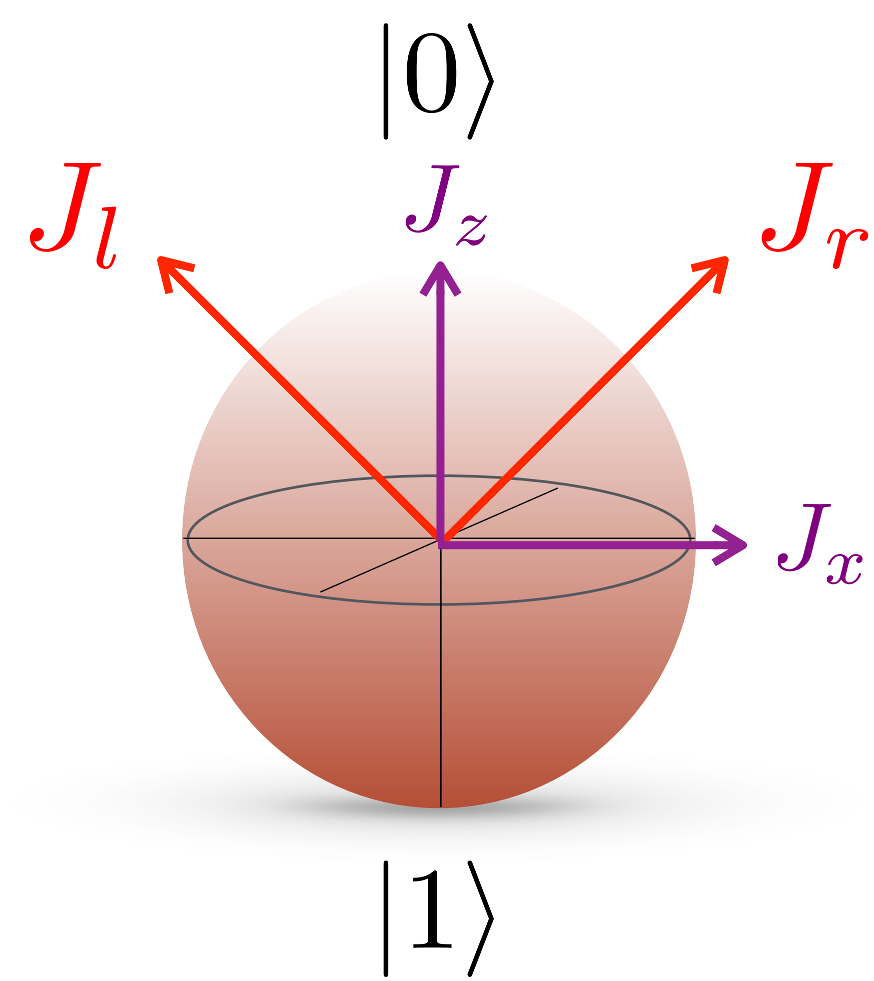
<table>
<tr><th>qubit</th><th>realization</th><th>spin</th><th>$|0\rangle$</th><th>$|1\rangle$</th></tr>
<tr><td>spin 1/2
Loss & DiVincenzo 1998</td><td>1 QD, 1 electron
Petta et al. 2005</td><td>$S = \frac{1}{2}$
$S_z = \pm\frac{1}{2}$</td><td>$|\uparrow\rangle$</td><td>$|\downarrow\rangle$</td></tr>
<tr><td>singlet-triplet
Levy 2002 Taylor et al. 2005
Hanson & Burkard 2007</td><td>2 QDs, 2 electrons
Petta et al. 2005</td><td>$S = 0, 1$
$S_z = 0$</td><td>$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$</td><td>$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$</td></tr>
<tr><td>exchange-only
DiVincenzo, Kempe, Bacon, Burkard, Whaley 2000
RF-exchange (RFX)
Medford et al.; Taylor et al.; Doherty et al., PRL 2013</td><td>3 QDs, 3 electrons</td><td rowspan="3">$S = \frac{1}{2}$
$S_z = \frac{1}{2}$</td><td rowspan="3">$|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle$</td><td rowspan="3">$2|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle$</td></tr>
<tr><td rowspan="2">spin-charge
(hybrid)</td><td>1 QD, 3 electrons
Kyriakidis & Burkard 2007</td></tr>
<tr><td>2 QD, 3 electrons
Shi et al. 2007</td></tr>
</table>

Exchange-only and resonant exchange (RX) qubits



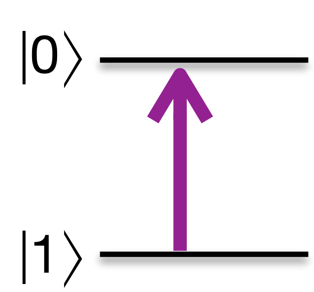
- exchange-only: *J_l=0, J_r=0* in idle state
  (low-bandwidth) pulse J_l or J_r for gates
  robust against charge noise (off state)





- resonant exchange (RX): J_l, J_r always "on"
  radio-frequency pulse for gates: J_x or J_z

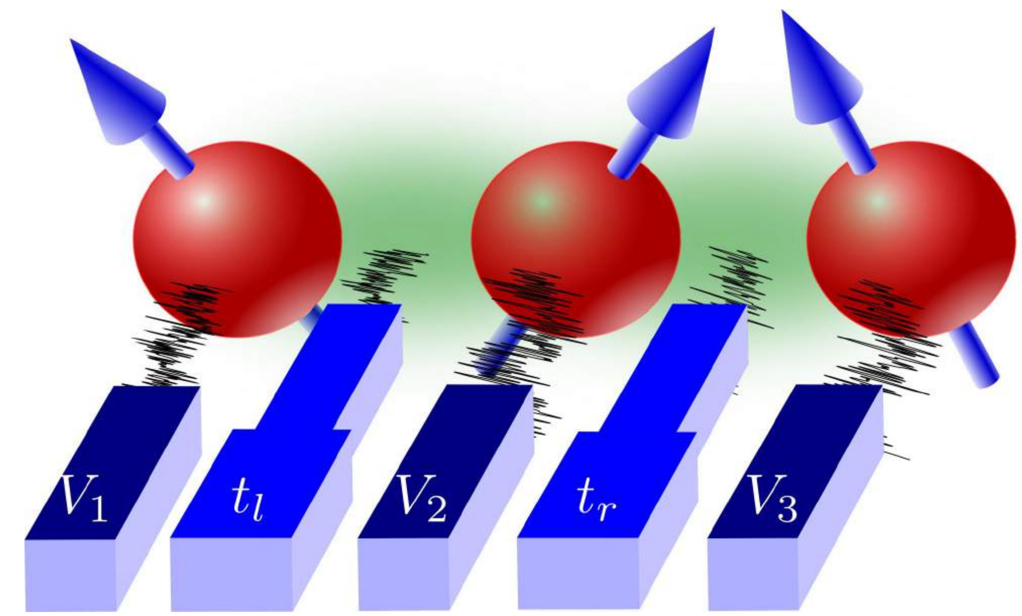
Medford *et al.*, PRL (2013)
Taylor *et al.*, PRL (2013)
Doherty and Wardrop, PRL (2013)



$$|0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{6}}(2|\uparrow\downarrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle)$$

Charge noise in the RX qubit



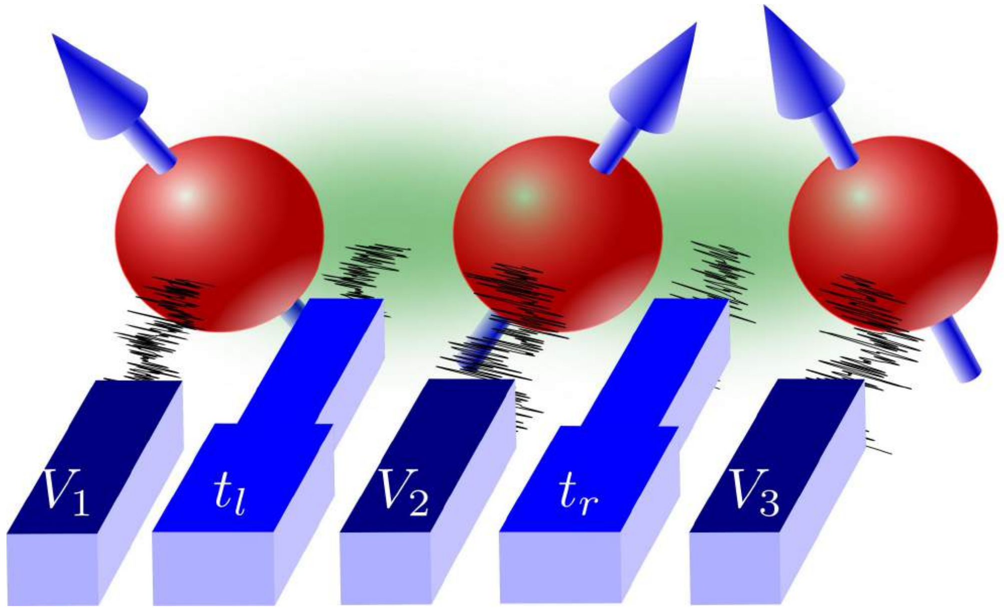
- J_l, J_r always “on” adds “charge” character to spin qubit
- (low-frequency) charge noise affects RX qubit
- model: independent noise affecting each quantum dot potential V_i and tunneling couplings t_i

M. Russ and G. Burkard, PRB 91, 235411 (2015)
M. Russ, F. Ginzel, and G. Burkard, PRB 94, 165411 (2016)

Dephasing

➤ longitudinal noise

$$\delta\omega_z = \frac{\partial\omega}{\partial V_1}\delta V_1 + \frac{\partial\omega}{\partial V_2}\delta V_2 + \frac{\partial^2\omega}{\partial V_1^2}\delta V_1^2 + \dots$$

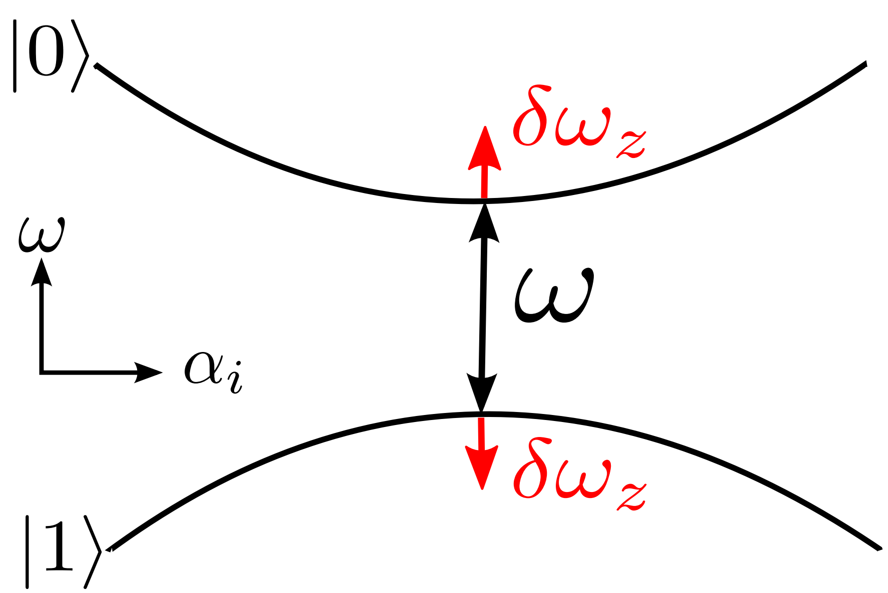


➤ Ramsey free decay

$$|+\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \xrightarrow{\text{time evolution}} |\Psi\rangle \equiv \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\phi(t)}|1\rangle\right)$$

$$\phi(t) \equiv \int_0^t \delta\omega_z(t')dt'$$ accumulated phase due to noise

$$\tilde{f} \equiv \left\langle e^{i\phi(t)} \right\rangle \xrightarrow{\text{Gaussian}} \boxed{e^{-\frac{1}{2}\left\langle \phi(t)^2 \right\rangle}} = e^{-(t/T_\phi)^2}$$



➤ "sweet spots" (α_i=V_1, V_2, V_3, t_l, t_r)

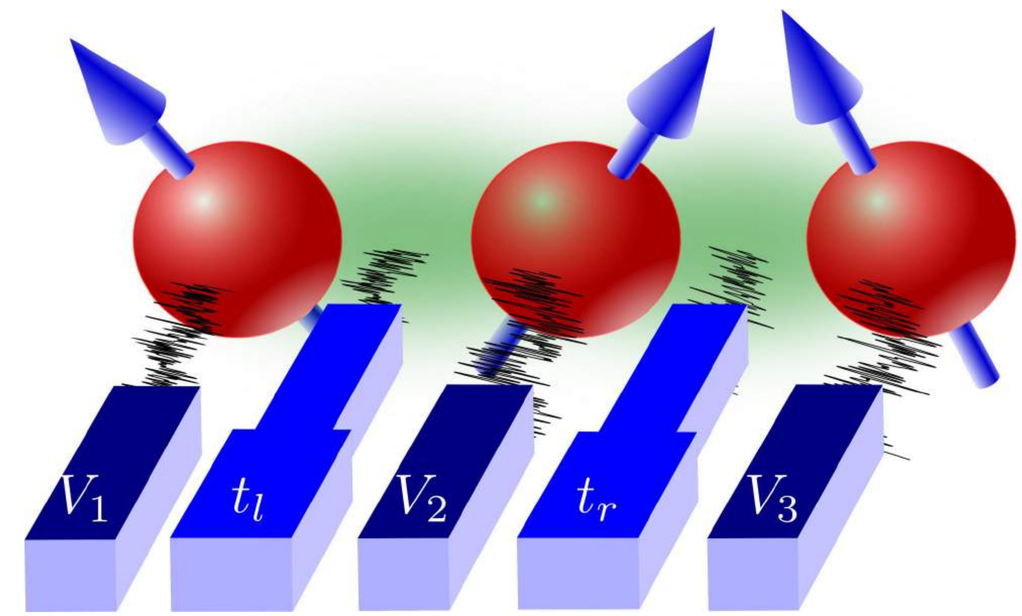
single $\frac{\partial\omega}{\partial\alpha_i} = 0$ mutliple $\frac{\partial\omega}{\partial\alpha_{i_1}} = \dots = \frac{\partial\omega}{\partial\alpha_{i_k}} = 0$

Model

- Hubbard Hamiltonian

potential in dot *i*

$$H_{\text{Hub}} = \sum_i \left[\frac{U}{2} n_i(n_i - 1) + V_i n_i\right] + \sum_{\langle i,j\rangle}\left[U_c n_i n_j + \sum_{\sigma=\uparrow\downarrow}\left(t_{ij} c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.}\right)\right]$$



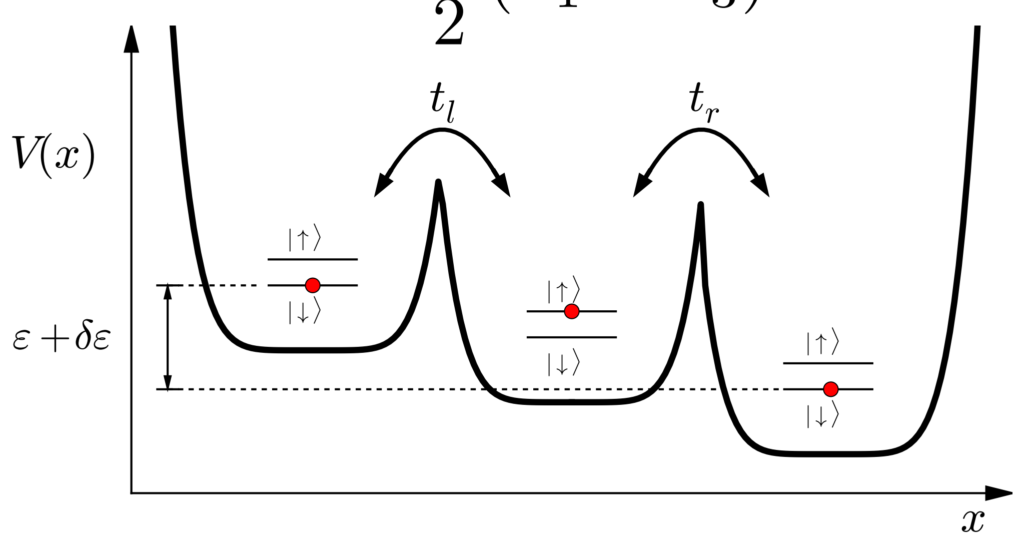
electron number operator $n_i = \sum_{\sigma=\uparrow,\downarrow} c^\dagger_{i,\sigma} c_{i,\sigma}$

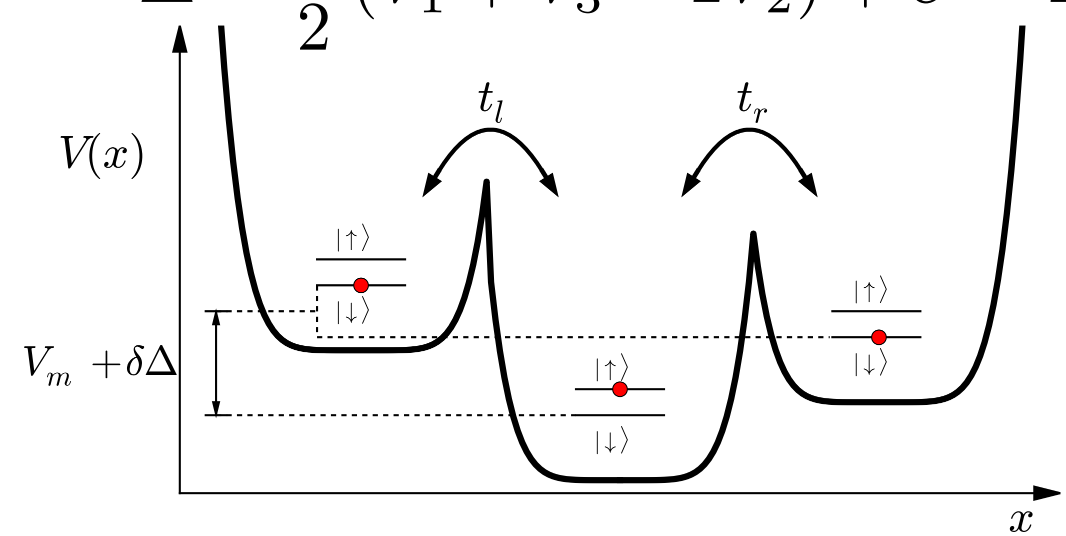
hopping between dot *i* and *j*

- two detuning parameters, affected by random fluctuations

$$\varepsilon = \frac{1}{2}(V_1 - V_3)$$

$$\Delta = \frac{1}{2}(V_1 + V_3 - 2V_2) + U - 2U_c$$





M. Russ and G. Burkard, PRB 91, 235411 (2015)

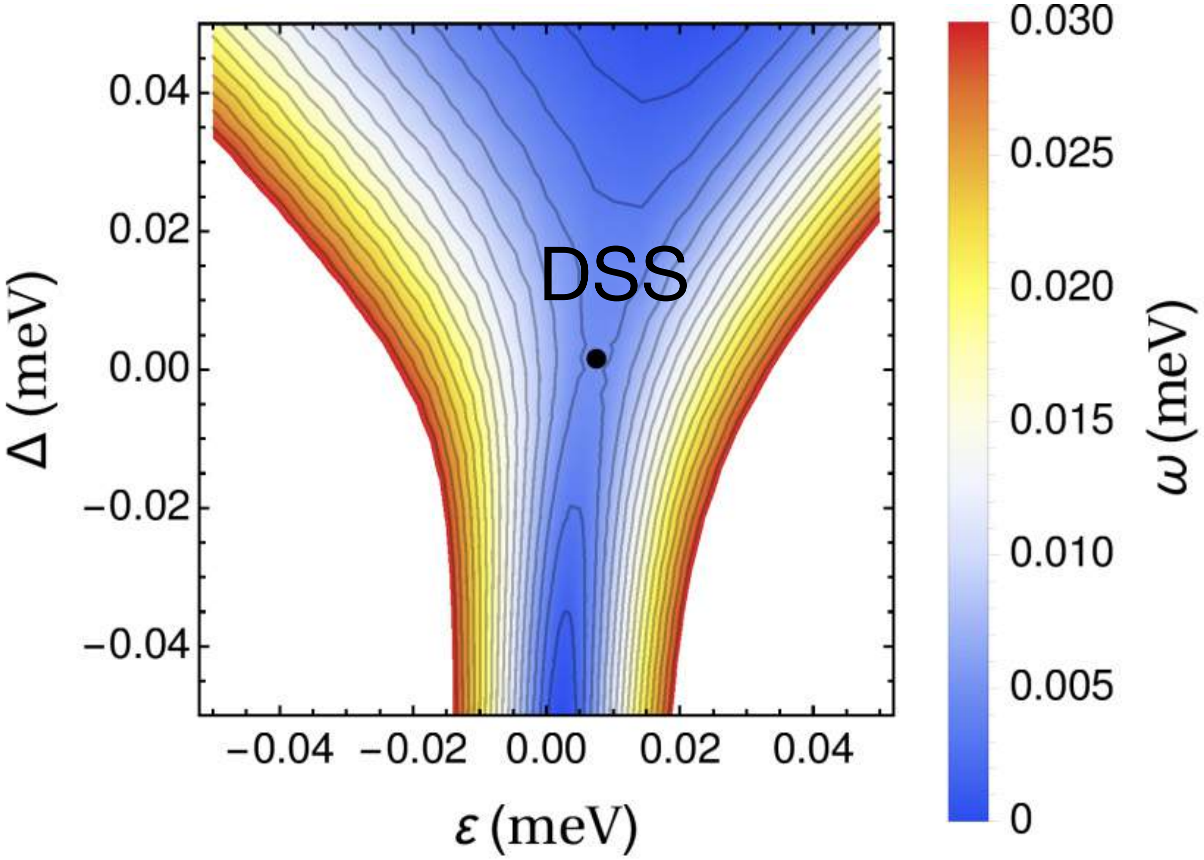
Charge noise in the RX qubit

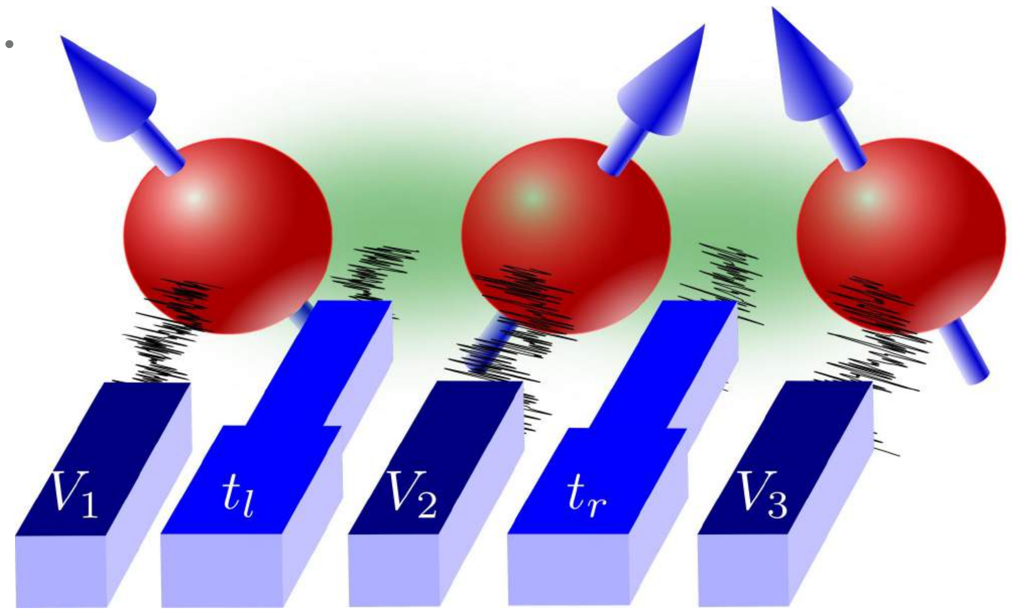
$$\varepsilon = \frac{1}{2}(V_1 - V_3)$$

$$\Delta = \frac{1}{2}(V_1 + V_3 - 2V_2) + U - 2U_c$$

Robustness against ε and Δ noise:
"double sweet spot" (DSS) in (ε,Δ) space?

$$\frac{\partial\omega}{\partial\varepsilon} = \frac{\partial\omega}{\partial\Delta} = 0$$



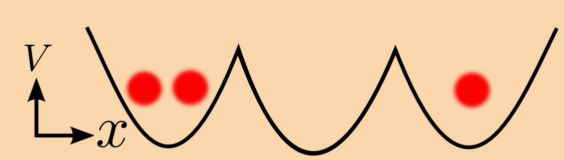


qubit

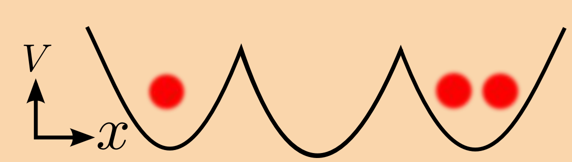
$$|0\rangle \equiv \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle\right)$$

$$|1\rangle \equiv \frac{1}{\sqrt{6}}\left(2\,|\uparrow\downarrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle\right)$$

$$|s_{1,1/2}\rangle \equiv |s\rangle_{11}\,|\uparrow\rangle_3 = |\uparrow\downarrow,\uparrow\rangle$$

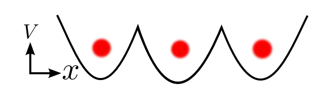


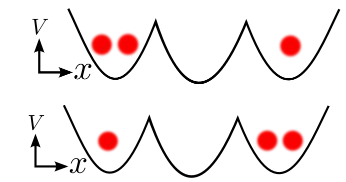
$$|s_{3,1/2}\rangle \equiv |\uparrow\rangle_1\,|s\rangle_{33} = |\uparrow,\uparrow\downarrow\rangle$$



M. Russ and G. Burkard, PRB 91, 235411 (2015)

Double sweet spot (DSS)





- diagonalize the Hamiltonian

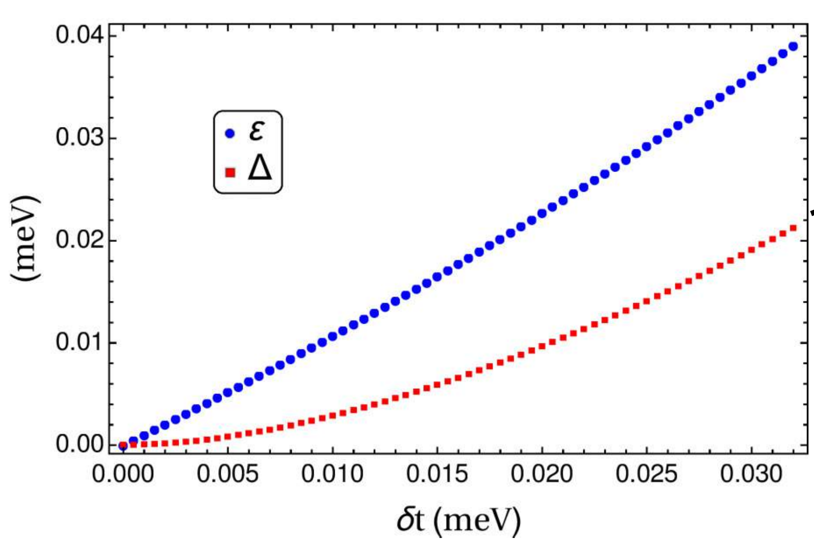
$$\bar{H} = \begin{pmatrix} 0 & 0 & t_l/2 & t_r/2 \\ 0 & 0 & \sqrt{3}t_l/2 & -\sqrt{3}t_r/2 \\ t_l/2 & \sqrt{3}t_l/2 & \Delta+\varepsilon & 0 \\ t_r/2 & -\sqrt{3}t_r/2 & 0 & \Delta-\varepsilon \end{pmatrix}$$

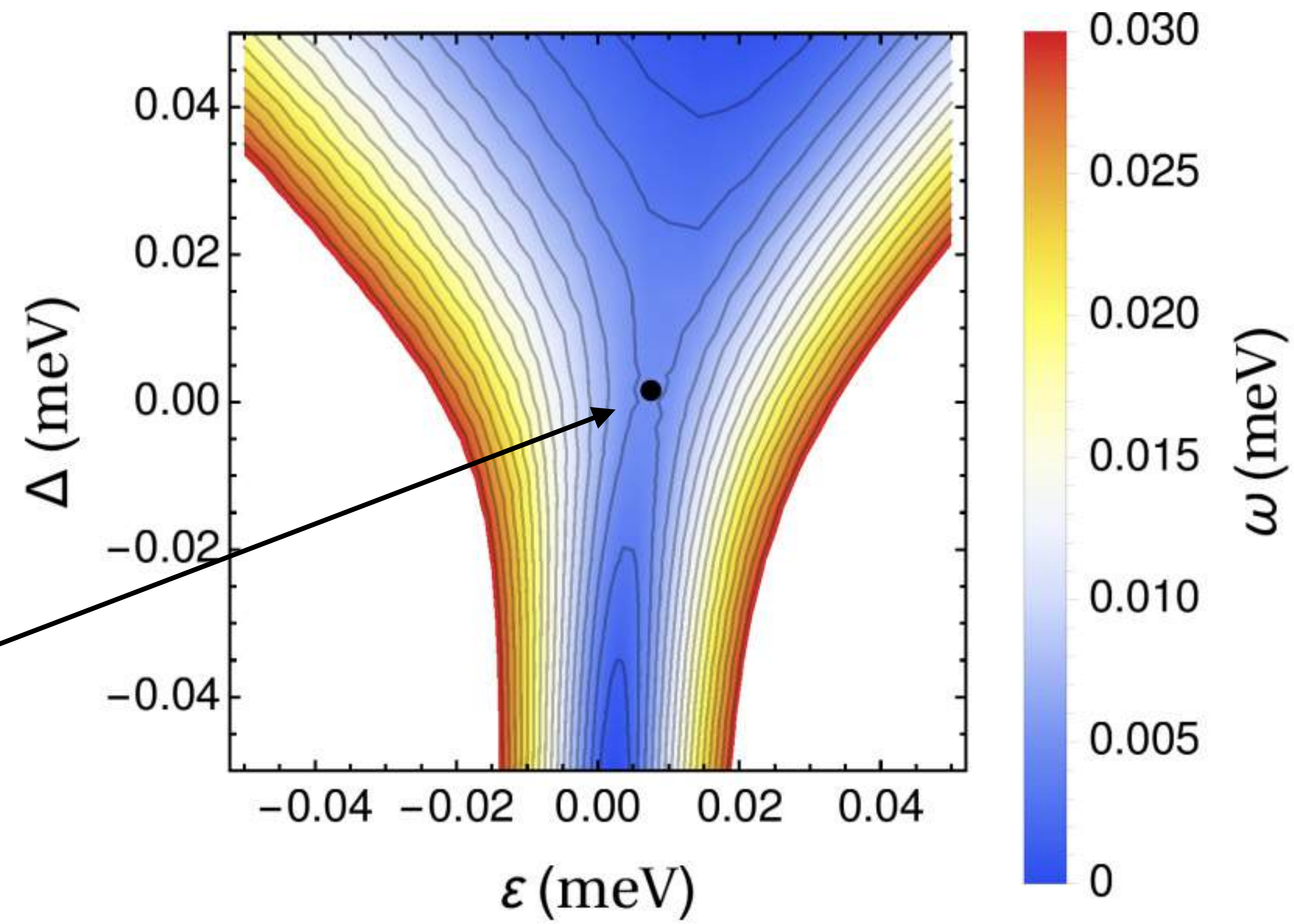
- qubit Hamiltonian

$$H_{\mathrm{RX}} = \frac{\hbar\omega}{2}\sigma_z$$

- exact double sweet spot wrt ε and Δ noise

$$\frac{\partial\omega}{\partial\varepsilon} = \frac{\partial\omega}{\partial\Delta} = 0$$





M. Russ and G. Burkard, PRB 91, 235411 (2015)

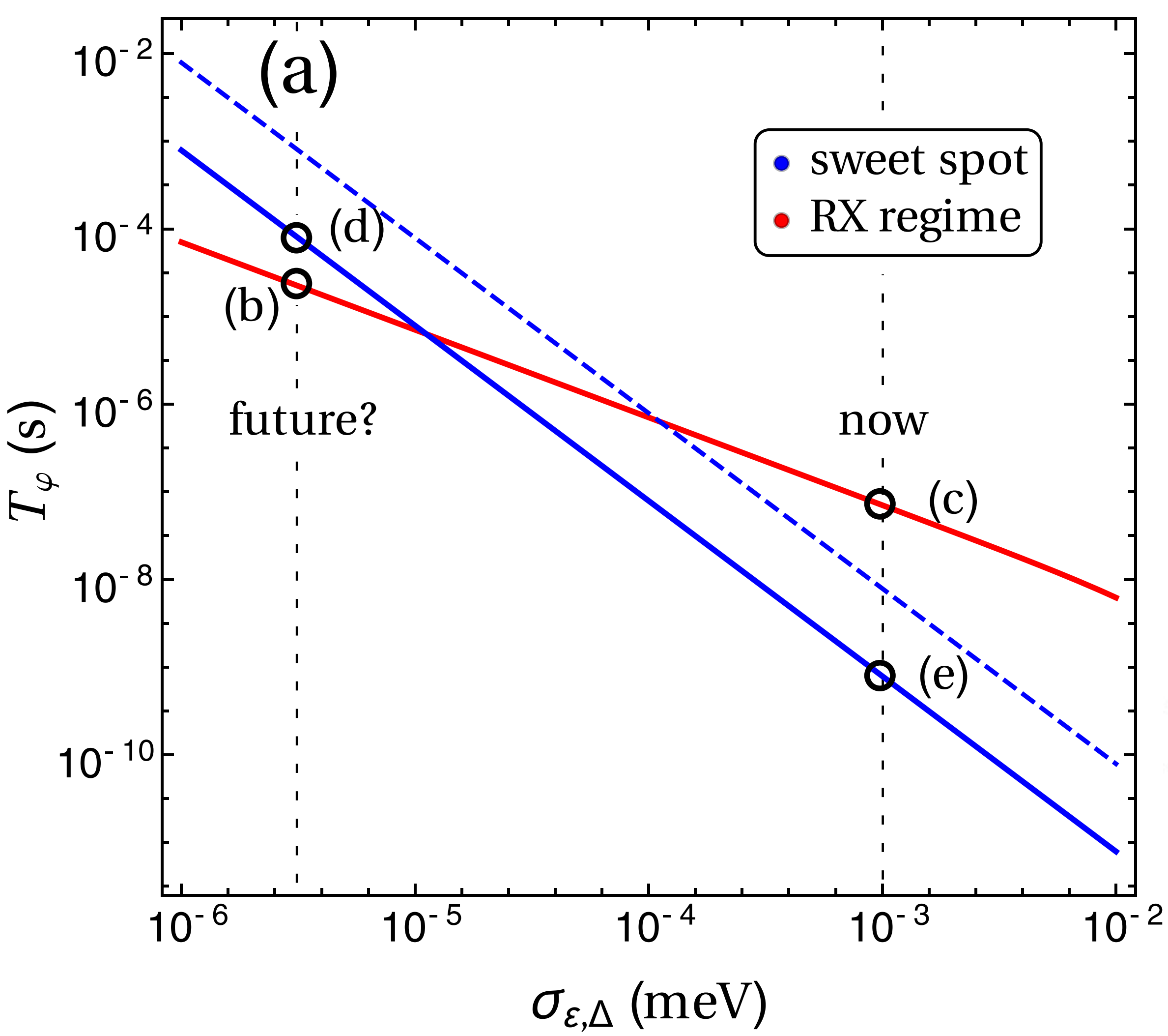
Dephasing at the double sweet spot

- accumulated phase

$$\phi(t) = \int_{t_0}^{t} dt' \delta\omega_z(t') \; = \int_{t_0}^{t} dt' \Big[\underbrace{\cancel{\omega_\varepsilon \delta\varepsilon(t') + \omega_\Delta \delta\Delta(t')}}_{} + \frac{1}{2}\omega_{\varepsilon,\varepsilon}\delta\varepsilon(t')^2 + \frac{1}{2}\omega_{\Delta,\Delta}\delta\Delta(t')^2 + \omega_{\varepsilon,\Delta}\delta\varepsilon(t')\delta\Delta(t') \Big]$$

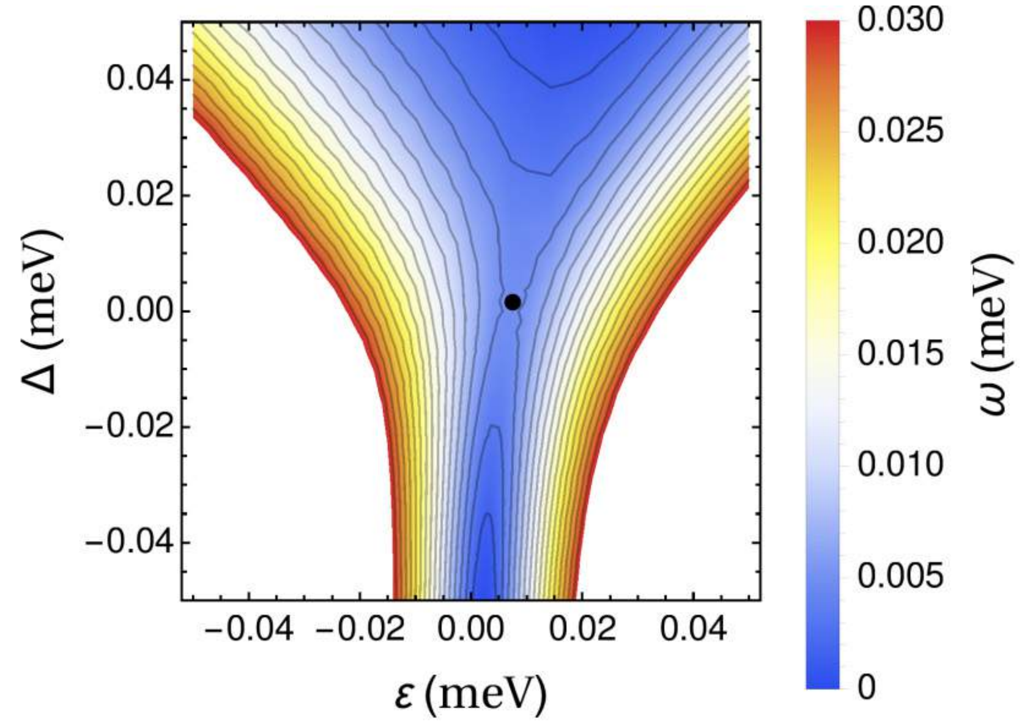
“sweet spot”

- dephasing time $\langle e^{i\phi(t)} \rangle \approx e^{-\langle \phi(t)^2 \rangle /2} \approx e^{-(t/T_\varphi)^2}$



$T_\varphi \sim 1/\sigma$

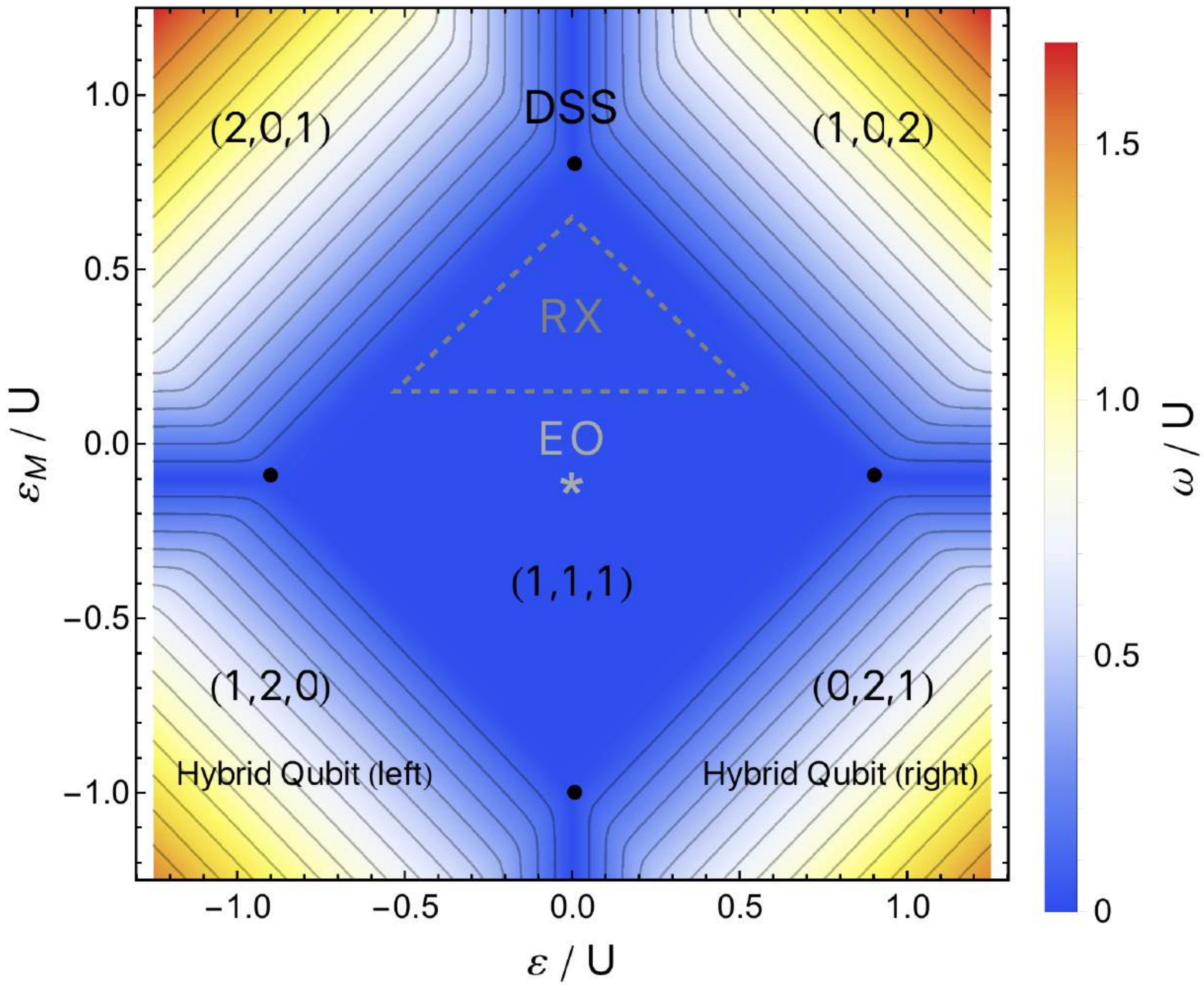
$T_\varphi \sim 1/\sigma^2$



M. Russ and G. Burkard, PRB 91, 235411 (2015)

Exploring the charge landscape

- in addition to (111), (201), and (102) states, include the (120) and (021) states (total of 6 states)
- exchange-only (EO) qubit @ $\varepsilon=\varepsilon_M=0$: AEON
  Y.-P. Shim & C. Tahan, PRB 93, 121410 (2016)
- include hybrid qubit: 3 electrons in 2 quantum dots
  Shi *et al.*, PRL (2012)
  Kim *et al.*, Nature (2014)
- double sweet spot (DSS) comes in 4 copies



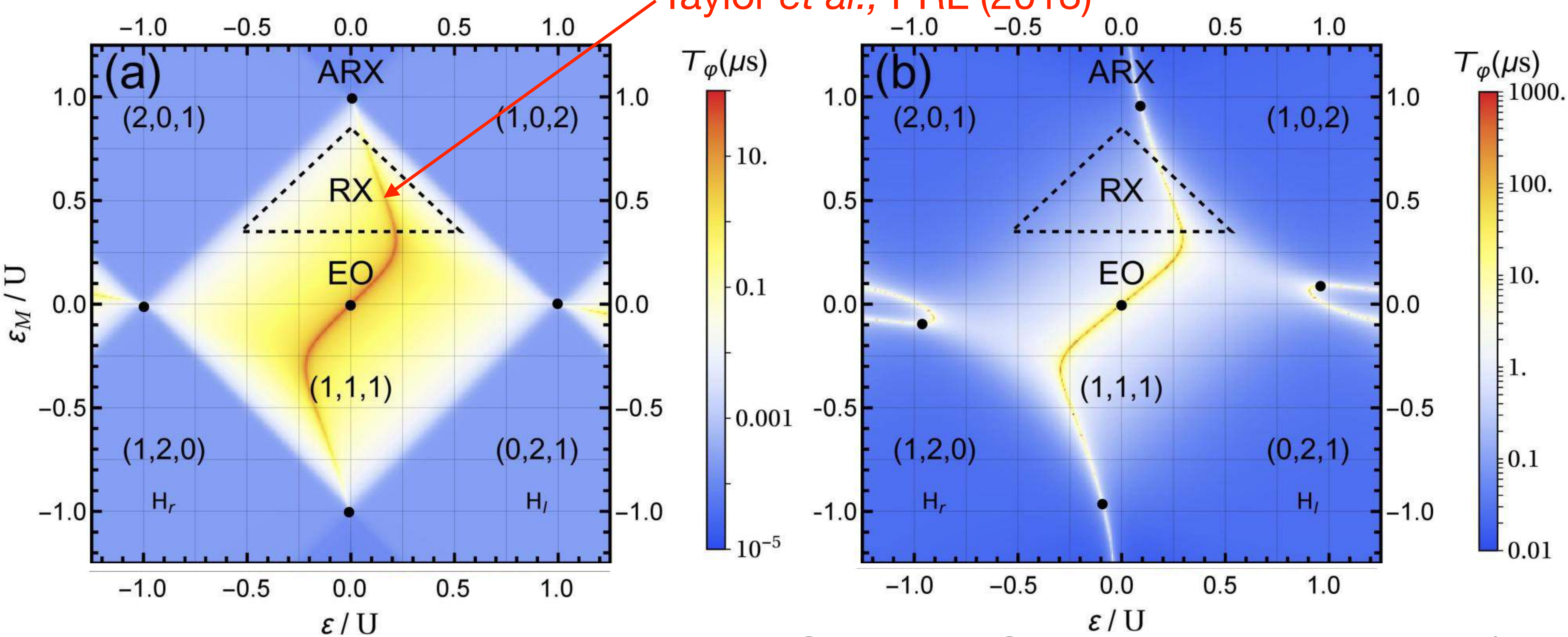
$$\varepsilon_M \equiv V_2 - (V_1 + V_3)/2 + U_C$$

M. Russ, F. Ginzel, G. Burkard, PRB 94, 165411 (2016)

Charge-noise induced dephasing

- e.g., only ε charge noise
- left: strong noise $A_\varepsilon=(1\ \mu\text{eV})^2$, weak tunneling ($t_l$ = 22 μeV, t_r = 15 μeV)
- right: weak noise $A_\varepsilon=(10^{-2}\ \mu\text{eV})^2$, strong tunneling ($t_l$ = 220 μeV, t_r = 150 μeV)

Taylor *et al.*, PRL (2013)

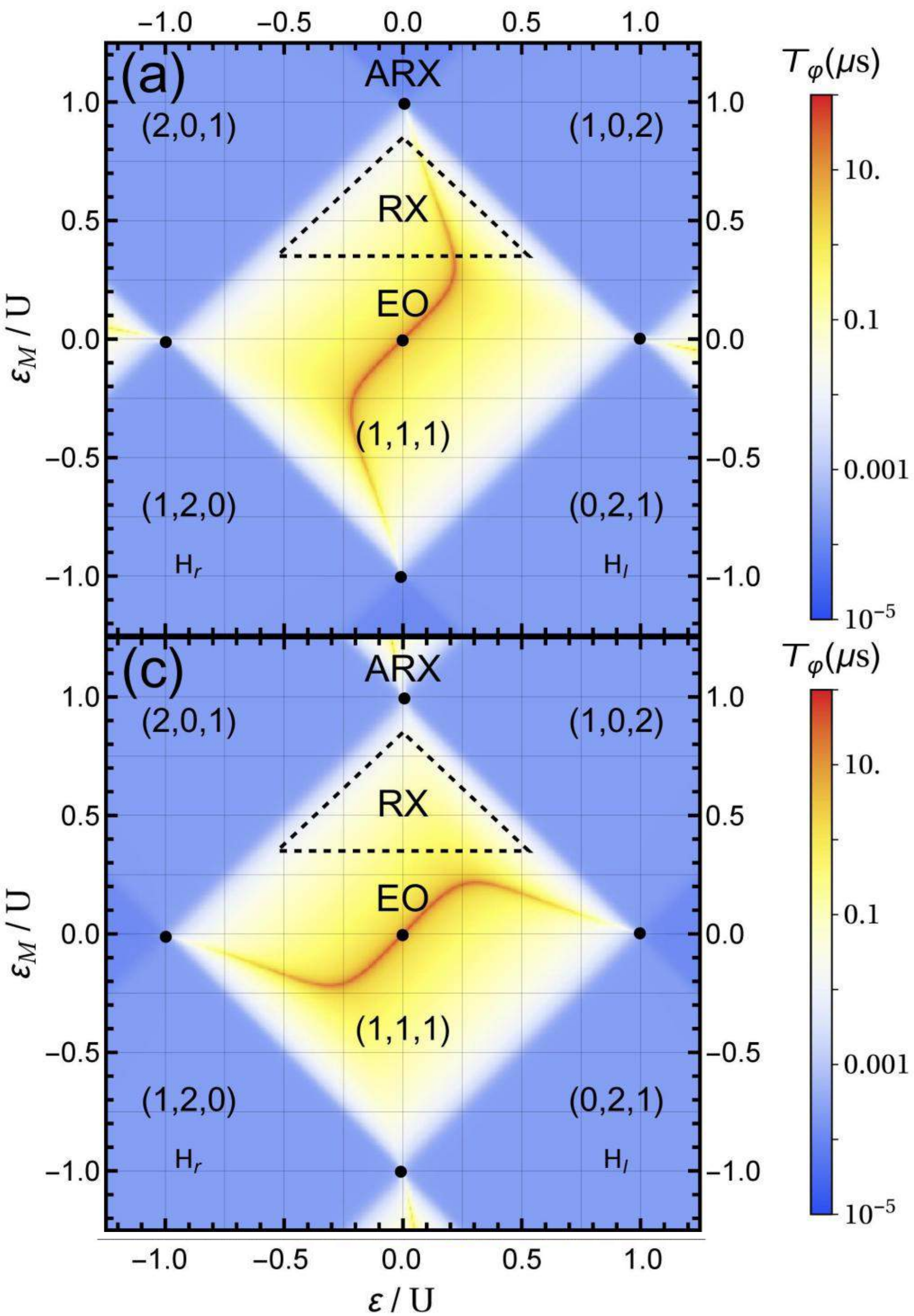


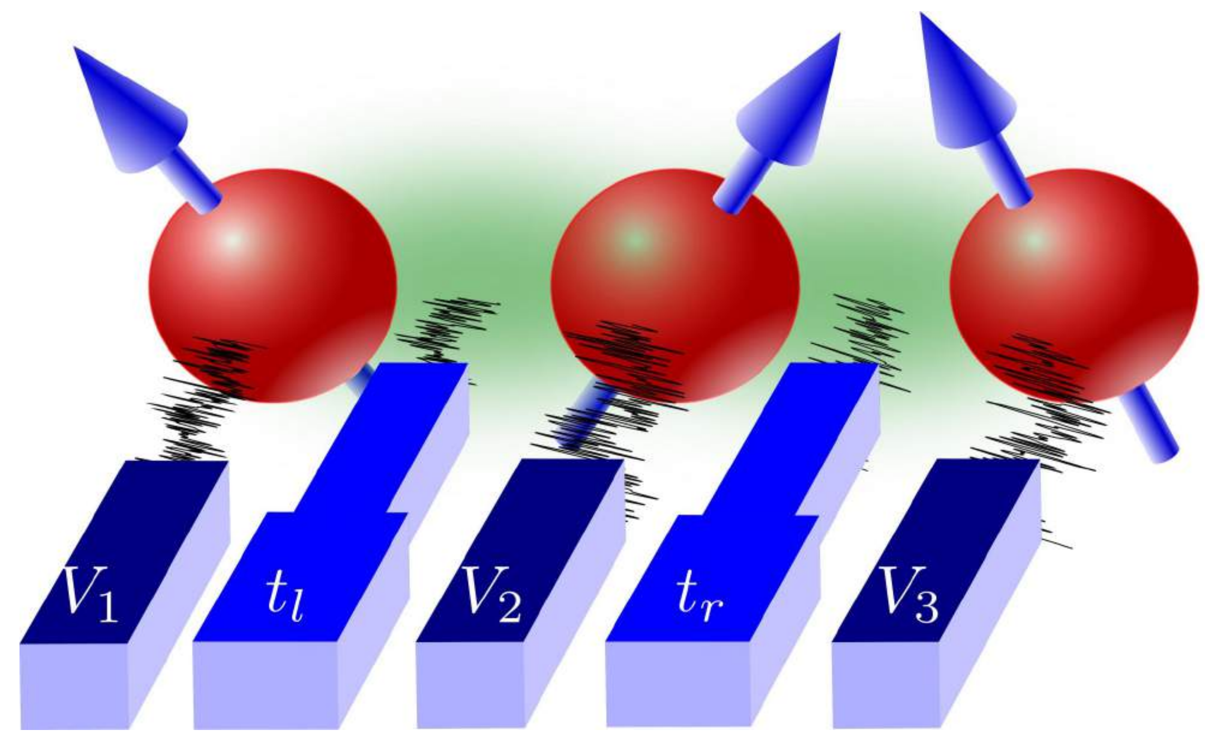
M. Russ, F. Ginzel, and G. Burkard, PRB 94, 165411 (2016)

Dephasing due to individual noise sources

ε noise

Δ / ε_M noise





$$\varepsilon \equiv (V_1 - V_3)/2$$

$$\varepsilon_M \equiv V_2 - (V_1 + V_3)/2 + U$$
$$\equiv \Delta + 2U_c$$

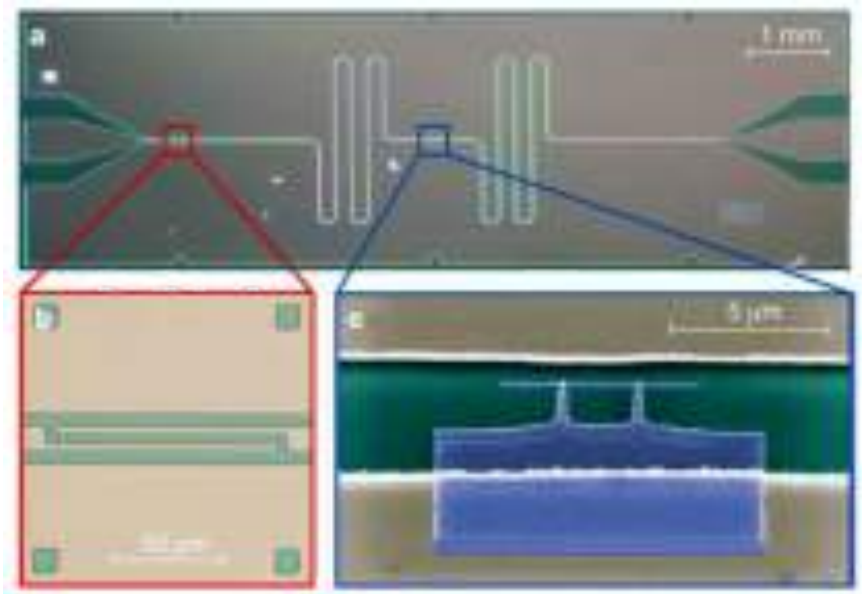
t_l = 22 μeV, t_r = 15 μeV

M. Russ, F. Ginzel, G. Burkard, PRB 94, 165411 (2016)

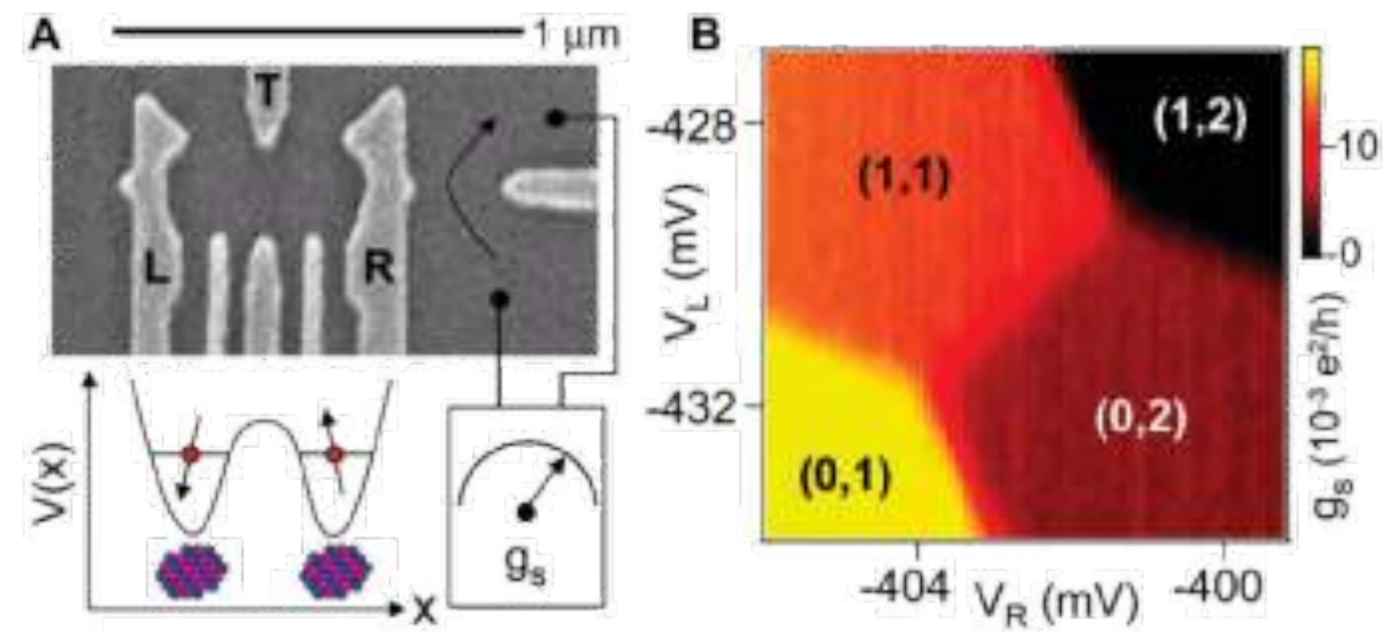
Cavity-mediated coupling between spin qubits

superconducting resonators
coupling remote qubits

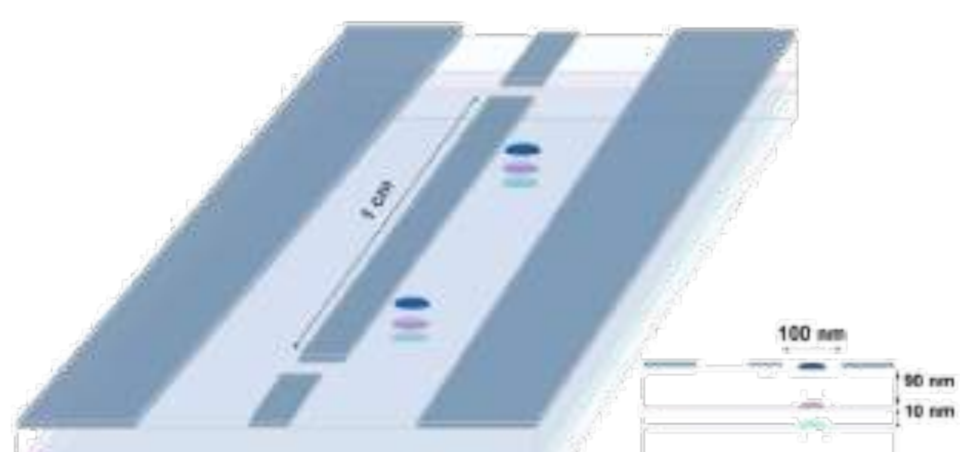
semiconductor spin qubits
long coherence



Wallraff *et al.*, Nature **431**, 162 (2004)



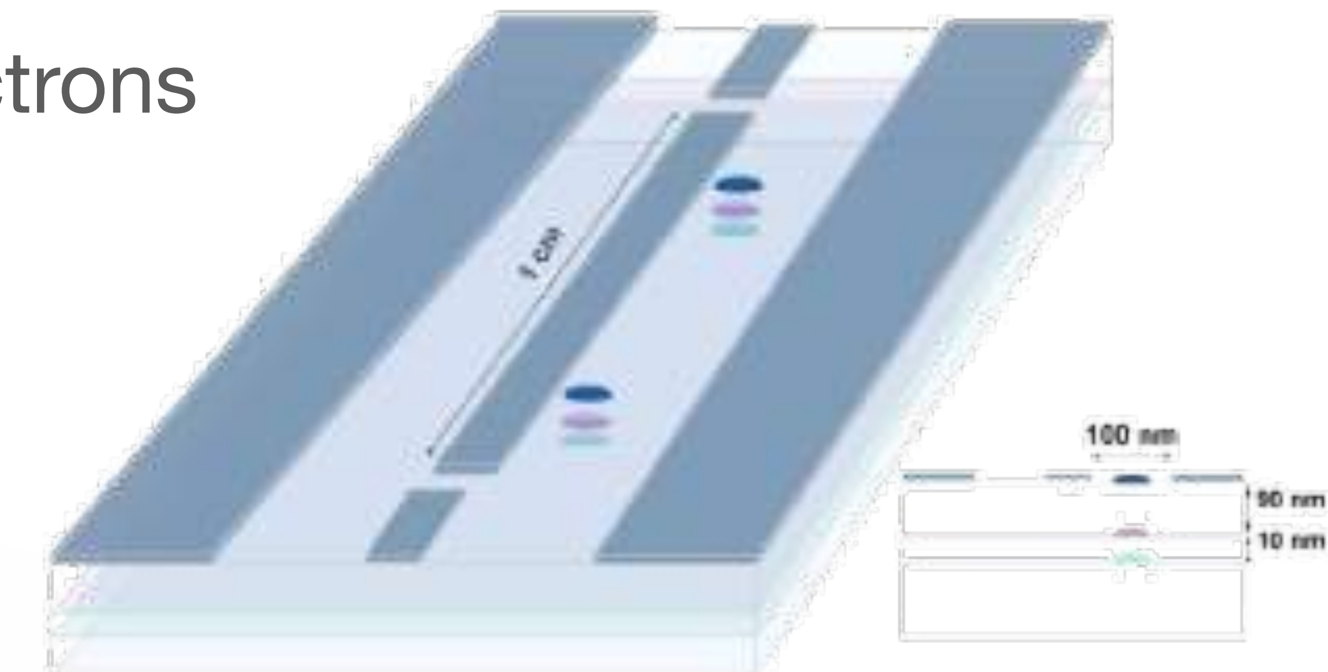
Petta *et al.*, Science **309**, 2180 (2005).



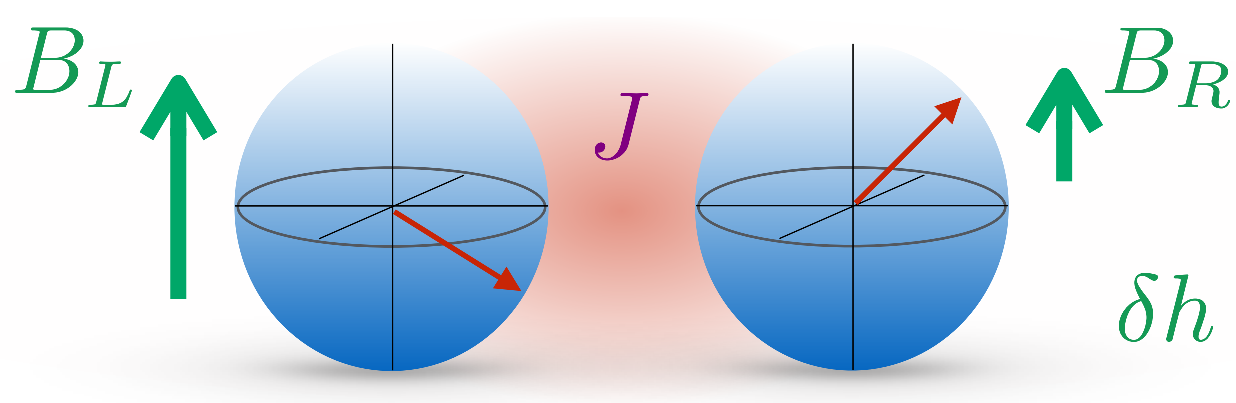
cavity-mediated long-distance interaction between spin qubits

Burkard & Imamoglu, Phys. Rev. B **74**, 041307R (2006)

Cavity-mediated coupling between spin qubits



- double quantum dot with 2 electrons
- electric dipole
- singlet-triplet qubit



$$\delta h = B_L - B_R$$

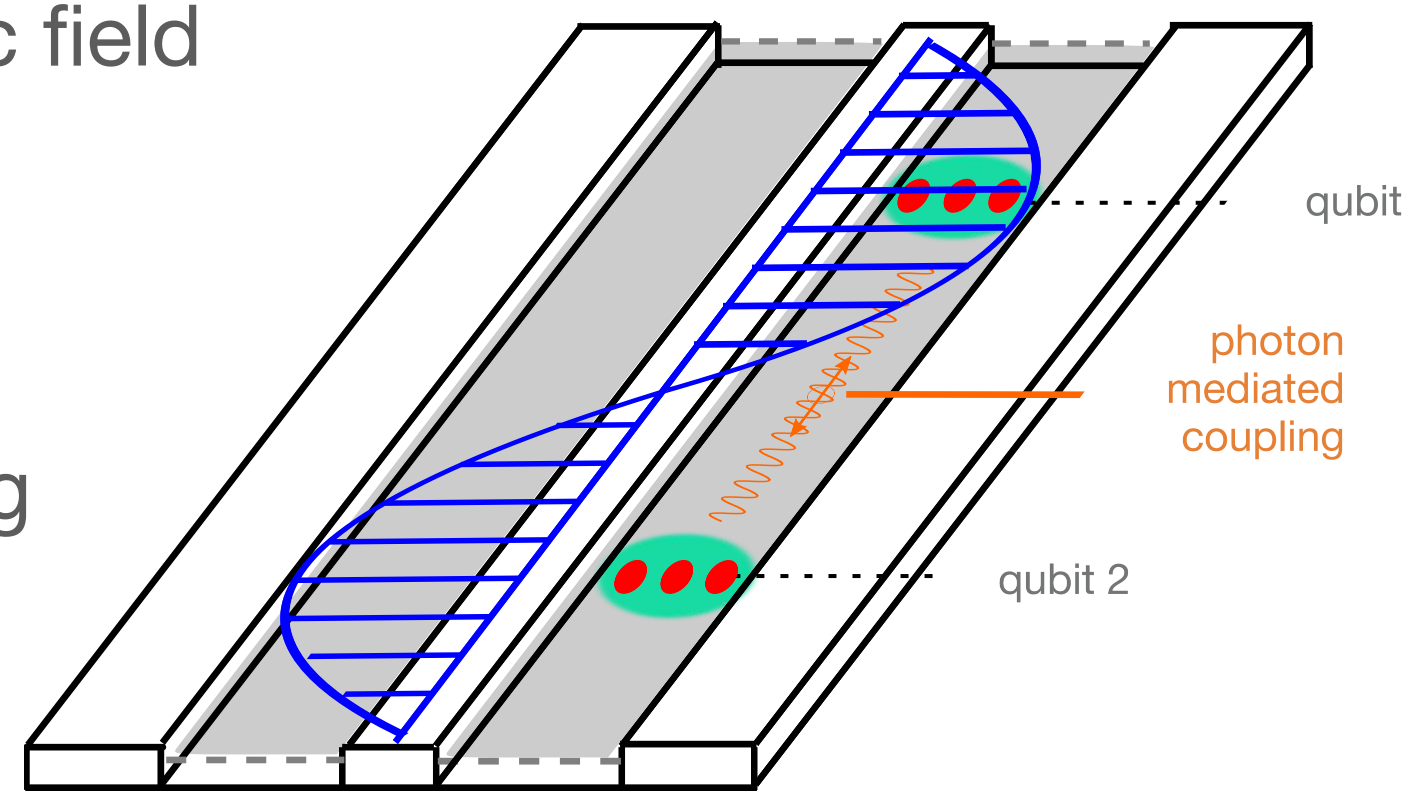
- need to break conservation to make electric dipole transition from singlet to triplet
- need to break spatial inversion symmetry (ε≠0) to make transition
- cavity coupling $g = eaE_0 \frac{J}{\hbar\omega} \frac{\varepsilon\,\delta h}{4U^2 - \varepsilon^2 - \delta h^2}$

Burkard & Imamoglu, Phys. Rev. B **74**, 041307R (2006)

Cavity-mediated coupling between spin qubits

- large electric dipoles of multi-QD spin qubits interacts with cavity electric field

  GB and Imamoglu, PRB 2006



- basis for two-qubit coupling

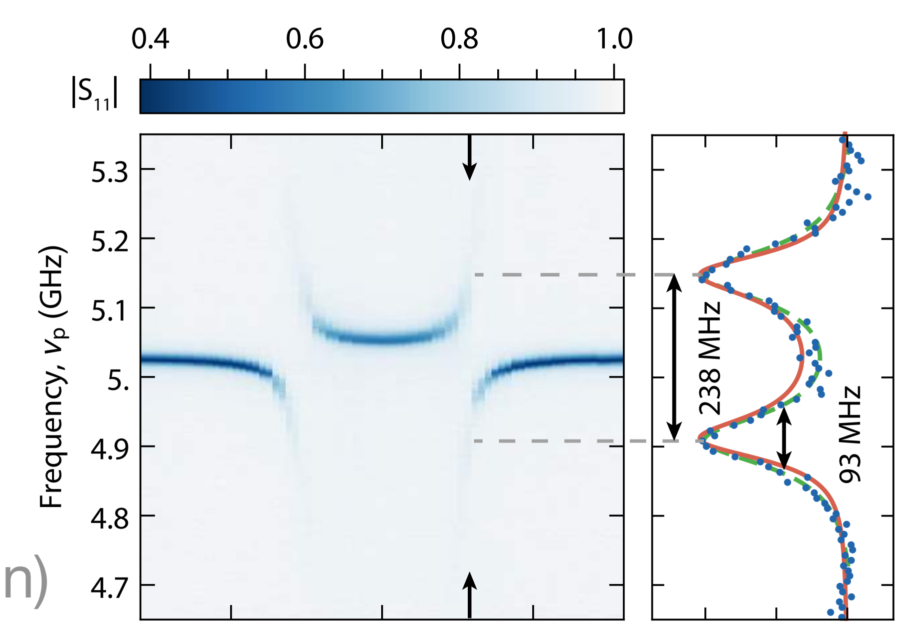
  Imamoglu *et al.*, PRL 1999

- cavity QED with RX qubits

  Russ and GB, PRB 2015

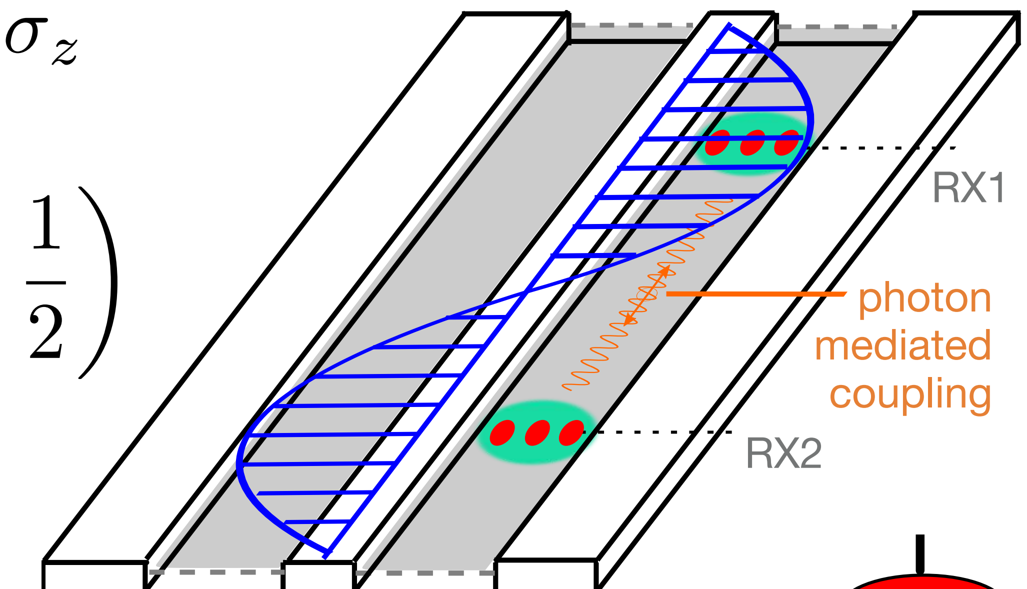
- experiments showing strong coupling

  X. Mi *et al.*, Science 2016 (Princeton: Petta group)
  L. E. Bruhat *et al.*, arXiv:1612.05214 (ENS: Kontos group)
  A. Stockklauser *et al.*, arXiv:1701.03433 (ETH: Wallraff/Ensslin)



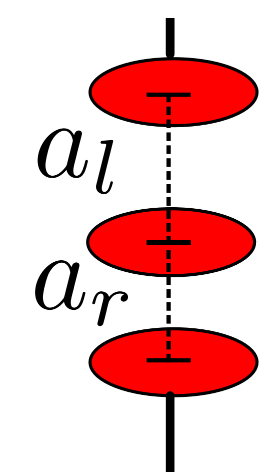
Cavity-mediated coupling of RX qubits

- 2 RX qubits in a microwave cavity $$H = \sum_{i=1,2} (H_i + H_{\mathrm{int},i}) + H_{\mathrm{cav}}$$
- quantum dot Hamiltonian $$H_i = \frac{\hbar\omega_{\mathrm{RX},i}}{2}\sigma_z$$
- cavity Hamiltonian $$H_{\mathrm{cav}} = \hbar\omega_{\mathrm{ph}}\left(a^\dagger a + \frac{1}{2}\right)$$



- qubit-cavity interaction $$H_{\mathrm{int},i} = g_s^i \sigma_x^i \left(a + a^\dagger\right)$$

$$J = (J_l + J_r)/2$$
$$j = (J_l - J_r)/2$$
$$\delta J \equiv \partial_\varepsilon J\,\kappa + \partial_\Delta J\,\kappa'$$
$$\delta j \equiv \partial_\varepsilon j\,\kappa + \partial_\Delta j\,\kappa'$$



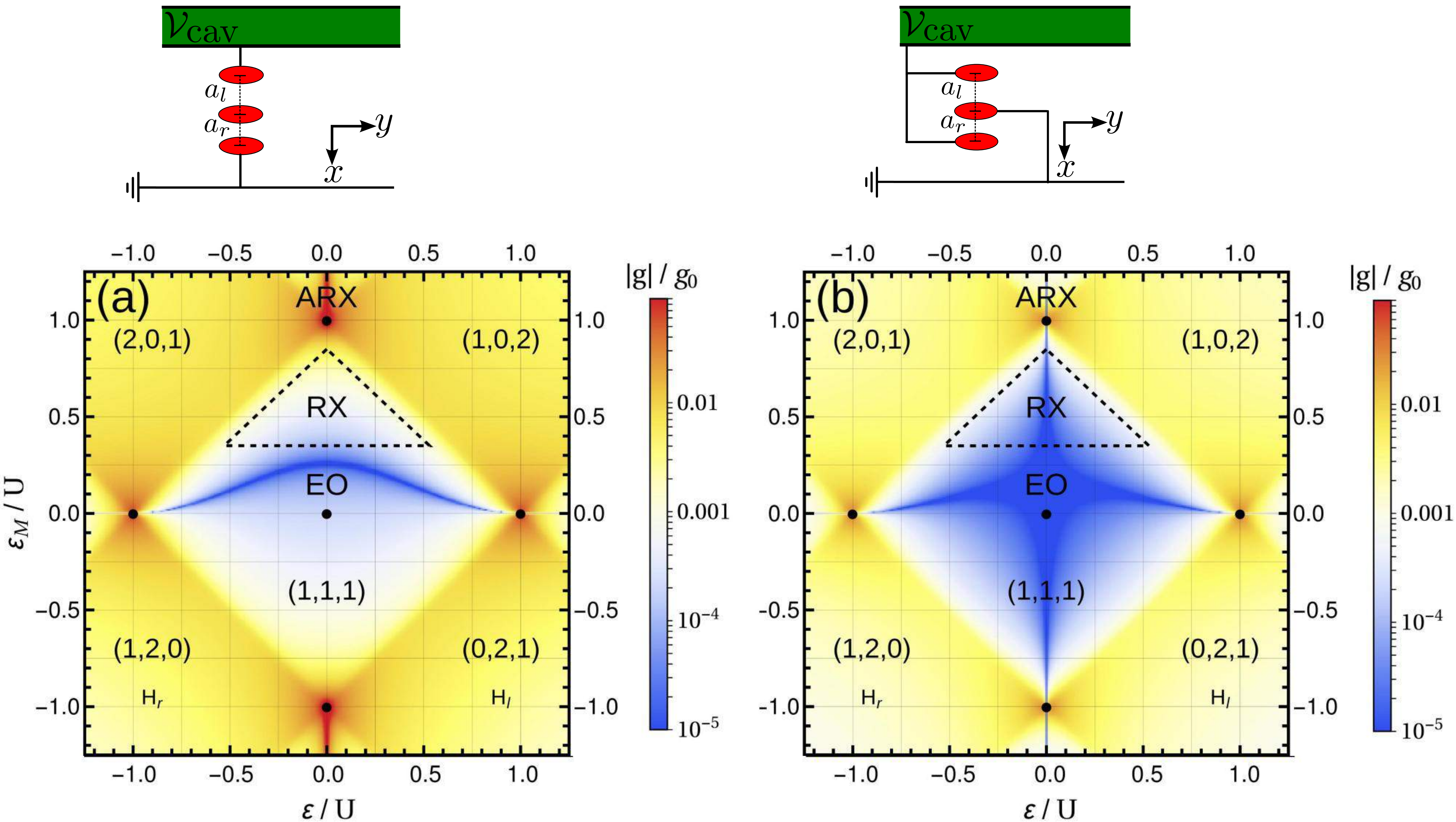
- simple model $$g_{\mathrm{s}} = \sqrt{3}(J\delta j - j\delta J)/\omega_{\mathrm{RX}}$$
- microscopic $$g_{\mathrm{r}} = -\frac{\sqrt{3}\,e\,E_0}{2\hbar\omega_{\mathrm{ph}}}\,\frac{t_l t_r \varepsilon}{\Delta^2 - \varepsilon^2}\tilde{a}_{\mathrm{rel}}\,(a_l - a_r)\,S_l S_r$$

M. Russ and G. Burkard, PRB 92, 205412 (2015)

Qubit-cavity coupling

$$g_0 = -e\mathcal{E}(a_l + a_r)$$

asymmetric coupling (ε)   symmetric coupling (ε_M)

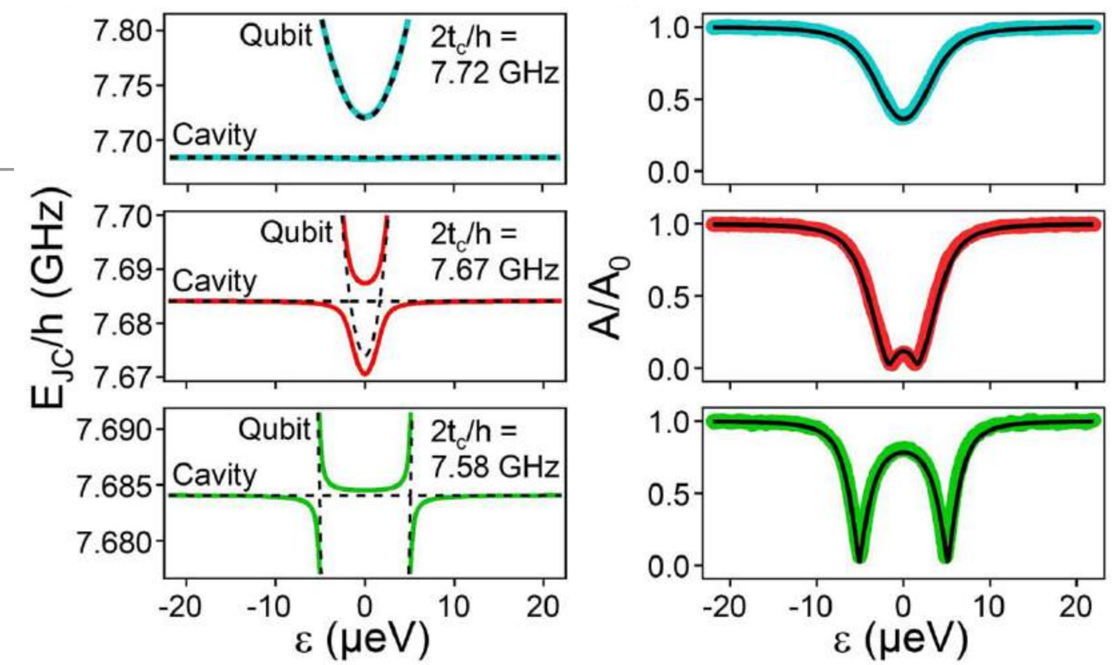


M. Russ, F. Ginzel, and G. Burkard, PRB 94, 165411 (2016)

Hybrid quantum systems: Strong coupling of spins in QDs to microwave cavities

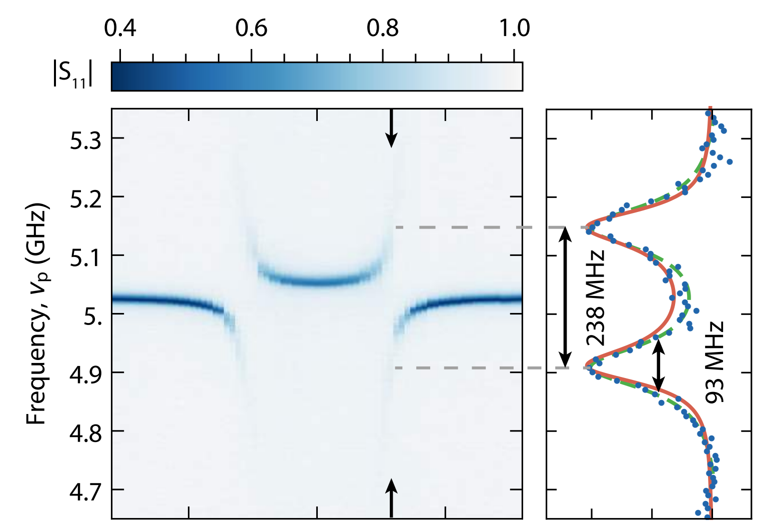
- Princeton
  - Petta group
  - Si double quantum dot
  - f ~ 8 GHz, g = 6.7 MHz, γ = 2.6 MHz, κ = 1.0 MHz, T = 10 mK

  X. Mi *et al.*, Science 10.1126/science.aal2469 (2016)



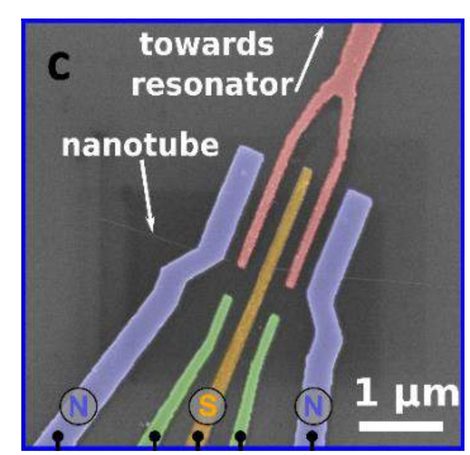
- ETH Zürich
  - Wallraff group
  - GaAs double quantum dot
  - f ~ 5 GHz, g = 119 MHz, γ = 80 MHz, κ = 12 MHz

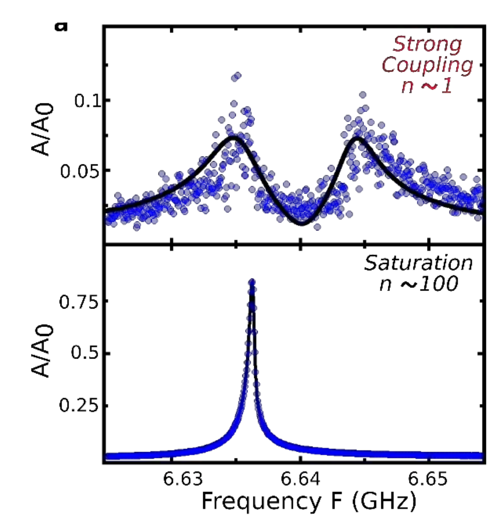
  A. Stockklauser *et al.*, arXiv:1701.03433 (2017)



- ENS Paris
  - Kontos group
  - Carbon Nanotube double quantum dot
  - f ~ 7 GHz, g = 5 MHz, γ = 2.6 MHz, κ = 0.6 MHz, T = 18 mK

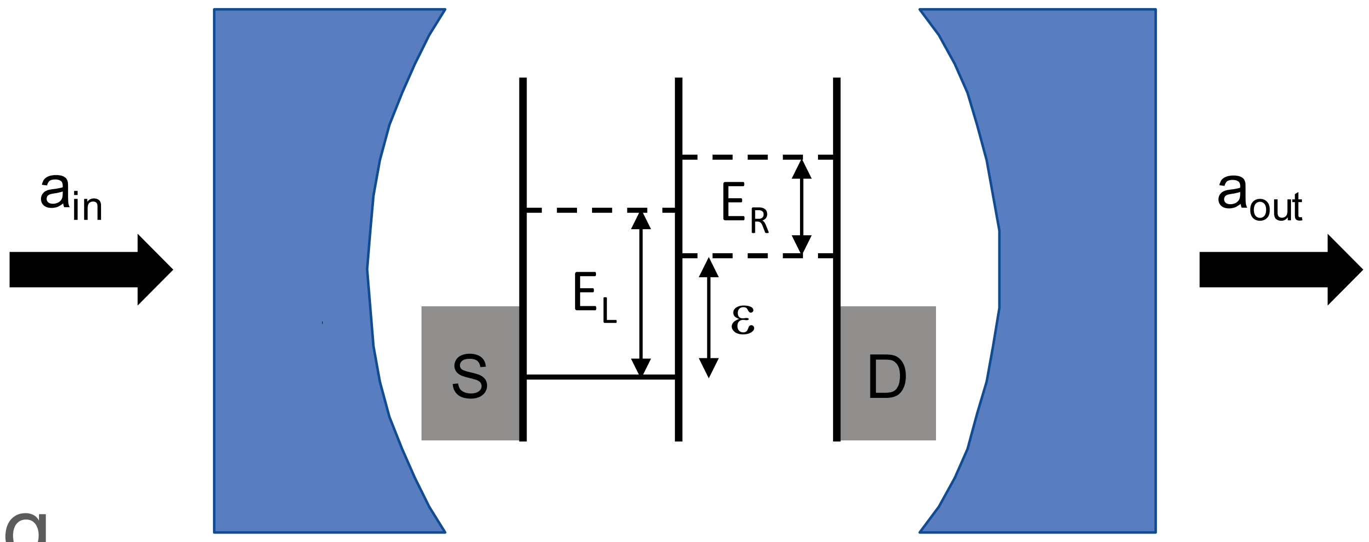
  L. E. Bruhat *et al.*, arXiv:1612.05214 (2016)



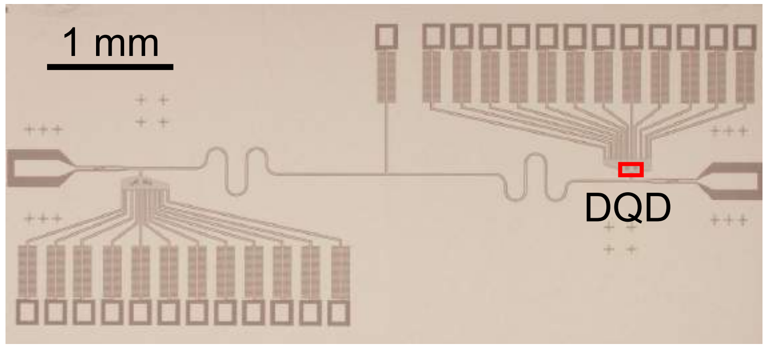


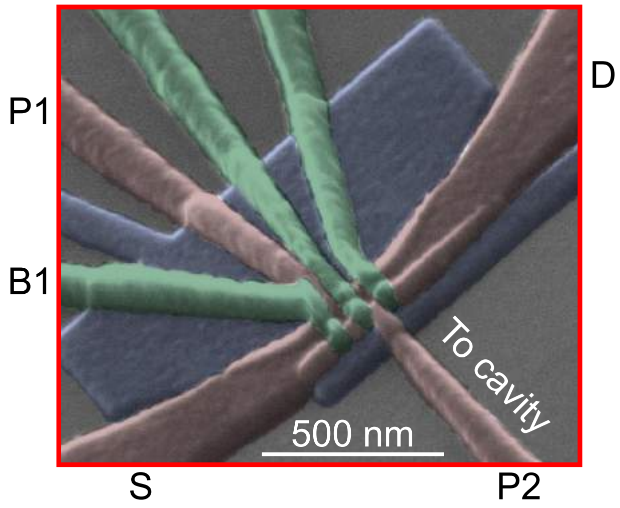
Probing QD valley physics in a microwave cavity

- unknown valley splitting in Silicon quantum dots (QDs)
- use double QD with 1 electron
- extract valley splitting
- extract intra/intervalley tunneling
- probe mw field at frequency ω_R



- input-output theory
  G. Burkard and J. R. Petta, PRB 94, 195305 (2016)





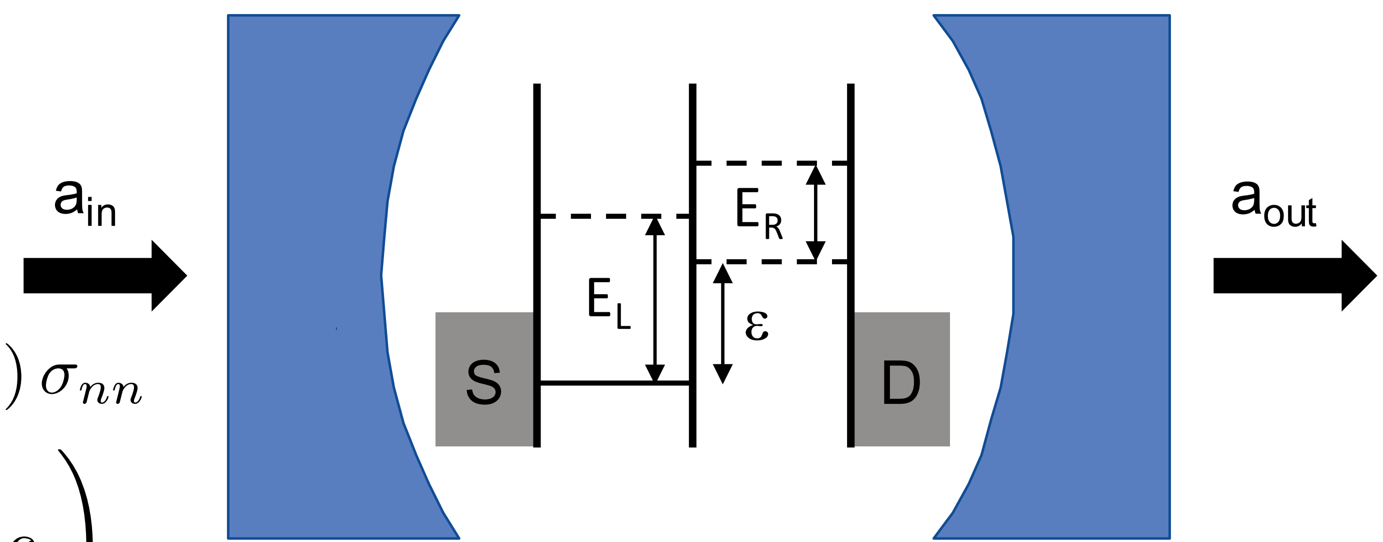
- experiment
  X. Mi, C. G. Peterfalvi, G. Burkard, J. R. Petta, arXiv:1704.06312 (2017)

"Input-output" theory

M. J. Collett and C. W. Gardiner, PRA (1984)
G. Burkard and J. R. Petta, PRB (2016)

probe freq.

- photons $H_C = (\omega_0 - \omega_R) a^\dagger a$
- electron $\sigma_{mn} = |m\rangle\langle n|$
- rotating frame $H_0 = \sum_{n=0}^{3} (E_n - n\omega_R)\, \sigma_{nn}$



$$H_I \simeq 2g_0 \left(a \sum_{n=0}^{2} d_{n+1,n} \sigma_{n+1,n} + \text{h.c.} \right)$$

cavity coupling el. dipole matrix elements

- equations of motion

$$\dot{a} = -i\Delta_0 a - \frac{\kappa}{2} a + \sqrt{\kappa_1} a_{\text{in},1} + \sqrt{\kappa_2} a_{\text{in},2} \; -2ig_0 \sum_{n=0}^{2} d_{n,n+1} \sigma_{n,n+1} \quad = 0$$

detuning
$\Delta_0 = \omega_0 - \omega_R$
cavity loss

$$\dot{\sigma}_{n,n+1} = -i(E_{n+1} - E_n - \omega_R)\sigma_{n,n+1} - \frac{\gamma}{2}\sigma_{n,n+1} - 2ig_0 d_{n+1,n}(p_n - p_{n+1})a \quad = 0$$

occupation probabilities

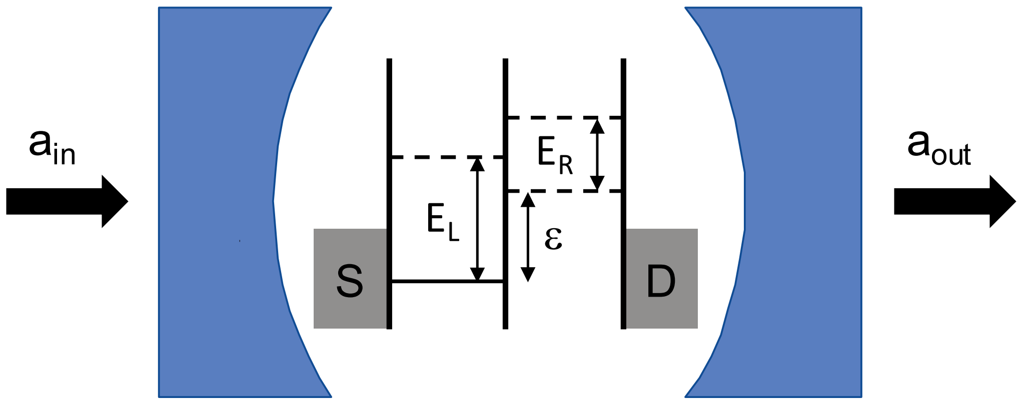
steady state

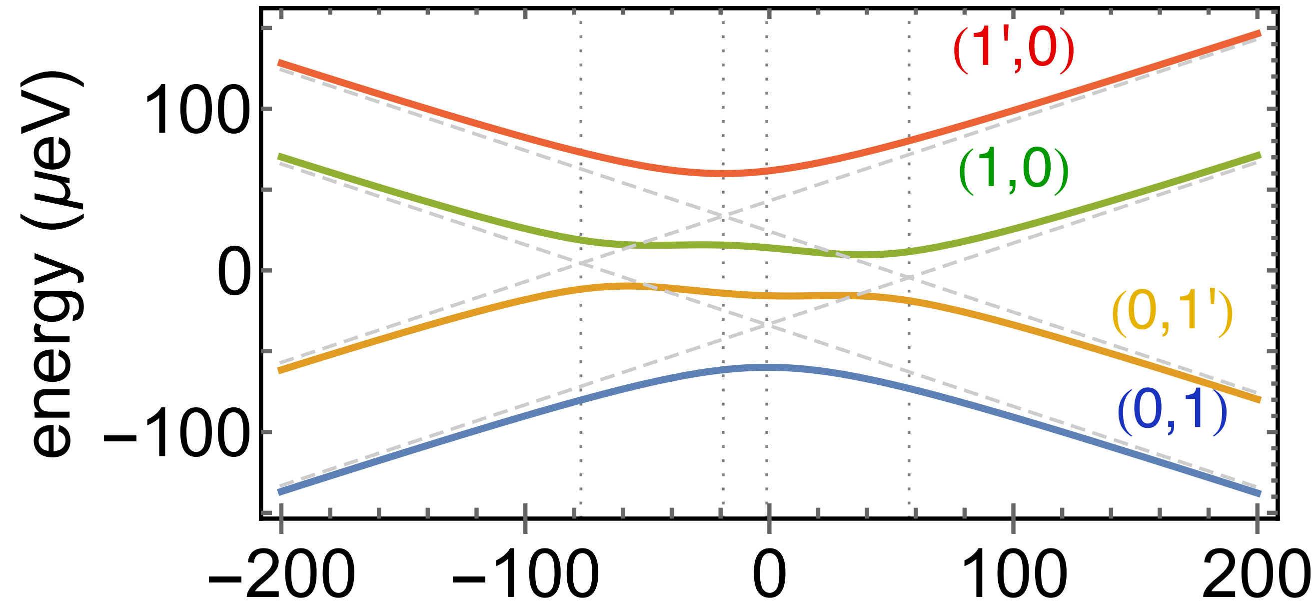
- transmission function

$$A = \frac{a_{\text{out}}}{a_{\text{in}}} = \frac{-i\sqrt{\kappa_1 \kappa_2}}{\Delta_0 - i\kappa/2 + 2g_0 \sum_{n=0}^{2} d_{n,n+1} \chi_{n+1,n}}$$

← electric susceptibility

Probing QD valley physics in a microwave cavity





- theory: microwave transmission

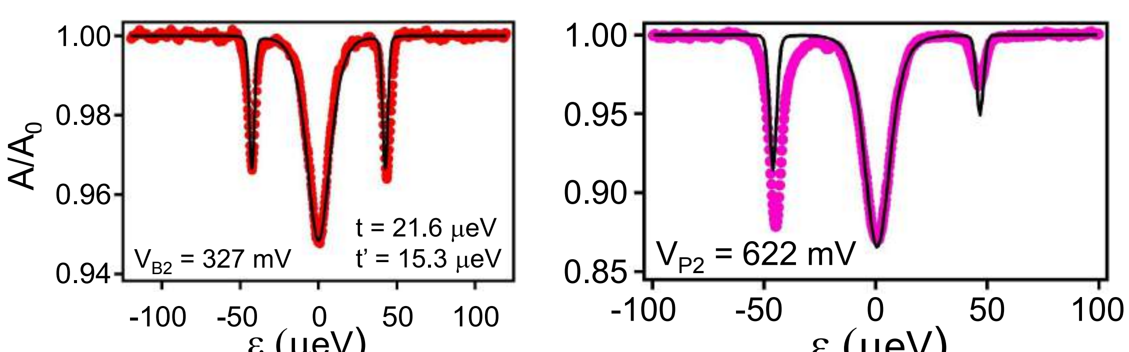
$$A = \frac{a_{\rm out}}{a_{\rm in}} = \frac{-i\sqrt{\kappa_1\kappa_2}}{\Delta_0 - i\kappa/2 + 2g_0\sum_{n=0}^{2} d_{n,n+1}\chi_{n+1,n}}$$

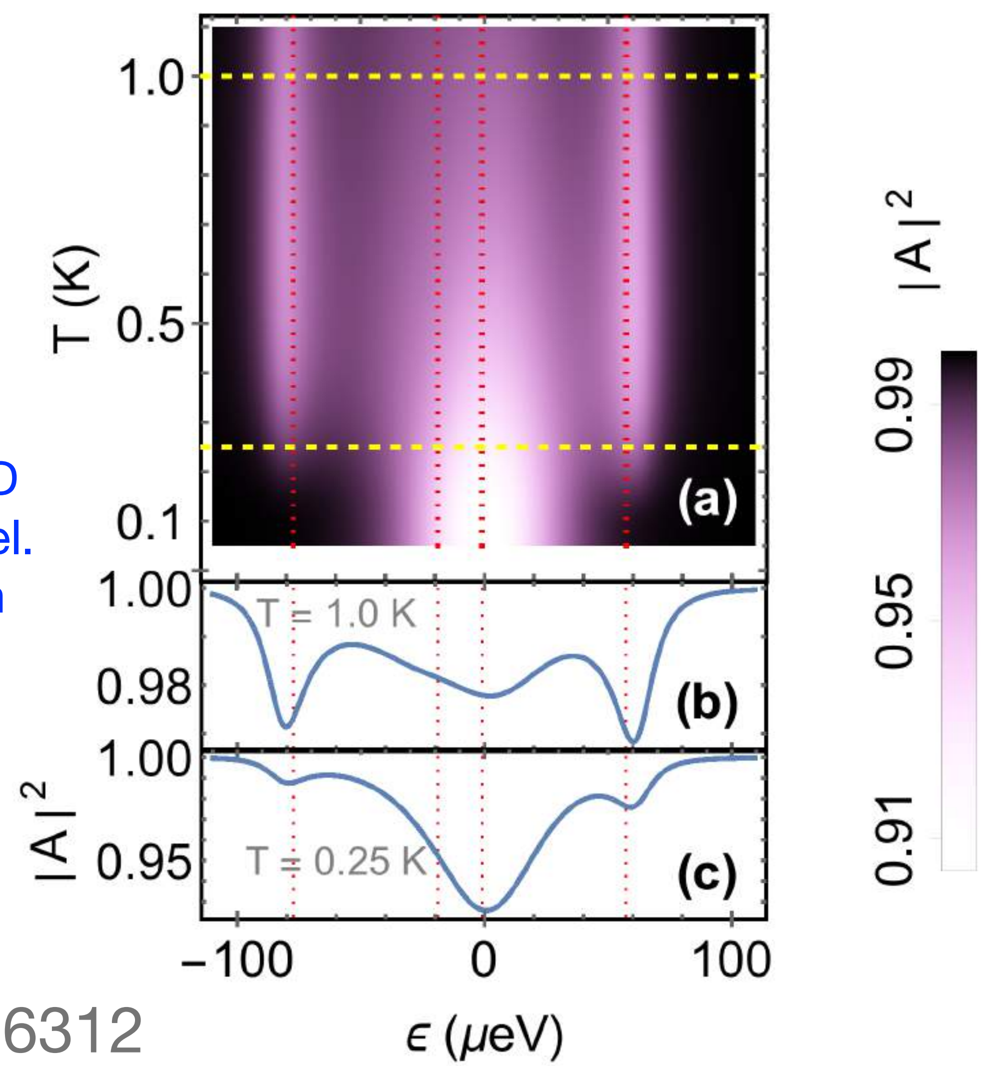
detuning   cavity loss   cavity coupling   electric susceptibility

$$\frac{-2g_0 d_{n+1,n}(p_n - p_{n+1})}{E_{n+1} - E_n - \omega_R - i\gamma/2}$$

E_n: energy levels of DQD
$d_{n,n+1}$: el. dipole matrix el.
p_n: (thermal) occupation

- experiment



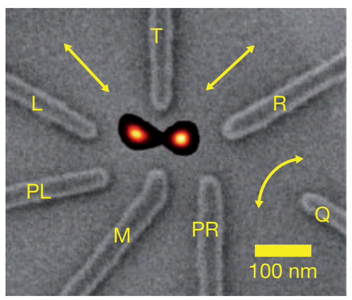


G. Burkard & J. R. Petta, PRB (2016)
X. Mi, C. G. Peterfalvi, G. Burkard, J. R. Petta, arXiv:1704.06312

Dilute nuclear-spin materials

Wikipedia

95% ^{28}Si, ^{30}Si (I=0)
5% ^{29}Si (I=1/2)



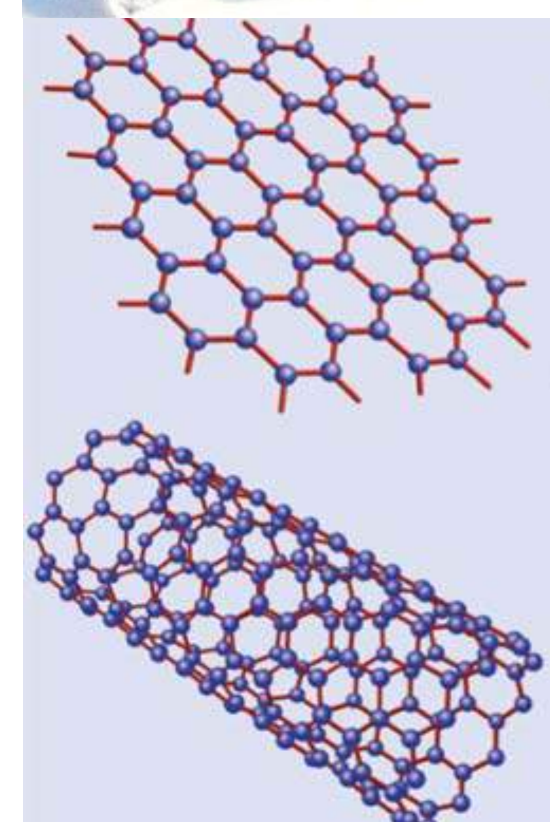
Maune *et al.*, Nature 2012

fraction of naturally occurring nuclear spin-free isotopes (I=0)

	II	III	IV	V	VI
		boron 5 **B** 0%	carbon 6 **C** 99%	nitrogen 7 **N** 0%	oxygen 8 **O** 99.9%
		aluminium 13 **Al** 0%	silicon 14 **Si** 95%	phosphorus 15 **P** 0%	sulfur 16 **S** 99.2%
molybdenum 42 **Mo** 73%	zinc 30 **Zn** 96%	gallium 31 **Ga** 0%	germanium 32 **Ge** 92%	arsenic 33 **As** 0%	selenium 34 **Se** 92%
tungsten 74 **W** 86%	cadmium 48 **Cd** 75%	indium 49 **In** 0%	tin 50 **Sn** 85%	antimony 51 **Sb** 0%	tellurium 52 **Te** 92%

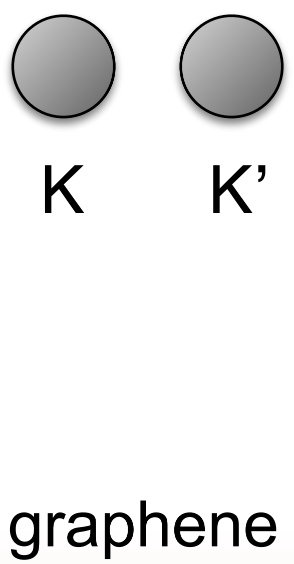
99% ^{12}C (I=0)
1% ^{13}C (I=1/2)

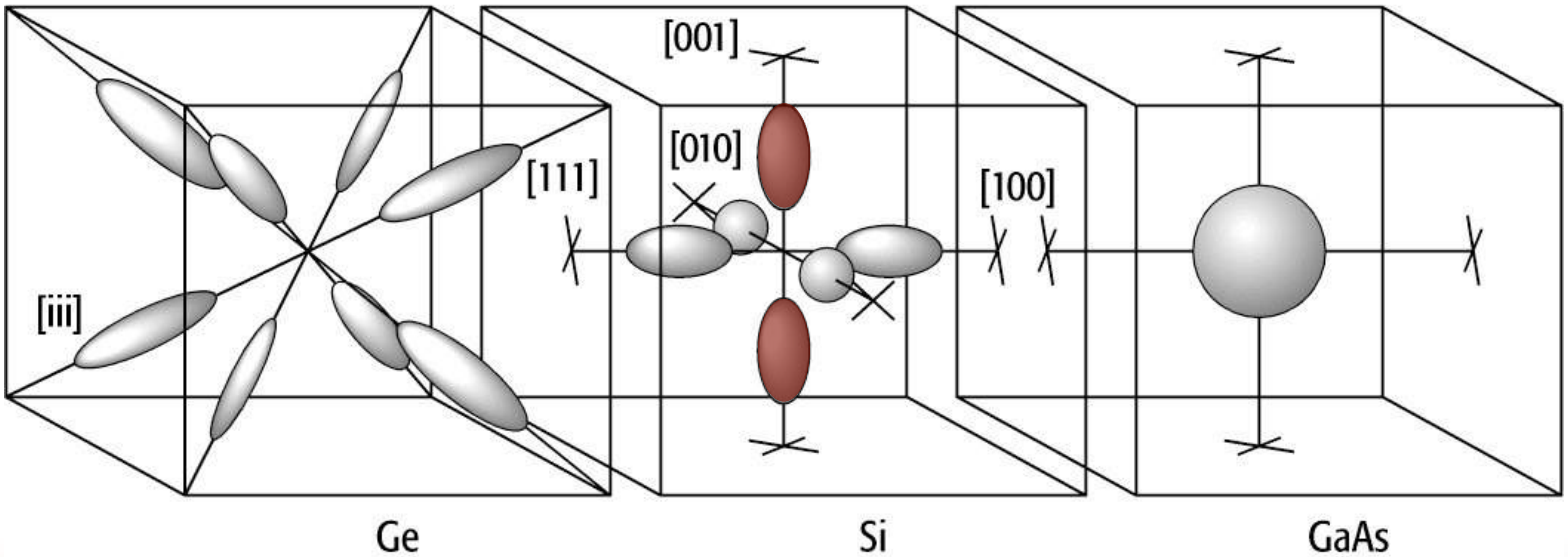
source: Wikipedia



Castro Neto et al., Rev. Mod. Phys, 2009

Valley degeneracy





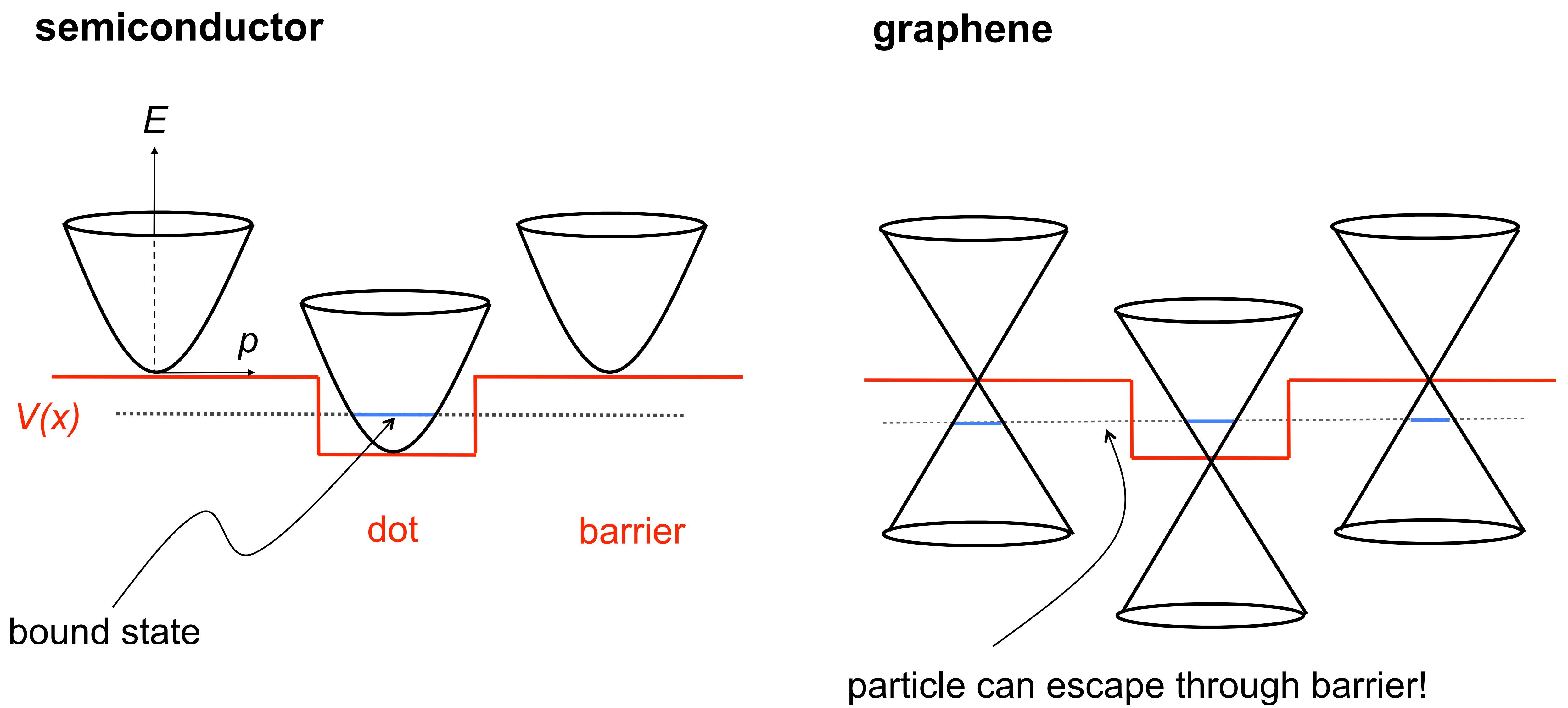
$$|\uparrow K\rangle, |\downarrow K\rangle, |\uparrow K'\rangle, |\downarrow K'\rangle$$

$$|\uparrow\rangle, |\downarrow\rangle$$

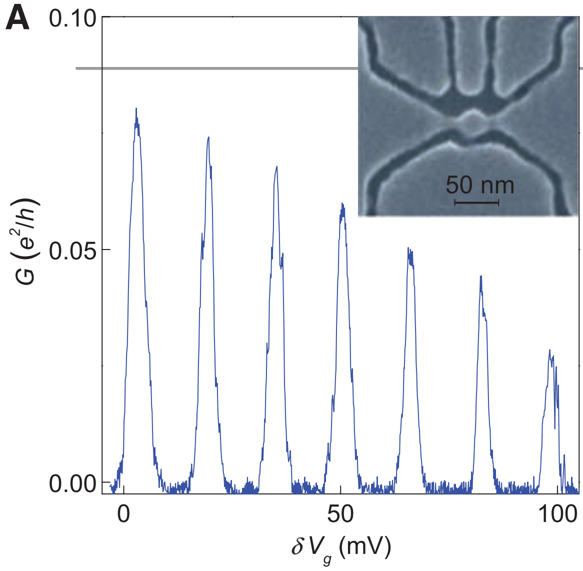
$$H = \frac{J}{8}\Big((\mathbf{S}_1 \cdot \mathbf{S}_2)(\tau_1 \cdot \tau_2) + \mathbf{S}_1 \cdot \mathbf{S}_2 + \tau_1 \cdot \tau_2\Big)$$

$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2$$

Problem 1: Klein tunneling (graphene)



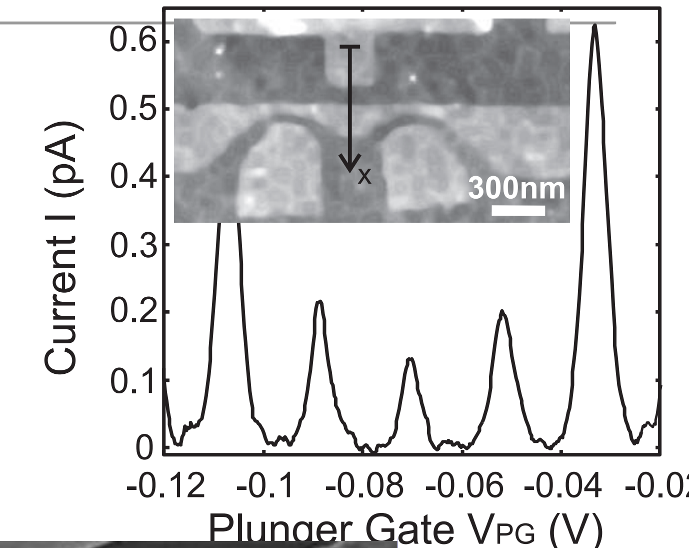
Quantum dots in graphene



experiment: nanostructured graphene

Ponomarenko *et al.*, Science 2008

Stampfer *et al.*, APL 2008

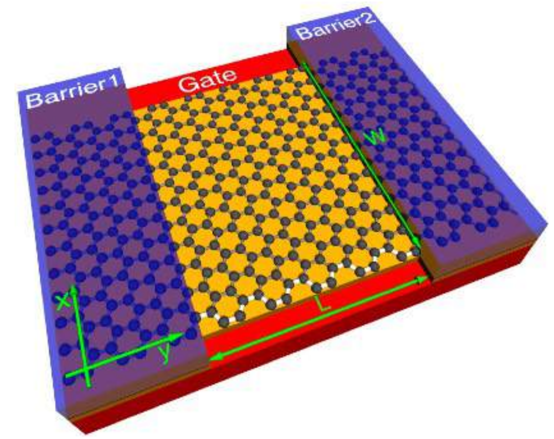


theory: graphene nanoribbon

boundary conditions: gap

electrodes: QD

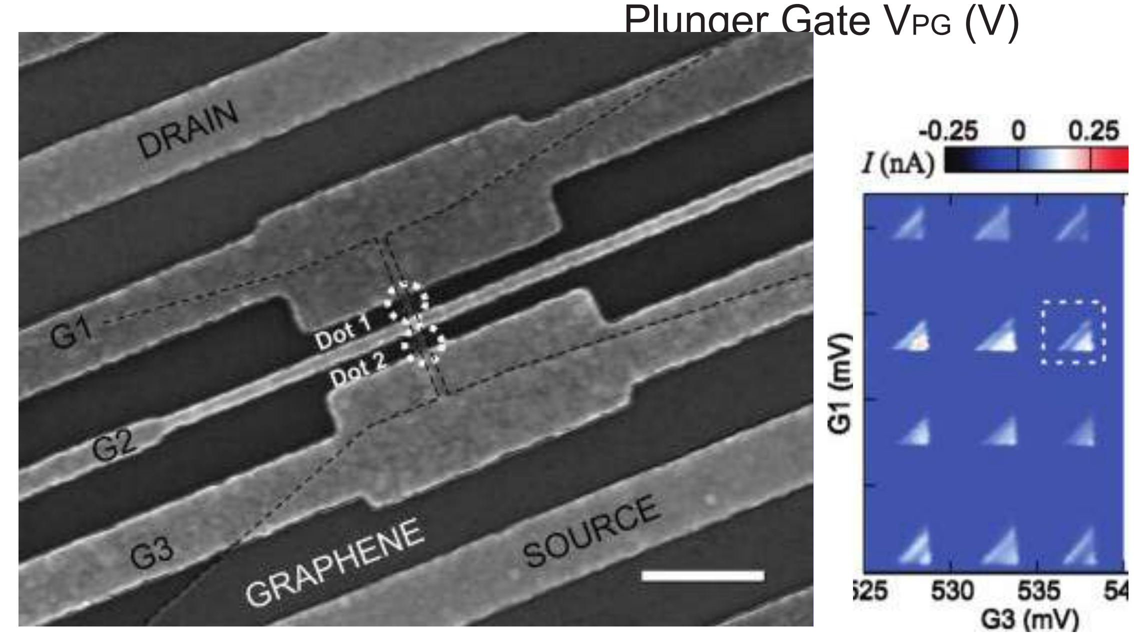
Trauzettel, Bulaev, Loss, GB, Nature Phys. 2007



experiment: graphene nanoribbon

gate-defined double quantum dot

X. L. Liu, D. Hug, L. M. K. Vandersypen, Nano Lett. 2010



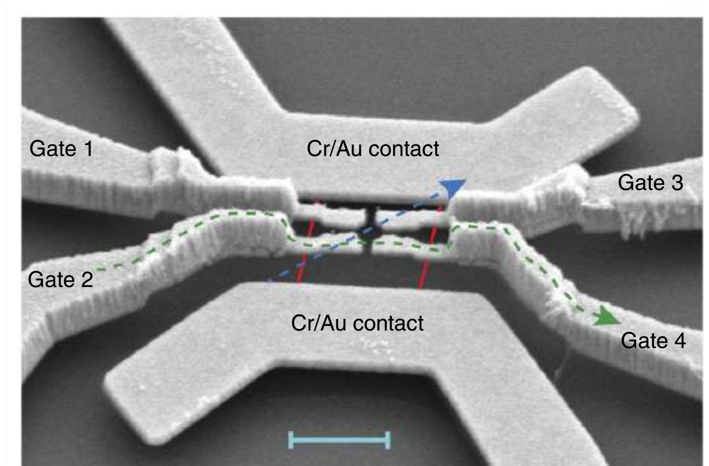
theory: gapped graphene (single or bilayer)

Milton Pereira *et al.*, Nano Letters 2007 (x-dependent doping)

Recher, Nilsson, GB, Trauzettel, PRB 2009 (lateral confinement)

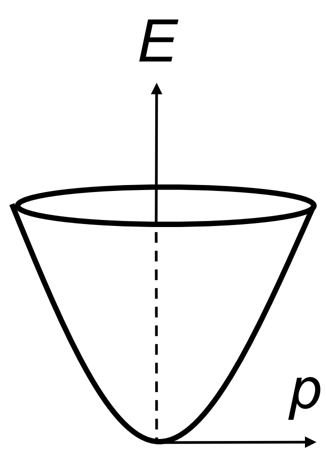
theory: gapped graphene (bilayer)

Allen *et al.*, Nature Communications 2012

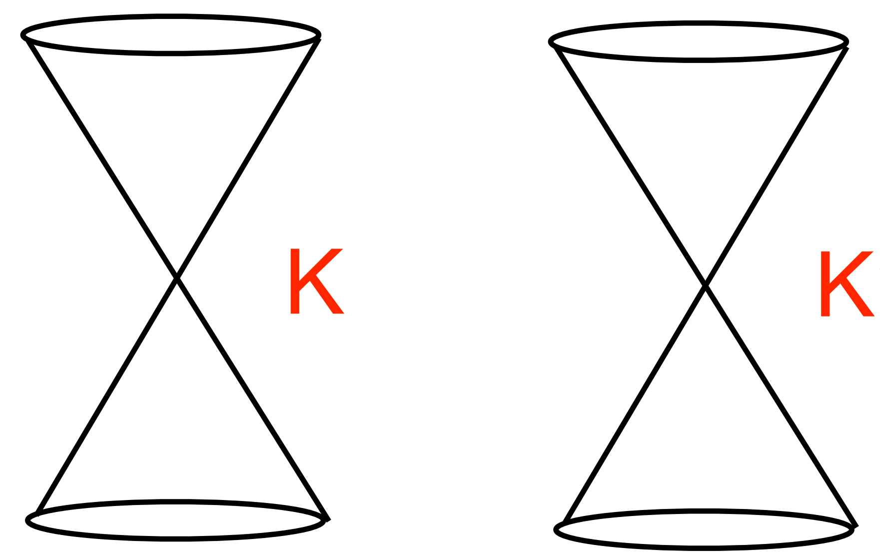


Problem 2: Valley degeneracy

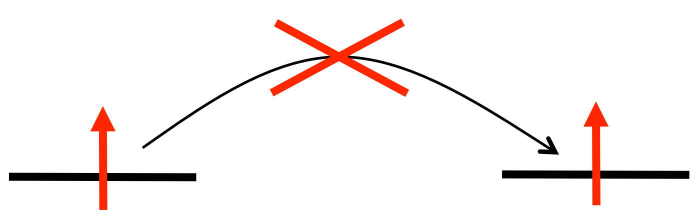
direct-gap semiconductor



graphene

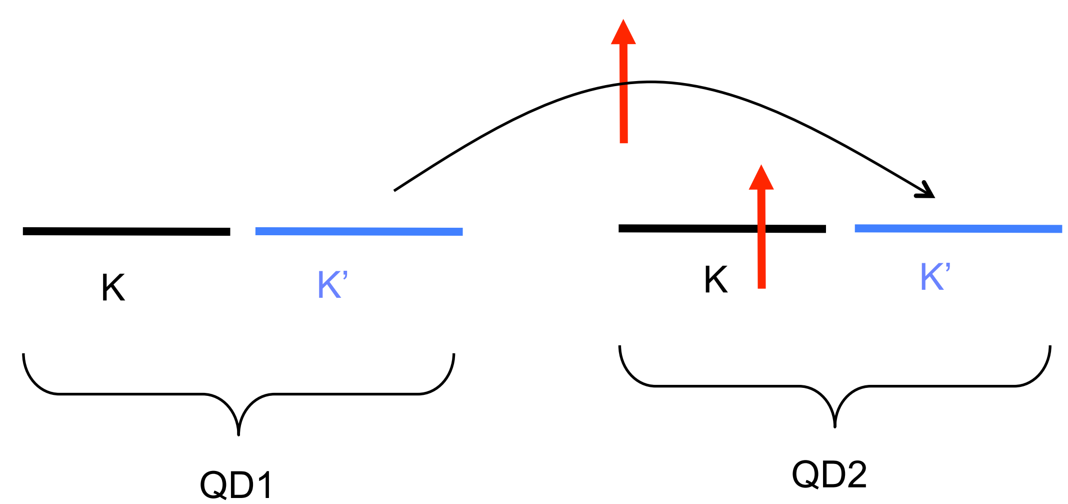


Pauli principle, exchange coupling



$$J = t^2/U$$

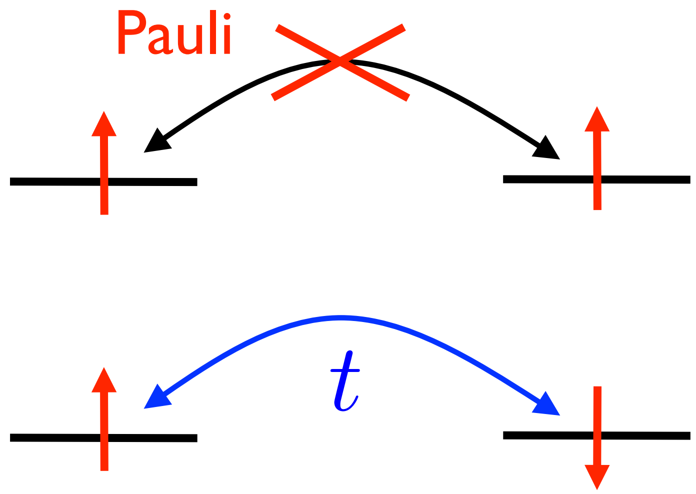
valley degeneracy



Exchange coupling

spin triplet
$s = 1$

spin singlet
$s = 0$



$$E_{\text{triplet}} = E_0$$

$$E_{\text{singlet}} = E_0 - 4t^2/U$$

$$J = E_{\text{triplet}} - E_{\text{singlet}} = -4t^2/U$$

$$H = -J P_{\text{singlet}} + E_0$$

$$P_{\text{singlet}} = |S\rangle\langle S| \overset{S^2 = s(s+1)}{=} \frac{1}{2}(2 - S^2) = 1 - \frac{1}{2}(S_1^2 + S_2^2 + 2\mathbf{S}_1 \cdot \mathbf{S}_2) = \frac{1}{4} - \mathbf{S}_1 \cdot \mathbf{S}_2$$

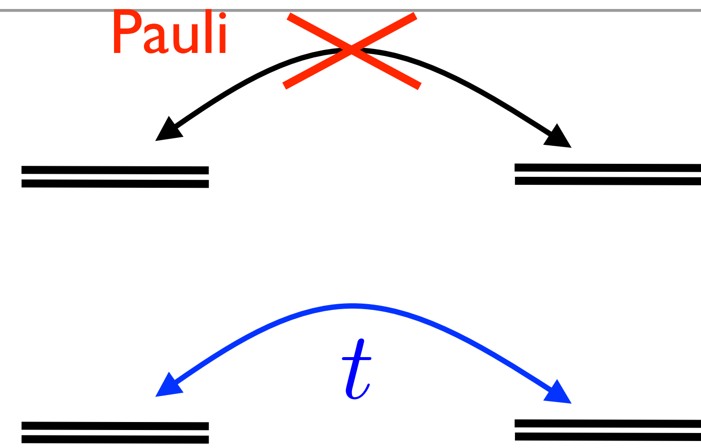
$$= P_{\text{as}}$$

$$\boxed{H = J\mathbf{S}_1 \cdot \mathbf{S}_2 + \cancel{\text{const.}}}$$

Exchange coupling with valley

supertriplet
(symmetric spin-valley states)

supersinglet
(anti-symmetric spin-valley states)



$$E_{\mathrm{striplet}} = E_0$$

$$E_{\mathrm{ssinglet}} = E_0 - 4t^2/U$$

$$J = E_{\mathrm{striplet}} - E_{\mathrm{ssinglet}} = -4t^2/U$$

$$P^{\mathrm{spin}}_{\mathrm{singlet}} = \frac{1}{4} - \mathbf{S}_1 \cdot \mathbf{S}_2$$

$$P^{\mathrm{spin}}_{\mathrm{triplet}} = 1 - P^{\mathrm{spin}}_{\mathrm{singlet}} = \frac{3}{4} + \mathbf{S}_1 \cdot \mathbf{S}_2$$

$$H = -JP_{\mathrm{ssinglet}} + E_0 = -JP_{\mathrm{antisym}} + E_0$$

$$P_{\mathrm{antisym}} = P^{\mathrm{spin}}_{\mathrm{singlet}} P^{\mathrm{valley}}_{\mathrm{triplet}} + P^{\mathrm{spin}}_{\mathrm{triplet}} P^{\mathrm{valley}}_{\mathrm{singlet}} = \frac{1}{8}\left((\mathbf{S}_1 \cdot \mathbf{S}_2)(\tau_1 \cdot \tau_2) + \mathbf{S}_1 \cdot \mathbf{S}_2 + \tau_1 \cdot \tau_2 - 3\right)$$

$$\boxed{H = \frac{J}{8}\Big((\mathbf{S}_1 \cdot \mathbf{S}_2)(\tau_1 \cdot \tau_2) + \mathbf{S}_1 \cdot \mathbf{S}_2 + \tau_1 \cdot \tau_2\Big)}$$

K. I. Kugel, D. I. Khomskii, JETP 1973

Exchange coupling with valley degeneracy

$$H = \frac{J}{8}\Big((\mathbf{S}_1\cdot\mathbf{S}_2)(\tau_1\cdot\tau_2) + \mathbf{S}_1\cdot\mathbf{S}_2 + \tau_1\cdot\tau_2\Big) \quad = -JP_{\text{as}}$$

$$U(\phi) = e^{-i\int_0^t dt'\,H(t')} = 1\!\!1 + \left(e^{i\phi}-1\right)P_{\text{as}} = \begin{cases} \text{SWAP}, & \phi=\pi \\ \sqrt{\text{SWAP}}, & \phi=\pi/2 \end{cases}$$

$\phi = Jt/\hbar$

$$\text{SWAP} = \text{SWAP}_{\text{spin}} \otimes \text{SWAP}_{\text{valley}}$$

$$\sqrt{\text{SWAP}} = \sqrt{\text{SWAP}_{\text{spin}} \otimes \text{SWAP}_{\text{valley}}} \neq \sqrt{\text{SWAP}_{\text{spin}}} \otimes \sqrt{\text{SWAP}_{\text{valley}}}$$

Loss-DiVincenzo sequence not directly applicable

solution: $\sqrt{\text{SWAP}_{\text{spin}}} = \sqrt{\text{SWAP}_{\text{spin}}} \otimes 1\!\!1_{\text{valley}} = e^{3i\pi/4}\left[U(\pi/4)\tau_{1x}U(\pi/4)\tau_{1z}\right]^2$

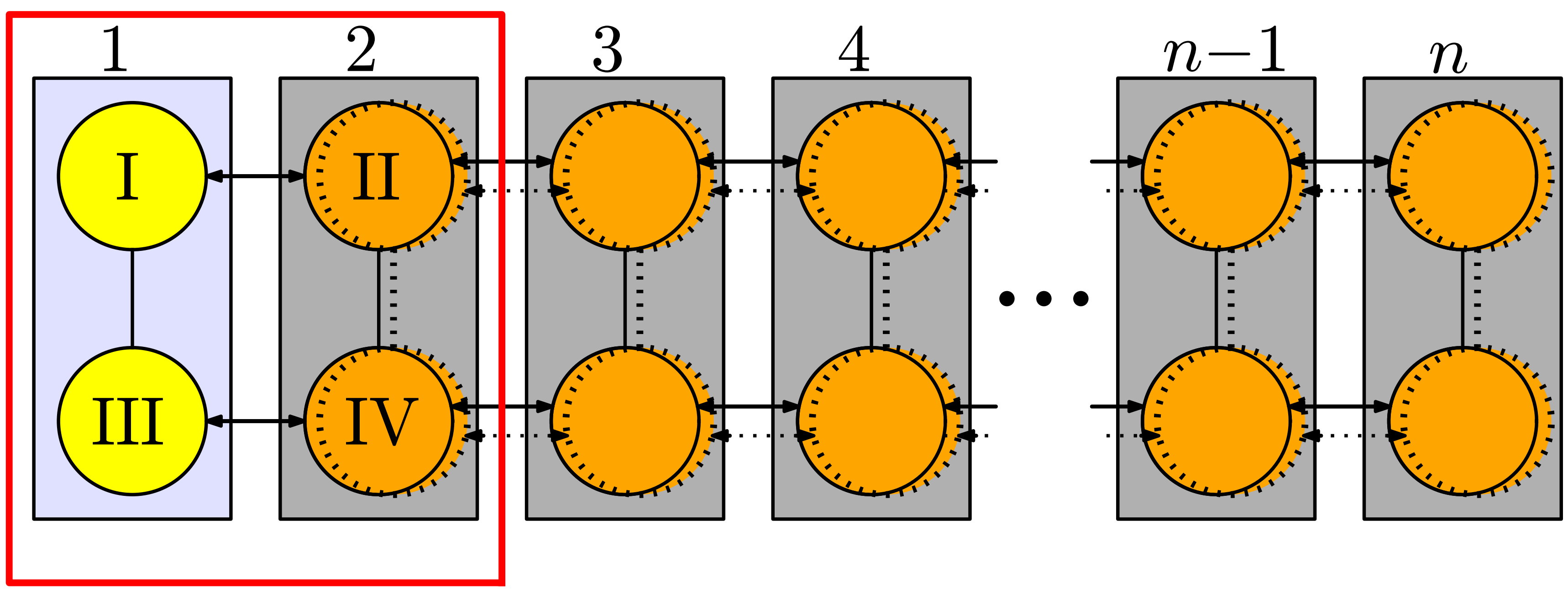
+ Loss-DiVincenzo sequence => CNOT

N. Rohling & GB, New J. Phys. 2012

crazy idea: use valley as qubits as well!

Exchange coupling with valley degeneracy

$$H = \frac{J}{8}\Big((\mathbf{S}_1 \cdot \mathbf{S}_2)(\tau_1 \cdot \tau_2) + \mathbf{S}_1 \cdot \mathbf{S}_2 + \tau_1 \cdot \tau_2\Big)$$

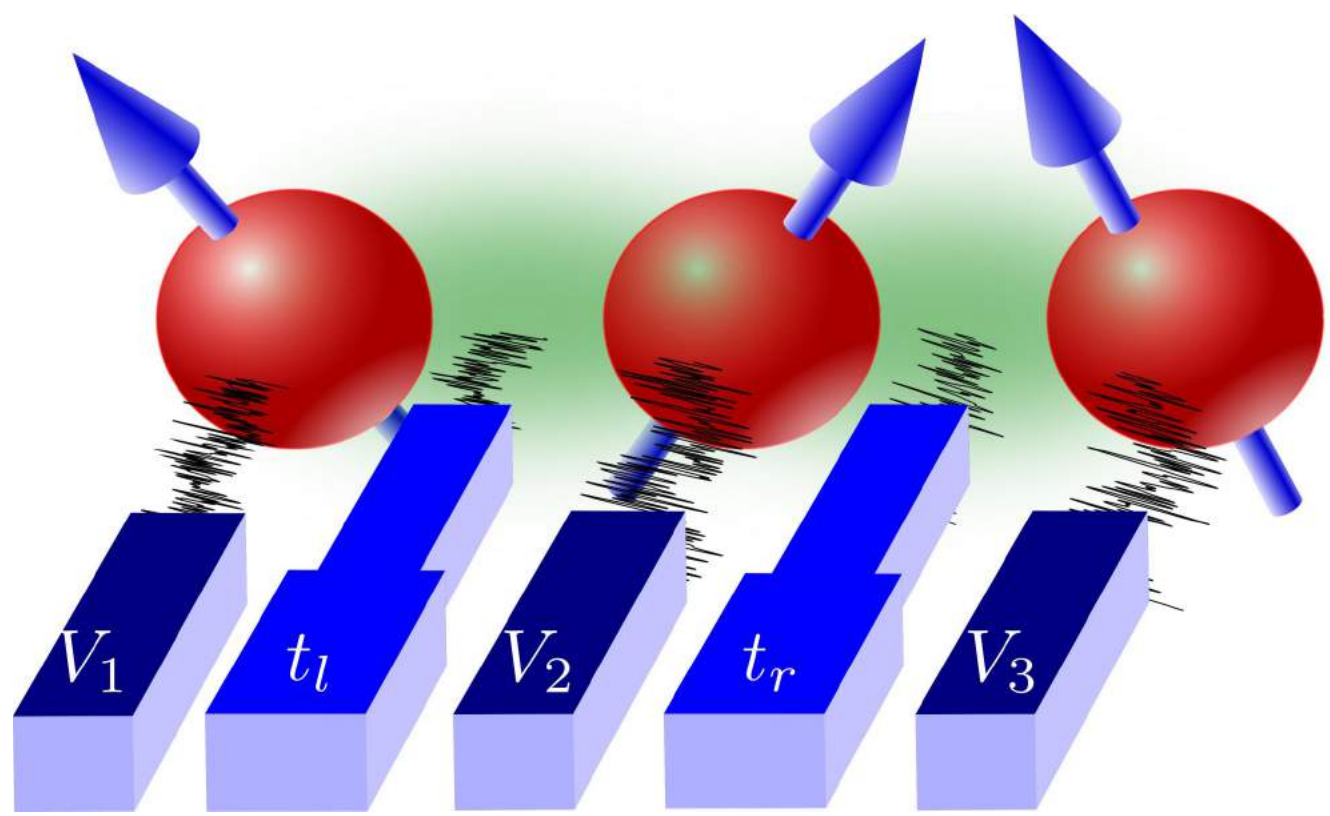


- universal QC using both spin and valley possible
- spin singlet-triplet T_0 qubit encoding

N. Rohling, M. Russ & GB, Phys. Rev. Lett. 113, 176801 (2014)

ICTS program on Open Quantum Systems
ICTS Bangalore, India
2017-07-27/28

Spin Qubits Theory



Guido Burkard

Department of Physics
University of Konstanz, Germany

lecture 3

Universität Konstanz

Spin Qubits

lecture 1

Introduction into Spin Qubits

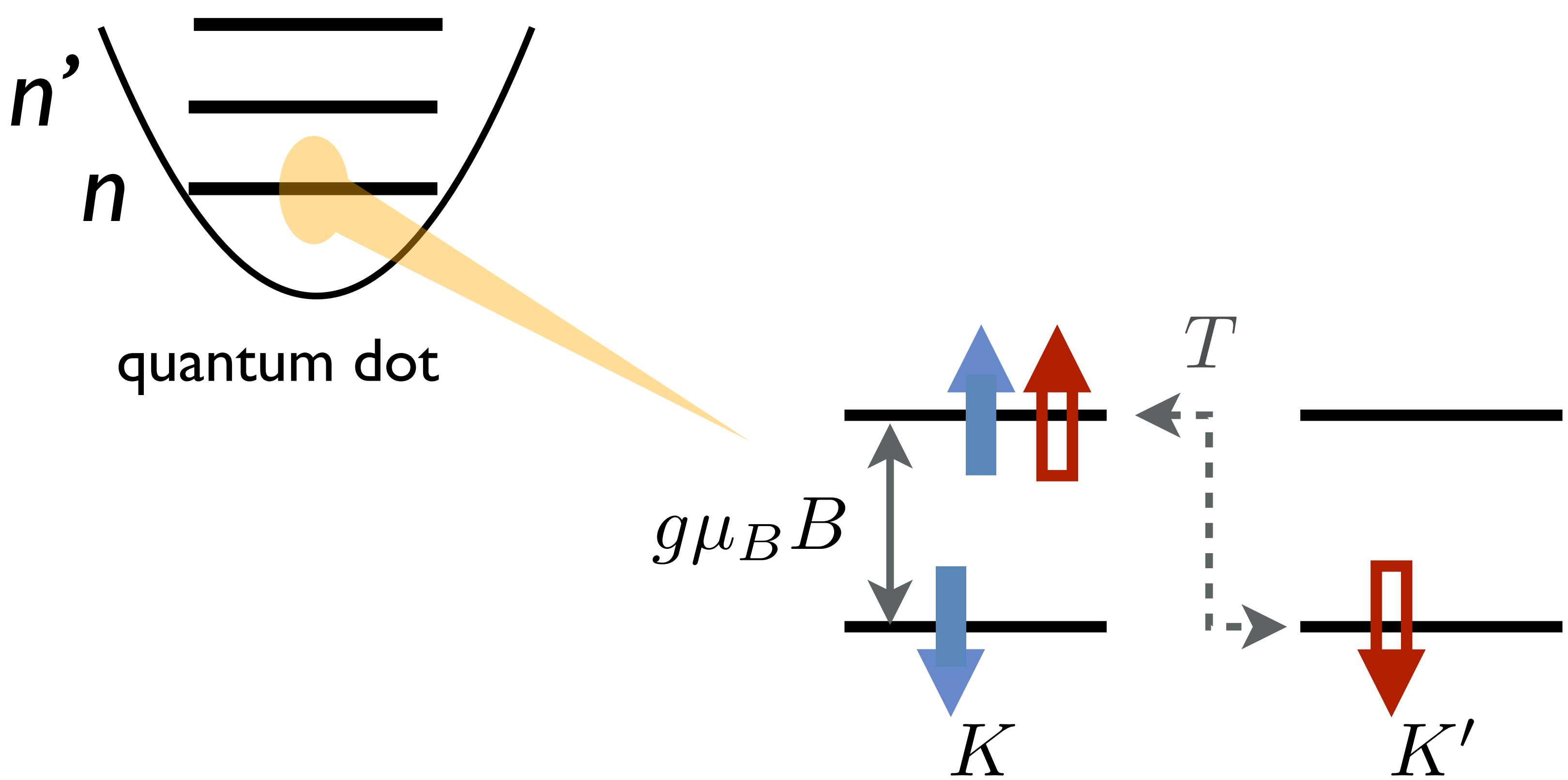
lecture 2

Multi-spin qubits
Coupling Spins to Electric Fields
Spin and Valley 1: Exchange

lecture 3

Spin and Valley 2: Spin Relaxation & Spin Blockade
Defect Spins

Spin relaxation in graphene QDs



in some graphene QDs, Si/Ge QDs: valley degeneracy (*K*, *K'*):

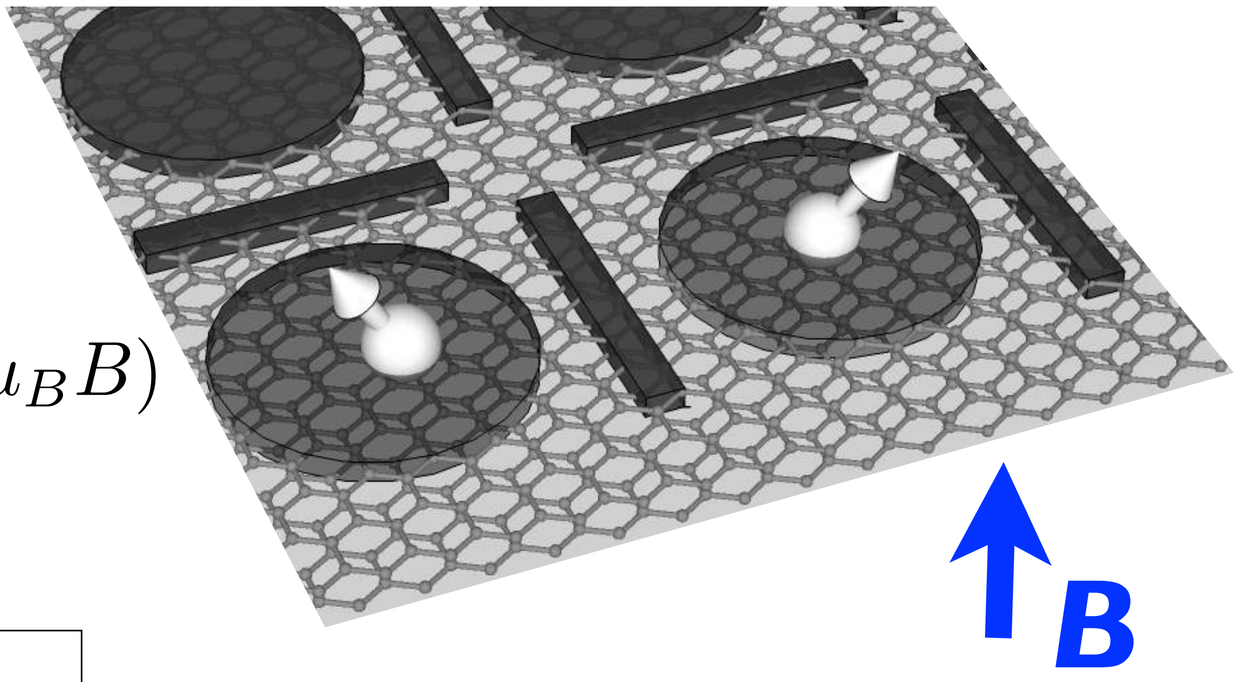
(1) time reversal invariance at *B=0* intact,
but Kramers pair resides in different valleys

(2) either: Kramers qubit
or: pure spin qubit in one valley

spin qubit
there is **no van Vleck cancellation**

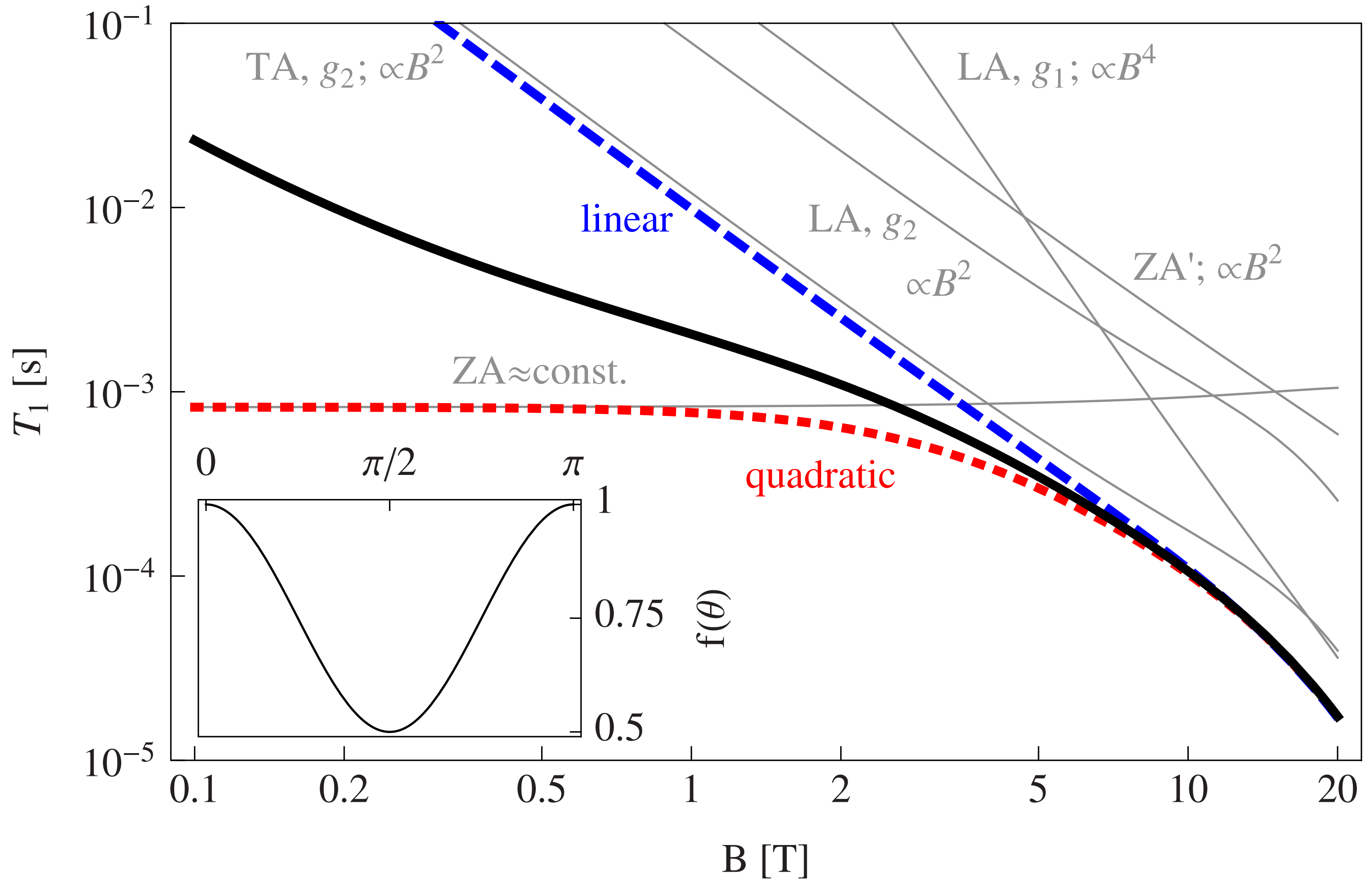
T_1 in circular graphene QDs

P. R. Struck & GB, PRB 2010



Fermi's Golden Rule

$$1/T_1 = 2\pi A \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \left|(H_{\mathrm{EPC}})_{nn}^{\uparrow\downarrow}\right|^2 \delta(\omega_{\mathbf{q}} - g\mu_B B)$$



Electron-phonon coupling matrix elements

$$(H_{\mathrm{EPC}})_{nn}^{\uparrow\downarrow} =$$

$$\sum_{n'\neq n}\left[\frac{(H_{\mathrm{SO}})_{nn'}^{\uparrow\downarrow}(H_{\mathrm{EPC}})_{n'n}}{E_n - E_{n'} - \frac{1}{2}g\mu_B B} + \frac{(H_{\mathrm{EPC}})_{nn'}(H_{SO})_{n'n}^{\uparrow\downarrow}}{E_n - E_{n'} + \frac{1}{2}g\mu_B B}\right]$$

(LA, TA)

$$i\Delta_{\mathrm{i}}\sqrt{1/A\rho\omega_{\mathbf{q}}}\,(q_x\langle\uparrow|s_x|\downarrow\rangle + q_y\langle\uparrow|s_y|\downarrow\rangle)\,\langle n|\sigma_z e^{i\mathbf{q}\cdot\mathbf{r}}|n\rangle$$

(ZA)

$U_0 = \Delta = 260$ meV $\quad R = 25$ nm

Spin relaxation time T_1

$$T_1 \propto (B^\alpha B^\beta B^\gamma)^{-2} B^{-\delta} \propto B^{-2(\alpha+\beta+\gamma)-\delta}$$

electron-phonon coupling

$$\alpha = \begin{cases} 0 & \text{piezo} \\ 1 & \text{other} \end{cases}$$

order dipole matrix element

$$\beta = \begin{cases} 0 & \text{zeroth order} \\ 1 & \text{first order} \end{cases}$$

phonon dimension & dispersion

$$\delta = \begin{cases} -2 & \text{2D phonons (quadratic)} \\ 0 & \text{2D phonons (linear)} \\ 1 & \text{3D phonons (linear)} \end{cases}$$

-1 1D phonons (linear)

time-reversal symmetry

$$\gamma = \begin{cases} 0 & \text{not Kramers qubit} \\ 1 & \text{Kramers qubit (van Vleck)} \end{cases}$$

3D GaAs quantum dot

$\alpha = 0,\ \beta = 1,\ \gamma = 1,\ \delta = 1$ $\quad T_1 \propto B^{-5}$

Amasha et al., PRL 2008.

3D Si quantum dot (valleys mixed)

$\alpha = 1,\ \beta = 1,\ \gamma = 1,\ \delta = 1$ $\quad T_1 \propto B^{-7}$

Xiao et al., arXiv:0909.2857

2D graphene QD (deformation potential)

$\alpha = 1,\ \beta = 1,\ \gamma = 0,\ \delta = 0$ $\quad T_1 \propto B^{-4}$

2D graphene QD (bond length change)

$\alpha = 1,\ \beta = 0,\ \gamma = 0,\ \delta = 0$ $\quad T_1 \propto B^{-2}$

2D graphene QD (direct coupling, ZA phonons)

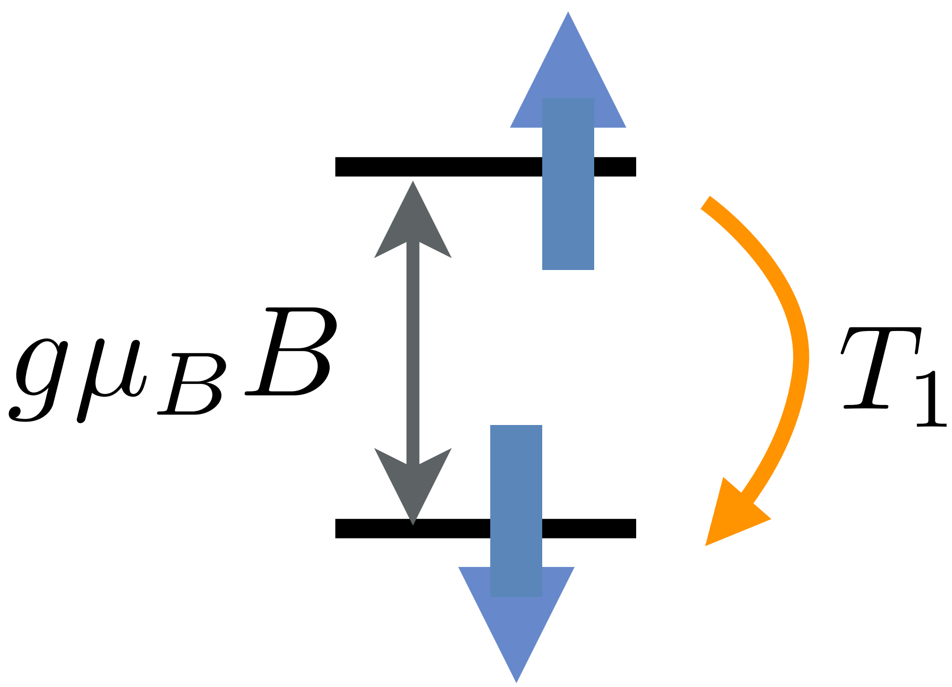
$\alpha = 1,\ \beta = 1,\ \gamma = -2,\ \delta = 0$ $\quad T_1 \propto B^{0}$

1D graphene QD (deformation potential)

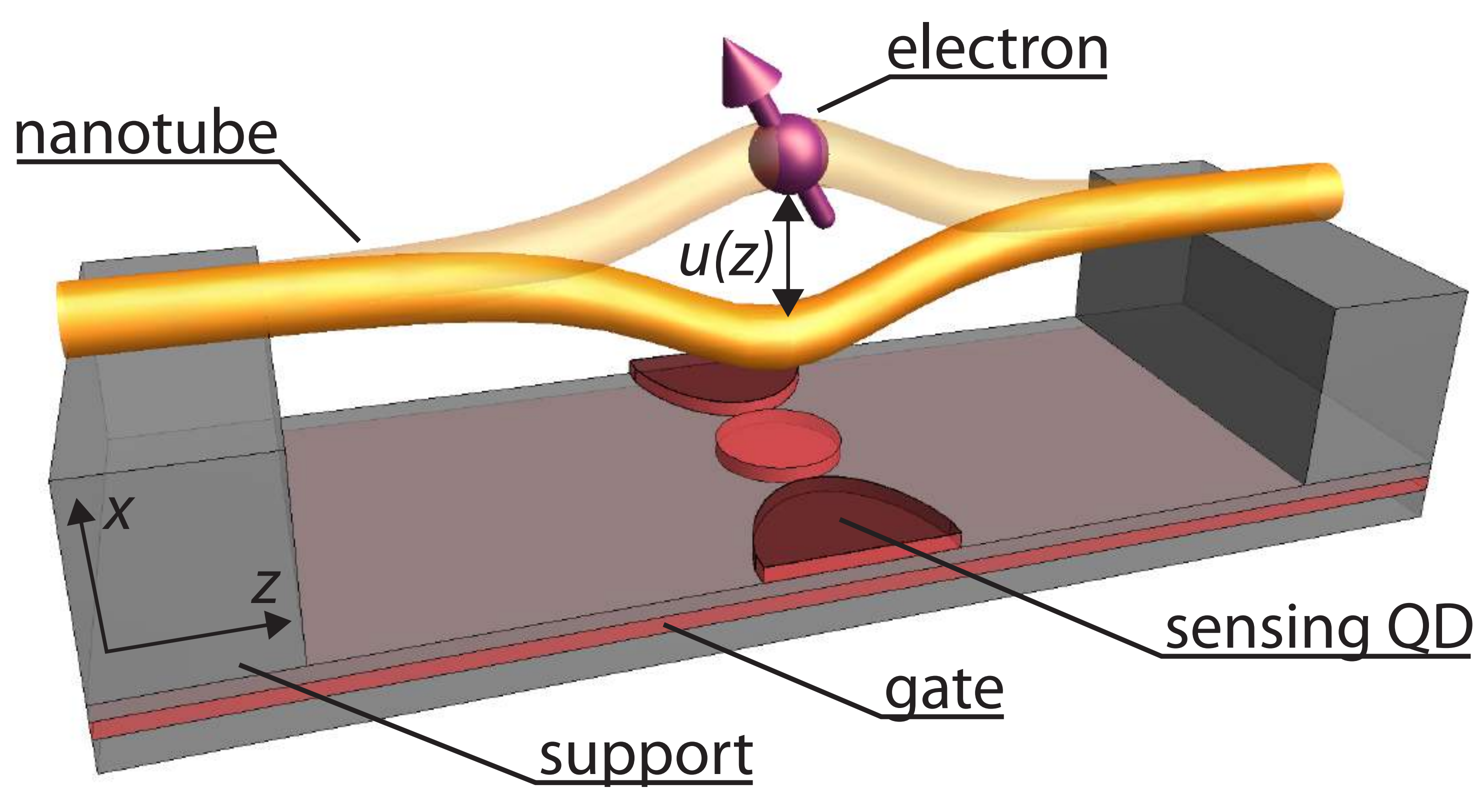
$\alpha = 1,\ \beta = 1,\ \gamma = 0,\ \delta = 0$ $\quad T_1 \propto B^{-5}$

M. Droth & GB, Phys. Status Solidi RRL **10**, 75 (2016)

Spin relaxation



- spin relaxation as spontaneous decay
- surrounding cavity modifies relaxation rate
- cavity = nanomechanical resonator

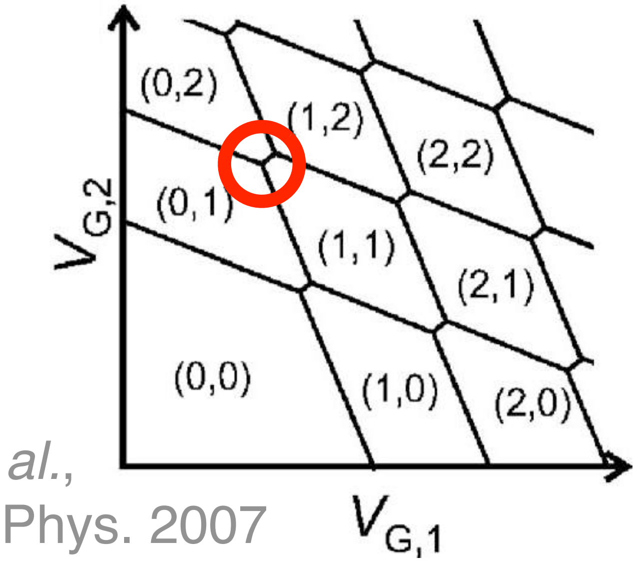


g ~ 0.49 MHz

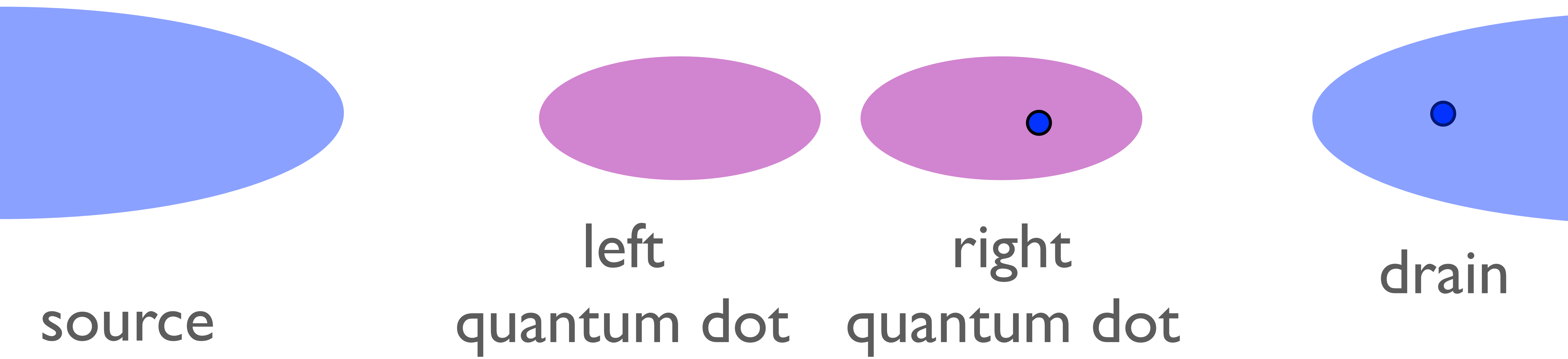
A. Pályi *et al.*, Phys. Rev. Lett. **108**, 206811 (2012)

Spin blockade

K. Ono, D. G. Austing, Y. Tokura & S. Tarucha, Science 2002
K. Ono and S. Tarucha, PRL 2004
Koppens *et al.*, Science 2005



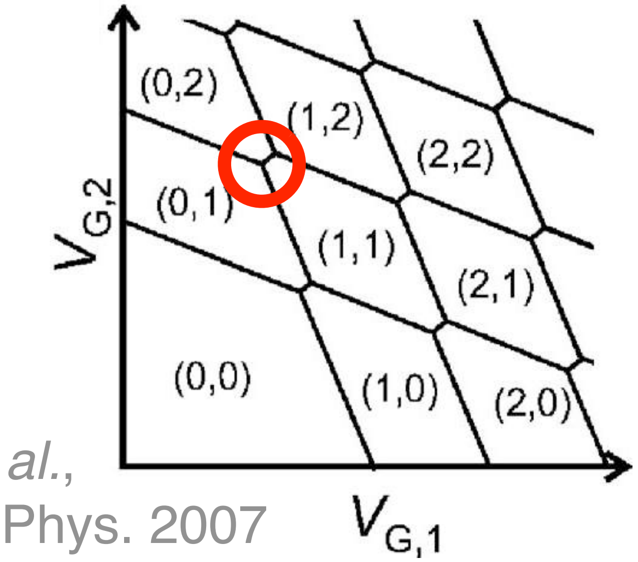
Hanson *et al.*,
Rev. Mod. Phys. 2007



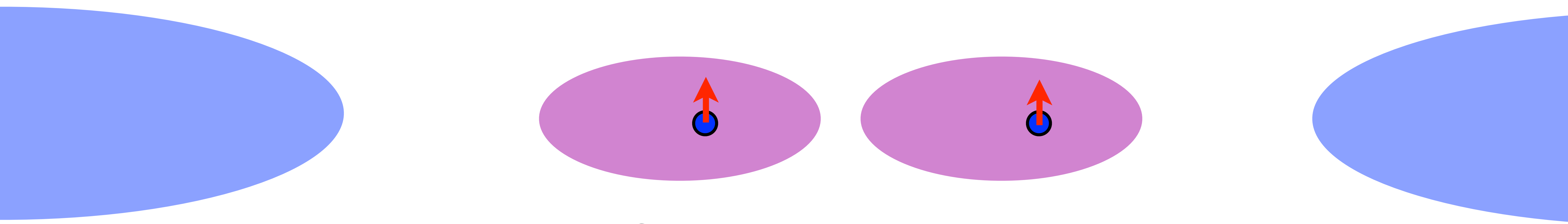
‘transport cycle’
(0,1)--(1,1)--(0,2)--(0,1)

Spin blockade

K. Ono, D. G. Austing, Y. Tokura & S. Tarucha, Science 2002
K. Ono and S. Tarucha, PRL 2004
Koppens *et al.*, Science 2005



Hanson *et al.*,
Rev. Mod. Phys. 2007



source

left
quantum dot

right
quantum dot

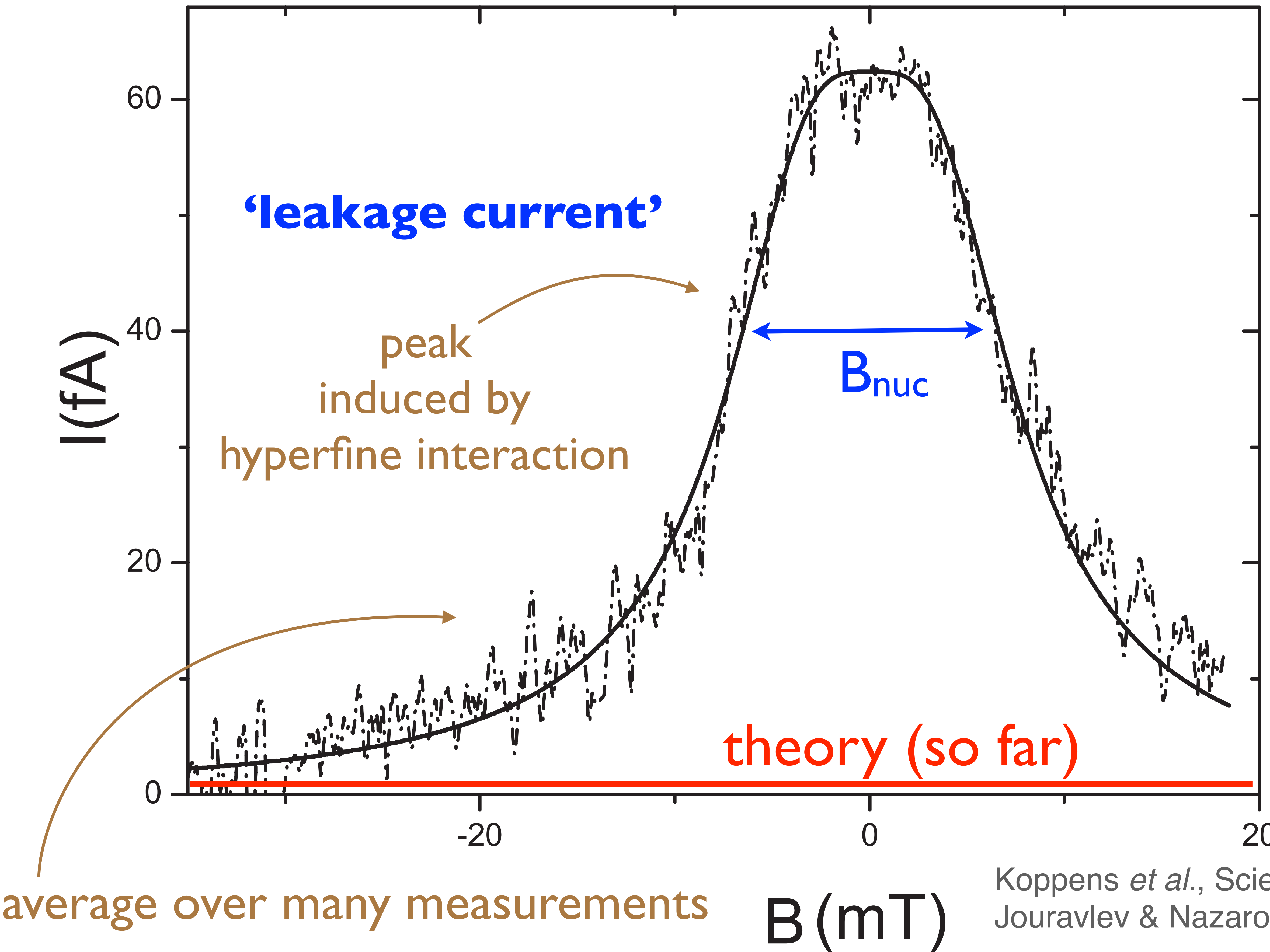
drain

‘transport cycle’

(0,1)--(1,1)×(0,2)--(0,1)

in steady-state:
current = 0

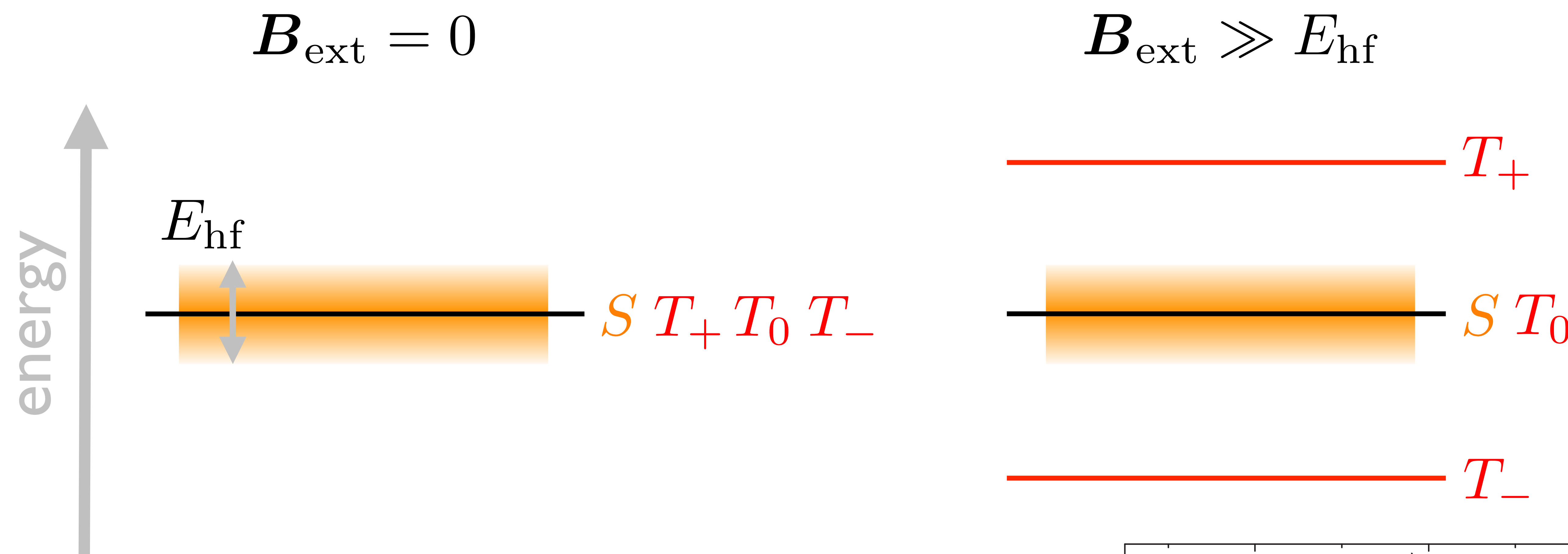
Spin blockade



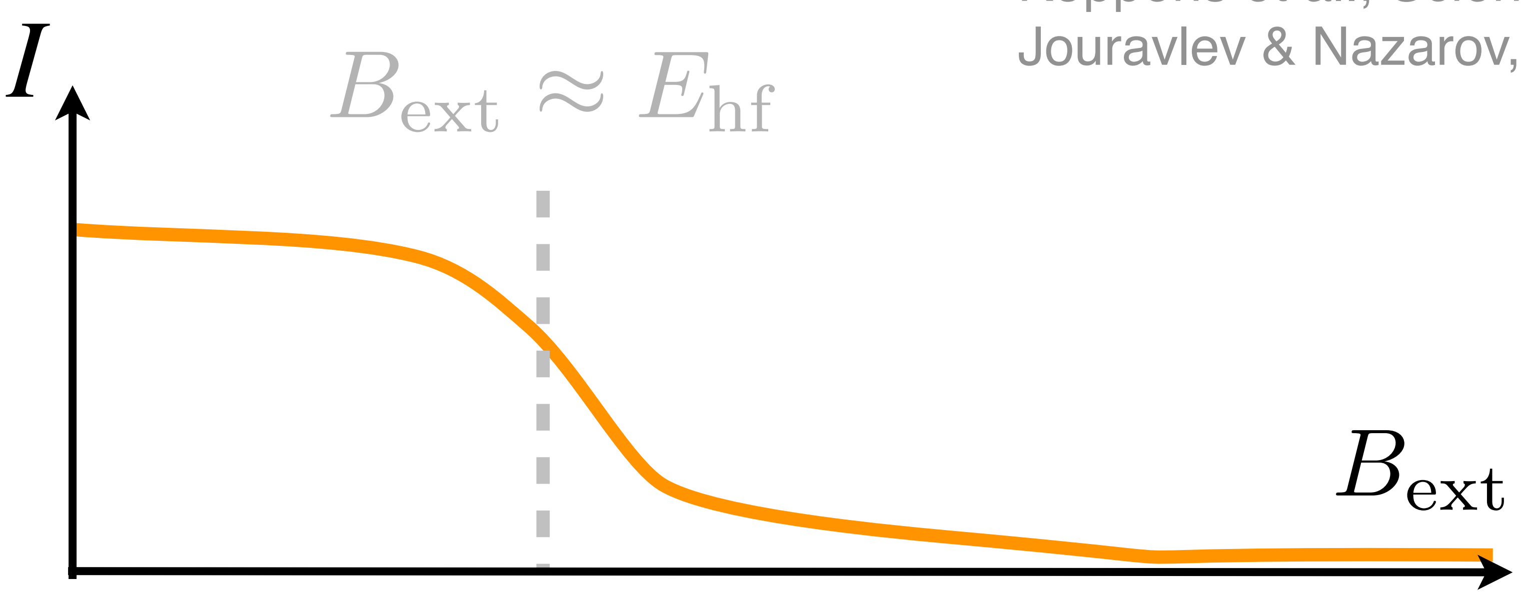
Koppens *et al.*, Science 2005
Jouravlev & Nazarov, PRL 2006

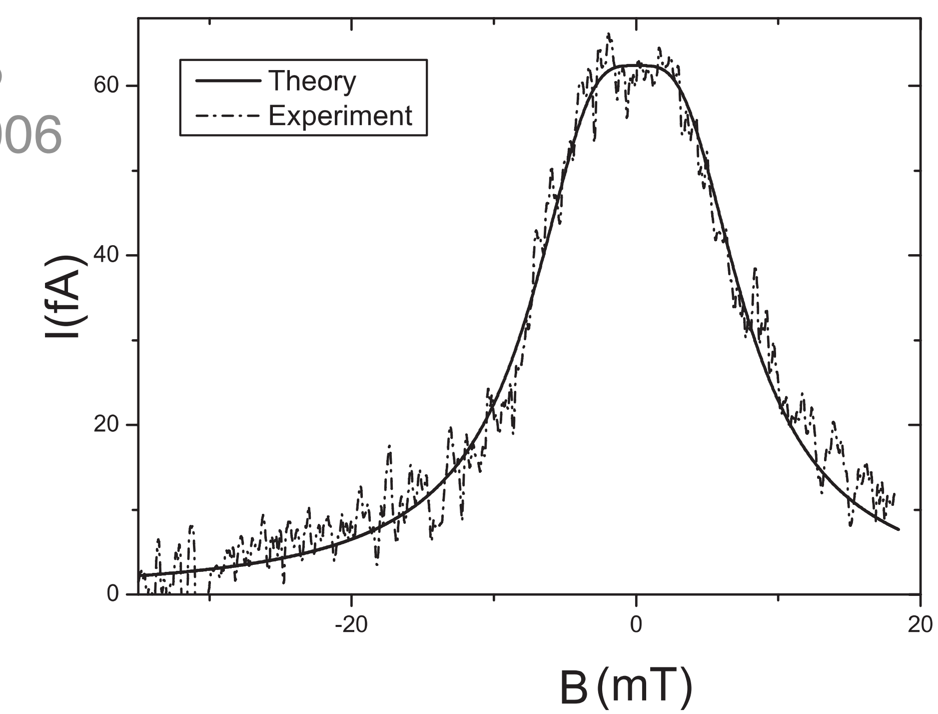
Spin blockade - magnetotransport

(1,1) energy diagram



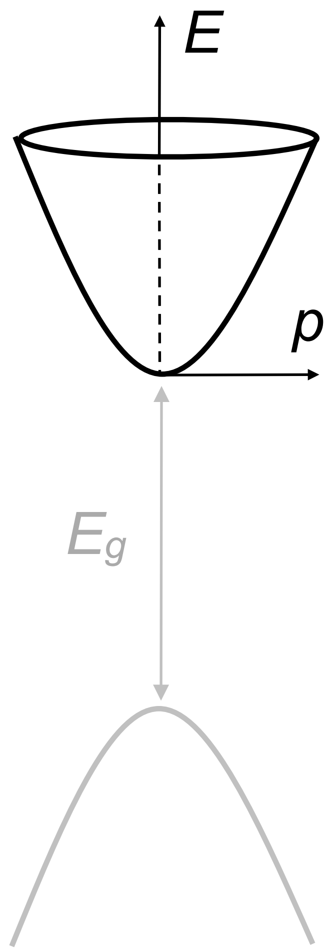
Koppens et al., Science 2005
Jouravlev & Nazarov, PRL 2006





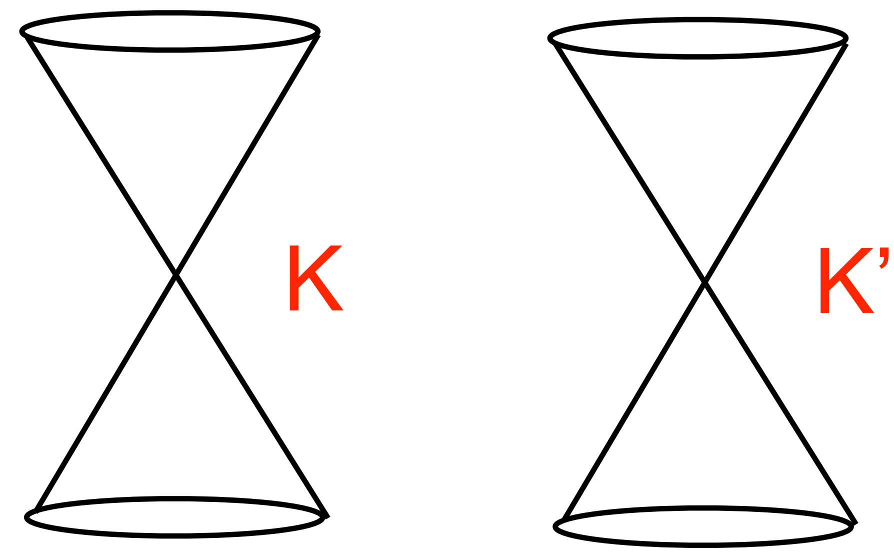
Graphene & CNTs: Valley degeneracy

direct-gap semiconductor

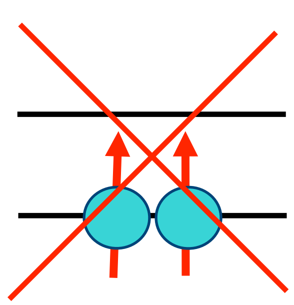


$$|\uparrow\rangle, |\downarrow\rangle$$

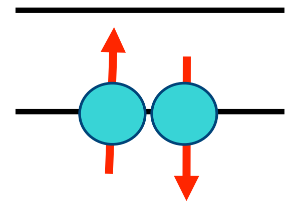
graphene / CNT



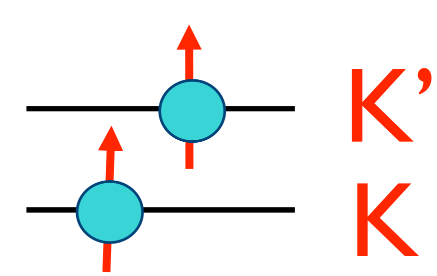
$$|\uparrow K\rangle, |\downarrow K\rangle, |\uparrow K'\rangle, |\downarrow K'\rangle$$



not possible



OK



OK

Pauli blockade in double QDs

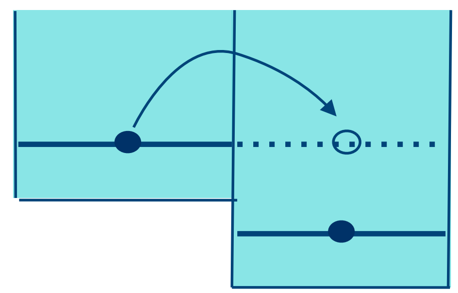
spin blockade (GaAs)

Ono *et al.*, Science 2002
Koppens *et al.*, Science 2005

$$|s, m_s\rangle$$

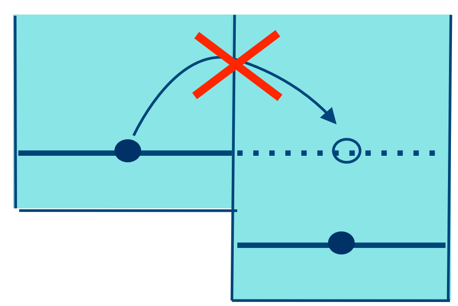
1 singlet

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



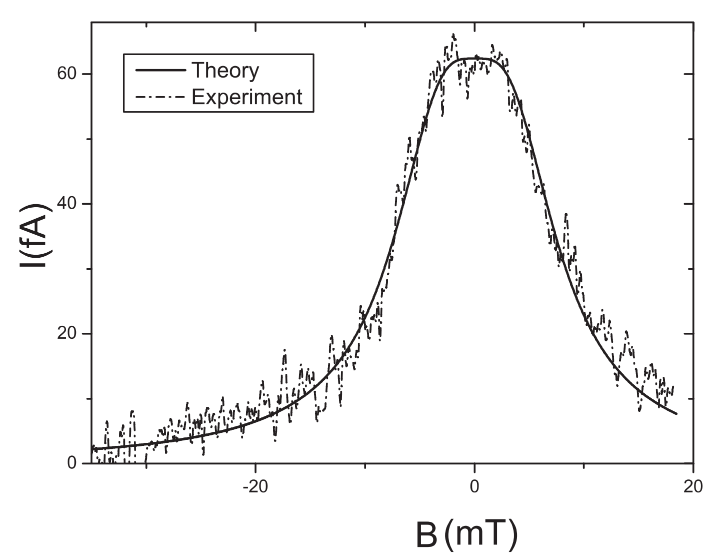
3 triplets

$$|1,1\rangle = |\uparrow\uparrow\rangle$$
$$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$
$$|1,-1\rangle = |\downarrow\downarrow\rangle$$



hyperfine interaction:
leakage current

Jouravlev & Nazarov, PRL 2006



spin-valley blockade (C)

Buitelaar *et al.*, PRB 2008
Churchill *et al.*, Nature Phys. 2009
Churchill *et al.*, PRL 2009
Pei *et al.*, Nature Nano 2012

6 supersinglets

$$|0,0\rangle_{\rm sp}|1,m_v\rangle_{\rm val}$$
$$|1,m_s\rangle_{\rm sp}|0,0\rangle_{\rm val}$$

10 supertriplets

$$|0,0\rangle_{\rm sp}|0,0\rangle_{\rm val}$$
$$|1,m_s\rangle_{\rm sp}|1,m_v\rangle_{\rm val}$$

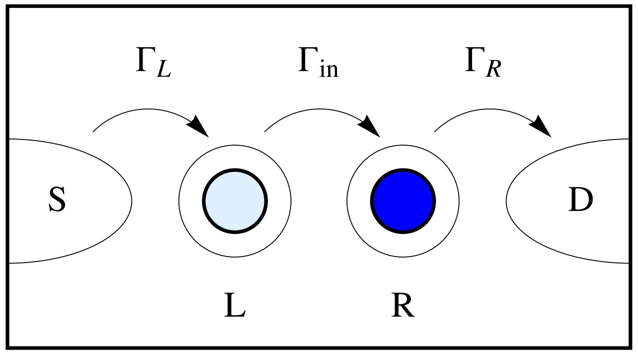
$$m_s, m_v \in \{1, 0, -1\}$$

spin **or** valley mixing
from **hyperfine**
or **non-magnetic atomic defect**:
leakage current

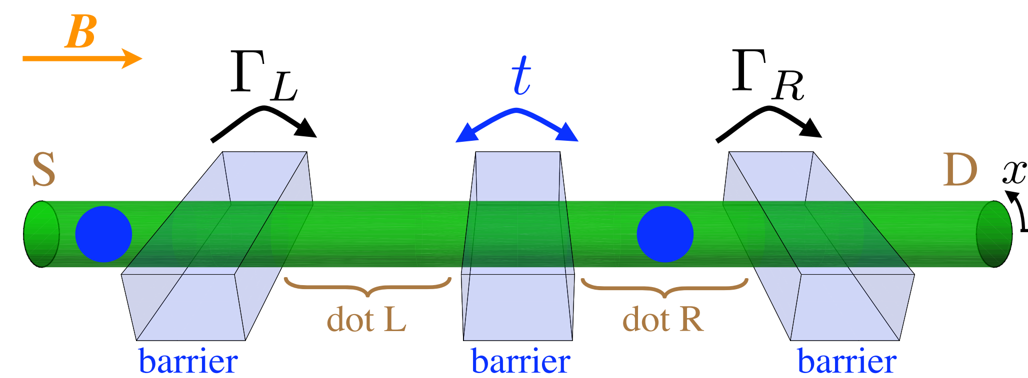
Pályi & GB, PRB 2009, PRB 2010

Transport model

Graphene case:
Palyi & Burkard PRB 2009



CNT case:
Palyi & Burkard, PRB 2010



Born-Markov
master equation +
secular approximation

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + D\rho$$

incoherent
tunneling processes

$\Gamma_L \gg \Gamma_R \gg \Gamma_{\rm in}$ ← rate hierarchy → $\Gamma_L \gg \Gamma_R$

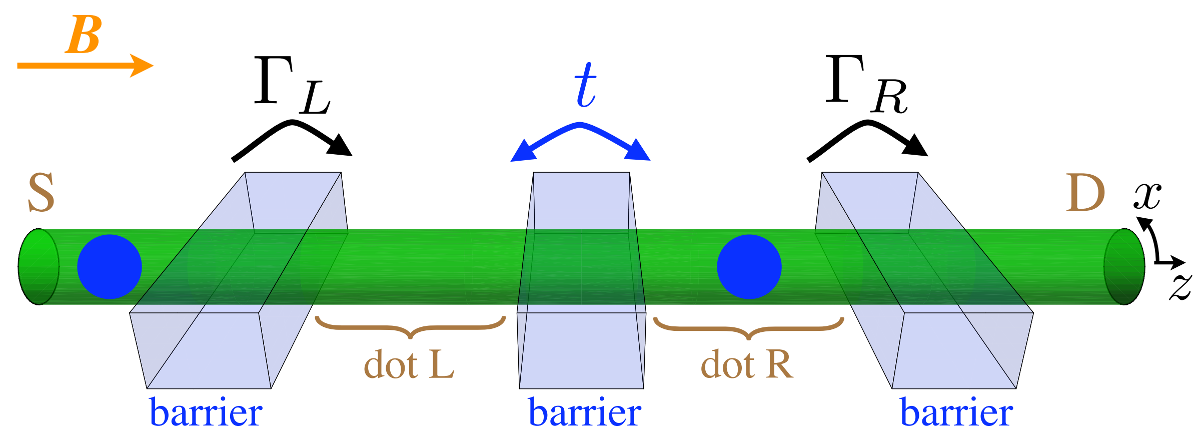
N	$H_{\rm so} = -\frac{\Delta_{\rm so}}{2}\sum_{d=L,R} s_{d,z}\tau_{d,z},$	Y
N	$H_{\rm dis} = \sum_{d=L,R}\left(b_{d,x}\tau_{d,x} + b_{d,y}\tau_{d,y}\right),$	Y
Y	$H_{\rm hfi} = \sum_{d=L,R} \boldsymbol{s}_d \cdot \sum_{i=0,x,y} \boldsymbol{h}_d^{(i)}\tau_i,$	N
Y	$H_{\rm magn} = \mu_B \boldsymbol{B}\cdot\sum_{d=L,R}\left(\frac{1}{2}g_s\boldsymbol{s}_d + \frac{1}{2}g_v\tau_{d,z}\hat{\boldsymbol{z}}\right),$	Y
N	$H_{\rm tun} = t\sum_{vs} d^\dagger_{Lvs}d_{Rvs} + \text{h.c.}$	Y

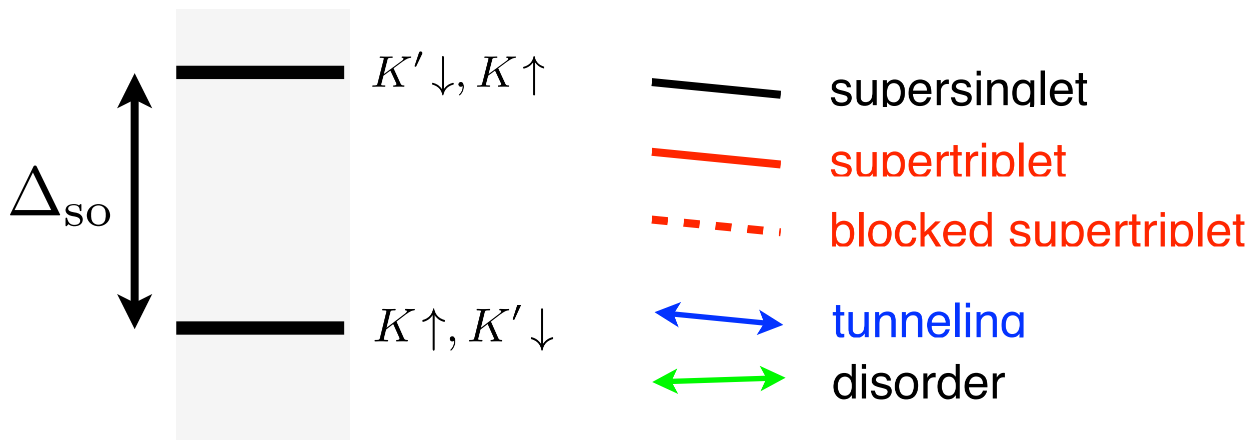
averaging for
Overhauser fields!

$$\langle I\rangle = e\Gamma_{\rm in}\left\langle \sum_{\alpha\in(1,1)}\rho_\alpha \sum_{j\in(0,2)} p_{j\leftarrow\alpha}\right\rangle$$

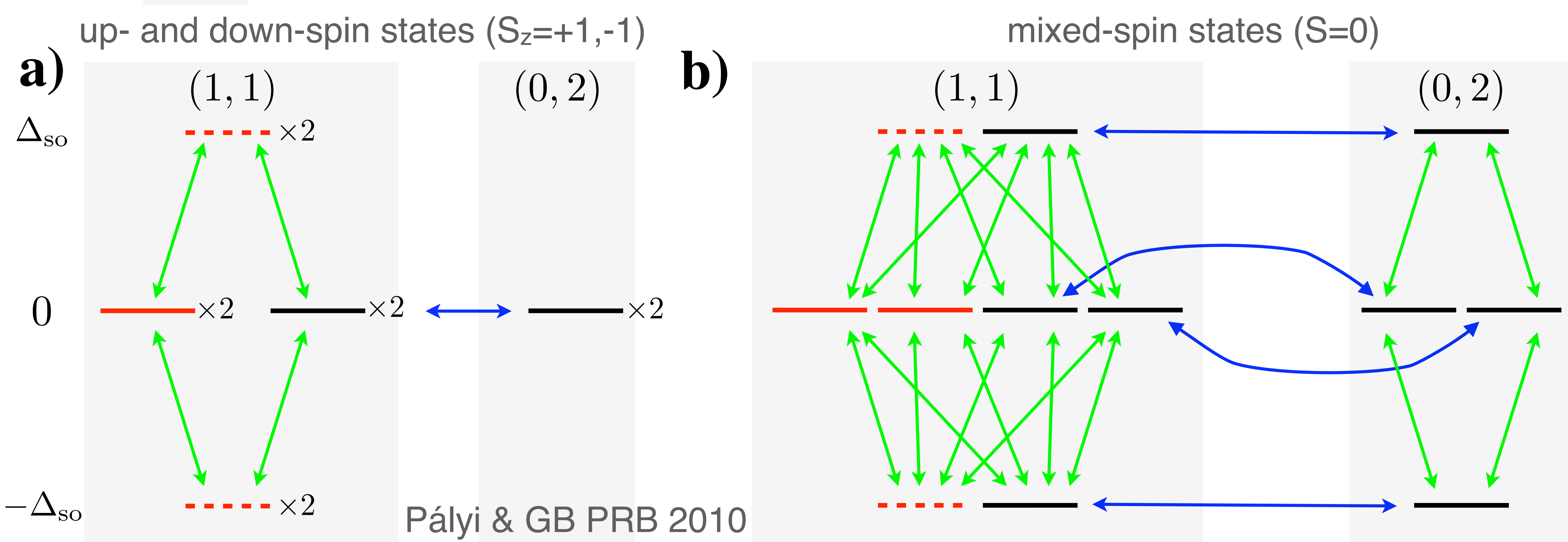
$$I = e\Gamma_R \sum_{\alpha\in 2e}\rho_\alpha \sum_{j\in(0,1)} p_{j\leftarrow\alpha}$$

Spin blockade in CNT dots



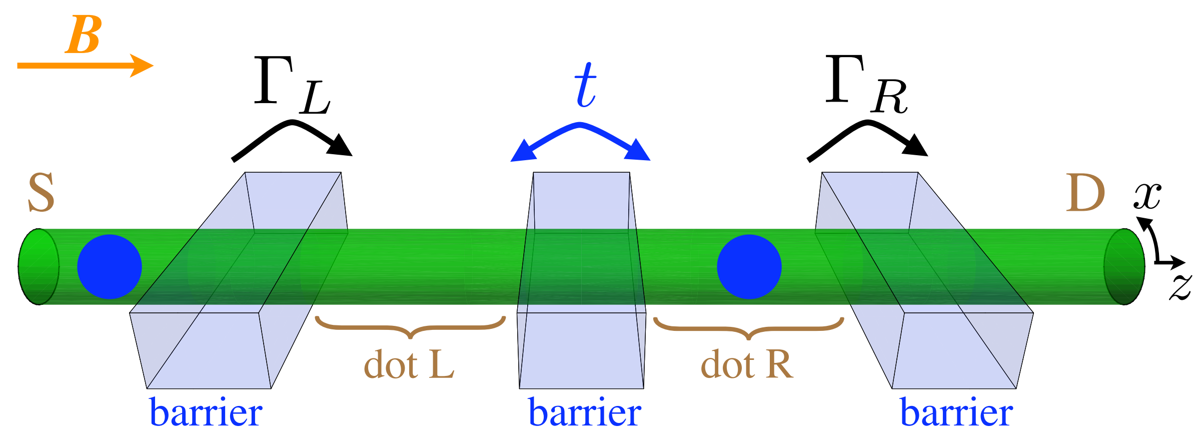


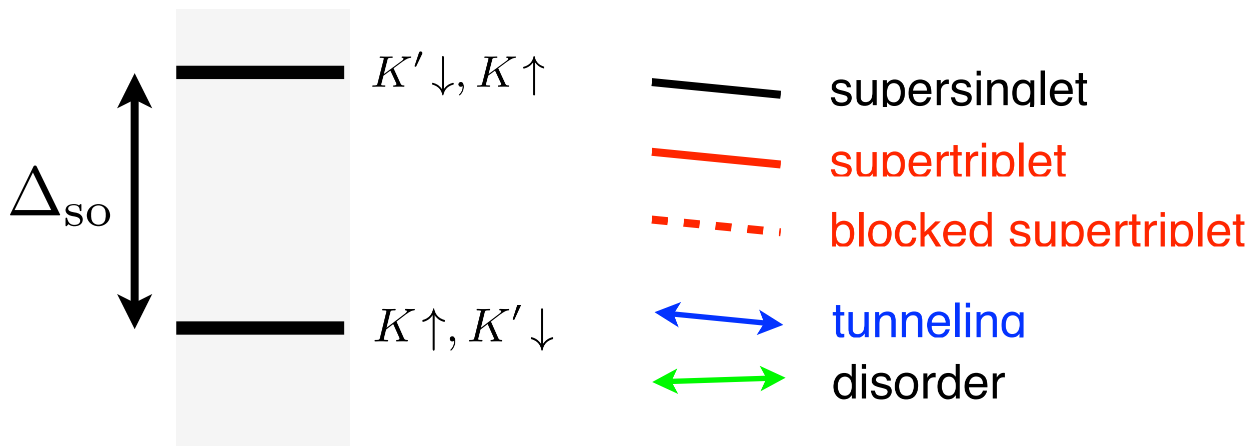
- effect of **disorder** on leakage current in CNTs?
- **spin-orbit splitting** taken into account
- coherent interdot tunneling



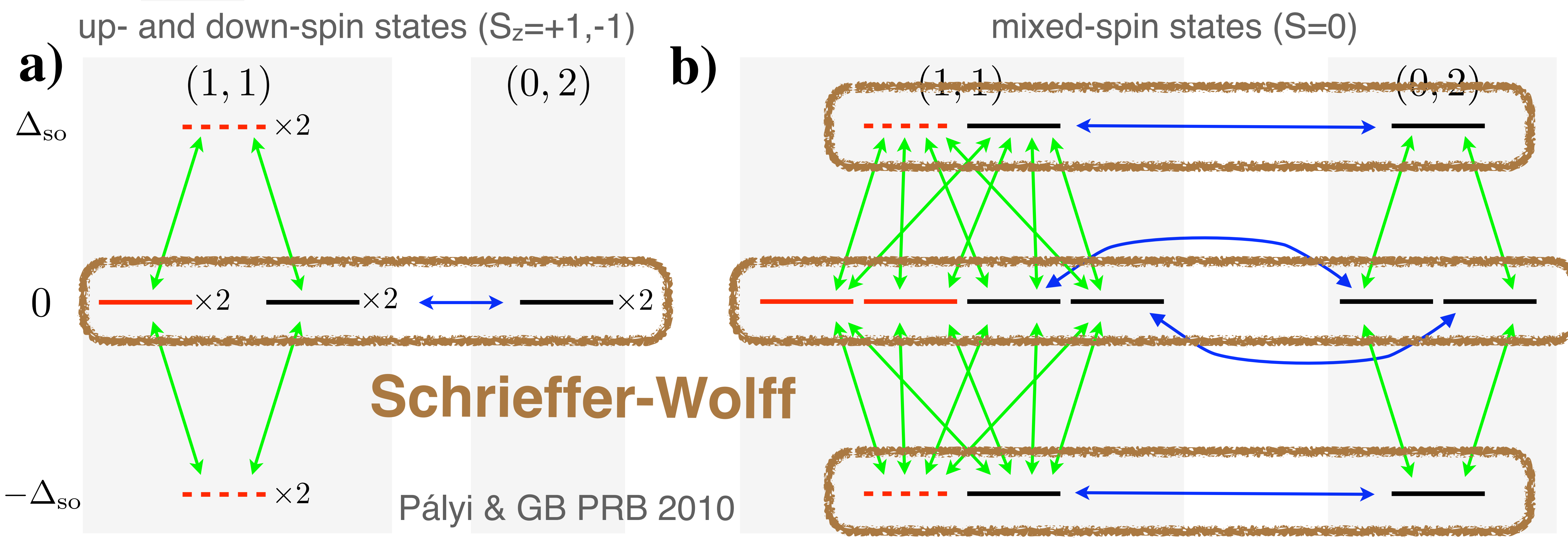
Pályi & GB PRB 2010

Spin blockade in CNT dots



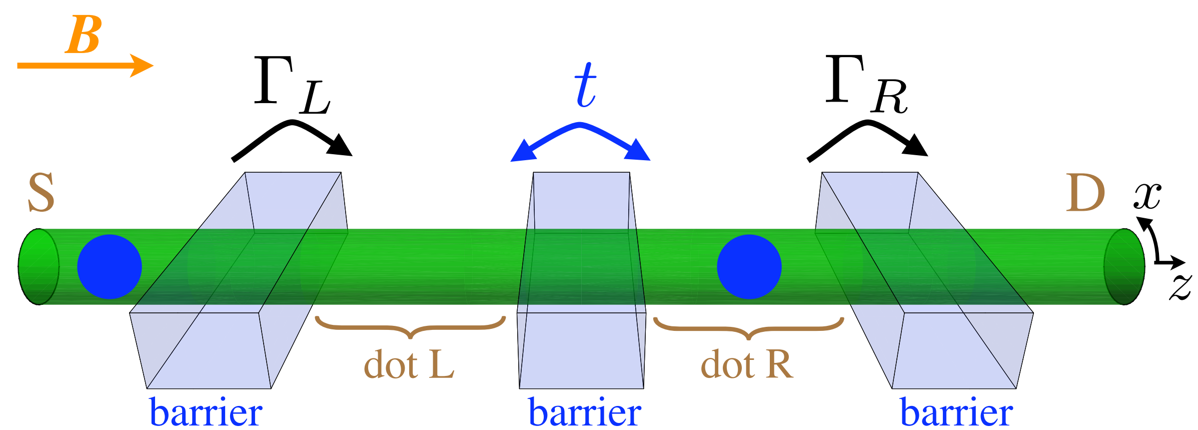


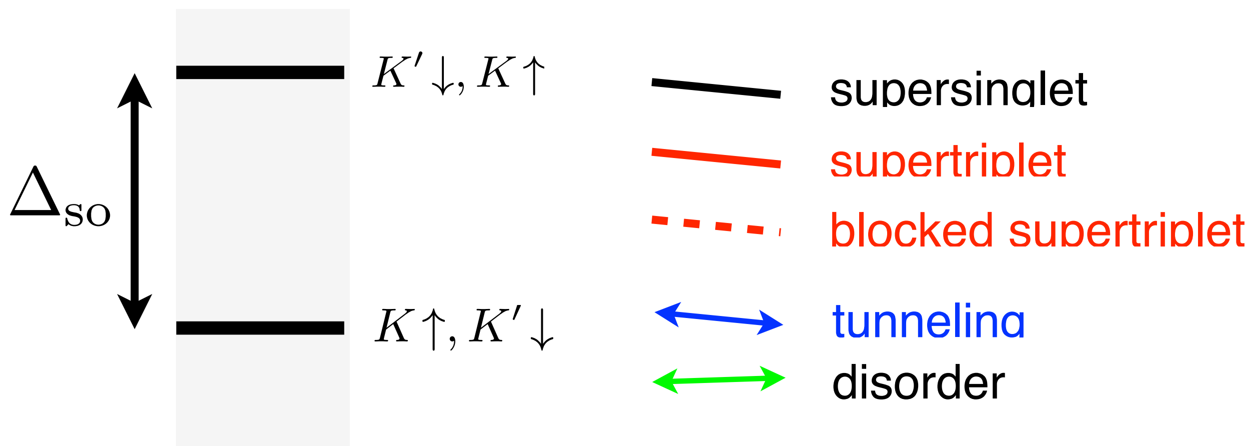
- effect of **disorder** on leakage current in CNTs?
- **spin-orbit splitting** taken into account
- coherent interdot tunneling



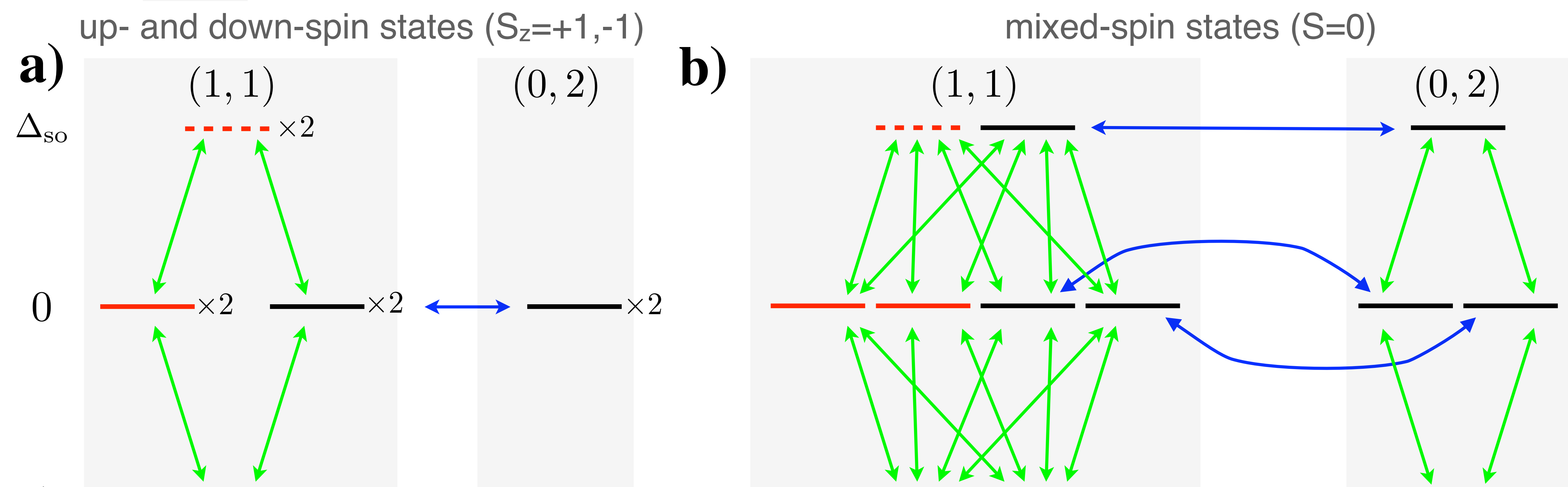
Pályi & GB PRB 2010

Spin blockade in CNT dots





- effect of **disorder** on leakage current in CNTs?
- **spin-orbit splitting** taken into account
- coherent interdot tunneling



Pályi & GB PRB 2010

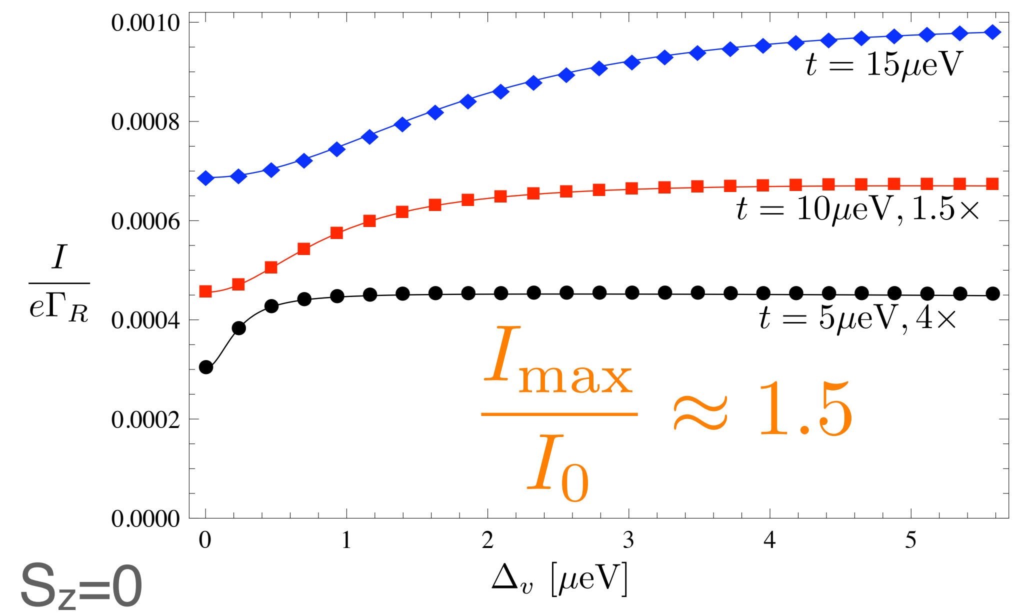
+ B field

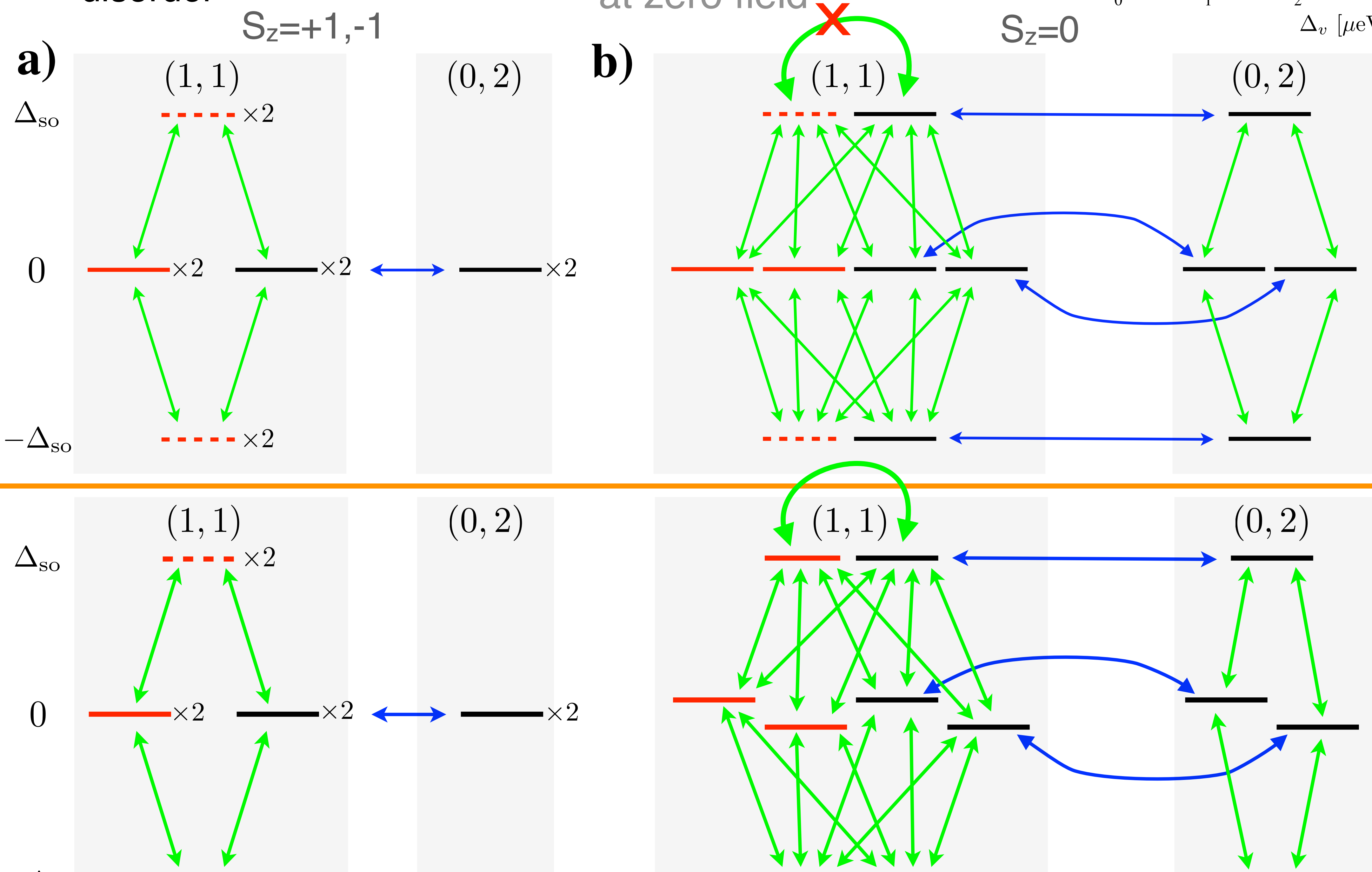
supersinglet
supertriplet
blocked supertriplet

tunneling
disorder

$|K\downarrow, K'\uparrow\rangle \pm |K'\uparrow, K\downarrow\rangle$

Schrieffer-Wolff:
no effective coupling
at zero field





zero
field

6
blocked
states

finite
field

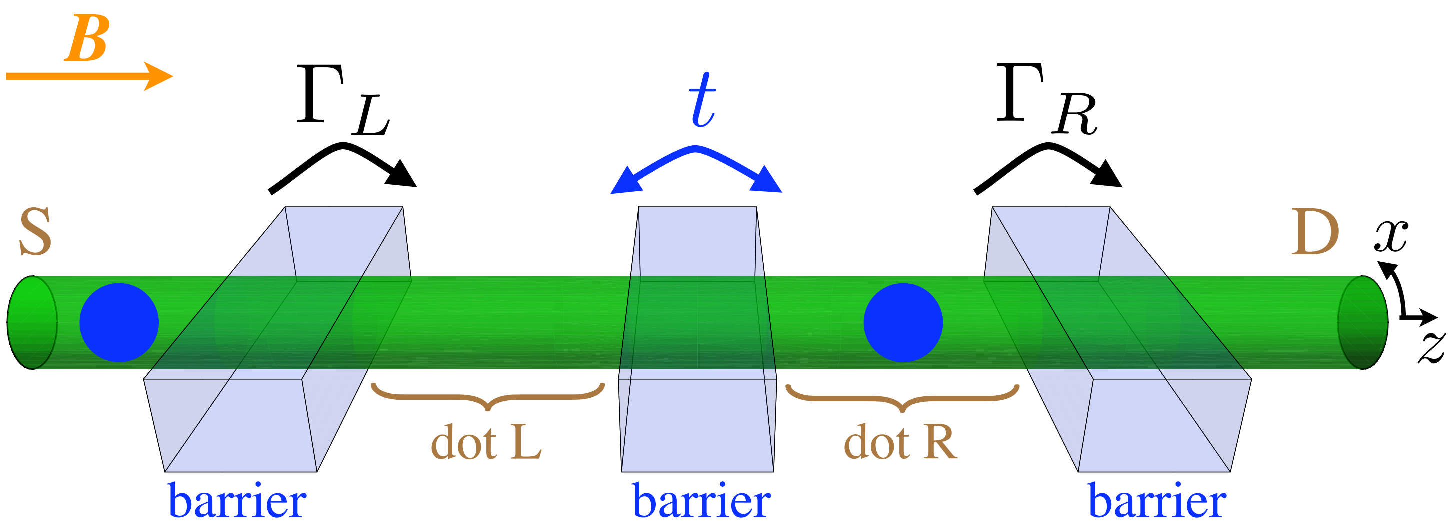
4
blocked
states

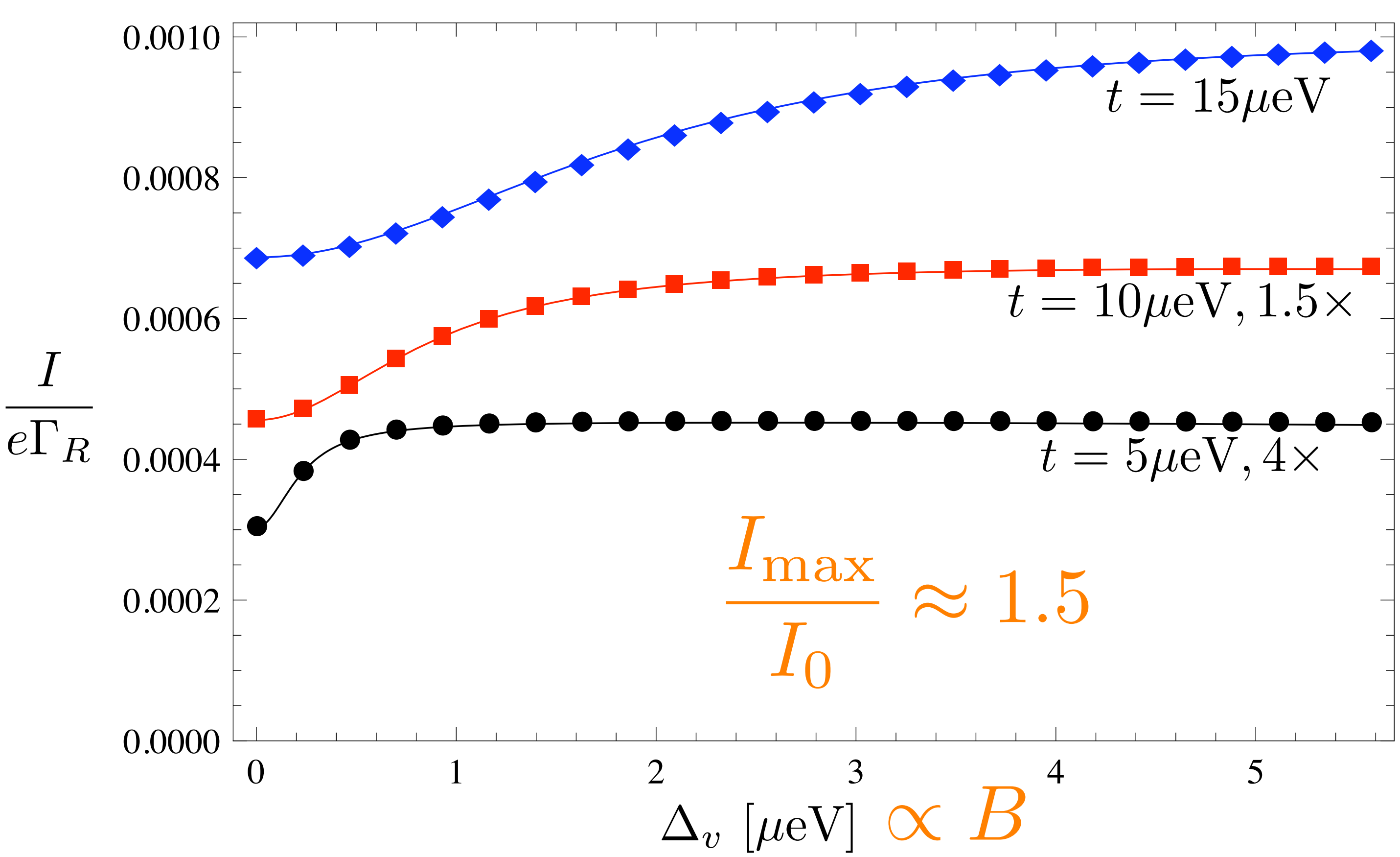
Pályi & GB PRB 2010

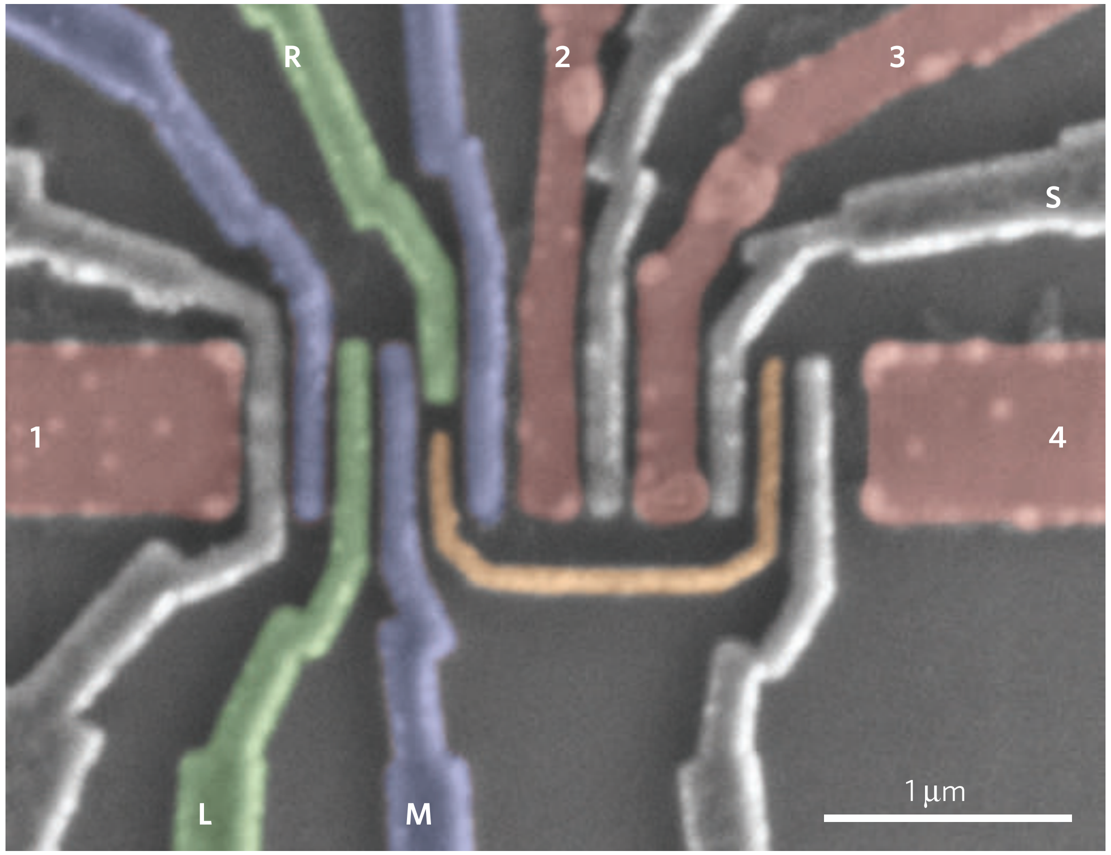
Spin blockade in CNT double dots

Pályi & GB, PRB 2010

- effect of **disorder** on leakage current in CNTs
- **spin-orbit splitting** taken into account
- coherent interdot tunneling





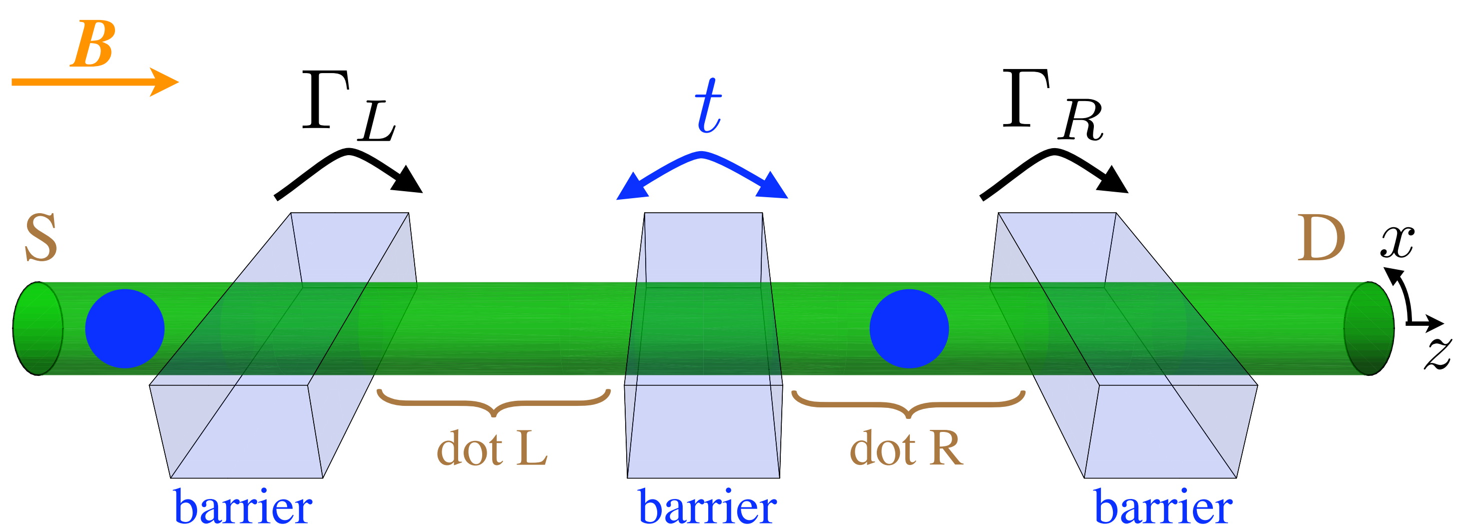


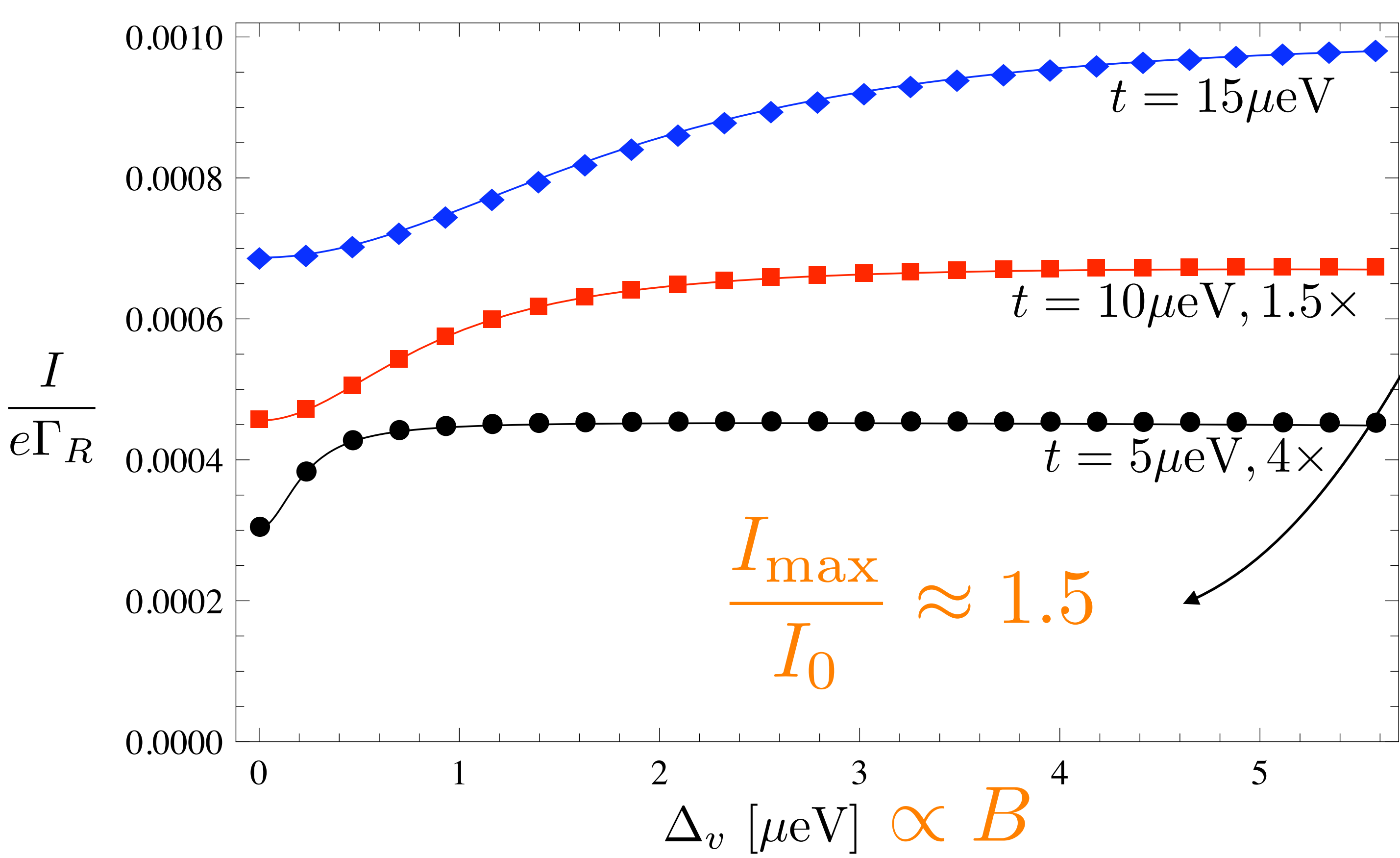
experiments:
Churchill *et al.*,
Nature Phys. 2009
PRL 2009

Spin blockade in CNT double dots

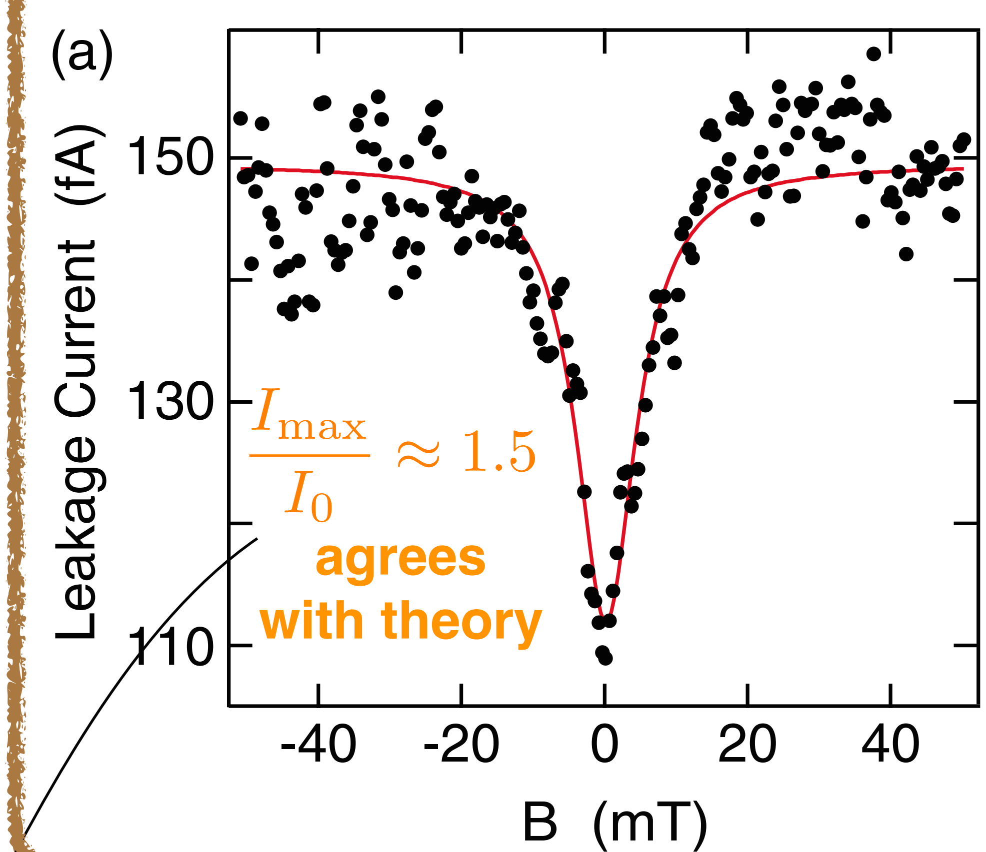
Pályi & GB, PRB 2010

- effect of **disorder** on leakage current in CNTs
- **spin-orbit splitting** taken into account
- coherent interdot tunneling

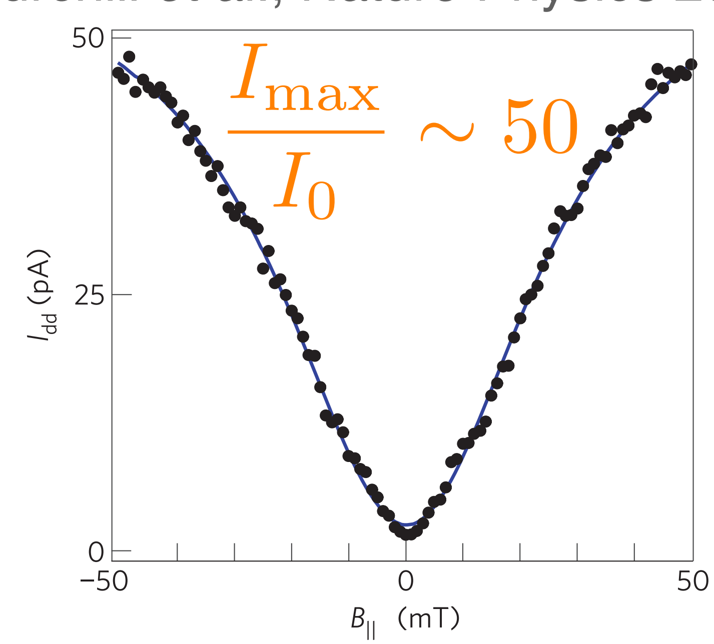




Churchill *et al.*, PRL 2009



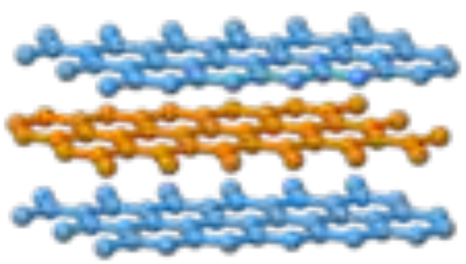
Churchill *et al.*, Nature Physics 2009

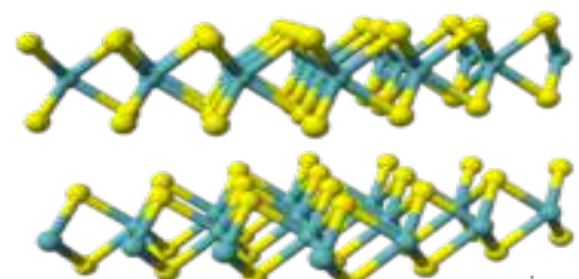


Transition-metal dichalcogenides MX_2

graphite

MoS_2





images: Wikipedia

Electric Field Effect in Atomically Thin Carbon Films

K. S. Novoselov,[1] A. K. Geim,[1*] S. V. Morozov,[2] D. Jiang,[1] Y. Zhang,[1] S. V. Dubonos,[2] I. V. Grigorieva,[1] A. A. Firsov[2]

22 OCTOBER 2004 VOL 306 SCIENCE

(2010) PHYSICAL REVIEW LETTERS

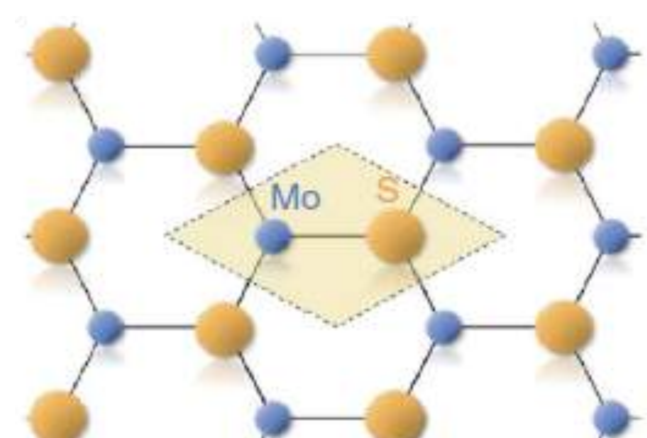
Atomically Thin MoS_2: A New Direct-Gap Semiconductor

Kin Fai Mak,[1] Changgu Lee,[2] James Hone,[3] Jie Shan,[4] and Tony F. Heinz[1,*]

PRL **105**, 136805 (2010)

M	X
molybdenum 42 **Mo** 73%	sulfur 16 **S** 99.2%
tungsten 74 **W** 86%	selenium 34 **Se** 92%
	tellurium 52 **Te** 92%

fraction of naturally occurring nuclear spin-free isotopes (I=0)



monolayer MoS_2, WSe_2, …:

"graphene with band gap and spin-orbit coupling"

k.p theory

Schrödinger equation for Bloch functions

$$\left(\frac{p^2}{2m}+V(r)\right)e^{i\mathbf{k}\cdot\mathbf{r}}u_{b\mathbf{k}}(\mathbf{r})=E_{b\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}u_{b\mathbf{k}}$$

Schrödinger equation for periodic part of the Bloch functions

$$\left(\frac{1}{2m}\underbrace{(\mathbf{p}+\mathbf{k})^2}_{p^2+2\mathbf{k}\cdot\mathbf{p}+k^2}+V(r)\right)u_{b\mathbf{k}}(\mathbf{r})=E_{b\mathbf{k}}u_{b\mathbf{k}}$$

small perturbation

K point: $\mathbf{k}=\mathbf{K}+\mathbf{q}$

Four-band k.p model

bands (even under horizontal mirror reflection)

representation of C_{3h} (crystal symmetry at K point)

$$\begin{array}{cc} & \begin{array}{cccc}|\Psi^{v}_{A'}\rangle & |\Psi^{c}_{E_1'}\rangle & |\Psi^{v-3}_{E_2'}\rangle & |\Psi^{c+2}_{E_2'}\rangle\end{array} \\ H_{\mathbf{kp}}= & \begin{pmatrix} \varepsilon_v & \gamma_3 q_- & \gamma_2 q_+ & \gamma_4 q_+ \\ \gamma_3 q_+ & \varepsilon_c & \gamma_5 q_- & \gamma_6 q_- \\ \gamma_2 q_- & \gamma_5 q_+ & \varepsilon_{v-3} & 0 \\ \gamma_4 q_- & \gamma_6 q_+ & 0 & \varepsilon_{c+2} \end{pmatrix}\end{array}$$

$q_\pm = q_x \pm i q_y$

band edge energies

A. Kormányos, V. Zólyomi, N. D. Drummond, P. Rakyta, GB, and V. I. Falko, Phys. Rev. B **88**, 045416 (2013)

Quantum dots in TMDCs (MX_2)

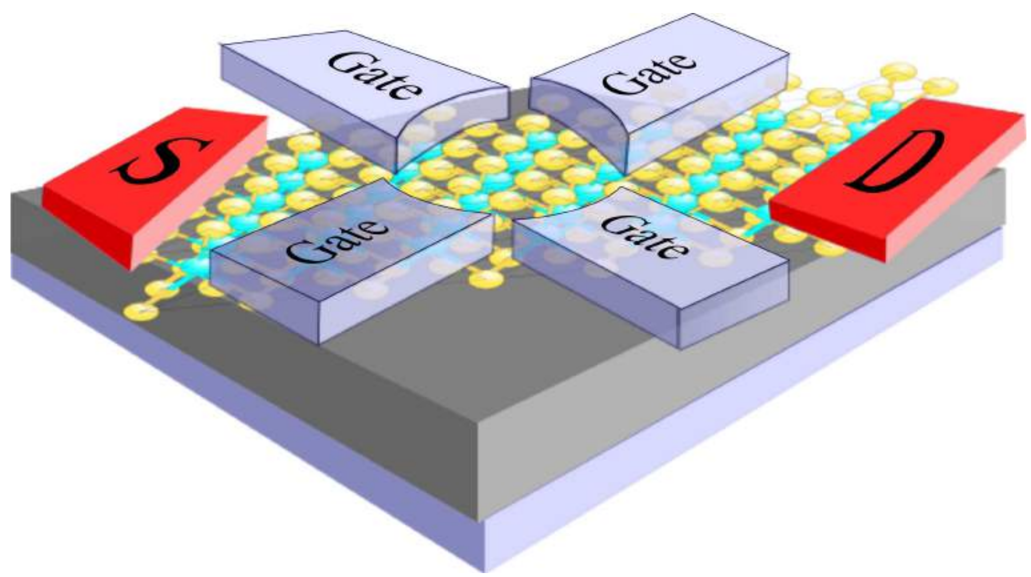
- use k.p model

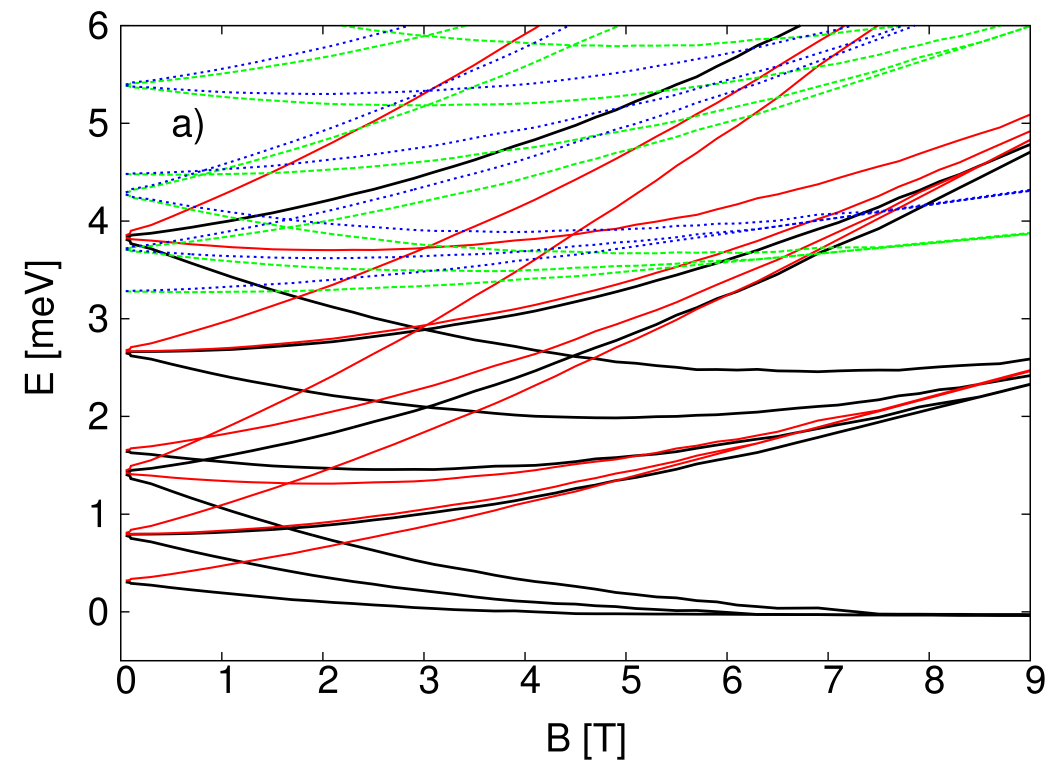
$$\tau s_z \Delta_{cb} \qquad \frac{\tau}{2} g_{vl}\mu_B B_z \qquad \frac{1}{2}\mu_B g_{sp}^{\perp} s_z B_z$$

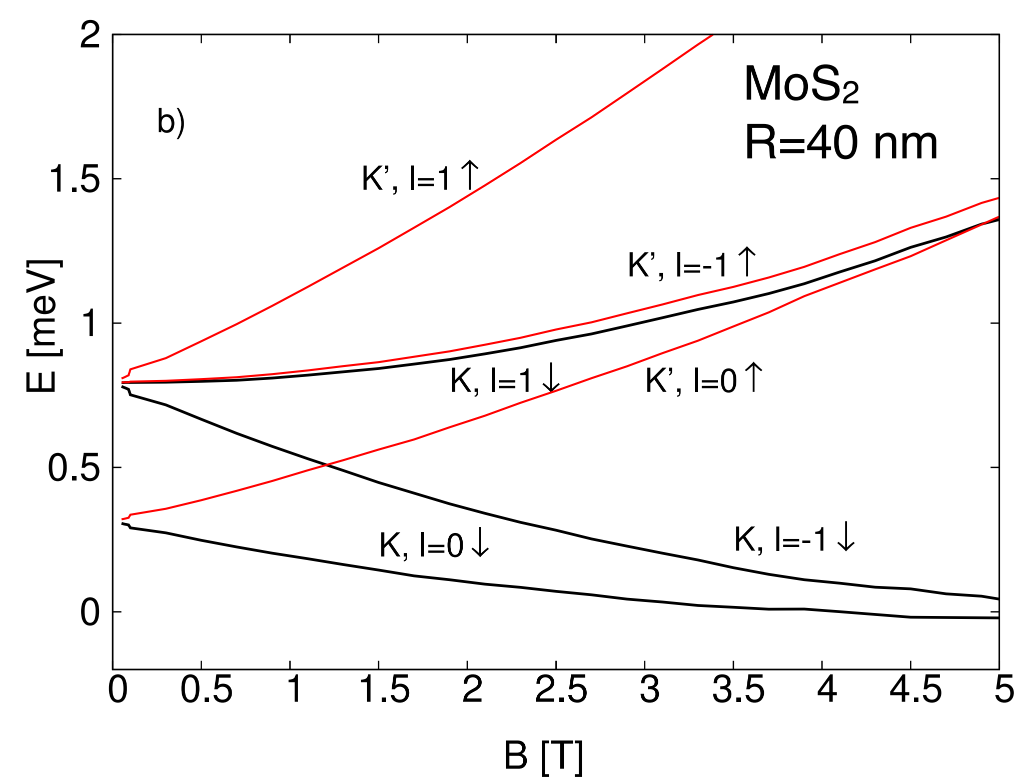
$$H_{dot} = H_{el}^{\tau,s} + H_{\rm so}^{\rm intr} + H_{vl}^{\tau} + H_{sp,tot} + V_{dot}$$

$$\frac{\hbar^2 q_+ q_-}{2m_{\rm eff}^{\tau,s}} + \frac{1}{2}\mathrm{sgn}(B_z)\hbar\omega_c^{\tau,s}$$

$$= \begin{cases} 0, & r < R \\ \infty, & r \geq R \end{cases}$$







- circular dot ⟶ angular momemtum quantum number l
- Bychkov-Rashba SOI splits crossings of l and l+1 levels with opposite spin and same valley
- spin and valley pairs at high B fields
- Kramers (spin-valley) pairs around *B=0*

A. Kormányos, V. Zólyomi, N. D. Drummond, & GB, Phys. Rev. X **4**, 011034 (2014)

Universität Konstanz

Spin Qubits: lecture 3

Spin and Valley 2

Defect Spins

Konstanz: Florian Hilser, Adrian Auer, Vladislav Shkolnikov, GB

Chicago: Brian Zhou, Chris Yale, Lee Bassett, Bob Buckley, Paul Jerger, Joseph Heremans, David Awschalom

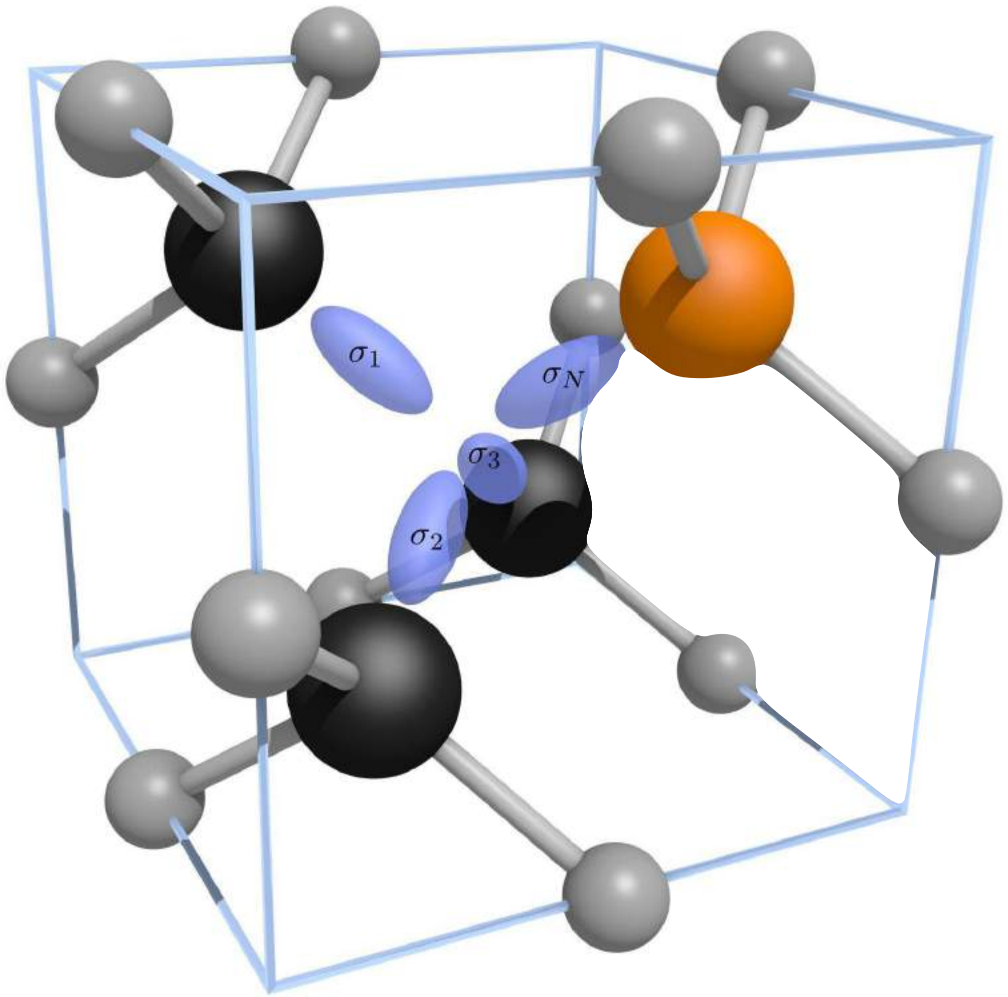
THE UNIVERSITY OF CHICAGO

Universität Konstanz

Spin qubits in diamond

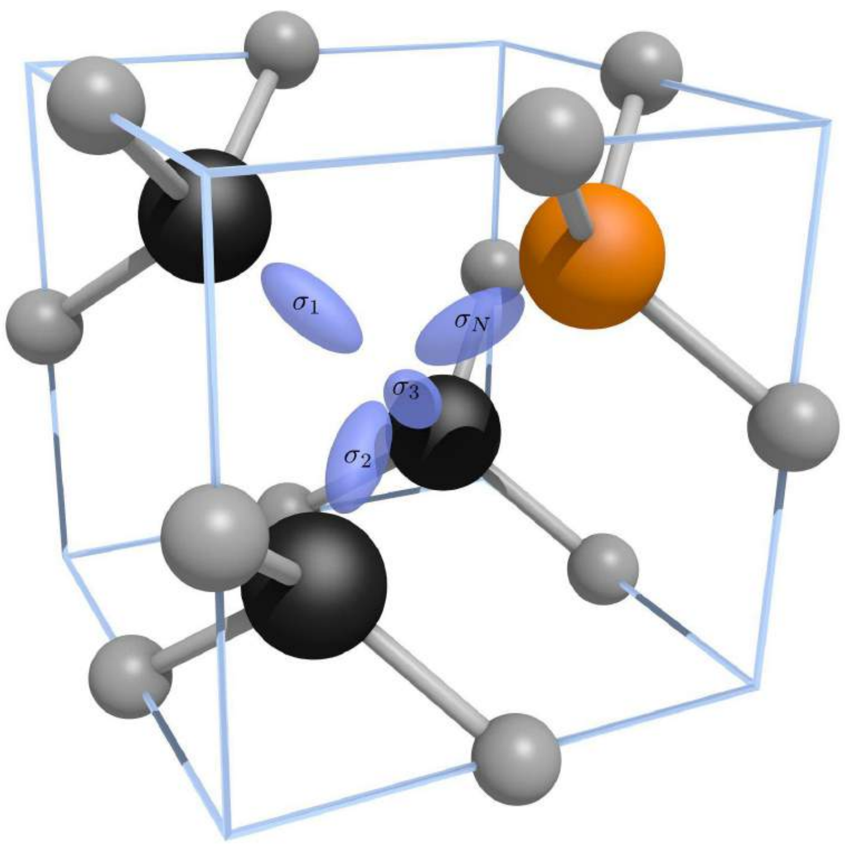
F. Jelezko et al., Phys. Rev. Lett. 93, 130501 (2004).
T. Gaebel et al., Nature Phys. 2, 408 (2006).

M. W. Doherty et al., Physics Reports 528, 1 (2013).

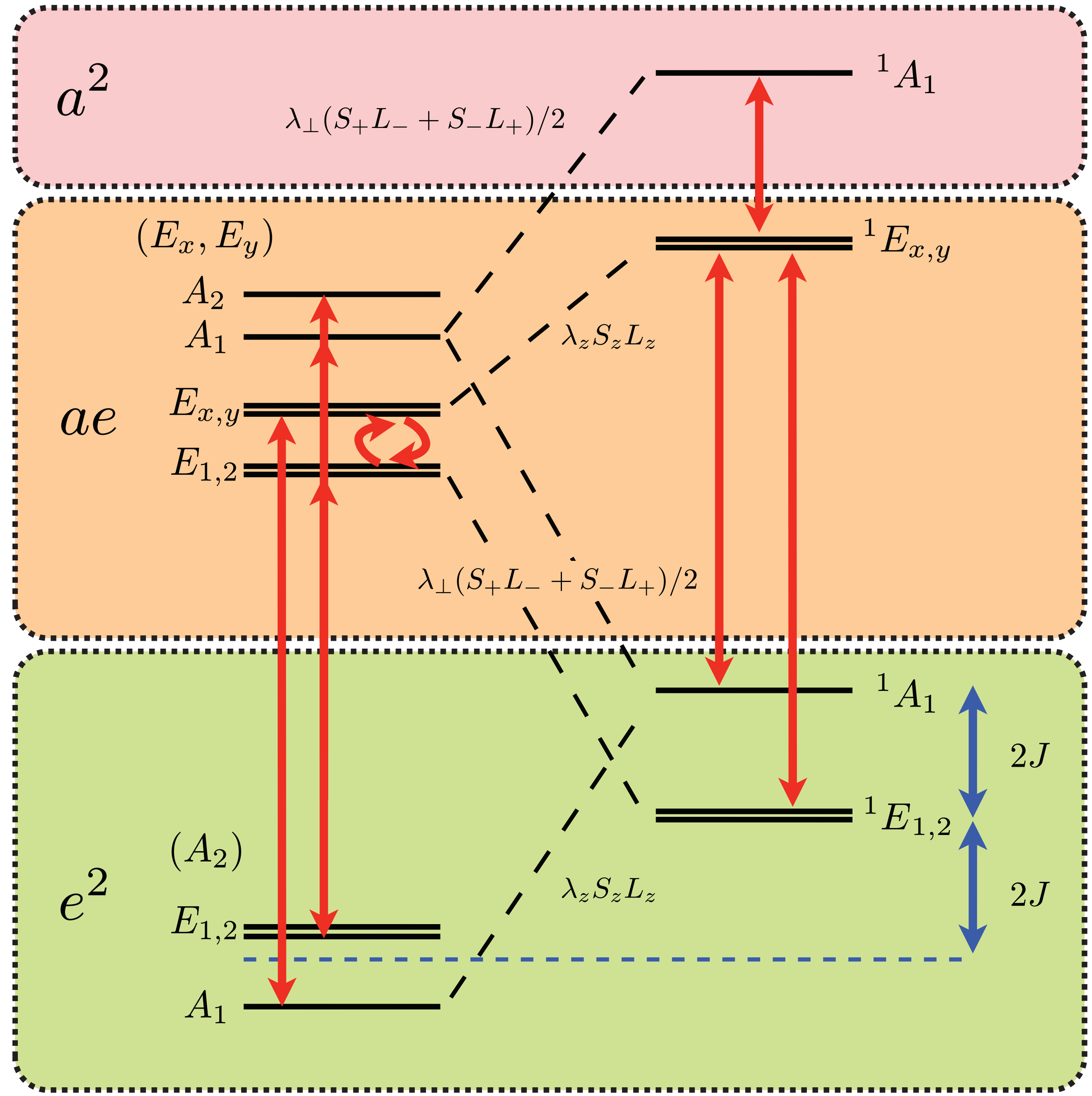


- nitrogen-vacancy (NV^-) center
- ground state spin triplet (S=1)
- coherence ~ ms @low-T, and ~ μs @RT
- optical preparation & readout
- microwave manipulation (ODMR)
- goal: fast, robust & quantum-coherent control of defect spin and charge

Nitrogen-vacancy center (NV$^-$) in diamond

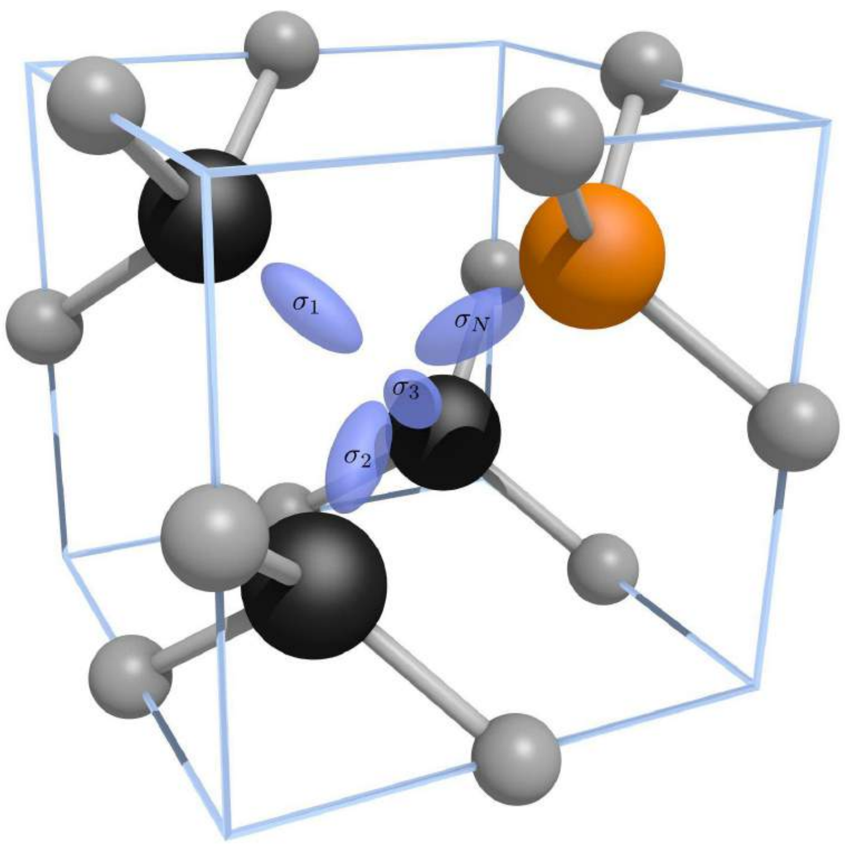


$$H = DS_z^2 + \gamma_e \mathbf{B} \cdot \mathbf{S}$$

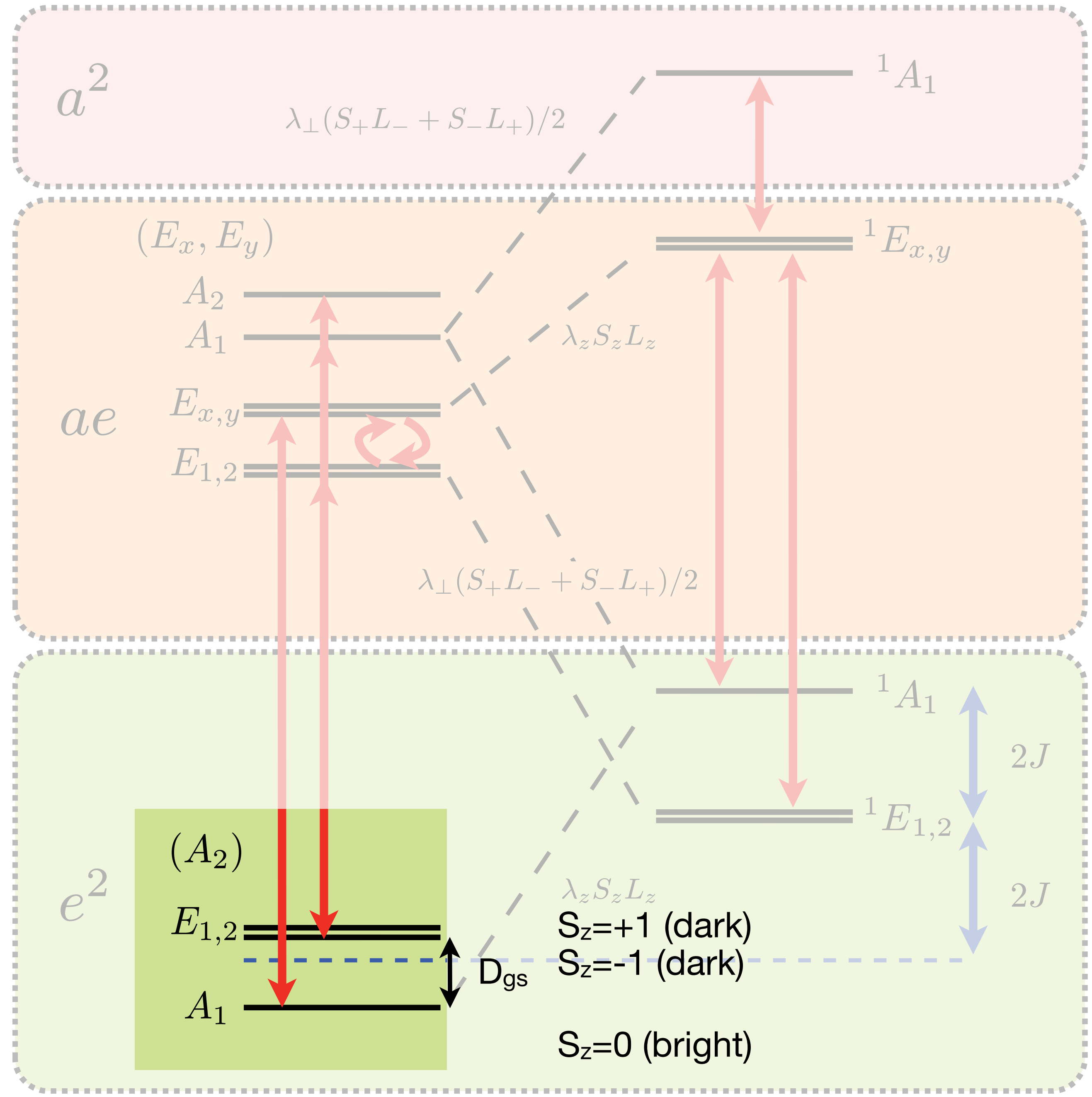


J. Maze et al., New J. Phys. 2011
A. Doherty et al., New J. Phys. 2011

Nitrogen-vacancy center (NV$^-$) in diamond

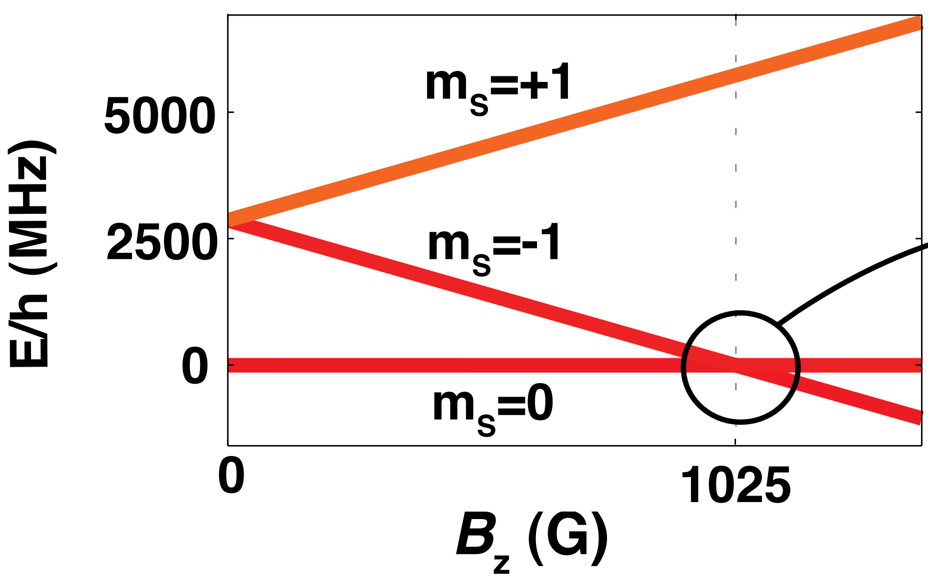


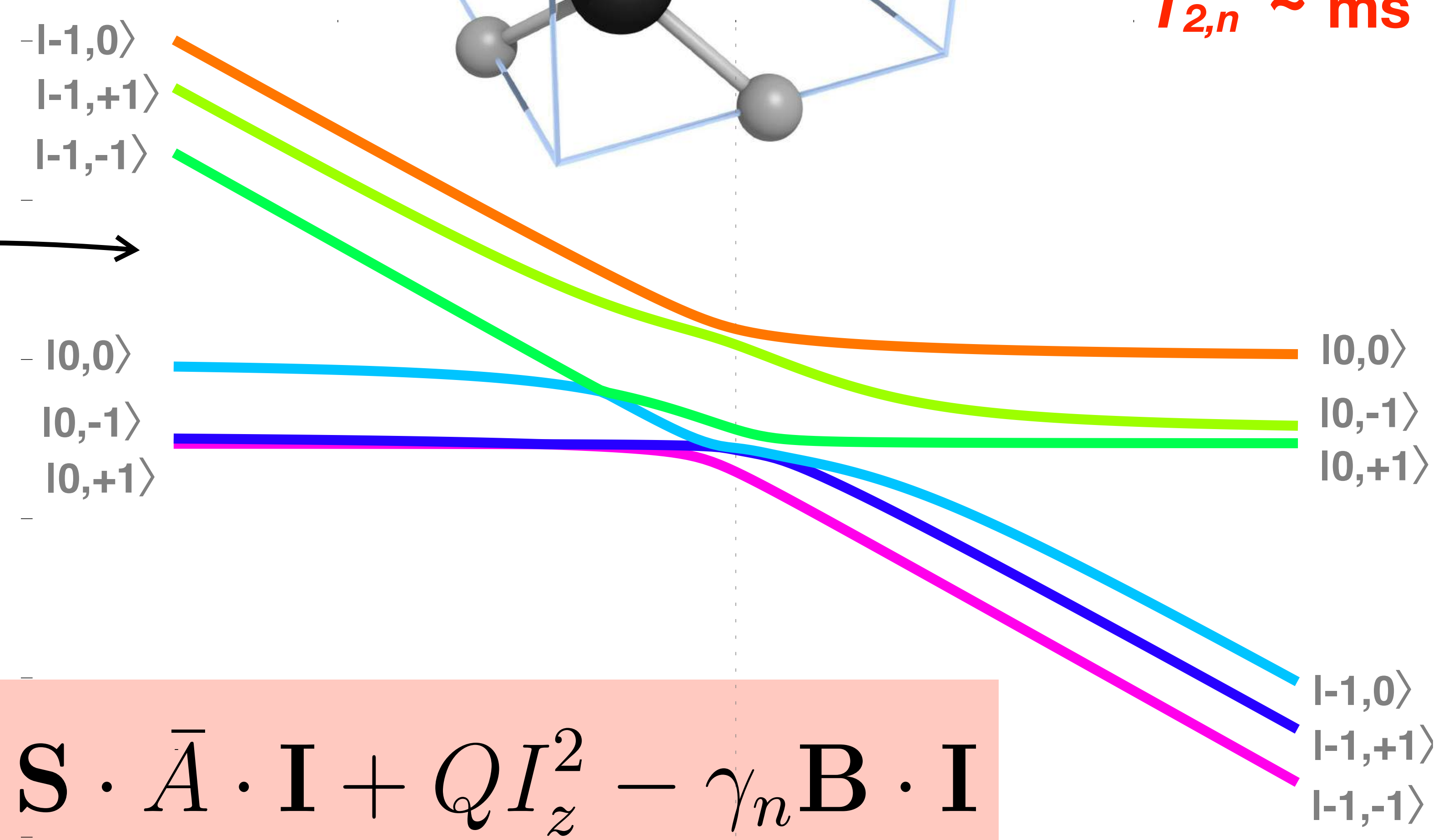
$$H = DS_z^2 + \gamma_e \mathbf{B} \cdot \mathbf{S}$$

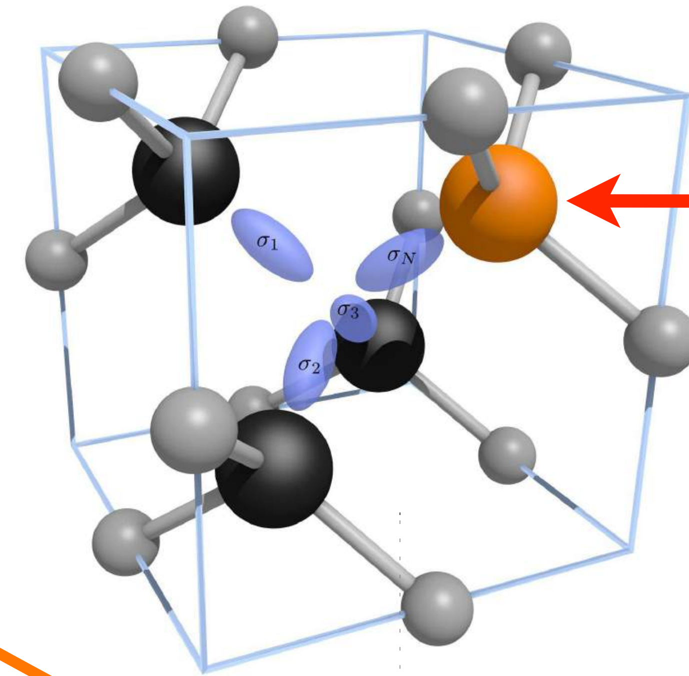


J. Maze et al., New J. Phys. 2011
A. Doherty et al., New J. Phys. 2011

Zeeman and hyperfine structure







^{14}N has nuclear spin 1

$T_{2,n}^{*}$ ~ ms

$$H = DS_z^2 + \gamma_e \mathbf{B}\cdot\mathbf{S} + \mathbf{S}\cdot\bar{A}\cdot\mathbf{I} + QI_z^2 - \gamma_n \mathbf{B}\cdot\mathbf{I}$$

hyperfine

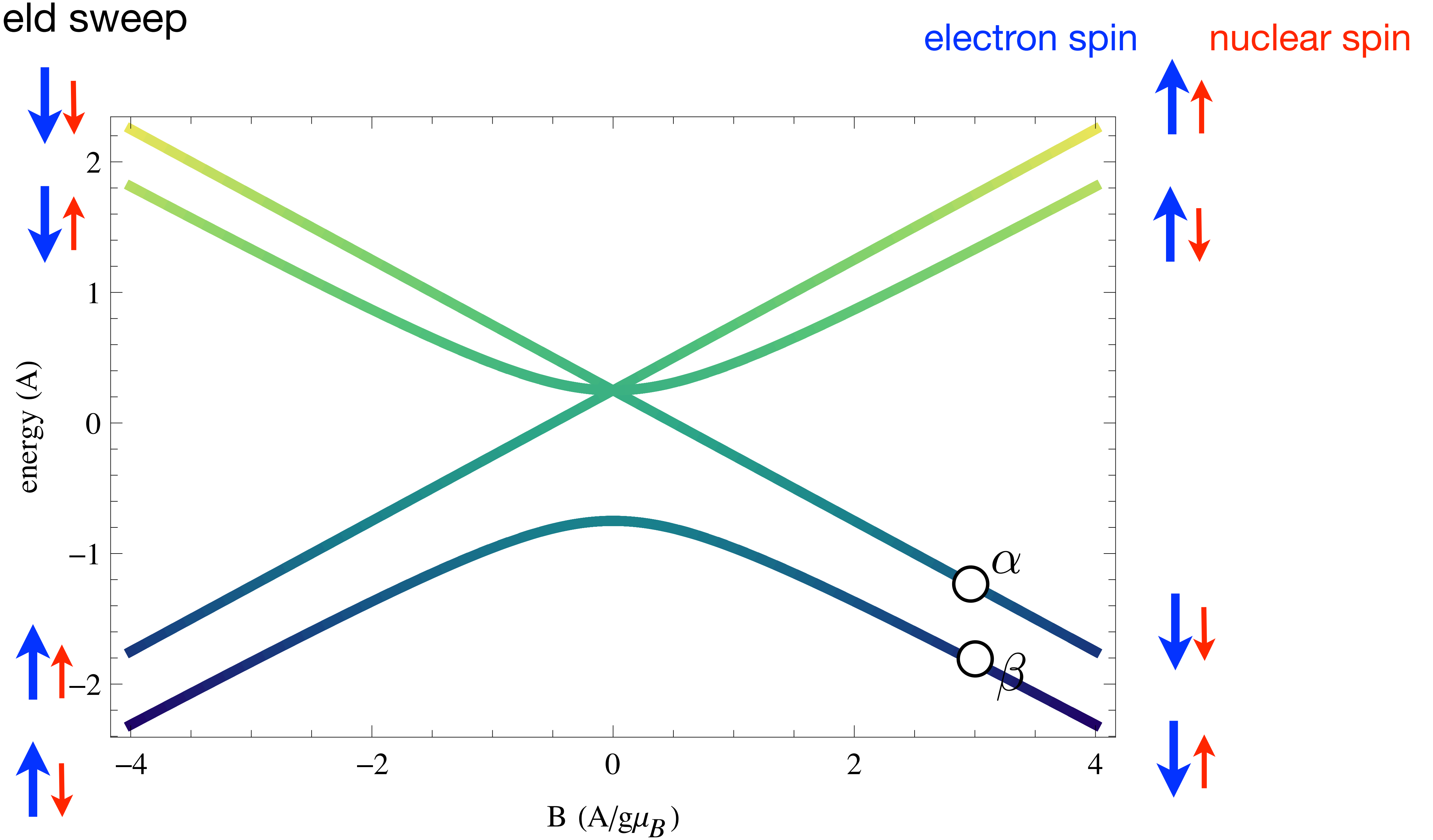
quadrupolar

nuclear Zeeman

$$\bar{A} = \begin{pmatrix} A_\perp & 0 & 0 \\ 0 & A_\perp & 0 \\ 0 & 0 & A_\parallel \end{pmatrix}$$

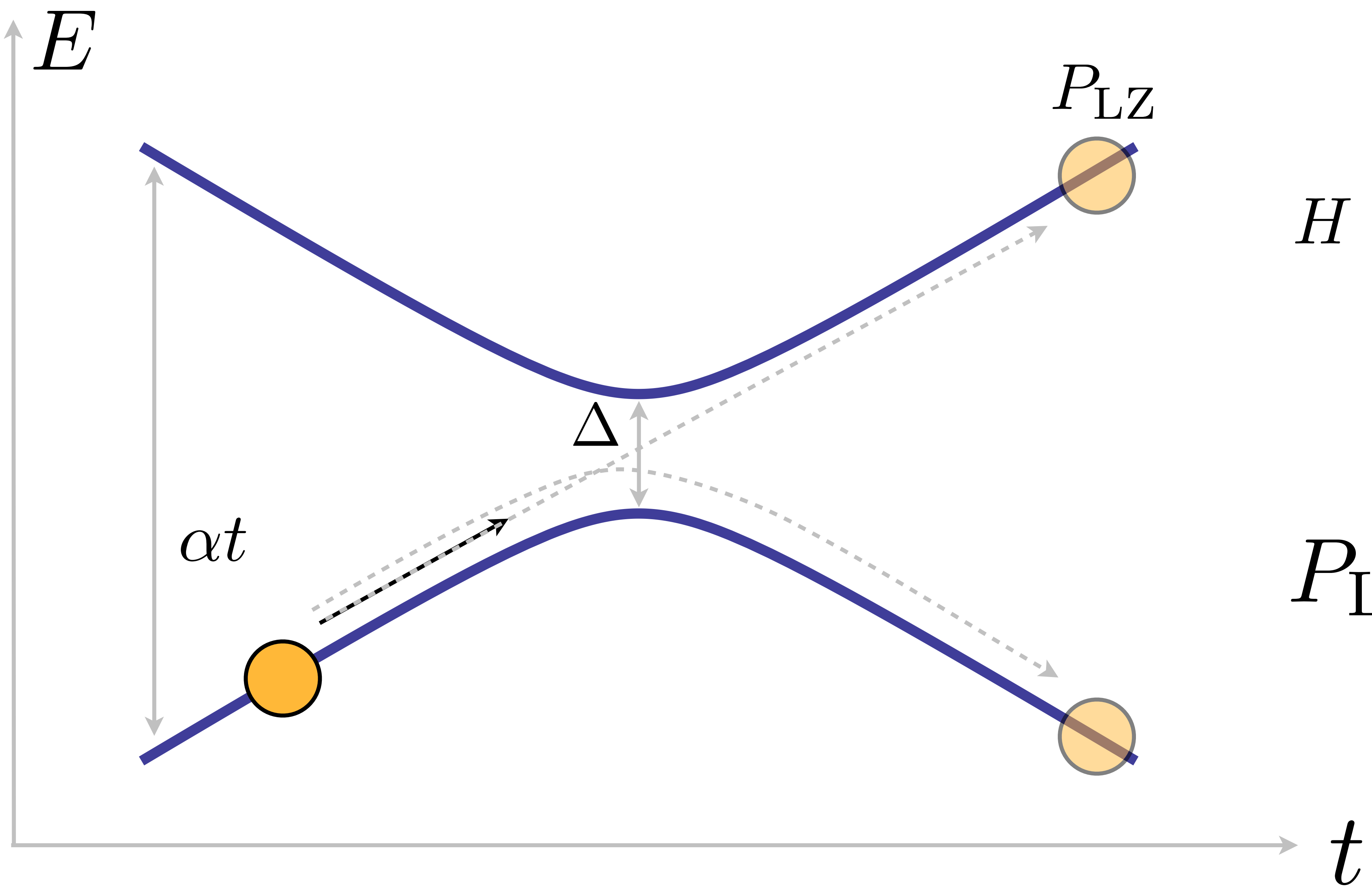
A nuclear quantum memory

- real NV with ^{14}N: two spins 1
- simpler example: two spins 1/2
- adiabatic B field sweep



- spin 1: unused states, quadrupolar splitting
- field misalignment, anisotropic hyperfine splitting
- non-adiabatic corrections?

Landau-Zener theory



$$H = \frac{1}{2}\begin{pmatrix} \alpha t & \Delta \\ \Delta & -\alpha t \end{pmatrix}$$

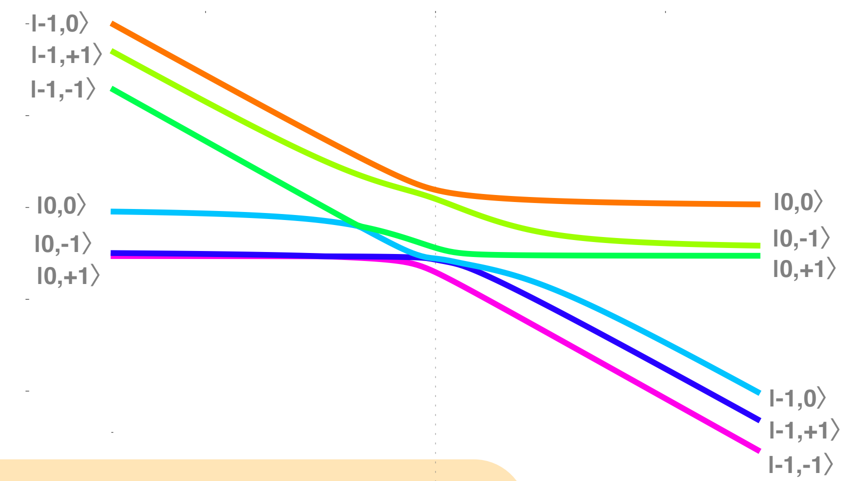
$$P_{\mathrm{LZ}} = e^{-\frac{2\pi|\Delta|^2}{\hbar\alpha}}$$

L. D. Landau, Phys. Z. **2**, 46 (1932)
C. Zener, Proc. R. Soc. London A **137** 696 (1932)
E. C. G. Stückelberg, Helv. Phys. Acta **5**, 370 (1932)
E. Majorana, Nuovo Cimento **9**, 43 (1932)

generalizations:

- finite-time sweep (Stückelberg oscillations)
- degenerate levels

Landau-Zener theory



- degenerate hyperfine level approximation

$$H = DS_z^2 + \gamma_e \mathbf{B}\cdot\mathbf{S} + \mathbf{S}\cdot\bar{A}\cdot\mathbf{I} + QI_z^2 - \gamma_n \mathbf{B}\cdot\mathbf{I}$$
$$\approx DS_z^2 + \gamma_e B_z(t) S_z + \gamma_e B_x S_x + \frac{A_\perp}{2}(S_+ I_- + S_- I_+)$$

- degenerate LZ problem

$$H = \begin{pmatrix} 0 & V \\ V^\dagger & \alpha t \mathbb{1} \end{pmatrix}$$

$$V = \begin{pmatrix} \gamma_e B_x/\sqrt{2} & 0 & 0 \\ A_\perp & \gamma_e B_x/\sqrt{2} & 0 \\ 0 & A_\perp & \gamma_e B_x/\sqrt{2} \end{pmatrix}$$

- Morris-Shore transform -> three independent LZ systems

$$\tilde{H} = U_{\mathrm{MS}} H U_{\mathrm{MS}}^\dagger = \begin{pmatrix} H_1 & 0 & 0 \\ 0 & H_2 & 0 \\ 0 & 0 & H_3 \end{pmatrix} \qquad H_k = \begin{pmatrix} 0 & \lambda_k \\ \lambda_k & \alpha t \end{pmatrix}$$

$$\gamma_e B_x/\sqrt{2} \ll A_\perp$$

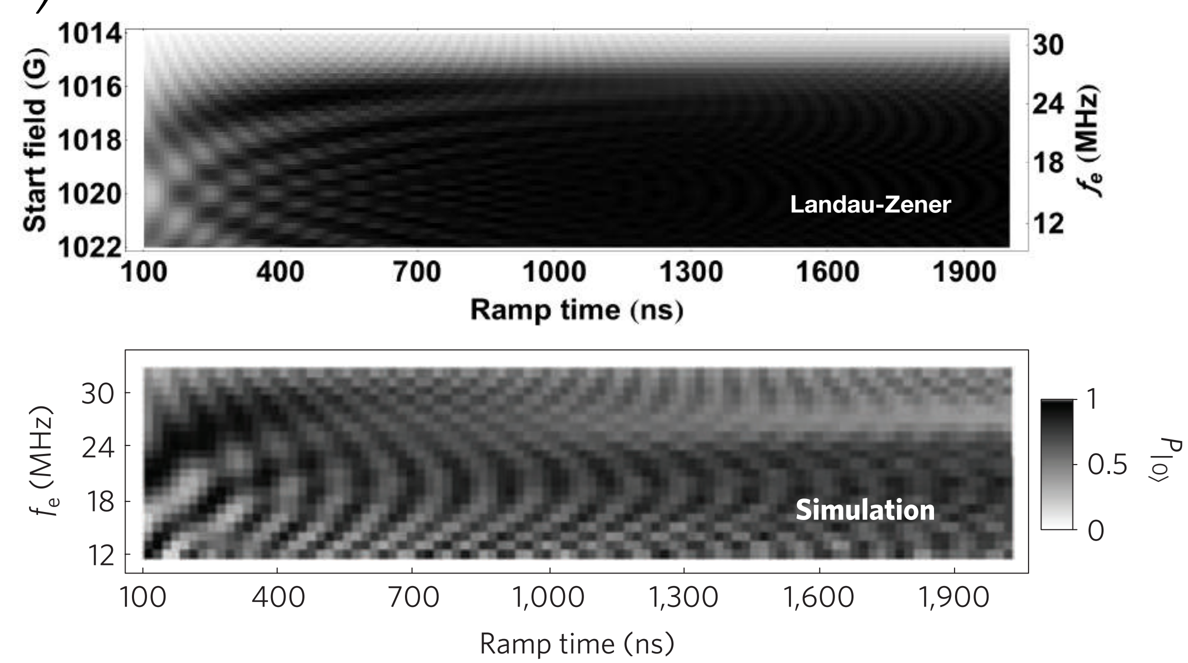
$$\lambda_{1,2} \approx A_\perp \sqrt{1 \pm \gamma_e B_x/(\sqrt{2} A_\perp)}$$

$$\lambda_3 \approx (\gamma_e B_x/\sqrt{2})^3 / A_\perp^2$$

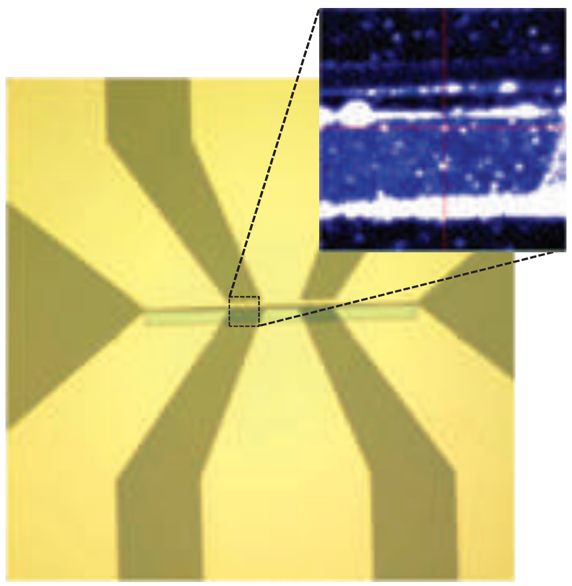
- finite-time LZ solutions

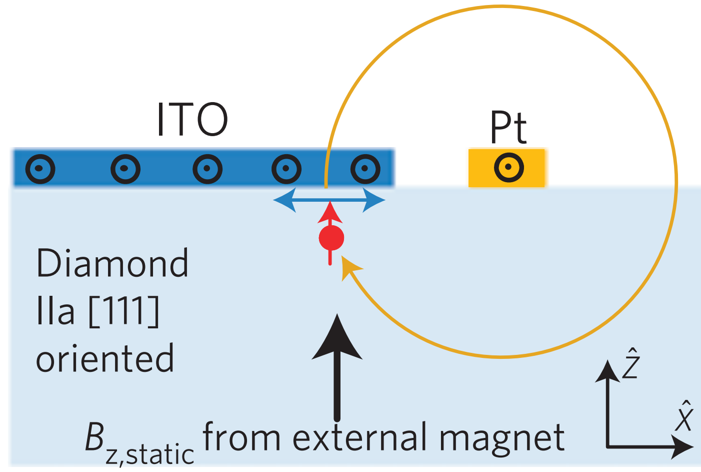
$$U_k = T\exp\left(-i\int_{t_i}^{t_f} H_k(t)dt\right) = \begin{pmatrix} a_k & b_k \\ -b_k^* & a_k^* \end{pmatrix}$$

$$a_k = \frac{\Gamma\left(1 - \frac{i\eta_k^2}{2}\right)}{\sqrt{2\pi}}\left[D_{\frac{i\eta_k^2}{2}}\left(\sqrt{2}e^{\frac{-i\pi}{4}}\tau_f\right) D_{\frac{i\eta_k^2}{2}-1}\left(\sqrt{2}e^{\frac{3i\pi}{4}}\tau_i\right) + D_{\frac{i\eta_k^2}{2}}\left(\sqrt{2}e^{\frac{3i\pi}{4}}\tau_f\right) D_{\frac{i\eta_k^2}{2}-1}\left(\sqrt{2}e^{-\frac{i\pi}{4}}\tau_i\right)\right]$$

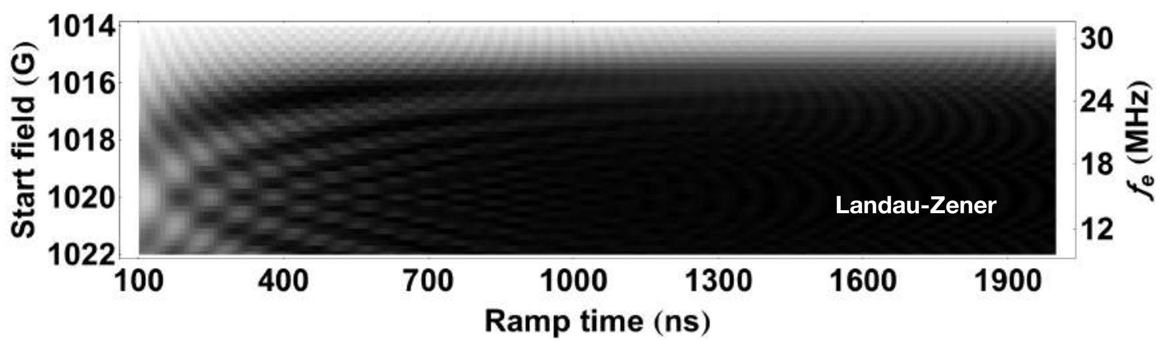


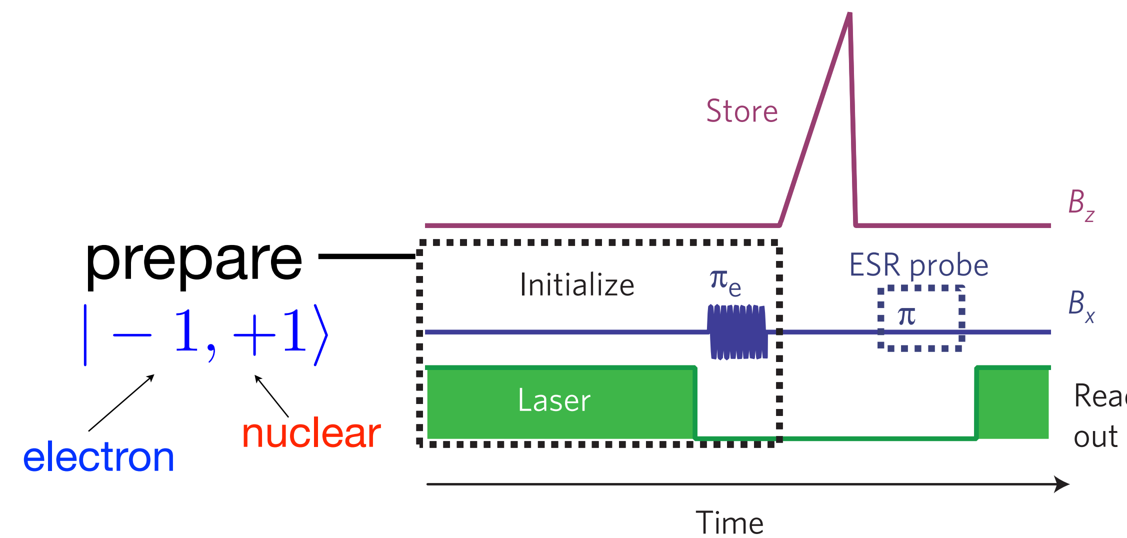
Experiment

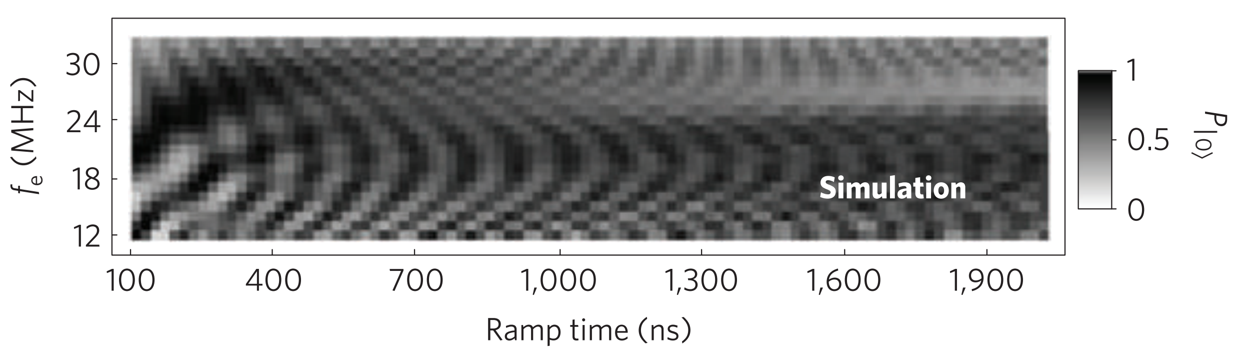


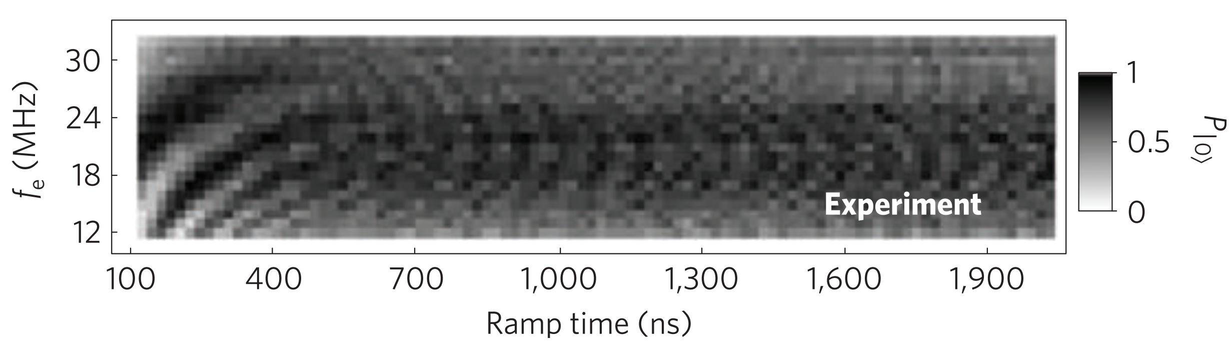


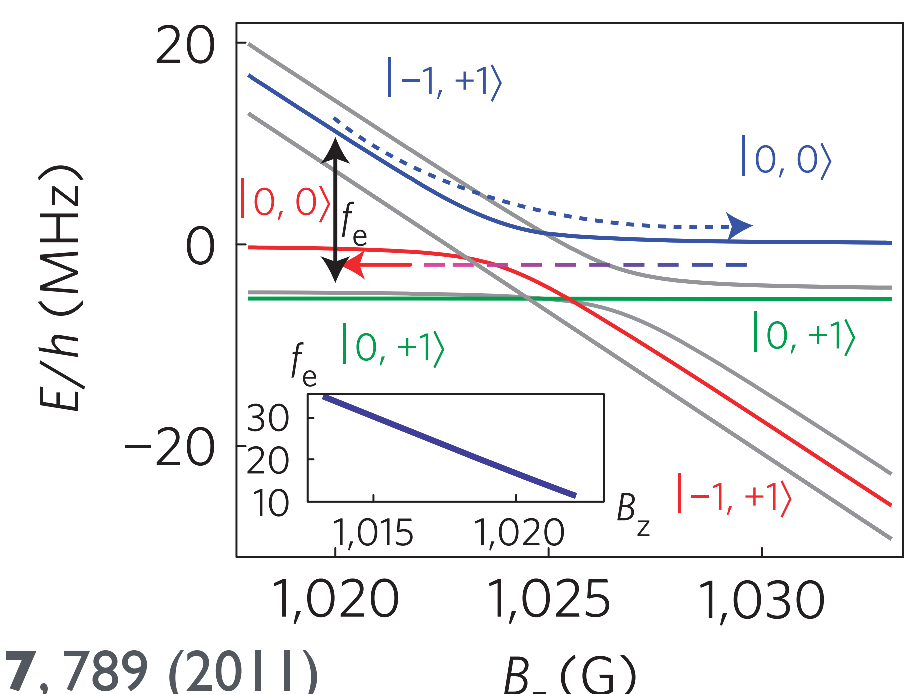
- single NV center
- room temperature
- $B = B_{magnet} + B_{wire}(t)$











G. D. Fuchs, G. Burkard, P.V. Klimov & D. D. Awschalom, Nature Phys. **7**, 789 (2011)

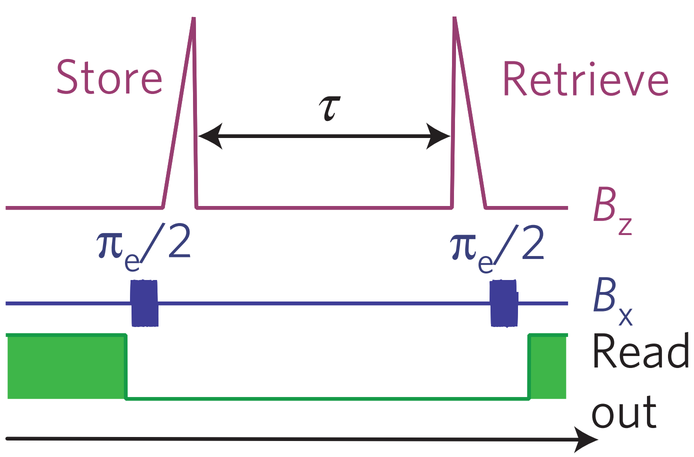
Ramsey fringes

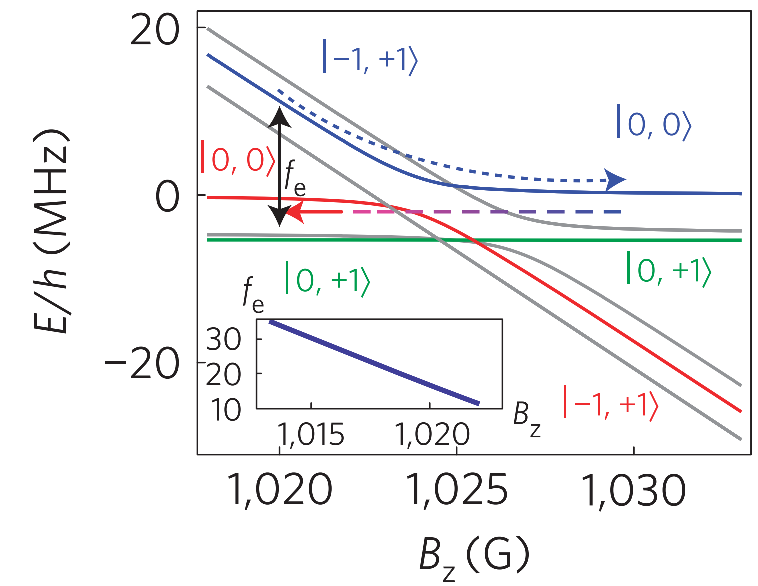
- coherent storage of qubits
- prepare $|0,+1\rangle + |-1,+1\rangle$

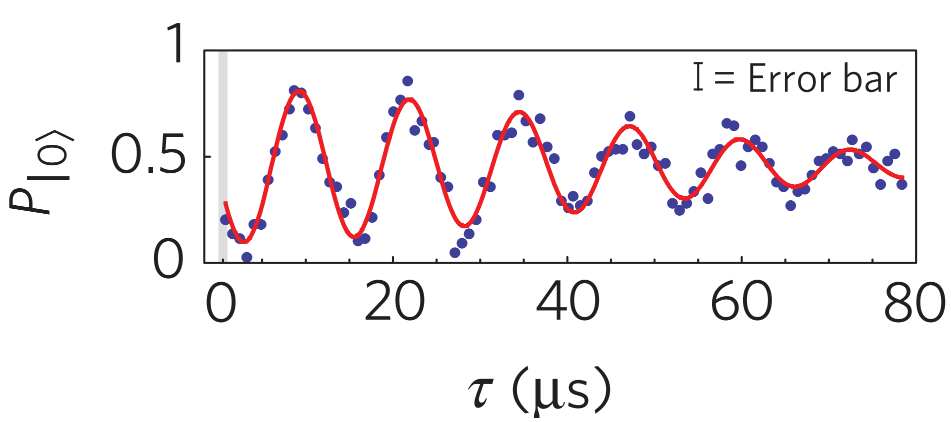
  electron nuclear

- store
- wait for time τ
- retrieve = (store)$^{-1}$
- measure projection onto

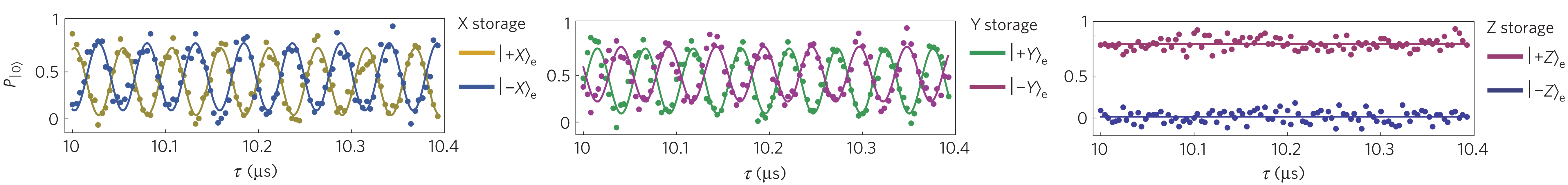
  $$|0,+1\rangle + |-1,+1\rangle$$







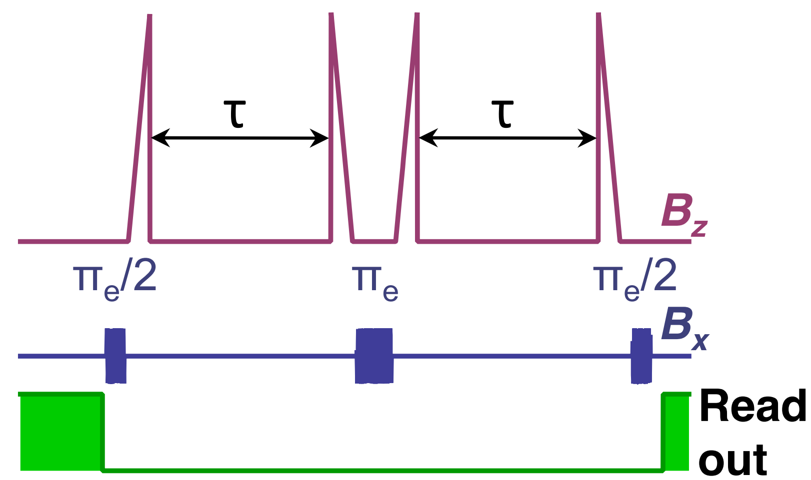
- store other qubit superposition states

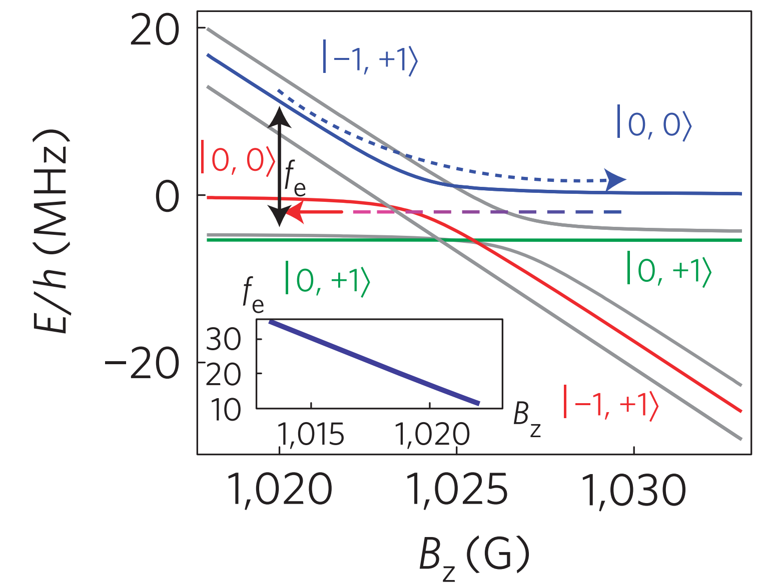


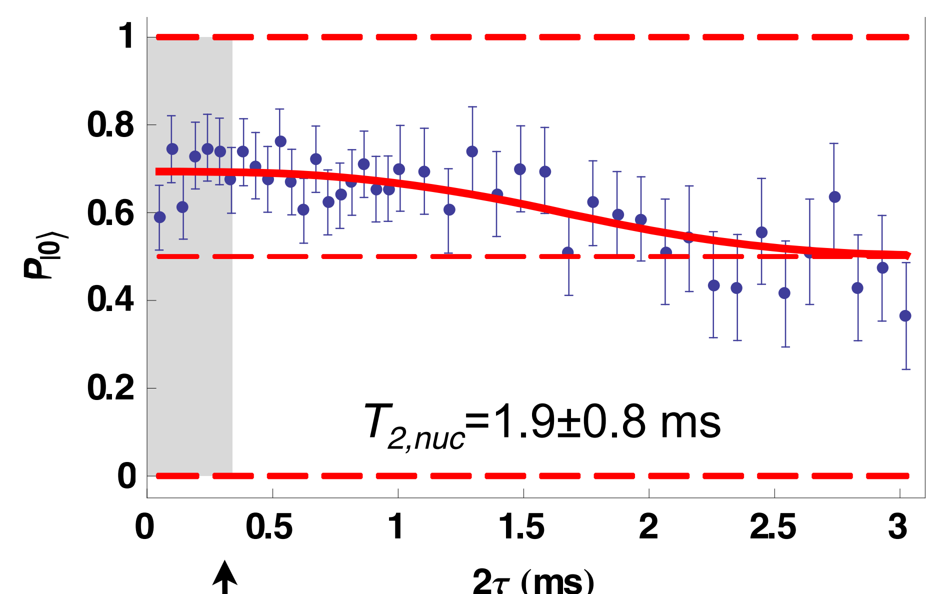
Spin echo

- prepare
- store $|0,+1\rangle + |-1,+1\rangle$
- wait for time T
- **retrieve = (store)$^{-1}$**
- **perform spin-echo π pulse**
- **store**
- wait for time T
- retrieve = (store)$^{-1}$
- measure projection onto

$$|0,+1\rangle + |-1,+1\rangle$$

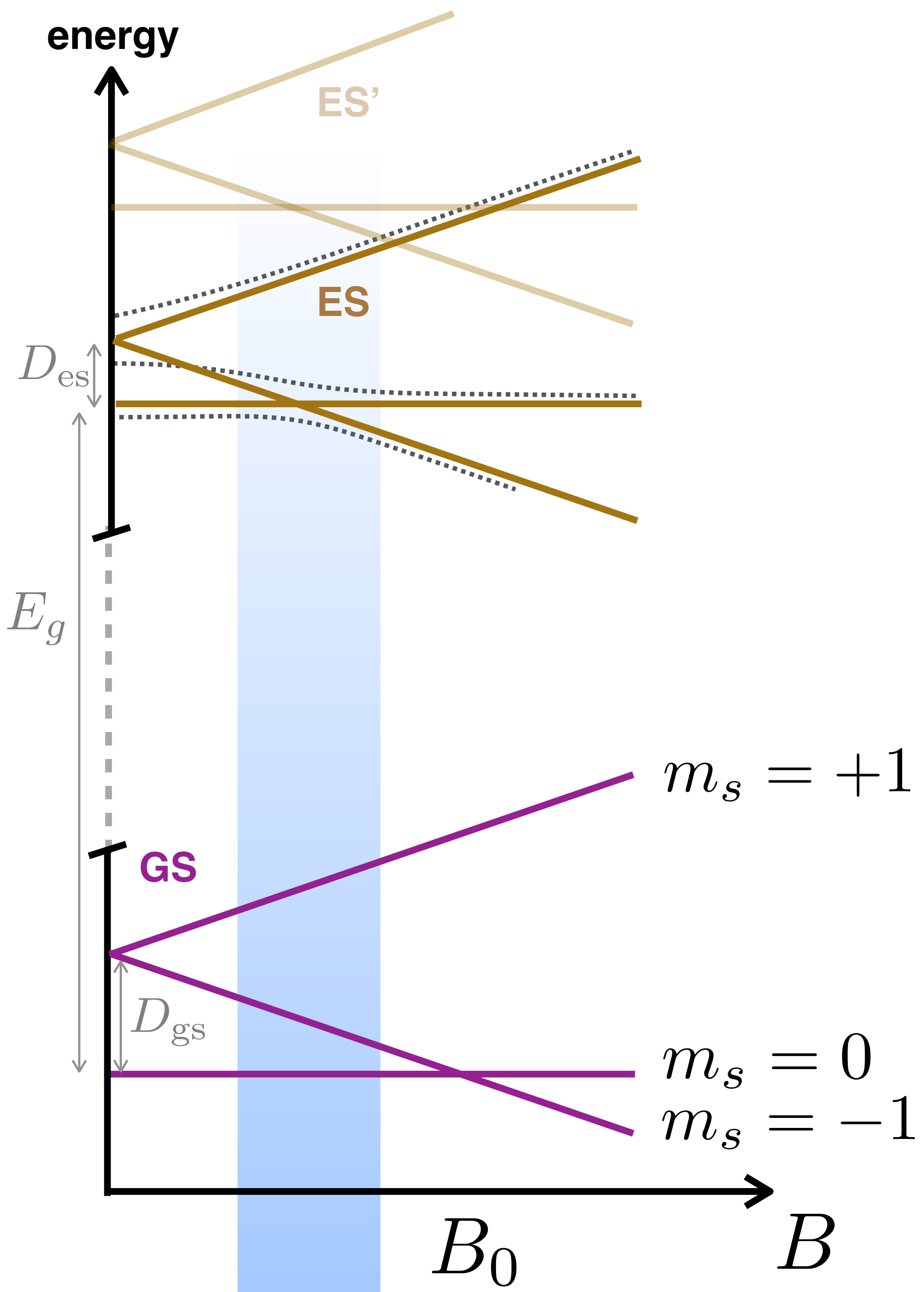




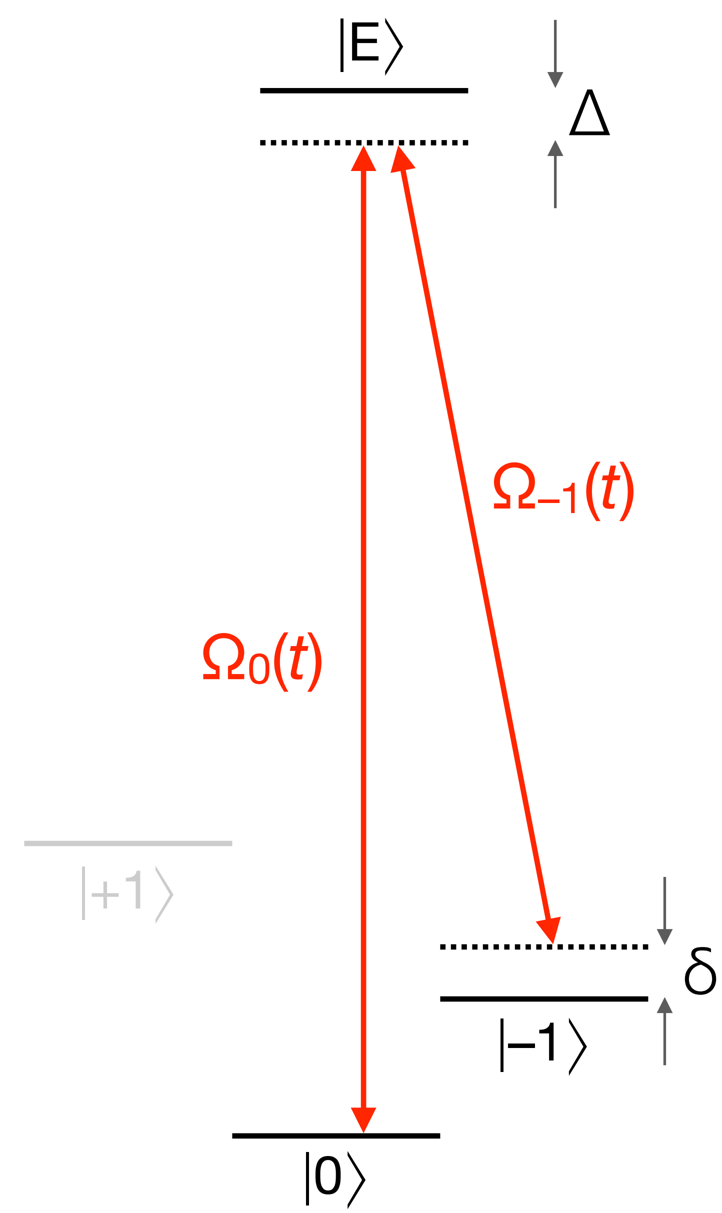


electron coherence time

Level structure of the NV center: Λ configuration



$$|E\rangle = |\alpha_1\rangle|S_z = -1\rangle + |\alpha_0\rangle|S_z = 0\rangle$$



F. Hilser and G. Burkard, Phys. Rev. B 2012

Optical control scheme

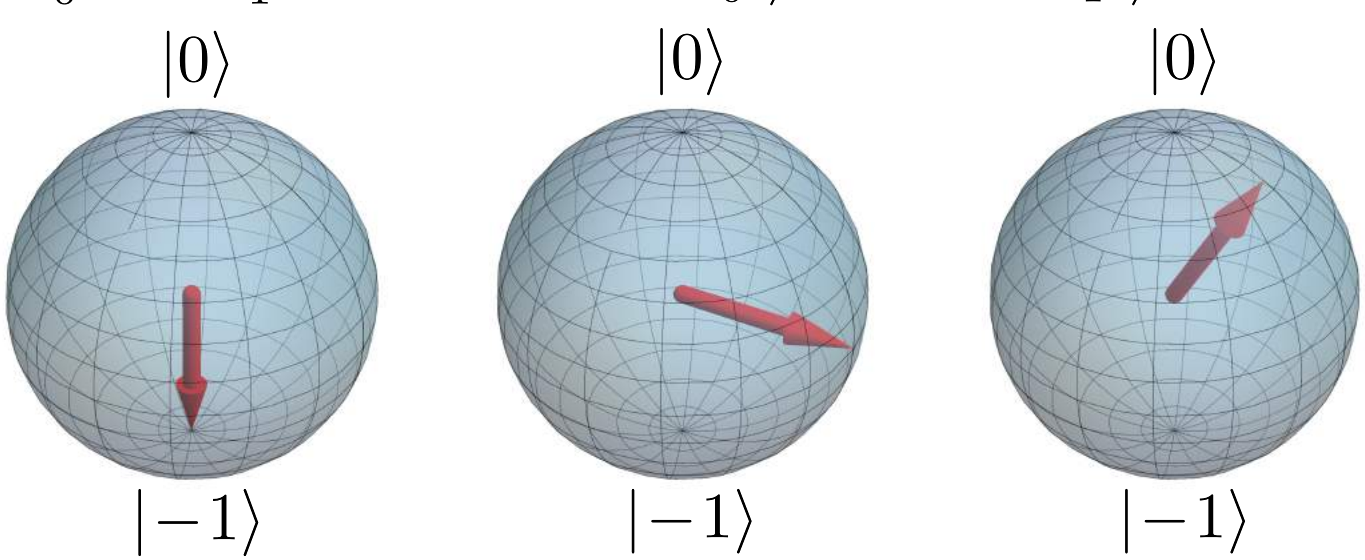
- virtual transitions via excited state (triplet) $|E\rangle = |\alpha_1\rangle|S_z = -1\rangle + |\alpha_0\rangle|S_z = 0\rangle$
- spin mixing at excited state level anticrossing due to transverse spin-spin interaction Δ_S
- effective transverse B-field $B_x = \Delta_S \frac{\Omega_{-1}\Omega_0}{\delta^2}$

$$B_z = -\frac{1}{2} g\mu_B (B - B_0) + \frac{3}{2}\frac{\Omega_{-1}\Omega_0}{\delta}$$

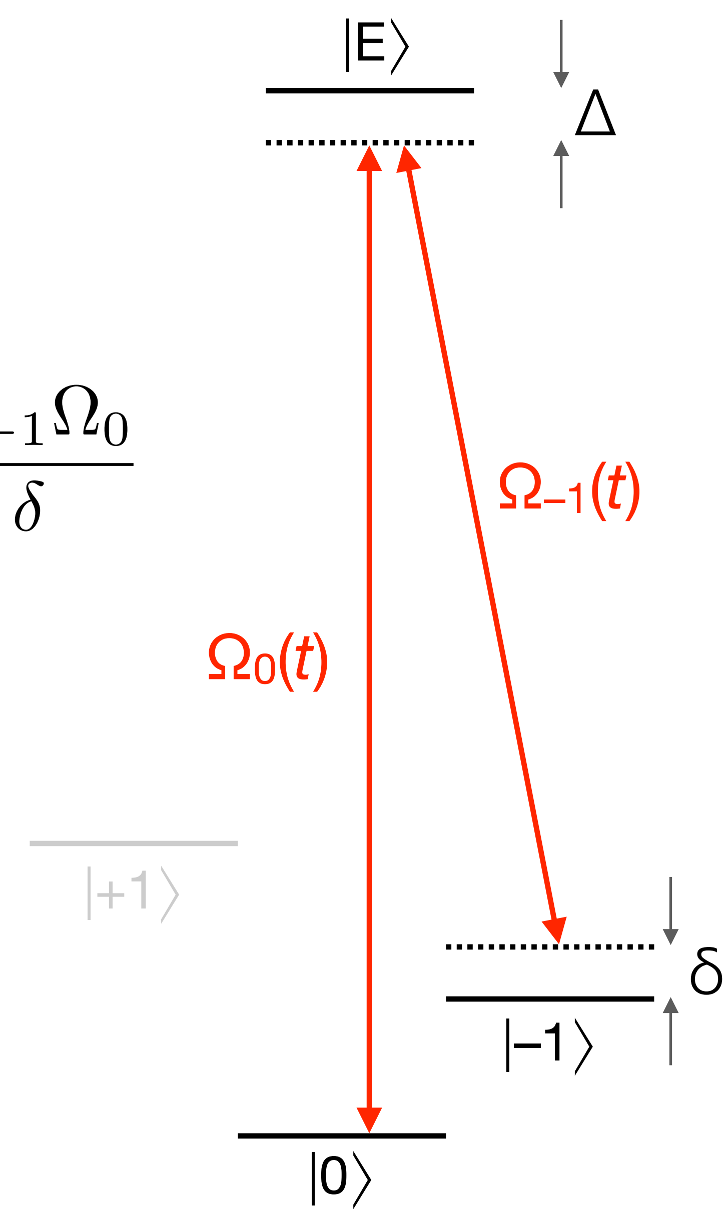
dark

with laser excitation

$\Omega_0 = \Omega_{-1} = 0$ $\Omega_0 \neq 0$ $\Omega_{-1} \neq 0$



$\delta = \delta^*$ $\delta = \delta^*/2$

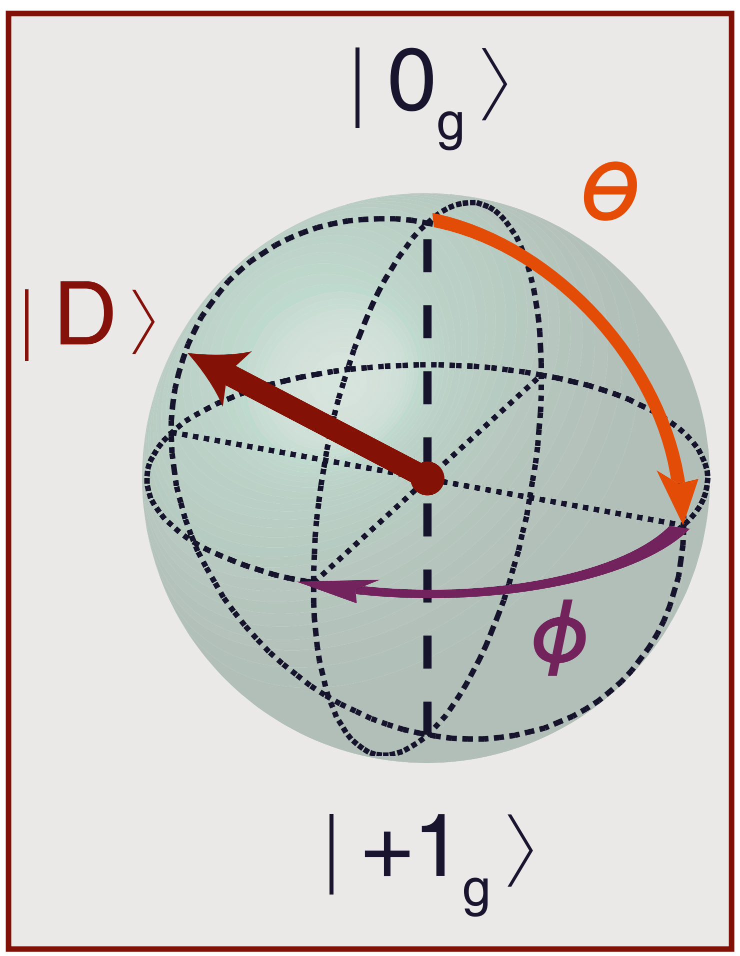


F. Hilser and G. Burkard, Phys. Rev. B 2012

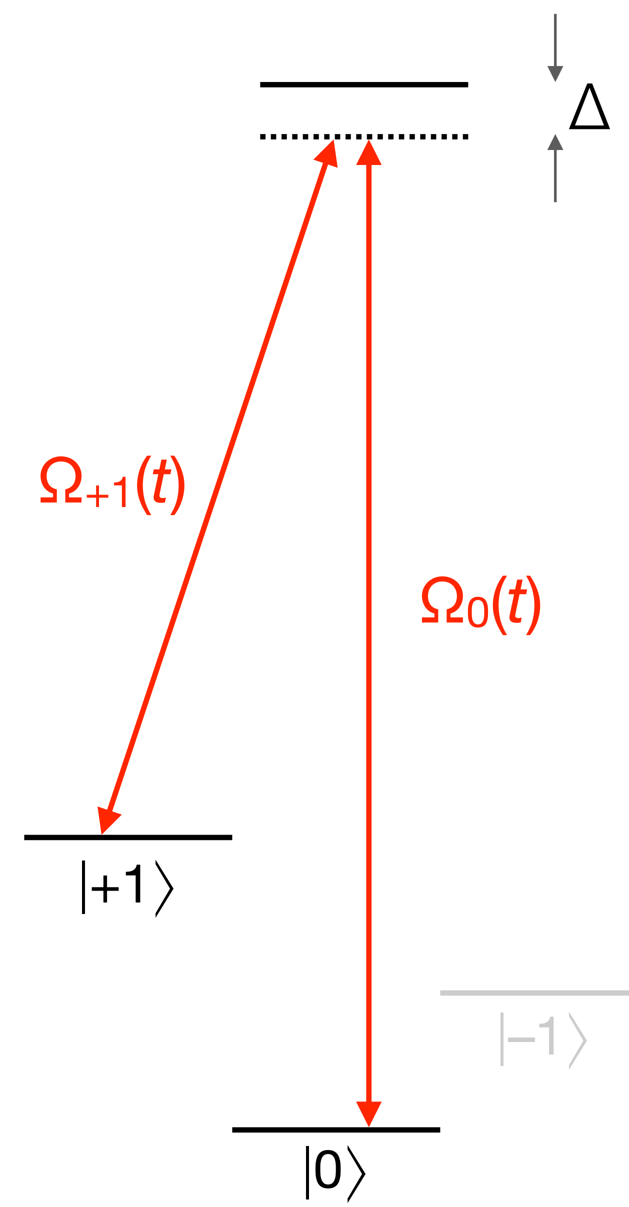
Coherent population trapping (CPT)

- dark state: $|D\rangle = \dfrac{1}{\sqrt{|\Omega_{+1}|^2 + |\Omega_0|^2}} \left(\Omega_{+1}|0\rangle - \Omega_0|+1\rangle\right)$

- can be any position on the Bloch sphere

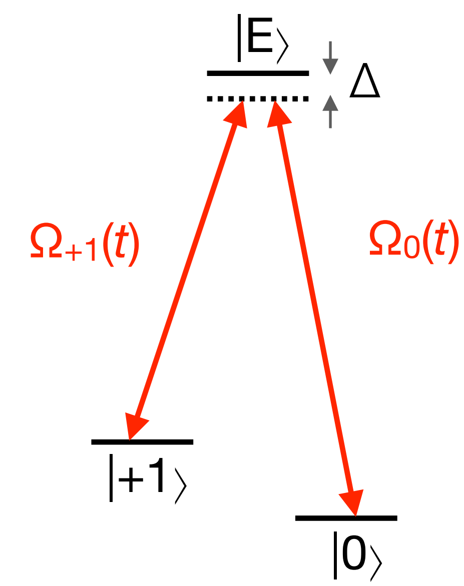


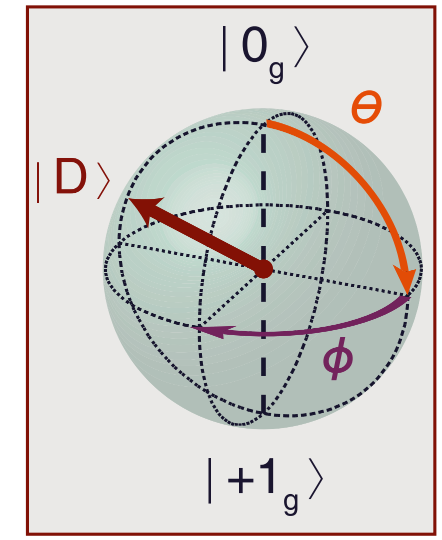
$$\Omega_{+1} = \cos(\theta/2)$$
$$\Omega_0 = e^{i\phi}\sin(\theta/2)$$



Spin (qubit) control with CPT

dark state $$|D\rangle = \frac{1}{\sqrt{|\Omega_{+1}|^2 + |\Omega_0|^2}} \left(\Omega_{+1}|0\rangle - \Omega_0|+1\rangle\right)$$

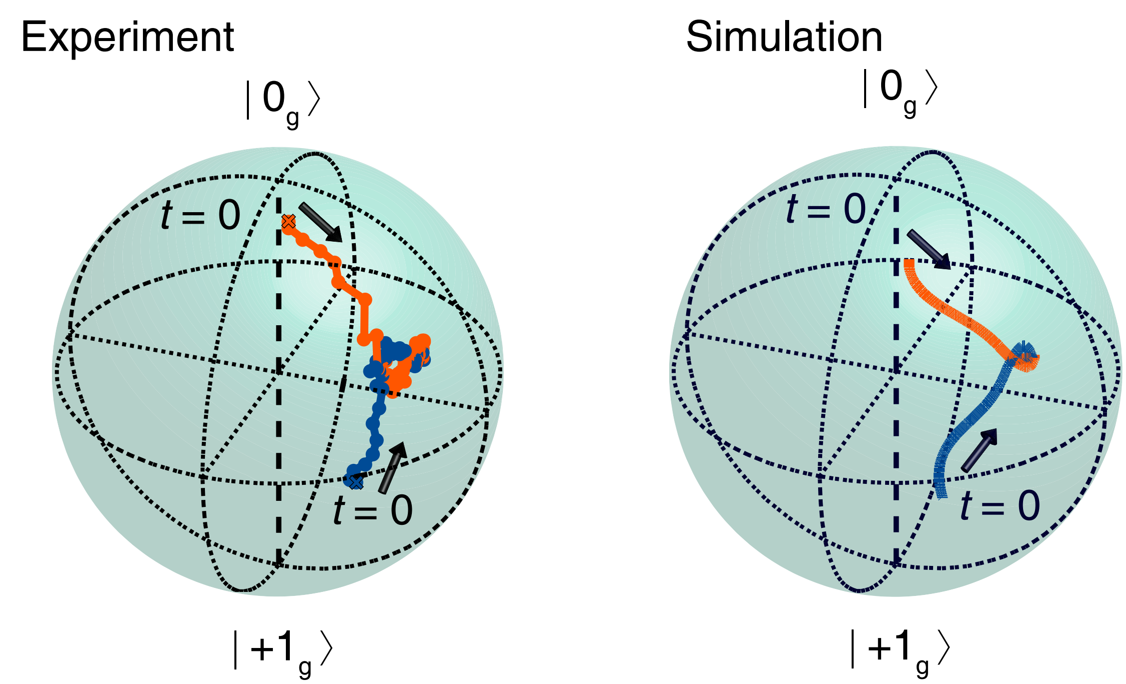




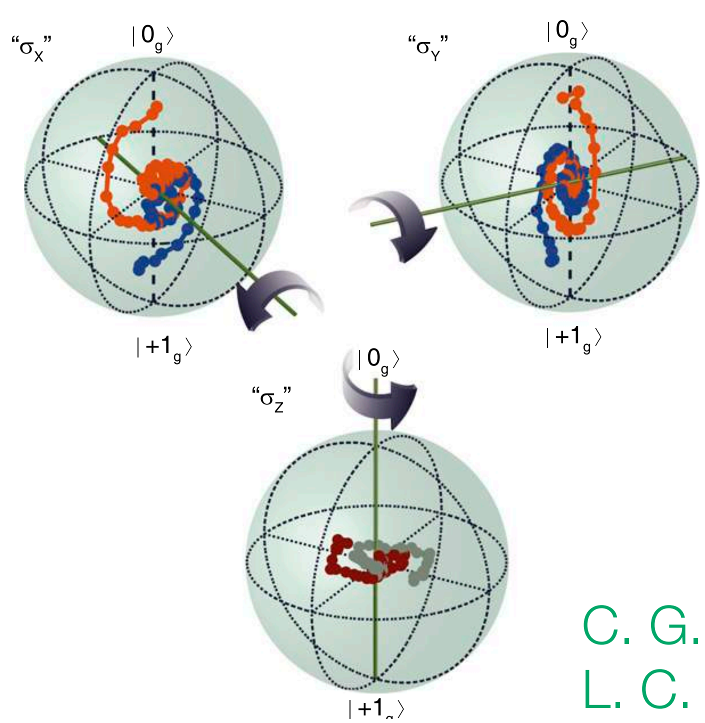
CPT for arbitrary-basis spin initialization

$$H = \sum_{\alpha} \varepsilon_\alpha |\alpha\rangle\langle\alpha| + \sum_{G=0,1} \sum_{E=L,R} h\left(\Omega_{GE} |E_{e1}\rangle\langle G_g| + h.c.\right)$$

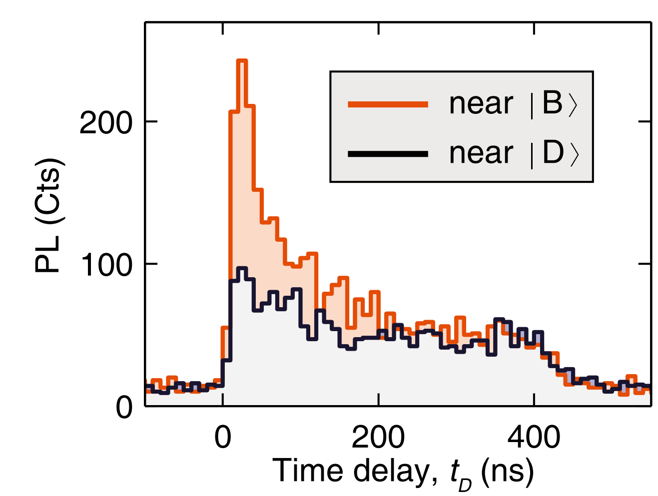
$$\dot{\rho} = i\,[\rho, H] + \sum_{\alpha,\alpha'} \Gamma_{\alpha\alpha'} \left(\sigma_{\alpha'\alpha}\rho\sigma_{\alpha\alpha'} - \frac{1}{2}\sigma_{\alpha\alpha}\rho - \frac{1}{2}\rho\sigma_{\alpha\alpha} \right) \equiv W\rho$$



CPT for arbitrary-axis spin rotation



CPT for arbitrary-basis spin readout



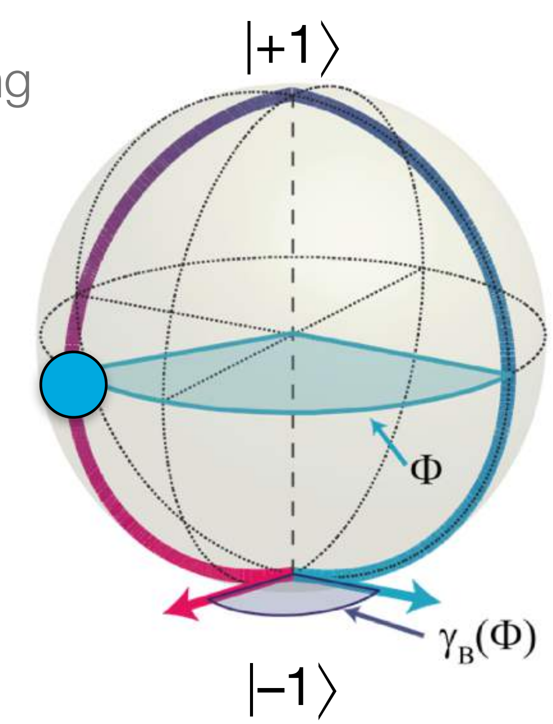
C. G. Yale, B. B. Buckley, D. J. Christle, G. Burkard, F. J. Heremans, L. C. Bassett, and D. D. Awschalom, PNAS 110, 7595 (2013)

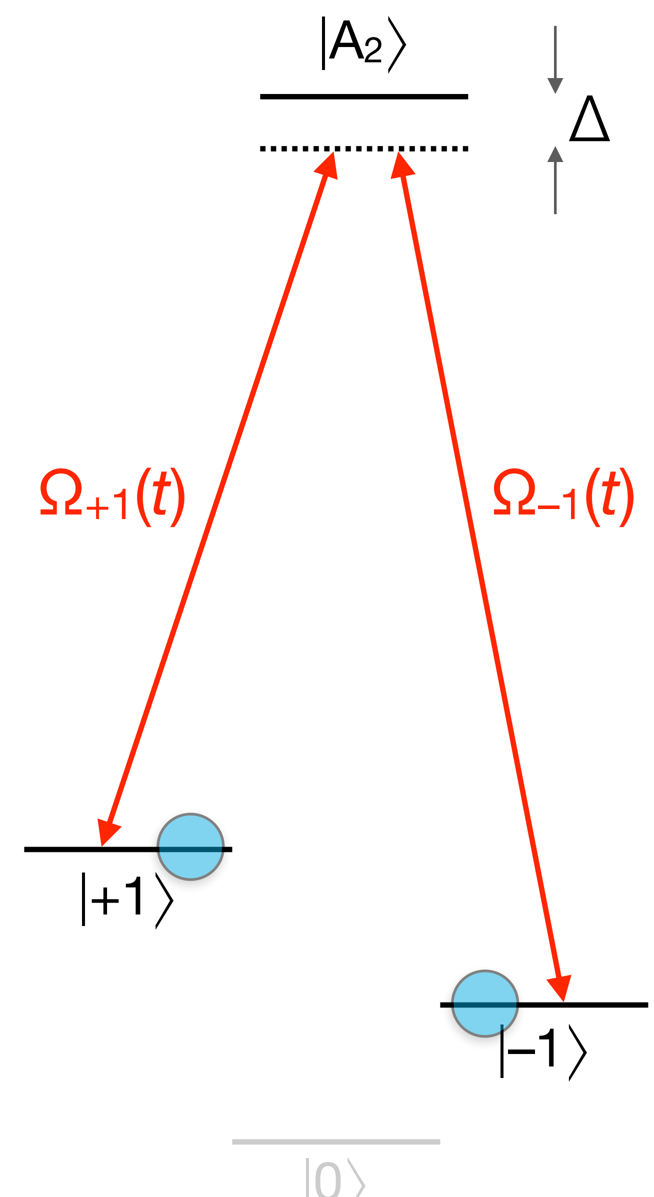
Stimulated Raman adiabatic passage (STIRAP)

- dark state $$|D(t)\rangle = \frac{1}{\sqrt{|\Omega_{+1}|^2 + |\Omega_{-1}|^2}} \left(\Omega_{+1}(t)|-1\rangle - \Omega_{-1}(t)|+1\rangle\right)$$

- adiabatically adjusting Rabi frequencies
  - arbitrary position on Bloch sphere
  - population transfer between $|+1\rangle$ and $|-1\rangle$

N. V. Vitanov *et al.*, Annu. Rev. Phys. Chem. 52, 763 (2001)
E. Togan *et al.*, Nature (2010)
D. A. Golter, K. N. Dinyari, H. Wang
PRA 2013





Stimulated Raman adiabatic passage (STIRAP)

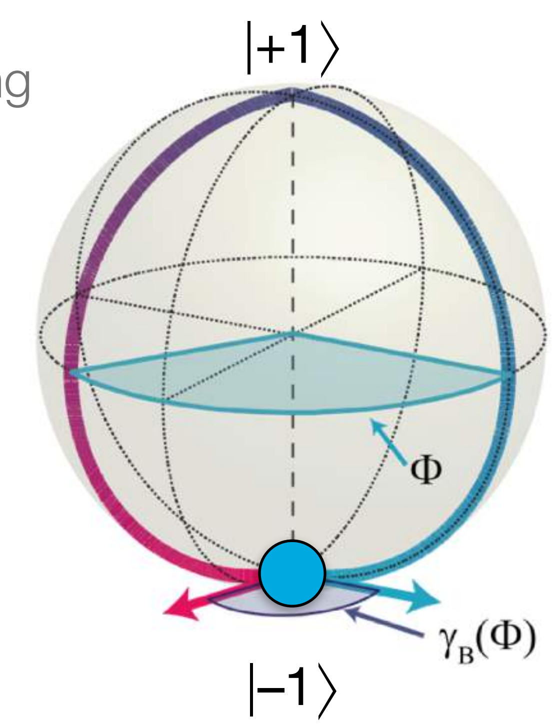
- dark state $|D(t)\rangle = \frac{1}{\sqrt{|\Omega_{+1}|^2 + |\Omega_{-1}|^2}} \left(\Omega_{+1}(t)|-1\rangle - \Omega_{-1}(t)|+1\rangle\right)$

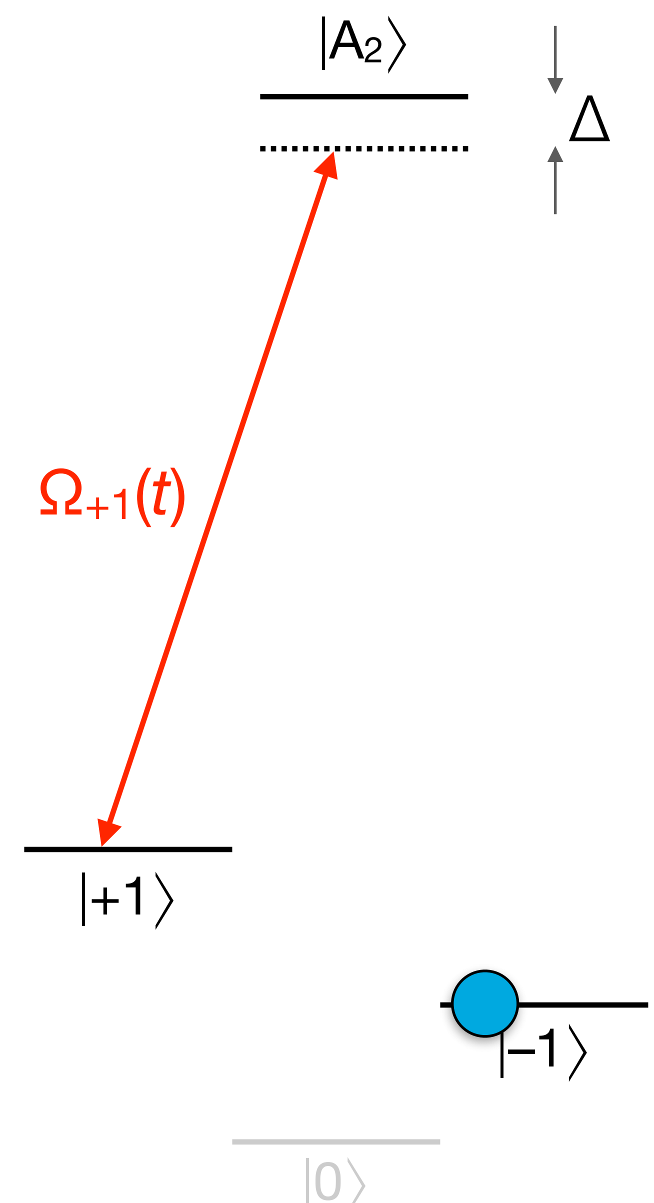
- adiabatically adjusting Rabi frequencies
  - arbitrary position on Bloch sphere
  - population transfer between |+1⟩ and |–1⟩

    N. V. Vitanov *et al.*, Annu. Rev. Phys. Chem. 52, 763 (2001)
    E. Togan *et al.*, Nature (2010)
    D. A. Golter, K. N. Dinyari, H. Wang
    PRA 2013

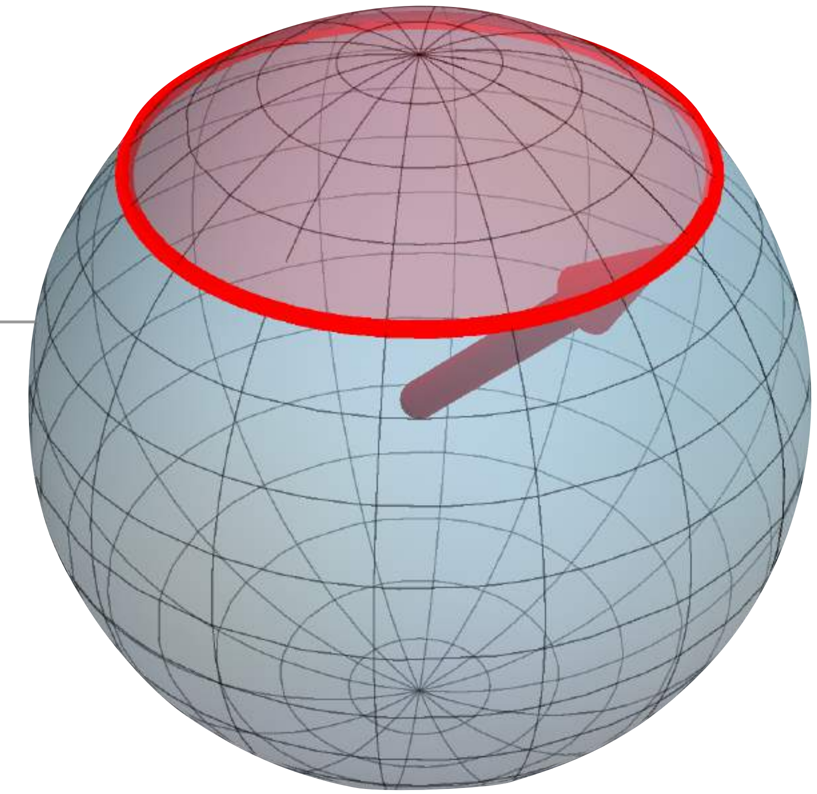
- Berry phase $\gamma_B(\Phi)$
  $|-1\rangle \mapsto e^{i\gamma_B}|-1\rangle$

  $\gamma_B(\Phi) = -\Phi$





Pancharatnam-Berry phase



- Hamiltonian $H = H(\mathbf{R}), \qquad \mathbf{R} = \mathbf{R}(t)$
- Dynamics $i\hbar \frac{\partial}{\partial t}|\psi\rangle = H(\mathbf{R}(t))|\psi\rangle$
- Instantaneous eigenstates $H(\mathbf{R})|n(\mathbf{R})\rangle = \epsilon_n(\mathbf{R})|n(\mathbf{R})\rangle$
- Adiabatic evolution

$$|\psi_n(t)\rangle = \exp\left[-i/\hbar \int_0^t \mathrm{d}t' \, \underbrace{\epsilon_n(\mathbf{R}(t'))}_{\text{dynamical phase}}\right] \exp(i\underbrace{\gamma_n(t)}_{\text{geometric phase}})|n(\mathbf{R}(t))\rangle$$

- Adiabatic evolution along closed loop in parameter space

$$\gamma_{\mathrm{B}} = i \oint_{\mathcal{C}} \mathrm{d}\mathbf{R} \cdot \langle n(\mathbf{R})|\nabla_{\mathbf{R}}|n(\mathbf{R})\rangle$$

geometric phase
(depends on the
geometry of path)

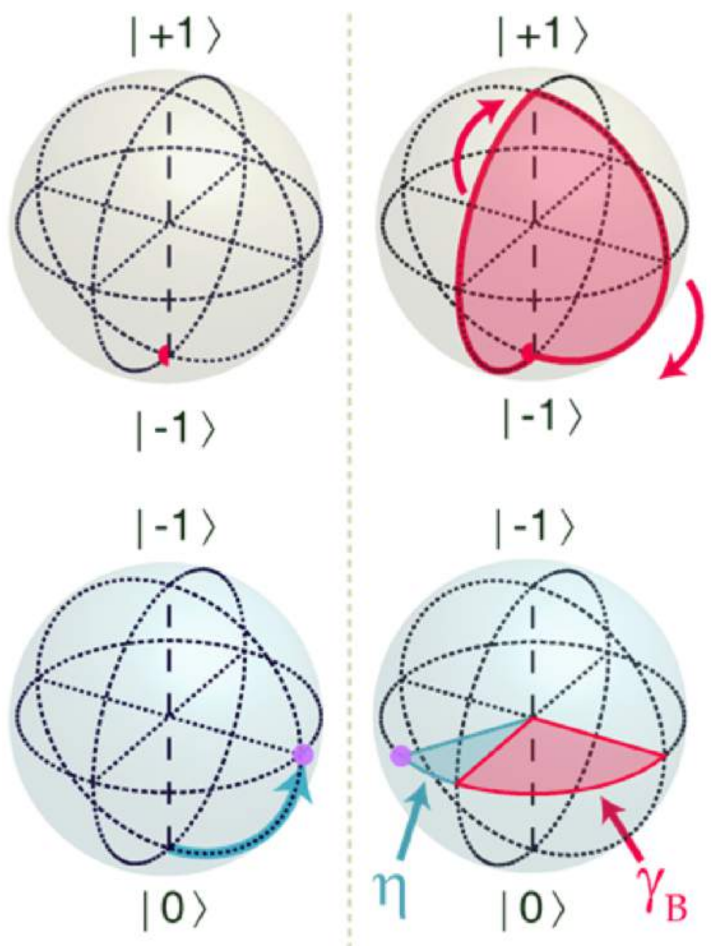
S. Pancharatnam, Proc. Indian Acad. Sci. A. **44**, 247 (1956)

M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984)

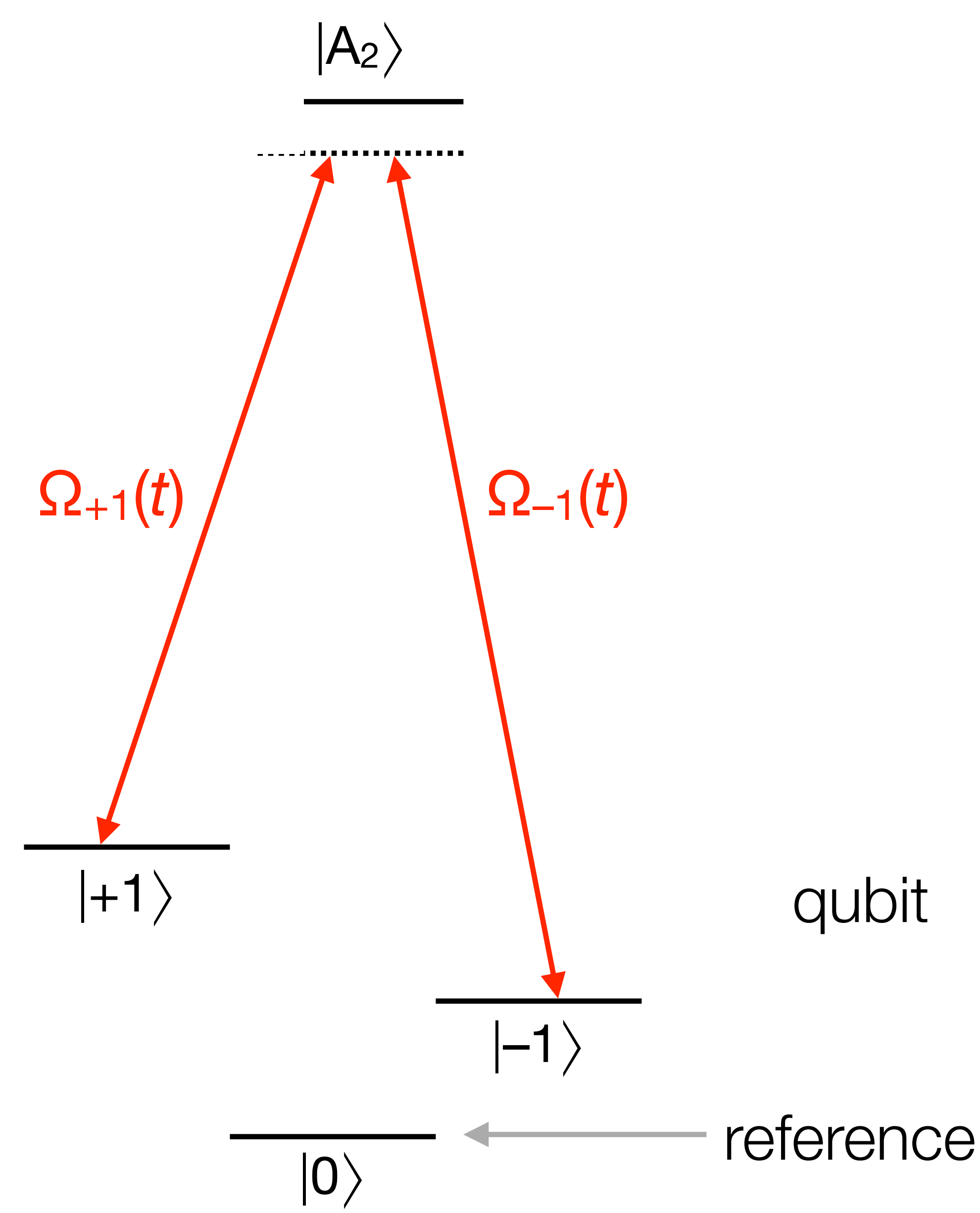
Optical accumulation of geometric phase

- Geometric phase $\gamma_B(\Phi)$

$$|-1\rangle \mapsto e^{i\gamma_B}|-1\rangle$$



$$|0\rangle + |-1\rangle \longrightarrow |0\rangle + e^{i\eta}\, e^{i\gamma_B}|-1\rangle$$



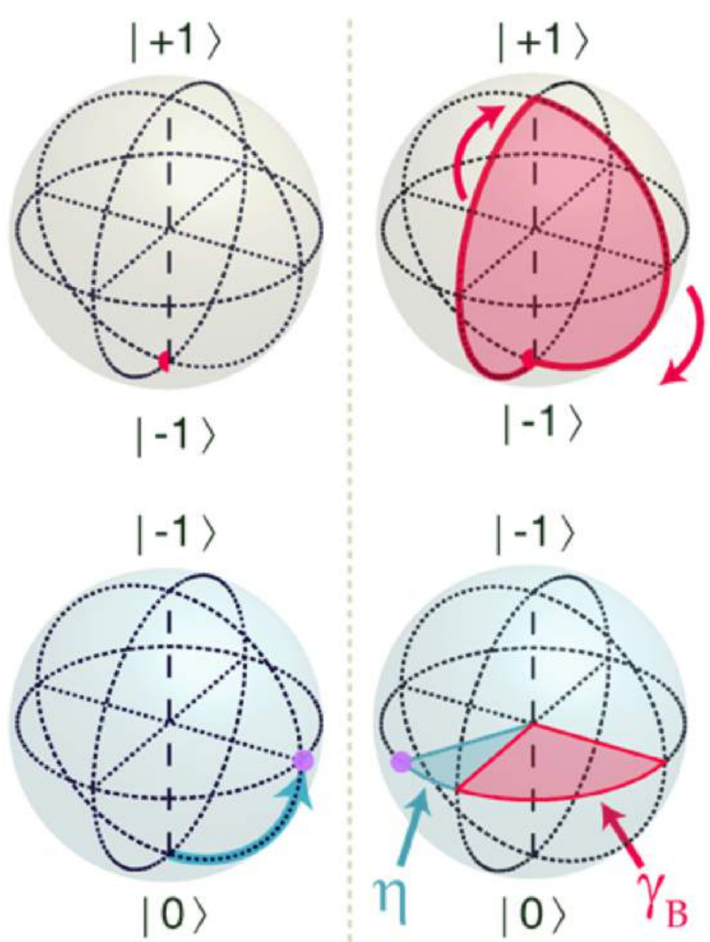
C.G. Yale, J.F. Heremans, B.B. Zhou, A. Auer, G. Burkard, D.D. Awschalom, Nature Photonics **10**, 184 (2016)

Optical accumulation of geometric phase

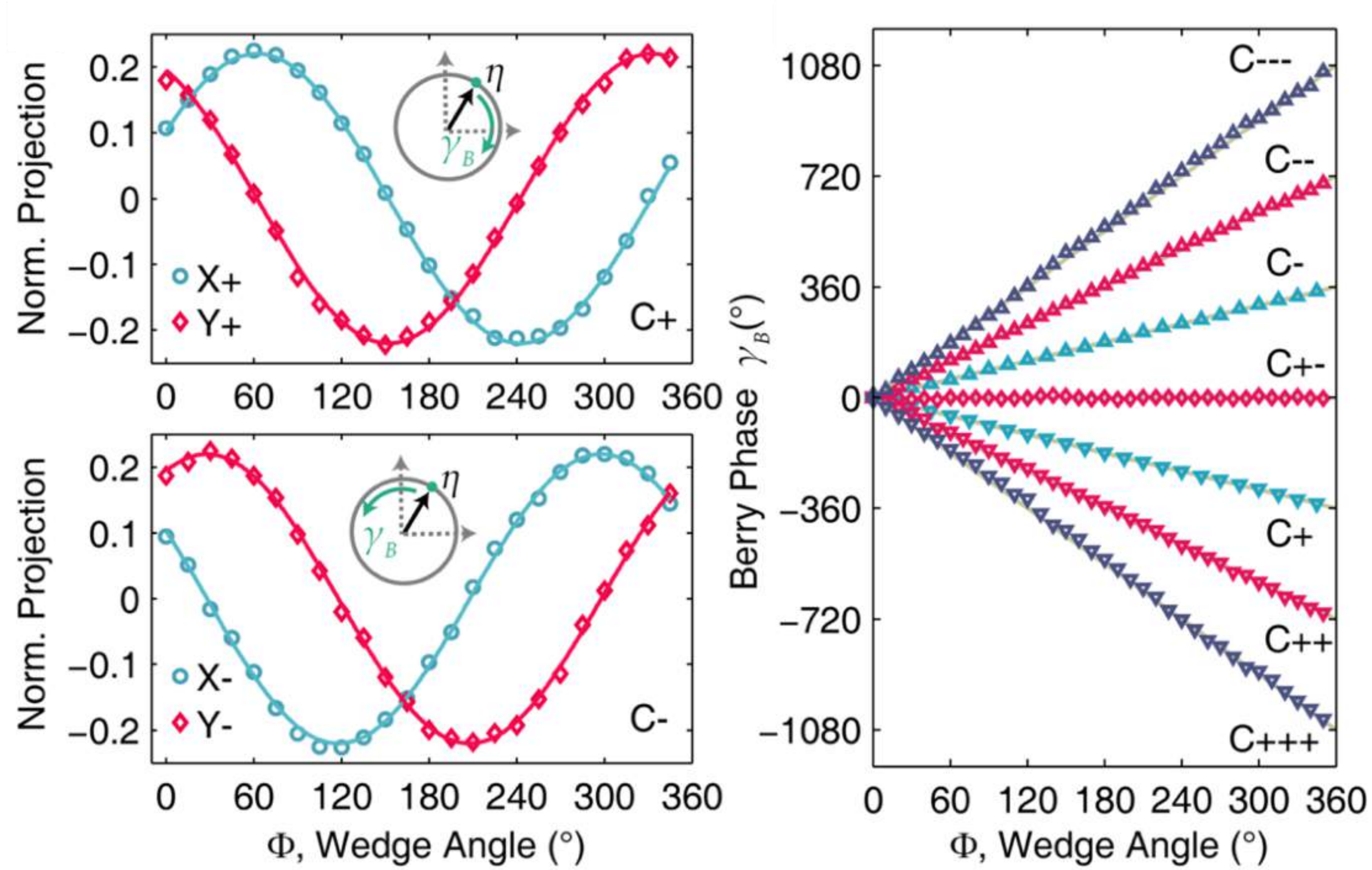
- Geometric phase $\gamma_{\mathrm{B}}(\Phi)$

$$|-1\rangle \mapsto e^{i\gamma_{\mathrm{B}}}|-1\rangle$$

- Experimental results



$$|0\rangle + |-1\rangle \longrightarrow |0\rangle + e^{i\eta}\, e^{i\gamma_{\mathrm{B}}}|-1\rangle$$



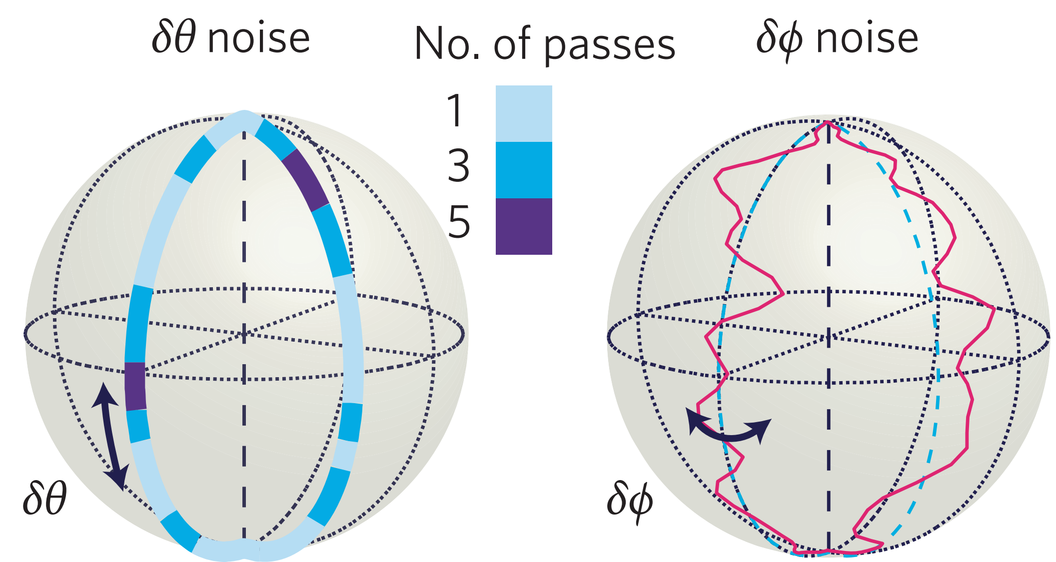
$$X_{\pm} = A\cos\left(\eta \pm \gamma_{\mathrm{B}}(\Phi)\right)$$

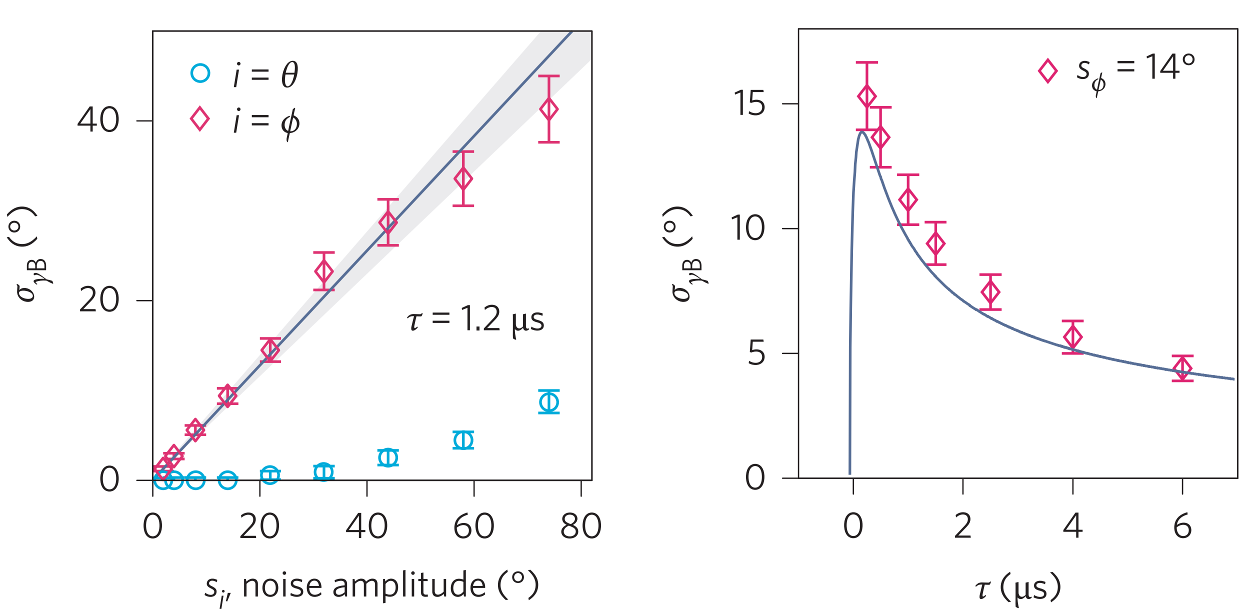
$$\gamma_{\mathrm{B}}(\Phi) = -\Phi$$

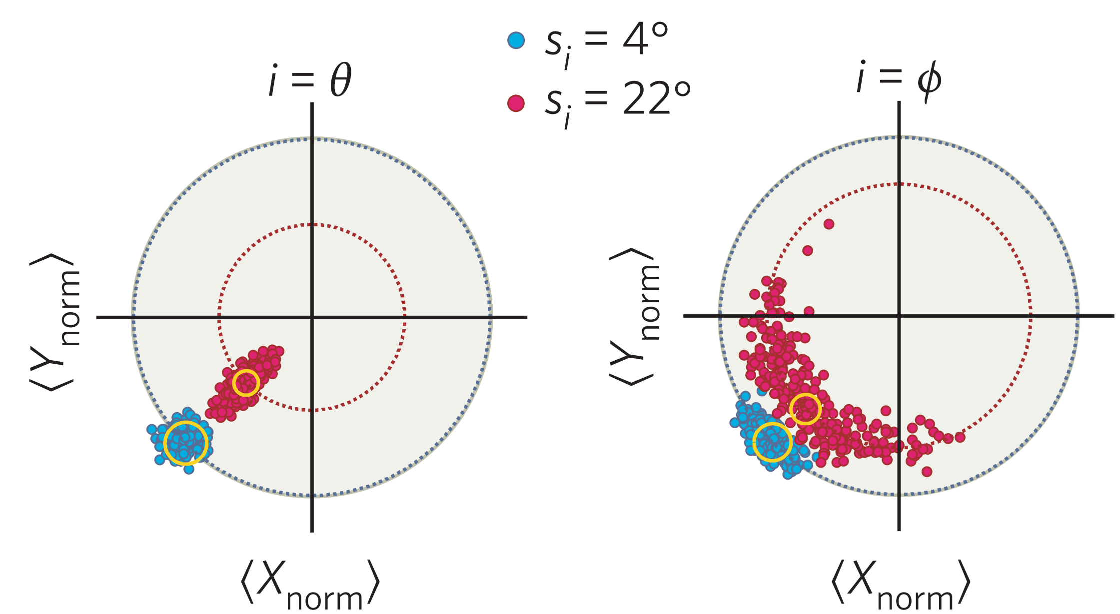
C.G. Yale, J.F. Heremans, B.B. Zhou, A. Auer, G. Burkard, D.D. Awschalom, Nature Photonics **10**, 184 (2016)

Robustness of the geometric phase against noise

- robust against δθ noise
- affected by δΦ noise

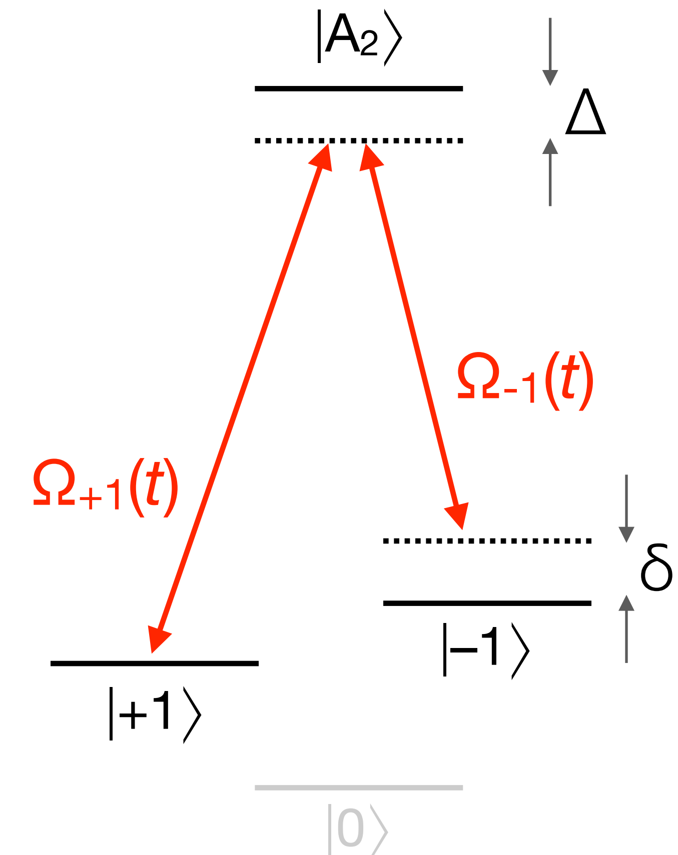






C.G. Yale, J.F. Heremans, B.B. Zhou, A. Auer, G. Burkard, D.D. Awschalom, Nature Photonics **10**, 184 (2016)

Geometric phase in open system



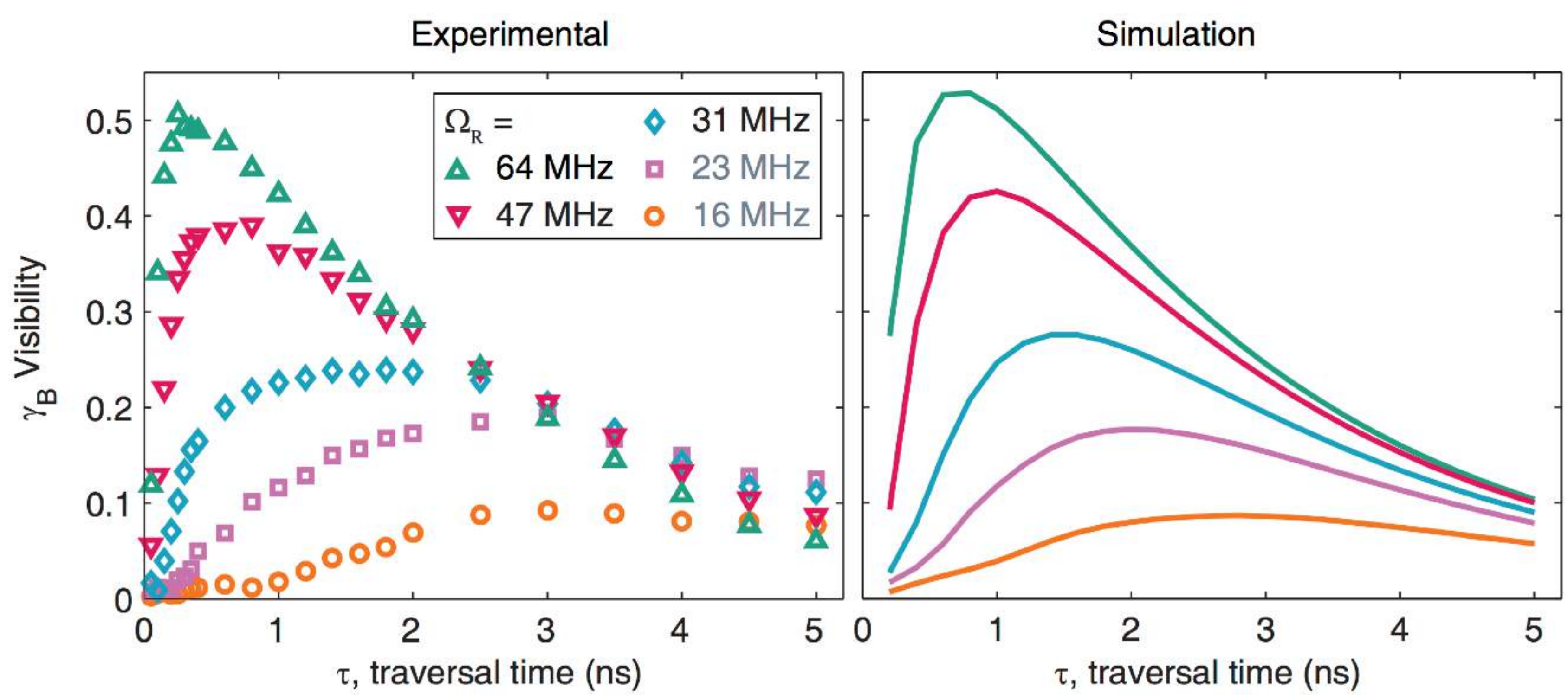
- Hamiltonian in the basis $|0\rangle, |-1\rangle, |+1\rangle, |A_2\rangle$

$$H(t) = \frac{h}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega_{-1}(t) \\ 0 & 0 & 2\delta & \Omega_{+1}(t) \\ 0 & \Omega^*_{-1}(t) & \Omega^*_{+1}(t) & 2\Delta \end{pmatrix}$$

- Master equation:

$$\dot{\rho}(t) = -i[H(t), \rho(t)] + \sum_k \left(L_k \rho(t) L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho(t) - \rho(t) \frac{1}{2} L_k^\dagger L_k \right)$$

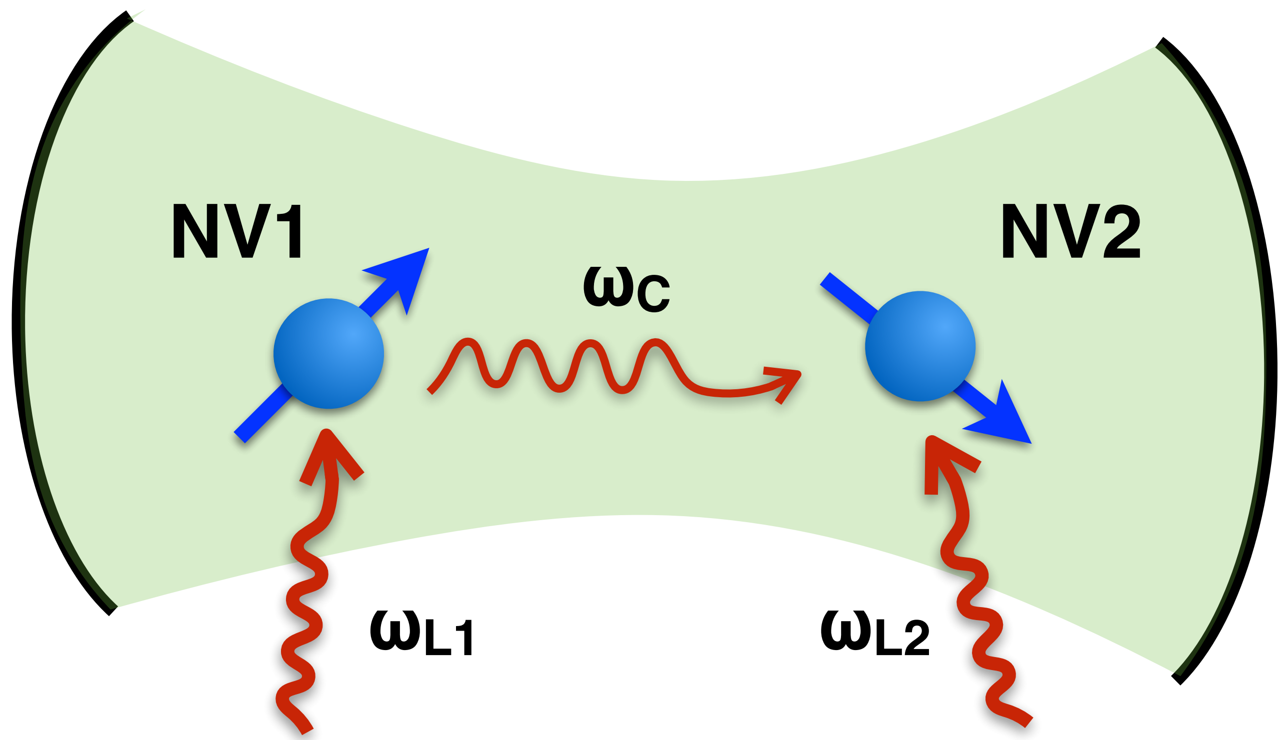
- Lindblad operators:
  - Relaxation
  - Spin dephasing
  - Orbital dephasing



C.G. Yale, J.F. Heremans, B.B. Zhou, A. Auer, G. Burkard, D.D. Awschalom, Nature Photonics **10**, 184 (2016)

Two-qubit gates between (remote) NV spins

- idea: optical cavity-mediated spin-spin interaction
- laser photons scatter at NV into cavity mode and back
- spin-dependent scattering

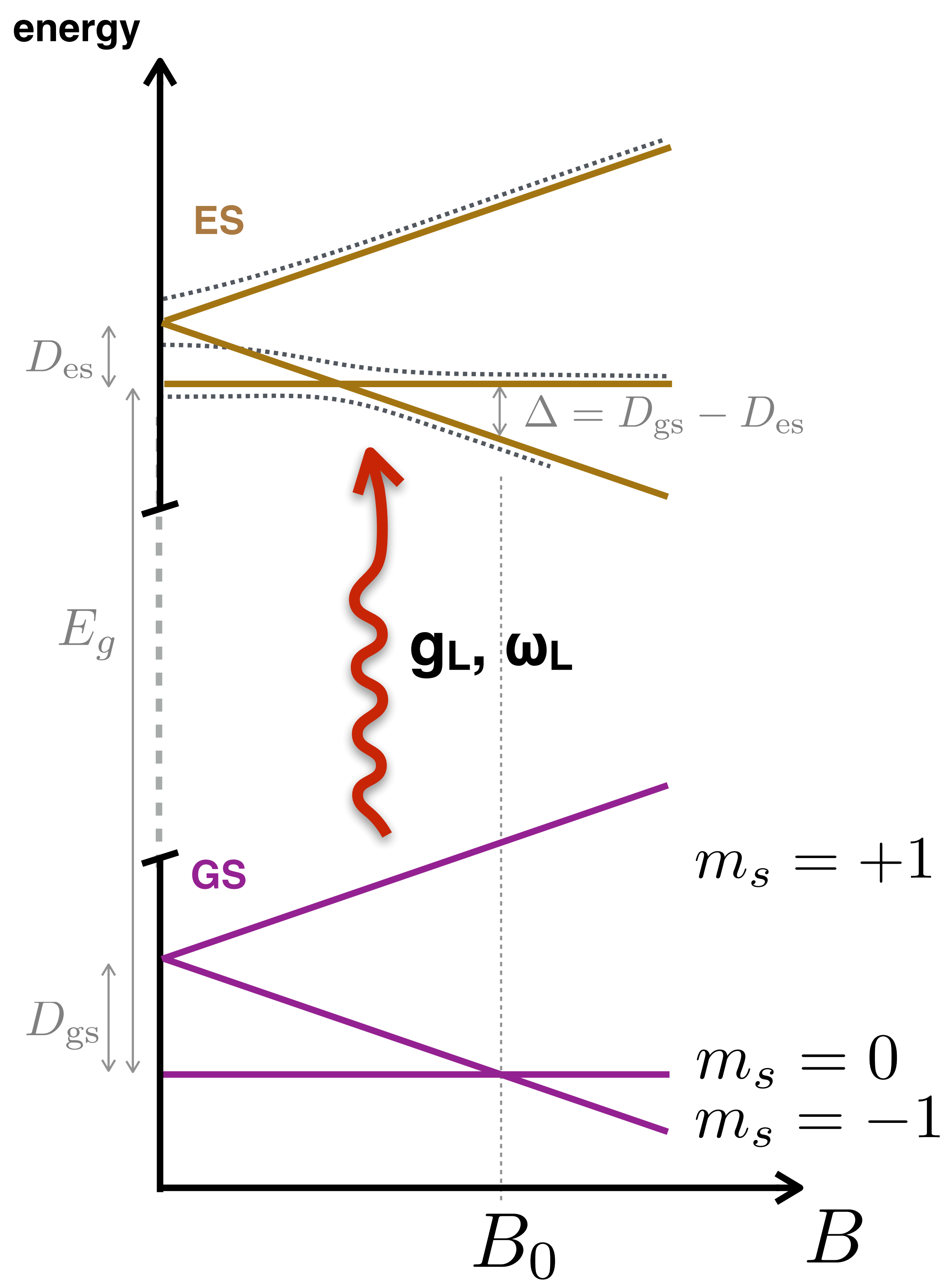


- two NVs: spin-dependent phase
- controlled-phase gate (CPHASE)

$$|0\rangle_{\rm control}|0\rangle_{\rm target} \rightarrow |0\rangle_{\rm control}|0\rangle_{\rm target}$$
$$|0\rangle_{\rm control}|1\rangle_{\rm target} \rightarrow |0\rangle_{\rm control}|1\rangle_{\rm target}$$
$$|1\rangle_{\rm control}|0\rangle_{\rm target} \rightarrow |1\rangle_{\rm control}|0\rangle_{\rm target}$$
$$|1\rangle_{\rm control}|1\rangle_{\rm target} \rightarrow -|1\rangle_{\rm control}|1\rangle_{\rm target}$$

G. Burkard, V. O. Shkolnikov, D. D. Awschalom, PRB 95, 205420 (2017)

Single nitrogen-vacancy center



- strained NV center

laser-induced transitions

ES

$$H_{\rm NV} = g\mu_B B S_z + \begin{pmatrix} E_g + D_{\rm es} S_z^2 & g_L^* e^{-it\omega_L} \\ g_L e^{it\omega_L} & D_{\rm gs} S_z^2 \end{pmatrix}$$

laser-induced transitions

GS

- in the co-rotating frame of the laser

$$H_{\rm NV} = g\mu_B B S_z + D S_z^2 + \frac{\Delta}{2} S_z^2 \tau_z + \frac{\delta_L}{2}\tau_z + g_L \tau_- + g_L^* \tau_+$$

laser detuning

$$\Delta = D_{\rm gs} - D_{\rm es}$$
$$D = (D_{\rm gs} + D_{\rm es})/2$$
$$\delta_L = E_g - \omega_L$$

$$\left.\begin{matrix} \tau_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ \tau_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{matrix}\right\}$$ orbital state (GS/ES)

- ES spin-spin interactions (dotted lines)

$$H_s = \frac{1}{2}(1+\tau_z)\left[\frac{\Delta_1}{2}\left(S_y^2 - S_x^2\right) + \frac{\Delta_2}{\sqrt{2}}\left(S_x S_z + S_z S_x\right)\right]$$

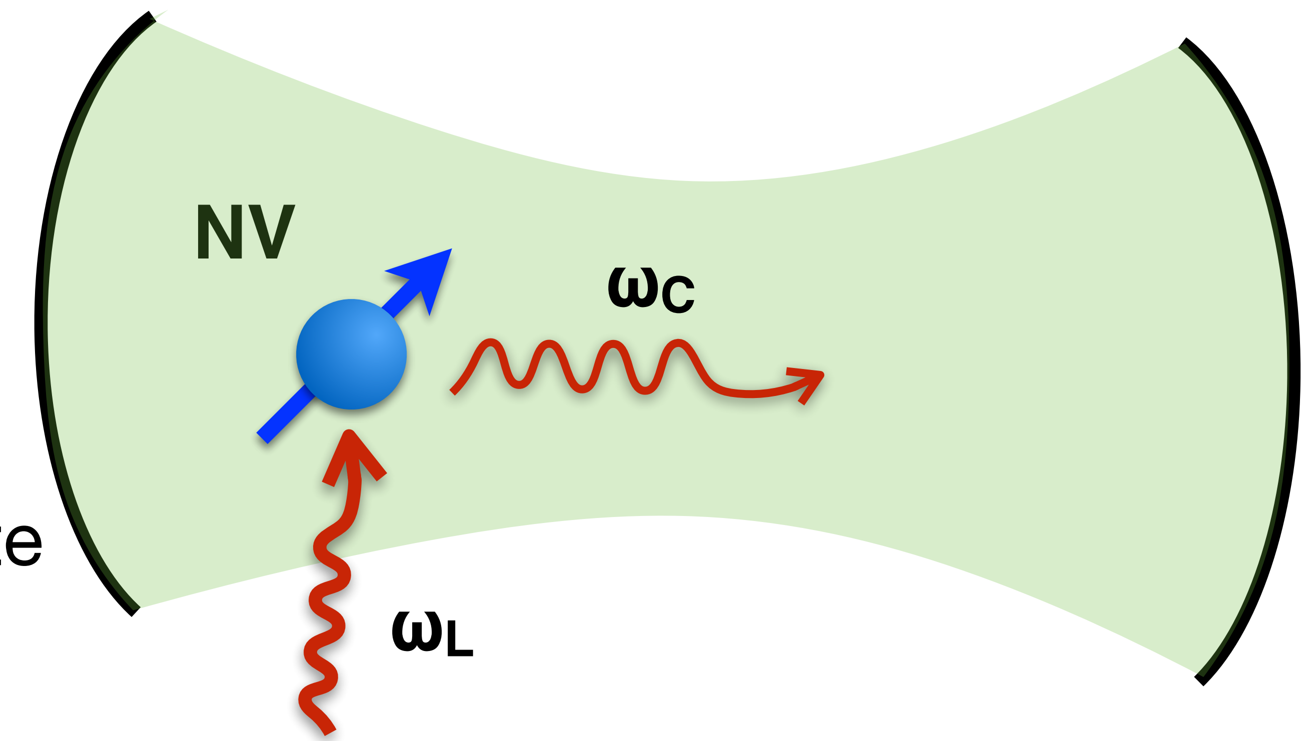
Coupling the NV center to a cavity mode

- off-resonant coupling to cavity

$$H = H_{\rm NV} + \delta_C a^\dagger a + g_C \left(\tau_+ a + \tau_- a^\dagger\right)$$

cavity detuning

GS→ES cavity phot. absorb.

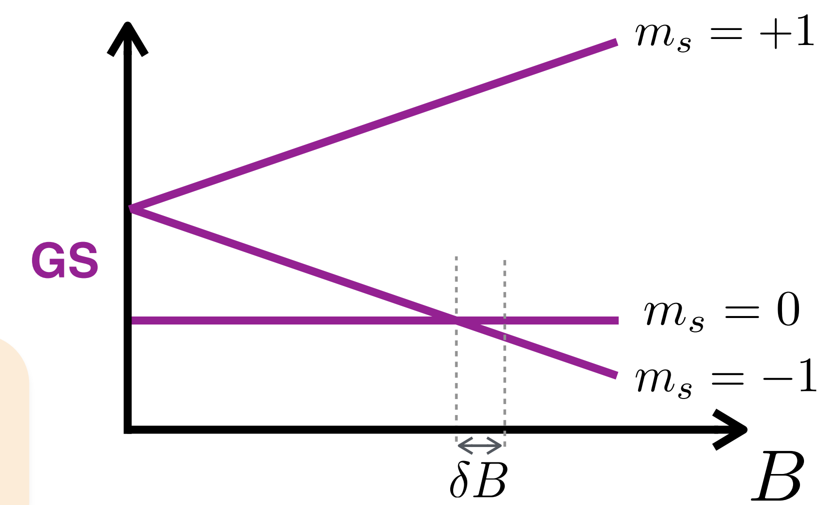


- eliminate intermediate NV excited state
(Schrieffer-Wolff transformation)

$$H = H_0 + V$$

g_L, g_C

$$H_{\rm eff} = e^{-S} H e^{S} = H_0 + \frac{1}{2}[S, V] + \cdots$$



- effective NV-cavity Hamiltonian

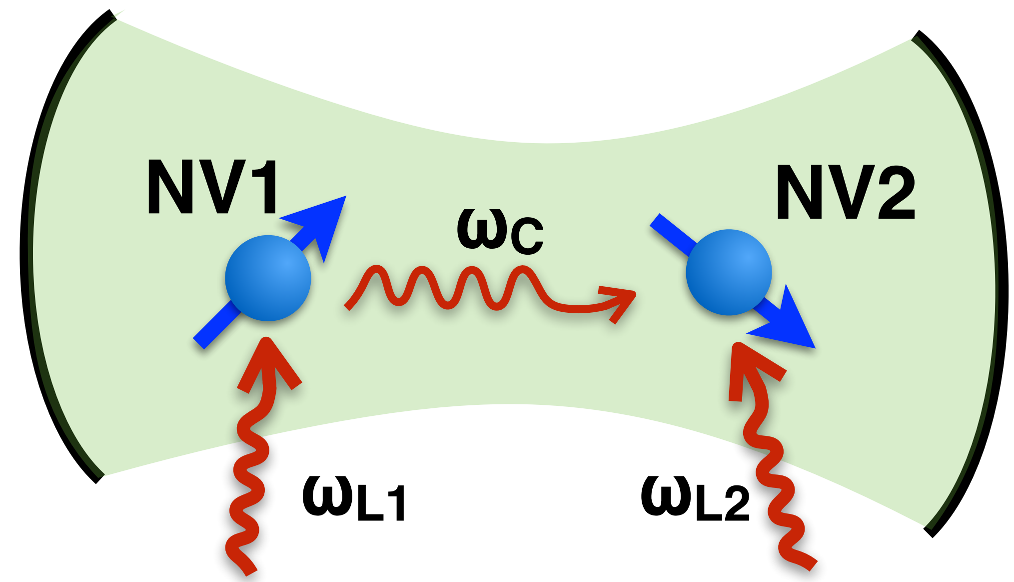
$$H_{\rm eff} = \delta_C a^\dagger a + \delta B |0\rangle\langle 0| + |0\rangle\langle 0| \left(g a + g^* a^\dagger\right)$$

$$g = g_L g_C \frac{\Delta}{\Delta^2 - (\delta_C/2)^2}$$

G. Burkard, V. O. Shkolnikov, D. D. Awschalom, PRB 95, 205420 (2017)

Cavity mediated coupling

- two NVs in a common cavity



- eliminate virtual cavity photon
(Schrieffer-Wolff transformation)
- effective two-qubit Hamiltonian

$$H_{2q} = \sum_{i=1,2} \left(-\frac{|g_i|^2}{\delta_C} + g\mu_B \delta B \right) |0\rangle_i \langle 0| - g_{12} |00\rangle\langle 00|$$

- qubit-qubit coupling strength

$$g_{12} \propto g_{L1} g_{L2} \cos(\phi_1 - \phi_2)$$

control: laser intensities & relative phase!

- generates controlled-phase gate $U_{\text{CPHASE}} = \mathbb{1} - 2|00\rangle\langle 00|$

G. Burkard, V. O. Shkolnikov, D. D. Awschalom, PRB 95, 205420 (2017)


School and conference on
Spin-based quantum information processing
http://www.uni-konstanz.de/spinqubits
Save the Date!
September 10-14, 2018
Konstanz, Germany
Steigenberger Inselhotel
organizers:
G. Burkard
D. Loss

Spin Qubits

lecture 1: Introduction (Exchange, Decoherence, Nuclear spins)

lecture 2: Multi-spin qubits, Coupling to E, Spin-Valley 1: Exchange

lecture 3: Spin-Valley 2: Spin Relaxation & Blockade, Defect Spins

Interesting directions

- cavity QED and hybrid quantum systems involving spins
- valley coherence
- coupling (remote) defect spins
- geometric two-qubit gates

Universität Konstanz